

ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 3

DEVELOPMENT OF MATHEMATICAL PROCEDURES AND MULTIPLE
CRITERIA FOR ASSEMBLY OF LARGE WORK GROUPS

January 16, 1956 — April 15, 1956

Paul S. Dwyer

Project 2413

U. S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
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Contract No.: AF 41(657)-9

Budget Project No.: 7-7713

Contract Title: Development of Mathematical Procedures and Multiple
Criteria for Assembly of Large Work Groups

Issuing Office: The Air Research and Development Command

Contractor: The Regents of The University of Michigan

Monitoring Agency: Director, Crew Research Laboratory, Air Force Per-
sonnel and Training Research Center, Randolph Field,
Texas

Principal Investigator: Dr. Paul S. Dwyer

Period: January 16, 1956 to April 15, 1956

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Research Laboratory	Up to 40 hours per month ¹
Graves, Patricia	Assistant in Research	Half time
Galler, Bernard A.	Instructor in Mathematics, Mathematics Consultant	Up to 40 hours per month ²

¹During this quarter, Dr. Dwyer worked full time on his university duties and his work on the project was limited to 40 hours per month.

²During this quarter, Dr. Galler worked full time on his university duties and his work on the project was limited to 40 hours per month.

ABSTRACT

This report provides information about the progress of the work on the project during its third quarter. It presents:

1. A statement of the general objectives as indicated in the specifications of the contract and as amplified during the July conference with Dr. Roby;
2. A statement of the objectives of the work of this quarter;
3. A discussion of the procedure used in carrying out the work of the quarter;
4. A statement of the general results obtained during the quarter;
5. A general discussion of the work on the contract to date with plans for the remaining term of the contract;
6. A summary statement.

I. OBJECTIVES

The general objectives of the work on this contract are the development of mathematical procedures for assembling individuals in large work units and employing multiple criteria for assembly. More specific objectives indicated in the contract include:

1. The extension of results obtained for 3- to 5-man group assemblies so as to secure the optimal assignment of individuals to groups of larger sizes;

2. The development of economical means of accomplishing data transformations and practical simplifications of the problem encountered in reducing grouped matrices;

3. The translation of these procedures into programs suitable for use with electronic digital computers;

4. The use of appropriate approximate solutions when the criteria for assembly are based on fallible scores and consideration should be given to errors in group scores;

5. The feasibility of simultaneously employing multiple criteria of classification such as may be desirable if both technical qualifications and social factors are to be considered in forming work groups.

Additional specific objectives were agreed on during the July conference with Dr. Roby. These include:

6. A revision of substantial portions of the 14-chapter report of the previous contract, necessitated by the development of improved techniques and theory in the later periods of the contract which were not incorporated in the draft presented;

7. Additional work directed toward the further analysis and improvement of the reduced grouped matrix transformations;

8. The identification of the basic mathematical problem in group assembly with the basic mathematical problem of other applied problems such as the general transportation problem;

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9. The determination of the extent of the relationship existing between the group-assembly problem and other related problems in linear programming;

10. An investigation of the effects of coarser groupings.

Objective 7 is closely related to objectives 1 and 2.

II. OBJECTIVES OF THE WORK OF THIS QUARTER

In general, the objectives of the work of this quarter were related to general objectives 1, 2, 3, 6, and 7 stated above. Since general objective 1 must be attained largely through the more specific techniques resulting from the attempt to meet objectives 2, 3, 6, and 7, the work of this quarter was closely associated with these latter objectives. More explicitly, the objectives of the work of this quarter, in the order of time spent (with the exception of 5), were:

1. Preparation of papers describing the results to date together with the selection and solution of problems suitable for purposes of illustration;

2. The continued improvement of the basic mathematical theory needed for the method of reduced matrices;

3. The development of economical means of accomplishing data transformations and practical simplifications of the problem encountered in reducing grouped matrices;

4. The translation of these procedures to programs suitable for use with electronic digital computers;

5. A revision of additional chapters of the 14-chapter report of the previous contract.

III. PROCEDURE

The procedure during this period may be indicated by the following facts:

1. The work on objectives 1, 2, 6, and 7 was carried on by Mrs. Graves and the project supervisor. They not only worked on the continued development of the improved theory, but they used the equipment in the Statistical Research

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Laboratory in translating this improved theory into suitable practical techniques. As indicated below, a major effort during this quarter was the selection of material and illustrations and the preparation and editing of the three Research Reports which describe the results of the first three quarters of the project.

