copy 6
Bradfield

EMB-8

AN EVALUATION OF THE DISCREPANCY BETWEEN EXPERIMENT AND THEORY FOR A TYPICAL PRESSURE DISTRIBUTION TEST AT A MACH NUMBER OF 1.93

by W.H. Dorrance

Report No. EMB-8

Copy No.________

AN EVALUATION OF THE DISCREPANCY BETWEEN EXPERIMENT AND THEORY FOR A TYPICAL PRESSURE DISTRIBUTION TEST AT A MACH NUMBER OF 1.93.

Prepared by W. H. Dorrance
W. H. Dorrance

Approved by F. W. Ross
F. W. Ross

November 29, 1948

engn
UMR 1039

TABLE OF CONTENTS

Introduction

Experimental pressure distributions over supersonic models are being conducted at a Mach Number of 1.93 at the University of Michigan Supersonic Wind Tunnel. Concurrently with these tests theoretical pressure distributions are being determined using the linearized theories, Taylor and Maccoll theory for cones and the three dimensional method of characteristics. The first of the series of tests were run on the simple model of a 20° included angle cone. An analysis is made in this report of the discrepancy present between theoretical and experimental values of pressure coefficient for the 20° cone model.

Summary

On the basis of the investigation reported herein the following conclusions are drawn regarding pressure test variables in the 20° cone test.

1. The effects of viscosity which are neglected by theory account for about 30% of the deviation in pressure coefficient values between experiment and theory.

2. Tunnel Mach Number variation is slight in the region of the cone model and had very little effect on the values of pressure coefficient.

3. The variation of $\frac{p_o}{q}$ and $\frac{p_o}{p_a}$ versus test section position along the centerline of the model can effect values of pressure coefficient significantly.

4. The effects of incorrect reading or deviation of angle of attack can be counteracted to a reasonable extent by averaging values of pressure coefficient diametrically opposed at each station.

This report examines the test parameters discussed above and evaluates their effects upon the experimentally determined values of pressure coefficient. On the basis of this analysis it was concluded that an error exists in the experimental values of pressure coefficient possibly due to erroneous recorded values of the pressure ratios $\frac{p_o}{q}$, $\frac{p_o}{p_a}$ and $\frac{p}{p_o}$.

Symbols

$C_p = \dfrac{p - p_a}{q}$ =	pressure coefficient
$C_{p_{T-M}}$ =	Taylor-Maccoll theoretical pressure coefficient
$K_D = \dfrac{C_{p_{T-M}}}{2}$ =	drag coefficient of cone given by M.I.T. tables
l =	length of cone model surface element
M =	free stream Mach Number
p =	local static pressure
p_a =	free stream static pressure
$q = \frac{\rho}{2} V^2$ =	free stream dynamic pressure
$R = \frac{\rho V l}{\mu}$ =	Reynolds Number of cone surface
V =	free stream velocity
α =	angle of attack
$\gamma = 1.405$ =	ratio of specific heats for air
ϕ =	angle of roll measured from vertical reference plane
θ_s =	semi-vertex angle of cone
ρ =	free stream density
μ =	coefficient of viscosity
I, II, III, IV, V =	meridian planes at 0°, 45°, 90°, 135° and 180° respectively

Discussion

A series of pressure distribution tests was run on a 20° included angle cone model at a Mach Number of 1.93 and was reported in Reference 1. Included in these tests were runs at about 0° angle of attack. The theoretical values of pressure coefficient as determined by the exact Taylor-Maccoll theory and the linearized theory were determined and compared with the experimental values. (Figure 1) Examination of the plot of pressure coefficient C_p versus axial station for the 20° cone reveals a discrepancy of $\Delta C_p \cong .022$. (Figure 1) An investigation was made into the test parameters that affect the surface pressures and the pressure coefficient.

Pressure coefficient as plotted in Reference 1 was based on the following expression.

$$(1) \quad C_p = \left(\frac{p}{p_a} - 1\right)\frac{2}{\gamma M^2}$$

where: $M = 1.93$ (average value)

$\gamma = 1.405$

In this formula $p/p_a = \frac{p}{p_o} \times \frac{p_o}{p_a}$

where $\frac{p_o}{p_a}$ = mean value at each station along the tunnel centerline.

Since this corrected value of $\frac{p_o}{p_a}$ was used it is believed that the largest source of deviation lies in the choice of the average value of $M = 1.93$. To check the value of C_p obtained by using formula (1) an alternative formula for C_p was used. This is:

$$(2) \quad C_p = \left(\frac{\frac{p}{p_o} - \frac{p_a}{p_o}}{\frac{q}{p_o}}\right)$$

In this formula the plots of the variation $\frac{p_a}{p_o}$ and $\frac{q}{p_o}$ along the length of the model were prepared by the wind tunnel group and used throughout the data reduction process. It was decided, however, to use a mean value of $\frac{q}{p_o}$ to reduce labor and time consuming calculations.

