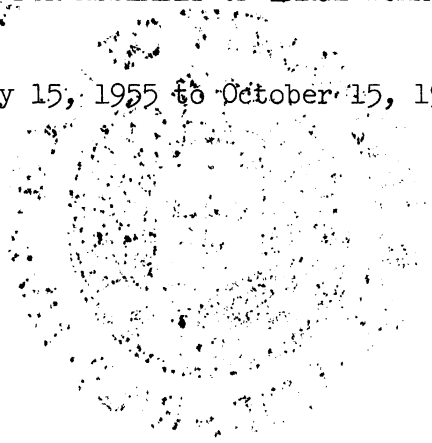
ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 1

DEVELOPMENT OF MATHEMATICAL PROCEDURES AND MULTIPLE CRITERIA FOR ASSEMBLY OF LARGE WORK GROUPS

July 15, 1955 to October 15, 1955



Paul S. Dwyer

Project 2413

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 41(657)-9

October 1955

Contract No.:	AF 41(657)-9
Budget Project No.:	7-7713
Contract Title:	Development of Mathematical Procedures and Multiple Criteria for Assembly of Large Work Groups
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of the University of Michigan
Monitoring Agency:	Director, Crew Research Laboratory, Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	July 15, 1955 to October 15, 1955

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Research Laboratory	Up to 40 hours per month[1]
Graves, Patricia (Taylor, Patricia)	Assistant in Research	Half time[2]
Graves, Glenn	Assistant in Research	Varied[3]

[1]During this quarter, Dr. Dwyer worked full time on his University duties and his work on the project was limited to 40 hours per month.

[2]Mrs. Graves began working on the project July 15, 1955.

[3]Mr. Graves began working on the project July 15, 1955, and terminated September 25, 1955, with the opening of classes at the University. He assisted in translating some of the methods to routines which can be performed on MIDAC. He worked a total of 31 hours on the project.

ABSTRACT

This report provides information about the progress of the work on the project during its first quarter. It presents:

1. A statement of the general objectives as indicated in the specifications of the contract and as amplified during the July conference with Dr. Roby;

2. A statement of the objectives of the work of the quarter;

3. A discussion of the procedure used in carrying out the work of the quarter;

4. A statement of the general results obtained during the quarter;

5. A general discussion of the work on the contract to date with plans for the next quarter;

6. A summary statement.

I. OBJECTIVES

The general objectives of the work on this contract are the development of mathematical procedures for assembling individuals in large work units and employing multiple criteria for assembly. More specific objectives indicated in the contract include:

1. The extension of results obtained for 3- to 5-man group assemblies so as to secure the optimal assignment of individuals to groups of larger sizes;

2. The development of economical means of accomplishing data transformations and practical simplifications of the problem encountered in reducing grouped matrices;

3. The translation of these procedures into programs suitable for use with electronic digital computers;

4. The use of appropriate approximate solutions when the criteria for assembly are based on fallible scores and consideration should be given to errors in group scores;

5. The feasibility of simultaneously employing multiple criteria of classification such as may be desirable if both technical qualifications and social factors are to be considered in forming work groups.

Additional specific objectives were agreed on during the July conference with Dr. Roby. These include:

6. A revision of substantial portions of the 14-chapter report of the previous contract, necessitated by the development of improved techniques and theory in the later periods of the contract which were not incorporated in the draft presented;

7. Additional work directed toward the further analysis and improvement of the reduced grouped matrix transformations;

8. The identification of the basic mathematical problem in group assembly with the basic mathematical problem of other applied problems such as the general transportation problem;

9. The determination of the extent of the relationship existing between the group-assembly problem and other related problems in linear programming;

10. An investigation of the effects of courser groupings.

Objective 7 is closely related to Objectives 1 and 2.

II. OBJECTIVES OF THE WORK OF THIS QUARTER

The specific objectives for the work of this quarter were:

1. The revision of Chapters I-VII of the extended report of the previous contract;

2. The development of improved methods for discovering the amount of the inconsistency of the equations of condition at each stage and the automatic determination of the next transformation therefrom;

3. The identification of the group-assembly problem with several generalizations of the transportation problem and the adaptation of the assembly-problem techniques for these transportation problems;

4. The further mechanization of the solution in the direction of its applicability to electronic digital computers.

III. PROCEDURE

The procedure used during this period may be indicated by the following facts:

1. A conference with Dr. Roby was held in July. This conference was for the purpose of giving the principal investigator orientation with reference to the need of the Crew Research Laboratory for results of the contract and of listing more detailed objectives of the work.

2. As indicated on the personnel page, the services of Patricia Graves (formerly Taylor) were secured. She was assigned the task of assisting with the study of Objectives II.1, II.2, and II.3.

3. The services of Glenn Graves were secured on a part time basis for assisting in the study of II.4.

4. Arrangements were made to have runs made on MIDAC. The bulk of the work of the quarter was done at the Statistical Research Laboratory, using equipment available there.

