RECORRELATION OF DATA FOR CONVECTIVE HEAT-TRANSFER BETWEEN CASES AND SINGLE CYLINDERS WITH LARGE TEMPERATURE DIFFERENCES

W. J. M. DOUGLAS
S. W. CHURCHILL

To Be Presented in Louisville, Kentucky
at the March 22, 1955 Meeting of the AIChE

IP-110

Prepaper for Limitied Distribution
by the College of Engineering Industry Program

February, 1955

engm

UMR0970

ABSTRACT

The data and correlations for the rate of convective heat transfer for gases flowing across a cylinder have been re-examined. The data obtained at large temperature differences between the gas and solid were found to demonstrate the dynamic dissimilarities indicated by dimensional analysis for heating and cooling. Recalculation of the original data of the early investigators indicated that the correlation presented by McAdams did not adequately represent the data. Despite theoretical dissimilarities, the reliable data for both heating and cooling were correlated satisfactorily by plotting hD/k_f versus DV_o/ν_f, where h is the heat transfer coefficient, D is the cylinder diameter, V_o is the free stream velocity, and k_f and ν_f are the thermal conductivity and kinematic viscosity, respectively, of the gas, evaluated at the arithmetic average of the free-stream and surface temperatures. Investigations of the effect of surface temperature variation, free-stream turbulence and the free-stream velocity profile were also reviewed.

INTRODUCTION

This paper presents the results of a comprehensive re-examination of the data and correlations for convective heat transfer for gases flowing across a cylinder, particularly with respect to large temperature differences between the gas and surface. Attention has been confined to the mean coefficient over the entire circumference.

Data obtained for convective heat transfer with small temperature differences between the fluid and surface have been successfully correlated in terms of the dimensionless groups, $DV\rho/\mu$, and hD/k, and $c_p\mu/k$ or combinations of these groups. For large temperature differences, the physical properties of the fluid, μ, c_p, k, and ρ may vary considerably from the bulk of the fluid to the transfer surface and the above dimensionless groups are not uniquely defined. Jakob (1) has pointed out that for complete dynamic similarity between two nonisothermal systems, the same ratios must exist between the significant physical properties at geometrically equivalent points. A lack of similarity will necessarily exist for any two fluids whose properties do not vary identically with temperature. A liquid and a gas fail in this respect, but two gases provide provide reasonable similarity.

For all gases the variation of μ, k and ρ with temperature is quite similar and can be approximated by the expression

$$\frac{\mu_2}{\mu_1} = \frac{k_2}{k_1} = \left(\frac{\rho_2}{\rho_1}\right)^{1/e} = \left(\frac{T_2}{T_1}\right)^{e} = \left(\frac{c_{p_2}}{c_{p_1}}\right)^{b} \qquad (1)$$

Nusselt (2) accordingly suggested that the dimensionless ratio (T_s/T_o) be included in correlations for convection with gases, i.e.,

$$\frac{hD}{k} = \phi\left(\frac{DV\rho}{\mu}, \frac{c_p\mu}{k}, \frac{T_s}{T_o}\right) \qquad (2)$$

For this expression hD/k, $DV\rho/\mu$, and $c_p\mu/k$ can be evaluated at T_s, T_o, or at integrated mean properties, since the properties can be converted from one temperature to another or to integrated mean values by multiplying by some power to T_s/T_o. Even for gases a similarity deficiency can be expected for heating and cooling under otherwise similar conditions, i.e., a single expression of the type of Equation (2) cannot completely define both the heating and cooling process.

Deissler (3) has derived expressions for convective heat transfer inside tubes with large temperature differences, using experimental velocity distributions and assuming an analogy between heat and momentum transfer. The bounding temperatures enter the expression in a complex manner, and different equations were obtained for heating and cooling. However, the calculated results for both heating and cooling are reasonably well represented when hD/k_f is plotted versus $DV_o\rho_f/\mu_f$. Comparable deviations have not been accomplished for convection outside tubes. For Reynolds numbers below perhaps 1.0, the flow pattern around a circular cylinder approaches potential flow. At higher values of the Reynolds number the boundary layer separates from the surface at about 80° from the forward stagnation point, and standing eddies are formed along the rear half of the cylinder. Beginning at a Reynolds number of about 1000, these eddies are shed periodically, and a turbulent wake is formed behind the cylinder. At very high Reynold numbers the boundary layer itself becomes turbulent and the point of separation shifts toward the rear of the cylinder. These phenomena and the Reynold numbers at which they occur are affected by the surface roughness of the cylinder, by the history and boundaries of the main stream, and by the presence of temperature gradients. No rigorous expression or general correlation has been formulated for isothermal flow around a body when a turbulent wake is formed. The further inclusion of a temperature gradient to yield heat-transfer information is therefore beyond present methods of analysis.

