

AN INVESTIGATION INTO RESONANCE IN RAMJET-TYPE BURNERS

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FOREWORD

The investigation reported herein is part of a program of research conducted at the University of Michigan for the United States Army Air Force under Contract W33-038 ac-21100 and is a continuation of the research previously reported.

This research was carried out under the direct supervision of R. B. Morrison and the faculty supervision of E. T. Vincent and J. W. Luecht.

ABSTRACT

The observed effects upon combustion of the resonance of the complex of gases in two simple ramjet-type burners are described. Records were obtained of the pressure fluctuations occurring within the burners as the position of the flame front was varied along the axis of the burner. Good agreement was found between the observed resonant frequency and the frequency predicted by assuming an organ-pipe type of resonance of the hot and cold gases.

Some of the factors which might lead to a feed-back mechanism are examined. It is concluded that the effect of pressure and temperature on flame speed is probably the most important factor.

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LIST OF SYMBOLS

c	=	speed of sound
γ	=	ratio of specific heats
n	=	frequency
L	=	total length of combustion chamber
x	=	distance along axis of combustion chamber
ρ	=	density
V_f	=	flame speed, the rate at which the flame front traverses the unburned gas in a direction normal to itself
A_f	=	area of flame
Q	=	heating value of fuel
m	=	mass flow
q	=	rate of heat release
t	=	time
y	=	particle displacement along axis of tube
A	=	constant in wave equation
v	=	particle velocity
P	=	pressure
P_0	=	atmospheric pressure

AN INVESTIGATION INTO RESONANCE IN RAMJET-TYPE BURNERS

INTRODUCTION

Resonance of the complex of gases in a ramjet-type combustor can generally be put into three categories: (1) that associated with the ignitor, (2) that related to the fuel injection system, and (3) that related to the geometry of the burner.* Generally, the first two types of resonance are the most violent, while the latter has a lesser effect. However, a burner was constructed which utilized a homogeneous fuel-air mixture and had no continuous ignitor. While this burner eliminated the first two causes, the resulting instability of burning was sufficient to cause premature blow-out of the flame. Because of this and other pernicious effects of resonance on ramjet operation,** it was felt that the causes and effects of resonance in a simple combustion chamber were worth investigating.

Initial investigation into this geometry-resonance, utilizing a 1-in. x 1-in. x 12-in. combustion chamber, revealed some interesting phenomena and indicated that the type of resonance was similar to that which occurs in an organ pipe.¹ A further investigation to substantiate the data on the above system and to obtain data on another system of different geometry was conducted from March to September, 1950. The results of this investigation are presented herein, with a correlation of the data and an

* These three types of resonance are discussed by Rayleigh⁵.

** Another group at the University of Michigan is presently studying the effect of periodic disturbances on the flow in and about a ramjet model.²

examination of the possible factors causing the resonant condition. It is felt that further research is necessary in order to understand the feed-back mechanism causing the resonant condition.

The experimentation reported was carried out at unburned mixture velocities from 10 to 210 ft/sec, the majority of the testing being done at 16 ft/sec. The fuel used was commercial propane.

DESCRIPTION OF APPARATUS

I. Burner Equipment

1. 1-in. x 1-in. x 24-in. Parallel-Walled Combustion Chamber.

A major portion of the experimentation reported herein was conducted on the equipment described in a previous report¹ with one modification, the use of a 24-in. combustion chamber in place of the 12-in. burner previously used. A photograph of the combustion chamber is shown in Fig. 1. The glass walls were made up of four 3-in. x 12-in. x 1/8-in. pieces of Vycor glass. The flame-holder position was made variable by means of the arrangement shown in Fig. 2. The majority of testing was accomplished at a jet velocity of 16 ft/sec. The fuel used was commercial propane.

2. Choked Inlet Combustion Chamber. This burner was made in two sections with a critical orifice at the inlet to the first section and a flame holder at the entrance to the second section (see Fig. 3). The critical orifice served to isolate acoustically the air source from the burner. The resulting system was then similar to a half-open organ pipe, i.e., closed at one end and open at the other. Both sections of the burner were constructed from standard 1-1/4-in. diameter black iron pipe. The upstream total pressure P_1 was maintained at 35 psi, sufficient to insure sonic flow through

