Articles 5

A DIAGRAMMATIC EXPOSITION OF THE LOGIC OF COLLECTION ACTION

John R. Chamberlin★

During the decade since the appearance of Mancur Olson's *The Logic of Collective Action*, a number of economists and political scientists have focused their efforts on further developing the model of group behavior analyzed by Olson.[1] While these efforts have contributed to both the precision and the generality of the theory of collective action, the theory remains a somewhat loosely connected set of verbal and graphical assertions about group behavior, and the central results of the theory have not all been demonstrated using the same analytical style. This paper attempts to remedy this deficiency in some major respects.[2] In addition to analyzing the relationship between group size and the voluntary provision of public goods, which is the focus of Olson's work, this paper carries out similar analyses of the relationship between the provision of public

*Assistant Professor of Political Science and Assistant Research Scientist, The Institute of Public Policy Studies, The University of Michigan. I am indebted to Paul N. Courant and Michael D. Cohen for several helpful discussions during the preparation and revision of this paper and to Howard Margolis for calling my attention to an error in an earlier draft of the paper. Research support was provided by The Institute of Public Policy Studies at The University of Michigan.

[1] The most important works which use the style of analysis of the present paper (indifference curve analysis) are those of Chamberlin (1974), McGuire (1974), and Olson and Zeckhauser (1966).

[2] The analysis here is restricted to a set of identical individuals, and thus does not address the issue of the equity of burden sharing discussed by Olson (1965) and Olson and Zeckhauser (1966). The analysis is also restricted to the case of pure public goods which are normal goods for all individuals. For analyses of the cases of inferior public goods and impure public goods, see Chamberlin (1974) and McGuire (1974).

goods and three other important parameters of the problem, the wealth of a group, the price of the public good, and the intensity of the individuals' preferences for the public good. The analysis is made particularly easy to follow because of a result discussed in the Appendix which enables the analysis of both Cournot and Pareto optimal behavior using the same graph. It is hoped that this method of presentation will contribute to a consolidation of the theory of collective action and will also make the theory more easily accessible to the average reader.

The basic model of group behavior used by Olson is one of Cournot behavior in the presence of a public good. In this model, each individual attempts to maximize his satisfaction, taking the behavior of others as given. The Cournot equilibrium is characterized by a set of actions based on a set of expectations converning others' actions that result in everyone's expectations being met. The Cournot equilibrium is in this sense equivalent to a Nash equilibrium if the problem is viewed as a non-cooperative game. Two properties of Cournot behavior which are of particular interest are the amount of the public good provided through such behavior and the difference between the amount provided through Cournot behavior and the amount which would be provided if the optimality conditions of the economic theory were to be met.

The optimality conditions require that the marginal costs of producing the public good be shared among its consumers in proportion to the marginal benefits received. From the infinite set of cost sharing plans which result in a Pareto optimal allocation a particular one will be used in the analysis below, that suggested by Lindahl. The Lindahl scheme is a quasi-market scheme, in which consumers are charged the same price for all units of the public good, just as in the case of a private good, although the prices charged consumers will normally be different because of different evaluations of marginal benefits. In addition, the Lindahl scheme does not involve lump sum redistribution of income.[3] The analysis below will compare the Cournot and Lindahl equilibria as a function of several important attributes of groups (size, wealth, price of the public good, and intensity of prefernce for the public good).

I. The Basic Model

Consider a group of n identical individuals, each endowed with an identical budget (of amount w.). There exist two goods, a private good Y with unitary price and a pure public good X with price p. The preferences of an individual are represented by the indifference curves as shown in Figure 1. The budget constraint is given by line AB, and the individual's optimal consumption is located at E, the point of tangency between the budget constraint and indifference curve I_2. Now suppose another individual were to provide an amount $\overline{x}$ of the public good. Because of the non-exclusion property of public goods, this amount is

[3]See Musgrave (1959) or Head (1974) for a discussion of the properties of the Lindahl equilibrium.

