ENGINEERING RESEARCH INSTITUTE
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QUARTERLY PROGRESS REPORT NO. 2

June 1, 1954, to August 31, 1954

DEVELOPMENT OF GENERALIZED MATHEMATICAL PROCEDURES FOR OPTIMUM ASSEMBLY OF POTENTIALLY EFFECTIVE COMBAT CREWS

PAUL S. DWYER

Project 2226

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
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June 1, 1954 to August 31, 1954

Contract No.:	AF18(600)-1050
Budget Project No.:	670-193
Contract Title:	Development of Generalized Mathematical Procedures for Optimum Assembly of Potentially Effective Combat Crews
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of the University of Michigan
Monitoring Agency:	Director, Detachment 4 (Crew Research Laboratory), Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	June 1, 1954, to August 31, 1954

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics Consultant in Statistical Laboratory	Full time*
Hubbell, Charles H.	Assistant in Research	Full time**
Lott, Fred W.	Assistant in Research	Half time***
Parker, Kathryn	Secretary	varied****

* During the three summer months, June 13 to September 13, Dr. Dwyer was working full time on this project. Beginning September 13 he will work full time on University duties and his work on the project will be limited to 40 hours per month.

** Mr. Hubbell started working full time on the project on June 7. He is terminating his work on the project with the opening of University classes in September.

*** Mr. Lott started working on the project on June 7. He is terminating his work on the project with the opening of college classes in September.

**** Miss Parker began work on the contract on August 24, about one week before the close of the period covered by this report. She is working half-time during the month prior to the opening of classes. She will become inactive on the project with the opening of University classes on September 20, except for holidays, etc., when she may be available for typing project reports.

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RESEARCH PROGRESS

1. General

Substantial results, described below, have been obtained during this second quarter and initial drafts of several portions of the proposed final report have been made.

An important phase of the work of the quarter was the further orientation received by the principal investigator during his trip to the Crew Research Laboratory. This resulted not only in a better general understanding of the problem and approaches to the solution desired by the monitoring agency, but also to a more specific outline of topics to be studied.

2. Outline of Topics

The outline of topics, drawn up after conferences at the Crew Research Laboratory in mid-June, is designed to specify important aspects of the project which should be studied. The outline of topics follows:

Outline of Topics as of June 16, 1954

1. The general group assembly problem. Relation to personnel classification problem and similar problems in linear programming.

2. The distribution of all possible assemblies. The mean and variance of the distribution. The third and fourth moments of the distribution. Estimates of the maximum and minimum assembly sums.

3. Transformations. Subtraction of constants from rows, columns, and layers. Deviate transformations. Extreme transformations.

4. Approximate solutions. Measures of the adequacy of an approximation.

5. Mathematical models for group scores.

6. Application of analysis of variance. Determination of mathematical model appropriate to empirical data.

7. Condensation of the problem by grouping:
   a. Reduction, in effect, of the number of classes by grouping.
   b. Reduction, in effect, of the number of personnel categories by grouping.

8. The group assembly problem as a problem in linear programming. Possibility of a dual and generalized conditions of solution. Inapplicability of simplex method, method of optimal regions, and method of interchange to the general problem.

9. The two-dimensional assembly problem. Theoretical properties and methods of solution with empirical data.

10. Successive application of two-dimensional techniques.

11. The three-dimensional assembly problem. Extensive treatment featuring groupings, results for empirical data, different mathematical models, analysis of variance, deviate transformations, and extreme transformations.

12. The general assembly problem. Mathematical models and analysis of variance. Method of marginal zeros. Determination of extreme assemblies. Use of machines.

3. Work on Specific Topics

All the listed topics have received attention during the quarter and some have been studied extensively. The results are substantial on topics 1, 2, 3, 4, 6, 9, 11, and 12 and, of these, they are essentially complete on topics 2, 3, 6, and 9. Initial drafts of reports on topics 2, 3, and 6, and on certain aspects of topics 1, 8, 11, and 12 have been prepared, and a draft of a report on topic 9 will be prepared soon. A copy of each of these reports will be given to Dr. Roby so that suggestions from the monitoring agency may be incorporated in later revisions.

