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A NOTE ON THE RELATION BETWEEN THE VECTOR MODEL AND THE UNFOLDING MODEL FOR PREFERENCES

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The Tucker [1960; see also Tucker and Messick, 1963] vector model has been known for some time to be a special case of the personal compensatory model for preferences [Coombs, 1964] and has also been shown to be (asymptotically) a special case of unfolding theory [Carroll, 1972]. These two more general theories are mutually independent. But there is another rather special relation between the vector model and unfolding theory which can occur under certain circumstances and which if not understood can seem quite confusing. Consider, for example, a set of candidates for promotion to academic tenure or for employment as research assistants. And, for purposes of illustration, let there be two attributes involved which are imperfectly correlated, say excellence of teaching and research productivity in the first instance or intelligence and technical training in the second. As both attributes are desired, more of either is better, and the linear vector model in E^2 would be a reasonable one for choosing among such a set of candidates.

In Figure 1 a sample space of possible candidates is portrayed and the vectors for two hypothetical "choosers", A and B. The rank order of a chooser's preferences predicted by the vector model is given by the rank order of the projections of the candidates on the corresponding vector, *i.e.*, the farther out a candidate projects on A's vector the more he is preferred by A.

Let us define an *efficient set* of candidates as a set in which each member of any pair exceeds the other on at least one attribute, *i.e.*, no member of the set is "dominated" by any other member. If the candidates were screened to select those along an arc segment *convex* to the origin and bounded by the points of tangency of the horizontal and vertical tangents, such as is illustrated in the figure, they would be members of a convex efficient set.

A's ideal candidate in a convex efficient set would be the candidate at the point of intersection of A's vector with the arc and A's preference order, being the order in which the candidates on the arc project on A's vector, would be generated by a single-peaked function over the set because if they are on the same side of the vector, the nearer they are on the arc to the ideal point the higher they will project on the vector. Similarly B's preference order over this same set would be generated by "folding" the arc at B's ideal point given by the point of intersection of B's vector with the arc defining this set. In general, the vector model always implies the unfolding model for a suitable set of stimuli such as indicated here.

Of course the choice process need not be restricted to two dimensions which are in an approach-approach conflict as used in this illustration. In general for an r-dimensional set of stimuli under the vector model any convex efficient set will yield an $(r - 1)$-dimensional space by unfolding. For example, a convex efficient set of stimuli in three desirable but imperfectly positively correlated dimensions would be those lying on a spherical triangle in the positive orthont on the surface of any spheroid in the space. The triangle would be a two-dimensional space unfolded. If some of the attributes were negatively correlated or were undesirable the spherical triangle would merely lie in a different orthont.

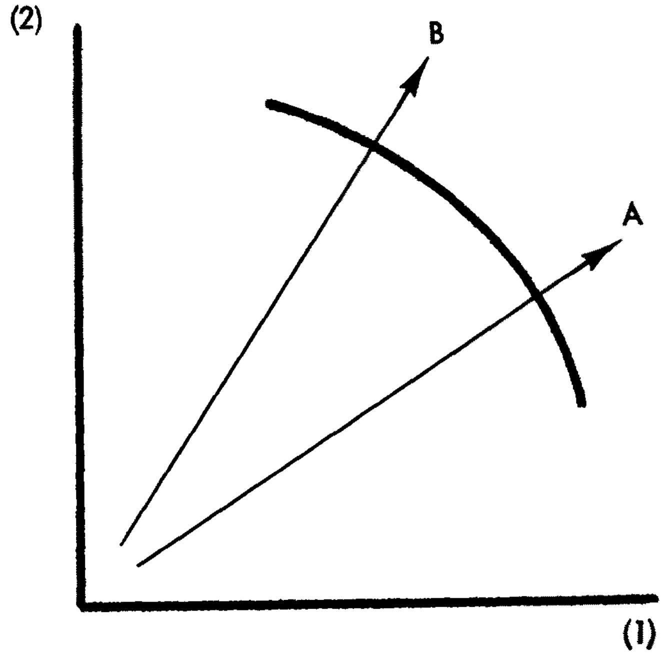

FIGURE 1.
A Convex Efficient Set of Stimuli

There are several ways of interpreting the implication of this relation between the two models which might be pointed out: 1) the conclusion one draws about which model is more appropriate may merely reflect the stimuli selected; 2) if the vector model fits in a particular instance, then a subset of stimuli exist such that the unfolding model will fit in one less dimension; 3) if the unfolding model fits in r-dimensions it may be worthwhile to see if the stimuli are best interpreted as vectors with $r + 1$ attributes, though, of course, it is not necessary that they should be.

This preliminary note is a consequence of a general theory of conflictual choice now in an extended gestation period.

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