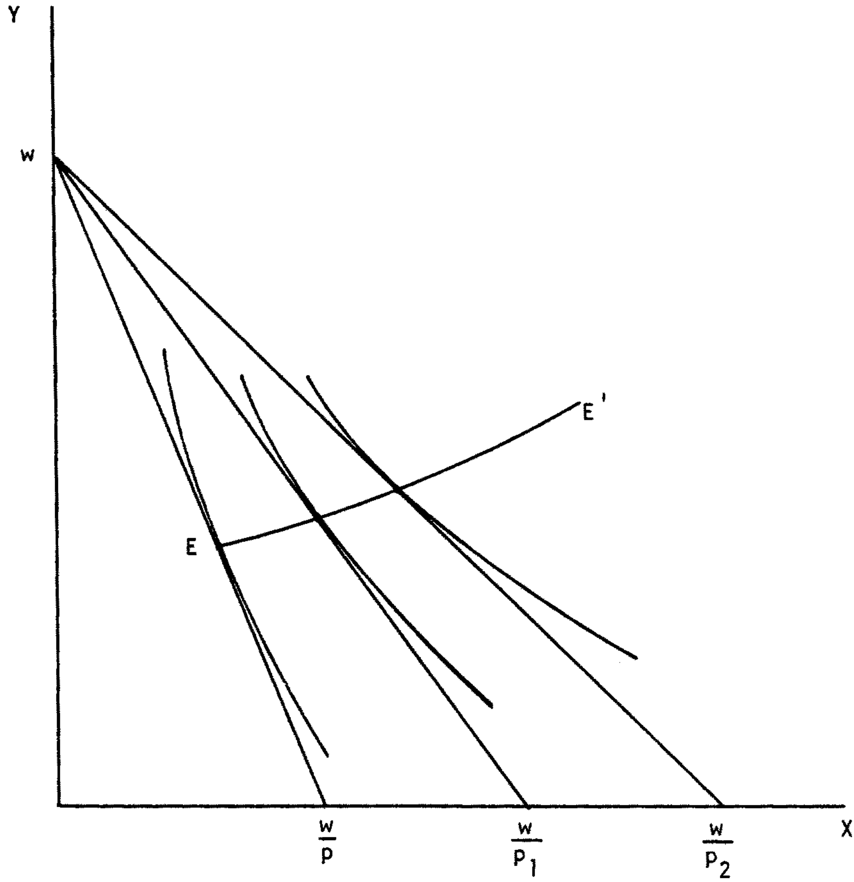


Figure 2. Price-Consumption Curve

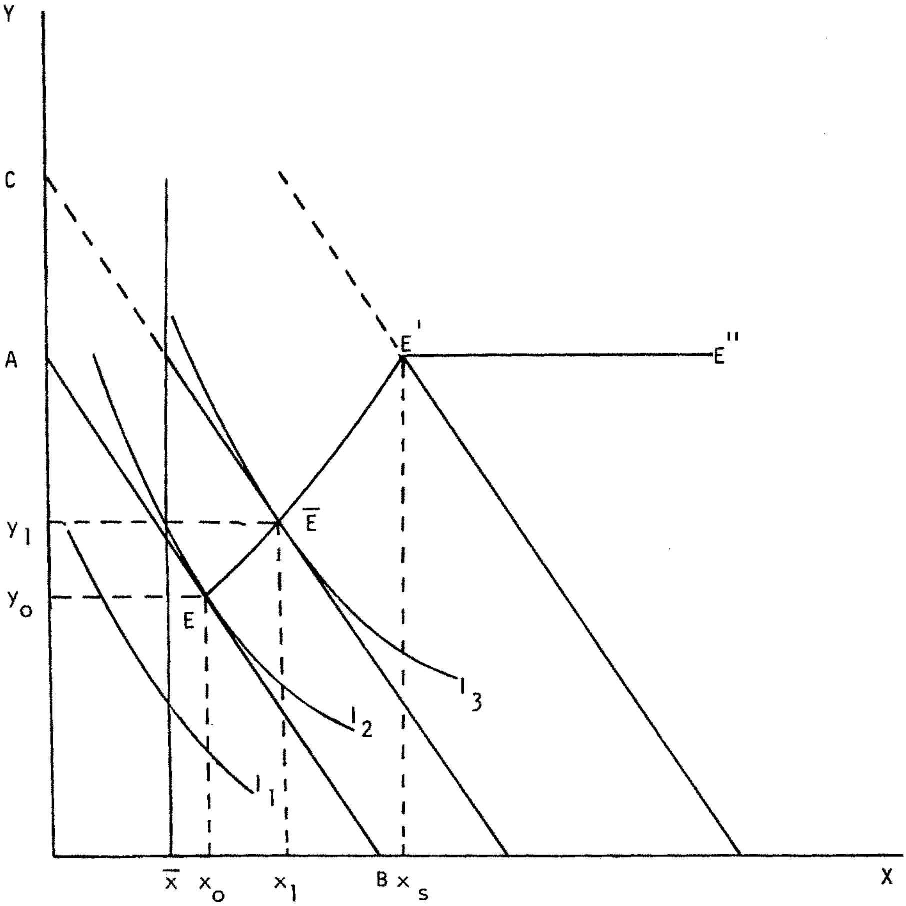


Figure 1. Income-Consumption Curve

automatically available to the first individual. This has the effect of shifting the vertical axis in Figure 1 to the right by an amount $\bar{x}$. The budget line is also shifted to the right by the amount $\bar{x}$, since the individual now consumes this amount of the public good without having to pay for it. This action by the other individual has the effect of increasing the first individual's income by an amount AC, and he is free to allocate his income as he chooses so long as he consumes at least an amount $\bar{x}$ of the public good. The individual's optimal consumption is now at $\bar{E}$, and he pays for an amount $(x_1-\bar{x})$ of the public good. If all possible amounts of the public good provided by others (up to an amount x_s) were considered, the locus of the individual's optimal consumption would be the line EE′, which is part of the individual's "income-consumption" curve. If others provide an amount of the public good greater than or equal to x_s, the individual will devote his entire budget to the purchase of the private good, as indicated by the horizontal line E′ E″. The analysis below will take into consideration only the portion EE′ of the locus of the individual's optima.

