Impact of Inspection Errors on Performance Measures of a Complete Repeat Inspection Plan

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Abstract

In quality assurance of critical multicharacteristic components complete repeat inspection plans have been proposed to guard against inspection errors. The optimal number of repeat inspections is determined using a model for the plan which minimizes the total expected cost, which consists of the cost of false acceptance, cost of false rejection and cost of inspection. In this paper performance measures for such plans are defined and then the statistical and economic impact of inspection errors on these measures is investigated. The impact of the errors is studied utilizing a computer software which implements the model of the plan. Then the levels of the errors are varied and the behavior of the performance measured are observed. Results indicate that the impact of type I and type II errors are significant and have considerable effect on complete repeat inspection plans. A methodology for incorporating the dynamic changes in the errors in the design of such plans is outlined.

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1.Introduction

A multicharacteristic critical component is a component having several characteristics and could cause a disaster or high cost upon failure. Such components supplied by vendors have to be inspected to determine their conformance to specified standards. A component will be rejected if any of its characteristics is found defective. There could be as many as 14 characteristics for which a component may fail [16]. Such components can be a part of an air craft, space shuttle or a complex gas ignition system. The quality requirements for critical components are tight and field failure should be kept to the minimum level. Due to the disastrous consequences from accepting a defective component a common practice in industry is to institute repeat (multiple) inspections. The reason for repeat inspections is that inspection is never perfect. There is always the possibility of false rejection (type I error) and false acceptance (type II error). In case of critical components the cost of inspection and the cost of false rejection are much less in order of magnitude than the cost of false acceptance. Because in case of accepting a defective component and mounting it on the system (such as an aircraft) will result in system failure which may cause the loss of the system and human lives. Therefore it is perceived and shown that repeat inspection is likely to reduce the expected cost of type I and type II errors and increase the cost of inspection; however the total expected cost which is the sum of the three costs is likely to reduce [9]. Realizing the need for repeat inspection, a model has been proposed [13] to determine the optimal number of repeat inspections and develop a repeat inspection plan to assure quality of critical components. In the literature it has been demonstrated that repeat inspection minimizes the probability of accepting a defective component [6].

Effect of inspection errors on single sampling plans have received considerable attention in the past. Ayoub et al [1] presented formula for the average out going quality (AOQ) and average total inspection (ATI) for a single sampling plan under inspection error. Latter Collins et al [4] relaxed the assumption of perfect replacement and allowed defective replacement in the formula of ATI. In [3] Case developed similar results for Dodge's Sampling plan for continuous production and in [5] Collins et al studied the effect of inspection errors on a single sampling plan. In [2] Bennett et al investigated the effect of inspection errors on a cost-based single sampling plans.

Another development which has significant impact on the quality control area is the increasing use of complete inspection plans. The development is mainly due to the growth in automatic manufacturing systems which makes complete inspection (100% inspection) inexpensive and reliable [14]. In [13] Raouf, Jain and Smith developed a minimum cost complete repeat inspection plan for multicharacteristic critical components. In [11] the model in [13] was simplified by Lee and simple optimality conditions for the model were obtained. Duffuaa and Raouf [10] developed three models for multicharacteristic critical components. The work in [6] extends the work in [13] by considering other objectives such as minimizing the probability of accepting a defective component.

All models which incorporate inspection error assume that type I and type II are fixed and known. This assumption is not true because these errors change with the incoming quality [8]. Repeat inspection means each characteristic is inspected more than once. After the first inspection the quality (probability that the characteristic is defective) of the characteristic changes. Therefore the characteristic arrives to the second inspection with different quality level. This new level of quality has different type I and type II errors. In other words inspectors commit different level of errors for different quality levels. This has not been incorporated in the models in the literature. To have a realistic inspection plan the change in the errors must be incorporated in the design of the inspection plan. In [15] Tang and Schneider studied the effects of inspection errors on a complete inspection plan developed based on the models in [14].

Maghsoodloo in [12] studied the effect of error on multistage sampling plan. The literature doesn't contain an extensive study that examines the effects of types I and II on the repeat inspection plan developed in [13].

The purpose of this paper is to investigate the statistical and economic impact of inspection errors on performance measures of a complete repeat inspection plan. the performance measures defined and used in this paper are average total inspection (ATI), expected total cost (ETC) and average outgoing quality (AOQ). Then a methodology is suggested to incorporate the dynamic change in type I and type II errors in the design of such plans.

The complete repeat inspection plan in this paper is the one proposed in [13], where incoming items are subjected to several cycles of inspections. The plan is applied as follows: An inspector inspects one particular characteristic for each component entering the inspection process and all the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the component is finally accepted. Here n is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those which are accepted in the nth cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., nth cycles.

The optimal inspection plan is obtained using a model which depicts the plan and minimizes the total expected cost, which consists of the cost of false acceptance (resulting from type II errors), the cost of false rejection (resulting from type I errors) and the cost of inspection. The model determines the optimal number of repeat inspections, n and hence determines the plan. The statistical and economic impact of type I and type II errors are obtained by conducting sensitivity analysis using the model in Section 2. Type I and type II errors were varied from .01 to 0.15 to observe their effect on the performance measures of the plan. The ranges used for the errors are the same as in [2].