EXPERIMENTAL DATA

Convective heat transfer for a gas stream flowing across a circular cylinder has been extensively investigated, but most of the work has been confined to air and to near-ambient temperatures. However, Benke[4] Kennelly and coworkers[5,6], King[7], and Hilpert[8] measured heat transfer to ambient air from wires with surface temperatures extending to 916°F, 1076°F, 1840°F, and 1915°F, respectively. At the other extreme, with cooled tubes, Reiher[9] and Vornehm[10] measured heat transfer from air at 489°F and 390°F, respectively, and Churchill and Brier[11] reported data for nitrogen up to 1800°F. Kilham[12] measured the total energy transfer from flames at about 3700°F to tubes at 2100 to 2800°F, but interpretation of his data in terms of convection coefficients does not appear justifiable.

SURFACE TEMPERATURE, TURBULENCE, AND VEOCITY PROFILE

Surface temperature distribution, the free-stream turbulence, and the free-stream velocity profile are generally thought to influence the heat-transfer rate. However, Krujilin[13] and Giedt[14] observed no difference in heat-transfer coefficients when the surface temperature distribution was varied. Comings, et al[15] made a comprehensive study of the effect of turbulence on convective heat transfer. They found that the heat-transfer rate increased 25 percent when the intensity of turbulence was increased from 1 to 7 percent at Reynolds number of 5800. At lower Reynolds numbers the effect of turbulence was less. Early experimenters did not recognize the importance of turbulence, and interpretation of their results in this context is impossible. Recent investigations have intentionally been carried out at levels of turbulence below the threshold level found by Comings. Most experimenters have made an attempt to obtain a uniform velocity profile, and no specific data have been reported for different profiles. However, Comings concluded that the high rates of heat transfer reported by McAdams for tube banks and by Goukhman and Reiher may have been due to high local velocities rather than true turbulence.

CORRELATION

The data for both heating and cooling with moderate temperature differences have been successfully combined in a general correlation by several authors, the errors associated with nonsimilarity apparently being small with respect to the experimental errors.

As noted previously, the dimensionless groups generally used for correlation of heat transfer are not uniquely defined in nonisothermal systems. Various authors have accordingly evaluated the physical properties at the surface temperature, T_s, the free-stream gas temperature, T_o, the arithmetic-mean temperature, $T_f = (T_s+T_o)/2$, the logarithmic-mean temperature, $T_{\ell m} = (T_o-T_s)/\ell n(T_o/T_s)$, the geometric-mean temperature, $T_{gm} = \sqrt{T_oT_s}$, and weighted arithmetic-mean temperature, $T_\beta = T_s + \beta(T_o-T_s)$. Additionally, mean properties have been use e.g., $\mu_a = (\mu_o+\mu_s)/2$ and $\mu_m = 1/(T_o-T_s)\int_{T_s}^{T_o} \mu dT$. Although the properties may vary considerable between T_o and T_s, numerical differences between the various mean properties and properties at the various mean temperatures are small and in most cases less than the experimental error in the heat-transfer rate data. In some instances individual physical properties have been evaluated at different temperatures, e.g., $DV_o\rho_o/\mu_f$ and $DV_o\rho_f/\mu_f$. For isothermal systems the Reynolds number is conveniently written as DG/μ. For nonisothermal systems the choice of DG/μ, as made by McAdams[16] and others, implies that the density is evaluated at the free-stream temperature, T_o, and thus sacrifices the above freedom in the evaluation of properties. Since there is no decisive theoretical justification for any of these forms, the choice should be based on the success and convenience of the correlation.

The data for flow of liquids across cylinders indicate that hD/k is proportional to $(c_p\mu/k)^{0.3}$. This same effect is generally assumed for gases. Since $c_p\mu/k$ is almost identical for air and nitrogen and since $(c_p\mu/k)^{0.3}$ varies only 0.2 percent between 0°F and 1800°D, it has not generally been included explicitly in the correlations.

For the data taken at high temperature differences, discrepancies have been observed which are attributable to nonsimilarity. Fishenden and Saunders[17] noted that the greatest deviations from their correlation occurred for data taken at the highest temperature difference. Hilpert[8] correlated his data and that of King[7] for high temperature differences by using integrated mean properties and including the temperature ratio T_o/T_s as an additional parameter. Churchill and Brier[11] used bulk gas properties and also found it necessary to include the temperature ratio as a parameter. The effect of the temperature ratio is not the same in the two correlations, however.