In addition to the individual's responses to increases in his income brought about by the provision of the public good by others, the analysis below will also make use of the individual's responses to variations in the price of the public good (brought about by an agreement among the individuals involved to share the costs of providing the public good). Figure 2 shows a portion of the "price-consumption" curve for the individual, which is the locus of the individual's optimal consumption as the price he is charged for the public good declines.

Figure 3 shows the relevant positions of the income and price-consumption curves and the original budget line.[4] In addition, a new budget line is shown, corresponding to a price of p/n for the public good. This budget line would result if the individuals were to share equally the costs of providing the public good. It is shown in the Appendix that the amounts of the public good provided under Cournot and Lindahl behavior are those associated with the allocations E_C and E_L, respectively. The Cournot equilibrium occurs where the income-consumption curve intersects the budget line (with price p/n for the public good), and the Lindahl equilibrium occurs where the price-consumption curve intersects this same budget line. The amounts provided at these equilibria are X_{E_C} and X_{E_L}, respectively. The degree of suboptimality associated with Cournot behavior is measured by $(X_{E_L} - X_{E_C})$.

The Cournot Equilibrium

The following properties of the Cournot equilibrium can be deduced from Figure 3:

[4]The analysis below is restricted to the case where neither good is an inferior good. This means that in Figure 3 B is above and to the right of A and that C is to the right of A. The price-consumption curve always lies below the income-consumption curve.

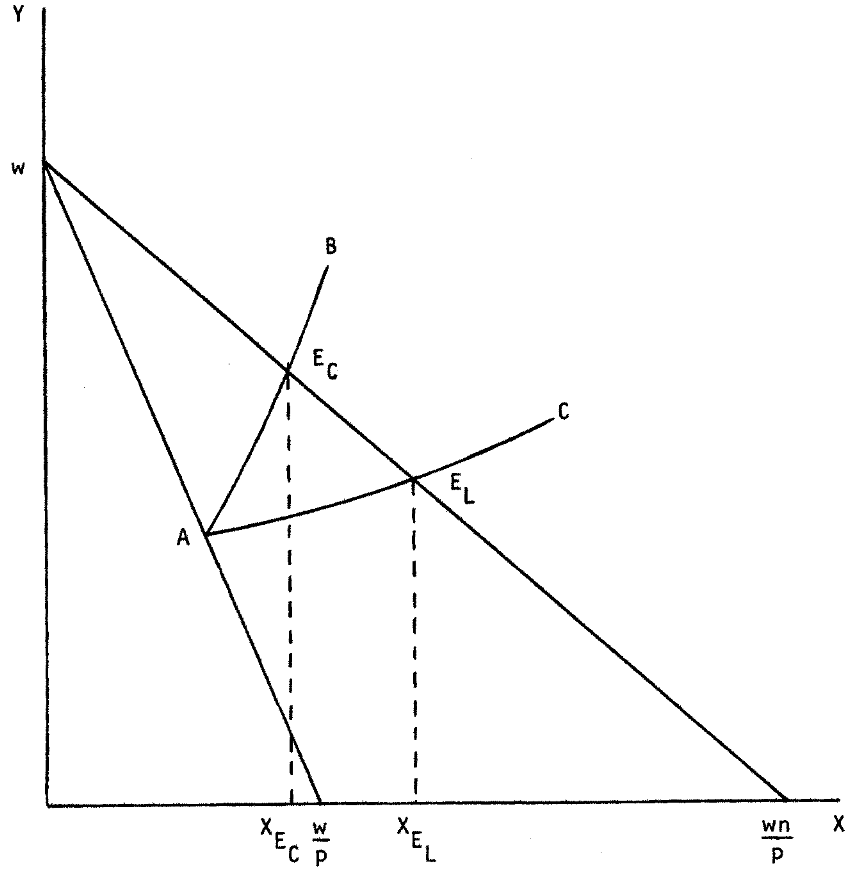


Figure 3. Cournot and Lindahl Equilibria

1) X_{E_C} increases as group size (n), wealth (w), and intensity of preference increase.[5]
2) X_{E_C} decreases as the price of the public good (p) increases.

With the exception of the relationship between group size X_{E_C}, these conclusions are obvious to anyone familiar with the basic model of consumer choice.[6] They are listed here only for completeness and because the discussion below will consider the relationship between these variables and the degree of suboptimality associated with Cournot behavior, as measured by $(X_{E_L} - X_{E_C})$.

[5] For the analysis below, one individual will be said to have a greater intensity of preference for the public good than another individual if the income-consumption and price-consumption curves for the first individual lie below and to the right of those for the second individual. That is, at all levels of wealth (w) and at all prices (p), the more intense individual will allocate a greater proportion of his budget to the public good.

[6] The result concerning the relationship between group size and X_{E_C} has been shown by Chamberlin (1974) and McGuire (1974).

