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REGENERATIVE HEAT EXCHANGER WITH HEAT-LOSS CONSIDERATION

E. K. Dabora
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Equation (6) on page 3 should read:

$$\theta = e^{-\beta\xi} \left[e^{-\eta} I_0(2\sqrt{\xi\eta}) + \int_0^{\eta} e^{-\eta} I_0(2\sqrt{\xi\eta}) d\eta \right]$$

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FOREWORD

Contract AF 18(600)-1199 calls for an investigation of the possibility of achieving a standing detonation wave. For this study, two regenerative heat exchangers of the pebble type have been designed and built by Personnel of the Aircraft Propulsion Laboratory. The excessive heat loss in the smaller one necessitated a closer study of its effect on this type of heat exchanger. Hence the analysis in this report came about.

TABLE OF CONTENTS

	Page
NOMENCLATURE	v
ABSTRACT	vii
OBJECTIVE	vii
I. INTRODUCTION	1
II. THE REGENERATIVE HEAT-EXCHANGER EQUATIONS	1
III. DIFFERENTIAL EQUATIONS IN NONDIMENSIONAL FORM	2
IV. SOLUTION OF THE DIFFERENTIAL EQUATIONS	3
V. DISCUSSION AND EVALUATION OF THE SOLUTION	4
VI. APPLICATION	6
VII. CONCLUSIONS	7
APPENDIX - DETAIL SOLUTION OF THE DIFFERENTIAL EQUATIONS	10
REFERENCES	15
DISTRIBUTION LIST	17

NOMENCLATURE

- C = Heat capacity of heat-exchanger bed material per unit volume, Btu/ft³-°F.
- c_p = Specific heat of fluid, Btu/lbm°F.
- G = Mass rate of flow per unit area, lbm/ft²-sec
- h = Heat-transfer coefficient between fluid and bed material, Btu/sec-ft²-°F.
- $I_0()$ = Modified Bessel function of the first kind and zero order.
- $\mathcal{L}()$ = Laplace transform of ().
- p = Variable of transformation.
- s = Heat-transfer surface area of bed material/unit volume of bed material, ft²/ft³.
- s_0 = Heat-loss surface area of exchanger/unit volume of bed material, ft²/ft³
- T = Fluid temperature, °F.
- T_i = Fluid inlet temperature, °F.
- t = Bed-material temperature, °F.
- t_0 = Bed-material temperature at zero time, °F
- t_s = Temperature of surroundings, °F.
- U_0 = Overall heat-transfer coefficient to surroundings based on area s_0 , Btu/sec-ft²-°F.
- V = Fluid velocity, ft/sec.
- x = Heat-exchanger length, ft.

NOMENCLATURE (Concluded)

Greek Letters

- α = $s_o U_o / sh$ nondimensional heat-transfer ratio.
- β = $1 + \alpha$.
- γ = $\alpha / \beta = \alpha / (1 + \alpha)$.
- δ = $(t - t_o) / (T_i - t_o)$ nondimensional bed temperature.
- δ_a = Nondimensional bed temperature for adiabatic heat exchanger.
- δ_{ss} = Steady-state nondimensional bed temperature.
- $\bar{\delta}$ = Laplace transform of δ .
- η = $sh \tau / C$ = reduced time.
- θ = $(T - t_o) / (T_i - t_o)$ nondimensional fluid temperature.
- $\bar{\theta}$ = Laplace transform of θ .
- K = $(t_s - t_o) / (T_i - t_o)$ nondimensional temperature of surroundings.
- ξ = $sh x / Gc_p$ reduced length of heat exchanger.
- ρ = Specific weight of fluid, lbm/ft^3 .
- τ = Time - sec.
- ψ = $c_p \rho / C$ ratio of heat capacities of fluid and bed material.

ABSTRACT

The differential equations describing the regenerative type heat exchanger with heat-loss consideration are solved by using Laplace transform. The solution is carried out for two cases: a) heating the bed from a uniform initial temperature equal to that of the surroundings with fluid at constant inlet fluid temperature, and b) cooling the bed from a uniform temperature with fluid at constant inlet temperature equal to that of the surroundings. Typical curves of the transient bed temperature are included for both cases.

The steady-state temperature in heating is shown to depend solely on the ratio of the heat loss and fluid heat capacity $[(s_0 U_0 / G c_p) \cdot x]$; therefore, for any heat-exchanger design where the maximum temperature that can be attained at the exit is of importance, this term should be carefully estimated.