The list of test variables which can affect the values of pressure coefficient as expressed by equations (1) and/or (2) and the experimental values of pressure are listed below with their ranges of variation.

1. Mach Number M
   $1.915 \leqq M \leqq 1.93$ Average $M = 1.93$ at model centerline

2. Stagnation to ambient pressure ratio $\frac{p_o}{p_a}$

   $6.980 \leqq \frac{p_o}{p_a} \leqq 7.030$ Average $\frac{p_o}{p_a} = 7.005$

3. Stagnation to dynamic pressure ratio $\frac{p_o}{q}$

   $2.68 \leqq \frac{p_o}{q} \leqq 2.695$ Average $\frac{p_o}{q} = 2.686$

4. Angle of attack variation

   $0^o \leqq \alpha \leqq .2^o$

5. Vertex angle variation

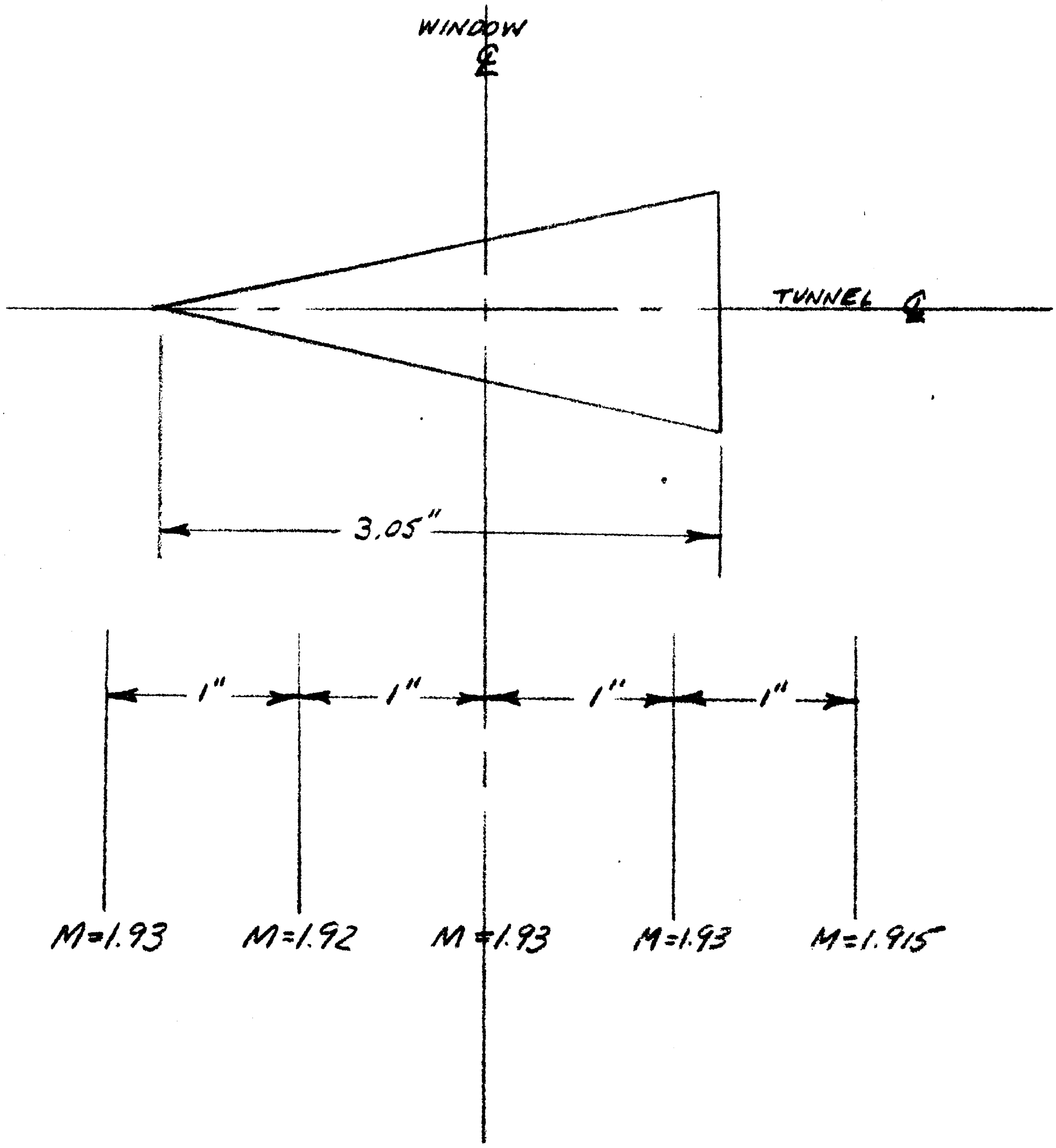
   $20^o00^1 \leqq 2\theta_s \leqq 20^o09^1$

6. Boundary layer increasing the effective cone angle.

Each of these parameters will be discussed in the following text.