The rest of the paper is organized as follows: The optimization model employed to develop the optimal complete inspection plan for multicharacteristic critical components is given first. Then the effect of the errors on the performance measurers of the inspection plan is investigated. Followed by a methodology for incorporating the error model in the design of the inspection plan. Finally conclusions are given.

2. Model Description

In order to investigate the impact of the errors on the inspection plan, it is necessary to describe the model which depicts the plan. The model is developed for components having several characteristics for inspection with incoming quality p_i, $i = 1, 2, \ldots, N$. A component is classified as nondefective only if all the characteristics met the quality specifications. Characteristics defective rates are assumed independent. The probabilities of type I error, P_{1i} and type II error P_{2i}, $i = 1, 2, \ldots, N$, are assumed to be known. Three different types of costs are considered: (1) cost due to false rejection of a nondefective component, C_r, (ii) cost due to false acceptance of defective component, C_a, and, finally, (iii) cost of inspection, C_i, $i = 1, 2, \ldots, N$. Prior to stating the model the following notations are defined. In the notation i ranges from 1 to N, and j from 1 to n.

Notation:

M_j Number of components entering the jth cycle of inspection.

N Number of characteristics in each component to be inspected.

P_i Probability of ith characteristic in the sequence of inspection being defective entering the inspection.

PG Probability of a component being nondefective entering the inspection.

PGC Probability of a component being defective entering the inspection, the complement of PG.

P_{1i} Probability of classifying the ith nondefective characteristic in the sequence of inspection as defective (type I error).

P_{2i} Probability of classifying the ith defective characteristic in the sequence of inspection as nondefective (type II error).

$P_i(j)$ Probability of ith characteristic in the sequence of inspection being defective entering the jth cycle of inspection.

$PG(j)$ Probability of a component being nondefective entering the jth cycle.

$PGC(j)$ Probability of a component being defective entering the jth cycle, complement of PG(j).

$M_{i,j}$ Expected number of components entering the ith stage of inspection in the jth cycle.

$PG_{i,j}$ Probability of a component being nondefective entering the ith stage of the jth cycle.

$FR_{i,j}$ Expected number of falsely rejected components in the ith stage of the jth cycle.

$FA_{i,j}$ Expected number of falsely accepted components in the ith stage of the jth cycle.

$CA_{i,j}$ Expected number of correctly accepted components in the ith stage of the jth cycle.

$R_{i,j}$ Rate of rejection of components due to the ith characteristics in the sequence of inspection in the jth cycle.

$A(j)$ Expected number of accepted components in the jth cycle.

$CFR(j)$ Cost of false rejection in the jth cycle.

CFA(j) Cost of false acceptance in the jth cycle.

CI(j) Cost of inspection in the jth cycle.

TCFR Total cost of false rejection.

TCFA Total cost of false acceptance.

TCI Total cost of inspection.

TA Total number of accepted components.

$E(tc)|_j$ Expected total cost per accepted component after j cycles of inspection

E ( ) Expected value of the argument inside the parentheses

2.1 Basic Relationships in the Model

The probability of the ith characteristics being defective will vary from cycle to cycle. The relationship between $P_i(j)$ and P_i is given below

$$P_i(1) = P_i \tag{1}$$

Using Bayes theorem

$$P_i(2) = P_iP_{2i}/[P_iP_{2i} + (1 - P_i)(1 - P_{1i})] \tag{2}$$

Similarly

$$P_i(3) = P_i(2)P_{2i}/[P_i(2)P_{2i} +(1 - P_i(2))(1 - P_{1i})] \tag{3}$$

and from the symmetry of expressions (2) and (3) we get

$$P_i(j) = P_i(j - 1) P_{2i}/[P_i(j - 1)P_{2i} +(1 - P_i (j - 1)(1 - P_{1i})] \tag{4}$$

The probability of a characteristic being defective changes in each cycle, hence the probability of a component being nondefective also changes. It is given below

$$PG = \prod_{i=1}^{N} (1 - P_i) \tag{5}$$

The probability of a component being defective is

$$PGC = 1 - PG \tag{6}$$

Clearly,

$$PG(1) = PG = \prod_{i=1}^{N} (1 - P_i(1)) \tag{7}$$

The probability of a component being nondefective entering the jth cycle is

$$PG(j) = \prod_{i=1}^{N} (1 - P_i(j)) \tag{8}$$

The probability of a component being defective entering the jth cycle is

$$PGC(j) = 1 - PG(j) \tag{9}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost of false acceptance of all the defective components

$$E(tc)|_{j=0} = C_a (1 - PG) \tag{10}$$

The expected total cost per accepted component, after n cycles of inspection, can be written as

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{11}$$

where TCFR, TCFA, TCI, and TA are as defined earlier.

2.2 cost Minimization Model

The objective of this model is to determine the optimal inspection plan for multicharacteristic components. The model minimizes the total cost per accepted component resulting from type I errors, type II errors, and cost of inspection. Given the basic relationships in the previous section, a mathematical expression for expected total cost per accepted component will be obtained. Our objective is to minimize this cost subject to the relationships governing this situation.

In order to derive the expected total cost of inspection after n cycles of inspections, analysis of cycle 1 of inspection is necessary. All the components entering cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. This is the first stage of inspection.