OBJECTIVE

The objective of this report is to present the solution of the regenerative heat-exchanger problem when heat loss is taken into consideration.

I. INTRODUCTION

The regenerative type heat exchanger has been subjected to extensive treatment within the last thirty years or so. Although it has been used in the steel industry for some time, the solution of the differential equations describing the problem has aroused interest only recently. This came about as a result of new applications of the regenerative exchanger, such as the rotary regenerator contemplated for use in automotive gas turbines^{1,2,3} and the pebble-bed regenerator for wind-tunnel application where simulation of flight stagnation temperature is necessary.^{4,5} Furthermore, the same type of differential equations are encountered in mass-transfer problems in the chemical industry.

Nusselt,⁶ Anzelius,⁷ Schumann,⁸ Furnas,⁹ Carslaw and Jaeger,¹⁰ Amundson,¹¹ and Churchill, Abbrecht, and Chu¹² have dealt with the solution of the differential equation, but with the exception of Amundson, no one considered any heat-loss effect. While Amundson does consider a very general case, the form of the solution is so involved that it could become time-consuming for engineering applications. In Ref. 12 two- and three-dimensional flows and an arbitrary initial temperature distribution in the exchanger as well as an arbitrary inlet temperature history are considered. Klinkenberg¹³ shows an approximate solution sufficiently accurate for engineering purposes.

The following analysis is confined to the case where the heat exchanger is initially at constant temperature and the fluid inlet temperature is constant. Because of heat-loss consideration, two separate solutions, one for heating and one for cooling, are necessary.

II. THE REGENERATIVE HEAT-EXCHANGER EQUATIONS

In developing the differential equation for the regenerative heat exchanger, the following assumptions are usually made:

1. The conductivity of the bed material is considered negligible in the direction of flow.
2. The conductivity of the material is very large in the direction transverse to the flow.
3. Heat transfer by conduction within the fluid is negligible.

4. The fluid is at uniform velocity across the bed.

5. End effects are negligible.

Assumption 2 implies that the temperature of the bed material is uniform at any plane perpendicular to the flow direction. Although for any heat-loss consideration this cannot be strictly true, the implication is that each plane is at some average temperature.

An additional assumption is made in this analysis: namely, that the overall heat-transfer coefficient to the surroundings U_0 is considered constant.

With these assumptions, the following two equations can be obtained:

$$-Gc_p \left[\frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial \tau} d\tau \right] = \left[(T - t)sh + (T - t_s)s_0 U_0 \right] dx \quad ; \quad (1)$$

$$C \frac{\partial t}{\partial \tau} = (T - t)sh \quad . \quad (2)$$

Equation (1) arises from a heat balance in an element of the heat exchanger: namely, the heat loss from the fluid is equal to the heat transfer to the bed material plus the heat loss to the surroundings. Equation (2) simply states that the heat transferred to the bed is equal to the heat stored in it.

III. DIFFERENTIAL EQUATIONS IN NONDIMENSIONAL FORM

The above two equations can be nondimensionalized by introducing the following parameters:

- $\xi = shx/Gc_p$ reduced length
- $\eta = sh\tau/C$ reduced time
- $\alpha = s_0\bar{U}_0/sh$ ratio of heat-transfer rate/ $^{\circ}F$ to surroundings over heat-transfer rate/ $^{\circ}F$ to bed.
- $\theta = (T-t_0)/(T_1-t_0)$ nondimensional fluid temperature
- $\delta = (t-t_0)/(T_1-t_0)$ nondimensional bed temperature
- $K = (t_s-t_0)/(T_1-t_0)$ nondimensional surroundings temperature
- $\psi = c_{pp}/C$ heat capacity ratio

If the mass flow is constant, and the heat-transfer coefficient, h , as well as the specific heats, are considered constant, ξ will depend on the length x and η will depend on the time τ , whereas α and ψ become constants. Noting that $d\tau = dx/V = \rho dx/G$, Eq. (1) becomes

$$-\left(\frac{\partial\theta}{\partial\xi} + \psi \frac{\partial\theta}{\partial\eta}\right) = \theta - \delta + (\theta - K)\alpha. \quad (1a)$$

But since in most cases ψ is small compared to unity, Eq. (1a) can be reduced to

$$\frac{\partial\theta}{\partial\xi} = \delta - (1 + \alpha)\theta + K\alpha. \quad (3)$$