The Effects of Mach Number on Pressure Coefficient.

The effect of Mach number variation in equation (1) is apparent when the smallest possible value of M in the region of the model is used to calculate pressure coefficient. The variation of M in the region of the model is shown in the following sketch.



The value of C_p based on M = 1.93 shown in Figure 1 is

$$C_p = .129$$

If the extreme value of M = 1.915 is taken the value of the mean C_p becomes,

$$C_p = .131$$

This yields a $\Delta C_p = .002$ and is not considered a significant change in C_p.

The Effects of $\frac{P_o}{P_a}$, $\frac{P_o}{q}$ and $\frac{P}{P_o}$ on Pressure Coefficient

The effects of variation in stagnation to ambient pressure ratio $\frac{P_o}{P_a}$ and stagnation to dynamic pressure ratio $\frac{P_o}{q}$ are discussed together. The ranges on these parameters are given below.

$$6.980 \leq \frac{P_o}{P_a} \leq 7.030$$

and,

$$2.680 \leqq \frac{p_o}{q} \leqq 2.695$$

Using extremal values of these parameters in the expression for pressure coefficient (2) an idea of the range of values in experimental C_p can be obtained.

$$C_p = \frac{p - p_a}{q} = \frac{\frac{p}{p_o} - \frac{p_a}{p_o}}{\frac{q}{p_o}}$$

The values of p/p_o from test data fell into the following range when values straying obviously from the mean were neglected.

$$.1890 \leqq \frac{p}{p_o} \leqq .1925 \qquad \text{Average } \frac{p}{p_o} = .1907$$

Using extremal values to determine the experimental range of C_p there results;

$$C_p \max = \frac{\left(.1925 - \frac{1}{7.030}\right)}{\frac{1}{2.695}} \cong .1327$$

$$C_p \min = \frac{\left(.1890 - \frac{1}{6.980}\right)}{\frac{1}{2.68}} \cong .12257$$

Hence, the experimental pressure coefficient lies within the range, Figure 1

$$.12257 \leqq C_p \leqq .1327$$

Using mean values of $\frac{p}{p_o}$ taken from experiment along with mean values of $\frac{p_o}{p_a}$ and $\frac{p_o}{q}$ will give a mean experimental value of C_p to compare with the mean value plotted on Figure 1 obtained in Reference 1 using equation (1). This is,

$$C_p \text{ mean} = \left(\frac{.1907 - \frac{1}{7.005}}{\frac{1}{2.686}}\right) \cong \underline{.1286}$$

This value is substantially the same as that found using expression (1) which was

$$C_p \text{ mean} = \left(\left(\frac{p}{P_a}\right)_{ave.} - 1\right) \frac{2}{1.405 \times \overline{1.93^2}} \cong \underline{.129}$$

Since mean values for $\frac{p}{P_a}$, $\frac{P_o}{P_a}$ and $\frac{P_o}{q}$ were taken throughout in the calculations to determine C_p mean above it is concluded the figure represented by $C_p = .129$ is as correct as can be reasonably determined using the data made available.

The Effects of Angle of Attack Deviation on Pressure Coefficient

There was a range of angle of attack variation which is given below.

$$0 \leq \alpha \leq .2^{\circ}$$

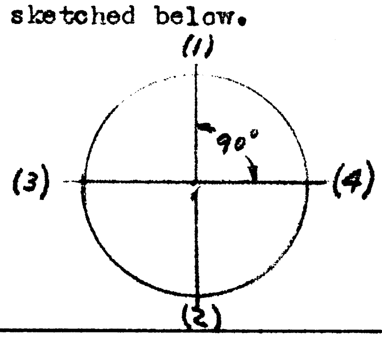
This range was obtained by measuring the angle between the centerline of the cone and a vertical reference line in the Schlieren photographs. Reference 3 has shown that taking the mean value of two pressure readings on a cone surface at two points diametrically opposed when the cone is at a slight angle of attack will yield the zero angle of attack value. Since readings were taken at 0° roll and 180° under similar conditions it is reasonable to assume under the hypothesis above that the mean value of P/P_o is close to the correct value. Thus the effect of slight angles of attack is ruled negligible.

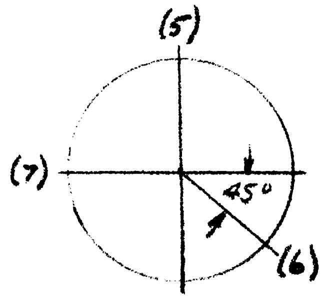
The Effects of Vertex Angle Deviation on Pressure Coefficient.