4. Results

The results can best be understood by an examination of the written reports. However, a few remarks may indicate something of their nature.

A. In topic 1, the problem of the different assemblies is identified with the problem of different permutation sets for general k as well as for k = 2. The formula for the number of assembly sums then follows easily.

B. Permutation sets are most useful in studying the moments of the distribution of all possible assemblies. Using them, we have developed the following:

(1) A formula for the standard deviation of all possible assembly sums for any k. (This includes the formula given by Votaw and Dailey when k = 2.)

(2) Formulas for the third central moment of all possible assembly sums when k = 2,3,4.

(3) A formula for the fourth central moment when k = 2.

Improved methods of calculation are outlined and illustrated.

C. The variance of the assembly sum is related to the calculational techniques used in the analysis of variance of a factorial problem. The formulas for standard deviation mentioned above are transformed to formulas involving the results of the analyses. Not only does this make possible the easy computation of the standard deviation from the results of well-known techniques, but also it makes possible the identification of those interactions which are making large or small contributions to the variation of the assembly sums.

D. Our research leads us to the conclusion that the use of transformations is a most effective means in reducing the problem both for k = 2 and for a larger k. In addition to the deviate transformation and the extreme transformation, an additional transformation has been found during this quarter which further simplifies the problem.

E. In every case, when k = 2, the successive use of these transformations reduces the problem to the simple one of selecting the solution from the zero terms of the reduced matrix.

F. When $k > 2$, it is not always possible to select from zero terms, but the additional transformations continue to simplify the problem. The present opinion is that this method is decidedly superior to any method, including the simplex, for problems with $k = 2$.

G. Punched-card methods have been worked out for finding the optimum solution from the matrix resulting from the transformations. These include the use of McBee cards for small problems and the use of IBM machines for problems of greater magnitude. Preliminary investigation indicates that electronic digital computers, such as MIDAC, can make this selection efficiently but a more thorough study of this will follow, along with studies of other topics not yet completed.

5. Proposed Outline for Final Report

It seems logical to use the outline of topics as the basis of a general report to cover all phases of the project. However, our results so far indicate that an order of presentation which differs somewhat from the outline of topics above is preferred. Accordingly there is proposed a tentative outline of the material for the general report. This outline should not be considered as final, since additional research may indicate an alteration. The proposed outline indicates only the nature and order of the presentation and does not, at this stage, organize the material within chapters.

A single asterisk after the chapter number indicates that the work on the chapter is nearing completion while a double asterisk indicates that a draft of the chapter has been completed.

Proposed Outline of Final Report by Chapters

Chapter	Contents
I*	The general group assembly problem Relation to personnel classification and similar problems Notation Use of permutation sets
II**	Transformations Subtraction of constants Deviate transformations Extreme transformations

Chapter	Contents
III**	The distribution of all possible assembly sums The mean and variance of the distribution for any k The third central moment for k = 2, 3, 4 The fourth central moment for k = 2 (Proofs to be supplied in Appendices)
IV**	Application of techniques of analysis of variance to the determination of σ_T for different values of k and n The determination of mathematical models appropriate to empirical data
V	Mathematical models for group scores
VI	Condensation by grouping Reduction, in effect, of the number of classes by grouping
VII	The group assembly problem as a problem in linear programming Possibility of a dual and generalized conditions of solution Extent of applicability of simplex method, method of bounding sets, method of interchange, and method of optimal regions to the general problem
VIII*	The two-dimensional assembly problem Use of extreme transformation followed by marginal zero transformations Determination of the solution from the reduced matrix
IX	Successive application of two-dimensional techniques
X	The three dimensional problem Use of extreme transformations followed by marginal zero transformations Determination of the solution from the reduced matrix
XI*	The general assembly problem Treatment similar to that of Chapter X except not so much detail