IV. RESULTS

The chief results of the work of the quarter are summarized below. The main topics in this summary of results correspond to the objectives of the work of this quarter, stated in Section II.

1. One major task undertaken during this quarter was a thorough revision of the material in Chapters I-VII of the extended report of the earlier project. In most cases, this revision consisted of a close examination of the content and form of this material, with a view to the elimination of errors, but in some cases there was complete revision of pages and sections. The amount of revision of this material was not extensive, since many of these chapters were written during the later stages of the earlier project. More extensive revision will be needed to incorporate the improved theory and techniques into the material of Chapters VIII-XI.

2. Another major task, and probably the most important one undertaken during the quarter, was the study and improvement of the techniques dealing with the reduced grouped matrix transformations. Though we were able to solve all problems previously, the routines demanded the application of intelligence at certain places. We have now improved and mechanized these routines (beginning with the reduced grouped matrix) so that they seem to be fully mechanized, and there is reason to expect that they can be adapted, without too many adjustments, to the use of electronic digital computers. In any case, the method is now more completely formalized. In general, the improvements are not yet written up, but we plan to incorporate them in the revision of Chapters VIII-XI of the extended report. However, certain drafts of the method as applied to the two-dimensional case have been written up and sent to several scientists who are experts in this area. Several comments have been made by these experts. As a result, we feel that the presentation of the revised Chapters I-XI will be nearly in its final form.

3. Another task undertaken during the quarter was the identification of generalized transportation problems with the mathematical problem of group assembly and the demonstration of the applicability of the general method of reduced matrices in solving alternative generalizations of the transportation problem.

The Hitchcock transportation problem, in which x_{ij} of the a_i unit packages at origin i are to be transported to destination j to be a part of the $x_{ij} < b_j$ packages to be delivered at destination j, calls for the determination of integral x_{ij} such that $\sum x_{ij}c_{ij}$ is to be minimized if c_{ij} is the cost of transporting one package from origin i to destination j. Thus, the transportation problem, as well as the personnel classification problem, is essentially a two-dimensional form of the general mathematical group-assembly problem, which is interpreted to include either maximization or minimization.

The transportation problem is immediately extendable to three dimensions. Suppose each package arrives at its destination only after passing through some distribution point, h. We denote the number of packages at the origin i by f_i, the capacity of the distribution point h by f_h, the number to be delivered to destination j by f_j, and the number of packages going from origin i to destination j through distribution point h to be x_{ijh}. The cost of delivery of a unit package for this route is c_{ijh}. It is desired to select the integral values of x_{ijh} so as to minimize the total cost $\sum x_{ijh}c_{ijh}$. The problem is thus mathematically the same as that of group assembly, $k = 3$, except that minimization of the linear form, rather than maximization of the linear form, is required. Thus the method of reduced matrices is immediately applicable to the three-dimensional problem. Similarly, this method is applicable to the generalization of the transportation problem in higher dimensions.

Other generalizations of the transportation problem, where $k = 3$, have been introduced. These involve different sets of specifications for the equations of condition. Schell has given a discussion of the nature of three of these sets of condition (in addition to the generalization described above), where $k = 3$, in his paper, "Distribution of a Product of Several Properties," which was presented to the Linear Programming Symposium sponsored by the National Bureau of Standards and the U.S. Air Force, at the Bureau of Standards, January 29, 1955. We have taken each of these generalized problems and have developed modifications of the method of reduced matrices for handling it. An important feature in the solution of each of these problems is the preliminary reduction to, and the use of, the completely reduced matrix for the general transportation problem (Case IV of Schell), which features f_i, f_j, and f_h. A report will be prepared on the use of the method of reduced matrices in solving these variations of the general transportation problem.

4. Another task which received some attention during the quarter was the continuation of the development of techniques suitable for automatic computation. In particular, attention was given to the automatic calculation on MIDAC of reduced grouped matrices for k = 2 problems with frequencies. As a result of the work, the programming was perfected so that we were able to obtain a reduced grouped matrix for the problem of Table 8.8 in 75 seconds. The completion of the solution resulted from the hand application of two reduced grouped matrix transformations.

V. DISCUSSION

The work on the contract is thus leading toward the desired goals. An improved theoretical basis of the reduced grouped matrix transformations was necessary before these could be completely mechanized. It now seems that a good basis has been made for this mechanization, so there will be considerable emphasis during the next two quarters on the solution of the various problems with electronic digital computers. Simultaneously, the results now available in the development of reduced grouped matrix transformations indicate a revision of Chapters VIII-XI of the extended report of the previous contract so as to incorporate the improved methods of reduced matrices as applied to general problems.

The development of automatic methods for the reduction and solution of the general problems, scheduled for the next two quarters, should be of great help in the study of the objectives involving suitable treatment of fallible scores and multiple criteria, scheduled for the last two quarters of the project.

VI. SUMMARY STATEMENT

It thus appears that the work of the quarter has accomplished the objectives for the quarter and has provided a suitable basis for the continuing work on the project.