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ELECTRICAL WIND PHENOMENA

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By

Harold C. Early
Haldon L. Smith
Daniel C. Lu

Approved by: W.G. Dow
W. G. DOW

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PERSONNEL

William G. Dow	Professor of Electrical Engineering	Supervisor
Harold C. Early	Project Engineer	1/2 time
Haldon L. Smith	Research Associate, student	3/4 time
Daniel C. Lu	Research Assistant, student	1/3 time
Edmund Kayser	Machinist and Technician	1/2 time
Priscilla L. Woodhead	Technical Illustrator, student	
Lynn R. Reeder	Secretary	

ABSTRACT

An electrical wind can be generated in an ionized gas by means of several different principles. Three of the most effective of these principles are: (1) The "motor" force exerted on an ionized gas which is conducting current transverse to a magnetic field; (2) electric field acceleration of ions in a region in which space charge is reduced by the presence of magnetically trapped electrons, as in a high-density ion source; and (3) the force due to thermal expansion when a gas is heated to an extreme temperature by an electrical discharge.

The laboratory work described in this report has been concerned primarily with the "motor" force produced by an electrical discharge in "crossed" electric and magnetic fields. This experimentation has involved power levels of five to ten kilowatts, and magnetic field strengths of three thousand to four thousand gauss. In most cases the gas pressure was in the range of five to one hundred microns (0.005 to 0.1 mm of Hg).

It is important, for the purpose of this investigation, to be able to determine the temperature and velocity of a stream of high-temperature ionized gas. A substantial part of the present work has been devoted to evaluating existing methods of instrumentation, and in devising new techniques of measurement. Significant progress in this direction has been made, but a great deal remains to be done.

ELECTRICAL WIND PHENOMENA

INTRODUCTION

There are various methods by which a very high velocity wind may be produced at low gas densities through the agency of an electrical discharge. An investigation of certain aspects of this phenomena is being conducted as a University of Michigan research project sponsored by the Office of Ordnance Research. This project is administered through the Engineering Research Institute of the University, and utilizes the facilities of the Department of Electrical Engineering.

The study of wind effects has involved experiments with gas discharges at unusually high electric and magnetic field strengths and power levels, and as a result, heretofore unreported phenomena have been observed. Electrically-produced forces in a gas are of significance from the standpoint of basic science, since they introduce new analytical relationships among fundamental variables. A study of the origin of pressure gradients relates the densities, velocities, and mobilities of ions and electrons which are moving through a plasma. The measurement and the interpretation of these effects promise to establish a new tool for obtaining fundamental information regarding charged-particle behavior.

This investigation is an outgrowth of a series of experiments made at the University of Michigan in the early part of 1949, in which a magnetically-driven arc was caused to rotate inside a cylinder and thereby generate a wind (Appendix III). These preliminary tests were made at gas pressures above five hundred microns (0.5 mm of Hg). The recent experiments in connection with the present O.O.R. contract have been conducted at pressures below one hundred microns. At these lower pressures the wind velocity associated with a given amount of electrical force is significantly increased, and certain aspects of the force-producing mechanism can thereby be studied more advantageously.

The first section of this report consists of a survey of a number of different principles by which directed momentum is imparted to the neutral molecules in an electrical discharge. This survey is primarily theoretical, but a limited amount of empirical verification has been performed. It is believed that this survey is reasonably complete; however, if any important force-producing mechanism has been omitted, the authors of this report would appreciate having it brought to their attention.

The second part of this report is concerned with the investigation of the wind effects associated with a low-density discharge in a strong transverse magnetic field. The choice of this particular objective for the experimental program was a difficult decision, since several of the alternatives mentioned in the survey looked equally attractive as topics for laboratory study.

The analytical interpretation of a new type of physical phenomenon requires an adequate amount of quantitative data. In the present instance, data of sufficient accuracy have been very difficult to obtain, so that considerable emphasis has been given to problems of instrumentation. A large part of the experimental work during the past year has been concerned with the problem of measuring the velocity and temperature of a low-density high-temperature stream of ionized gas. Encouraging progress has been made recently in the development of new measurement techniques. In the course of further work, information will become available which will provide a basis for a more analytical study than is now possible.

SURVEY OF MECHANISMS FOR PRODUCING WIND IN AN IONIZED GAS

There are a number of different principles by which directed momentum can be imparted to the neutral molecules in a gas discharge. These principles can be classified according to three basic methods by which the force or pressure gradient is produced. The three types of forces are: electrostatic force on charged particles in an electric field; magnetic force associated with the motion of charged particles through a magnetic field; the force associated with the thermal expansion of a gas which is heated by an electrical gas discharge.

This survey is intended to convey an impression as to the relative effectiveness of various electrically-produced mechanisms for accelerating a stream of gas. For this reason, the discussion will be accompanied by a number of numerical calculations which indicate the order of magnitude of the effects which are involved.

Re Electrostatic Forces

In the usual type of steady state gas discharge, the net electrostatic force on a neutral plasma is zero. The directed momentum transferred to the gas by the ions is equal and opposite to that transferred by the electrons.

Since it is often assumed that the ions transfer more momentum than the electrons, the reason for the equality of momentum exchange will be illustrated in the following: Consider the forces on a volume element of a homogeneous conducting plasma. If this volume is surrounded by a region sufficiently large so that boundary effects can be neglected, then both ions and electrons will be moving through the gas with steady state terminal drift velocities. All of the force which the electric field exerts on the charged particles is continuously communicated to the gas as a whole. In this case, the total electrostatic force on all of the positive ions in a unit volume is $N_i q_i E$, where N_i is the ion density, q_i is the charge on the ion and E is the electric field strength. The net force on all of the electrons in the volume is $-N_e q_e E$, where N_e is the electron density and q_e is the electron charge. Since $N_i = N_e$ and $|q_e| = |q_i|$, the two forces are equal and opposite. Therefore, the total momentum imparted to the gas by the electrons is equal and opposite to the momentum imparted by the ions.

There are some exceptions to the above generalization, particularly when a magnetic field is involved, but in most situations the comparatively small mass of the electron is completely compensated for by its higher velocity and collision rate.

Force Produced by Ion Beams

A number of devices have been described in the literature, by which high velocity ion beams can be produced. A beam of ions is not exactly a "wind" in the usual sense of the word, but the principle by which an ion beam is produced represents an effective mechanism for obtaining directed momentum by means of electrostatic forces. The momentum of an ion beam might be utilized to obtain a wind of neutral molecules, either by deionizing the beam by recombinations, or by an interchange of momentum with neutral particles.

A numerical example will illustrate the magnitudes involved in a device of this type. Ion currents as high as one ampere per square centimeter have been reported.(2) Suppose such a beam of nitrogen ions, having a kinetic energy corresponding to 1500 electron volts per particle, were transformed by recombination into a beam of neutral molecules. The resultant stream would have a velocity of 10^7 cm per second and a density of 6×10^{11} molecules per

cm^3. This molecular density would correspond to a gas at room temperature at a pressure of 2×10^{-5} mm of Hg.

Some idea as to the effectiveness of the ion accelerator in exerting the accelerating force on its beam, may be gained from the fact that the momentum created per second to produce such a beam is 3000 dynes per square centimeter of beam cross-sectional area. This magnitude of force is significantly high compared to other methods described in this survey.

Some of the possibilities associated with an ion beam wind generator can be illustrated by discussing the proposed design shown in Fig. 1. The positive ions are produced in a Phillips Ionization Gage (P.I.G.) type discharge apparatus, having a hole in the cathode. The mechanism of this type of discharge is described in a later section of this report. The fundamental principle involved is that the magnetic field acts as a sort of "cage" in which electrons are trapped. These trapped electrons oscillate back and forth, moving parallel to the magnetic field, between the two cathodes, and by their presence neutralize the space charge produced by the high positive ion current to the cathodes. This type of apparatus would be most effective below perhaps 1 micron of mercury pressure. If a hole is present in one cathode, an ion beam can be brought out through the hole.(2) Such a high intensity ion beam has a strong tendency to disperse, due to its own space charge, although the magnetic field tends to collimate the beam and restrain the dispersion.

It is necessary either to neutralize or to de-ionize the beam before excessive dispersion has occurred. One method of neutralizing the beam might be to increase the energy of the oscillating electrons in the P.I.G. discharge with an r-f field, so that part of them would have enough energy to escape from the potential trap between the cathodes and pass out of the hole along with the ion beam. This process would result in the production of a high velocity beam

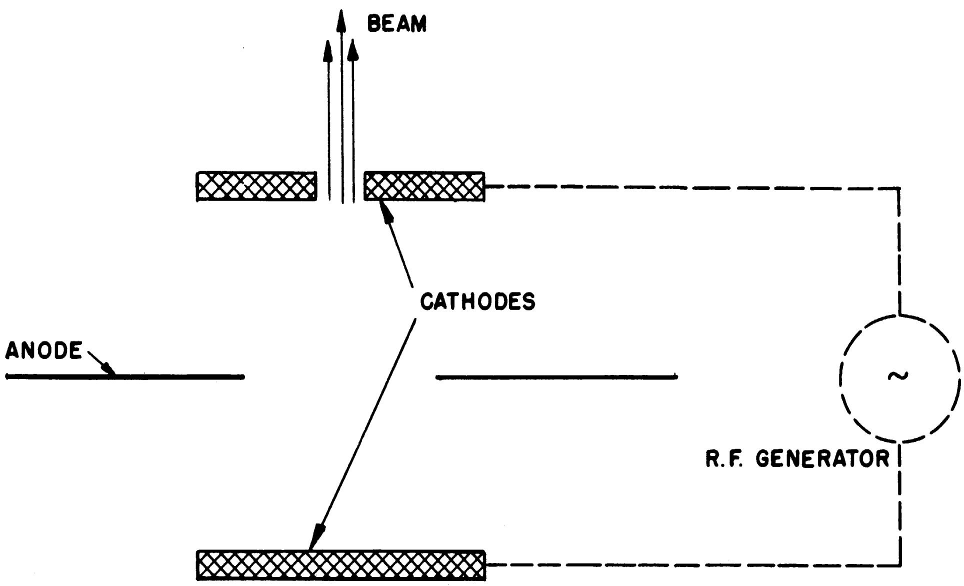


FIG. 1

PROPOSED METHOD OF PRODUCING HIGH VELOCITY BEAM OF NEUTRAL PLASMA, UTILIZING A P.I.G. DISCHARGE WITH HOLE IN CATHODE of neutral plasma, but since volume recombination of ions and electrons is very ineffective at these low pressures, the plasma beam could not be expected to de-ionize itself into a wind of neutral particles.

A possible method of producing a beam of neutral molecules, might be to arrange a source of negative ions so that these negative ions could be injected into the positive ion beam. Volume recombination between the positive and negative ions would take place and a neutral beam would result. In this case, the most difficult problem is to produce a source of negative ions of sufficient intensity.

Another method of transforming an ion beam into a molecular beam is to utilize collision processes. This mechanism is discussed in the next section of this report.

Re Transfer of Directed Momentum Between Charged Particles and Neutral Molecules

Several of the methods of producing wind, to be considered here, depend on a collision mechanism by which an electric force exerted on charged particles, results in a stream motion of neutral molecules. An efficient momentum transfer of this type requires that the neutral molecules receive a maximum of momentum in the direction of the force, and a minimum of momentum in random directions. A kinetic theory analysis, based on elastic spherical molecules, shows some significant facts regarding the nature of this mechanism.

One case of special interest involves a collision between an ion and a neutral molecule of the same mass. Suppose this ion is moving in the "x" direction and collides with a stationary molecule. Since the exact "head on" type of collision is rare, most of the collisions will be at various glancing angles. In reference 9, and also in Appendix I, dealing with the persistence of forward motion, it is shown that, on the average, a spherical molecule will receive half of the original "x" directed momentum of the ion, which means that the average ion retains half of its original "x" directed momentum. On the basis of energy exchange, the average resultant "x" directed motion of the molecule represents 1/3 of the original ion energy; the average "x" directed motion of the ion also represents 1/3 of the original energy. The remaining 1/3 appears as energy due to the lateral motion which is on the average, shared equally between the two particles. These relations imply that a substantial part of the original energy of the ion is converted to random thermal energy.

It would thus at first thought appear that a wind produced by this type of momentum exchange would necessarily be associated with a high gas temperature, and therefore a low molecular velocity ratio (i.e., stream velocity divided by characteristic thermal velocity).

A molecular beam, nearly homogeneous as to velocity, may be obtained by self-elimination of the molecules having incorrect velocities. To consider this possibility, refer to Fig. 2 which illustrates a possible means of converting an ion beam into a high velocity molecular beam. This could be accomplished by introducing laterally a supply of low velocity neutral molecules into the path of the ion beam. The ions are collected electrically after they have delivered momentum to the "wind" of neutral molecules which results.

In the region where the collisions are taking place, the random motion is of the same order of magnitude as the "wind" motion. The term "molecular velocity ratio" is not applicable in this region since the random velocities are not Maxwellian. It can be shown that if the neutral molecules in this region were to collide among themselves so as to establish a Maxwellian distribution, the molecular velocity ratio would have a value of $\sqrt{3/2}$.

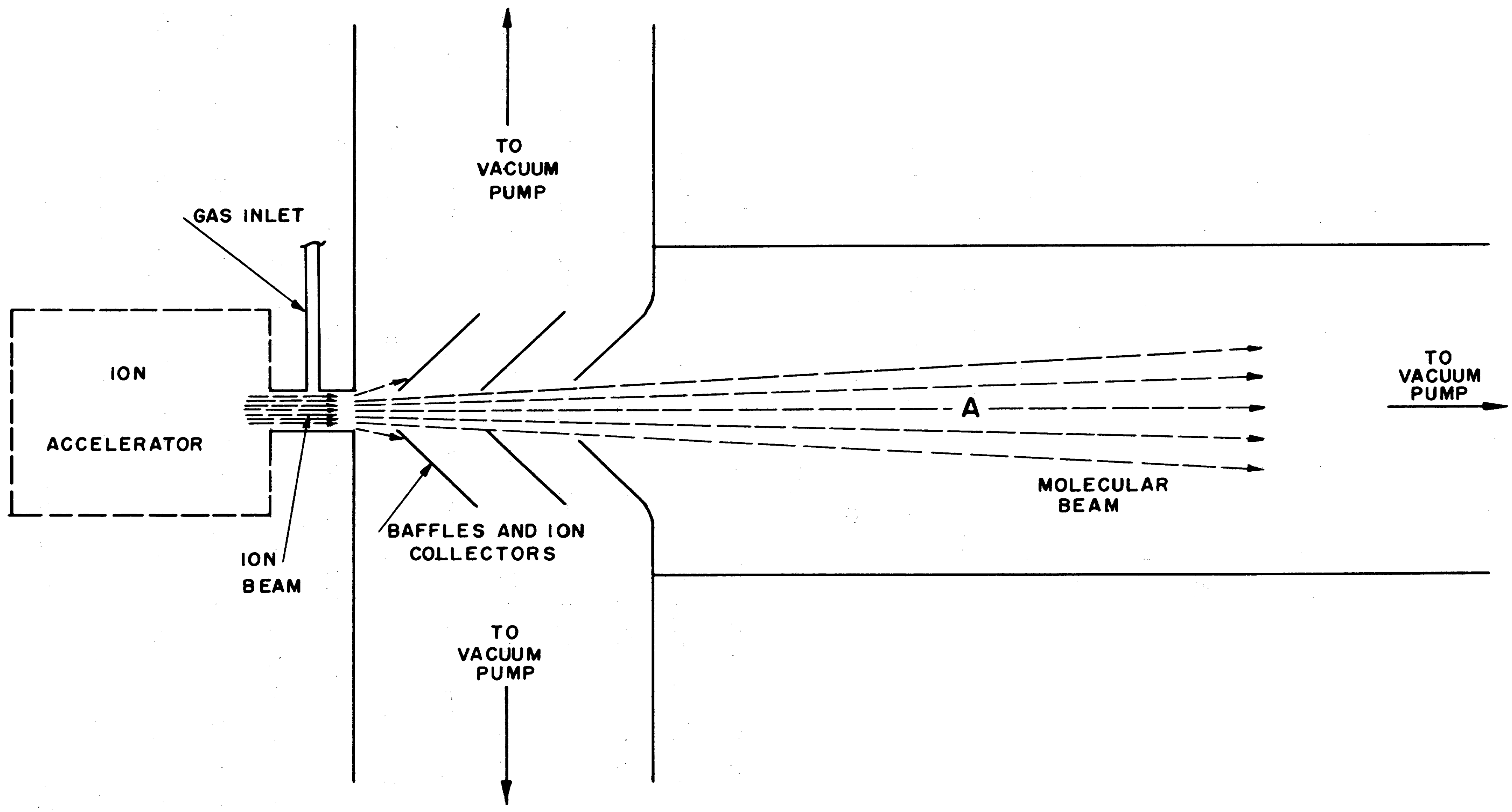


FIG. 2 PROPOSED DEVICE FOR CHANGING ION BEAM TO MOLECULAR BEAM

In the region at "A" (Fig. 2) which is some distance removed from the location of the collisions, the molecules are moving as a beam because the molecules with transverse motion have been eliminated. This type of molecular beam would differ in two significant respects from a molecular beam obtained by allowing molecules to escape from a furnace through collimating apertures; as has been done at the University of California. The first difference is that the velocity of the molecules could be made very much higher than is obtainable from a furnace. The second difference is that the beam could be made nearly homogeneous as to velocity. The uniform velocity condition might be obtained as follows.

The ion beam can be made nearly homogeneous as to velocity because all the ions can be accelerated through the same potential difference. All the neutrals which are hit "head on" by the attacking ions will all receive the <u>same momentum</u> and proceed in the <u>same direction</u> as the ion beam. Since the ionic and the molecular masses are assumed equal, these molecules will proceed with the same velocity as the attacking ions. Molecules which have glancing collisions from an attacking ion will be deflected away and removed from the axis of the ion beam. If the region along the extended axis of the ion beam is to consist mostly of molecules which have had "head on" collisions, it is important that most of the ions experience not more than one collision before being collected. An electric field and/or a magnetic field could be used to effectively remove all ions from the path of the molecular beam. By proper adjustment of the density of neutral molecules in the collision region it would presumably be possible to meet approximately the "not more than one collision per ion" requirement.

It thus appears theoretically possible to produce homogeneous beams of neutral molecules having extremely high velocities.

Force Due to Discharge in Crossed Electric and Magnetic Fields

An electric current through a conducting gas in a transverse magnetic field produces a force in the $\vec{E} \times \vec{H}$ direction. Much of the experimental work during the past year has involved this principle. Various considerations as to the effectiveness of this method of producing wind will be discussed later in this report.

From an overall macroscopic standpoint, the total force developed on the gas equals (magnetic field strength) x (current) x (electrode spacing). This relation implies that the only limit on the amount of accelerating force which can be developed, is the amount of current which can be passed between the electrodes. This maximum current limit is very high. Experiments have been performed with d-c currents up to 30 amperes and pulsed currents from capacitor discharges up to 1,000 amperes, and the voltage drop across the discharge has not significantly increased.

These high currents are associated with extremely high gas temperatures where the density of the gas becomes low and the viscosity becomes very large. If any confining walls are present, the loss of momentum to these walls sets a limit to the stream velocity which can be achieved. The loss of momentum to the walls might be eliminated, if the accelerating force were applied while the gas was passing out of a nozzle into a large vacuum chamber.

An illustrative numerical calculation regarding the force produced in this manner must be based on a very arbitrary assumption as to current. In one particular experimental situation, the gas pressure was 0.5 mm of Hg, the current was 20 amperes, the spacing between the electrodes was 10 cm, and the magnetic field was 5000 gauss. This produced a force on the air stream of 100,000 dynes. The cross-section of the air stream was approximately 100 cm^2 and the accelerating force was approximately 1000 dynes per cm^2.

Force Due to Thermal Expansion

One method of producing a high velocity stream of gas by means of an electrical discharge, is to utilize a principle similar to a rocket motor; except that instead of heating the gas stream by chemical combustion, an electrical discharge could be used to heat the expanding gas. Experience gained in connection with the present investigation has shown that the maximum gas temperatures obtainable from an electrical discharge can be greatly increased by utilizing a magnetic field. If an air stream were heated, in this manner, to an extremely high temperature and allowed to expand through a nozzle, very high stream velocities could be produced. The only apparent limit to these temperatures and velocities is the melting point of the materials exposed to the heat of the gas. This limitation could be alleviated by pulsed or intermittent operation.

Consideration of these melting point limitations led to the conjecture that the thermal expansion of an ionized gas might be converted into a stream or jet without the need of walls and nozzles, and the streaming action might be achieved by means of a magnetic field. Since the mobility of charged particles is far greater in directions parallel to the magnetic field than crosswise to the field, the diffusion of ions and electrons out of the discharge tends to be parallel to the flux lines. This flow of ions and electrons and the resultant collisions could be expected to produce a motion of the gas as a whole. Furthermore, if the random motion of an ion is approximately equally distributed among the three translational degrees of freedom, (motion in the x and y directions while spinning around the flux lines, and motion in the z direction while travelling parallel to the flux lines), then, if the ions are allowed to do work while expanding in the z direction, there should be a conversion of thermal energy into streaming action. This hypothesis would predict that a discharge in a magnetic field would be characterized by neutral air molecules moving in towards the discharge transverse to the magnetic field, and being "pumped out" of the discharge parallel to the field due to the higher mobility in this direction.