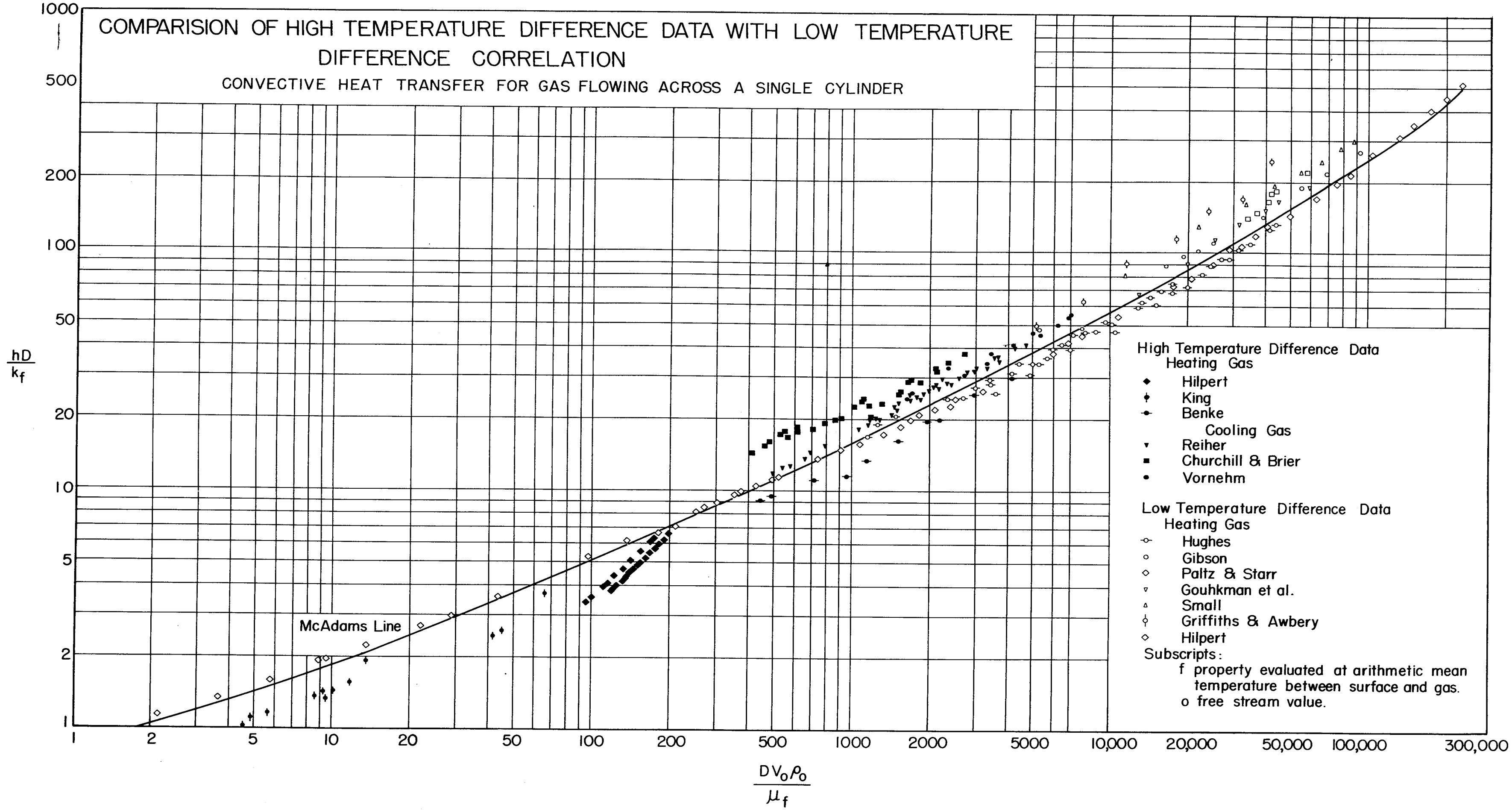
McAdams[16] included almost all the data in a plot of hD/k_f versus DG/μ_f. However, examination of the original sources of the data indicates that the results of several investigators may have been misinterpreted. The data of Hilpert[8] and Vornehm[10] were originally expressed in terms of hD/k_m and DV_o/ν_m, the data of Benke[4] in terms of hD/k_f and DV_o/ν_f and the data of Reiher[9] in terms of hD/k_m and $DV_o'\rho_m/\mu_m$, where V_o' is a modified free-stream velocity to take into account wall effects. These data have apparently been plotted directly by McAdams as hD/k_f and DG/μ_f, although DG/μ_f differs as much as 60 percent from DV_o/ν_m, DV_o/ν_f, and $DV_o\rho_m/\mu_m$. Jakob[1] and Eckert[18] also appear to have misinterpreted Hilpet's data, but less seriously, reporting, respectively, that arithmetic-mean properties and properties at the arithmetic-mean temperature were used.