Equation (2) is simply reduced to:

$$\frac{\partial\delta}{\partial\eta} = \theta - \delta. \quad (4)$$

IV. SOLUTION OF THE DIFFERENTIAL EQUATIONS

The above two simultaneous equations are solved for the following two cases: a) heating of bed with constant fluid inlet temperature, with bed being originally at uniform temperature equal to the temperature of the surroundings; b) cooling of bed with fluid at constant temperature equal to that of the surroundings and bed being originally at uniform temperature. It is clear that $K = 0$ for case (a), and $K = 1$ for case (b). In addition, the boundary conditions corresponding to both cases are:

$$\begin{aligned} \theta(0, \eta) &= 1 & \eta > 0 \\ \delta(\xi, 0) &= 0 & 0 < \xi < \xi_1. \end{aligned}$$

It can be shown (see Appendix) that for case (a)

$$\delta = e^{-\alpha\xi} \int_0^\eta e^{-(\eta + \xi)} I_0(2\sqrt{\xi\eta}) d\eta \quad (5)$$

and

$$\theta = e^{-\alpha\xi} \left[e^{-\eta} I_0(2\sqrt{\xi\eta}) + \int_0^\eta e^{-\eta} I_0(2\sqrt{\xi\eta}) d\eta \right], \quad (6)$$

and for case (b)

$$\delta = 1 - e^{-\gamma\eta} + (1 - \gamma) e^{-\gamma\eta} \int_0^\eta e^{-\beta\xi - \eta(1 - \gamma)} I_0(2\sqrt{\eta\xi}) d\eta \quad (7)$$

and

$$\theta = 1 - (1-\gamma) \left[e^{-\gamma\eta} - e^{-\eta-\beta\xi} I_0(2\sqrt{\eta\xi}) \right] + (1-\gamma)^2 e^{-\gamma\eta} e^{-\beta\xi} \int_0^\eta e^{-\eta(1-\gamma)} I_0(2\sqrt{\eta\xi}) d\eta \quad (8)$$

where

$$\gamma = \frac{\alpha}{1+\alpha} \quad \text{and} \quad \beta = (1+\alpha).$$

V. DISCUSSION AND EVALUATION OF THE SOLUTION

With the exception of the factor $e^{-\alpha\xi}$, Eqs. (5) and (6) are the familiar solutions for the bed temperature and the fluid temperature, respectively, as shown in the literature.⁷⁻¹⁰ In fact, they represent a more general case with the adiabatic case ($\alpha = 0$ and therefore $e^{-\alpha\xi} = 1$) being a special one. It can easily be shown that Eqs. (7) and (8) will reduce to Eqs. (5) and (6), respectively, when α is set to zero in both sets of equations. This is tantamount to saying that the solution for both heating and cooling cycles are the same when no heat loss is considered (see Fig. 1).

Evaluation procedure of the solution will be limited to that of δ , since Eq. (4) when rearranged as

$$\theta = \frac{\partial\delta}{\partial\eta} + \delta \quad (4a)$$

will provide the solution for θ once δ is known as a function of η .

The integral in Eq. (5) has been evaluated in (8) for values of ξ up to 10 and in (9) for values of ξ up to 500. A numerical method used in Ref. 3 implicitly gives the value of this integral. Since this integral is unity when $\eta \rightarrow \infty$, the steady-state bed temperature can be written as

$$\delta_{ss} = e^{-\alpha\xi} \quad (9)$$

[This of course can be checked by noting that for the steady-state solution $\delta_{ss} = \theta_{ss}$, and that Eq. (3) reduces to $d\delta_{ss}/d\xi = \alpha\delta_{ss}$ for the heating cycle.]

Thus

$$\frac{\delta}{\delta_{ss}} = \int_0^\eta e^{-(\eta+\xi)} I_0(2\sqrt{\xi\eta}) d\eta \quad (10)$$