The theoretical value of C_p for a 20° vertex angle cone at a Mach Number 1.93 was extrapolated from tables given in Reference 4. These tables were assembled from calculations based on the accurate Taylor-Maccoll theory for cones. According to Reference 4 such extrapolation is permissible. The value of C_p thus determined and plotted in Figure 1 is given below.

$$C_{P_{T-M}} = .10672$$

Since it is impractical to expect the true cone angle of the model to be exactly 20° a series of measurements was made of the vertex angle. Readings were taken in the planes illustrated below as the model was clamped rigidly in a lathe chuck. Readings (1) and (2) were taken in planes diametrically opposed. Readings (3) and (4) were also taken 180° apart and 90° from the plane of readings (1) and (2). Readings (5), (6), and (7) were taken in the orifice planes of the model as sketched below.





The mean of readings (1), (2), (3), and (4) was taken and compared with the mean of readings (5), (6) and (7). On the basis of these readings it was decided that the true vertex angle was

$$2\theta_s = 20^\circ\ 6^1$$

Using this value for $2\theta_s$ the theoretical value of C_p for $M = 1.93$ was determined again. This value was

$$C_{p_{T-M}} = .10762$$

The difference between this value and the mean experimental value was attributed to boundary layer growth. This difference is,

$$\Delta C_p \cong .1286 - .1076 \cong .021$$

Effect of Boundary Layer on Pressure Coefficient

On the basis of the experimental value of $C_p = .1286$ the effective cone angle can be determined for $M = 1.93$. Using the Taylor-Maccoll theory the apparent effective cone angle is,

$$(\theta_s)_{\text{effective}} = 11.18^\circ$$

From which the apparent $\Delta\theta_s$ due to boundary layer is determined. This is

$$\Delta\theta_s = (\theta_s)_{\text{effective}} - (\theta_s)_{\text{measured}} = 11.18^\circ - 10.05^\circ$$

$$\Delta\theta_s = 1.13^\circ$$

On the basis of this apparent $\Delta\theta_s$ a boundary layer thickness can be determined. This is, for the cone model, where:

l = length of cone surface element $\cong$ 3.10"

δ = boundary layer thickness at end of cone element

$$\delta \cong l \tan \Delta\theta_s = 3.10 \times \tan 1.13^\circ$$

$$\delta \cong .0608''$$

It is apparent that the boundary layer thickness δ must be this value if the experimental values of C_p are to be consistant with the theoretical values of C_p.

References 5 and 6 have determined that boundary layer formulas for laminar and turbulent flow are unaffected by Mach Number variation in the low supersonic range. Using the familiar expressions for boundary layer thicknesses for laminar and turbulent flow over flat plates an order of magnitude of the thickness of the boundary layer to be expected over the cone can be determined. These formulas are given below including the numerical calculations.

The Reynolds Number of the wind tunnel test section is given as:

$$R/ft \cong 3.92 \times 10^6/ft$$

The Reynolds Number of the cone model is then;

$$R = 3.92 \times 10^6 \times 1 = 3.92 \times 10^6 \times \frac{3.10}{12} = 1.012 \times 10^6$$

<u>For laminar flow</u>

$$\delta = \frac{5.2 \times 1}{\sqrt{R}} = \frac{5.2 \times 3.10}{1.006 \times 10^3} = .01602''$$

for which

$$\Delta\theta_s = \tan^{-1} \frac{.01602}{3.10} \cong .293^\circ$$

and

$$\theta_{s_{effective}} = \Delta\theta_s + \theta_s = .29 + 10.05 \cong 10.34^\circ$$

the theoretical pressure coefficient for this cone angle is: (Figure 2)

$$C_{P_{T-M}} \cong .11302$$

<u>For turbulent flow</u>

$$\delta = \frac{.376\ 1}{(R)^{1/5}} = \frac{.376 \times 3.10}{(10)^{1.2} \times (1.012)^{.2}} = .0737''$$

for which

$$\Delta\theta_s \cong \tan^{-1} \frac{.0737}{3.10} \cong 1.36^\circ$$

and

$$\theta_{s_{effective}} = \Delta\theta_s + \theta_s = 1.36 + 10.05 = 11.41^\circ$$

The theoretical pressure coefficient for this cone angle is: (Figure 2)

$$C_{P_{T-M}} = .1329$$

Thus the experimental value of $C_p \cong .129$ lies between the value for pressure coefficient including an allowance for a laminar boundary layer and the value for pressure coefficient including an allowance for turbulent boundary layer. (Figure 2) That is,

$$C_{P_{T-M}} + (\Delta C_p)_{\text{laminar B.L.}} \cong .113$$

$$(C_p)_{\text{mean experimental}} \cong .129$$

$$C_{P_{T-M}} + (\Delta C_p)_{\text{turbulent B.L.}} \cong .133$$

Examination of a typical Schlieren photograph of the cone tests indicates that the boundary appears to be laminar. Experiments run by the NACA reported in Reference 5 at Reynolds Numbers including and exceeding the Reynolds Number of the cone test showed that the boundary layer was laminar over bodies of revolution placed in a Mach Number 1.5 stream. Experiments reported in Reference 7 reported boundary layer thicknesses over models at Mach Numbers tabulated below.