The high-temperature difference data of Hilpert, Vornehm, Reiher, King, Benke, and Churchill and Brier are replotted in Figure 1 in the form suggested by McAdams, along with the low-temperature difference data of Hilpert, Hughes[19], Gibson[20], Paltz and Starr[21], Griffiths and Awbery[22], Goukhman[23], and Small[24]. For all high-temperature experiments hD/k_f and DG/μ_f were calculated from the original data. Except in the cases of Benke and Vornehm, where only derived results were available, the raw experimental data of these authors were used along with thermal conductivity values given by Stops[25] and viscosity given by Tribus and Boelter[26]. Wherever possible, the high-temperature data were checked for internal consistency with respect to velocity and diameter and for experimental conditions. The data of Kennelly, Wright, and van Bylevelt[5] were rejected because of the anomolous results reported for the various wire diameters. The data of Kennelly and Sanborn(6) were not included in the correlation because the exceptionally high heat-transfer rates reported are apparently due to their unique experimental technique. Representative values only were selected from the very extensive data of King(7). To distinguish the high- for the low-temperature difference data on Figure 1, open symbols are used for data taken at temperature differences of less than 150°F and solid symbols for data taken at temperature differences greater than 300°F. It is apparent that the coordinates of Figure 1, while satisfactory for correlation of low-temperature difference data, are inadequate to correlated that taken at high-temperature differences. It may also be noted that all high-temperature cooling data lie above McAdams' line, while heating data fall below.

Numerous other forms were tested. None completely eliminated the effect of temperature difference. As expected, inclusion of the temperature ratio as a parameter permitted individual correlation of the heating and cooling data but failed to yield a general correlation. The most satisfactory correlation was obtained when hD/k_f was plotted versus DV_0/ν_f, as shown in Figure 2. Data are included for gas temperatures ranging from 60°F to 1800°F and for surface temperatures from 70°F to 1915°F. A curve recommended for design was arbitrarily drawn through the data. The deviations from the curve appear to be random with respect to high- and low-temperature differences and to heating and cooling, and are undoubtedly due in part to undefined variations in the free-stream turbulence level and velocity profile as well as to experimental errors.

CONCLUSIONS

The dynamic dissimilarity indicated by dimensional analysis for the heating and cooling of gas in flow across a single cylinder is apparent in the data taken at large temperature differences. The fluid dynamics of flow across a cylinder is insufficiently defined to yield a theoretical expression for the dissimilarities. The correlation given by McAdams(16), where hD/k_f is plotted against DG/μ_f, does not adequately represent the high temperature difference data. As suggested by Nusselt(2), the data for heating only and for cooling only can be correlated by the introduction of the ratio of the free-stream temperature to the surface temperature as a parameter. Despite theoretical objections, a satisfactory working correlation for both heating and cooling is obtained when hD/k_f is plotted versus DV_0/ν_f. The correlation presented in Figure 2 is probably representative for low intensities of turbulence in the free stream, and the results of Comings et.al(15) can be used to correct for high levels of turbulence.



COMPARISION OF HIGH TEMPERATURE DIFFERENCE DATA WITH LOW TEMPERATURE DIFFERENCE CORRELATION
CONVECTIVE HEAT TRANSFER FOR GAS FLOWING ACROSS A SINGLE CYLINDER
$\frac{hD}{k_f}$
1000
500
200
100
50
20
10
5
2
1
1
2
5
10
20
50
100
200
500
1000
2000
5000
10,000
20,000
50,000
100,000
300,000
$\frac{DV_o \rho_o}{\mu_f}$
McAdams Line
High Temperature Difference Data
Heating Gas
Hilpert
King
Benke
Cooling Gas
Reiher
Churchill & Brier
Vornehm
Low Temperature Difference Data
Heating Gas
Hughes
Gibson
Paltz & Starr
Gouhkman et al.
Small
Griffiths & Awbery
Hilpert
Subscripts:
f property evaluated at arithmetic mean temperature between surface and gas.
o free stream value.