To evaluate δ , one notes that the integral is the solution for the adiabatic case, i.e.:

$$\delta_a = \int_0^\eta e^{-(\eta+\xi)} I_0(2\sqrt{\xi\eta}) d\eta \quad (11)$$

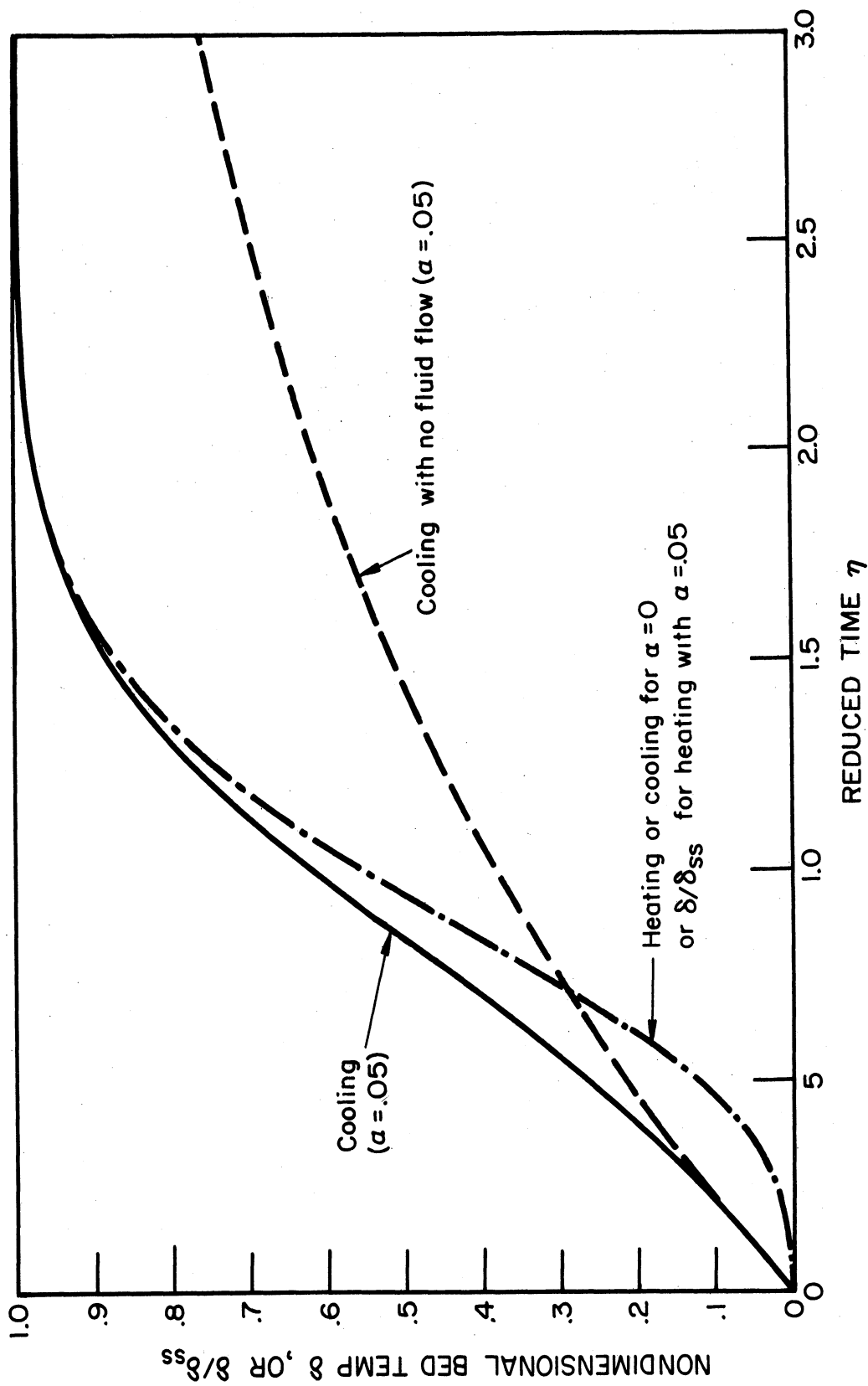


Fig. 1. Transient bed temperature in cooling and heating at $\xi = 10$.

Hence

$$\delta = \delta_a e^{-\alpha \xi}, \quad (12)$$

which can be evaluated once α and ξ are known.

This is the solution for the heating part of the cycle, and is shown in Fig. 1 for $\xi = 10$. For the cooling part, δ has to be determined from Eq. (7). When α is known, the integral in this equation can be evaluated graphically if the modified Bessel function is known. For arguments up to 10, this function has been evaluated in Ref. 14, but for larger arguments an approximate evaluation due to Furnas¹⁵ can be used. Figure 1 shows the variation of δ of Eq. (7) with η for $\xi = 10$ and $\alpha = .05$. The graphical method is used to determine the integral.