Model 1058	$M = 1.86$	$\delta = .020" - .033"$
Model 22-588	$M = 1.76$	$\delta = .04"$
Model 11058	$M = 1.96$	$\delta = .02" \rightarrow .04"$

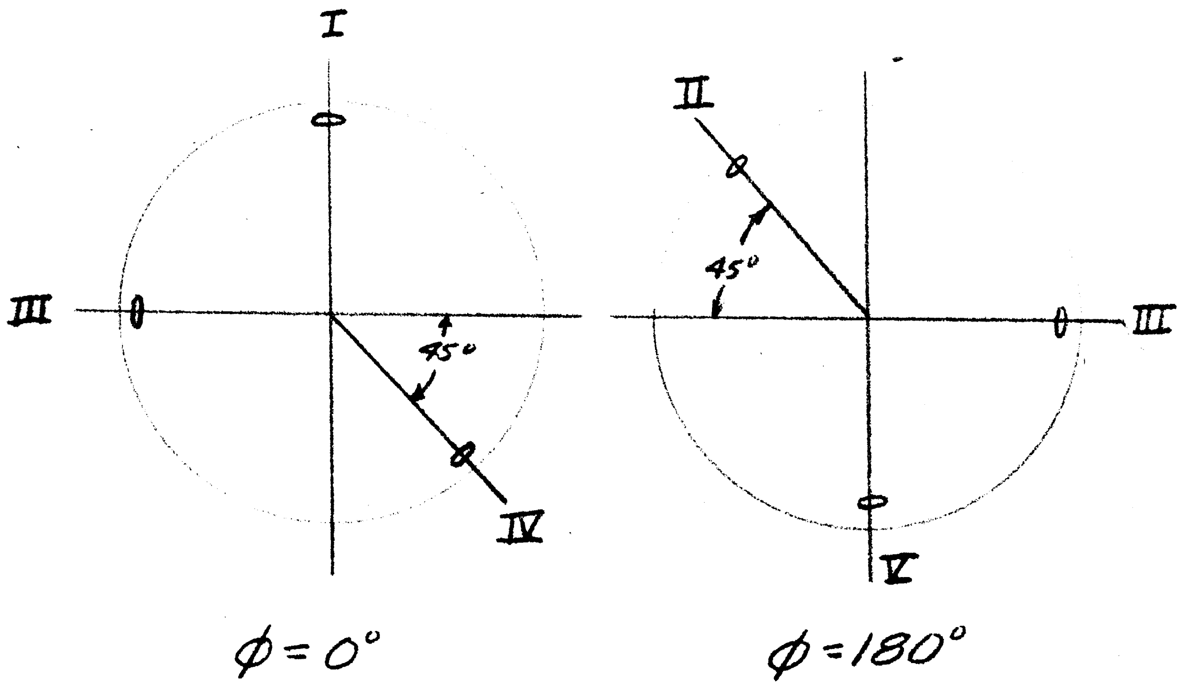
Experiments over a cone in a Mach 1.75 free jet reported in Reference 9 found boundary layer thicknesses indicated below.

20° cone	$M = 1.75$	$\delta \cong .0298"$
30° cone	$M = 1.75$	$\delta \cong .0248"$
40° cone	$M = 1.75$	$\delta \cong .0294"$

These values are near the value for a laminar boundary layer over the surfaces tested. On the basis of this information and the analysis above it is concluded that the boundary layer over the cone model was nearly laminar in character.

A Check on Test Data Consistancy

A check was made to determine if the discrepancy in pressure coefficient was consistant in more recent tests. Pressure taps on the 19.66° conical nose of another model tested after the cone model at $M = 1.93$ supplied data useful for comparison purposes. The orifices on this nose were located as shown in the following sketch.



Readings were taken at meridian planes I, II, III, IV, and V for seven different runs at about zero angle of attack and averaged to yield a mean value. Table 1 gives the values of C_p for these readings. The mean value of experimental C_p was

$$C_p \cong .123$$

The Taylor-Maccoll value for a 19.60° cone was,

$$C_{p_{T-M}} = .10396$$

To this theoretical value of C_p must be added an allowance for boundary layer effects and other effects included in the increment apparently present in the previous cone test made in similar circumstances. The nose of the biconic model was the same length as the cone model so that the same discrepancy should be apparent if a consistant discrepancy is present. This value of ΔC_p was found to be $\Delta C_p \cong .020$.