Stage 1 in cycle 1: Number of components entering this stage is

$$M_{1,1} = M_1 \tag{12}$$

The probability of a component being nondefective is

$$PG_{1,1} = PG \tag{13}$$

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1}PG_{1,1}P11 \\ &= M_1PGP_{11} \end{aligned} \tag{14}$$

E (number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1P_{21} + (1 - PG_{1,1} - P_1)(1 - P_{11})] \\ &= M[P_1P_{21} + (1 - PG - P_1)(1 - P_{11})] \end{aligned} \tag{15}$$

E (number of correctly accepted components) is

$$\begin{aligned} CA_{1,1} &= M_{1,1}PG_{1,1}(1 - P_{11}) \\ &= MPG\,(1 - P_{11}) \end{aligned} \tag{16}$$

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component in order to classify it as defective or nondefective.Stage 2 of the first cycle

$$\begin{aligned} M_{2,1} &= FA_{1,1} + CA_{1,1} \\ &= M_1[P_1P_{21} + (1 - P_1)(1 - P_{11})] \end{aligned} \tag{17}$$

Using equation (8), we get,

$$PG_{2,1} = (1 - {}^2P_1) \prod_{i=2}^{N} (1 - P_i)$$

Substitute the value of ${}^2P_1$, the following formula is obtained,

$$= PG(1 - P_{11})/[P_1P_{21} + (1 - P_1)(1 - P_{11})] \tag{18}$$

$$\begin{aligned} FR_{2,1} &= M_{2,1}PG_{2,1}P_1 \\ &= M_1PG(1 - P_{11})P_{12} \end{aligned} \tag{19}$$

$$FA_{2,1} = M_{2,1}[P_2P_{22} + (1 - PG_{2,1} - P_2)(1 - P_1$$

$$= M_1[P_2P_{22} + (1 - PG_{2,1} - P_2)(1 - P_{12})] \tag{20}$$

$$CA_{2,1} = M_{2,1}PG_{2,1}(1 - P_{12})$$

$$= M_1 PG \prod_{i=1}^{2} (1 - P_{1i}) \tag{21}$$

By symmetry, we can obtain stage N of the first cycle

$$M_{N,1} = M_1 \prod_{i=1}^{N-1} [P_i P_{2i} + (1 - P_i)(1 - P_{1i})] \tag{22}$$

$$PG_{N,1} = PG \prod_{i=1}^{N-1} [(1 - P_{1i}) / (P_i P_{2i} + (1 - P_i)(1 - P_{1i}))] \tag{23}$$

$$FR_{N,1} = M_1 PG \prod_{i=1}^{N-1} (1 - P_{1i}) P_{1N} \tag{24}$$

$$FA_{N,1} = M_1 \prod_{i=1}^{N-1} [P_i P_{2i} + (1 - P_i)(1 - P_{1i})]$$

$$x[P_N P_{2N} + (1 - PG_{N,1} - P_N)(1 - P_{iN})] \tag{25}$$

$$CA_{N,1} = M_1 PG \prod_{i=1}^{N} (1 - P_{1i}) \tag{26}$$

This completes one cycle of inspection, and the result of this cycle is described by the following equations.

Number of accepted components after completing the first cycle is,

$$A(1) = FA_{N,1} + CA_{N,1} \tag{27}$$

Cost of false rejection is

$$CFR(1) = C_r \sum_{i=1}^{N} (FR_{i,1}) \tag{28}$$

Cost of false acceptance is

$$CFA(1) = C_a(FA_{N,1}) \tag{29}$$

Cost of inspection is

$$CI(1) = \sum_{i=1}^{N} C_i M_{i,1} \tag{30}$$

E (total cost per accepted components after one cycle of inspection is)

$$E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/A(1) \tag{31}$$

where CFR(1), CFA(1), CI(1), and A(1) are given by equations (28), (29), (30) and (27), respectively.

Before proceeding to the second cycle, it was shown in [8] that the manner in which characteristics are ordered for inspection affects the cost of inspection; in [14] a rule is given and proved to be optimal for minimizing the cost of inspection within each inspection cycle. To minimize the cost of inspection, this rule should be applied in each cycle. The rule states: at inspection cycle j, compute the ratio $C_i/R_{i,j}$ for all i. The formula for $R_{i,j}$ is given in equation (36). Then, for each component, first inspect the characteristic with the least ratio and, lastly, inspect the one with the highest ratio. This rule ensures that CI(j) is minimized within cycle j.

From the analysis of cycle 1, it can easily be seen that after this cycle we can compute the new values of $P_i(2)$, PG(2), M_2 and proceed in the same manner as in the first cycle to compute the cost of false rejection, cost of false acceptance and cost of inspection. Hence, by symmetry the results of the nth cycle can be obtained.

$$A(n) = FA_{N,n} + CA_{N,n} \tag{32}$$

$$CFR(n) = C_r \sum_{i=1}^{N} (FR_{i,N}) \tag{33}$$

$$CFA(n) = C_a (FA_{N,n}) \tag{34}$$

$$CI(n) = \sum_{i=1}^{N} C_i M_{i,n} \quad (35)$$

The ratio used to determine the optimal ordering of characteristics in the nth is $C_i/R_{i,n},\ i = 1, ..., N$ where

$$R_{i,n} = P_i(n)(1 - P_{2i}) + (1 - P_i(n))P_{1i} \quad (36)$$

After n cycles of inspection we must determine the total cost of inspection per accepted component, is determined from the total cost of false rejection the TCFR, total cost of false acceptance TCFA, and the total cost of inspection given below.