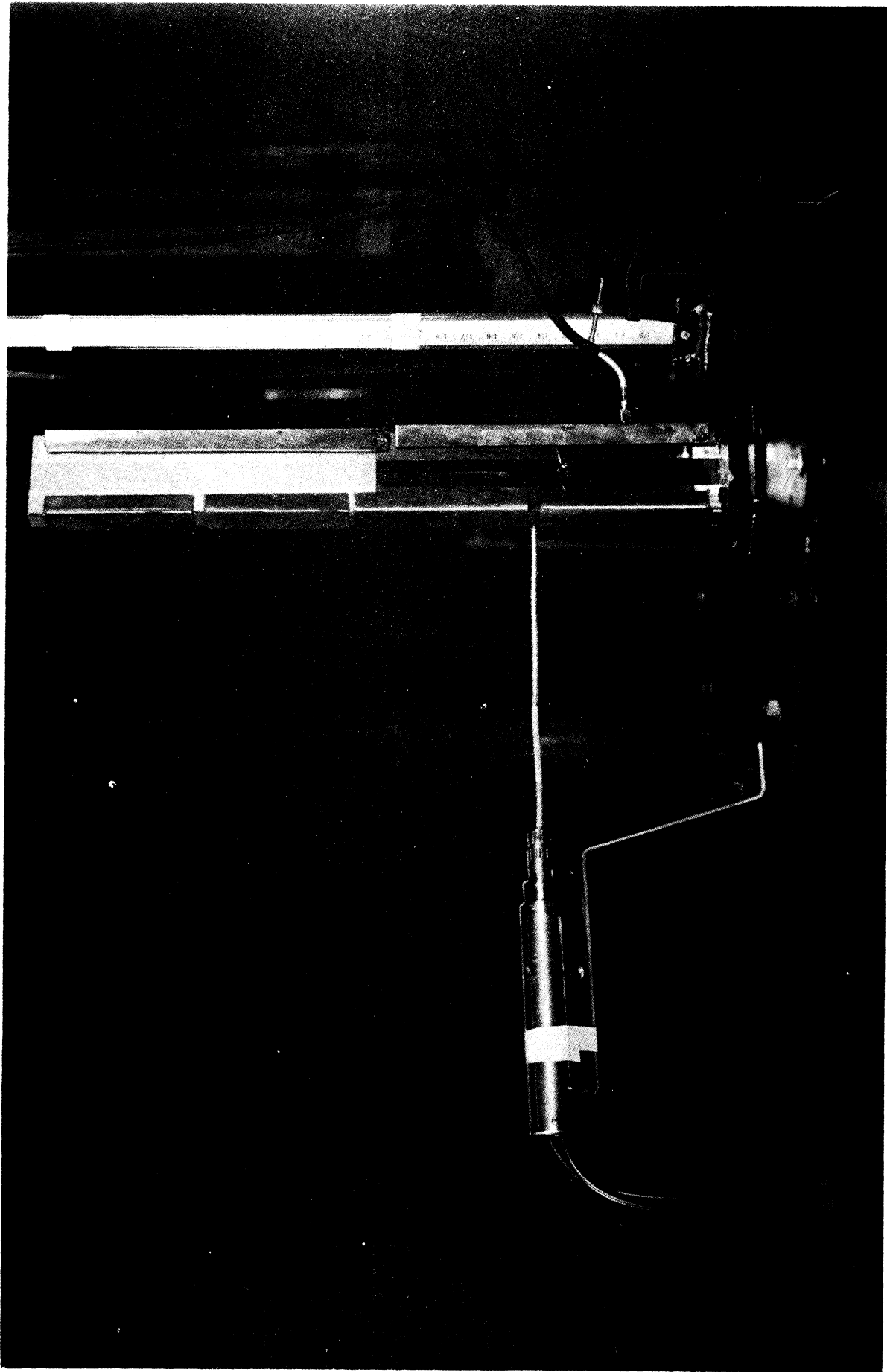


FIG. 1 PHOTOGRAPH OF 1" x 1" x 24" PARALLEL-WALL COMBUSTION CHAMBER,
SHOWING INSTALLATION OF MASSA MICROPHONE

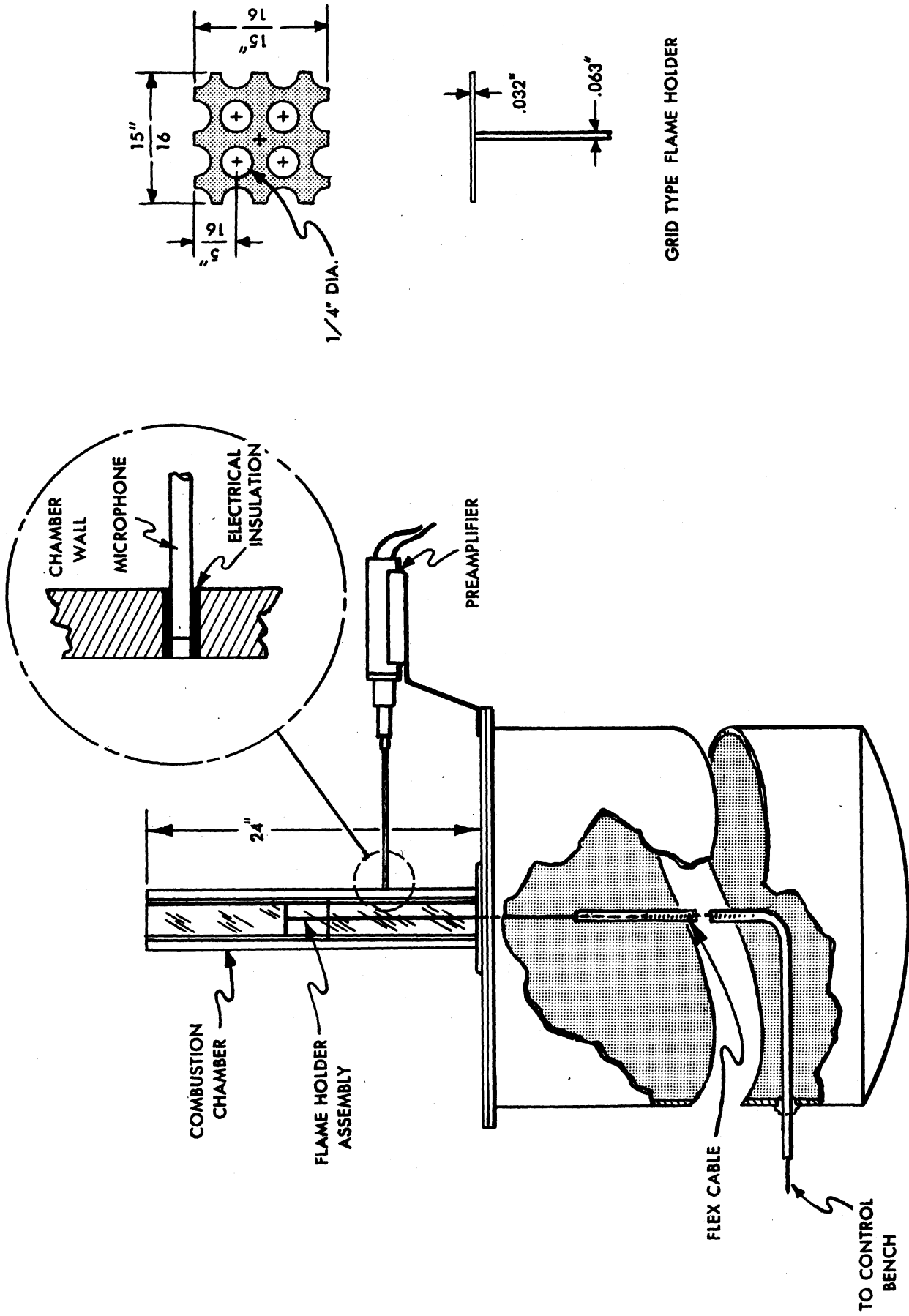
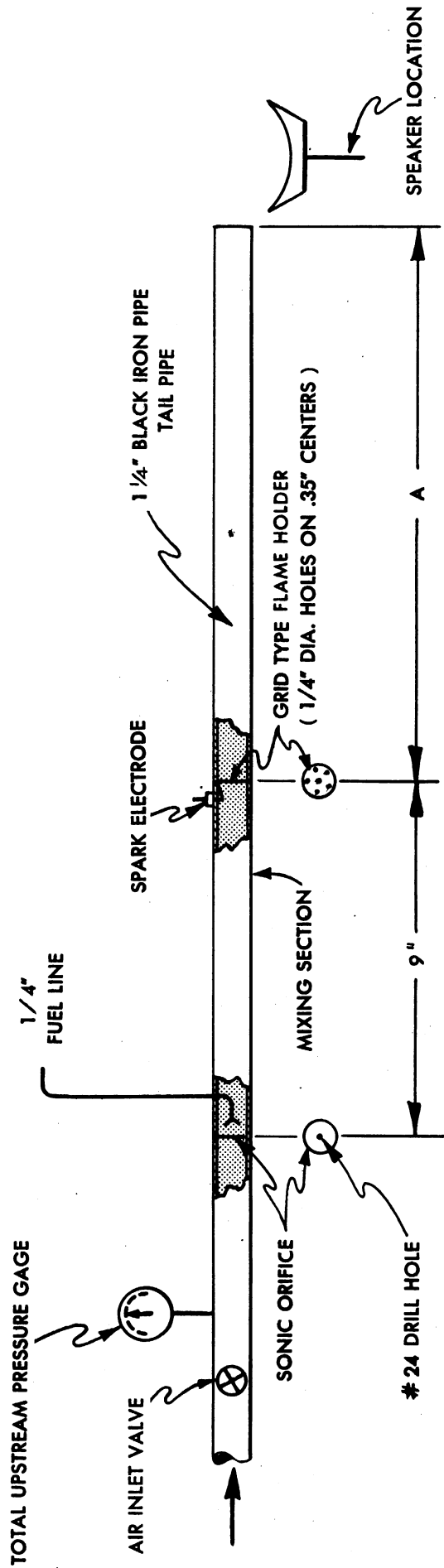


FIG. 2 FLAME-HOLDER & MICROPHONE ARRANGEMENT
IN 1" x 1" x 24" BURNER



A = 8 1/2, 14, 24 3/4

FIG. 3 CHOKED INLET BURNER

the orifice. Most experiments were conducted at a jet velocity of 20 ft/sec.