This hypothesis was tested with the apparatus shown in Fig. 3 . The electrode geometry is the same as in the revolving plasma except that the diameter of the anode ring was reduced to the point where rotational wind effects were not prominent. Thin slivers of mica were used as wind indicators and were placed around the discharge and over the hole in the end plate, where it was expected there would be a wind blowing.

The tests were made at 3000 gauss and at pressures from 1-50 microns. Not the slightest trace of any wind could be consistently detected. When the current to the electrodes was pulsed by discharging a 12μfd, 3000 volt capacitor, there was a strong blast of wind through the hole. Also there was a visible blast of luminous gas through the hole, but the luminous plasma did not spread out perpendicular to the field. This was some indication that the desired effect did exist on a transient basis. However, the steady state effect did not promise to be a potent method of generating wind.

Force on a Gas Discharge Caused by Momentum Transfer to Walls

When a gas discharge takes place inside a confining tube, a force is exerted which tends to make the pressure at the anode end greater than the pressure at the cathode end. This differential pressure can be employed to create

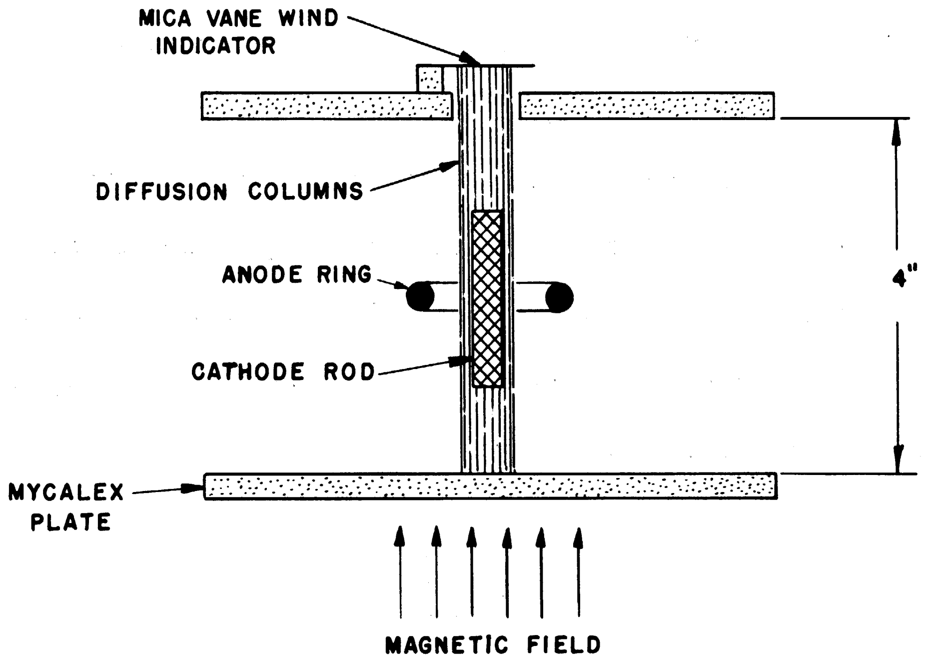


FIG. 3 EXPERIMENT TO DETECT WIND PRODUCED BY THERMAL EXPANSION OF IONIZED GAS

a wind, provided proper provision is made for the return air flow.

The pressure gradient is caused by the ambipolar diffusion of ions and electrons to the walls of the tube. Since the wall of the tube is a dielectric, the diffusion current consists of equal numbers of ions and electrons. When an ion strikes the wall, it has on the average, a component of velocity towards the cathode end of the tube, which is given to the wall on impact. Similarly the electrons give up to the wall a component of momentum towards the anode. However, the ions have a larger drift momentum (in spite of a lower velocity) and the net result is a force on the wall parallel to the wall surface and in the direction of the discharge axis. The approximate relative magnitudes of ion momentum as compared to electron momentum can be seen by means of Compton's mobility equations, which are:

$$V_i = \frac{8\sqrt{2}}{3\pi} \frac{q_i}{m_i} \frac{\ell_i}{\overline{C}_i} E \tag{1}$$

$$V_e = \frac{8}{3\pi} \frac{q_e}{m_e} \frac{\ell_e}{\overline{C}_e} E \quad , \tag{2}$$

where q_i, m_i, ℓ_i, $\overline{C}_i$ are respectively the ionic charge, mass, mean free path and average total velocity; and q_e, m_e, ℓ_e, $\overline{C}_e$ are the same quantities for the electrons.

The ratio of momenta (due to drift velocities) of the ions and electrons is:

$$\frac{m_i v_i}{m_e v_e} = \sqrt{2} \cdot \left(\frac{\ell_i}{\ell_e}\right) \cdot \left(\frac{\overline{C}_e}{\overline{C}_i}\right) \quad . \tag{3}$$

The ratio of mean free paths for ions and electrons is dependent on field strength and other variables but an order of magnitude value is:

$$\frac{\ell_i}{\ell_e} = \frac{1}{4\sqrt{2}} \quad .$$

The ratio of average thermal velocities is:

$$\frac{\bar{C}_e}{\bar{C}_i} = \sqrt{\frac{T_e}{T_i}} \quad .$$

In some low pressure discharges the electron temperature may be of the order of 50,000°K, while the gas temperature may be of the order of 400°K. If these approximations are substituted in Eq 3, then

$$\frac{m_i v_i}{m_e v_e} = \sqrt{2}\ \frac{1}{4\sqrt{2}}\sqrt{\frac{50,000}{400}} \cong 3 \quad .$$

It thus appears reasonable, that under certain conditions, the average longitudinal momentum of the ions striking the wall is several times larger than that of the electrons striking the wall. When these particles strike the wall they deliver all their tangential momentum to the surface and are re-emitted in a random manner. Hence there is a net force developed, tending to push the wall towards the cathode and a corresponding reacting force on the gas in the opposite direction. Since the net electrostatic force on the plasma is essentially zero, a pressure gradient is developed which tends to move the gas toward the anode.

The pressure gradient produced in this manner has been shown to be of significance in small capillary tubes where wall effects are prominent, but the action is not significant in larger diameter tubes.

A rough calculation of this effect in a typical low pressure discharge is of interest as an illustration of the magnitudes involved. The numerical values used,are known to be representative values for a particular set of experimental conditions.

A mercury vapor discharge takes place inside a long hollow glass tube 6 cm in diameter. The conditions are as follows:

gas pressure = 48 microns

total current = 10 amps

longitudinal voltage gradient = 0.4 volts/cm

ion current to wall = 4×10^{-3} amps/cm^2

gas temperature = 700°K

ionic mean free path = 0.15 cm

ion drift velocity = 13,000 cm/sec.

For simplicity, the electron momentum carried to the wall will be neglected and the average drift component of ion momentum delivered to the wall will be considered as equivalent to the force developed in the gas.

Wall force/cm^2 = (no. ions/sec.) x (average drift velocity) x (mass)

Substituting the experimental values,

$$\text{force} = \left(\frac{4 \times 10^{-3}}{1.6 \times 10^{-19}}\right) \times (13{,}000) \times (3.3 \times 10^{-22})$$

$$= 0.107 \text{ dynes/cm}^2 \text{ of wall} \quad .$$

If the discharge tube were a meter long the total force developed would be only about 200 dynes.

This phenomenon would be of much greater magnitude in discharges involving a transverse magnetic field, where the drift velocities and the random ion currents are very much larger. Several experiments were made in an attempt to confirm this, but the action was completely overshadowed by other effects due to the magnetic field. A more complete investigation of this effect appears warranted.

Electrostatic Accelerator

The most obvious method of generating an electric wind is by injecting ions, (either positive or negative ions, but not both), into a gas in the presence of an electric field. This is the basis of the "corona breeze" effect which is associated with the partial breakdown of air around a pointed electrode. This method is both simple and straightforward, but the forces which can be produced in this manner are very small compared to the forces which can be produced by other methods.

The essential difficulty with an ion accelerator is the severe limitation of the ion density due to space charge. Since the ions are all of the same sign, any appreciable concentration throughout a volume develops a high potential. As an illustration of the severity of this space charge problem, consider the following numerical calculation based on the situation shown in Fig. 4: An air stream charged with positive ions is blowing through a cylindrical glass pipe. The magnitude of the radial electric gradient caused by a relatively small space charge density may be evaluated easily. Assume that the tube is long enough so that end effects can be neglected. Also assume that the tube is 20 cm in diameter, and contains a uniform ion density of 6×10^8 ions per cm^3, which is equivalent to a charge density of 10^{-10} coulombs/cm^3.

Poission's equation in cylindrical coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{dV}{dr}\right) = -\frac{\rho}{\epsilon_0}$$

If the space charge density, ρ , is assumed constant, this equation

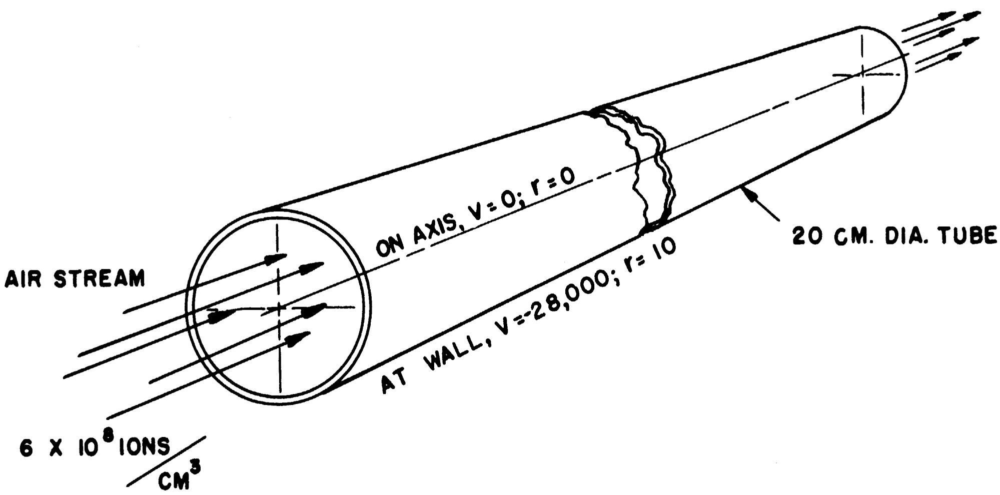


FIG. 4

SPACE CHARGE EFFECTS IN AIR STREAM CONTAINING POSITIVE IONS is easy to integrate

$$V = -\frac{\rho}{\epsilon_o}\frac{r^2}{4} + C_1 \log r + C_2 \quad .$$

At the axis of the cylinder, $r = 0$ and $dV/dr = 0$; let us also set $V = 0$ at $r = 0$. With these boundary conditions, it can readily be shown that

$C_1 = 0$, $C_2 = 0$.

Therefore:

$$V = -\frac{\rho}{\epsilon_o}\frac{r^2}{4} \; ,$$

$$\frac{dV}{dr} = -\frac{\rho}{\epsilon_o}\frac{r}{2} \quad .$$

Substituting $\rho = 10^{-4}$ coulombs/m^3, $r = 0.1$ meter. Then at the wall of the tube,

$$V = -\ 28{,}000 \text{ volts}$$

$$\frac{dV}{dr} = -\ 5.6 \times 10^5 \text{ volts/meter} \quad .$$

If this uniform ion density were to exist in the tube, the wall would be at a potential 28,000 volts- **lower** than the axis, and the radial potential gradient at the wall would be 5600 volts per cm. Any tendency that the ions would have to move along the tube would be completely masked by the radial dispersive force outward. Furthermore, this radial gradient would greatly exceed the breakdown strength of the low density air in the tube. Conceivably, this radial gradient difficulty could be alleviated somewhat by some trick involving wire screen grids, but a workable scheme for doing this without excessively impeding the air flow appears unlikely.

If a longitudinal electric field of 100 volts/cm were applied to accelerate the ions, the force exerted on the gas = $N_i q_i$ E newtons/$m^3 \cong$ 1 newton/m^3 or 10^{-1} dynes/cm^3. This accelerating force is quite small compared to forces produced by other methods.

INVESTIGATION OF DISCHARGE IN CROSSED ELECTRIC AND MAGNETIC FIELDS

General

Much of the experimental program during the past year has been concerned with discharges in crossed electric and magnetic fields. These experiments are very unusual in that they involve a combination of low gas pressure, strong magnetic field and a high power level. Some very interesting phenomena have been observed which have not been reported anywhere in the literature. The study of wind effects promises to be a new tool for finding out basic information regarding the mechanism of gas discharges.

In order to achieve significant wind effects, the experiments were conducted at power levels of five to ten kilowatts. The heat generated by this high rate of energy input created experimental difficulties, but operation at this power level was essential since the production of high air velocity--even at very low air density--requires substantial amounts of energy. As an illustration of power requirements, the acceleration of the ion beam described on page 5, requires 1.5 kilowatts of power per/cm^2 of beam cross-sectional area.

Most of the experiments were conducted at gas pressures of 5 - 100 microns. The choice of this pressure range was dictated by the fact that the accelerating force which could be exerted on the gas stream was of the order of a few ounces. Calculations as to momentum and viscosity relations indicate that air densities in this range, or lower, are necessary if high stream velocities are to be obtained from this type of wind generator.

As a convenient means of studying the "crossed fields" type of discharge, an experimental arrangement called the "revolving plasma" was employed. One type of electrode geometry for producing the "revolving plasma" is illustrated in Fig. 9. The discharge takes place between the center cathode post and a surrounding anode ring. An axial magnetic field produces a "motor" force on the gas which causes it to revolve. This electrode arrangement was very suitable for studying cathode phenomena, instrumentation problems, and determining various experimental facts. As a wind generator, the velocity is limited by the high gas temperature (and the resultant high viscosity) and centrifugal force. Present information indicates that some aspects of the "crossed fields" type of wind phenomena can be studied more advantageously with the linear flow arrangement which is described later in this report.

The number one problem in this investigation has been the development of instrumentation to determine quantitative data about the magnitude of the effects which are observed. The experimental conditions are so unrelated to other types of gas discharges (or wind tunnels) that all attempts to adapt existing instrumentation techniques have been unsuccessful. At the present time new methods of instrumentation are being developed and it is expected that especially significant quantitative information will be obtained in the course of further work.

Equipment

Fig. 5 is a general view of the experimental equipment which was assembled for the experiments with the revolving plasma. In the foreground is the electromagnet with an aluminum (nonmagnetic) vacuum box placed between the pole pieces. The electromagnet is designed for intermittent operation at a low duty cycle, and, as a result, much higher field strengths are produced than could be obtained otherwise from a magnet of this size. The aluminum vacuum box has inside dimensions of 9" x 24" x 28". The entire front end is covered with a removable thick glass window through which the discharge may be observed.

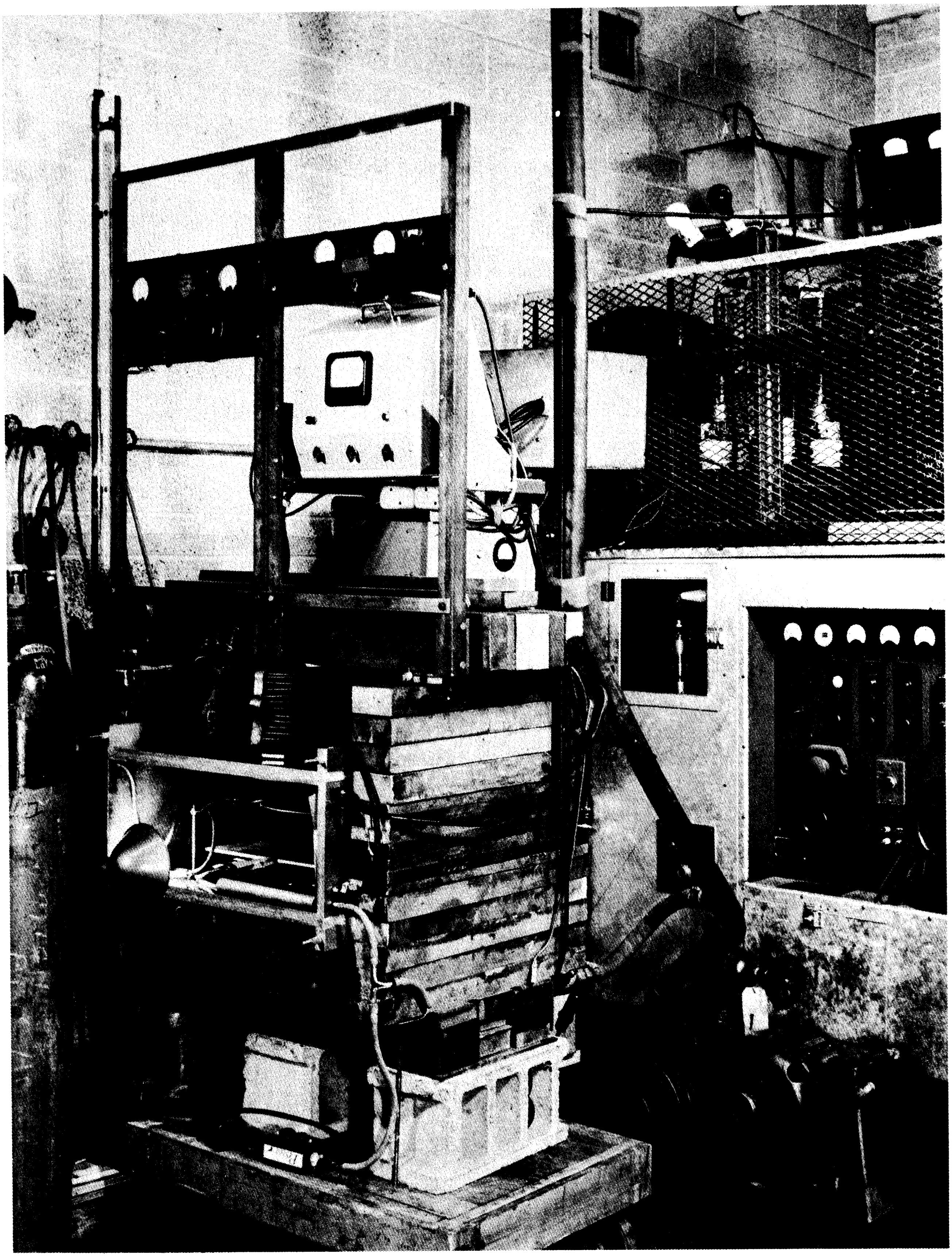


FIG. 5
GENERAL VIEW OF EXPERIMENTAL APPARATUS

On the rack above the electromagnet are mounted the alphatron and thermocouple vacuum gages and the booster-pump controls. At the extreme left is a cylinder of dry nitrogen which is continuously bled into the system to maintain any desired pressure. In the background to the right may be seen the two high-voltage rectifier power supplies. The lower unit is a converted RA-38 variac-controlled power supply which will deliver currents up to 1 ampere at voltages up to 15,000 volts. The upper unit consists of a "pole" type distribution transformer and four type 869B mercury-vapor rectifier tubes which can produce 10 amperes of current at 2,000 volts.

Fig. 6 is a rear view of the magnet and vacuum chamber, and shows the vacuum pumps and the magnet power supplies. The vacuum system consists of a Kinney type CVD 556 forepump used in conjunction with a Distillation Products, Inc. type MB 100 vapor booster pump. These pumps provide usable working pressures of less than 1 micron and a pumping speed of 100 liters/sec. This high speed is very desirable, since it permits an equilibrium pressure to be maintained, even when nitrogen is bled into the system at a fairly rapid rate. Such a continual exchange of gas helps to reduce contamination due to outgassing of the dielectric surfaces. Most of the tests were made in dry nitrogen. Air could not be used because the presence of the discharge caused chemical combination of the nitrogen and oxygen. These nitrogen compounds caused rapid deterioration of the oil in the vacuum system.

The magnet power is supplied from two war-surplus "turret trainer" d-c power supplies, which can be seen in the background of Fig. 6. They can supply up to 400 amperes of current.

Fig. 7 is a plot showing the variation of magnetic field across the vacuum chamber. This plot was taken with a magnet current of about 200 amperes.