$$TCFR = \sum_{j=1}^{n} [CFR(j)] \quad (37)$$

$$TCFA = CFA(n) \quad (38)$$

$$TCI = \sum_{j=1}^{n} [CI(j)] \quad (39)$$

Total accepted components

$$TA = A(n) \quad (40)$$

The above equations (1) through (40) provide the basic relationship for the model; the purpose is to find the value of n which minimizes the expected total cost per accepted component. The above model can be stated as find n which minimizes equation (41).

$$\text{Min } E(tc)|_{j=n} \quad (41)$$

The following is an algorithm which can be used for finding a local optimal n.

Step 1 Determine the PG and $E(tc)|_{j=0}$ from equations (5) and (10); respectively, set $j = 1$.

Step 2 Compute $P_i(j)$, $PG(j)$, $PG_{N,j}$, M_j and $C_i/R_{i,j}$ for i = 1, 2, ..., N using equations (4), (8), (23), (27), (36), respectively. Arrange the ratios $C_i/R_{i,j}$ (i = 1, 2, ..., N) in order of decreasing magnitude. This is the optimal sequence of inspection for the jth cycle and has been shown in [13].

Step 3 Rearrange the probabilities P_i, P_{1i}, P_{2i}, and the inspection cost C_i according to the optimal sequence obtained in step 2.

Step 4 Compute A(j), CFR(j), CFA(j), and CI(j) using equations (32) (33), (34), and (35) respectively.

Step 5 Compute TCFR, TCFA, TCI, and TA from equations (37), (38), (39), and (40) respectively.

Step 6 Compute $E(tc)|_j$, using equation (11).

Step 7 If $E(tc)|_j$ is less than $E(tc)|_{j-1}$, set j = j + 1 and go to step 2; otherwise STOP (n = j - 1).

3.0 Impact of the Inspection Errors on the Performance Measures

The model stated above depicts the plan and is employed in this section to investigate the impact of the inspection errors (types I and type II errors) on ATI, ETC and AOQ. To examine the impact of type I and type II errors, a batch of 100 components, each with three characteristics were used in the model and the algorithm stated above is used to determine the optimal inspection plan. The probability each characteristic being defective P_i is taken to be 0.1, 0.2, 0.3, for i = 1,2,3 respectively. The cost of inspection C_i = \$100 for i = 1,2,3. The cost of false acceptance is \$ 100000 and the cost of false rejection is \$500. Type I and type II were varied from 0.00 to .15. The above algorithm that determines the optimal parameters of the model was implemented on IBM personal computer and the model was run 36 times using different pairs of type I and type II errors. For each run type I and type II errors are assumed the same for all characteristics and referred to as E_1 and E_2. The model determines the optimal number of repeat inspections and simulates the inspection process for each pairs of errors. The results of the 36 runs are shown in Table 1. Table 1 shows the pairs of errors and the optimal values for ETC, AOQ which is (1-PG) and A(n) for each pair of errors. Also the Table shows the total cost of inspection CI(n), the cost of false acceptance CFA(n) and the total cost of false rejection TCFA(n). All costs per accepted component. Next we examine the Table and determine the impact of the errors on the performance measures of the inspection plan.

3.1 Impact of Inspection Errors on Average Total Inspection (ATI)

The average total inspection (ATI) is defined as the total number of inspection conducted in the optimal inspection plan. For a batch of M components ATI is computed as:

$$ATI = \sum_{j=1}^{n} \left(M_j \sum_{k=1}^{N} \prod_{i=1}^{k-1} \left[{}^{j}P_i P_{2i} + \left(1 - {}^{j}P_i\right)\left(1 - P_{1i}\right) \right] \right)$$

Figure 1 shows the relationships between type I error E_1 and ATI for fixed values of type II errors, E_2 . The fixed values for type II errors are .05, .1 and .15. It is observed that ATI decrease in a Linear fashion as type I error increase. Similarly Figure 2 shows the relationship between ATI and the type II errors for fixed values of type I error. ATI increases in a piece wise linear fashion as type II errors increase.

It is concluded that type II errors increases the inspection load while type I error reduces the inspection load in complete repeat inspection plans.

3.2 Impact of Inspection Errors on Expected Total Cost (ETC)

The expected total cost ETC is the optimal cost for the inspection plan. It is the sum of the inspection cost, the cost of false acceptance and the cost of false rejection per accepted component.