3. Flame-Holder Configuration. Except where otherwise noted, the flame holder was a simple grid, constructed from 0.031-in. thick sheet steel with 1/4-in. holes drilled, as shown in Fig. 2.

II. Pressure Pickup and Recording Instruments

1. Speaker Arrangement. A 7-in. permanent magnet speaker was located at the exit of the burner (see Fig. 3). The output from this speaker was fed through a coil and then to one beam of a Dumont Type 279 two-beam oscilloscope. A 120-cps timing pip, which was fed into the other beam, was used as a base for measuring frequency. A Fairchild camera, adapted for use on the above oscilloscope, recorded the traces made on the scope. More complete data concerning these units are given in Appendix III. The accuracy of frequency measurements by this method is ± 15 cps. A sample record is shown in Fig. 4a.

2. Massa Sound Pressure Measurement System. The Massa equipment consisted of an extremely sensitive microphone, a preamplifier, and a power supply. Another standard amplifier plus the oscilloscope, camera, and 120-cycle pulser were used with the Massa instrument. This instrument gives a record of pressure versus time which was undistorted from 50 cycles to approximately 250 kilocycles. More complete information about this pickup is given in Appendix III. A photograph of the installation of the microphone in the combustion chamber is shown in Fig. 1. A sample pressure record along with the timing pip and a record of the pressure calibration is shown in Fig. 4b. The microphone was mounted flush in the combustion-chamber wall as shown

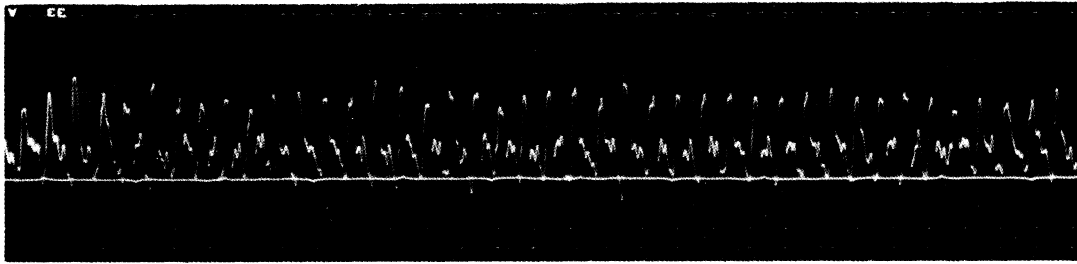
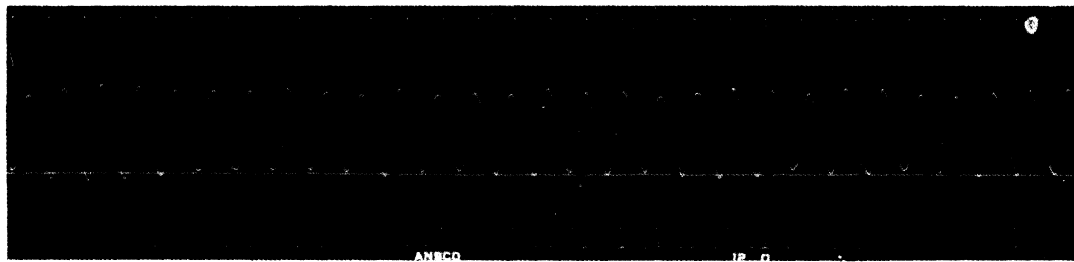
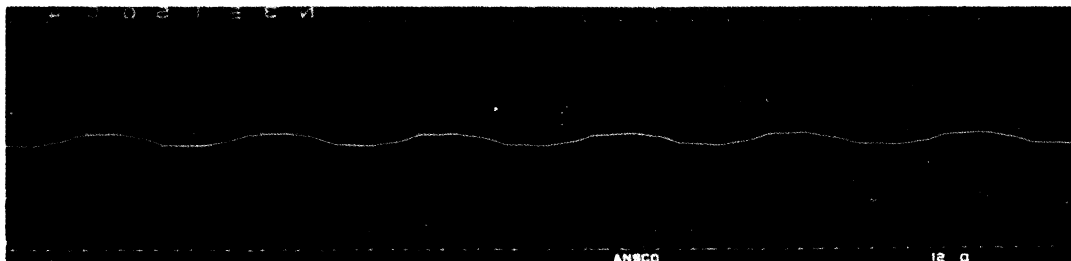


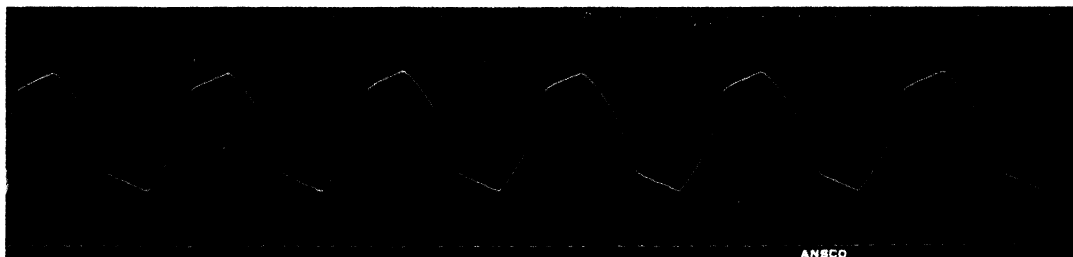
FIG. 4a SAMPLE RECORD OF FREQUENCY PICKED UP BY SPEAKER PLACED AT EXIT OF 24 in. RECTANGULAR BURNER. RUN No. 87c. FREQUENCY - 420 cps



PRESSURE TRACE - 300 CPS, .75 PSI PEAK TO PEAK



.10 PSI CALIBRATION TRACE



1.0 PSI CALIBRATION TRACE

FIG. 4b SAMPLE RECORD OF PRESSURE VARIATION IN 24 in. RECTANGULAR BURNER PICKED UP BY MASSA MICROPHONE MOUNTED FLUSH WITH BURNER WALL, LOCATED 6 1/2 in. FROM CHAMBER ENTRANCE.

in Fig. 2. Again, the accuracy of the frequency measurements was ± 15 cps. The accuracy of the pressure-measurement system and the calibration system as a unit was checked by the controls group at the Willow Run Research Center and was found to be ± 20 per cent.

RESULTS AND DISCUSSION

1. Description of Burning in 1-in. x 1-in. Doubly Open Burner

Burning at low velocity (ca. 16 ft/sec) in the 1-in. x 1-in. x 12-in. parallel-walled burner was described in a previous report.¹ This burning generally resulted in a tone being emitted from the combustion chamber and, if a rod-type flame holder was used, a wave motion on the flame front. The use of a grid-type flame holder also usually resulted in a musical tone being given off; however, at times the flame became very agitated, the intensity of the tone increased, and the pureness of the tone was lost.