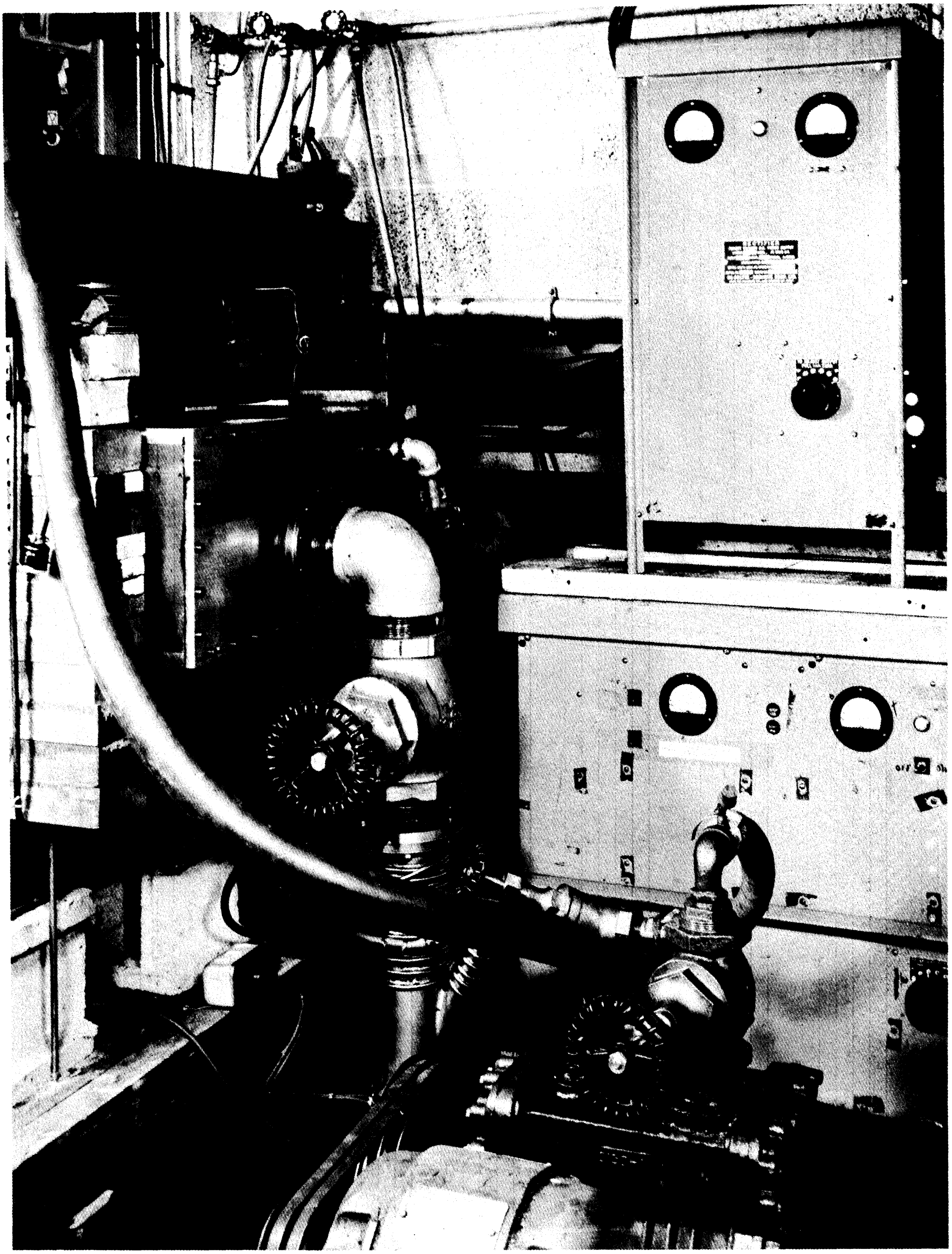


FIG. 6

REAR VIEW OF MAGNET AND BOX SHOWING
VACUUM PUMPS AND MAGNET POWER SUPPLIES

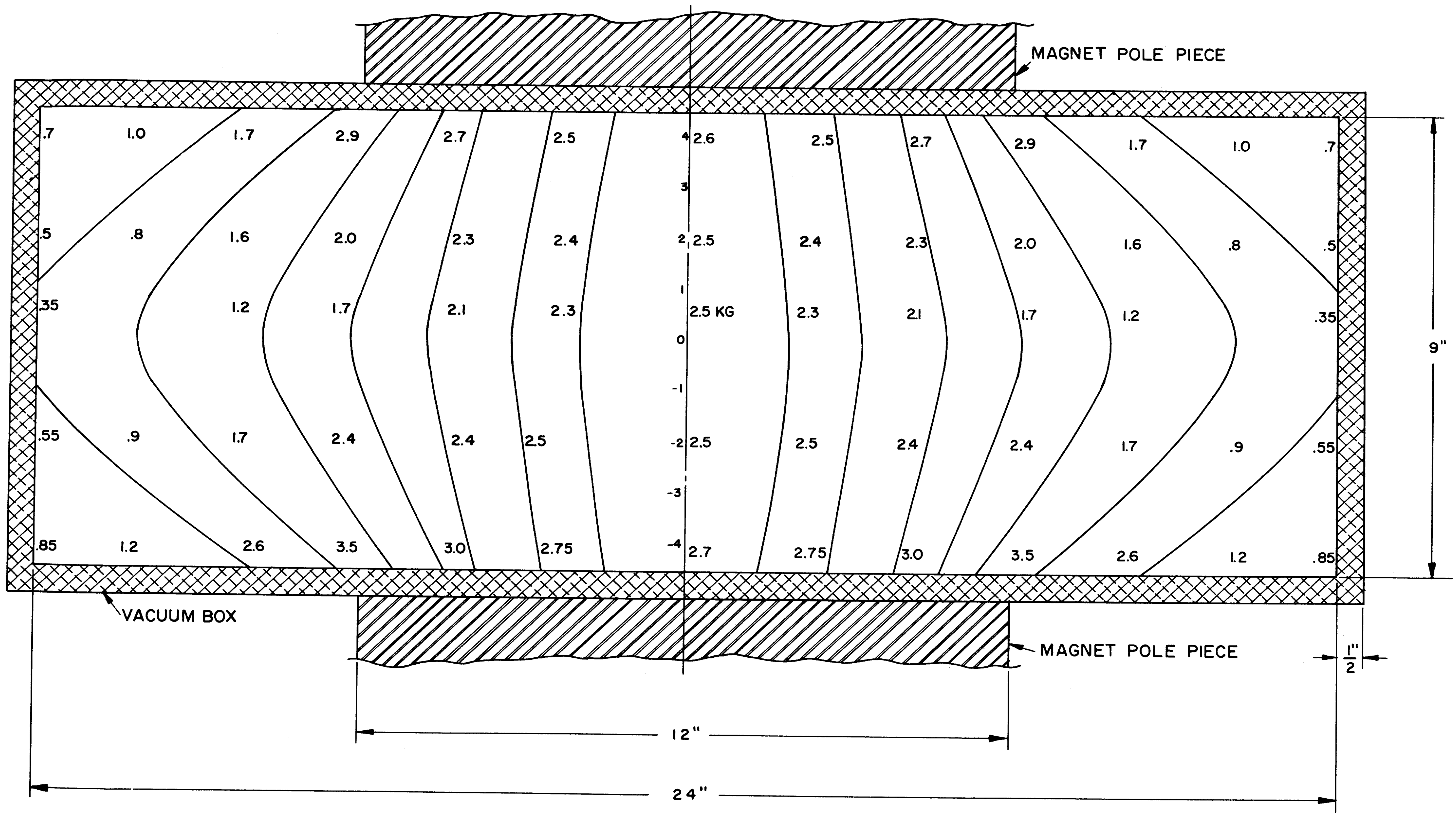


FIG. 7
CROSS-SECTION OF MAGNET AND VACUUM BOX SHOWING
FRINGING OF MAGNETIC FIELD AND FLUX DENSITY IN KILOGAUSS

The pole pieces were 12 inches in diameter and the flux gap was 10 inches. Most of the experiments were conducted with this approximate value of field strength. By increasing the current to 400 amperes and decreasing the flux gap to 7 inches, fields up to 5,000 gauss could be obtained.

Description of "Revolving Plasma"

Fig. 8 is a photograph taken through the glass window of the vacuum chamber which shows the apparatus of Fig. 9 in operation at a gas pressure of 80 microns. In this photograph the luminosity is greater than normal because of the cathode sputtering which was coloring the discharge. The discharge in pure nitrogen gas is pale blue and quite transparent, especially at lower pressures. The barrel-shaped region of greatest luminosity is defined by the anode ring and the magnetic flux lines which fringe outward between the pole pieces of the magnet. The tendency of the discharge to collimate along the magnetic flux lines is quite pronounced in any region where the transverse electric gradient is not too large.

At gas pressures below about 100 microns the luminosity spreads uniformly over the region between the top and bottom plates. This is in contrast to the behavior at pressures of 500 microns and above where the luminous region resembles a disk about 1/2 inch thick which is located midway between the top and bottom surfaces of the chamber.

Due to the large amount of power dissipated in the discharge, water cooled electrodes were necessary. Electric wires were insulated with glass tubing. Due to the poor dielectric strength of air at low pressures, any crack or poor joint in the tubing would result in a short circuiting electric arc. Mycalex plates were used as insulation in some locations, while in other locations exposed to columns of ambipolar diffusion, Vycor glass (which is nearly

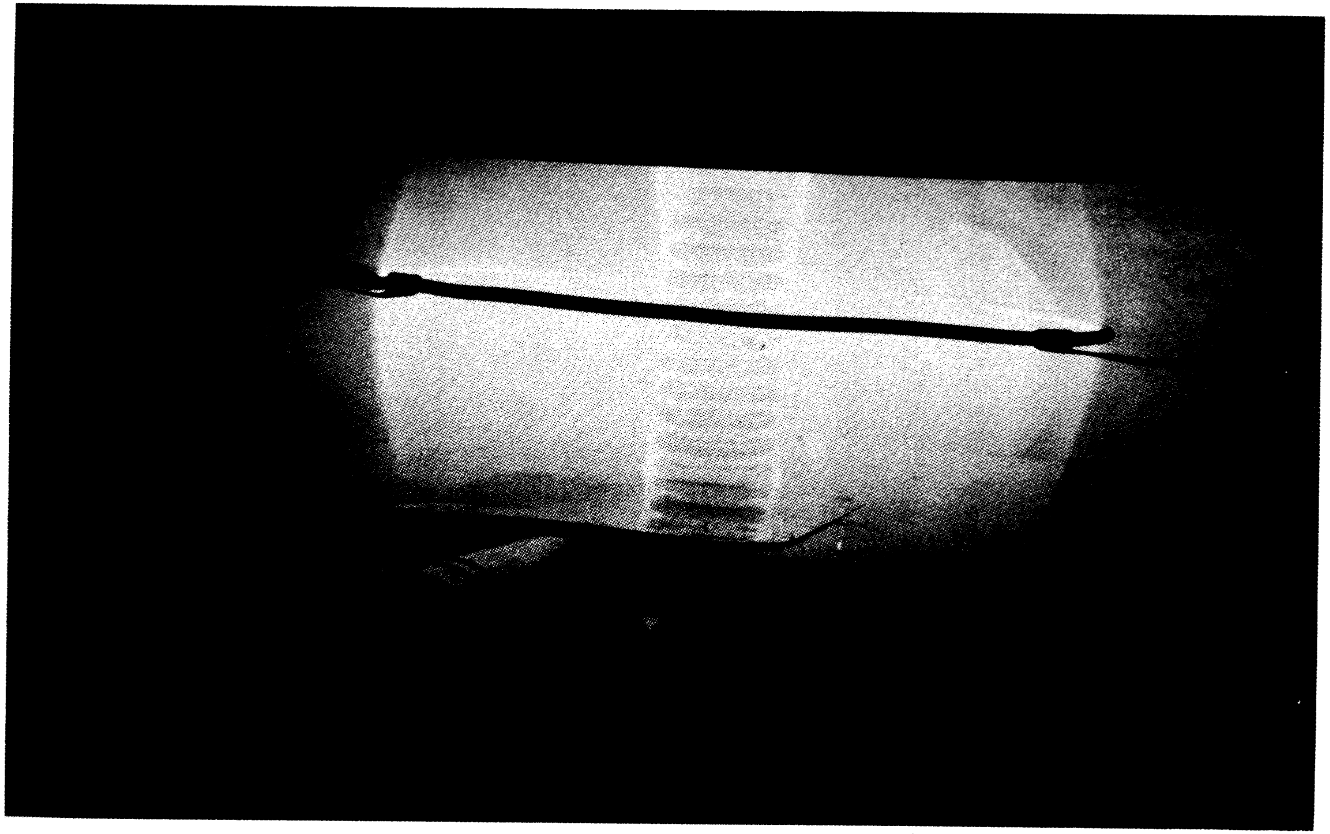


FIG. 8
"REVOLVING PLASMA" DISCHARGE IN OPERATION. PHOTOGRAPH TAKEN THROUGH VACUUM BOX WINDOW WITH ELECTRODE GEOMETRY SHOWN IN FIG. 9.

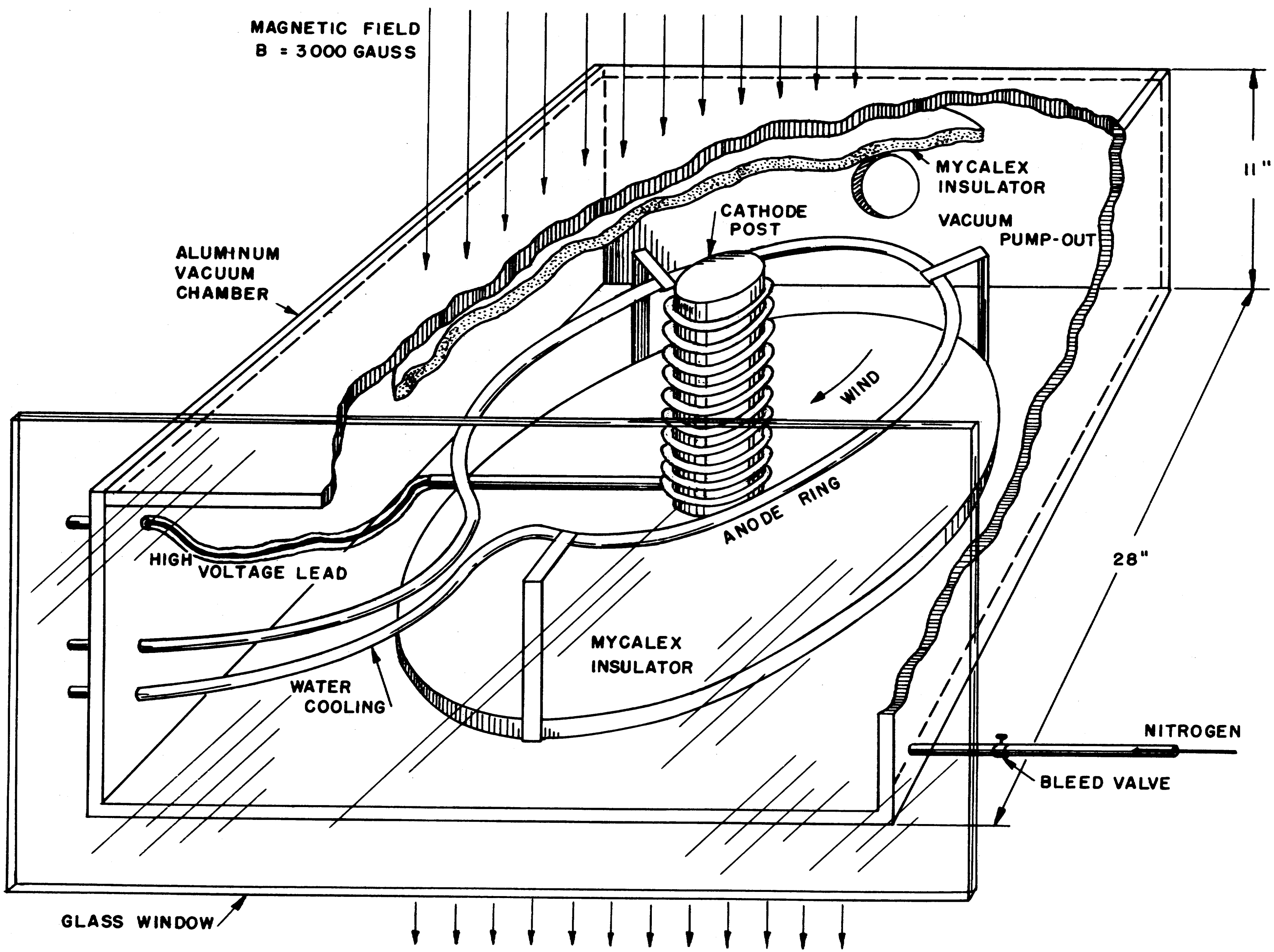


FIG. 9 SKETCH OF THE VACUUM BOX AND ELECTRODE ARRANGEMENT USED IN TESTS OF THE "ROTATING DISCHARGE" TYPE OF WIND GENERATOR

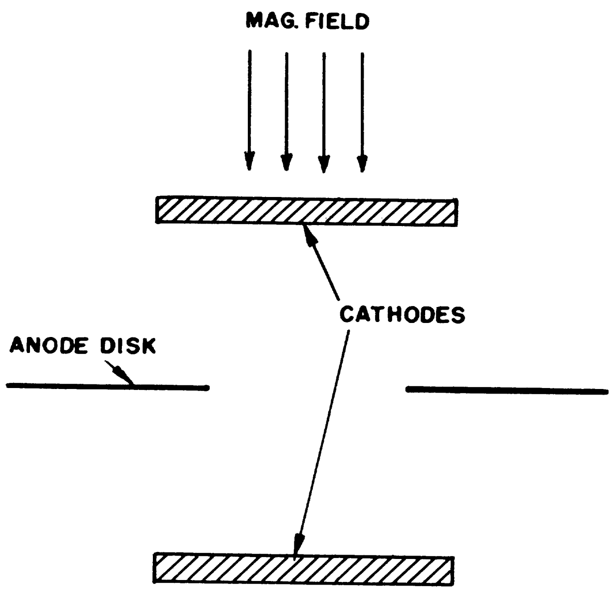
pure quartz) was used. While the discharge was operating, the dielectric surfaces heated very rapidly. Much of this heat was due to the recombination of ions and electrons on the surfaces.

The heating of these dielectric surfaces caused pressure conditions in the box to change due to the liberation of gas. For this reason, the power was ordinarily turned on for intervals of a few seconds to a minute at a time. In taking data regarding electrical conditions, readings which were taken after the discharge was turned on for 3 to 10 seconds gave reasonably reproducible results. Pressure measurements were much less satisfactory because the response time of the vacuum gage was ordinarily too slow to keep up with the changing pressures in the discharge. This equipment could be set up for continuous operation, but much more elaborate water cooling provisions would be required.

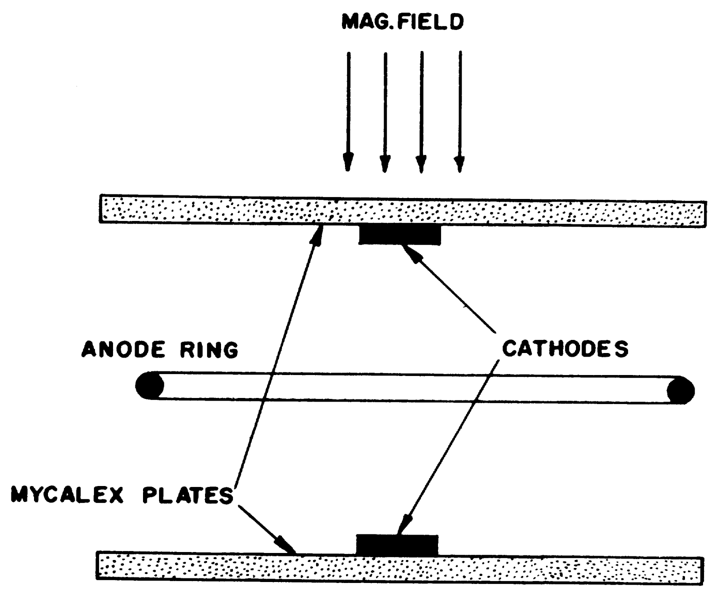
The discharge is characterized by violent plasma oscillations which cover a wide range of frequencies. When the power supply voltage is maintained constant the current will fluctuate, and when the current is maintained constant (with chokes and/or pentodes) the voltage will fluctuate. Plasma probe measurements are related to the time average values of the noise voltages which have high frequency fluctuations as large as the average value.

<u>The Phillips Ionization Gage (P.I.G.) Discharge</u>

The revolving plasma resembles the Phillips Ionization Gage type of discharge in certain respects. Further, the P.I.G. mechanism was used as a cathode for the revolving plasma apparatus, in a number of experiments. Thus, a review of the essential characteristics of the P.I.G. discharge is pertinent to a discussion of the revolving plasma. The electrode geometries of these two devices can be compared in Fig. 10(a) and Fig. 10(b).



(a.)



(b.)

FIG. 10

ELECTRODE GEOMETRY
ASSOCIATED WITH THE PHILLIPS
IONIZATION GAGE

ELECTRODE GEOMETRY OF THE
"REVOLVING PLASMA" UTILIZING
THE P.I.G. CATHODE MECHANISM

A significant difference between the revolving plasma and Phillips Ionization Gage, as shown in Fig. 10, is that the revolving plasma apparatus has an anode of much larger diameter. This means that the discharge current has a longer radial path length, and thus the force in the $\vec{E} \times \vec{H}$ direction is much greater. Another difference is that the revolving plasma has dielectric end plates, so that the diffusion current to these walls removes equal numbers of ions and electrons. In the Phillips Ionization Gage the end plates are just the cathode surfaces; therefore, only positive ions are removed at these surfaces.

The P.I.G. mechanism permits a cold cathode glow discharge to be maintained at gas pressures far lower than is otherwise possible. The magnetic field acts as an "electron trap", and greatly reduces the mobility of the electrons in directions perpendicular to the field. The electrons in the discharge oscillate back and forth between the cathodes while slowly draining off laterally to the anode. This greatly increases the path length of the electrons. Hence, the probability of an electron having an ionizing collision before reaching the anode is greatly increased, and the gas density required to sustain the discharge is reduced by a large factor. The emission of electrons from the cathode by positive ion bombardment is not necessarily an important factor in sustaining the discharge. Ordinarily the cathode current is almost entirely positive ion current.

The current density of positive ions to the cathodes can be surprisingly high. This is because the electrons oscillating in the "magnetic trap" tend to neutralize the space charge, especially near the cathodes, thus permitting a high density of ion current without the usual space charge limitation. Measurements of the energy with which the positive ions strike the cathode, reference 2, page 352, indicate that the average energy is approximately the same as the total voltage across the discharge. This suggests that the ions originate in a plasma which is at about the same potential as the anode.