The Degree of Suboptimality

Since in Figure 3 the curve AC must always lie below curve AB, it is always true that X_{E_C} is less than X_{E_L}. That is, as Olson argued, Cournot behavior results in the suboptimal provision of a public good. The following conclusions concerning the degree of suboptimality can be deduced from Figure 3:

3) the more elastic the demand for the public good, the greater the degree of suboptimality ($X_{E_L} - X_{E_C}$).

4) the more income elastic the demand for the public good, the less the degree of suboptimality.

II. Variations in Group Size

Figure 4 shows the case of two groups of sizes n_1 and n_2, where $n_1 < n_2$. The four equilibria are shown, where the superscripts refer to the groups. Another of Olson's central conclusions follows from Figure 4:

5) $(X_{E_L}^1 - X_{E_C}^1) < (X_{E_L}^2 - X_{E_C}^2)$; that is, the larger the group, the more suboptimal the provision of the public good.[7]

III. Variations in Wealth, Price and Intensity of Preference

Figures 5, 6 and 7 show the cases of two groups with different levels of wealth, prices for the public good and intensities of preference respectively.[8] In each case, the nature of indifference maps is such that no general conclusions can be reached concerning the relationship between these parameters and the degree of suboptimality associated with Cournot behavior. That is, in Figure 5, the price-consumption curve A′C′ may be drawn quite arbitrarily, subject only to the constraint that it lie between curves AC and A′B′. This freedom allows one to draw A′C′ in such a way that the degree of suboptimality may be either greater or smaller for the wealthier group. In Figure 6, the income-consumption curve A′B′ has the same property. Figure 7, showing different intensities of preferences, allows an even greater arbitrariness in drawing the curves. Thus, in contrast with the case involving variation in group size, there exists no general relationship between

[7]An additional assumption is necessary for this result, namely that in Figure 3, for all values of n, the slope of AB at E_C is greater than the slope of AC at E_L. The result follows directly from this assumption. The assumption is always met if demand for the public good is price-elastic, but there could be cases of inelastic demand in which the assumption is not met. I have been unable to determine whether there exists some "reasonable" class of preferences which fails to meet the assumption.

[8]In the analysis of interest group behavior, the price of the public good indicates the rate at which individuals can transform private goods into effective political activity. Different prices facing two groups would be an indication of the fact that the group facing the lower price is more influential than the other group (can achieve the same change in the political outcome using fewer private resources).

