ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 3

September 1, 1954, to November 30, 1954

DEVELOPMENT OF GENERALIZED MATHEMATICAL PROCEDURES FOR OPTIMUM ASSEMBLY OF POTENTIALLY EFFECTIVE COMBAT CREWS

PAUL S. DWYER

Project 2226

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF18(600)-1050

December, 1954

No. 3

QUARTERLY PROGRESS REPORT NO. 3

September 1, 1954, to November 30, 1954

Contract No.:	AF 18(600)-1050
Budget Project No.:	670-193
Contract Title:	Development of Generalized Mathematical Procedures for Optimum Assembly of Potentially Effective Combat Crews
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of the University of Michigan
Monitoring Agency:	Director, Detachment 4 (Crew Research Laboratory), Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	September 1, 1954, to November 30, 1954

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Laboratory	[1]
Hubbell, Charles H.	Assistant in Research	Full time[2]
Lott, Fred W.	Assistant in Research	Half time[3]
Rider, Leonard	Assistant in Research	Half time[4]
Taylor, Patricia	Assistant in Research	Half time[5]
Parker, Kathryn	Secretary	Varied[6]

[1]During the summer months, June 13 to September 13, Dr. Dwyer worked full time on the project. Beginning with September 13, he worked full time on his University duties and his work on the project was limited to 40 hours per month.

[2]Mr. Hubbell terminated his work on the project with the opening of University classes in September.

[3]Mr. Lott terminated his work on the project with the opening of classes in September.

[4]Mr. Rider started work on the project September 18, 1954.

[5]Miss Taylor started work on the project September 23, 1954.

[6]Miss Parker worked half time on the project for about a month prior to the opening of classes in September. She has not worked on the project since September.

DEVELOPMENT OF GENERALIZED MATHEMATICAL PROCEDURES FOR OPTIMUM ASSEMBLY OF POTENTIALLY EFFECTIVE COMBAT CREWS

RESEARCH PROGRESS

1. General

Substantial results, described below, have been obtained during this third quarter and initial drafts of three chapters of the proposed detailed report have been made.

An important aspect of the work of the quarter resulted from the conference with Dr. Roby in New York City in early September. As a result of this conference, at which there was an opportunity to examine the results of the work of the second quarter and to plan for the future work of the project, agreement was reached as to future plans. The general outline of topics drawn up at the conference at the Crew Research Laboratory in mid-June was reaffirmed. It was indicated that the official final report, specified in the contract, should be a nontechnical report of not more than 20 to 35 pages indicating the main results of the research. Since the results of the contract study are quite extensive, it seemed wise that they be made available in an additional detailed report not specified in the contract. This detailed report, which is hereafter known as the extended report, is taking the form of a treatise of some fourteen chapters and several appendices.

The plan is that a single copy of this extended report is to be placed in the hands of Dr. Roby at the terminus of the contract. Additional copies of this extended report, if they are needed, will be the responsibility of the Crew Research Laboratory. This extended report will contain the detailed methods with appropriate derivations and extensive illustrative material. The official report, specified in the contract, will be in the nature of a shorter and less technical version of the material in the extended report.

2. Outline of Topics

The outline of topics, drawn up at the conference in mid-June and reaffirmed at the conference in September, is next presented. This outline of topics, which is repeated from the report of the second quarter, is presented here because it serves as the basis of the work on the contract and the material which will appear in the official report and the extended report. Reference is made in the next section of this quarterly report to some of the numbered items of this outline of topics, which follow:

1. The general group assembly problem. Relation to personnel classification problem and similar problems in linear programming.

2. The distribution of all possible assemblies. The mean and variance of the distribution. The third and fourth moments of the distribution. Estimates of the maximum and minimum assembly sums.

3. Transformations. Subtraction of constants from rows, columns, and layers. Deviate transformations. Extreme transformations.

4. Approximate solutions. Measures of the adequacy of an approximation.

5. Mathematical models for group scores.

6. Application of analysis of variance. Determination of mathematical model appropriate to empirical data.

7. Condensation of the problem by grouping:
   a. Reduction, in effect, of the number of classes by grouping.
   b. Reduction, in effect, of the number of personnel categories by grouping.