FIGURE I

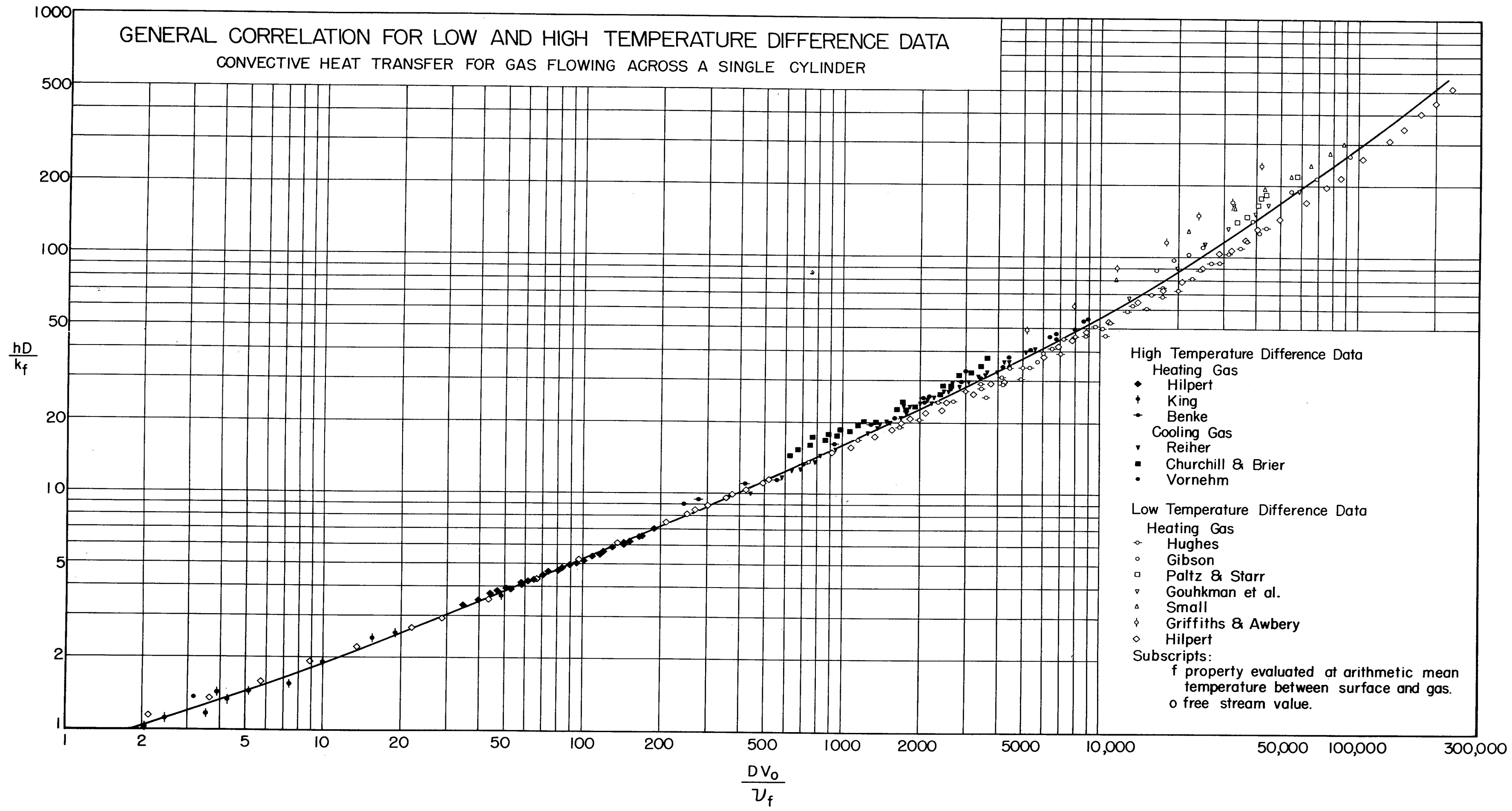

GENERAL CORRELATION FOR LOW AND HIGH TEMPERATURE DIFFERENCE DATA
CONVECTIVE HEAT TRANSFER FOR GAS FLOWING ACROSS A SINGLE CYLINDER
$\frac{hD}{k_f}$
1000
500
200
100
50
20
10
5
2
1
1
2
5
10
20
50
100
200
500
1000
2000
5000
10,000
50,000
100,000
300,000
$\frac{DV_o}{\nu_f}$
High Temperature Difference Data
Heating Gas
Hilpert
King
Benke
Cooling Gas
Reiher
Churchill & Brier
Vornehm
Low Temperature Difference Data
Heating Gas
Hughes
Gibson
Paltz & Starr
Gouhkman et al.
Small
Griffiths & Awbery
Hilpert
Subscripts:
f property evaluated at arithmetic mean temperature between surface and gas.
o free stream value.