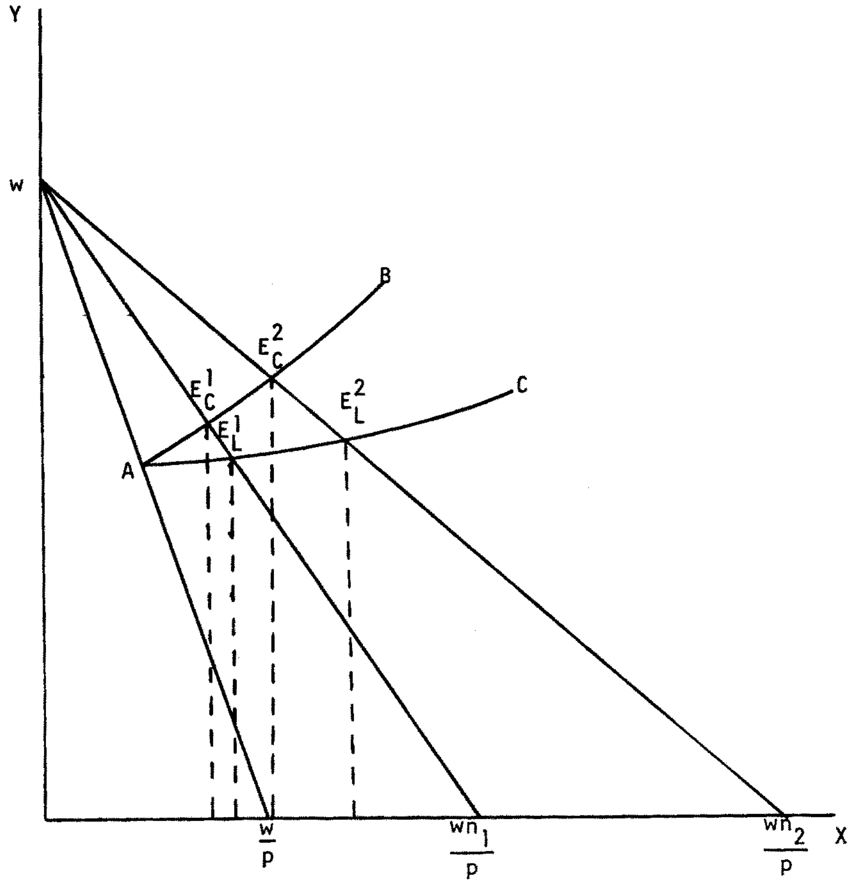


Figure 4. Variation in Group Size

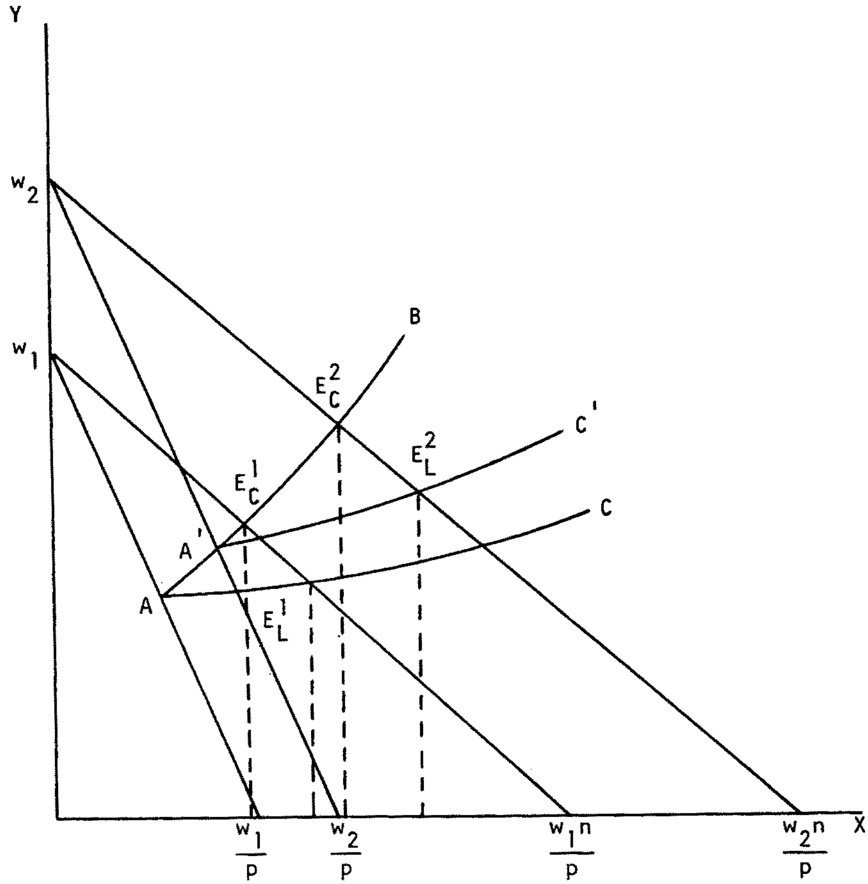


Figure 5. Variation in Wealth

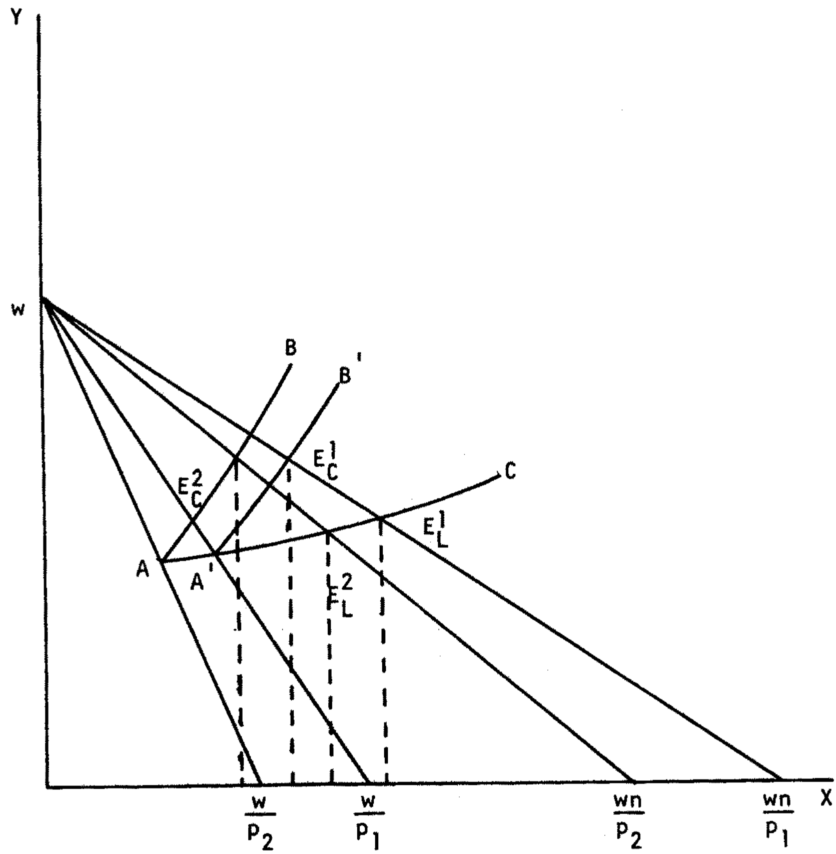


Figure 6. Variation in Price

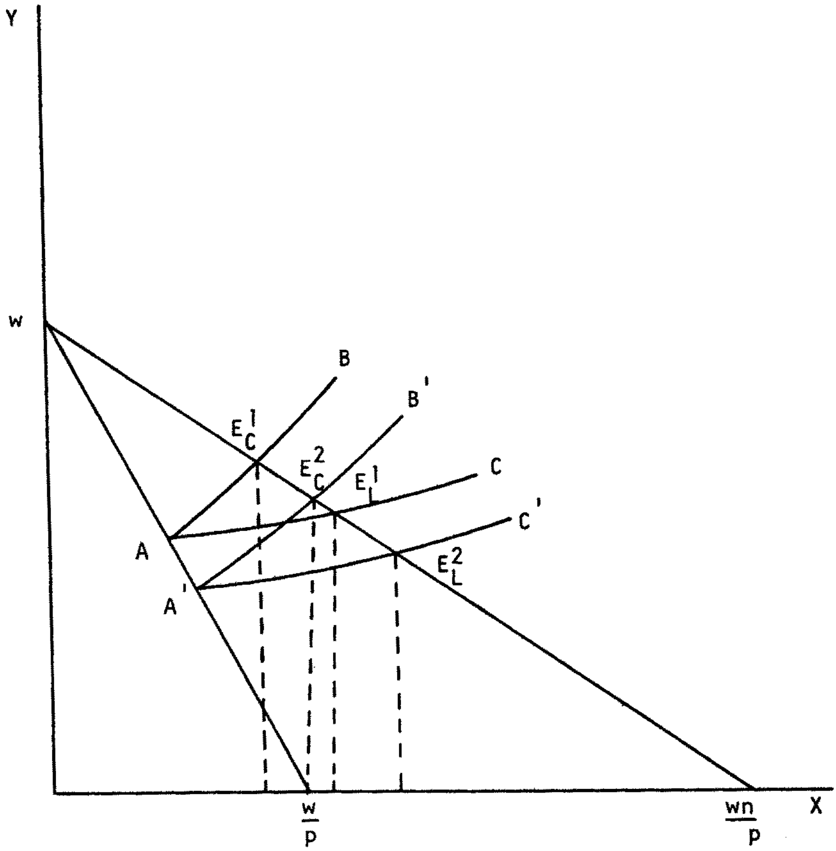


Figure 7. Variation in Intensity of Preference

wealth, price, and intensity of preference and the degree of suboptimality associated with Cournot behavior.

IV. Analysis of a Special Case

It is possible to find classes of indifference maps for which the ambiguity discussed above can be resolved. One such class consists of preferences which are representable by linear logarithmic utility functions, which take the form

$$u_i(X,Y) = k(\Sigma X_i)^{\alpha}(Y_i)^{\beta}$$

where X_i and Y_i are the amounts of the public and private goods consumed by the i^{th} individual, and k, α and β are positive constants. These preferences are characterized by constant income elasticity (income-consumption curve is a straight line through the origin) and unitary price elasticity (horizontal price-consumption curve). If this utility function can be used to represent preferences, then an analytic solution of the problem can be substituted for the diagrammatic analysis used above. In this case, the utility function is maximized subject to the budget constraint