A third curve in Fig. 1 shows the variation of δ (in the cooling case) when the fluid flow is zero. This δ can be determined directly from Eq. (7), since $\xi \rightarrow \infty$ as $G \rightarrow 0$ and the integral vanishes for zero flow. Thus, for no flow:

$$\delta = 1 - e^{-\gamma \eta}. \quad (13)$$

It is notable to see that when α is small this equation becomes:

$$\delta \approx 1 - e^{-\alpha \eta}, \quad (13a)$$

which reduces to

$$\frac{t - t_0}{t_s - t_0} = 1 - e^{-\frac{s_0 U_0 \tau}{C}}, \quad (13b)$$

or

$$\ln \frac{t_0 - t_s}{t - t_s} = \frac{s_0 U_0}{C} \tau, \quad (13c)$$

which is the classical solution for the transient temperature of a body originally at uniform temperature suddenly immersed in an atmosphere of different temperature and where no temperature gradient within the body is assumed.

Figure 1 clearly shows the difference in cooling and heating of the exchanger bed when $\xi = 10$ and $\alpha = .05$. The third curve for cooling with no flow is shown for comparative purposes. Of course similar curves can be obtained for any other values of ξ and α .

VI. APPLICATION

In many cases it is desirable to know the time required for the bed to reach a steady-state condition in heating, and, when reached, the maximum temperature that can be attained at the exit of the heat exchanger.

By knowing ξ , the design engineer can easily find η when δ reaches a steady state; for example, when $\xi = 10$, Fig. 1 shows that η should be greater than 26 for the bed temperature to reach a steady state. Thus, since η is a function of time, the time required to reach steady state can be found.

When the conductivity of the insulating material around the heat exchanger is known, the value of α can be estimated, and since ξ is known, the maximum temperature that can be attained can easily be found from Eq. (9). For example, when ξ is 10 and $\alpha = .05$, $\delta_{ss} = .606$. In Fig. 2, the familiar exponential curve is plotted, showing the effect of the product $\xi\alpha$ on the steady-state temperature. Thus in estimating the maximum attainable temperature in a particular heat exchanger, the product of $\xi\alpha$ becomes an important parameter that should be carefully evaluated.

In the cooling part of the cycle, the time required to keep the exit temperature within certain limits is often important. Thus η can be evaluated from curves such as the one shown in Fig. 1. For examples, when $\xi = 10$ and $\alpha = .05$ η should be less than 4 to keep δ within 80% of the original uniform temperature of the bed.

It should be pointed out that the theoretical prediction on the heating part of the cycle has been substantially justified by experiments conducted on two pebble-bed-type heat exchangers at the Aircraft Propulsion Laboratory of The University of Michigan. The data will be presented in another report that should be forthcoming in the near future.

VII. CONCLUSIONS

1. A solution of the heat-transfer equation in a regenerative heat exchanger with heat loss has been presented. The solution is presented in two parts: a) for heating of bed material, and b) for cooling of bed material.
2. The temperature of both the solid bed and the fluid in the heating part of the cycle is found to be different by a factor of $e^{-\xi\alpha}$ from the corresponding values for known ξ and η , when no heat loss is taken into account. Thus the steady-state temperature becomes a function of $\xi\alpha = (s_o U_o x)/(Gc_p)$ only.
3. The temperatures in the cooling part of the cycle do not have as simple a correspondence as those of the heating part.
4. An example of the transient temperature variation in heating and cooling has been presented in a graphical form to show the difference between the solutions for the two cases.

