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# Turbulent Heat Transfer for Pipe Flow with Uniform Heat Generation

T.Y. Na, Dearborn and J.P. Chiou, Detroit

Abstract. An Analytic solution is presented of the problem of turbulent heat transfer in pipes with internal heat generation and insulated wall by applying a recently-developed eddy conductivity model. The results agree closely with available experimental data for a wide range of Prandtl number (0.02-10.5).

Turbulente Wärmeübertragung in Flüssigkeitsströmungen durch Rohre bei gleichmäßiger Wärmeerzeugung

Zusammenfassung. Eine analytische Lösung des Problems der turbulenten Wärmeübertragung in Rohren mit innerer Wärmeerzeugung und isolierender Wandung wird vermittels eines neu entwickelten Modells für das Wirbelleitvermögen angegeben. Die Ergebnisse stimmen mit den zugänglichen experimentellen Werten innerhalb eines ausgedehnten Bereiches der Prandtl-Zahl (0.02-10.5) befriedigend überein.

## Nomenclature

| | |
|---|---|
| $A^+$ | damping factor for eddy viscosity |
| a | pipe radius |
| $a^+$ | dimensionless pipe radius, $a\sqrt{\tau_w/\rho}/\nu$ |
| $B^+$ | damping factor for eddy conductivity |
| $C_p$ | constant pressure specific heat |
| k | thermal conductivity |
| l | mixing length |
| $l^+$ | dimensionless mixing length, $l\sqrt{\tau_w/\rho}/\nu$ |
| Nu | Nusselt number |
| Pr | Prandtl number |
| Pe | Péclet number, Pe = RePr |
| p | static pressure |
| q | heat flux |
| Q | rate of heat generation |
| Re | Reynolds number, $u(2a)/\nu$ |
| r | radial coordinate |
| St | Stanton number |
| T | temperature |
| u | axial velocity |
| $u^+$ | dimensionless axial velocity, $u/\sqrt{\tau_w/\rho}$ |
| x | axial coordinate |
| y | transverse coordinate normal to wall |
| $y^+$ | $y\sqrt{\tau_w/\rho}/\nu$ |
| $\alpha$ | thermal diffusivity |
| $\mu$ | viscosity |
| $\varepsilon_m$, $\varepsilon_h$ | eddy viscosity and eddy conductivity, respectively |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| $\tau$ | shearing stress |

Subscripts

| | |
|---|---|
| B | bulk |
| h | heat |
| m | momentum |
| o | wall |

## Introduction

Recent advances in the formulation of turbulent eddy viscosity and eddy conductivity have led to many useful contributions in the field of turbulent flows. One of such contributions is in turbulent heat transfer in channel flows. The purpose of this work is to solve the problem of turbulent heat transfer in pipes with internal heat generation and insulated wall [1-4] by applying a recently-developed eddy conductivity model [5, 6]. It will be shown that the results from the theory agree very closely with available experimental data for a wide range of Prandtl number (0.02-10.5).

## Analysis

Consider the heat transfer of a Newtonian fluid inside a smooth and insulated circular pipe. Heat is generated uniformly in the fluid. The flow is considered turbulent and steady with the temperature and the velocity profile fully-developed. Under these conditions the momentum and energy equations can be written as:

$$\frac{dp}{dx} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\mu + \rho\varepsilon_m)\frac{\partial u}{\partial r}\right] \tag{1}$$

$$\rho C_p u \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(k + \rho_p\varepsilon_h)\frac{\partial T}{\partial r}\right] + Q\ . \tag{2}$$

The boundary conditions are:

$$r = 0: \quad \frac{\partial u}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0$$

$$r = a: \quad u = 0, \quad \frac{\partial T}{\partial r} = 0$$

where $\varepsilon_m$ and $\varepsilon_h$ are the eddy diffusivities of momentum and energy respectively and "a" is the radius of the pipe.