Cathodes

A high current discharge in a strong magnetic field presents a very difficult cathode problem. At gas pressures above a couple of hundred microns, the cold cathode glow mechanism is quite effective and almost any piece of metal of any shape will serve as a cathode; however, the sputtering is usually quite objectionable. A cylindrical post of copper or aluminum, 2 inches in diameter and 5 inches high, will provide a discharge current of many amperes. At lower pressures, the secondary emission mechanisms of glow cathodes become less and less effective, and other types of cathodes have to be used. In the pressure range of 20 - 100 microns a glow cathode will operate only as long as a film of corrosion or oxide is present. As this film is sputtered away, the voltage required to sustain the discharge increases until many thousands of volts are required to maintain a very small current. A cathode suitable for operation under these unusual conditions had to be obtained before the experimental program could be continued.

Thermionic Cathodes

Thermionic cathodes have certain significant advantages, but when operating in a strong magnetic field several difficulties are encountered. Heated tungsten cathodes will emit well in nitrogen or other inert gases for a wide range of pressures, including pressures too low for other types of cathodes. The cathode fall of potential is also less for thermionic cathodes than for any other type. This type generally has proven most suitable for pressures below 20 microns.

The difficulties with the thermionic cathodes are caused by the strong ion bombardment and high currents required to heat them. The thoriated or oxide types cannot be used because of the heavy ion bombardment, and relatively heavy pure tungsten wire is required. A heavy tungsten filament requires a large amount of heater power to maintain its emitting temperature. This causes excessive heating of the surrounding gas and dielectric surfaces. One filament which was tested consisted of a circular loop of 30 mil tungsten wire about 3 inches in diameter. This wire required 500 watts of power at close to 30 amperes of current to maintain an emitting temperature.

Another difficulty with heated cathodes is the interaction between the heater current and the magnetic field. The filament wire must be very carefully supported in order to withstand the strong mechanical force which results from this interaction. If the filament is heated with alternating current, the vibration is quite severe and will fracture the wire at any weak spot. Even when direct current is used many supports are necessary, and there is the added problem of providing a high current d-c filament supply at a high voltage with respect to ground potential.

Recent experience has shown that the operation of a thermionic filament is much improved when it is mounted close to metal plates which act as ion collectors. The reason for using this combination can be explained as follows: consider the situation where the cathode is only a tungsten filament and the top and bottom of the discharge are bounded by dielectric walls. The filament is negative with respect to the plasma so that it is surrounded by a positive ion sheath. The ratio of electron current to ion current across this sheath is approximately given by the relation $\frac{J_e}{J_i} = \sqrt{\frac{m_i}{m_e}}$ where m_i and m_e are the masses of the ion and of the electron. This is true provided the filament emission and the plasma random ion density are high enough so that both currents across the sheath are space charge limited. This ratio of J_e/J_i is about 200:1 so that the filament removes relatively few ions from the discharge. However, a sizeable fraction of the drift current in the plasma is due to positive ions. These ions are not collected at the filament, but the filament does provide an ample supply of electrons to neutralize the space charge of the ions as fast as they arrive in the vicinity of the filament. Since volume recombination is not an effective process at these low pressures, the ion and electron densities build up. This results in a high concentration gradient which produces a high density ambipolar diffusion current to the walls. This diffusion stream is strongly collimated by the magnetic flux lines so that the regions of the wall on which these flux lines terminate are subjected to terrific heating from the recombination energies. This action is potent enough to "drill" a hole through a one-quarter inch quartz plate in less than a minute.

A tungsten filament cathode in combination with metal ion collectors has been tested in connection with the linear flow air accelerator, Fig. 31 The metal cathode plates above and below the filament remove the ions but tend to reflect electrons back into the plasma as in the P.I.G. discharge. So far, this cathode has operated satisfactorily (aside from problems associated with power dissipation and interaction effects). All of the reasons for its successful operation have yet to be thoroughly investigated.

P.I.G. Cathodes

The Phillips Ionization Gage mechanism provides a suitable cathode for pressures between 0.1 and 20 microns. It is capable of furnishing more current for this pressure range than any other type of cathode except the thermionic. The chief limitations of the P.I.G. cathode are its high cathode fall of potential and the resulting high rate of sputtering. The cathode fall of potential is typically 400 - 500 volts. If the discharge is running at 6 amperes, this represents several kilowatts of power loss. The heat generated on the cathode surfaces can be removed by water cooling, but the heat generated in the gas is much more objectionable because this increases the gas temperature and visocosity. The sputtering is objectionable since it produces a metal vapor of unknown properties. This tends to confuse experimental results.

Secondary-Emission Cathodes

At gas pressures below 100 microns, the glow discharge type of cold cathode has two limitations: 1) The current density available becomes too small since the secondary-emission mechanism of a glow type cathode results in a current density which varies directly as the square of the pressure. 2) The high cathode fall of potential causes very rapid sputtering of the cathode material and represents a large power loss in the form of heat. An investigation of secondary-emission cathodes was undertaken in the hope that a material could be found which would not have these limitations. Cathodes have been developed for use in flash tubes which have a high resistance to sputtering. They consist of sintered tungsten, impregnated with a low work function material such as barium oxide. Unfortunately these cathodes have been made only in very small sizes for experimental purposes (by Germershausen and Edgerton at M.I.T.), and are not available commercially.

Efforts were made to try to devise cathodes of this type. One test involved a piece of very porous tungsten which was impregnated with barium oxide. Another cathode, which showed some promise at first, was a sintered mixture consisting of tungsten powder, barium oxide, and a binder. Various mixtures were tried containing 2 to 10 per cent barium oxide with about 5 per cent copper powder as a binder, the remainder being tungsten. This mixture was pressed into slugs 5/16 inch in diameter by 3/8 inch long under a pressure in excess of 20 tons per sq. in. The slugs were then fired in a hydrogen atmosphere furnace at 1300°C.

None of these cathodes were very successful. Some of them showed resistance to sputtering, but they exhibited a strong tendency to form small arc spots which were very erratic and irregular and released foreign vapors into the discharge. The presence of an arc spot caused a strong diffusion column along the magnetic flux lines which would burn holes in the dielectric plates, in the same manner as the thermionic filaments previously described.

One secondary-emission cathode which did prove relatively successful for certain tests consisted of a chunk of magnesium alloy. This cathode had a tendency to form a great many tiny arc spots which were constantly dancing over the surface, creating the effect of a relatively uniform current distribution. The emission was quite satisfactory as long as the surface was coated with oxide, but as this oxide gradually disappeared from the surface the voltage drop greatly increased.

Hybrid Cathodes

Several types of cold cathodes were tried which utilized the "electron trap" action of the magnetic field but were not otherwise similar to the Phillips Gage.

One of these cathodes was a cylindrical post shown in Fig. 11. The electric field at the cathode surface is radial, but because of the presence of the vertical magnetic field, the electrons emitted from the cathode revolve around the cylinder in cycloidal paths. The electrons are thus retained in the vicinity of the cathode. The effects of positive-ion space charge and sheath

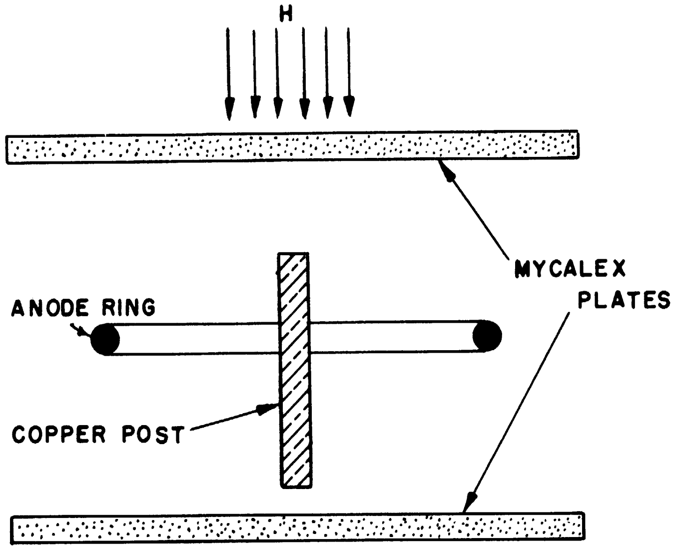


FIG. 11

CYLINDRICAL POST CATHODE

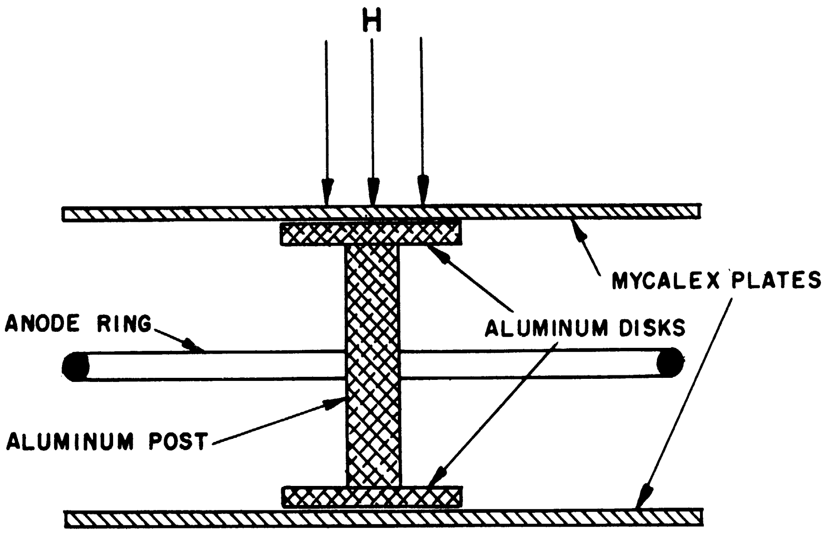


FIG. 12 "POST" CATHODE WITH METAL END PLATES formation which are troublesome with other geometries of secondary-emission cathodes, are greatly reduced as a result of this trapping or constraining action.

With this cylindrical cathode, there is an additional emission mechanism which may be important. This geometry is similar to that in a magnetron, where the back-bombardment of the cathode by electrons is a significant factor in obtaining large cathode emission. This back-bombardment mechanism of the magnetron cathode is very effective in liberating electrons, quite aside from the heating effect. In the analogous "revolving discharge" situation, there are strong plasma oscillations which seem capable of causing electrons to back-bombard the cathode with considerable energy.

The experiments with the cylindrical-post cathode have been inconclusive as to the importance of the back-bombardment mechanism. In one particular experiment, at 30 microns pressure, a cathode was tested which consisted of a tungsten rod about 1/2 inch in diameter and 4 inches long. This provided several amperes of emission when oriented parallel to the magnetic field, but the emission dropped to a fraction of an ampere when the cylinder was turned crosswise to the field.

Fig. 12 illustrates a cylindrical-post cathode to which end plates have been added. These end plates are a definite improvement since they serve to reflect electrons back into the discharge as in the P.I.G. mechanism. These electrons, which would otherwise be lost due to ambipolar diffusion, help to neutralize the positive-ion space charge. This arrangement combines the function of the Phillips Ionization Gage type of cathode and the cylindrical-post cathode mentioned above.

The "totem pole" cathode illustrated in Fig. 13 has special advantages at higher pressures where the mean free path is so short that the P.I.G.

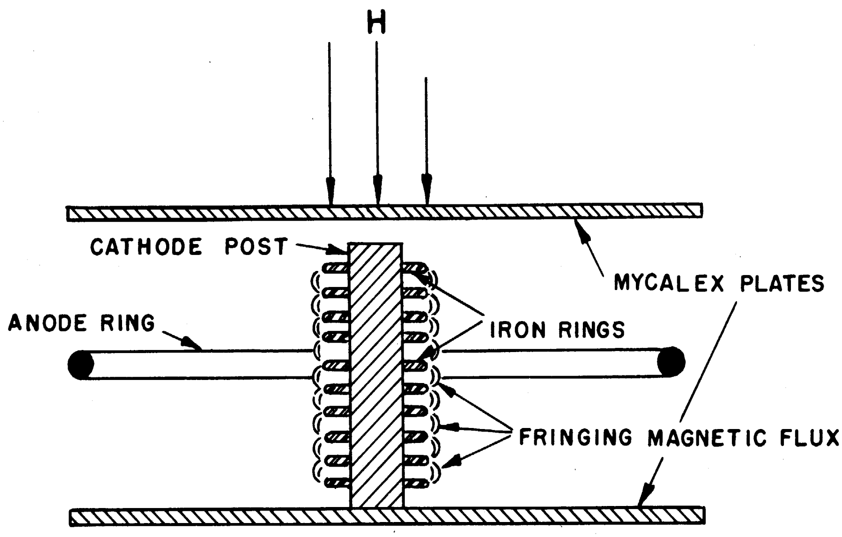


FIG. 13 "TOTEM POLE" CATHODE

arrangement of Fig. 10 is less efficient. The rings on this cathode are made of magnetic material, and the magnetic field fringes out between them as shown in the figure. The electrons oscillate back and forth along these fringing flux lines and produce ionization. The iron rings thus act as a series of P.I.G. cathodes. Since the area for collecting positive ions is effectively increased, the current density of positive ions and the cathode fall of potential are correspondingly decreased.

Total Voltage-Current-Pressure Relations

A theoretical interpretation of the basic mechanisms associated with electrical wind effects requires quantitative experimental data. In the present investigation, certain types of data have been particularly difficult to obtain because of instrumentation problems. As a result, the analytical explanation is handicapped by a lack of experimental facts. The voltage-current-pressure relations associated with the revolving plasma are an exception in that they can readily be obtained in a consistent and reproducible manner. The over-all voltage is a summation of several different effects, and does not indicate the relative magnitudes of the various components. For instance, the total voltage between the electrodes is the sum of the cathode fall of potential, the plasma drop and the anode fall of potential. The relative contribution of these individual components can be expected to vary as the conditions are changed. Therefore, this data is not as meaningful as might be desired. A number of significant facts can be obtained from it, however, and it constitutes a means of cross-checking the validity of other types of measurements, such as some of the plasma probe data given in the next section. The quantitative interpretation of the overall voltage curves will become more meaningful as more analytical information becomes available.

The relations involving the total potential drop are given in the following figures:

Fig. 14	Voltage drop vs. amperes, for different values of pressure	"totem pole" cathode, nitrogen gas
Fig. 15	Voltage drop vs. pressure for different values of current	"totem pole" cathode, nitrogen gas
Fig. 16	Voltage drop vs. pressure for different values of current (nitrogen gas.)	"P.I.G." type cathode
Fig. 17	Voltage drop vs. pressure for different values of pressure (hydrogen gas.)	"P.I.G." type cathode

The voltage-current-pressure relations in Figs. 15, 16, and 17 were plotted on the coordinates of potential drop vs. pressure, since this method of presentation illustrated the trends more clearly. Another reason for using these coordinates is that the more conventional plot of current vs. voltage causes many of the curves to lie on top of one another at some constant slope. This tendency can be seen in Figure 14 for pressures above 80 microns.

These curves demonstrate several interesting facts. There is a large voltage drop at very low pressures and an approach to a constant voltage condition at high pressures. At high pressures an increase in current requires a smaller proportionate increase in voltage than is required at low pressures.

A comparison of the total voltage drop under the same conditions of current and pressure, provides an excellent means for evaluating the various types of cathodes described in the previous section. Fig. 15 and 16 are examples of this type of comparison between the "totem pole" and P.I.G. cathodes. Examination of these figures shows that the P.I.G.

arrangement is more effective at low pressures, while at pressures above 100 microns the voltage drop with the "totem pole" is about 350 volts less than with the P.I.G.. Other cathodes were compared in this same way. Most of the secondary-emission and hybrid types described previously, gave overall characteristics very similar to those of the "totem pole", except that the overall voltage drop was, in general, somewhat greater.

Fig. 17 is a plot of the overall voltage relations when hydrogen gas is substituted in place of the nitrogen usually used. Although the data for hydrogen gas is incomplete, it is interesting to note that there is a crossover in these curves which does not appear when nitrogen is used.

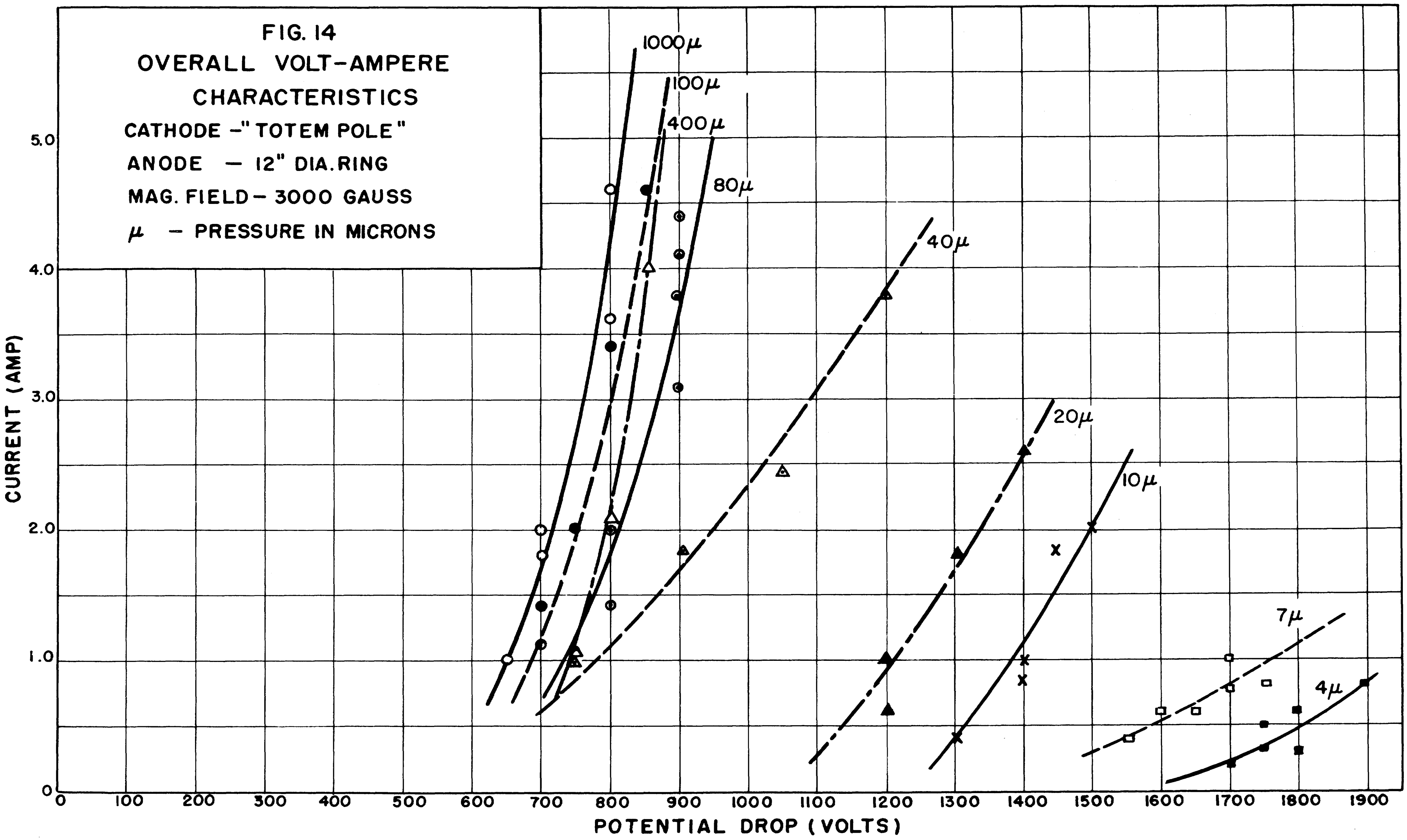


FIG. 14
OVERALL VOLT-AMPERE CHARACTERISTICS
CATHODE -"TOTEM POLE"
ANODE — 12" DIA. RING
MAG. FIELD - 3000 GAUSS
μ - PRESSURE IN MICRONS
1000μ
100μ
400μ
80μ
40μ
20μ
10μ
7μ
4μ
CURRENT (AMP)
POTENTIAL DROP (VOLTS)