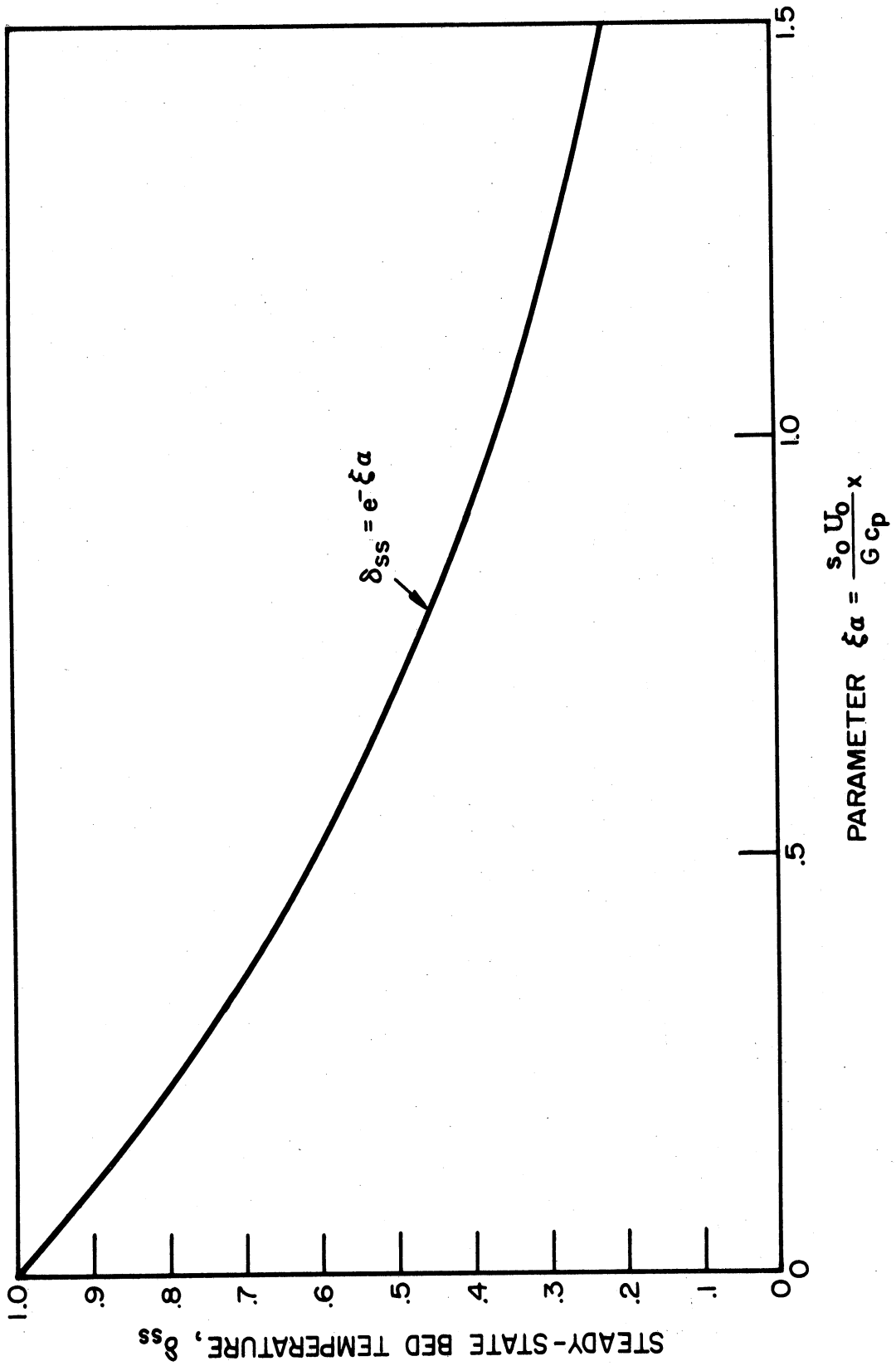


Fig. 2. Steady-state bed temperature vs. the parameter $\xi\alpha$.

5. The reasonable assumption that the overall heat-transfer coefficient to the surroundings is constant turns out to be the key to the simplicity of the solution in this analysis as compared to that of Amundson.¹¹

APPENDIX

DETAIL SOLUTION OF THE DIFFERENTIAL EQUATIONS

The solution of Eqs. (3) and (4) is presented here for the two cases mentioned on p. 3.

In case (a), $K = 0$ and Eq. (3) becomes

$$\frac{\partial \theta}{\partial \xi} = \delta - (1 + \alpha)\theta \quad (3a)$$

Equation (4) is rewritten here for convenience:

$$\frac{\partial \delta}{\partial \eta} = \theta - \delta \quad (4)$$

These two equations are to be solved for the following boundary conditions:

$$\theta(0, \eta) = 1 \quad \eta > 0 \quad (14)$$

$$\delta(\xi, 0) = 0 \quad 0 < \xi < \xi_1 \quad (15)$$

Taking Laplace transform with respect to η , Eqs. (3a) and (4) become, respectively,

$$\frac{d\bar{\theta}}{d\xi} = \bar{\delta} - (1 + \alpha)\bar{\theta} \quad (16)$$

$$p\bar{\delta} - \delta(0) = \bar{\theta} - \bar{\delta} \quad (17)$$

Introducing Eq. (15) into (17), the latter reduces to

$$p\bar{\delta} = \bar{\theta} - \bar{\delta} \quad (18)$$

The remaining boundary conditions, Eq. (14) becomes

$$\bar{\theta}(0, p) = 1/p \quad (19)$$

By eliminating $\bar{\delta}$ in Eq. (16) and (18), we get

$$\frac{d\bar{\theta}}{d\xi} - \left(\frac{1}{1+p} - 1 - \alpha \right) \bar{\theta} = 0 \quad (20)$$