The differential equation for the solution of the velocity distribution can be obtained from Eq.(1) as:

$$\frac{\rho(\nu + \varepsilon_m)}{\tau_w} \frac{\partial u}{\partial r} = \frac{r}{a} \tag{3}$$

where the shear stress at wall, $\tau_w$, is given by

$$\tau_w = -\frac{a}{2}\frac{dp}{dx}. \tag{4}$$

In terms of the usual dimensionless notation,

$$y^+ = \frac{yv^*}{\nu}, \quad a^+ = \frac{av^*}{\nu}, \quad u^+ = \frac{u}{v^*}$$

where $y = a - r$, $v^*$ is the frictional velocity,

$$v^* = \sqrt{\frac{\tau_w}{\rho}}$$

and $\nu$ is the kinematic viscosity, Eq.(3) becomes:

$$\left(1 + \frac{\varepsilon_m}{\nu}\right)\frac{du^+}{dy^+} = 1 - \frac{y^+}{a^+}. \tag{5}$$

For the case of constant heat flux on the wall, it is well-known that

$$\frac{\partial T}{\partial x} = \frac{\partial T_0}{\partial x} = \frac{dT_B}{dx} \tag{6}$$

where $T_0$ is the wall temperature and $T_B$ is the bulk temperature defined by

$$T_B - T_0 = \frac{1}{\pi a^2 \bar{u}} \int_0^a 2\pi r u(T - T_0)dr. \tag{7}$$

In Eq.(7), $\bar{u}$ is the average velocity defined by

$$\bar{u} = \frac{1}{\pi a^2}\int_0^a 2\pi r u dr \tag{8}$$

Combining Eqs.(2) and (6) and integrating over r, we obtain:

$$\rho C_p \frac{dT_B}{dx}\int_0^r u r dr = r(k + \rho C_p \varepsilon_h)\frac{\partial T}{\partial r} + \frac{r^2 Q}{2}. \tag{9}$$

When Eq.(9) is evaluated at the wall, the result is:

$$\rho C_p \frac{dT_B}{dx}\int_0^a u r dr = \frac{a^2 Q}{2}. \tag{10}$$

By dividing Eq.(9) by Eq.(10) and introducing

$$\theta = \frac{T - T_B}{T_0 - T_B} \tag{11}$$

and the dimensionless variables $y^+$, $u^+$, and so on, we get:

$$\frac{k(T_0 - T_B)}{a^2 Q} = \frac{2}{a^{+2}\,\overline{u^+}}\int_0^{a^+}(a^+ - y^+)u^+ \times$$

$$\times \left\{ \int_0^{y^+} \frac{2\int_{y^+}^{a^+} u^+(a^+ - y^+)dy^+ - \left(1 - \frac{y^+}{a^+}\right)^2}{a^{+2}\,\overline{u^+}(a^+ - y^+)\left(1 + \Pr\frac{\varepsilon_h}{\nu}\right)} \right\} dy^+ \tag{12}$$

where the dimensionless form of the average velocity, defined in Eq.(8), is given by:

$$\overline{u^+} = \frac{2}{a^{+2}}\int_0^{a^+}(a^+ - y^+)u^+ dy^+. \tag{13}$$

Equations (5) and (12) are the equations for the calculation of $u^+$ and $k(T_0 - T_B)/a^2 Q$, respectively, if the eddy viscosity and the eddy conductivity are known. The expressions chosen here are those given by Na and Habib [5] and Habib and Na [6], which are:

$$\frac{\varepsilon_m}{\nu} = (a^+)^2\left\{.4\frac{y^+}{a^+} - .44\left(\frac{y^+}{a^+}\right)^2 + .24\left(\frac{y^+}{a^+}\right)^3 - .6\left(\frac{y^+}{a^+}\right)^4\right\}^2 \times \{1 - \exp(-y^+/A^+)\}^2 \frac{\partial u^+}{\partial y^+} \tag{14}$$

and

$$\frac{\varepsilon_h}{\nu} = (a^+)^2 \left\{ .4 \frac{y^+}{a^+} - .44 \left(\frac{y^+}{a^+}\right)^2 + .24 \left(\frac{y^+}{a^+}\right)^3 - \right.$$

$$\left. - .6 \left(\frac{y^+}{a^+}\right)^4 \right\}^2 \times \{ 1 - \exp(-y^+/A^+) \} \times$$

$$\times \left\{ 1 - \exp\left(-y^+ \sqrt{Pr}/B^+\right) \right\} \frac{\partial u^+}{\partial y^+} \tag{15}$$

where

$A^+ = 26$

and

$$B^+ = \sum_{i=1}^{5} C_i (\log Pr)^{i-1}$$

with

$C_1 = 34.96$
$C_2 = 28.79$
$C_3 = 33.95$
$C_4 = 6.33$
$C_5 = -1.186$

These expressions have been applied to the cases of turbulent heat transfer in pipes with constant wall heat flux [5] and with constant wall temperature [6]. Results from these studies have shown that the predicted results agree very closely with experimental data for a wide range of Prandtl numbers (0.01 to 14.).