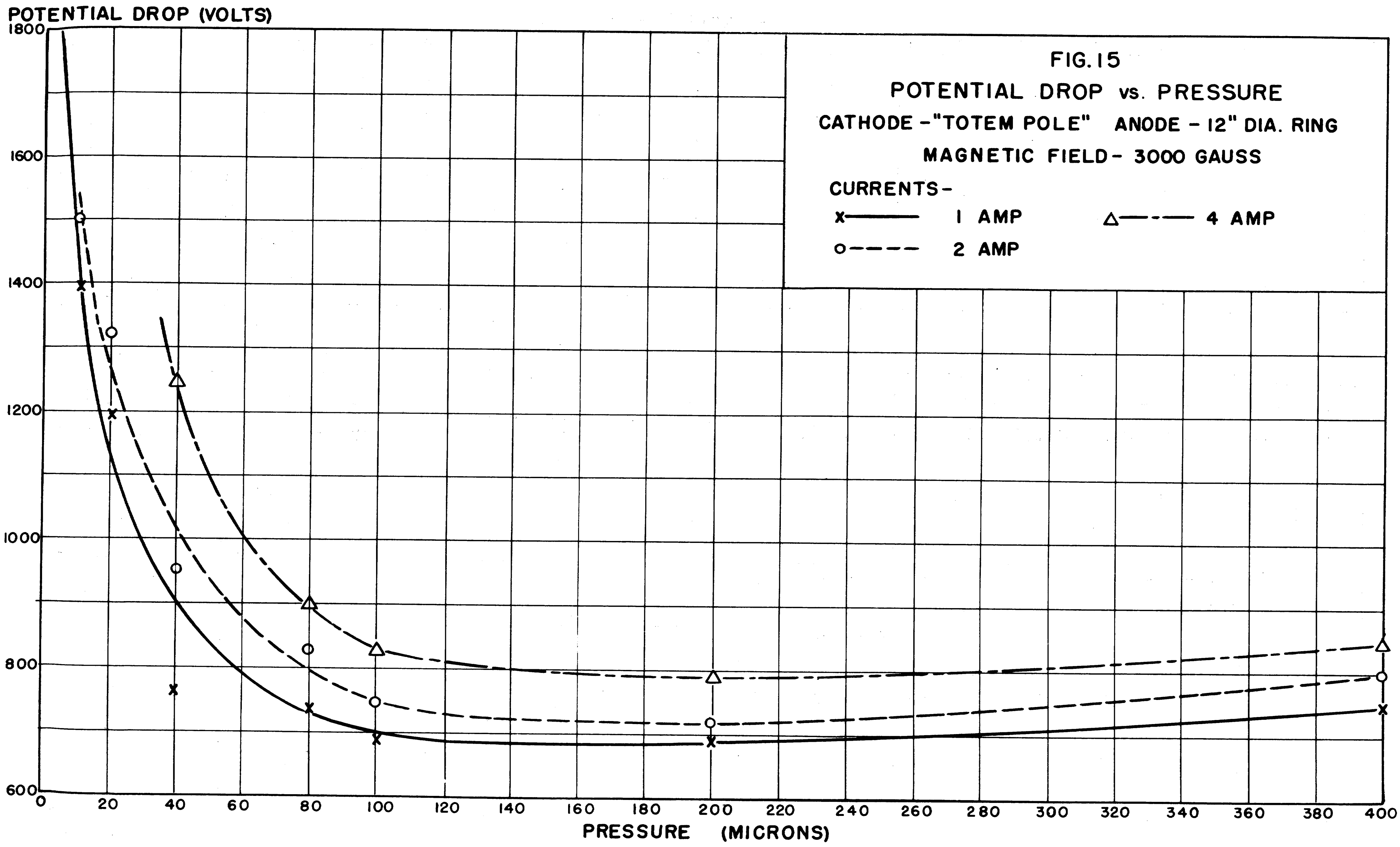


FIG. 15
POTENTIAL DROP vs. PRESSURE
CATHODE - "TOTEM POLE" ANODE - 12" DIA. RING
MAGNETIC FIELD - 3000 GAUSS
CURRENTS -
1 AMP
2 AMP
4 AMP
POTENTIAL DROP (VOLTS)
1800
1600
1400
1200
1000
800
600
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
380
400
PRESSURE (MICRONS)

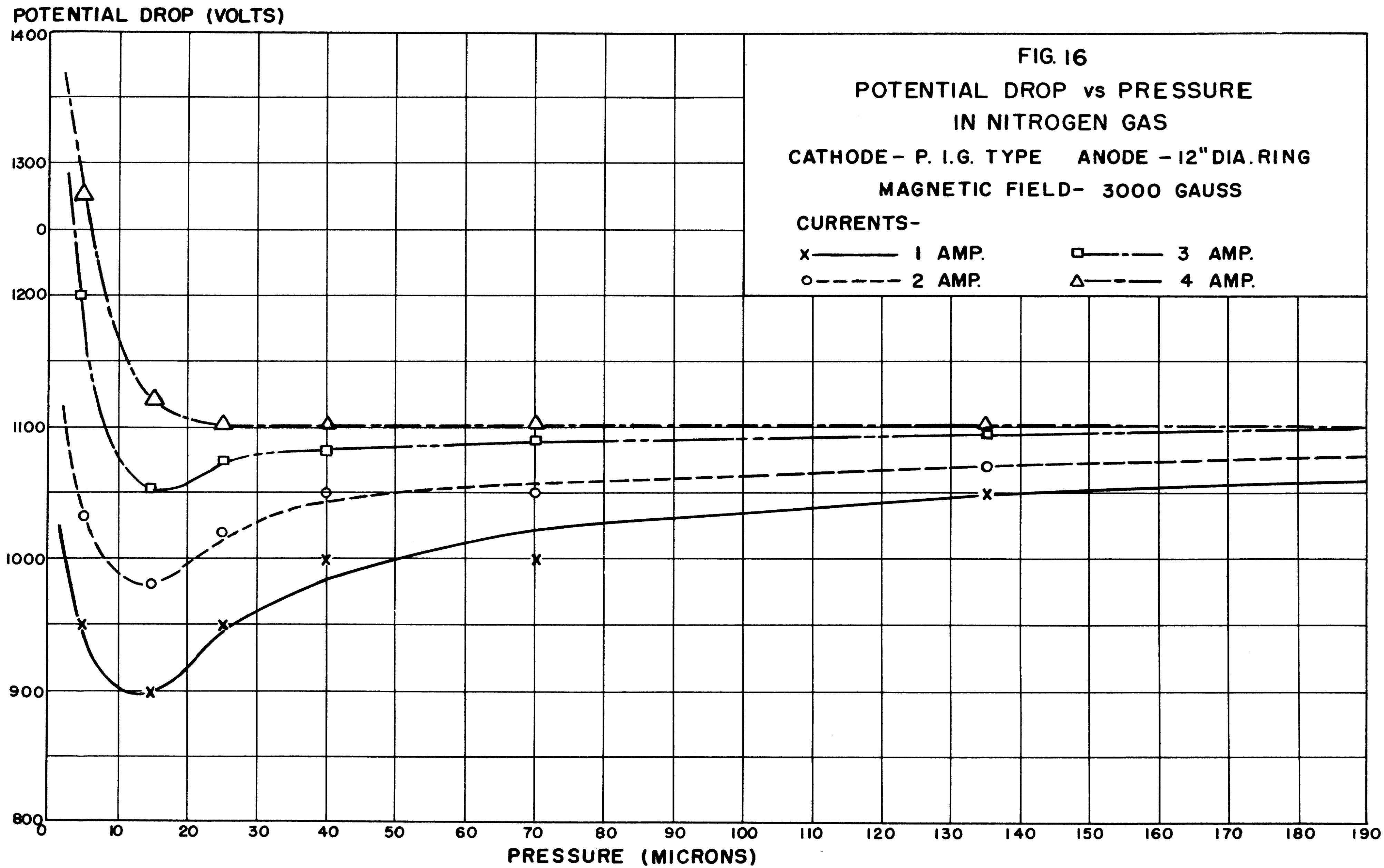
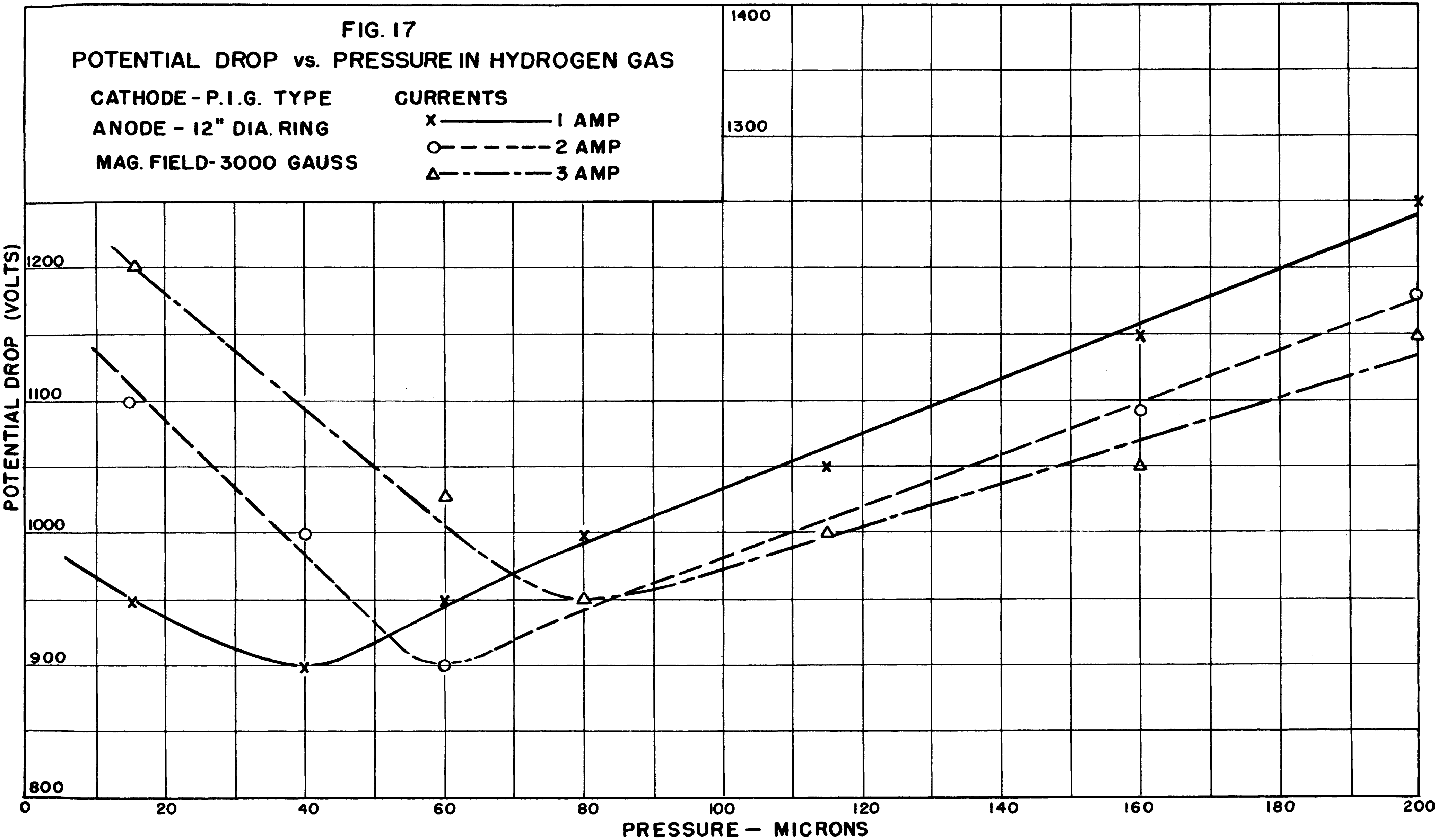


FIG. 16
POTENTIAL DROP vs PRESSURE
IN NITROGEN GAS
CATHODE - P. I. G. TYPE ANODE - 12" DIA. RING
MAGNETIC FIELD - 3000 GAUSS
CURRENTS -
1 AMP.
2 AMP.
3 AMP.
4 AMP.
POTENTIAL DROP (VOLTS)
1400
1300
0
1200
1100
1000
900
800
PRESSURE (MICRONS)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190

FIG. 17

POTENTIAL DROP vs. PRESSURE IN HYDROGEN GAS



Plasma Probe Measurements

Electrical probe measurements were taken to obtain specific information about the voltage gradients and the variation of potential throughout the discharge. The interpretation of probe data is much more difficult when a magnetic field is involved. However, much useful information which does not require a complete analysis can be obtained from such measurements.

Description of Probe and Circuit

The probe data was obtained by means of the planar probe shown in the sketch in Fig. 22. It is made of 50 mil. tantalum ribbon, and has an area of 1.8 cm^2. The probe was supported at the end of a glass tube, and the surface of the exposed area was always oriented perpendicular to the electric field. Data was obtained for probe positions along a radial line. For all of these measurements a P.I.G. cathode was used. The type of cathode has little bearing on the radial plasma gradient or the anode fall of potential, but the gradients in the cathode region, of course, depend on the type of cathode.

The electrical circuit used in connection with the probe is shown in Fig. 18. All voltages were referred to the anode potential (ground) purely for convenience of instrumentation. In plotting potential distribution curves, "floating probe" (zero probe current) potentials were used. This was the voltage read on the voltmeter when the potentiometer shown in Fig. 18 was adjusted for zero current as indicated by the ammeter. The significance of these potentials and the use of the 0.5μf capacitor will be discussed later.

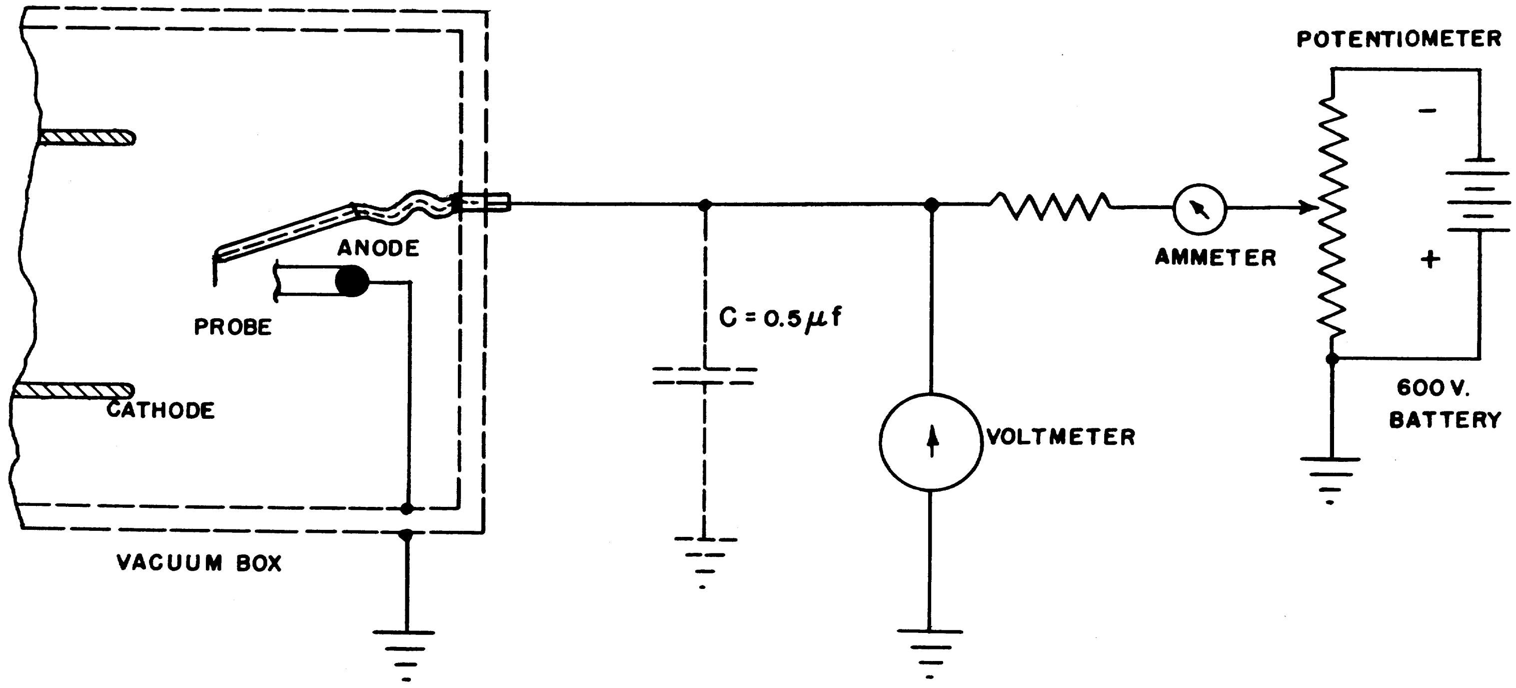


FIG. 18
PROBE CIRCUIT

Probe Current-Voltage Curves

In order to interpret the data obtained from these measurements, it was necessary to determine the relation between the probe voltage and the current to the probe. Figs. 19 and 20 show the current-voltage curves obtained at different gas pressures. For these tests the probe was located midway between the cathode and anode. The probe voltages were originally recorded with respect to the anode or ground potential, but in plotting the curves, the "zero current" or "floating" potential was used as a reference voltage. This method of presenting the data provided a better comparison of the effect of changing pressures than was the case when the anode potential was taken as a reference.

The probe characteristics (Figs. 19 and 20) have the same general form as those obtained in a discharge in the absence of a magnetic field. The outstanding difference is that when the field is present the probe current increases more gradually for positive probe potentials and shows no rapid rise in current, as is usually the case, when the probe reaches plasma potential. The Langmuir type of probe analysis is not applicable when a magnetic field is present, since other factors become prominent which far overshadow the usual effects. The most important of these factors is the presence of plasma oscillations. A study of these oscillations is quite involved; however, a qualitative consideration of these fluctuations is important for a correct interpretation of potential distribution plots. The characteristics presented in Fig. 19 and 20 represent time averages of these rapidly varying voltages.

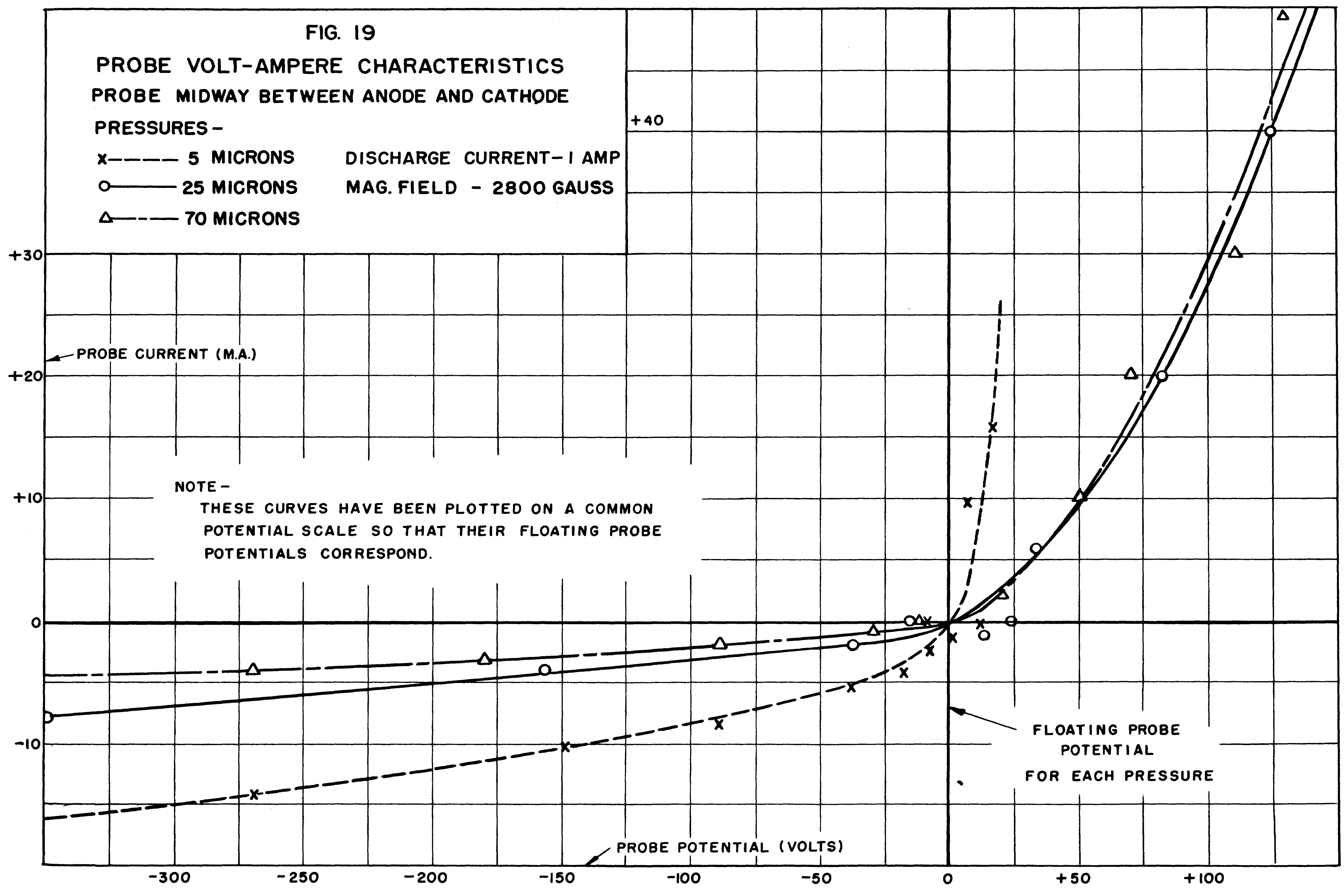


FIG. 19
PROBE VOLT-AMPERE CHARACTERISTICS
PROBE MIDWAY BETWEEN ANODE AND CATHODE
PRESSURES-
5 MICRONS
25 MICRONS
70 MICRONS
DISCHARGE CURRENT- I AMP
MAG. FIELD - 2800 GAUSS
NOTE-
THESE CURVES HAVE BEEN PLOTTED ON A COMMON POTENTIAL SCALE SO THAT THEIR FLOATING PROBE POTENTIALS CORRESPOND.
PROBE CURRENT (M.A.)
PROBE POTENTIAL (VOLTS)
FLOATING PROBE POTENTIAL FOR EACH PRESSURE
+40
+30
+20
+10
0
-10
-300
-250
-200
-150
-100
-50
0
+50
+100