Table 1. Effect of the Inspection Errors on the Inspection Plan

No.	E_1	E_2	1 - PG	ETC	CI	CRF	CFA	A(n)	n*
1.	0.00	0.00	0.0000	448.42	448.42	0.00	0.00	50	1
2.	0.00	0.01	0.0001	758.88	750.98	0.00	7.90	50	2
3.	0.00	0.03	0.0007	826.87	755.84	0.00	71.03	50	2
4.	0.00	0.05	0.0020	957.47	760.33	0.00	197.14	50	2
5.	0.00	0.10	0.0008	1155.44	1076.51	0.00	78.93	50	3
6.	0.00	0.15	0.0027	1357.23	1091.21	0.00	266.02	50	3
7.	0.01	0.00	0.0000	473.74	458.43	15.31	00.00	49	1
8.	0.01	0.01	0.0001	820.38	781.25	31.06	8.06	48	2
9.	0.01	0.03	0.0007	889.86	786.33	31.02	72.47	48	2
10.	0.01	0.05	0.0020	1023.17	791.01	47.30	201.14	48	2
11.	0.01	0.10	0.0008	1267.37	1138.74	47.30	81.34	47	3
12.	0.01	0.15	0.0026	1475.85	1154.50	47.21	274.14	46	3
13.	0.03	0.00	0.0000	527.40	479.56	47.84	00.00	46	1
14.	0.03	0.01	0.0001	995.97	847.33	100.25	8.39	42	2
15.	0.03	0.03	0.0008	1028.56	852.88	100.19	75.49	42	2
16.	0.03	0.05	0.0021	1167.55	857.98	100.05	209.51	42	2
17.	0.03	0.10	0.0007	1523.28	1279.25	157.56	86.47	38	3
18.	0.03	0.15	0.0030	1746.05	1297.40	157.24	291.41	38	3
19.	0.05	0.00	0.0000	585.45	502.27	83.18	00.00	43	1
20.	0.05	0.01	0.0001	1110.69	921.76	180.17	8.75	37	2
21.	0.05	0.03	0.0008	1186.60	927.85	180.05	78.71	37	2
22.	0.05	0.05	0.0012	1331.62	933.41	179.79	218.41	37	2
23.	0.05	0.10	0.0009	1830.09	1444.97	293.07	92.05	32	3
24.	0.05	0.15	0.0013	2068.55	1465.96	292.42	310.17	32	3
25.	0.10	0.00	0.0000	752.97	567.10	185.87	00.00	37	1
26.	0.10	0.01	0.0001	1604.34	1153.79	440.80	9.75	27	2
27.	0.10	0.03	0.0009	1689.67	1161.53	440.45	87.69	27	2
28.	0.10	0.05	0.0022	1851.59	1168.51	439.77	243.31	27	2
29.	0.10	0.10	0.0073	2587.43	1182.62	436.57	968.24	27	2
30.	0.10	0.15	0.0036	3192.83	2040.47	787.71	364.65	20	3
31.	0.15	0.00	0.0000	960.20	646.03	314.17	00.00	31	1
32.	0.15	0.01	0.0092	1876.31	642.07	311.27	922.97	31	1
33.	0.15	0.03	0.0001	2409.45	1486.23	824.92	98.30	19	2
34.	0.15	0.05	0.0028	2591.30	1495.10	823.48	272.72	19	2
35.	0.15	0.10	0.0108	3413.72	1512.35	816.78	1084.59	19	2
36.	0.15	0.15	0.0242	4745.27	1522.40	805.78	2417.09	20	2

Figure 3 shows the relationship between type I error E_1, and ETC for fixed values of type II error. The fixed values for type II errors are 0.03, 0.05, 0.1 and 0.15. Similarly Figure 4 provides the relationship between type II error and ETC for fixed values of type I error. The Figures show that both errors results in an increase in ETC. However at values $\geq .05$, type I error has a more drastic effect on ETC than type II errors. This is apparent from the slope of the graph in Figure 4.

3.3 Impact on Average Outgoing Quality (AOQ)

The performance measure AOQ is defined as:

$$AOQ = \frac{\text{number of defective components after inspection}}{\text{total number of accepted components}}$$

Figure 5 shows the relationship between type I error and AOQ for fixed values of type II error. The fixed values for type II error are 0.05, 0.1 and 0.15. Similarly Figure 6 shows the relationship between type II error and AOQ for fixed values of type I error.

It is observed that both errors increase AOQ, in other words, they deteriorate the quality of the accepted component. However if AOQ reaches a high level the model adjust the optimal number of repeat inspection to attain lower values of AOQ.

The above analysis implies that type I and type II errors have significant impact on the performance measures of complete repeat inspection plans. It has been documented in [11] that type I and type II errors are functions of incoming quality. In complete repeat inspection plan the incoming quality for each cycle of inspection changes, and therefore the inspection errors also change. This dynamic change in type I and type II errors must be incorporated in the design of the inspection plan. Next a methodology is outlined for incorporating the dynamic change in the errors in the design of the inspection plan.

4.0 Methodology for Incorporating Error Change in the Complete Inspection Plan

The optimal complete inspection plan is developed using the optimization model presented in Section 2. Type I and type II errors are assumed known and fixed. In reality these errors are functions of incoming quality. In [10] a model is developed to express type I error (P_{1i}) and type II error (P_{2i}) as a function of incoming quality.

$$P_{1i} = f_{1i}(P_i) \tag{42}$$

$$P_{2i} = f_{2i}(P_i) \tag{43}$$

The above function can be incorporated in the model given in Section 2. After the inspection of each characteristic the error is updated for each characteristic, using equations (42) and (43) before the next inspection. In this fashion the model incorporates the change in type I and type II errors and hence the resulting inspection plan.