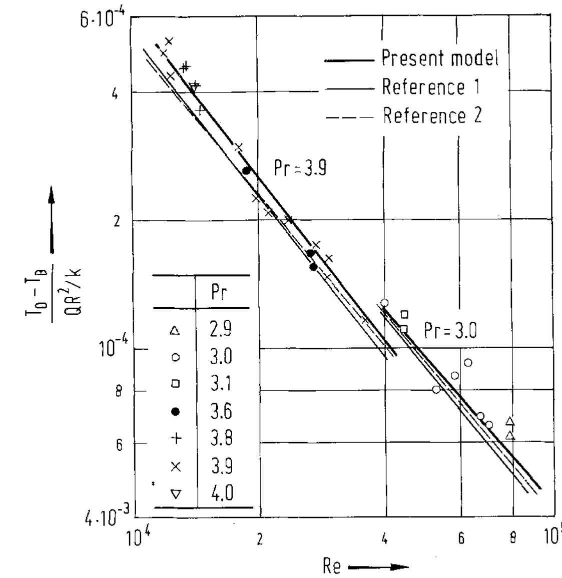


Fig.1. Comparison of results with experimental data (Pr = 3.9)

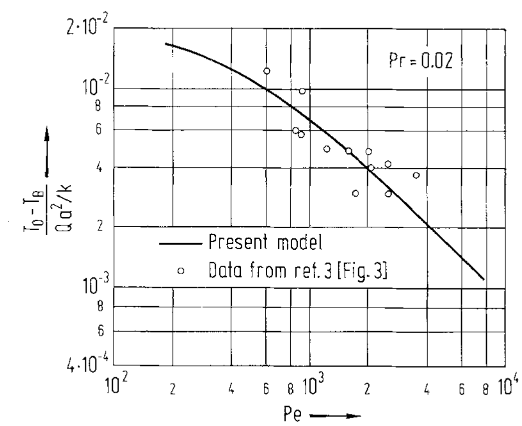


Fig.2. Comparison of results with experimental data (Pr = 0.02)

## Comparison with Experimental Data

Experimental data available in the literature on turbulent flow in pipes with heat generation are limited to the works of Kinney and Sparrow [1], Poppendiek [4] and Muller [3]. The data taken by Kinney and Sparrow [1] used salt water and are for the Prandtl number range of 2.9 to 4.0. Poppendiek [4] used sulfuric acid solution in his work and the Prandtl number range is 4.6-10.5. Muller [3] 's work is on liquid metals where he used mercury (Pr = 0.02) as the working medium.

Figure 1 shows the comparison of $k(T_0 - T_B)/a^2Q$ with experimental data for Pr = 3.0 and 3.9. Also shown in this figure are the theoretical curves of Kinney and Sparrow [1] and Chung and Thomas [2]. The agreement is seen to be very close. Comparison is also made with experimental data taken by Muller for turbulent flow of liquid metals (Pr = 0.02) with heat generation. The results are shown in Fig.2. Again, the agreement is excellent. For higher Prandtl numbers, only limited data were given by Poppendick [4] for the range of Pr between 6.0 and 8.7. They are shown in Fig.3, together with the analytical curve based on the present model (for Pr=7).

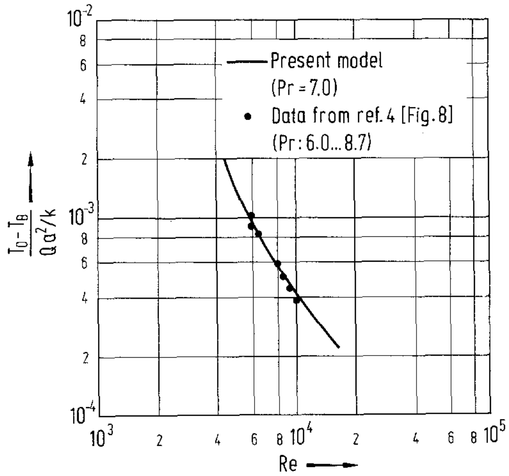


Fig.3. Comparison of results with experimental data (Pr = 6 to 6.8)

Even though each data point on this figure represents slightly different Prandtl numbers, they appear to be quite close to the theoretical curve based on the average Prandtl number of 7.

The above shows that the model used in this work gives excellent prediction of heat transfer in pipe flow with heat generation in the turbulent regime for a wider range of Prandtl numbers than other available theories.

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Prof. T.Y. Na
University of Michigan-Dearborn
Dearborn, Michigan, 48128, U.S.A.

Prof. J.P. Chiou
University of Detroit
Detroit, Michigan, 48221, U.S.A.


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