FIGURE 2

NOMENCLATURE

C - Heat capacity, Btu/(hr)(pound mass)
D - Cylinder diameter, ft
G - Mass velocity, (pound mass)/(hr)(sq. ft)
h - Heat transfer coefficient, Btu/(hr)(0°F)(sq. ft.)
k - Thermal conductivity, Btu/(hr)(0°F)(ft)
ln - Natural logarithm of
T - Absolute temperature, °R
V - Velocity, ft/hr
ρ - Density, (pounds mass)/(cu. ft.)
ν - Kinematic viscosity, (sq. ft.)/(hr)
ϕ - Function of
μ - Viscosity, (pound mass)/(hr)(ft)

Subscripts

a - Arithmetric average property
f - Arithmetric average temperature
gm - Geometric mean
lm - Logarithmic mean
m - Intergrated mean
o - Free stream
p - Constant pressure
s - Surface

BIBLIOGRAPHY

1. Jakob, M., Heat Transfer, Vol. I, John Wiley and Sons, New York (1949).

2. Nusselt, W., Gesund.-Ing, 38, 477 (1915).

3. Deissler, R. G., Nat. Advisory Comm. Aeronaut. Tech. Note 2242 (Dec. 1950); Ibid., Techn. Note 2410 (July 1951); and Trans. Am. Soc. Mech. Engrs., 73, 101 (1951).

4. Benke, R., Arch. Wärmewirt., 19, 287 (1938).

5. Kennelly, A. E., Wright, C. A., and van Bylevelt, J. S., Trans. Am. Inst. Elect. Engrs., 28, 363 (1909).

6. Kennelly, A. E., and Sanborn, H. S., Proc. Am. Phil. Soc. 53, 55 (1914).

7. King, L. V., Trans. Roy. Soc., (London), A214, 373 (1914).

8. Hilpert, R., Forsch. Gebiete Ingenieurw., 4, 215 (1933).

9. Reiher, H., Forschungsarb. Gebiete Ingenieurw., No. 269, 1 (1925).

10. Vornehm, L., reported by Ulsamer, J., Forsch. Gebiete Ingenieurw., 3, 94 (1932).

11. Churchill, S. W. and Bier, J. C., preprinted for St. Louis Meeting of A.I.Ch.E. (Dec. 1953).

12. Kilham, J. K., Third Symposium on Combustion Flame and Explosion Phenomena, Paper No. 98, p. 733, Williams and Wilkins, Baltimore (1949).

13. Krujilin, G., and Schwab, B., Tech. Phys. U.S.S.R., 2, 312 (1935).

14. Giedt, W. H., Trans. Am. Soc. Mech. Engrs. 71, 375 (1949).

15. Comings, E. W., Clapp, J. T., and Taylor, J. F., Ind. Eng. Chem., 40, 1076 (1948).

16. McAdams, W. H., Heat Transmission, 3rd Edition, McGraw-Hill, New York, 1954.

17. Fischenden, M., and Saunders, O., The Calculation of Heat Transmission, His Majesty's Stationery Office, London (1932).

18. Eckert, E. R. G., Introduction to the Transfer of Heat and Mass, McGraw-Hill Book Co., New York (1950).

19. Hughes, J. A., London Phil. Mag., 31, 118 (1916).

20. Gibson, A. H., London Phil. Mag., 47, 324 (1924).

21. Paltz, W. J. and Starr, C. E., Thesis in Chem. Eng'g., M.I.T., 1931.

22. Griffiths, E., and Awbery, J. H., Proc. Inst. Mech. Engrs. 125, 319 (1933).

23. Gouhkman, A., Joukovsky, V., and Loiziansky, L., Techn. Phys. U.S.S.R., 1, 221 (1934).

24. Small, J., London Phil, Mag., 19, 251 (1935).

25. Stops, D. W., Nature, 164, (1949).

26. Tribus, M., and Boelter, L. M. K., Nat. Advisory Comm. Aeronaut., War-time Report, ARR-W9, (Oct. 1942).