The burning which occurred in the 1-in. x 1-in. x 24-in. burner generally resulted in the following pattern as the flame holder (a grid) was moved from the chamber exit toward the entrance. When the flame holder was located at the exit, the flame emitted no tone. As the flame was moved further upstream, a musical tone was emitted, and upon further movement upstream the flame began to have a high-frequency oscillation, first about its undisturbed position and then about the flame holder, the flame holder becoming heated. When the flame holder was located 13-in. from the entrance, the flame was observed to oscillate as much as 1 in. upstream of the flame holder. As the flame holder was moved further upstream, the amplitude of oscillations appeared first to decrease and, upon further movement, to

increase, giving an amplitude of observed flame oscillation plot such as is shown in Fig. 5.

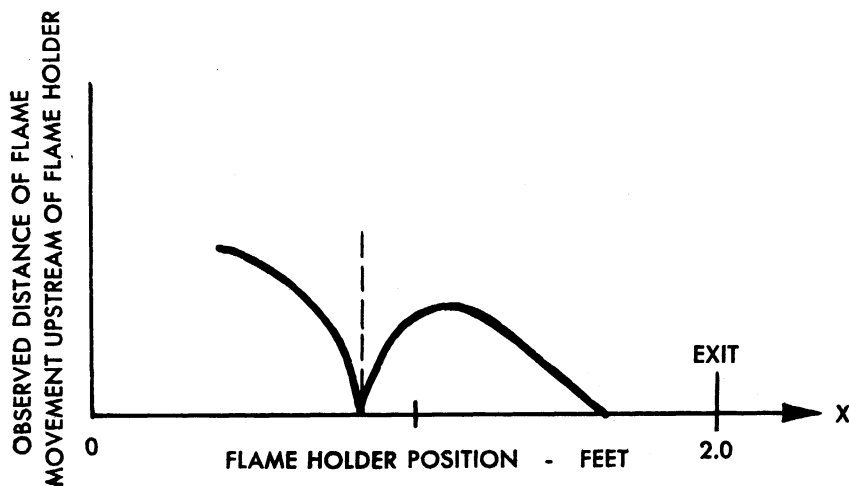


FIG. 5 FLUCTUATION OF FLAME UPSTREAM OF FLAME-HOLDER

Any vents or leaks in the combustion chamber reduced the amplitude of the flame fluctuations and altered the resulting tone frequency considerably. A leak caused by a 1/16-in. x 1-in. crack in the glass (effected by separating the upper and lower glass walls) was sufficient to cause a flame which was previously oscillating as much as 1/4 in. upstream of the flame holder to not oscillate at all.

The effect of fuel-air ratio on the amplitude of flame oscillations was not investigated in any great detail. It was noted, however, that the resonance was most severe at fuel-air ratios slightly greater than stoichiometric. The flame was not prone to resonate at extreme rich or lean mixtures. As will be pointed out later in the report, varying the fuel-air ratio changed the resonant frequency of the burner.

2. Correlation of Resonant Frequency Data from Doubly Open Burner with Theoretical Predictions

In Appendix I of a previous report¹ an expression was derived for the resonant frequency of a doubly open pipe in which an interface separated regions of hot and cold gases. This expression, Eq 1,

$$- c_1 \gamma_1 \tan \frac{2\pi n(L-x)}{c_2} = c_2 \gamma_2 \tan \frac{2\pi n x}{c_1} \quad (1)$$

where

- 1 refers to unburned gas
- 2 refers to burned gas
- c = velocity of sound
- L = combustion-chamber length
- n = resonant frequency
- x = distance of flame holder from combustion-chamber entrance
- γ = ratio of specific heats ,

gives the resonant frequency of this system as a function of position of the interface (x) and is plotted in Fig. 6.*

The frequency of the tone emitted from the burner was determined in two ways, described on page 6. The frequency data obtained by means of the speaker located at the exit of the burner and by means of the Massa microphone placed inside the burner are shown in Fig. 6. All data were obtained at a mixture velocity of 16 ft/sec and with a grid-type flame holder. Data could not be obtained in the region $0 < x < 0.25$ because of the danger of flash-back into the settling tank nor in the region $1.55 < x < 2.0$ for reasons explained later in the report.

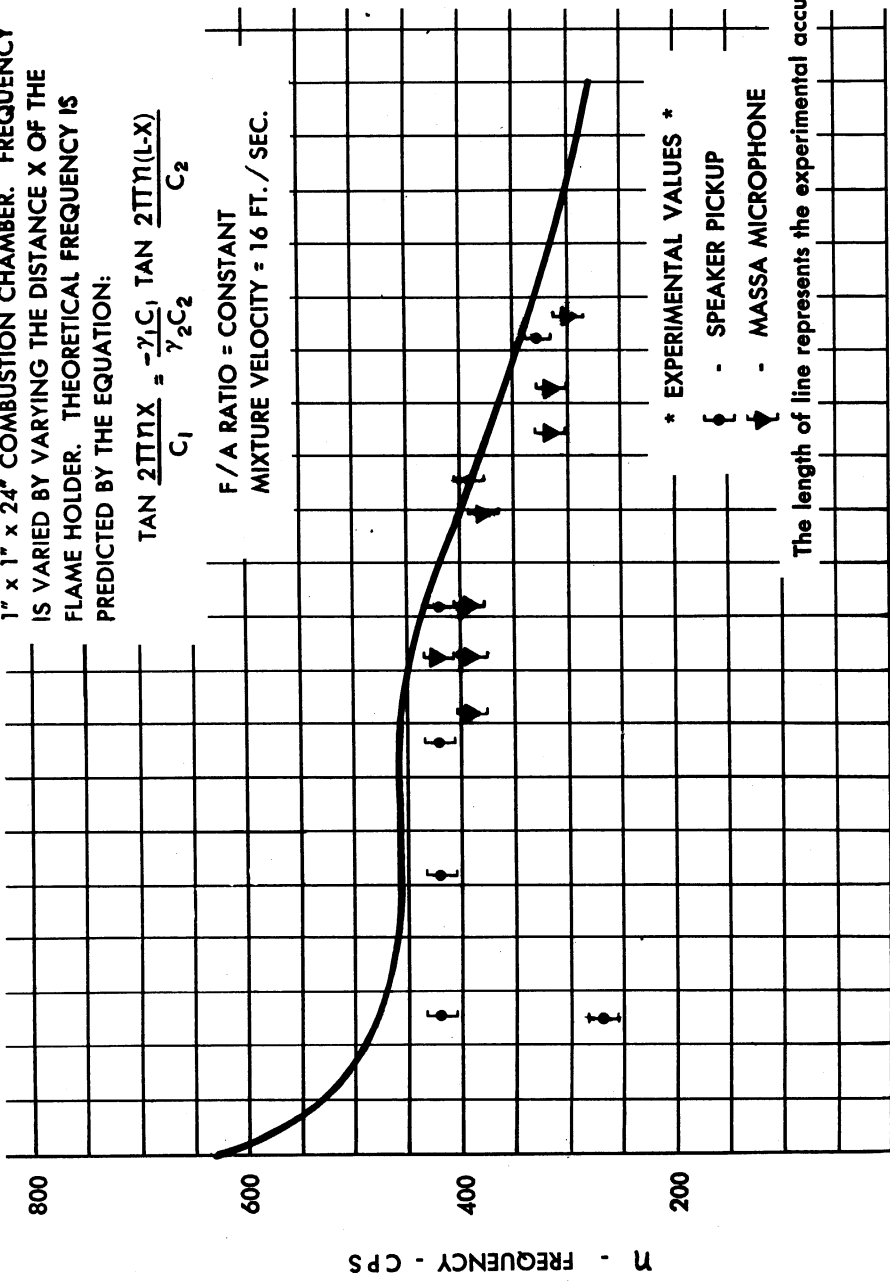
As shown in Fig. 6, the experimental data and theoretical predictions agree quite well. There are several factors which might cause

* It was assumed for the calculations that $\gamma_1 = \gamma_2 = 1.4$; $c_1 = 1120$ ft/sec; and $c_2 = \sqrt{5} \times 1120$ ft/sec.