$$pX_i + Y_i \leq w_i .$$

This yields the following provisions of public goods in the cases of Cournot (X_{E_C}) and Lindahl (X_{E_L}) behavior:

$$X_{E_C} = \frac{\alpha wn}{p(\alpha+\beta n)}$$

$$X_{E_L} = \frac{\alpha wn}{p(\alpha+\beta)} .$$

The degree of suboptimality is then given by

$$\Delta = X_{E_L} - X_{E_C} = \frac{\alpha\beta wn(n-1)}{p(\alpha+\beta)(\alpha+\beta n)} .$$

It is then easy to show that

$$\frac{\partial\Delta}{\partial n} > 0$$

$$\frac{\partial\Delta}{\partial w} > 0$$

$$\frac{\partial\Delta}{\partial p} < 0 ,$$

that is, that the degree of suboptimality is an increasing function of group size and wealth, and a decreasing function of price.

For the analysis of the relationship between intensity of preference and the degree of suboptimality, it is useful to define

$$\gamma = \frac{\alpha}{\beta}$$

yielding

$$\Delta = \frac{\gamma k}{(\gamma+1)(\gamma+n)}$$

where $k = \frac{wn(n-1)}{p}$. γ is then a measuryof intensity of preference. One then finds that

$$\frac{\partial \Delta}{\partial \gamma} = \frac{k(n-\gamma^2)}{(\gamma+1)^2(\gamma+n)^2}$$