Adding this to the theoretical value $C_{p_{T-M}}$ will give a value for C_p which should be close to the experimental mean value. The two values are given below.

$$(C_p)_{\text{experimental mean}} \cong .123$$

$$C_{p_{T-M}} + (\Delta C_p)_{B.L.} = .1040 + .0205 \cong .1245$$

The two values agree closely and the conclusion is made that the same discrepancy appears in both tests.

Summary of Conclusions

Because a larger boundary layer thickness then can reasonably be expected from a near laminar boundary layer is needed to increase the effective cone angle to a value necessary to yield the experimental pressure coefficient it is concluded that the discrepancy between experimental and theoretical pressure coefficients cannot be completely accounted for by viscous effects. The error between the experimental value and the theoretical value based on a laminar boundary layer is in the region of $\Delta C_p \cong .013$. The experimental scatter about the mean value of experimental C_p is $+\Delta C_p \cong .004$ and $-\Delta C_p \cong .006$. As illustrated in Figure 1.

On the basis of this analyses a table of percentage error can be assembled for the cone test. The percents are expressed as percent of mean experimental pressure coefficient.

Percent discrepancy between experimental pressure coefficient and Taylor-Maccoll pressure coefficient.	16.3%
Percent discrepancy between experimental pressure coefficient and theoretical pressure coefficient corrected for a laminar boundary layer.	12.1%
Percent error possible using extremal experimental values of $\frac{P_o}{q}$, $\frac{P}{P_o}$, and $\frac{P_o}{P_a}$	(+3.19% (-4.7%

On the basis of this evidence it is concluded that some unforeseen quantity is affecting the pressure data. This quantity may be present in the form of erroneous recordings of pressure ratios $\frac{P_o}{P_a}$, $\frac{P}{P_o}$, and $\frac{q}{P_o}$.

References

1. "Pressure Distribution and Force Coefficients on a 20° Cone Using Experimental and Theoretical Methods at Mach Number 1.93", by W. H. Dorrance, University of Michigan Report EMB-5.

2. "Final Calibration Report on the Mach 2 Configuration, Excluding Balance System", by W. H. Curry, University of Michigan, U.M.E.R.I. Supersonic Wind Tunnel Report WTM-45.

3. "Pressure Distributions Over a Cylinder with Conical or Hemispherical Nose", by T. L. Cronvich, Applied Physics Laboratory, The Johns Hopkins University Report TG-10-4.

4. "Supersonic Flow of Air Around Cones", by Z. Kopal, Massachusetts Institute of Technology Report TR-1.

5. "Experimental Investigation of the Effects of Viscosity on the Drag of Bodies of Revolution at a Mach Number of 1.5", by Chapman and Perkins, NACA RM A7A31a.

6. "Experiments on Drag of Revolving Discs, Cylinders and Streamline Rods at High Speeds", by Theodorsen and Rezier, NACA ACR L4F16.

7. "Data on Boundary Layer Thickness for Models", by A. C. G. Mitchell, Applied Physics Laboratory, The Johns Hopkins University, Report APL-CF-118-N

8. "Experimental Determination of Pressure on the Surface of a Cone in a Supersonic Stream", by F. K. Hill, Applied Physics Laboratory, The Johns Hopkins University, Report APL-CF-530-N.

9. "Pressure Survey on Cones in the Two-Dimensional 1.75M Free Jet at F.G.S.", by F. K. Hill, Applied Physics Laboratory, The Johns Hopkins University Report APL-CF-716-N.

EXPERIMENTAL AND THEORETICAL VALUES OF PRESSURE COEFFICIENT FOR 20° CONE INCLUDING RANGE OF EXPERIMENTAL SCATTER AT $\alpha = 0°$ $M = 1.93$