RESONANT FREQUENCY OF A PROPANE - AIR FLAME BURNING FROM A GRID TYPE FLAME HOLDER IN A 1" x 1" x 24" COMBUSTION CHAMBER. FREQUENCY IS VARIED BY VARYING THE DISTANCE X OF THE FLAME HOLDER. THEORETICAL FREQUENCY IS PREDICTED BY THE EQUATION:

$$\tan \frac{2\pi n x}{c_1} = \frac{\gamma_1 c_1}{\gamma_2 c_2} \tan \frac{2\pi n (L-x)}{c_2}$$

F/A RATIO = CONSTANT
MIXTURE VELOCITY = 16 FT. / SEC.



X DISTANCE FROM COMBUSTION CHAMBER ENTRANCE - FT. L

FIG. 6

the discrepancies which were obtained between the experimental and theoretical values. A factor governing the vertical shift between the two curves is the assumption made for the speed of sound, c_1 and c_2 , and for σ_1 and σ_2 ; Table 1 shows the change in frequency for two different fuel-air ratios (the change in fuel-air ratio effectively changes the speed of sound in the burned gases). Another error might arise from the fact that as a result of the large jet speed compared to the flame speed, the flame front (interface) was not normal to the walls of the burner as was assumed in the derivation of Eq 1. The effect of this error on the resonant frequency is not fully understood. Another error would result from the end effects of the pipe, which were not accounted for in Eq 1.

In general, the agreement between the experimental and theoretical values indicates quite conclusively that the type of resonance is that of a doubly open organ pipe, resonating at its fundamental frequency. This is the same conclusion as that arrived at in the earlier report¹ for the 1-in. x 1-in. x 12-in. burner.

TABLE 1

THE EFFECT OF A CHANGE IN F/A RATIO ON THE RESONANT FREQUENCY

<u>Flame Holder Position</u> <u>x, ft</u>	<u>F/A Ratio</u> <u>by Weight</u>	<u>Frequency</u> <u>cps</u>
0.516	0.0682	420
	0.0594	240
0.758	0.0682	420
	0.0594	375
1.25	0.0682	390
	0.0594	330
1.52	0.0682	330

It was thought at one time that perhaps the type of resonance occurring in the combustion chamber was that of a Helmholtz resonator. A simple calculation shows that the resonant frequency of such a system (the plenum chamber being the cavity and the combustion chamber the neck) is about 15 cps, much lower than any frequency observed.

3. Correlation of Observed Flame Movements with Theoretical Predictions

As was stated previously in the report and shown in Fig. 5, the flame was observed to oscillate about the flame holder, at times moving as much as 1 in. upstream of the flame holder. This observation indicated that the velocity amplitude due to the standing wave set up in the burner was at times greater than the average mixture velocity (ca. 16 ft/sec). By means of the equations previously derived¹, it was possible to calculate the velocity amplitude due to the standing wave at any point in the burner for a given pressure amplitude. The pressure amplitude was obtained with the Massa sound pressure equipment. Calculations for the velocity amplitude at the flame holder were made in Appendix I for two cases: one, in which the flame was observed to fluctuate about the flame holder, and the other, in which it was not. In the first case, the pressure amplitude observed was of sufficient magnitude to cause the velocity amplitude calculated at the flame holder to be greater than the mixture velocity at that point; hence, the flame should have moved upstream, as it was observed to do. In the second case, the velocity amplitude calculated was less than the mixture velocity; hence, the flame should not have fluctuated about the holder, and it was observed not to do so. These calculations would indicate that the equations describing the resonant condition are accurate to some degree.

It was noted that a velocity node occurs somewhere near the middle of a doubly open pipe when the pipe is excited at its fundamental frequency, giving a velocity amplitude (or particle displacement) versus distance plot such as shown in Fig. 7. As shown previously, the amplitude of the flame oscillations decreased until the flame appeared quiescent at the node as the interface (or flame front) was moved toward this nodal point. An attempt was made to utilize this fact to increase the blowoff velocity of the flame holder by removing the flame from the disturbed region. However, as the mixture velocity was increased, the flame lengthened (hence and interface was no longer at one value of x) and uniqueness of the nodal point was lost, resulting in very little increase in blowoff velocity being obtained.

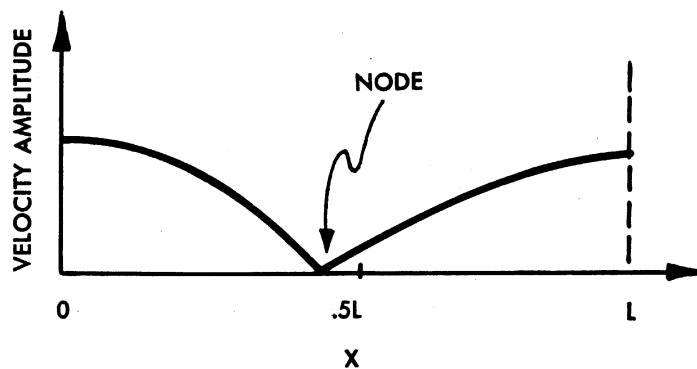


FIG. 7 PLOT OF THEORETICAL VELOCITY AMPLITUDE ALONG COMBUSTION CHAMBER, WHEN RESONATING AT FUNDAMENTAL FREQUENCY

4. Description and Results of Burning in Choked Inlet Burner

The choked inlet combustion chamber was designed to simulate acoustically a half-open organ pipe. The combustion occurring in this burner was quite similar to that described for the doubly open burner. During resonance the flame fluctuated about the flame holder, the flame holder becoming heated. A speaker located at the exit of the burner, as shown in Fig. 3, was used in conjunction with the recording oscilloscope and 120-cps pulser previously mentioned. This system allowed the determination of the frequency of the tone emitted from the burner.

By varying the length of the burner, it was found possible to obtain several frequencies and so correlate the experimental with the theoretical frequency predicted by Eq 2* below:

$$\frac{\gamma_2}{c_2} \tan \frac{2\pi n x}{c_2} = \frac{\gamma_1}{c_1} \cot \frac{2\pi n(L-x)}{c_1} \quad (2)$$

where 1 refers to unburned gas
2 refers to burned gas
c = velocity of sound
L = combustion-chamber length
n = resonant frequency
x = distance from combustion-chamber exit
 γ = ratio of specific heats

These are shown in Table 2 on the following page. It was found that frequencies corresponding to overtones of the fundamental were excited when the burning was exceptionally rough; these are also shown in Table 2.