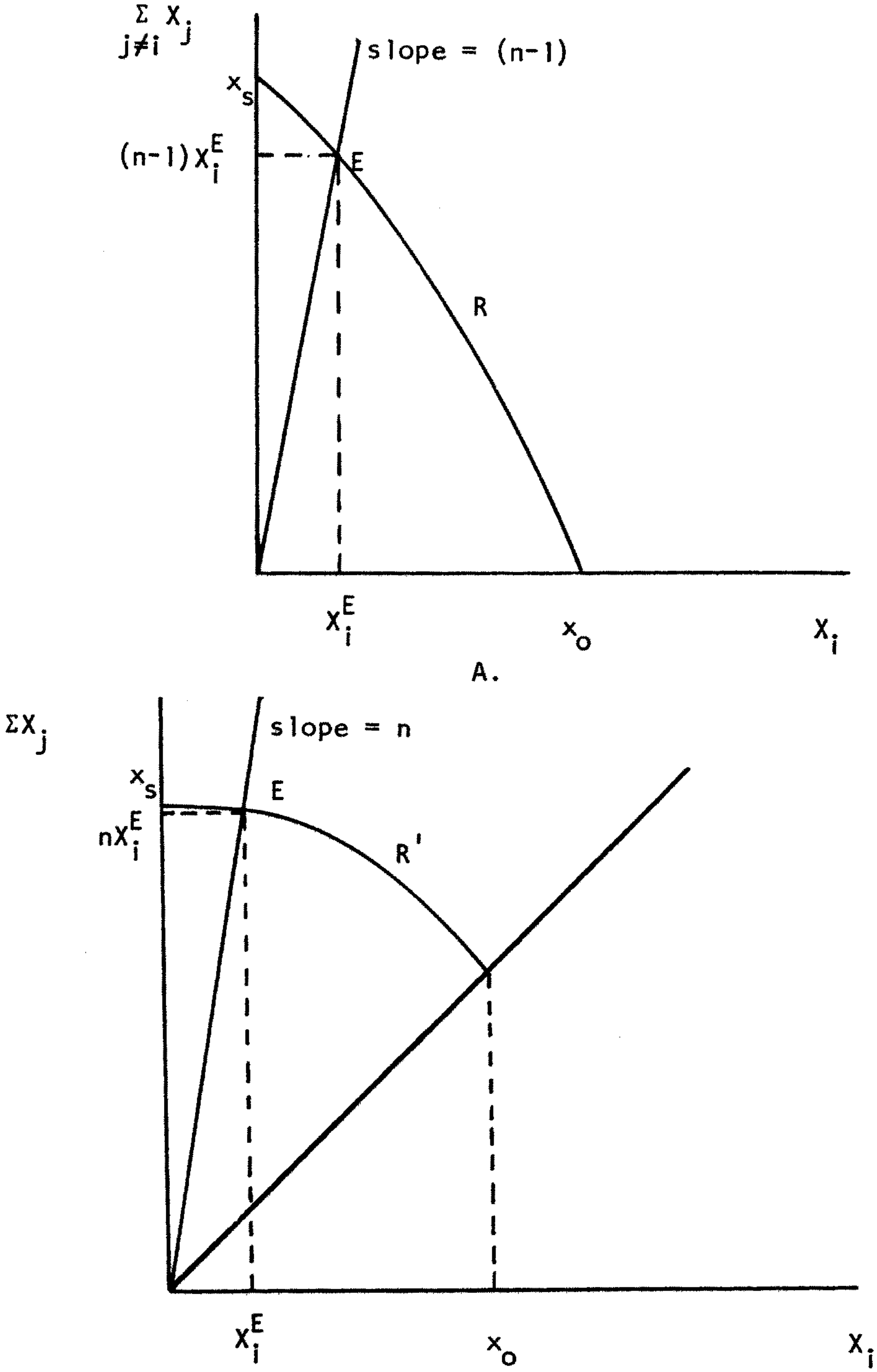
Thus $\frac{\partial \Delta}{\partial \gamma} > 0$ if $n > \gamma^2$, that is, if $n > (\frac{\alpha}{\beta})^2$ or $\frac{\alpha}{\beta} < \sqrt{n}$. For large groups it seems reasonable that this inequality will hold, and that the degree of suboptimality will be an increasing function of intensity of preference. Cases for which the inequality does not hold will presumably be of a life-or-death nature.

Thus, for the case of preferences representable by linear logarithmic utility functions, the degree of suboptimality increases as group size and group wealth increases and decreases as the price of the public good increases. The relationship between the degree of suboptimality and intensity of preference for the public good is not completely determined, and depends upon the relationship between group size and the intensity preference. The condition derived above, however, suggests that the degree of suboptimality can usually be expected to increase as intensity of preference increases.[9]

V. Summary

This paper has investigated the relationships between two properties of the provision of public goods through Cournot behavior (the amount of the public good provided and the degree of suboptimality relative to Pareto optimal behavior) and several important parameters of groups (size, wealth, price of the public good, and the intensity of preference for the public good). It has been shown that the amount of the good provided through Cournot behavior increases as the size,

[9] Prafulla Joglekar has suggested as an alternative definition of the degree of suboptimality the ratio Δ/X_{E_L}; that is, the difference between the amounts of the good provided through Lindahl and Cournot behavior expressed as a percentage of the amount of the good provided through Lindahl behavior. For the case of linear logarithmic utility functions, the degree of suboptimality defined in this fashion increases as group size increases, decreases as intensity of preference increases, and is unaffected by changes in price and wealth. The choice of which definition one uses is arbitrary; the original measure was used here because it can be more easily depicted in the diagrammatic argument presented above.