2. Dr. Galler has continued to work on the translation of the procedures into techniques suitable for electronic digital computers. Considerable progress has been made in programming the general method of reduced matrices for MIDAC. He has also programmed an approximate method using deviates, $k \leq 7$, on the IBM 650 which was installed in the Statistical Research Laboratory last month.

IV. RESULTS

The chief results of this quarter are summarized below. Since, for the most part, these results are described in much more detail in the Research Reports, they are only mentioned here.

1. The major effort of this quarter was the preparation of the three Research Reports. The preparation of the first of these is a major effort and contains a description of the improved method of reduced matrices as it now stands. Though minor changes, and perhaps additions, may be indicated before the end of the work on the project, we feel that the statement of the method, as described in this report, is now well established.

Research Report No. 2, in which the general method is adapted to hand computation, is not as fundamental as Research Report No. 1. However, in addition to translating the general theory into specific numerical techniques, it presents the general theory in which the inconsistent linear forms at one iteration are determined from the inconsistent linear forms of the previous iteration without the necessity of applying the equations of condition formally.

The third report deals with the adaptations of the methods to electronic computers. This is essentially a preliminary report since the bulk of the work on this aspect of the contract will be done during the summer when both Dr. Galler and the project supervisor will be working full time.

Also, the informal distribution of the December, 1955, draft of the paper "The Solution of the Hitchcock Transportation Problem with the Method of Reduced Matrices" has led to an invitation from the editor of the Naval Research Logistics Quarterly to submit the manuscript for publication.

2. The improvement in the mathematical theory, leading to more economical

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means of performing the data transformations, has continued. In particular, the compact presentation of the determination of the inconsistencies, described in Research Report No. 2, is a major improvement, especially for hand calculation.

3. The application of the general method to problems to be worked by hand has continued. These results are described and illustrated in Research Report No. 2.

4. Much progress has been made, though this aspect of the project is not yet near completion, in adapting the general method of reduced matrices to machine calculation. The use of the homogeneous transpose system of equations, described in Research Report No. 3, has been of great help in this. Also, the approximate solution based on deviates, obtained with the IBM 650 and described in Research Report No. 3, seems very satisfactory.

5. During the quarter we also completed the revision of the extended report of the previous project and mailed the material for Chapters XI, XII, XIII, and XIV to Dr. Roby.

V. DISCUSSION

The results of this quarter, like those of the previous ones, seem to lead to the desired goals. On the theoretical side, we have found improvements in the method which have been beyond our earlier expectations and which seem to make possible solutions by hand of problems of reasonable size in a reasonable length of time. The solution by machine methods is progressing and we expect to be able to make effective demonstrations of the power of the method of reduced matrices during the summer months, when we can work full time on the project. This can be most effective if the use of a machine such as UNIVAC or the IBM 704 can be made available to us.

The work of the remaining five months of the contract will feature

1. The continued development of machine methods;
2. The study of general objective 4;
3. The study of general objective 5;
4. Further study on the topics indicated by objectives 8, 9, and 10.

VI. SUMMARY STATEMENT

In the main, the objectives set for the first three quarters have been obtained as is shown by the Research Reports. The state of the general theory is now very satisfactory, though additional study may lead to additional improvements. The machine solution is not quite as far developed as we would like, but there is every indication that the method is well adapted to machine calculation and that the machine solutions will be very satisfactory. All in all, we feel that the work on the project is progressing in a satisfactory manner.