* See Jost³, page 105, for derivation of this equation. This expression is similar to that derived by Dunlap¹ except that different boundary conditions, owing to the burner being closed at one end, were imposed upon the solution to the wave equation.

TABLE 2

COMPARISON OF OBSERVED RESONANT FREQUENCY WITH THAT PREDICTED BY EQ 2

<u>Distance x, in.</u>	<u>Observed Frequency</u>	<u>Theoretical Frequency Fundamental</u>	<u>Theoretical First Overtone</u>	<u>Error, Per Cent</u>
8-1/2	408	310	810	31.5
	810	310	810	0.0
14	555	270	625	11.2
	562	270	625	10.1
24-3/4	216	208		3.9

Appendix II shows the method used for calculation of the fundamental and overtone frequencies.

5. An Examination of the Conditions Allowing Vibrations to be Maintained

It was found that burners of the types described in this report nearly always resonated, the flame fluctuating more in some cases than in others. It was also found that the use of a grid-type flame holder generally resulted in the flame oscillating more than when the rod or sphere flame holders were used. This and other observations* might lead to the conclusion that the resonance-forcing mechanism is tied up directly with the flame-holding mechanism (e.g., production and destruction of vortices behind the flame holder). Perusal of the literature showed, however, that the resonant condition (i.e., when a tone was emitted and the flame front oscillated) had been observed in a combustion chamber in which there was no flame holder.⁴ This result would seem to indicate that while the flame

* The location of the critical distance was described by Dunlap¹, page 27.

holder has an effect on the amplitude of resonance, it is not the factor governing the cause of resonance.

Probably the most important factor that would lead to a feedback mechanism is the dependence of flame speed upon pressure and temperature. It has been found by other experimenters that an increase in ambient pressure causes a decrease in flame speed and an increase in ambient temperature an increase in flame speed for a propane-air flame.* Such an effect is shown in Fig. 8.

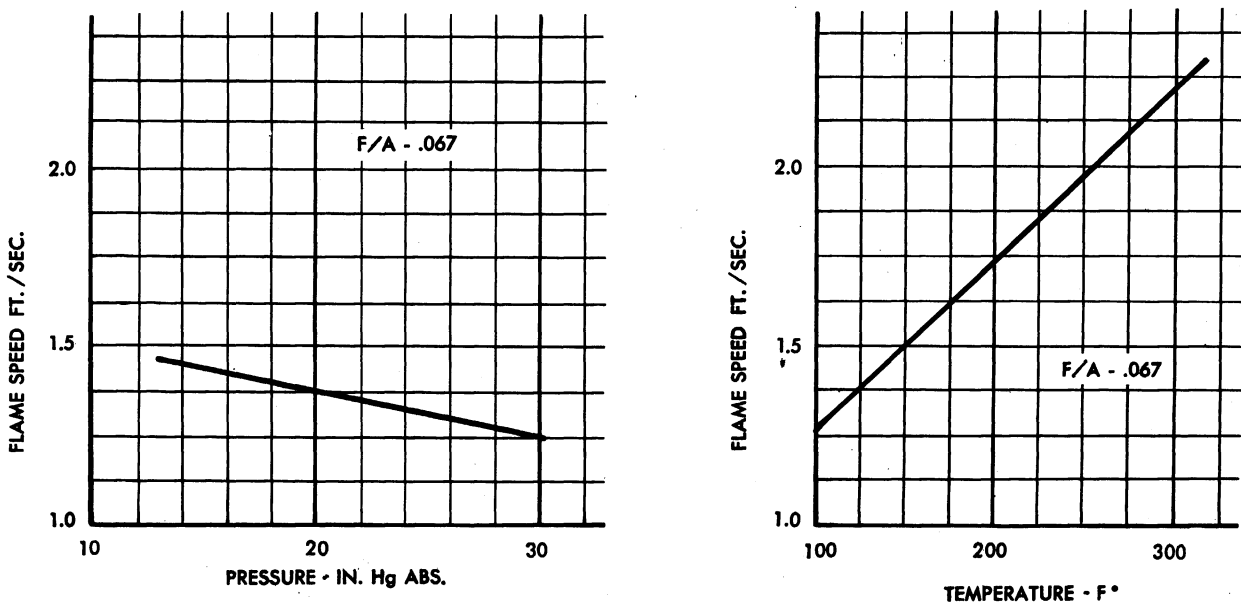


FIG. 8 EFFECT OF AMBIENT PRESSURE & TEMPERATURE ON THE FLAME SPEED OF A PROPANE-AIR FLAME

* Unpublished data obtained by R. E. Cullen, University of Michigan.

Standing sound waves in the combustion chamber produce a variation in pressure and temperature at the flame front with time. Due to the slope of the curves in Fig. 8 an increase in pressure and the associated isentropic rise in temperature will result in a net increase in flame speed. Since

$$q = Q_m = Q_p A V_f$$

and V_f and ρ increase with an increase in P , then an instantaneous increase in the heat release will result when the pressure is greater than ambient in the combustion chamber. In the same manner, the heat release will decrease when the pressure is less than ambient. That is, the heat release is increased at high pressure and decreased at low pressure. It is this condition that is cited by Rayleigh as being necessary for the maintenance of vibrations by heat.* Hence, resonance is apt to occur in any burner in which the flame is located in a region of varying pressure.

The above explanation also shows why the flame was not observed to resonate when it was placed at the extreme ends of the burner. That is, the pressure nodes, which theoretically occur at the ends, did not allow the flame speed to change and hence did not give a variation in the rate of heat release with time. Even at a small distance away from the ends, the pressure variation is so small and results in such a small unsteady heat release that any vibration initiated cannot be maintained.

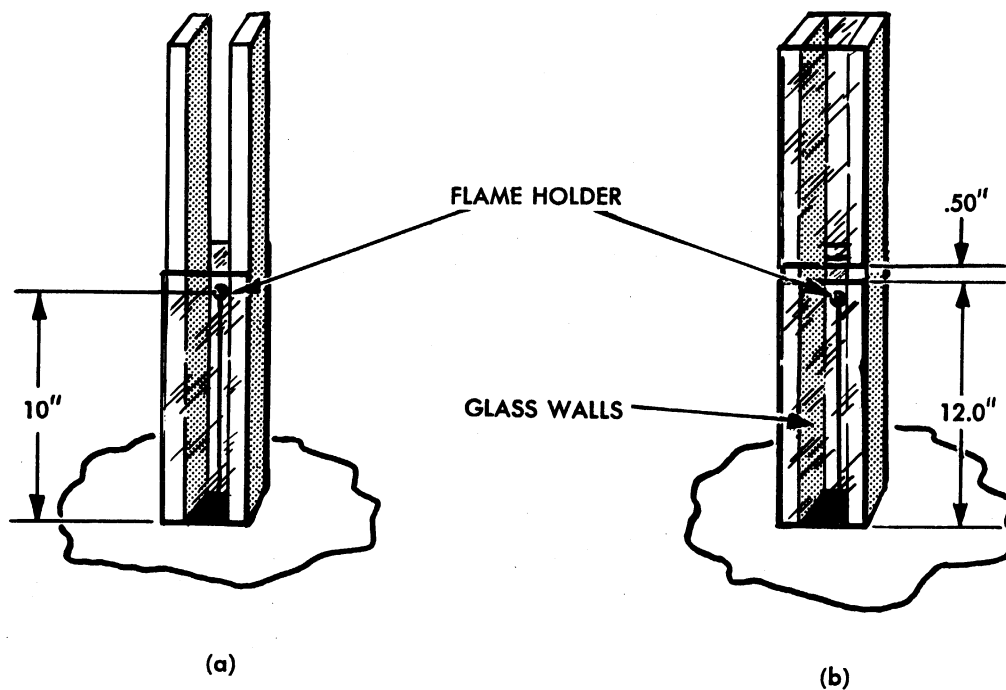
The question as to why the condition cited by Rayleigh is most amenable to maintenance of vibrations should be further investigated, and it is recommended that this be done. It can be concluded, however, that