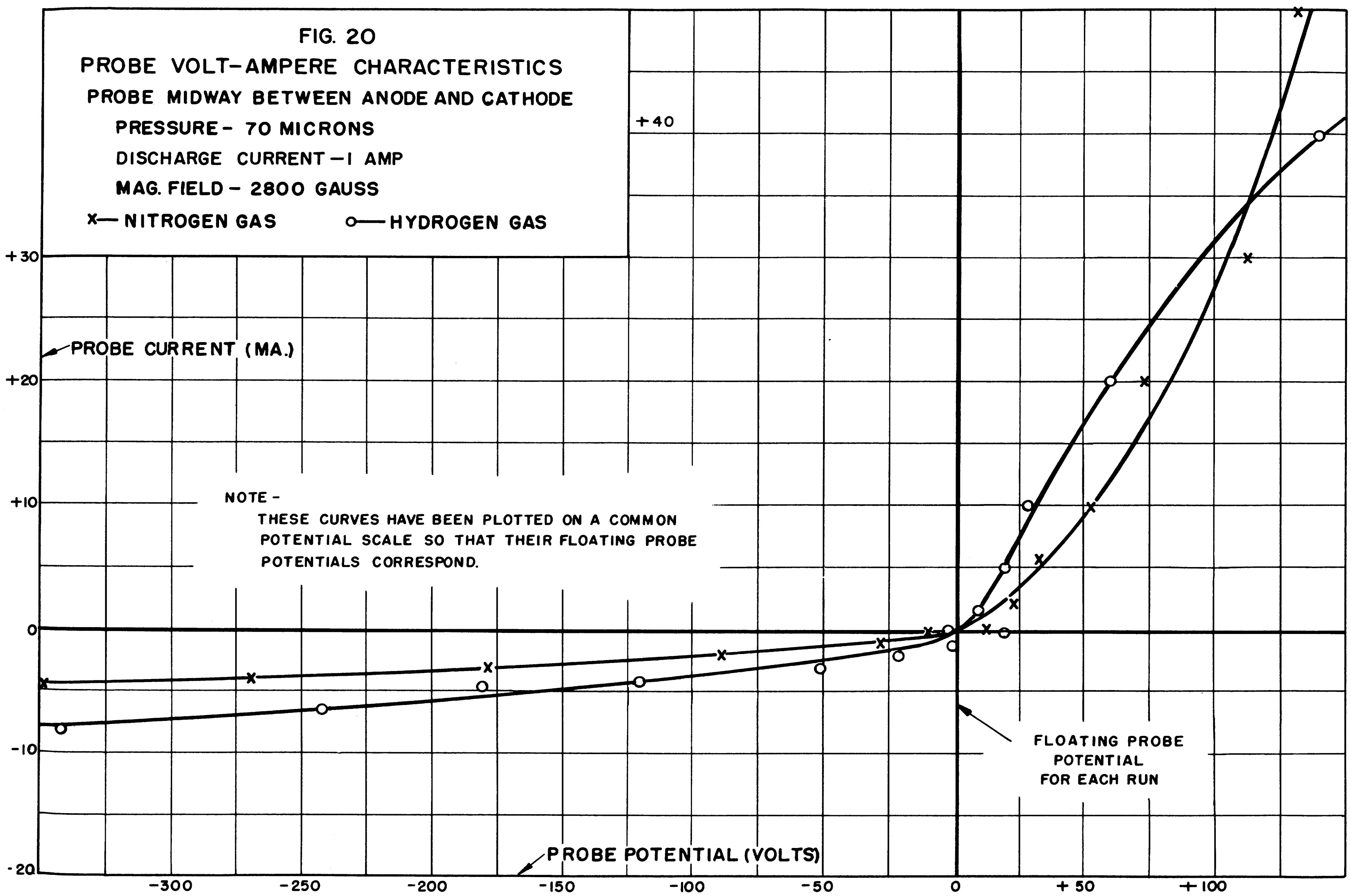


FIG. 20
PROBE VOLT-AMPERE CHARACTERISTICS
PROBE MIDWAY BETWEEN ANODE AND CATHODE
PRESSURE - 70 MICRONS
DISCHARGE CURRENT - 1 AMP
MAG. FIELD - 2800 GAUSS
x— NITROGEN GAS
o— HYDROGEN GAS
PROBE CURRENT (MA.)
+40
+30
+20
+10
0
-10
-20
NOTE -
THESE CURVES HAVE BEEN PLOTTED ON A COMMON POTENTIAL SCALE SO THAT THEIR FLOATING PROBE POTENTIALS CORRESPOND.
FLOATING PROBE POTENTIAL FOR EACH RUN
PROBE POTENTIAL (VOLTS)
-300
-250
-200
-150
-100
-50
0
+50
+100

Equivalent Circuit of Probe

The interpretation of probe data is facilitated by substituting an equivalent circuit for the probe and plasma, which will simulate the observed behavior. One possible circuit for this purpose is shown in Fig. 21. The elements inside the dotted box represent the probe and plasma. Since the current-voltage relations of Figs. 19 and 20 resemble diode characteristics, the essential feature of the equivalent circuit is a diode-rectifier. The resistances R_1 and R_2 are merely details which enhance the equivalence. R_1 determines the slope of the curve for negative probe potentials, and R_2 influences the slope for positive probe potentials. The capacitor, C, outside of the dotted box, represents the stray wiring capacity of the probe lead and extra capacity which was, at times, added to the circuit, such as the 0.5 μf capacitor shown in Fig. 18. I_o and V_o correspond to the current and voltage read on the meters shown in Fig. 18. In the presence of plasma oscillations (noise), this circuit acted as a peak charging rectifier. If V_o was adjusted so that the average probe current, I_o, was zero (floating probe), then V_o was the average voltage across the capacitor C. If the capacitor was very small, V_o approximated the average plasma potential. If, on the other hand, C was quite large, V_o was some average value of the peak negative plasma fluctuations.

This effect was strikingly demonstrated by placing a 0.5 μf capacitor between the probe and ground. This caused the floating probe potential to be about 75 volts more negative than when no capacitor was used. Thus, it is reasonable to assume that the noise amplitude was somewhat greater than 150 volts peak to peak. This was roughly the same order of magnitude as the average potential with respect to the anode. These results were

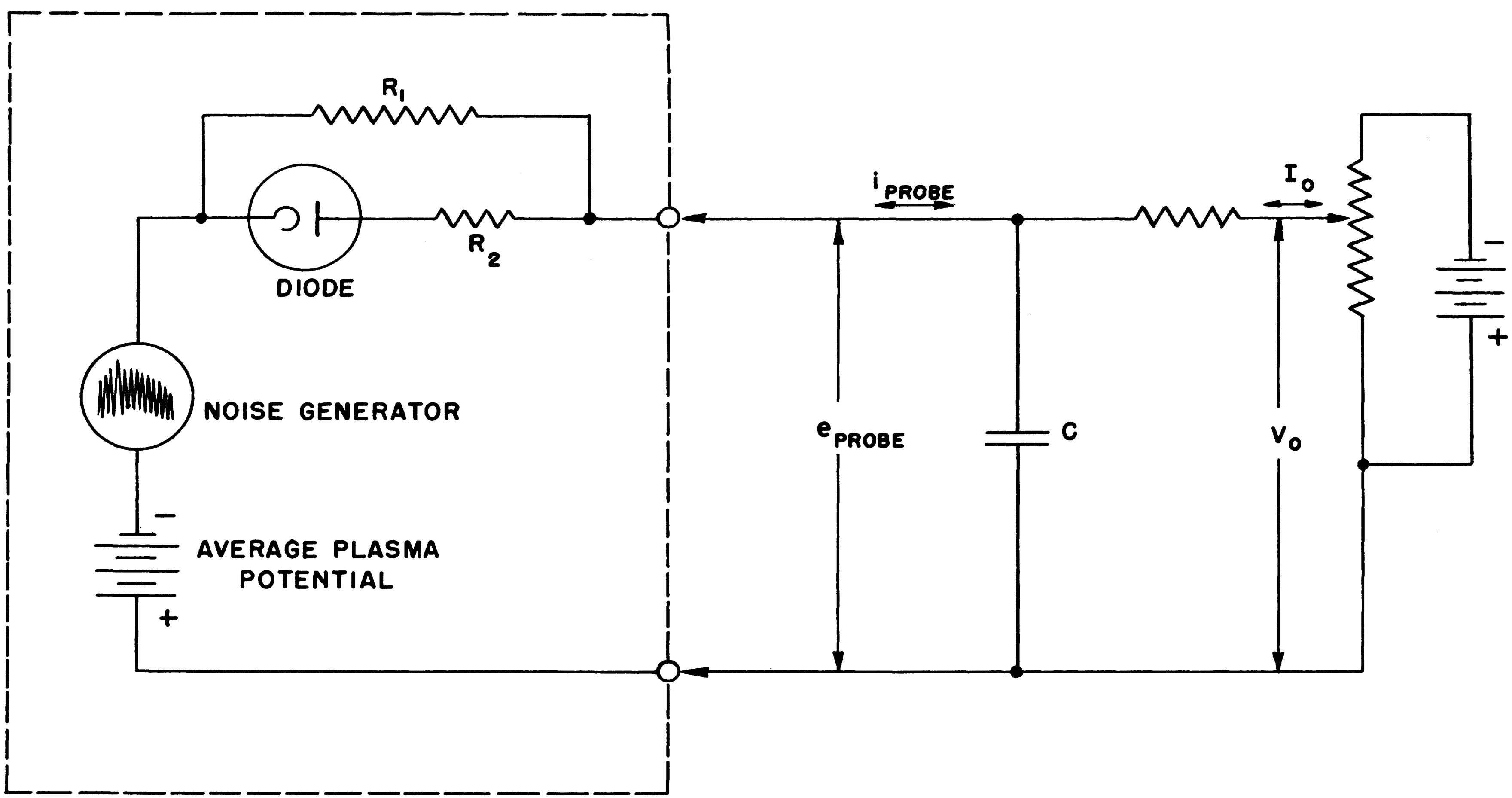


FIG. 21
EQUIVALENT PROBE CIRCUIT

confirmed by oscillographic observations. The relation between the plasma potential and the floating probe potential is influenced much more by plasma oscillations than by the electron temperature, which is usually the case.

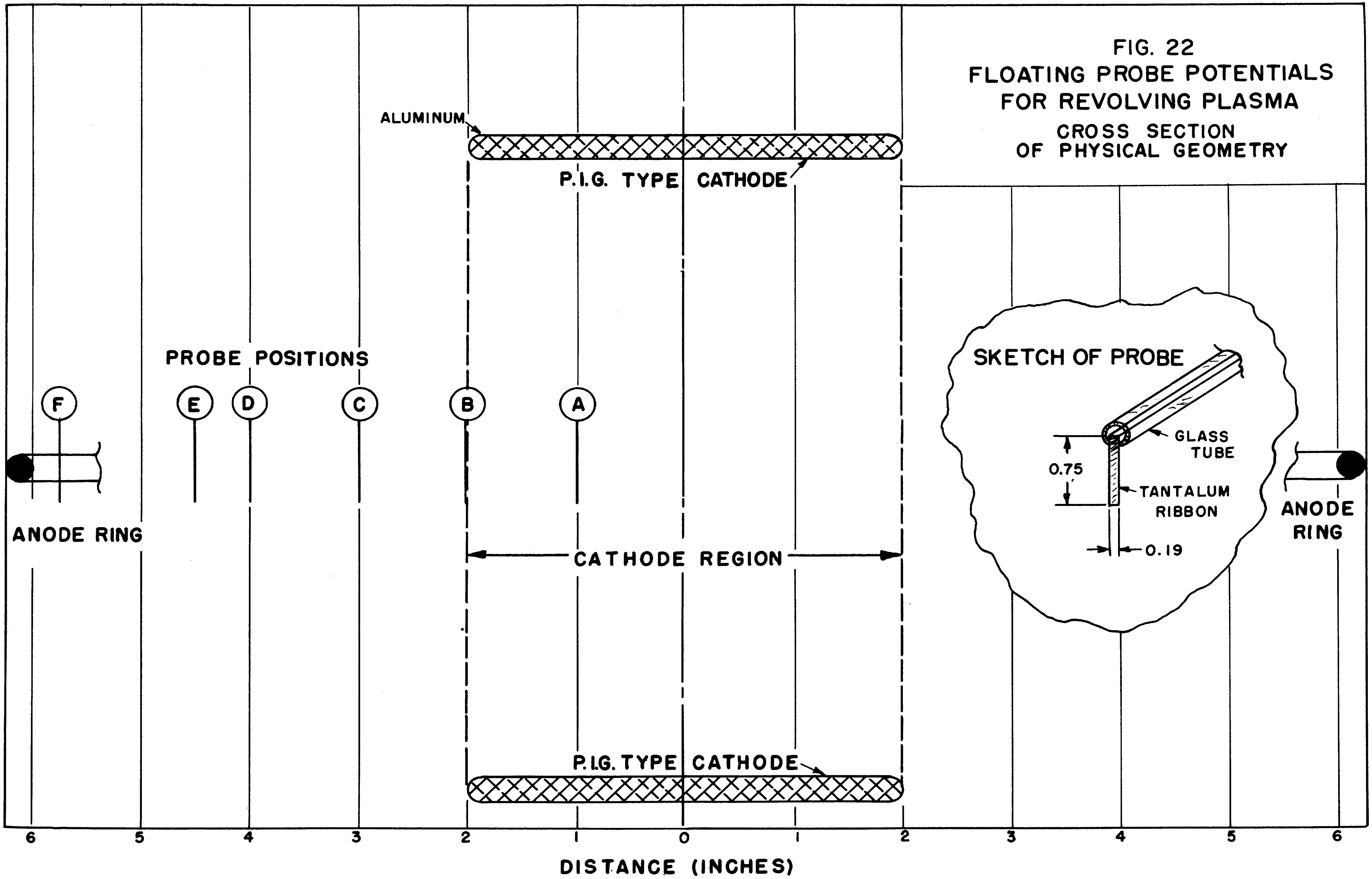
Potential Distribution Curves

An interpretation of the floating probe potential distributions shown in Figs. 23 and 24, can be made on the basis of the above considerations. If the stray wiring capacity of the probe circuit is small, the floating probe potential will be slightly negative, but not very different than the average space potential. A comparison of floating probe potentials for different locations, pressures, and discharge currents is meaningful, provided the effect due to the noise is relatively constant from one situation to the next. The error introduced by neglecting the changes in the character of these fluctuations, thereby assuming a constant "correction factor" , is small compared with the voltages and gradients being measured.

Fig. 22 is a cross section through the discharge and shows the positions in which the probe was placed for these measurements. Fig. 23 is a plot of potential versus probe position. The values obtained for four different pressures can be seen directly below the corresponding probe positions of Fig. 22. Fig. 24, which is similar to Fig. 23, is a comparison of the potentials for two different values of discharge current.

The anode fall of potential can be seen in the extreme left of Figs. 23 and 24 (between the anode and the probe position "F.") The indicated voltage drop is probably greater than actually existed, since the voltages measured by the probe were more negative than the average plasma potential, for the reasons given above. One interesting feature of this

FIG. 22
FLOATING PROBE POTENTIALS
FOR REVOLVING PLASMA
CROSS SECTION
OF PHYSICAL GEOMETRY



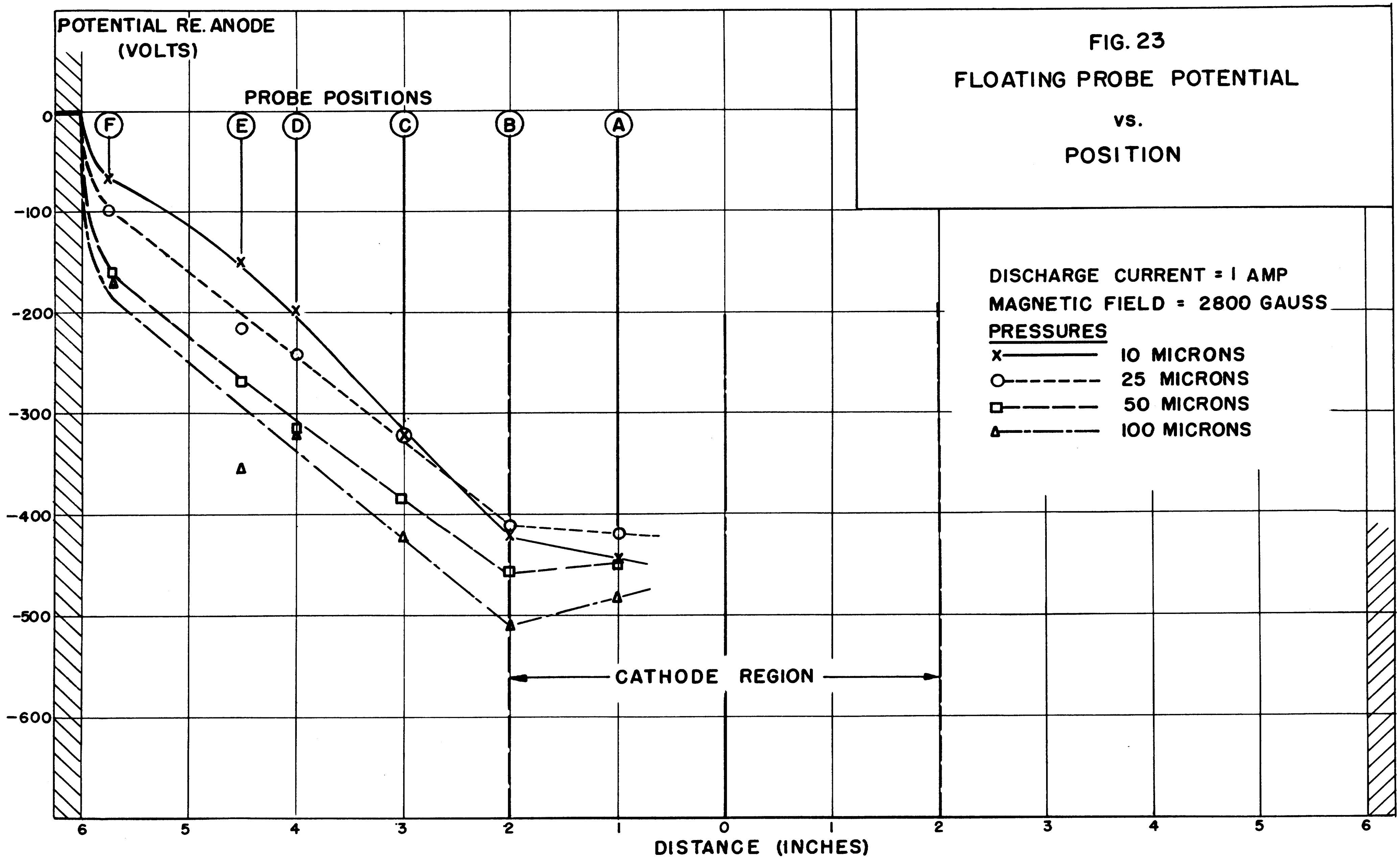


FIG. 23
FLOATING PROBE POTENTIAL
vs.
POSITION
POTENTIAL RE. ANODE (VOLTS)
PROBE POSITIONS
F
E
D
C
B
A
DISCHARGE CURRENT = 1 AMP
MAGNETIC FIELD = 2800 GAUSS
PRESSURES
10 MICRONS
25 MICRONS
50 MICRONS
100 MICRONS
CATHODE REGION
0
-100
-200
-300
-400
-500
-600
6
5
4
3
2
1
0
1
2
3
4
5
6
DISTANCE (INCHES)

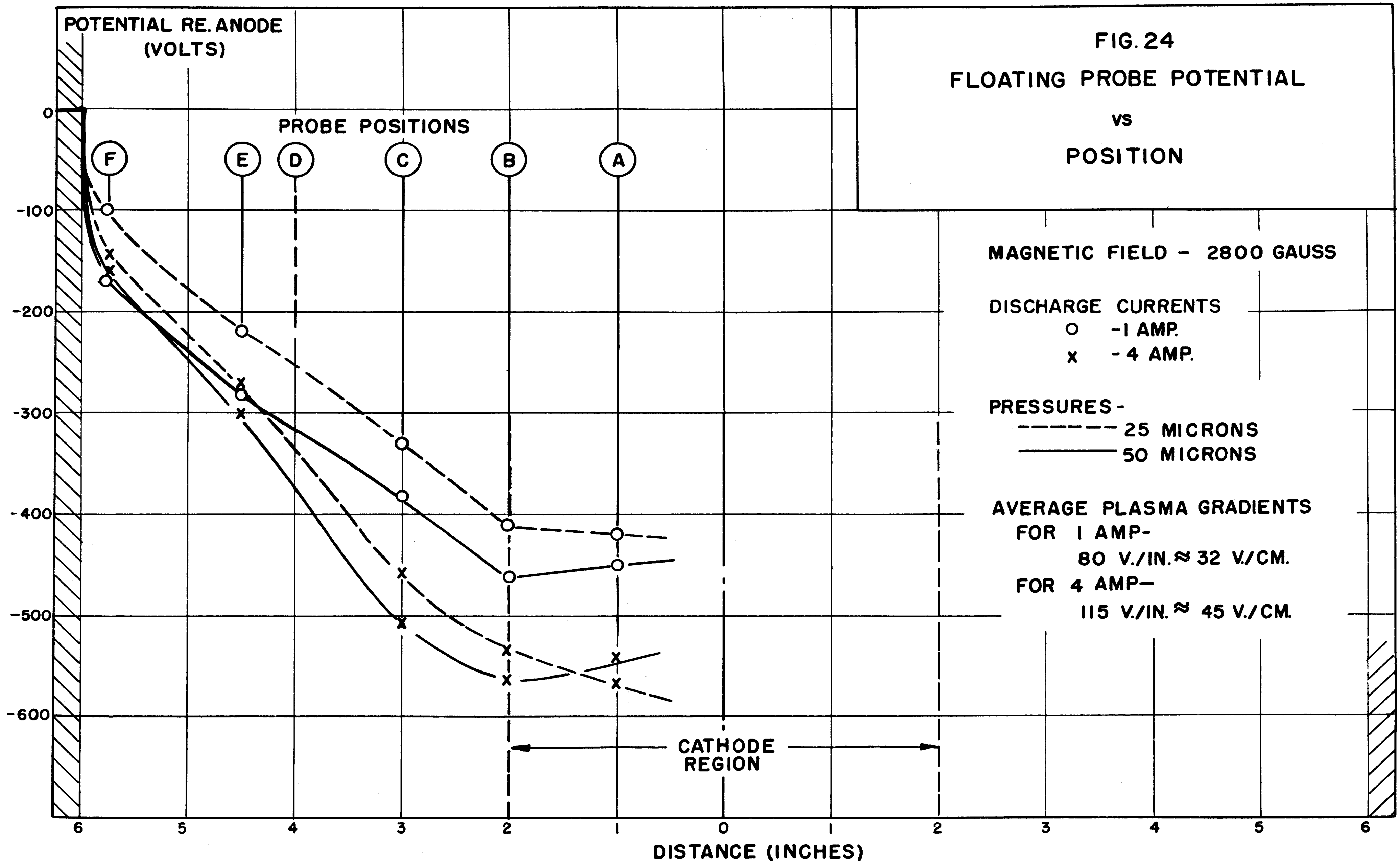


FIG. 24
FLOATING PROBE POTENTIAL
vs
POSITION
POTENTIAL RE. ANODE
(VOLTS)
0
-100
-200
-300
-400
-500
-600
PROBE POSITIONS
F
E
D
C
B
A
MAGNETIC FIELD - 2800 GAUSS
DISCHARGE CURRENTS
O -1 AMP.
x -4 AMP.
PRESSURES -
25 MICRONS
50 MICRONS
AVERAGE PLASMA GRADIENTS
FOR 1 AMP-
80 V./IN. ≈ 32 V./CM.
FOR 4 AMP-
115 V./IN. ≈ 45 V./CM.
CATHODE
REGION
6
5
4
3
2
1
0
1
2
3
4
5
6
DISTANCE (INCHES)

gradient adjacent to the anode is its variation with pressure. In some cases, the variation of anode fall of potential is sufficient to account for the change in total discharge voltage, observed in the voltage-pressure characteristics (Figs. 15 and 16) for pressures above the minimum potential point.