8. The group assembly problem as a problem in linear programming. Possibility of a dual and generalized conditions of solution. Inapplicability of simplex method, method of optimal regions, and method of interchange to the general problem.

9. The two-dimensional assembly problem. Theoretical properties and methods of solution with empirical data.

10. Successive application of two-dimensional techniques.

11. The three-dimensional assembly problem. Extensive treatment featuring groupings, results for empirical data, different mathematical models, analysis of variance, deviate transformations, and extreme transformations.

12. The general assembly problem. Mathematical models and analysis of variance. Method of marginal zeros. Determination of extreme assemblies. Use of machines.

3. Work on Specific Topics

The major attention during the third quarter was directed to the actual solution of the assembly problem when k=2, k=3, and k>3 with the values of $G_{ijh\ldots}$ known. Certain approximate solutions were also devised so major work on topics 4, 9, 10, 11, and 12 was accomplished during the third quarter of the contract. Initial drafts for the extended report on topics 9, 10, and 11 have been prepared. These will be typed soon and a copy will be given to Dr. Roby so that suggestions from the monitoring agency may be incorporated in a later revision.

4. Results

The results of the work of this quarter can best be understood by an examination of the initial drafts of the chapters of the extended report. However, a few remarks may indicate something of the nature of the new results.

A. In topic 9, the use of the method of marginal zeros has been perfected and extended. Different types of marginal zero transformations have been discovered. These transformations have led to certain matrices known as the reduced matrix and the completely reduced matrix. Special consideration has been given to the general problem expressed in frequency form. For this purpose transformations have been introduced which lead routinely to the reduced grouped matrix and then to the completely reduced grouped matrix from which the solution can be obtained. We feel confident that this method of marginal zero transformations will eventually replace the simplex method and other methods of handling grouped k=2 problems, with the exception of the detailed method of optimal regions applied to results of the row deviate transformation which seems most efficient in handling quota form problems with a small number of job categories. A brief nontechnical presentation, with illustrations, of the method of marginal zeros, when k=2, was given at the Invitational Conference of the Educational Testing Service at New York City on October 30 under the title "Multiple Assignments of Men to Jobs." A copy of the manuscript for this talk, which might be considered as a supplement to this quarterly report, was sent to the Crew Research Laboratory early in October.

B. The work on topic 10 has been completed and is written in the proposed Chapter IX of the extended report. We have found the successive application of two-dimensional techniques useful not only in problems in which the statement of the problem demands the successive use of these techniques, but also in obtaining approximate answers to general problems. We have found a way of integrating these methods with the results of the analysis of variance worked out during the second quarter of the contract. In this way a selection can be made of the particular orders in which the successive two-dimensional problems can be applied most successfully.

C. A major portion of the work of the period was spent on topics 11 and 12. The work on these topics, which we consider to be the very essence of the work on the contract, is almost complete and an initial draft of an extensive Chapter X of the extended report has been prepared. Considerable attention has been given to the study of the general problem with grouped data and routine techniques have been worked out for transforming the grouped matrix to the reduced grouped matrix. It is not always possible to transform further to a completely grouped matrix but we have been able to obtain the solutions with the additional use of systems of simultaneous equations. We are quite pleased with our results in this area though we feel that the methods can be perfected with further study.

5. Proposed Outline for the Extended Report

The fourteen chapters of the extended report are indicated in the outline following. This is a revision of the outline appearing in the second quarterly report. The outline is given in more detail for these chapters which have been written.

The extent of the work on each chapter is indicated. A single asterisk indicates that some work has been done on the subject matter of the chapter. A double asterisk indicates that considerable work is completed, while a triple asterisk indicates that at least a preliminary draft of the chapter is finished.

Chapter	Contents
I**	The general group assembly problem 1. Relation to personnel classification and similar problems 2. Notation 3. Use of permutation sets