CAN SETUP REDUCTION RESULT IN WORSE PERFORMANCE?

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Abstract

In a recent paper, Sarkar and Zangwill argued that there exist production systems for which cutting setup times leads to an increase in inventory and therefore worse performance. In this note, we show that it is easy to construct policies that allow idling that are guaranteed not to result in higher inventories when average setup times are cut. In particular, we show that an optimal policy (in the class of policies that allow idling) cannot have this behavior. We therefore argue that cutting average setup times never leads to worse performance, provided one employs policies that take advantage of this decrease.

1. Introduction

Scheduling algorithms employed in practice are designed and implemented based on many factors, including the simplicity of implementation and the robustness of the algorithm to unmodeled effects and disturbances. In practice, the definition of optimality for a scheduling algorithm may be impossible to identify. It is not too surprising then that unexpected and seemingly counterintuitive examples will arise on occasion. It is not always a simple matter, however, to identify the source of an anomaly. In a recent paper, Sarkar and Zangwill (1991) argued that cutting setup times can lead to an increase in WIP and therefore to higher costs. Furthermore, Zangwill (1992) and Zangwill and Sarkar (1994) argued that these results require a reevaluation of Japanese inventory theory, which prescribes reduction of setup times.

In Sarkar and Zangwill's model, a machine produces n different items. The machine makes an item from kits of supply parts that arrive according to a Poisson process. The arrival rate for kits of type i is λ_i. The time to make from a kit each item of a given type, i, is an independent and identically distributed random variable S_i for $i = 1, \ldots, N$. If the machine is set up to produce items of type i, and management decides to switch to producing items of type j, a setup of (random) duration D_j is required. The durations of successive setups of the same type are independent, identically distributed random variables.

In Sarkar and Zangwill's model, the machine produces the items in a cyclic and exhaustive manner. First, the machine is set up to make item 1, then the machine keeps making item 1 until all kits for item 1 (including those that arrived after the machine was set up) are exhausted. Next, the machine is set up to produce item 2 (regardless of whether a kit for item 2 is available at that point). Then, all kits for item 2 are exhausted, and the machine is set up for item 3 and so on. After finishing all kits for the last item, the cycle starts again. Thus, the machine uses an exhaustive polling policy to process the different items. Under this policy, Sarkar and Zangwill (1991) have shown that cutting average setup times can lead to an increase in inventory.

The purpose of this note is to show that for a production system of the sort considered by Zangwill that provided idling is used appropriately, decreasing the average setup times (ceteris paribus) cannot cause higher inventory costs. Either the use of idling with the original policy will decrease (or maintain) inventory costs, or better performance can be accomplished with a more effective policy.

2. Discussion

We claim that Sarkar and Zangwill's examples do not actually show that setup reduction can be harmful. To see this, consider a manufacturer who has been using the above described cyclic exhaustive policy. This manufacturer succeeds in reducing the mean setup time for an item from ED_i to $ED_i' < ED_i$, where the subscript denotes the product class. The variance of the setup time may also have been reduced so we assume that $VarD_i' \leq VarD_i$. In all of Sarkar and Zangwill's examples, the manufacturer ends up with higher inventory levels because he/she keeps using the same policy. However, there is no reason why the manufacturer should in fact continue to use the same policy. Since the setup times have changed, the manufacturer can also decide to use a more appropriate policy, namely one that takes advantage of the lower setup times. In general, it is advantageous to consider the class of policies that allows the server to idle. We note that Federgruen and Katalan (1993) have proposed and analyzed such policies.

Consider, for the sake of argument, the class of cyclic, exhaustive policies analyzed in Sarkar and Zangwill (1991) and extend this class to allow idling. Suppose that after decreasing the setup times to the new lower levels the manufacturer implements a policy such that every time the machine is setup for item i, the setup will be followed by a (random) idle time I such that the sum of the reduced setup time and the idle time is equal in distribution to the original setup time. Thus, we have mean $EI = ED_i - ED_i'$ and variance $VarI = VarD_i - VarD_i'$. Under this policy the moments of the combined setup and idle periods (the time that the machine is stopped whenever a setup occurs) have not changed. In fact, it suffices to match only the first two moments, since expected WIP levels under the exhaustive cyclic policy depend only on the mean and variance of the combined setup and idle periods (see for example equation 2.2.8 in Sarkar and Zangwill (1991)). Thus, the expected WIP levels after the setup time reduction will be exactly the same as that before the setup reduction. We note that in practice, implementing an idle time with mean EI and second moment $EI^2 = VarI + (EI)^2$ is straightforward. For example, an idle time of $zB(p)$ can easily be implemented, where z is a real number and $B(P)$ is a Bernoulli random variable with probability p. That is, with probability p, the machine will idle for duration z, and with probability $1 - p$, the machine will not idle. It is straightforward to show that $p = (EI)^2/EI^2$ and $z = EI^2/EI$. In particular, this indicates that if only the mean has been decreased with no change in the variance, then an idle time equal to the decrease in the mean can be implemented to ensure that the mean WIP level remains the same after the change.

We have shown above that it is possible to do at least as well after the setup reduction as before. The question that remains to be answered is whether it is possible to decrease WIP levels further. Since the scheduler is able to choose any idling policy, and not the one described above that idles to match the given setup mean and variance, it may be possible to improve WIP levels even further. We demonstrate this with an example similar to the ones in Sarkar and Zangwill (1991) and Zangwill (1992).