The radial plasma gradient for various pressures and currents is shown in Figs. 23 and 24. This is probably very close to the true picture since the effect of the noise should be relatively constant for different probe positions. The magnitude of this gradient is important in connection with the discussion in the following section. This electric field strength is related to the power input per unit volume of the discharge and thus is an indication of the power used in generating wind. It is interesting to note that this gradient is substantially constant over a wide range of pressures. As is to be expected, the gradient increases with increasing current.

The plasma gradients parallel to the magnetic field from the mid-plane of the discharge toward the end surfaces were also observed. Midway between the cathode and anode this gradient was about 1 to 1.5 v/cm. Thus an equipotential surface in the plasma has a roughly cylinderical shape.

The cathode potential is not shown in Figs. 23 and 24. Because of the geometry of a P.I.G. cathode, this feature is difficult to show in this type of potential profile.

Trajectories of Ions and Electrons

The potential distribution curves presented earlier, tend to convey the impression that the drift motion of the ions and electrons is determined by steady state, constant, potential gradients. Actually, the random "noise" voltage which is picked up by a measuring probe, has about the same magnitude as the d-c voltage. The presence of fluctuating plasma voltage gradients makes an analysis of particle motion very difficult.

A good deal of useful information can be obtained by ignoring the random fluctuations, and considering the behavior of a charged particle when it is in a vacuum; also, when it is in an environment of only neutral molecules. A study of the motion of ions and electrons on this basis, furnishes valuable qualitative concepts and explains a number of experimental facts. As soon as better methods of instrumentation have been developed, more quantitative studies will be possible.

The following numerical calculations of charged particle behavior are based on typical values of the electric and magnetic field strengths associated with the revolving plasma experiments. It is assumed that the charged particles are in a uniform electric field between parallel plates (instead of the cylindrical geometry of the revolving plasma).

Magnetic field strength = 3000 gauss = 0.3 weber per sq. meter

Electric field strength = 5000 volts per meter

Velocity with which a charged particle (electron or ion), moves transverse to electric field (in the absence of collisions) $= \frac{E}{B} = 1.67 \times 10^4$ meters/sec

Kinetic energy of an ion when moving at a velocity of $\frac{E}{B}$	=	40 electron volts
Kinetic energy of an electron when moving at velocity of $\frac{E}{B}$	=	7.8×10^{-4} electron volts
Angular velocity of the rotational motion of an ion in magnetic field	=	$\frac{Bq_i}{m_i} = 1.03 \times 10^{6}$ radians/sec
Angular velocity of the rotational motion of an electron in a magnetic field	=	$\frac{Bq_e}{m_e} = 5.3 \times 10^{10}$ radians/sec

The motion of an ion or electron in the presence of crossed electric and magnetic fields, and in the absence of collisions, is characterized by:

(1) A velocity component transverse to the electric field equal to $\frac{E}{B}$, where E is the electric field strength and B is the magnetic field strength.

(2) A rotational component which takes place at the "cyclotron frequency" defined by $\frac{1}{2\pi} \frac{Bq}{m}$, where q is the electric charge and m is the mass. The radius of the rotational orbit is dependent on the initial conditions and the kinetic energy with which the particle started its motion.

(3) A motion parallel to the magnetic field which is not affected by the magnetic field.

Fig. 25 is an illustration of the cycloidal trajectories of an ion in electric and magnetic fields. These trajectories are drawn to scale and are

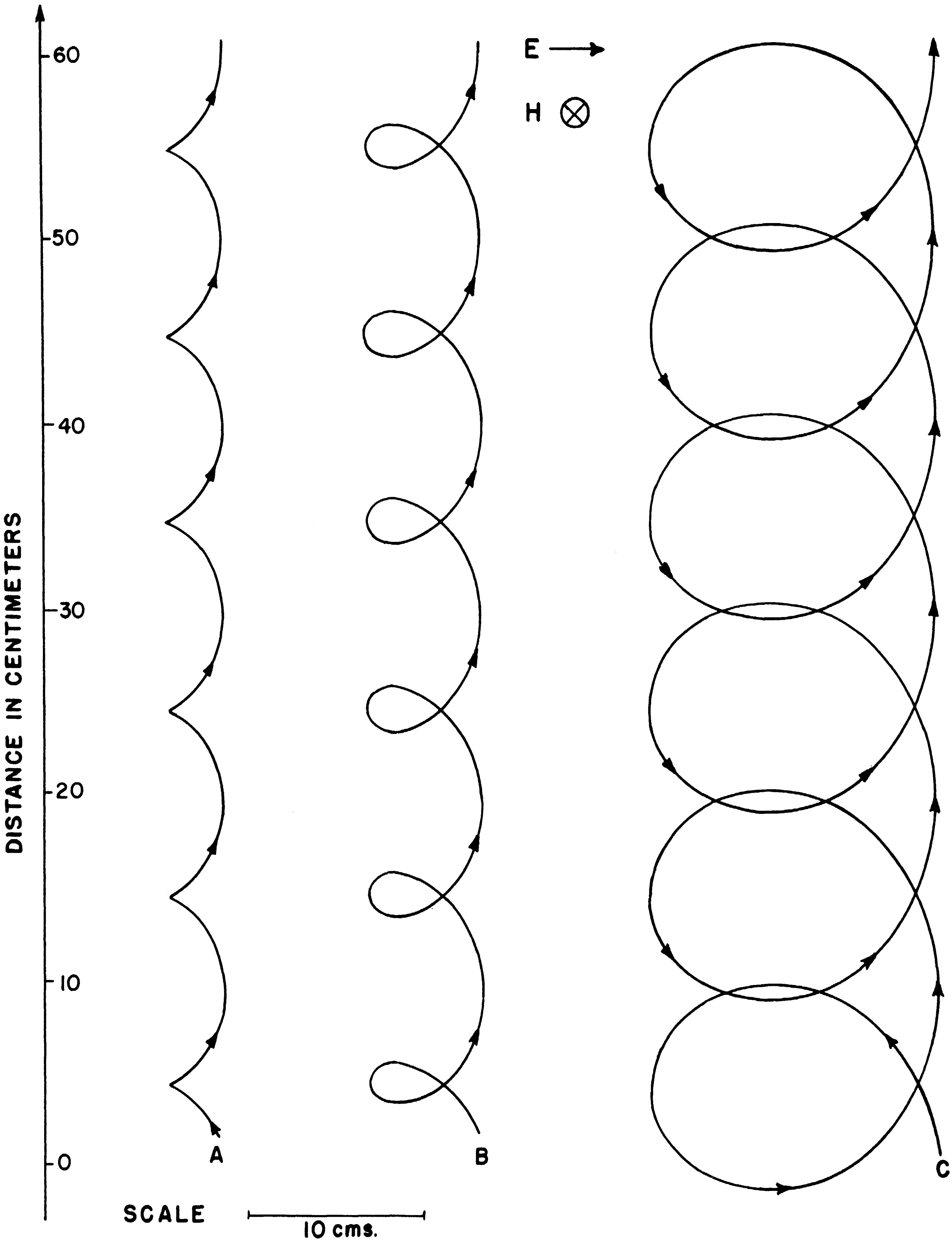


FIG. 25

TRAJECTORIES OF NITROGEN ION, N_2^+, IN ELECTRIC FIELD OF 50 VOLTS/cm, AND TRANSVERSE MAGNETIC FIELD OF 3000 GAUSS quantitatively representative of the field strengths used in the revolving plasma experiments. Curve "A" represents a nitrogen ion which starts from rest. It has a component of velocity of 1.67×10^4 meters/sec in the "E cross H" direction (perpendicular to both E and H), and a rotational velocity of 1.67×10^4 meters/sec. Curve "B" represents an ion with a rotational velocity twice as large or 3.33×10^4 meters/sec. In curve "C" the rotational velocity is five times as large as in curve "A", or 8.33×10^4 meters/sec. In all three cases the velocity in the "E cross H" direction is constant.

The electrons tend to follow cycloidal trajectories along equipotentials in the same direction as the ions, but the radius of rotation is very much smaller. In the case of an electron starting from rest, the radius of rotation is 3.2×10^{-5} cm. If the electron has six electron volts of energy, the radius is still only 2.8×10^{-3} cm, and the electron makes 5050 orbital revolutions while moving one centimeter in the "E cross H" or transverse direction.

In a vacuum, the net motion of an ion or an electron is entirely transverse to the electric field. In an actual plasma, the presence of collisions and plasma oscillations permits a component of motion in the direction of the electric gradient. In all cases, a net drift motion or "drain" parallel to the electric field is associated with a mechanism for the dissipation of energy.

In the case of electrons, the transfer of energy by collisions is very much less effective than for ions. Part of the energy loss associated with electron "drain" is utilized in producing excitation and ionization. There is not enough information available as yet, to evaluate the magnitude of the electron "drain" in the revolving plasma. However, it does appear that a substantial fraction of the total electrode current is due to electron motion parallel to the electric field. (This statement is supported by the fact that there is no large anode drop of potential, as there would be if the positive ions were the only important current carriers.)

In the case of ions, the energy loss at each collision is so large that the explanation as to where the ion energy is dissipated, is more amenable to mechanistic interpretation. An approximate calculation as to the relative magnitudes of the two components of ion drift motion can be made by means of two simplifying assumptions. This calculation will be pertinent to a subsequent discussion of tangential current flow and radial pressure variation in the revolving plasma. The two assumptions are:

(a) The average ion velocity is very high compared to that of neutral molecules, so that the neutral molecule is relatively stationary by comparison. Then, on the average, the ion looses half of its kinetic energy at the end of each mean free path.

(b) The radius of curvature of the ion path is sufficiently large so that one mean free path is only a fraction of one cusp of the cycloid, and there is no important change in the direction of motion between collisions.

These conditions can be evaluated by means of Fig. 25. Let us assume that the mean free path is 2 cm. Then the average ion energy, even when starting from rest, is many electron volts compared to a fraction of an electron volt for the average neutral molecule. Also, 2 cm. corresponds to a small fraction of one rotation. On the basis of these approximations, it is believed that the ion drift in the direction of the electric gradient can be calculated roughly, by neglecting the effect of the magnetic force. The ion drift velocity towards the cathode, in the absence of a magnetic field, can be computed by means of a relation given in Reference (1), page 484. According to this reference, when the velocity of the ion is large compared to the thermal velocity of the gas, the ion drift velocity is approximately given by the relation

$$v_i \cong \frac{5.93 \times 10^5}{\sqrt{m_g/m_e}} \times 0.59 \sqrt{E\ell_g} \text{ meters/sec.}$$

where m_g/m_e is the ratio of the masses of the ion and electron;

ℓ_g is the mean free path.

For a nitrogen N_2^+ ion, this relation reduces to

$$v_i \cong 1550 \sqrt{E\ell_g} \text{ meters/sec.}$$

The average magnetic force on this ion which causes a drift in the "E cross H" direction is

$$Bq_i v_i = 1550\, Bq_i \sqrt{E\ell_g}$$

where q_i is the charge of the ion.

The force due to the electric field which causes a drift towards the cathode is $q_i E$. It is reasonable to assume that the flow is viscous in nature, so that the ratio of the drift velocities is approximately equal to the ratio of the forces. Hence,

$$\frac{v_{(\text{electric})}}{v_{(\text{E cross H})}} \cong \frac{q_i E}{1550\, Bq_i \sqrt{E\ell_g}} \cong \frac{E}{1550\, B \sqrt{\ell_g}}$$

In the conditions associated with the revolving plasma experiments, a value of 0.02 meters for ℓ_g in the above relations makes the two components of drift velocity approximately of equal magnitude. This value of ℓ_g corresponds to nitrogen gas at 2000°K and 18 microns pressure, and is roughly equivalent to the experimental conditions previously reported.

This calculation indicates that the component of ion drift velocity in the direction of the electric field produces a force in the gas towards the cathode which has, in some cases, the same magnitude as the force exerted by the ions on the air stream.

Efficiency of "Crossed-Fields" Discharge as a Wind Generator

The efficiency with which electrical energy is converted into kinetic energy of mass motion of the gas stream can be approximately evaluated. This calculation indicates the order of magnitude of the maximum efficiency of the fundamental electrical process. If the approximate efficiency is known, the molecular velocity ratio can be estimated.

The following discussion will be based on the type of experimental situation shown in Fig. 26, where the force due to the discharge is used to accelerate air in a linear manner, into a vacuum chamber. In this situation, it is obvious that the heating of the air, while it is expanding out of the throat-shaped electrodes, will have an important thermodynamic effect on the conditions in the air stream. The overall efficiency is closely dependent on the velocity and density of the air entering and leaving the region of the discharge. In order to keep this present discussion as general as possible and to avoid arbitrary assumptions as to the thermodynamic effects, it will be convenient to consider only the electrical factors which are operating inside a small volume, $\Delta x \, \Delta y \, \Delta z$, which remains stationary with respect to the electrodes. This volume element is small enough so that the percentage changes of velocity and density across it are quite small. However, in describing the effects inside this small volume, it will be convenient to use dimensions as if it were a unit volume, but it must be noted that the volume element is very much smaller than a cubic meter. In this simplified analysis, the stream velocity, U, is independently determined by factors outside the volume element.

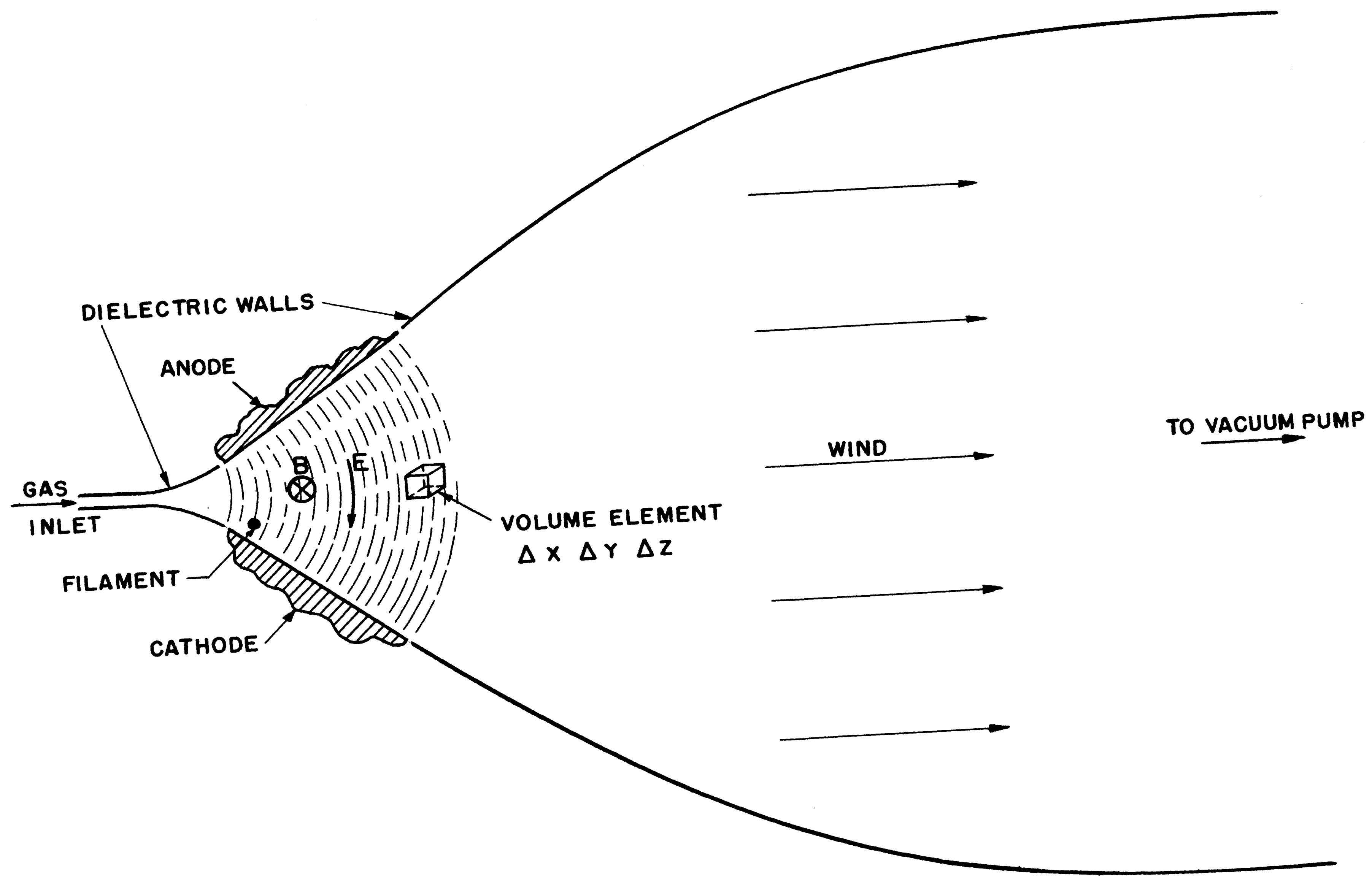


FIG. 26 DESIGN OF AIR ACCELERATOR USING THE "CROSSED—FIELDS" DISCHARGE AND THE "LINEAR-FLOW" ELECTRODE ARRANGEMENT.

The efficiency of the electrical processes within this unit volume can be defined as:

$$\eta = \frac{\text{(force on air stream) x velocity of stream)}}{\text{(electric power input)}}$$

The magnetic force exerted on the air stream is given by

$$F = B\ J \text{ newtons}$$

where B is flux density in webers/m^2, and J is current density in amperes/m^2.

The electric power input is

$$P_{in} = E\ J \text{ watts}$$

where E is the electric gradient.

The useful power delivered to the air stream is equal to the product of force and stream velocity

$$P_s = B\ J\ U \text{ watts}$$

where U is the stream velocity.

Hence, the efficiency is

$$\eta = \frac{B\ J\ U}{E\ J} = \frac{U}{E/B}$$

The ratio E/B is the velocity with which the ions and electrons move in the "E cross H" direction, in a vacuum. In a low density stationary gas, the ion velocity is less than the E/B value in a vacuum, but mobility calculations indicate it is usually a substantial fraction of the limiting E/B value; of the order of 30 to 50 percent. When $E/B \gg U$, the neutral molecules are relatively stationary compared to the ions. Since the denominator of the expression for efficiency is relatively constant for all small values of U (as is indicated by the experimental data), the efficiency is proportional to the stream velocity.

If the stream velocity U is observed to be substantially increased, then the value of E/B must also have increased because Ampere's law requires that the passage of current through the volume be accompanied by a magnetic force. The only way this force can be exerted is for the voltage to rise high enough so that the ions and electrons have sufficient velocity to deliver momentum to the stream. It would thus appear that in the expression for efficiency, there is some limiting maximum efficiency, above which any increase of U in the numerator is accompanied by a corresponding increase of E in the denominator.

Experimental values of E/B are of the order 1.7×10^4 meters/sec. If the stream velocity is 1700 meters/sec, the value of efficiency as derived above, is 10 percent. This implies that the molecules of the stream are receiving nine times as much random thermal energy as directed kinetic energy due to mass motion.

Tangential Current

In the study of the revolving discharge, there is one type of experiment which would furnish very important information regarding the drift motion of the charged particles. This experiment involves a measurement of the net current flowing in the "E cross H" direction. The ion and electron drift motions perpendicular to both the electric and magnetic fields are both in the same direction; however, since these particles carry charges of opposite sign, the two types of current tend to cancel, and the net transport of electric charge depends on which component is predominate. The volume densities of positive and negative charges are very nearly equal, so that it might appear that the (presumed) higher mobility of the electrons would result in a predominance of electron current. Perhaps this is the case, but it can also be mentioned that the electron motion is almost entirely in the form of orbital rotation, so that the drift motion between collisions is severly restricted. A measurement of the net current which is flowing transverse to the electric field, would be of considerable help in explaining the basic mechanisms.