* Rayleigh⁵, page 226.

the dependence of flame speed on pressure and temperature is probably the governing factor in the organ-pipe type of combustion-chamber resonance.

OTHER OBSERVATIONS

(1) A flame holder (1/4-in. sphere) was mounted 10 in. from the entrance of the 24-in. combustion chamber. With the upper glass walls removed (see Fig. 9), the flame blew off the flame holder at a jet velocity of 228 ft/sec. With the upper glass walls on but separated from the lower (see Fig. 9), the flame blew off at 60.5 ft/sec. With the glass walls joined so that all vents were closed, the flame blew off at 15 ft/sec. This experiment showed very graphically the effect of the resonant condition on blow-off velocity, or flame stability.



**FIG. 9 COMBUSTION CHAMBER CONFIGURATION FOR TESTING EFFECT
OF RESONANCE ON BLOW-OFF VELOCITY**

(2) A jacketed, perforated tail pipe (Fig. 10) was used in place of pipe A (Fig. 3) in an attempt to reduce the amplitude oscillation of the burned gases. By this scheme it was found possible to reduce the oscillation to some degree. Removal of the jacket eliminated the oscillation completely.

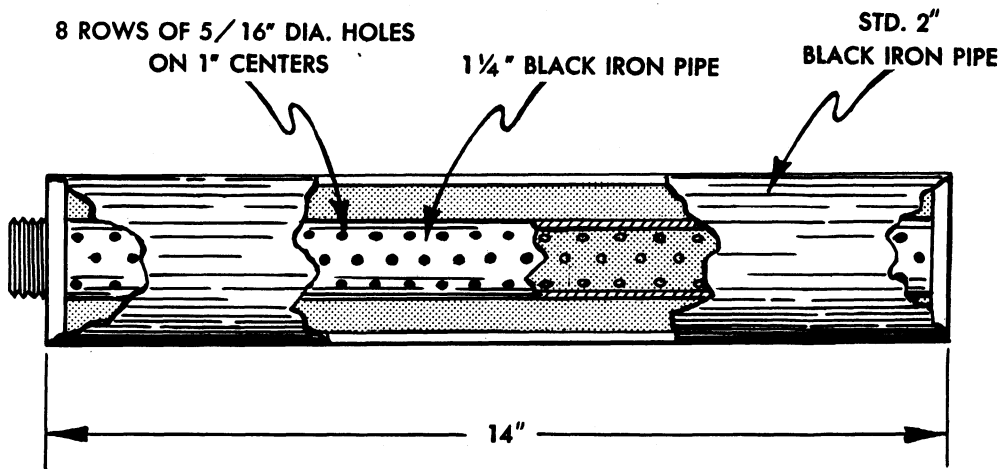


FIG. 10 PERFORATED TAIL-PIPE WITH JACKET USED WITH CHOKED INLET BURNER

APPENDIX I

CALCULATION OF THEORETICAL VELOCITY AMPLITUDE IN

A 1-IN. x 1-IN x 24-IN. BURNER

FROM OBSERVED PRESSURE AMPLITUDES

1. Derivation of the Equation

It can be shown¹ that the theoretical particle movement due to standing sound waves in a tube is given by

$$y = A \cos \frac{2\pi nx}{c_1} \cos 2\pi nt \quad (1a)$$

$$v = \text{particle velocity} = \frac{\partial y}{\partial t} = 2\pi nA \cos \frac{2\pi nx}{c_1} \sin 2\pi nt \quad (2a)$$

$$|v| = \text{particle velocity amplitude} = 2\pi nA \cos \frac{2\pi nx}{c_1}; \quad (3a)$$

$$\text{also, } P - P_0 = \text{excess pressure} = P_0 \gamma \frac{\partial y}{\partial x} = \bar{P},$$

$$\text{so } \bar{P} = P_0 \gamma \frac{2\pi nA}{c_1} \sin \frac{2\pi nx}{c_1} \cos 2\pi nt.$$

$$\text{Maximum excess pressure} = |\bar{P}| = \frac{2\pi nA}{c_1} \sin \frac{2\pi nx}{c_1} \quad (4a)$$

Then for Eqs 3a and 4a:

$$\begin{aligned} \frac{|\bar{P}|_a}{|v|_b} &= \frac{\text{Pressure amplitude measured at } x = a}{\text{Velocity amplitude measured at } x = b} \\ &= \frac{P_0 \gamma}{c_1} \frac{\sin (2\pi nx_a/c_1)}{\cos (2\pi nx_b/c_1)} = \rho c \frac{\sin (2\pi nx_a/c_1)}{\cos (2\pi nx_b/c_1)} \end{aligned}$$

2. Calculation

a) Case in which flame was observed to fluctuate about flame holder. Run No. 126.

$$\begin{aligned}x_a &= 0.541 \text{ ft} & c_1 &= 1120 \text{ ft/sec} \\x_b &= 1.193 \text{ ft} & n &= 375 \text{ cps} \\|\bar{P}|_a &= 0.8 \text{ psi} & \rho &= 0.073 \text{ lb/ft}^3\end{aligned}$$

$$\begin{aligned}|v|_b &= \frac{|\bar{P}|_a \cos(2\pi n x_b / c_1)}{\rho c \sin(2\pi n x_a / c_1)} = \frac{0.8 \cdot 144 \cdot 32.2 \cos(2\pi \cdot 375 \cdot 1.193 / 1120)}{0.073 \cdot 1120 \sin(2\pi \cdot 375 \cdot 0.541 / 1120)} \\&= 4.54 \frac{\cos 2.51}{\sin 1.14} = 45.4 \frac{0.809}{0.909} = 40.4 \text{ ft/sec}\end{aligned}$$

Mixture velocity was 16 ft/sec. Due to blockage of grid the velocity through the grid was 33.5 ft/sec. Since the velocity amplitude due to the standing wave was greater than the velocity through the grid, the flame should move upstream as it was observed to do.

b) Case in which flame was not observed to fluctuate about flame holder. Run No. 123.