B. Figure 8. Reaction Curves

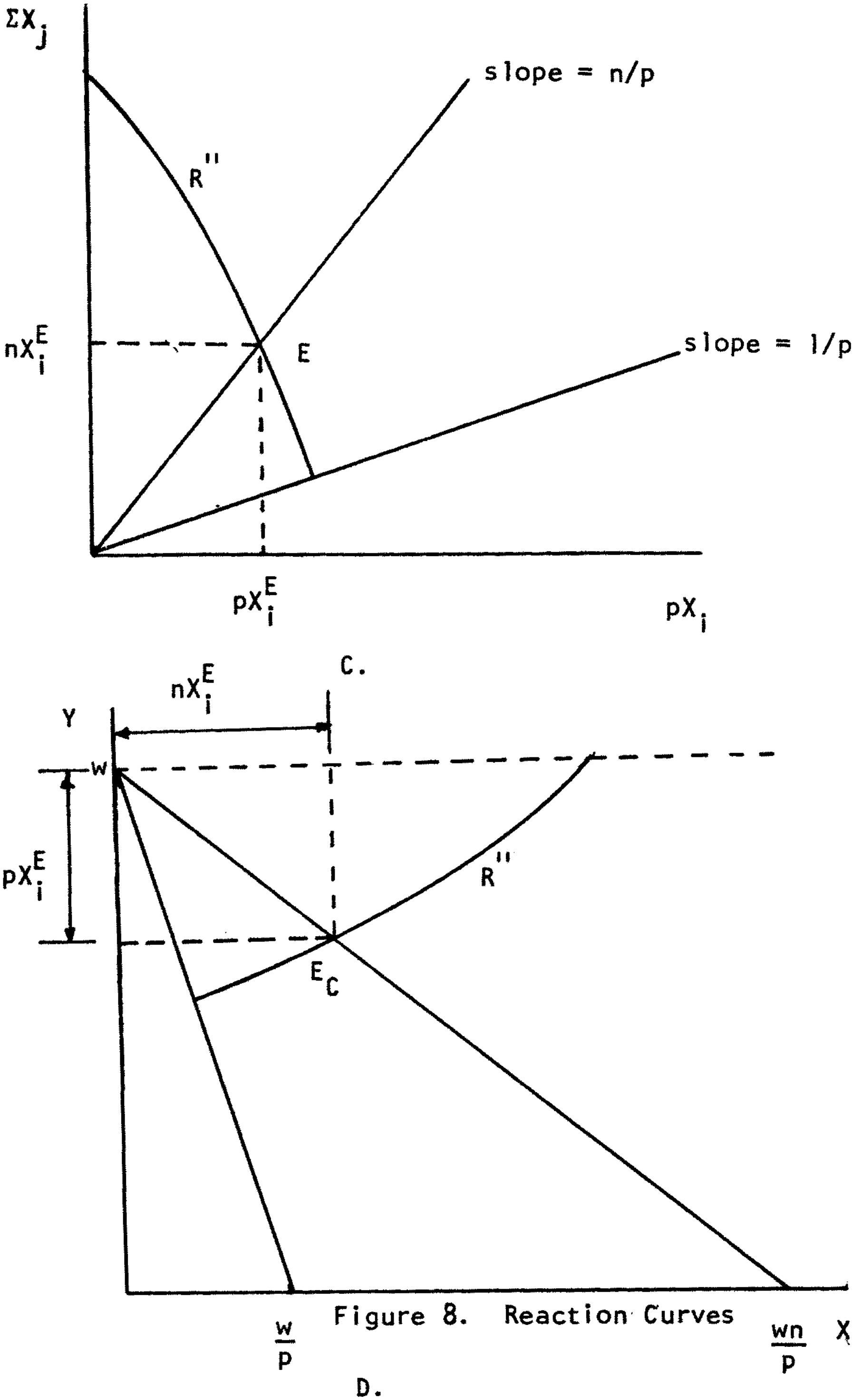


Figure 8. Reaction Curves

with slope n/p intersects the transformed reaction curve R″. Finally Figure 8D is derived by rotating Figure 8C 90° in a clockwise direction. When a new horizontal axis is added, it will be noted that Figure 8D is identical to Figure 3. The transformed reaction curve R″ is the income-consumption curve, and E_C (in Figures 3 and 8D) is the Cournot equilibrium since the total amount of the public good provided ($X_i = nX_i^E$) is (n/p) times the amount of money (pX_i^E) spent by the individual on the public good, as in Figure 8C.

REFERENCES

Chamberlin, John. "Provision of Collective Goods as a Function of Group Size," *American Political Science Review*, 68 (June, 1974), 707-716.

Head, John. *Public Goods and Public Welfare*. Durham, North Carolina: Duke University Press, 1974.

McGuire, Martin. "Group Size, Group Homogeneity and Aggregate Provision of a Pure Public Good under Cournot Behavior," *Public Choice*, 18 (Summer, 1974), 107-126.

Musgrave, Richard. *The Theory of Public Finance*. New York: McGraw-Hill, 1959.

Olson, Mancur, Jr. *The Logic of Collective Action*. Cambridge: Harvard University Press, 1965.

______________, and Zeckhauser, Richard. "An Economic Theory of Alliances," *Review of Economics and Statistics*, 48 (August, 1966), 266-279.