Consider a firm that produces two products. The two products have identical process requirements. The arrivals are Poisson with rates $\lambda_1 = \lambda_2 = 5$ units per minute. The service rates are $\mu_1 = \mu_2 = 50$ per minute. The service time variances are $Var(S_1) = Var(S_2) = 0$. The mean setup times are $ED_1 = ED_2 = 200$ minutes and the variance of the setup times are $VarD_1 = 3092$ and $VarD_2 = 3092$. It is straightforward to compute using equation (2.2.8) in Sarkar and Zangwill that the expected waiting time for both types of jobs equals 232.7 minutes.

Suppose that the firm were able to develop (through their Just-In-Time and Kaizen efforts) a new process for their setups which would decrease their mean setup times from 200 minutes to 5 minutes for each setup although this new process would not change the variance of the setup times. If the firm implements this new process and keeps using the same cyclic exhaustive policy, this would *increase* the mean waiting times to 314.8 as predicted by Sarkar and Zangwill. So, it appears that decreasing setup times would indeed cause an increase in waiting times and WIP levels. However, if the firm told its workers to idle for exactly 15 minutes after each setup (or to perform during that time some other useful task such as maintenance), then the mean waiting times would decrease to 99.8. Hence, the decrease in setup times would in fact cause the mean WIP levels and waiting times to decrease more than 50%.

This example clearly demonstrates that the increase in WIP levels is in fact due to the continued use of the policy by Sarkar and Zangwill. A decrease in setup times cannot cause an increase in WIP levels as long as the firm switches to a policy that takes advantage of this decrease.

3. Analysis

Our arguments above may seem to depend upon the company's use of the cyclic exhaustive polling policy prior to the improvement in setup times. Suppose instead that the firm initially used an "optimal" policy. We do not completely specify the objective function as this would depend on the firm's preferences. Some firms might choose to minimize the mean weighted waiting times for jobs. As Zangwill and Sarkar (1994) have pointed out, however, this might cause the mean waiting times of different types of jobs to vary considerably and might be undesirable. In that case, the firm might want to add, for example, a condition that the mean waiting time ratio for any two types of jobs not exceed a given constant. Therefore, we assume only that the objective to be minimized is a nondecreasing function of the waiting times (or WIP levels) for any type of job. If the firm uses policies that are "optimal" with respect to their objective, can a decrease in mean setup times (or mean process times) without changes in the other central moments of the distribution cause an increase in WIP levels? The following theorem and corollary state that this cannot be the case. Theorem 1 addresses a more general class of setup or service time reductions based on stochastic dominance of the distributions.

Theorem 1: *For the stochastic multi-item production model described above, assume that the objective is to maximize over the class of* idling policies *the expected cost of policy g, $J_g \triangleq J(E\{W_1^g\}, E\{W_2^g\}, \ldots, E\{W_N^g\})$, where $E\{W_i^g\}$ is the expected waiting time in queue i under policy g and where J is a nondecreasing function of its arguments. For $i = 1, 2, \ldots, N$, a decrease in the setup time (respectively, service time) from D_i (resp. S_i) to D_i' (resp. S_i') such that $D_i \geq_{st} D_i'$ (resp. $S_i \geq_{st} S_i'$) will result in an equal or lesser expected cost under an optimal policy.*

Proof: To begin, consider D_i and an optimal policy g for the original system. To show that there exists a system with setup times for class i distributed according to D_i' and a policy g' which performs at least as well as the original system, we construct a system coupled to the original that preserves the appropriate behavior of the new system. The idea is to use idling of the server in the new system to match the behavior of the original system.

Along any sample path ω of the stochastic process, the setup times in queues other than i, interarrival times, and service times take on the same realization in the new system as in the old.

Consider the kth setup of queue i in the original system, which has setup time realization denoted by $D_i(k,\omega)$. For the new system, the corresponding setup is coupled to the realization of the original system using

$$D'_i(k,\omega) = F^{-1}_{D'_i}(F_{D_i}(D_i(k,\omega))), \tag{3.1}$$

where F_{D_i} (F'_{D_i}) is the cumulative distribution function (C.D.F.) of D_i (D'_i), as is described in Lemma 8.2.1 of Ross [2]. Because $D'_i(k,\omega) \leq D_i(k,\omega)$ along a set of realizations of probability one, we employ idling appropriately along each sample path to make the evolution of the original system under g identical to that of the new system under g'. To see this, we may think of this idle period in the new system as being associated with the setup time and as dependent upon its realization. Let this kth idle period be denoted by $I'_i(k,\omega) = D_i(k,\omega) - D'_i(k,\omega) \geq 0$. Thus, it is clear that along any ω, the policy g' can reconstruct (at any instant) the state of the original system under g, resulting in equal mean waiting times and thus equal performance. Because g' may not necessarily be optimal under the reduced setup times, the optimal policy will perform at least as well as g'.

The case of reduction in service time can be handled as above. In the case where the setup or service time of more than one class is reduced, recursively apply the above argument one class at a time. □

Corollary 1: *For a system as described in Theorem 1, if for $i = 1,2,\ldots,N$, $D'_i = D_i - r_i$ (respectively, $S'_i = S_i - q_i$) for some $r_i \geq 0$, ($q_i \geq 0$) the optimal cost will not increase.*

Proof: The result follows from the fact if the only difference between two random variables is their mean, one stochastically dominates the other. □

The above result clearly shows that reduction of mean processing times under any "optimal" policy that allows idling cannot cause an increase in WIP levels. We have therefore shown that the paradoxical behavior observed by Sarkar and Zangwill is only a characteristic of the policy they analyze and that any optimal policy that allows idling cannot exhibit this behavior.

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