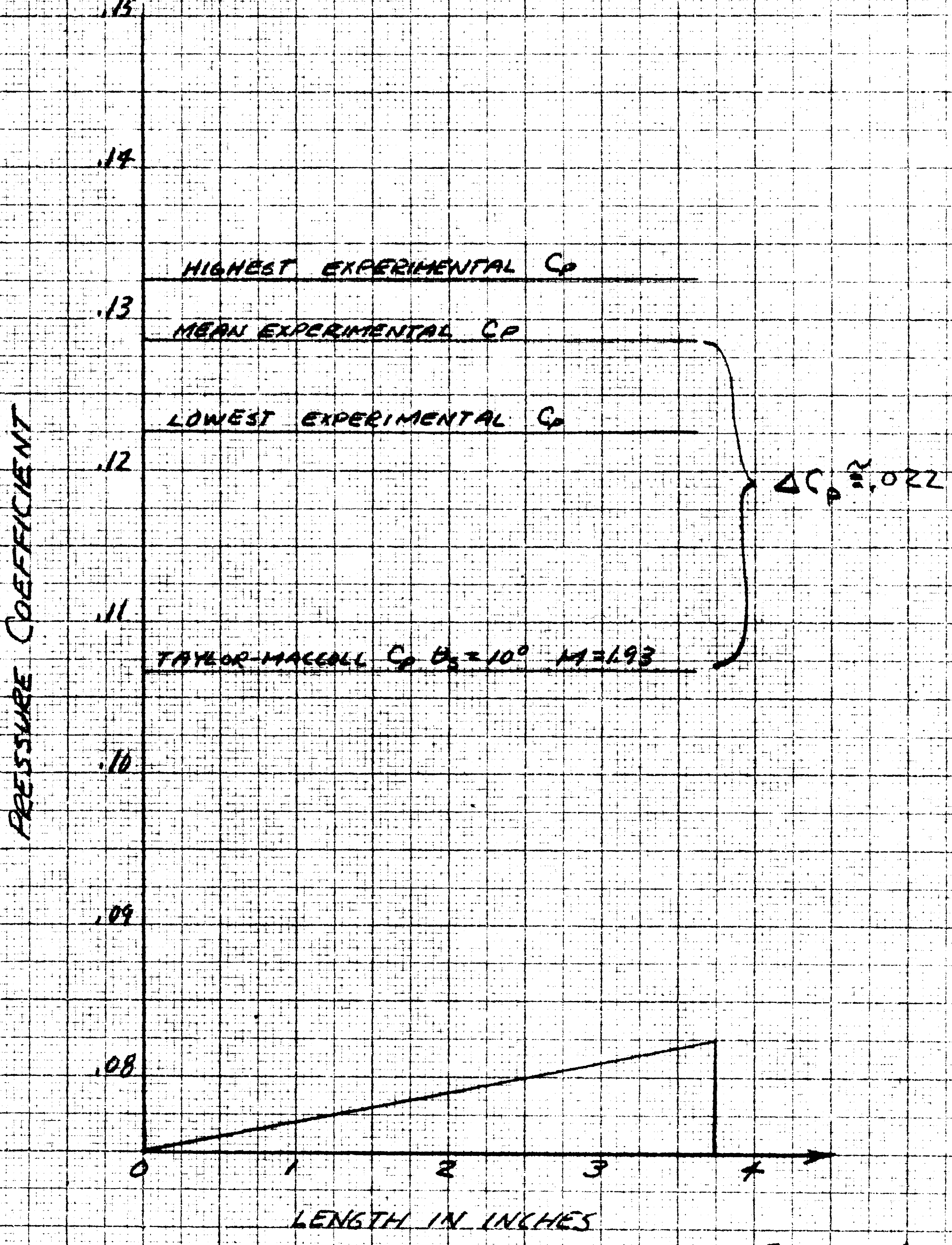


FIGURE 1

EXPERIMENTAL AND CORRECTED THEORETICAL PRESSURE COEFFICIENT FOR 20.10° CONE $\alpha = 0°$ $M = 1.93$

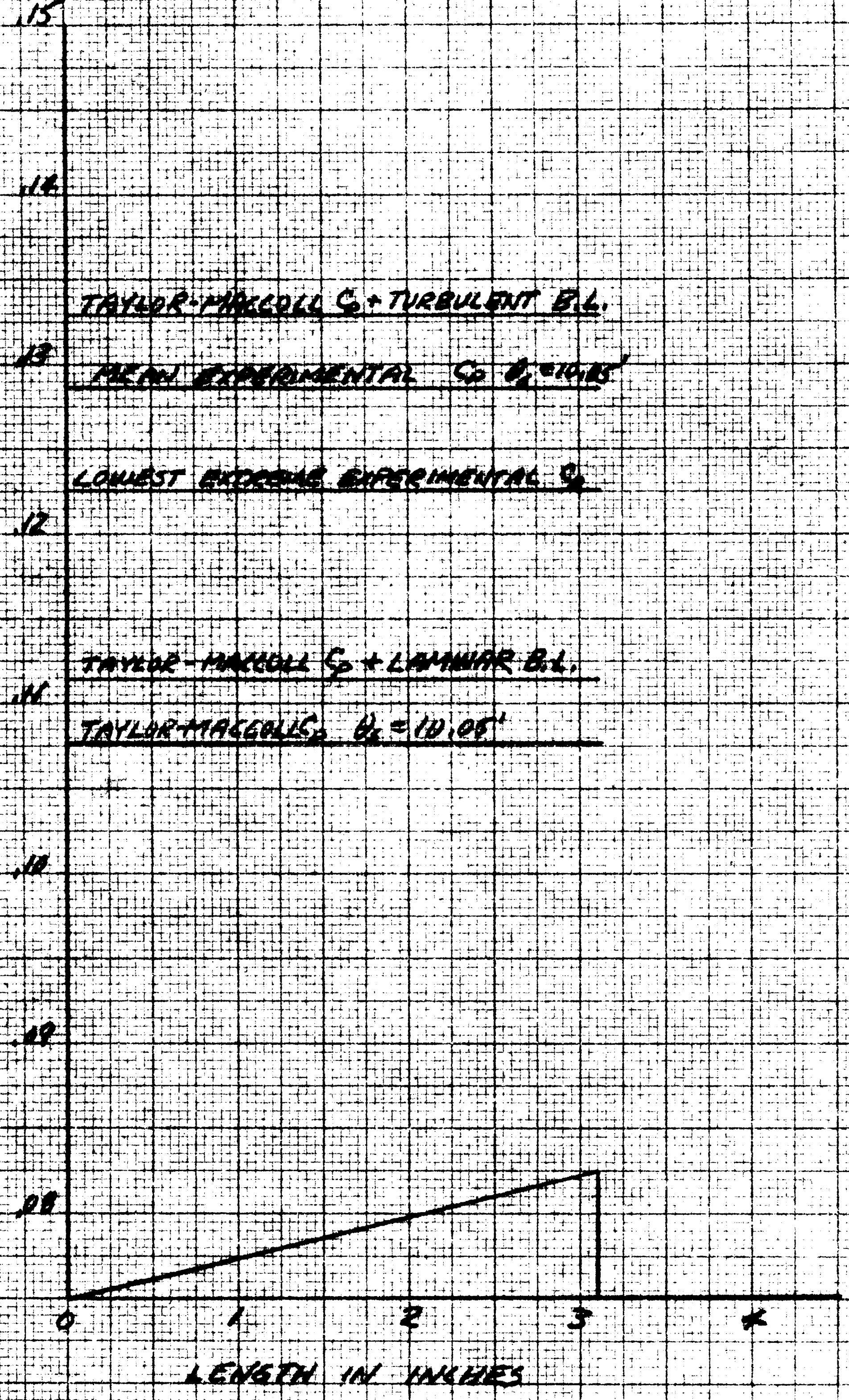
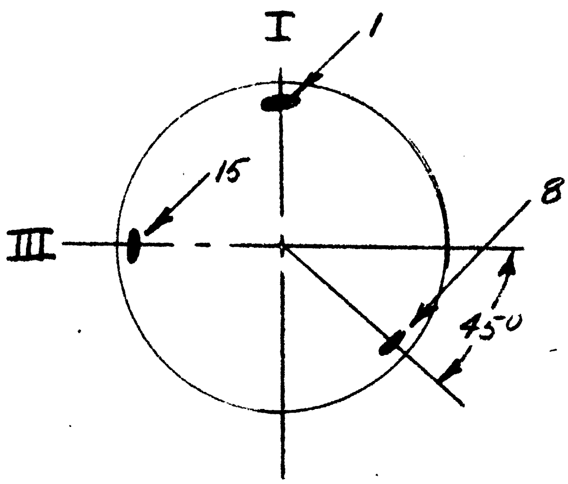


FIGURE 2

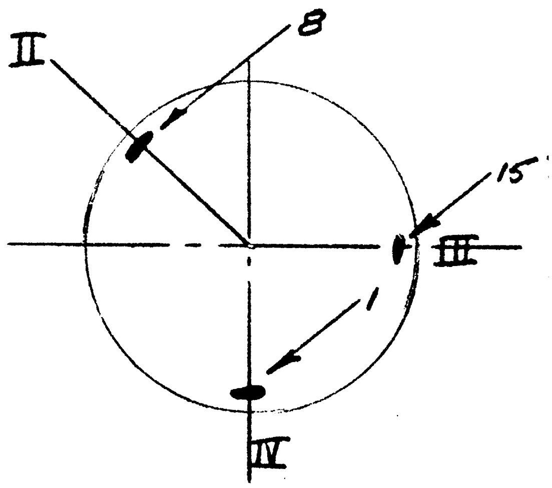
TABLE 1

EXPERIMENTAL VALUES OF PRESSURE COEFFICIENT FOR 19.66° CONE MODEL $M=1.93$ $\alpha \cong 0°$

RUN No.	TAP	C_p	
48-11-2-1	1	.1289	
"	8	.1217	$\phi = 0°$
"	15	.1279	
48-11-2-2	1	.1281	
"	8	.1209	$\phi = 0°$
"	15	.1249	
48-11-2-13	1	.1150	
"	8	.1058	$\phi = 0°$
"	15	.1179	
48-11-2-14	1	.1187	
"	8	.1050	$\phi = 0°$
"	15	.1179	
48-11-5-9	1	.1174	
"	8	.1257	$\phi = 180°$
"	15	.1445	
48-11-5-10	1	.1190	
"	8	.1198	$\phi = 180°$
"	15	.1330	
48-11-5-21	1	.1268	
"	8	.1163	$\phi = 180°$
"	15	.1249	



$\phi = 0°$



$\phi = 180°$

UNIVERSITY OF MICHIGAN
3 9015 02653 5875