Equation (20) can be solved with the boundary condition (19), yielding:

$$\bar{\theta} = \frac{1}{p} e^{-\left(1 + \alpha - \frac{1}{1+p}\right)\xi} \quad (21)$$

or

$$\bar{\theta} = e^{-(1+\alpha)\xi} \frac{1}{p} e^{\left(\frac{1}{1+p}\right)\xi} \quad (21a)$$

Using Eqs. (18) and (21a), $\bar{\delta}$ can be found as

$$\bar{\delta} = e^{-(1+\alpha)\xi} \frac{1}{p(1+p)} e^{\left(\frac{1}{1+p}\right)\xi} \quad (22)$$

In Ref. 10, it is noted that

$$\frac{1}{p} e^{\xi/p} = \mathcal{L} [I_0(2\sqrt{\xi\eta})],$$

from which it can be shown¹⁶ that

$$\frac{1}{1+p} e^{\xi/(1+p)} = \mathcal{L} [e^{-\eta} I_0(2\sqrt{\xi\eta})]. \quad (23)$$

Noting from Ref. 16 that

$$\frac{1}{p} \bar{f}(p) = \mathcal{L} \left[\int_0^\eta f(\eta) d\eta \right], \quad (24)$$

we get for the inverse transform of $\bar{\delta}$

$$\delta = e^{-(1+\alpha)\xi} \int_0^\eta e^{-\eta} I_0(2\sqrt{\xi\eta}) d\eta. \quad (25)$$

To find θ , we note that

$$\frac{1}{p} = \frac{1}{p(p+1)} + \frac{1}{p+1};$$

$$\therefore \bar{\theta} = e^{-(1+\alpha)\xi} \left[\frac{1}{p(p+1)} + \frac{1}{p+1} \right] e^{\left(\frac{1}{1+p}\right)\xi}. \quad (21b)$$

Hence from (23) and (24) we get:

$$\theta = e^{-(1+\alpha)\xi} \left[\int_0^\eta e^{-\eta} I_0(2\sqrt{\xi\eta}) d\eta + e^{-\eta} I_0(2\sqrt{\xi\eta}) \right]. \quad (26)$$

Equations (25) and (26) are the same as (5) and (6), respectively. In case (b), $K = 1$ and Eq. (3) becomes

$$\frac{\partial \theta}{\partial \xi} = \delta - (1 + \alpha) \theta + \alpha \quad (3b)$$

The boundary conditions remain the same as in case (a).

Taking Laplace transform with respect to η again, and applying the boundary condition, Eq. (15), we get

$$\frac{d\bar{\theta}}{d\xi} = \delta - (1 + \alpha) \bar{\theta} + \frac{\alpha}{p} \quad (27)$$

$$p\bar{\delta} = \bar{\theta} - \bar{\delta} \quad (28)$$

for Eqs. (3b) and (4), respectively.

The last two equations when combined, give

$$\frac{d\bar{\theta}}{d\xi} = \frac{\bar{\theta}}{1 + p} - \beta \bar{\theta} + \frac{\alpha}{p}, \quad (29)$$

where $\beta = \alpha + 1$, which can be solved for $\bar{\theta}$ for the boundary condition of Eq. (19).

$$\bar{\theta} = \frac{1}{p} \left[1 + \frac{\alpha}{\frac{1}{1+p} - \beta} \right] e^{-\left(\frac{1}{1+p} - \beta\right)\xi} - \frac{\alpha}{p \left(\frac{1}{1+p} - \beta\right)} \quad (30)$$

From Eq. (28) it follows that

$$\delta = \frac{1}{p+1} \left[\frac{1}{p} \left(1 + \frac{\alpha}{\frac{1}{1+p} - \beta} \right) e^{-\left(\frac{1}{1+p} - \beta\right)\xi} - \frac{\alpha}{p \left(\frac{1}{1+p} - \beta\right)} \right] \quad (31)$$