5.0 Conclusion

The main purpose of this paper is to demonstrate and quantify the effect of type I and type II errors on the performance measures of a complete inspection plan. This is accomplished by observing the changes in ATI, ETC and AOQ under varied values of the errors. It was observed ATI decreases as type I error increases. This is in line with intuition, since an increase in the probability of rejecting good components will reduce the number of components to be inspected in subsequent cycles of inspections. On the other hand as type II error increases ATI increases. Therefore higher values of type II error increase the inspection load in the plan.

Both errors increase ETC, but type I error increases ETC at a faster rate especially at values $\geq .05$. AOQ is higher for higher values of type I and type II errors, however it was observed that as AOQ reaches a high level the model adjust the optimal n to obtain a lower value for AOQ. It can be concluded that type I and type II errors have significant effect on the performance measures of repeat inspection plans and a methodology for incorporating the dynamic changes in the inspection errors in the design of the plan is needed. Such a methodology is proposed and outlined in this paper.

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Figure 1. Effect Of Type I Error On Average Total Inspection (ATI)

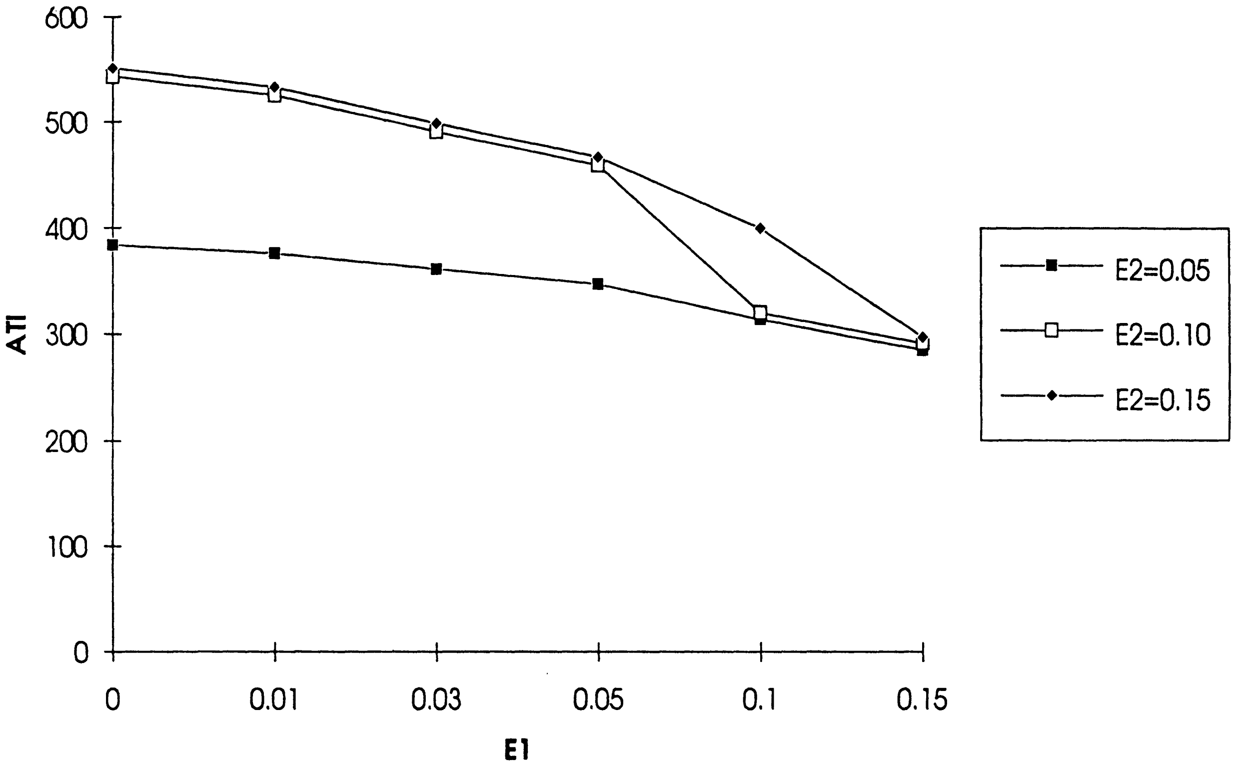

Figure 2. Effect Of Type II Error On Average Total Inspection (ATI)