$$\begin{aligned}x_a &= 0.541 \text{ ft} & c_1 &= 1120 \text{ ft/sec} \\x_b &= 1.55 \text{ ft} & n &= 300 \text{ cps} \\|\bar{P}|_a &= 0.37 \text{ psi} & \rho &= 0.073 \text{ lb/ft}^3\end{aligned}$$

$$|v|_b = \frac{|\bar{P}|_a \cos(2\pi n x_b / c_1)}{\rho c \sin(2\pi n x_a / c_1)} = \frac{0.37 \cdot 144 \cdot 32.2 \cos(2\pi \cdot 300 \cdot 1.55 / 1120)}{0.073 \cdot 1120 \sin(2\pi \cdot 300 \cdot 0.541 / 1120)}$$

$$= 21.0 \frac{\cos 2.61}{\sin 0.91} = 21 \frac{0.636}{0.789} = 17 \text{ ft/sec}$$

Again, the velocity through the grid was 33.5 ft/sec. Since the velocity amplitude due to the standing sound wave was, then, less than the velocity through the grid, the flame should not move upstream. This was observed to be so.

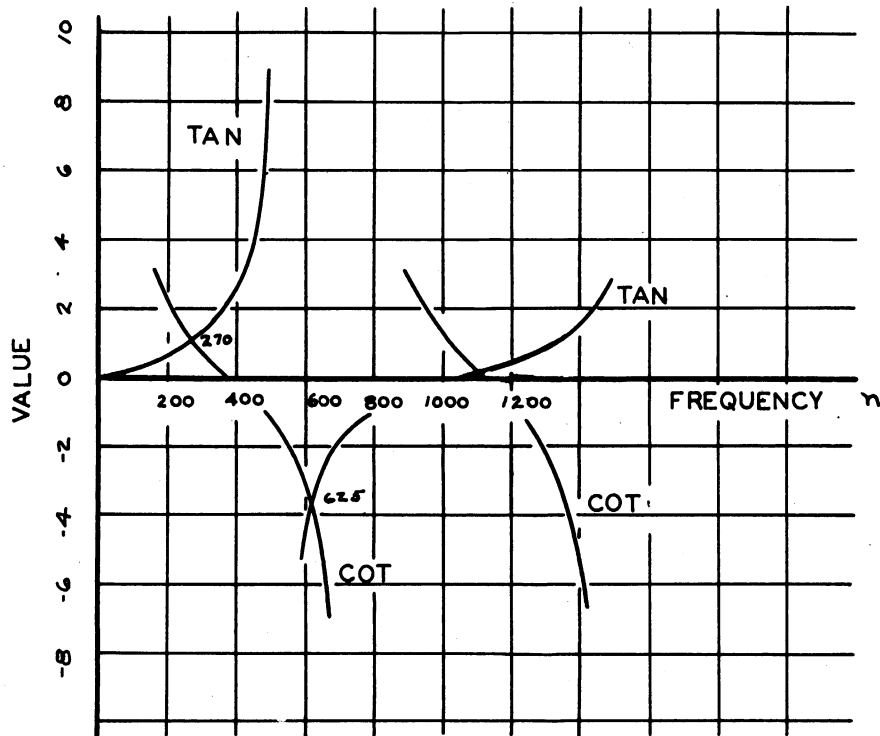


FIG. 11 PLOT OF TAN AND COT FUNCTIONS IN EQUATION 2

APPENDIX II

CALCULATION OF THE FUNDAMENTAL AND OVERTONE FREQUENCIES FROM EQ 2

Eq 2 can be most easily solved by plotting both the tangent and cotangent functions for a fixed value of x versus the frequency n .

$$\frac{\delta_2}{c_2} \tan \frac{2\pi n x}{c_2} = \frac{\delta_1}{c_1} \cot \frac{2\pi n (L-x)}{c_1} \quad (2)$$

The intersection of the two functions is, then, the solution to the equation. A plot of these functions for $x = 14$ in. is shown in Fig. 11. The first intersection of the curves gives the fundamental frequency, the second intersection the first overtone, etc.

APPENDIX III

DESCRIPTION OF SOUND PRESSURE MEASUREMENT SYSTEM AND ASSOCIATED EQUIPMENT

1. Block Diagram of Pressure Measurement System

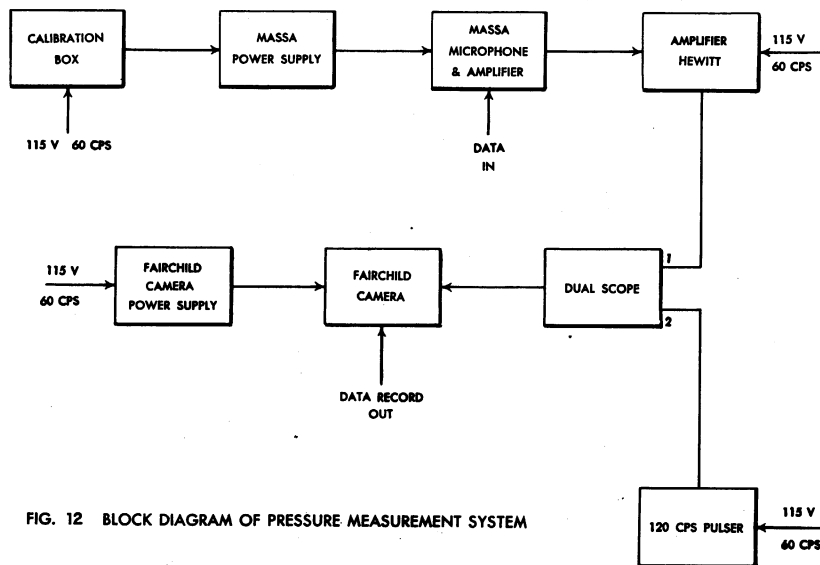


FIG. 12 BLOCK DIAGRAM OF PRESSURE MEASUREMENT SYSTEM

2. Data Concerning Components

- a) Massa sound pressure microphone system, Model GA-1007, made by Massa Laboratories, Inc., Cleveland

Microphone Model M-123 S, Serial No. 2 (employs ADP crystals)

Microphone diameter = $3/16$ in.

Pressure sensitivity = 4 microvolts/degree/cm²

Frequency response range flat from 50 cps to 250 KC

Microphone linear to pressure magnitudes of several million degrees/cm²

Preamplifier Model M-114B

Power Supply Model M-116D

b) Calibration Box

This calibration unit was built by UMERI Controls Group and has voltage outputs corresponding to 0.0001, 0.001, 0.01, 1.0, and 10.0 psi.

c) Dumont Dual-Beam Cathode Ray Oscilloscope, made by Dumont Laboratories, Inc., Passaic, N. J.

Type 279, Serial No. 159

d) Fairchild 35 mm Camera

The Fairchild camera is a special camera made for use with the above oscilloscope.

Type 314, Serial No. 113, Maximum film speed = 60 in./sec

e) 120-Cycle Pulser Unit

This unit, built by the UMERI Controls Group, gives voltage pulses of 120 cps. These pulses were put into the oscilloscope on one beam and recorded on the film.

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1/20/98 0900

1/24/ 11:00 a.m.