Now

$$\begin{aligned} \frac{1}{p} \left[1 + \frac{\alpha}{\left(\frac{1}{1+p} - \beta\right)} \right] &= \frac{1}{p} \left[1 + \frac{\alpha(1+p)}{1 - \beta(1+p)} \right] \\ &= \frac{1}{p} \left[1 + \frac{\alpha(1+p)}{1 - (1+\alpha)(1+p)} \right] \\ &= \frac{1}{p} \left[1 - \frac{\alpha(1+p)}{\alpha + \beta p} \right] = \frac{1}{p} \left[1 - \frac{\gamma(1+p)}{\gamma + p} \right], \quad (32) \end{aligned}$$

where $\gamma = \alpha/\beta$ but

$$\frac{\gamma(1+p)}{p(\gamma+p)} = \frac{1}{p} - \frac{1-\gamma}{\gamma+p} ; \quad (33)$$

$$\therefore \frac{1}{p} \left[1 + \frac{\alpha}{\left(\frac{1}{1+p} - \beta\right)} \right] = \frac{1-\gamma}{\gamma+p} . \quad (34)$$

Also,

$$\frac{\alpha}{p\left(\frac{1}{1+p} - \beta\right)} = -\frac{\gamma(1+p)}{p(\gamma+p)} = -\frac{1}{p} + \frac{1-\gamma}{\gamma+p} . \quad (35)$$

Hence

$$\bar{\theta} = \frac{1-\gamma}{\gamma+p} e^{\left(\frac{1}{1+p} - \beta\right)\xi} + \frac{1}{p} - \frac{1-\gamma}{p+\gamma} . \quad (36)$$

By simple algebra, one can show that

$$\bar{\delta} = \left(\frac{1}{p+\gamma} - \frac{1}{p+1}\right) e^{\left(\frac{1}{1+p} - \beta\right)\xi} + \frac{1}{p} - \frac{1}{p+\gamma} . \quad (37)$$

Now

$$\frac{1}{(p+\gamma)} - \frac{1}{(p+1)} = \frac{1-\gamma}{(p+\gamma)(p+1)} ; \quad (38)$$

$$\therefore \bar{\delta} = \frac{1-\gamma}{(p+\gamma)(p+1)} e^{\left[\frac{1}{(1+p)} - \beta\right]\xi} + \frac{1}{p} - \frac{1}{p+\gamma} . \quad (39)$$

From Ref. 16

$$\bar{f}(p) \bar{g}(p) = \mathcal{L} \left[\int_0^\eta f(\eta - Z) g(Z) dZ \right] ;$$

$$\begin{aligned} \therefore \frac{1}{(p+\gamma)(p+1)} e^{\frac{1}{1+p}} &= \mathcal{L} \left[\int_0^\eta e^{-\gamma(\eta - Z)} e^{-Z} I_0(2\sqrt{\xi Z}) dZ \right] \\ &= \mathcal{L} \left[e^{-\gamma\eta} \int_0^\eta e^{-Z(1-\gamma)} I_0(2\sqrt{\xi Z}) dZ \right] . \quad (40) \end{aligned}$$

Hence

$$\delta = (1-\gamma) e^{-\gamma\eta} e^{-\beta\xi} \int_0^\eta e^{-\eta(1-\gamma)} I_0(2\sqrt{\eta\xi}) d\eta + 1 - e^{\gamma\eta} . \quad (41)$$

To solve for θ , we note from Eq. (38)

$$\frac{1}{p + \gamma} = \frac{1 - \gamma}{(p + \gamma)(p + 1)} + \frac{1}{p + 1} ; \quad (38a)$$

then

$$\bar{\theta} = e^{-\beta\xi} \left[\frac{(1 - \gamma)^2}{(p + \gamma)(p + 1)} + \frac{(1 - \gamma)}{p + 1} \right] e^{\left(\frac{1}{1 + p}\right)\xi} + \frac{1}{p} - \frac{1 - \gamma}{p + \gamma} . \quad (42)$$

Again by use of Eqs. (40) and (23) we can get

$$\theta = (1 - \gamma)^2 e^{-\gamma\eta - \beta\xi} \int_0^\eta e^{-\eta(1-\gamma)} I_0(2\sqrt{\eta\xi}) d\eta + (1 - \gamma) e^{-(\eta + \beta\xi)} I_0(2\sqrt{\eta\xi}) + 1 - (1 - \gamma) e^{-\gamma\eta} . \quad (43)$$

Equations (42) and (43) are identical to Eqs. (7) and (8), respectively.

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