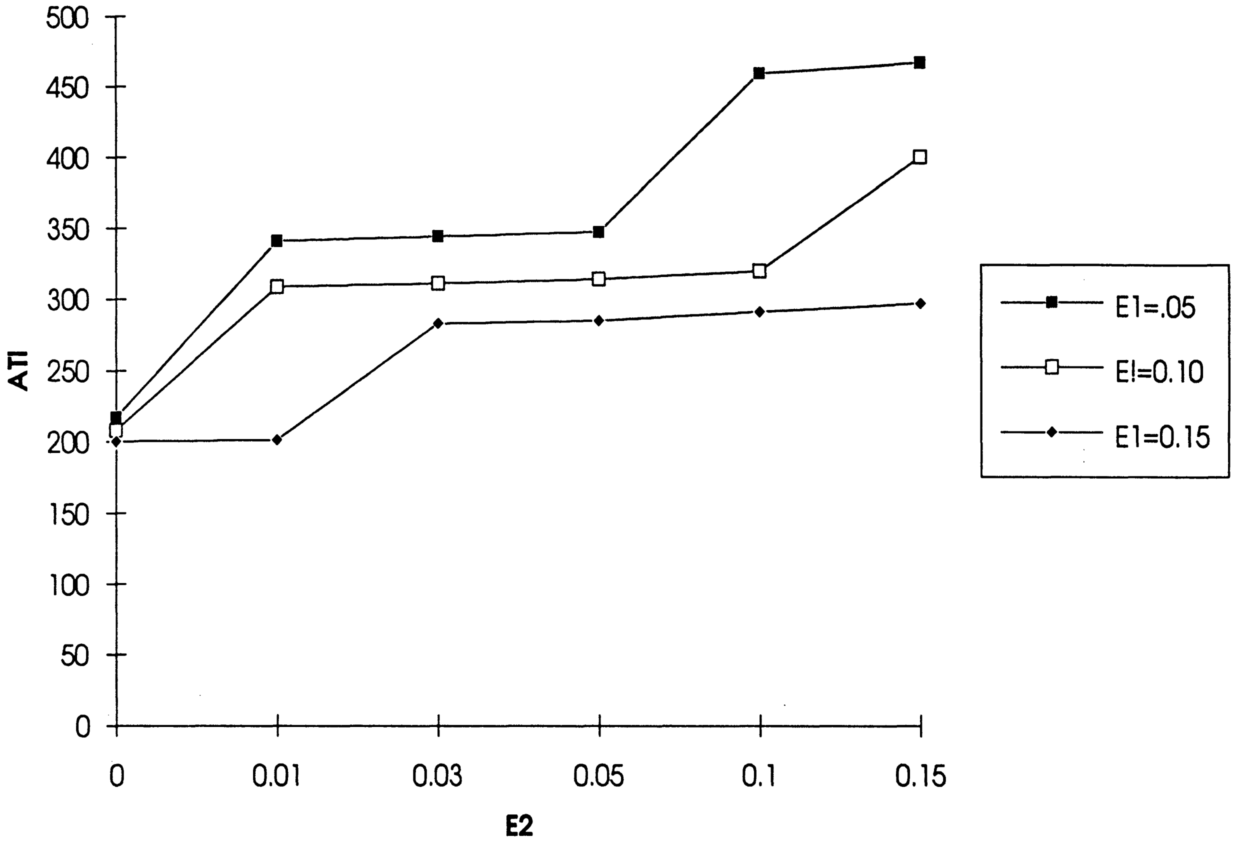

Figure 3. Effect Of Type I Error On Expected Total Cost (ETC)

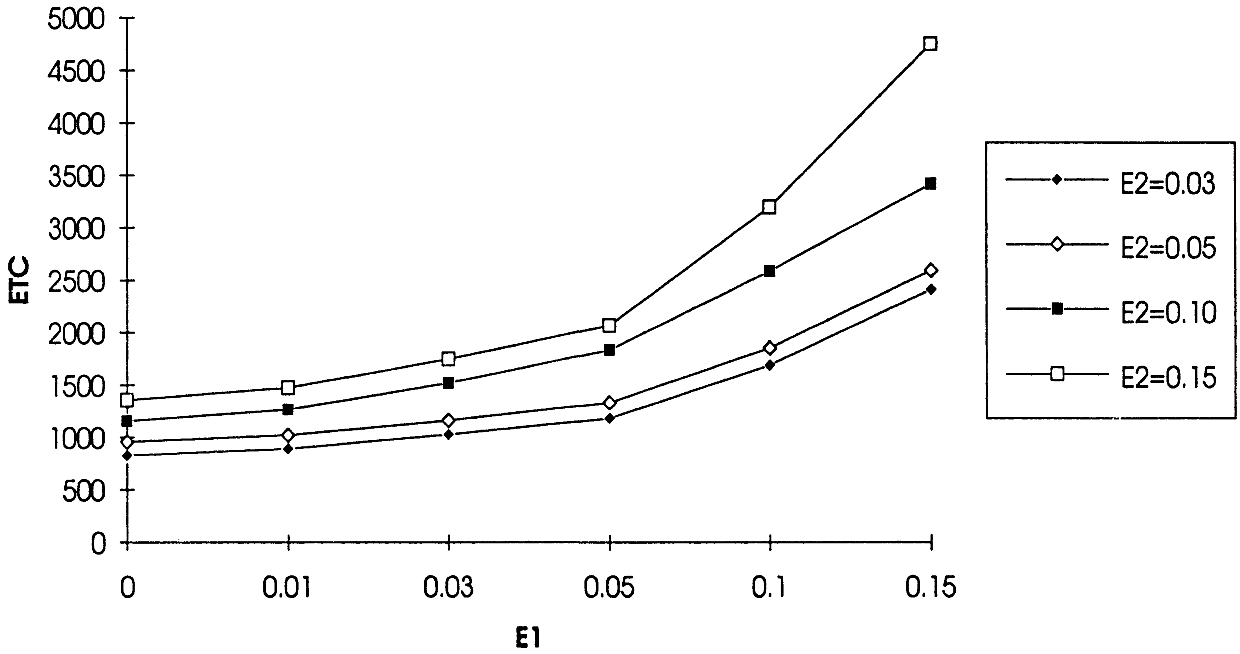

Figure 4. Effect Of Type II On Expected Total Cost (ETC)

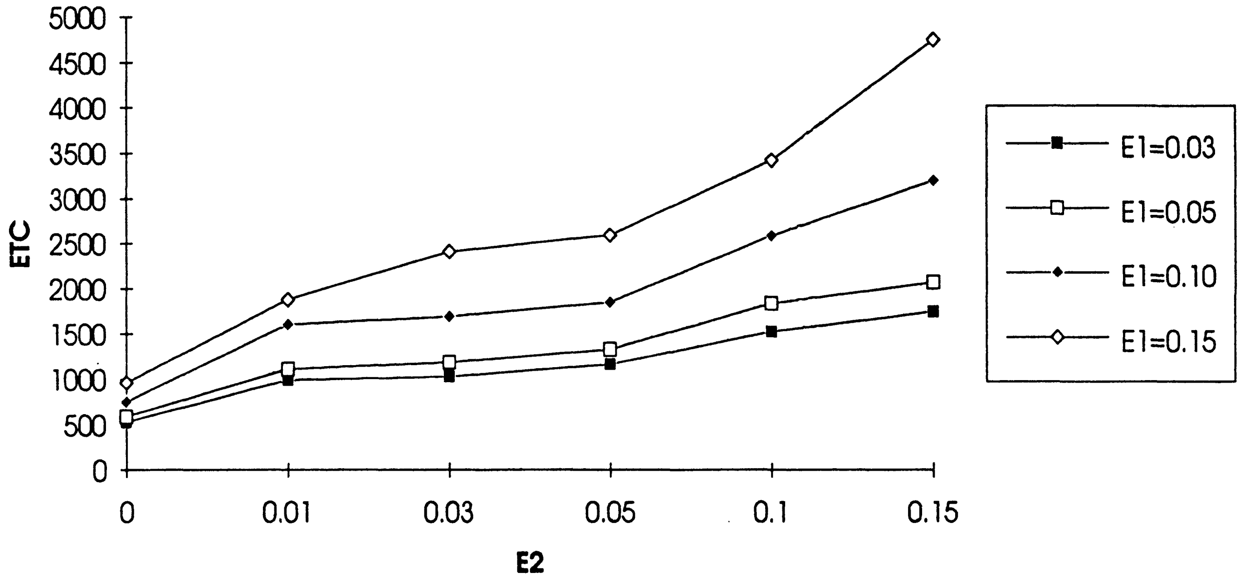

Figure 5. Effect Of Type I Error On Average Outgoing Quality (AOQ)

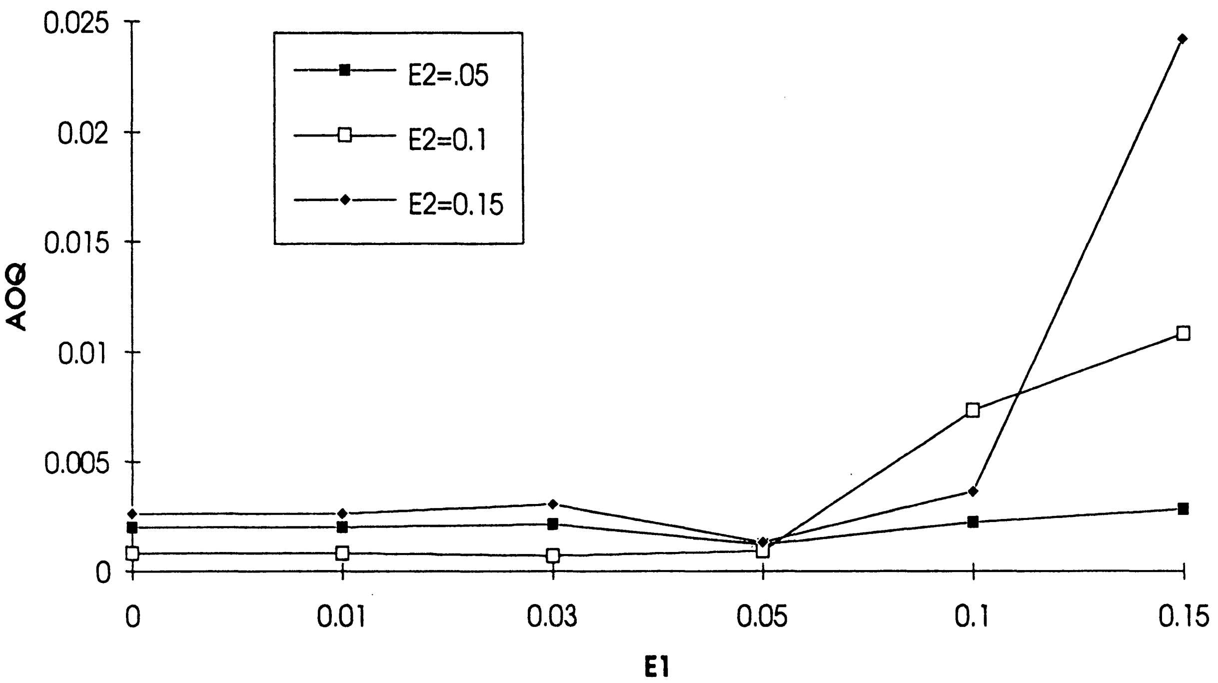

Figure 6. Effect Of Type II Error On Average Outgoing Quality (AOQ)