In the revolving plasma, this net transverse or "tangential" current is of special significance, because it is related to factors which produce a radial force or pressure on the gas. This radial pressure may be an important factor in counteracting the centrifugal force which tends to throw the neutral molecules out of the revolving discharge. The existence of the radial pressure can be explained either on the basis of a macroscopic or a mechanistic point of view, and the conclusions are the same.

From the macroscopic standpoint, the flow of a net current through the gas in a tangential or circumferential direction reacts with the magnetic field to produce a force. If the cathode is at the center and if the tangential current due to electrons predominates over that due to ions, the result is a force toward the cathode.

The mechanistic explanation is as follows: The movement of ions toward the cathode transfers momentum to neutral molecules, while the motion of electrons toward the anode transfers momentum in the opposite direction. It was shown on Page 4 that, in the absence of a magnetic field, the momentum due to these two types of motion is usually equal and opposite, and the net force is zero. However, it is unlikely that this is the case when a strong magnetic field is present. It seems reasonable that the proportion of ion current to electron current is substantially larger when the magnetic field is present, and the momentum given to the gas in the direction of the cathode is greater than the momentum toward the anode.

Experiments were undertaken to simultaneously measure the pressure in the center of the discharge and at the circumference, to determine the magnitude of the radial pressure gradient. The measured pressure at the edge was approximately twice the value at the center. When the electrical polarity was reversed so that the anode was in the center, there was no definite indication of a change in this pressure ratio. Hence, the experiments do not yet confirm the hypothesis of a force toward the cathode. The experimental measurements however, involved uncertainties due to temperature gradients which are discussed in the section under instrumentation.

During the past year much time and effort have been spent in attempting to obtain an electrical measurement of the tangential current. Although these tests have not been successful, a description of some of the problems encountered may be of interest to other workers in this field.

Experimental Measurement of Tangential Current

In the revolving plasma the tangential current can be considered as flowing in a closed loop inside a torus or doughnut-shaped region. An obvious method of measuring this current is to utilize some magnetic effect such as inductive coupling. Of several schemes that were considered, the most satisfactory method involved the use of a pickup coil surrounding the discharge, and arranged in such manner as to produce a maximum amount of inductive coupling with the tangential component of current. The voltage induced in the pickup coil is related to the tangential current by the equation $e = M\,di/dt$, where M is the coefficient of mutual inductive coupling, and di/dt is the rate of change of current.

The experimental procedure involved two separate measurements as follows:

(1) a measurement of the value of M between the plasma and the pickup coil;

(2) a measurement of the induced voltage as the tangential current was varied.

The value of M was determined by a separate calibration experiment in which a solid conductor was substituted for the gaseous plasma. The mutual inductance between this solid conductor and the pickup coil was then determined in a conventional manner. The second measurement could not be carried out in a straightforward manner since the tangential component of current could not be controlled directly. The current to the electrodes (radial current) could be controlled easily, but the relation between radial and tangential currents was not known. It was assumed that this difficulty could be avoided by turning the discharge current completely on or off while observing the induced voltage pulse on the oscilloscope. Then, if the induced voltage waveform were integrated over the time interval, the steady state value of tangential current could be calculated.

Experimental Apparatus

Fig. 27 is an illustration of the apparatus which was assembled to investigate the measurement of tangential current. The anode for the discharge was a copper cylinder. Apickup coil of ten turns was wound around it. In order to prevent this cylinder from acting as a short circuiting turn, it was split lengthwise and an insulating segment of mica was inserted, as shown in Fig. 27. The electrical connections to the anode and cathode were brought out together, at a point opposite to the split in the anode. This was done in order to minimize any extraneous pickup from currents flowing in these leads

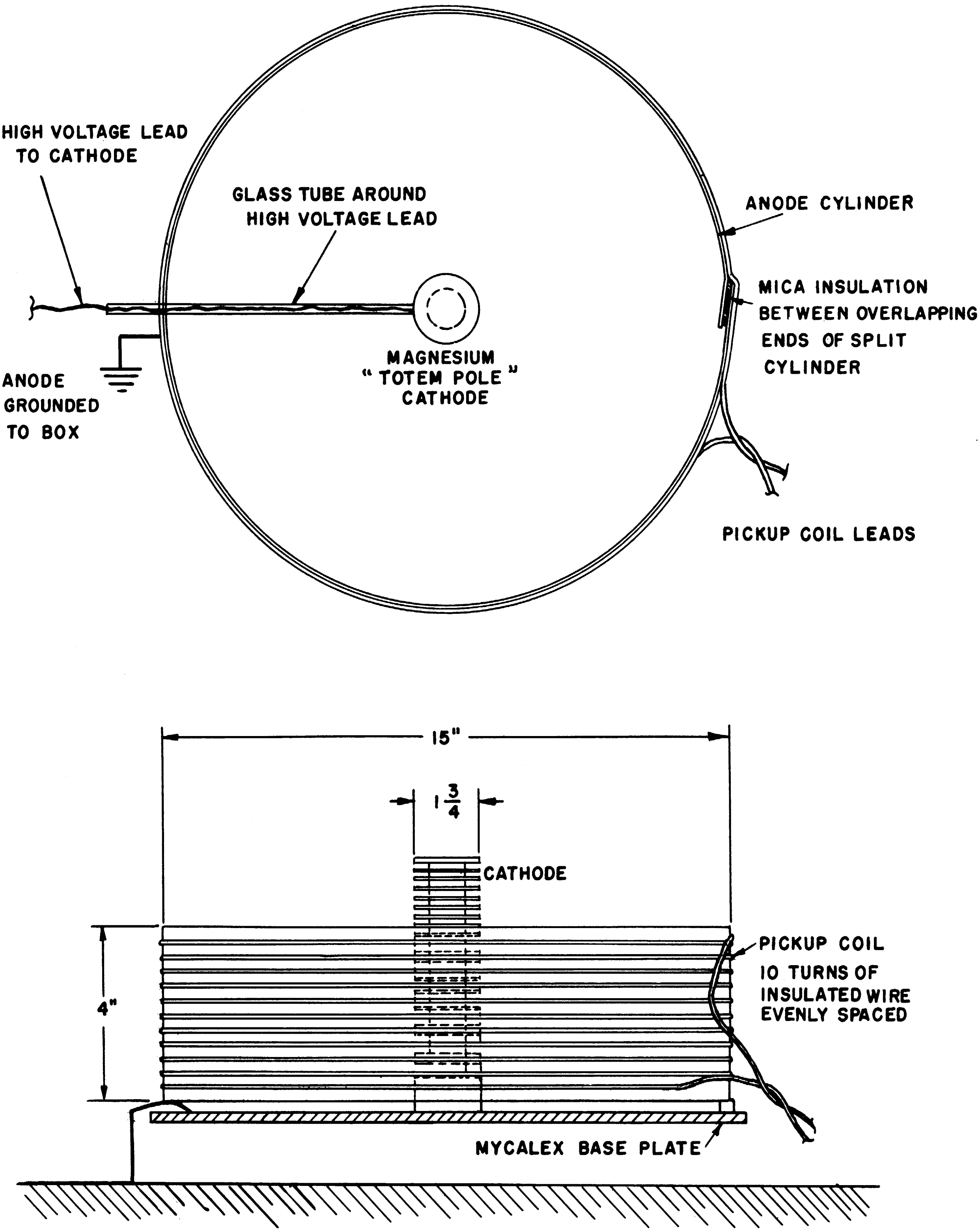


FIG. 27

APPARATUS FOR OBSERVING TANGENTIAL CURRENT and in the cylinder. The voltage generated in the pickup coil was observed on a Tecktronix type 513-D oscilloscope.

It was also necessary to observe the electrode current (radial current component) on the oscilloscope, and to modulate this current in a known manner. A two-ohm shunt or "current viewing resistor" was placed in the ground return from the vacuum box, and provided a convenient means for monitoring the radial current. This current was controlled by varying the grid bias on two type 813 pentode tubes. These were placed in the high voltage lead between the power supply and the cathode, so that the current could be turned on or off, or modulated in any desired fashion. Pentodes were used for this purpose because of their desirable constant current characteristics.

Measurement of Mutual Inductance

The tangential component of current can be regarded as consisting of many individual ring-shaped current elements distributed throughout a doughnut-shaped region. It is obvious that these elements will not all be equally effective for inducing a voltage in the pickup coil. This variation in coupling was investigated by placing a single-turn loop of copper wire inside the anode cylinder, and measuring the mutual inductance, M, between this loop and the pickup coil as the diameter and position of the loop were changed. For the pickup coil shown in Fig. 27, there was very little change in the value of M when a particular loop was moved about, except when it was at the ends of the cylinder where the value of M dropped by about 20 percent.

The values of M for this arrangement were determined experimentally by measuring the voltage induced in the pickup coil for a known current in the primary loop. An audio oscillator and a high fidelity audio amplifier were used to produce a sinusoidal current in the primary. The voice coil terminals of the amplifier provided a low impedance source and furnished a current of several amperes at frequencies as high as 100 Kc.

Another method for measuring M, which did not prove satisfactory, involved an exponential decay of current in the primary loop. This produced an abrupt change in the induced voltage at the start of the exponential, which resulted in "ringing" in the pickup circuit. This "ringing" was a consequence of the inductance of the pickup coil and the input capacity of the scope, and could be eliminated only by considerable damping resistance.

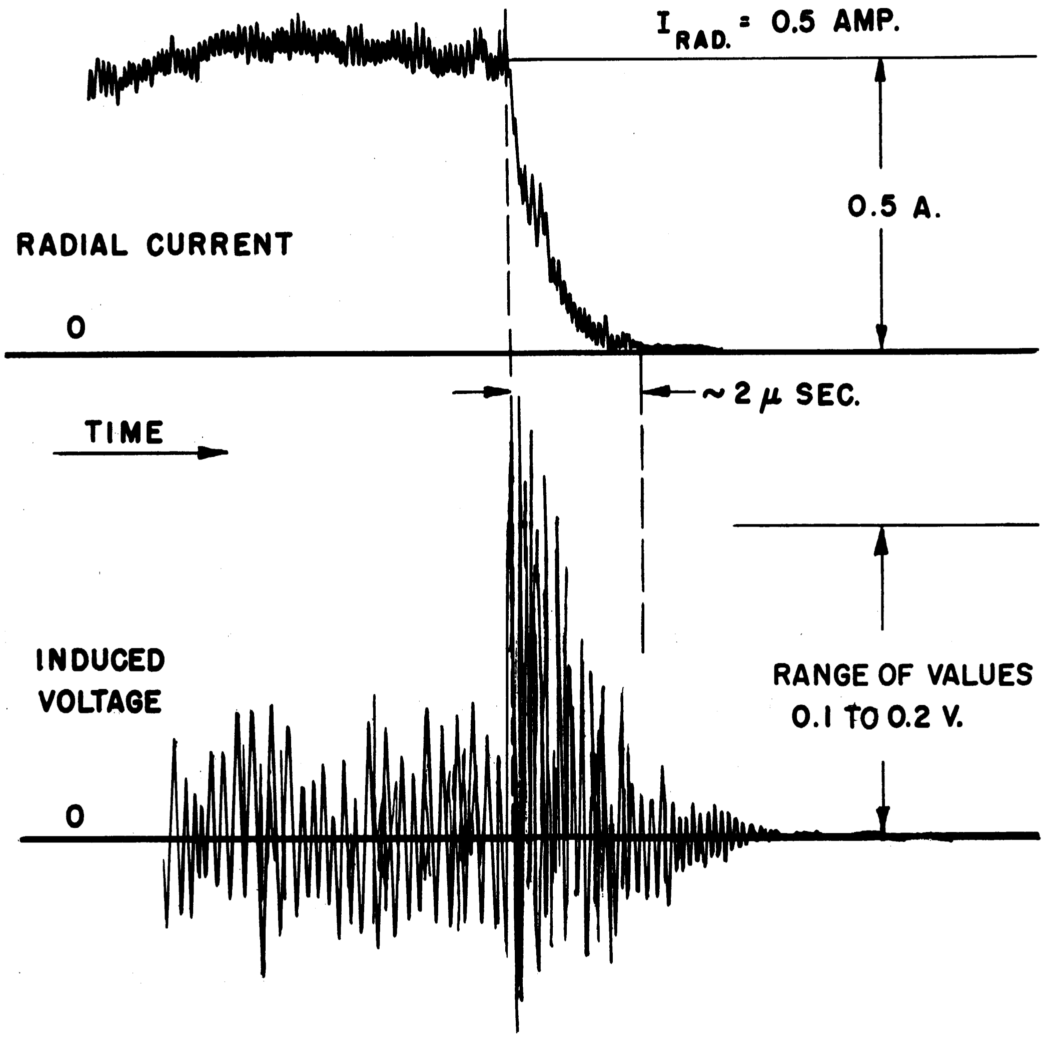
The experimental values of M derived from these experiments ranged from 1.0 to 1.5 microhenrys for the ten-turn pickup coil. The smaller values were at the higher frequencies. This value of M was smaller than the value obtained when the measurements were made outside the vacuum box. In order to minimize the shielding or short-circuiting effect of the metal box, the anode cylinder and pickup coil were positioned midway between the top and bottom of the chamber.

Experimental Results

Before any attempt was made to analyze the voltages induced in the pickup coil, careful observations were made as to the fluctuations of the current and voltage. When the power supply employed an LC filter, which would simulate a constant voltage power supply, it was found that the current would fluctuate wildly. Similarly, when an RL filter combination was employed, to simulate a constant current power supply, the voltage would then fluctuate in the same random manner. For this experiment, it seemed more desirable to obtain a constant current condition for the discharge (radial) current. The most satisfactory results were achieved by using two type 813 pentodes in parallel, which were connected in series with the power supply. With these tubes it was possible to obtain a steady discharge current of over an ampere, which had less than 50 milliamperes of 120 cycle ripple and less than 25 milliamperes of random "noise".

The voltages induced in the pickup coil were observed on the oscilloscope while the discharge was running with the current maintained relatively steady by means of the pentodes. Even with this constant radial current, there was a large amount of "noise" or "hash" induced in the pickup coil. The "noise" was quite random and contained a wide range of frequencies and amplitudes, indicating that the tangential current is not directly related to the radial current, but is subject to the random fields or "plasma oscillations" known to be present. Evidence given in the literature[(2)] substantiates that this type of discharge contains plasma oscillations of large amplitude and thus is a potent source of noise. The presence of this unavoidable and excessive noise presents a serious problem for the successful measurement of tangential current.

Attempts were made to measure the tangential current in the hope that the desired induced voltage might be of greater amplitude than the "noise". In one series of tests, the radial current was interrupted by abruptly biasing the grids of the pentodes below cutoff. A small condenser was placed across the switch contacts so that the radial current decayed to zero exponentially. Fig. 28 is a sketch of the waveforms of the radial current and also the voltage induced in the pickup coil, as observed on the oscilloscope. The same difficulties existed here as in the case of an earlier use of an exponential decay of current for measuring M. "Ringing" in the pickup circuit in addition to the excessive "noise" made these measurements of dubious value. If the observed effects were truly due to the tangential current, the values given in Fig. 28 indicate that the tangential current has roughly the same order of magnitude as the radial current.



$$I_{TAN} = \Delta i = \frac{E}{M}\Delta t$$

E = 0.1 TO 0.2 VOLTS

$M = 1 \times 10^{-6}$ HENRY

$\Delta t = 2 \times 10^{-6}$ SEC.

$I_{TAN} \approx$ 0.2 TO 0.4 AMP.

FIG. 28

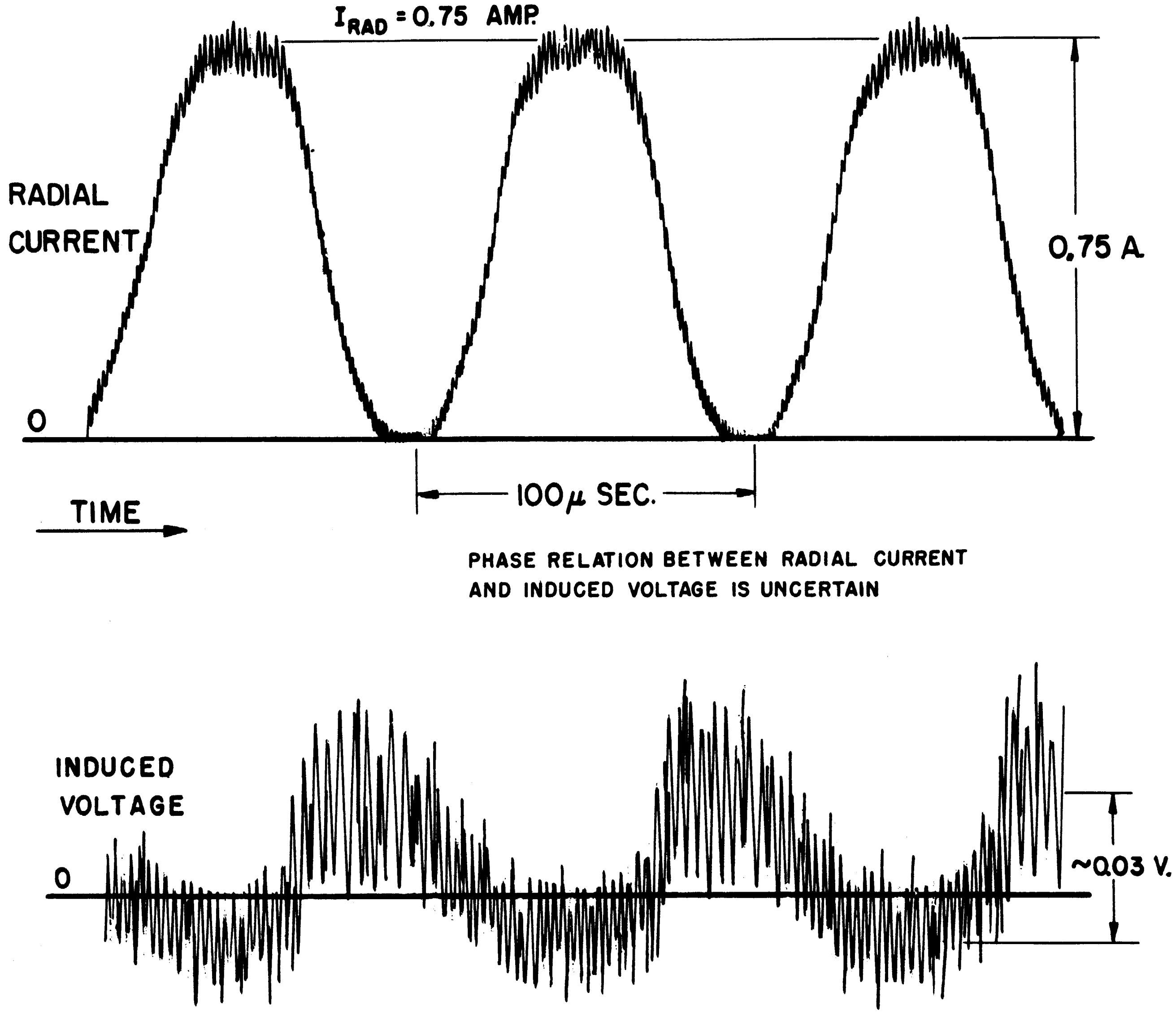
OSCILLOSCOPE WAVEFORMS RESULTING FROM EXPONENTIAL DECAY OF RADIAL CURRENT

Measurements were made with a sine-wave modulation of the radial current, but these were only slightly more meaningful. With this method, an oscillator and amplifier were used to drive the grids of the pentodes. The amplitude of drive was adjusted so as to bring the radial current to zero on the negative half cycles. It was desirable to use as high a frequency as possible so as to make the value of the induced voltage large. However, too high a frequency would presumably not permit the tangential current to "slow down" completely to zero during the negative half cycles, because of inertia effects due to the momentum of the ions and the gas. Rough calculations based on momentum considerations indicate that this "decay time" has an order of magnitude of about 100 microseconds. Fig. 29 shows sketches of the waveforms observed when the radial current was modulated at a frequency of 10 Kc. The experimental values illustrated in this figure indicate that the tangential current is about half of the radial current. In view of all of the uncertainties and extraneous effects known to be present, this figure is an indication of the order of magnitude only.

The amplitude and character of the noise were considerably affected by the pressure in the box and by the strength of the magnetic field. Above 20 microns of pressure, the magnitude of the noise became so great that it was impossible to make any measurements. The wave forms shown in Figs. 28 and 29 were taken at pressures between 10 and 20 microns. Increasing the magnetic field strength also increased the amplitude of the noise.

Suggestions for Further Work

Certain improvements would be necessary in order to obtain a quantitative measurement of tangential current. A more effective method of controlling and modulating the radial current would be essential. The use of pentodes



$$I_{TAN} = \frac{E}{M\omega} \; \frac{\sin \omega t}{\cos \omega t}$$

$$E \approx 0.03 \text{ VOLTS}$$

$$M = 1.2 \times 10^{-6} \text{ HENRY}$$

$$\omega = 63 \times 10^{3} \text{ RAD./SEC.}$$

$$\underline{I_{TAN} \approx 0.4 \text{ AMP}}$$

FIG. 29

OSCILLOSCOPE WAVEFORMS RESULTING FROM SINE WAVE MODULATION OF RADIAL CURRENT is probably the right answer but considerable effort should be made to optimize the conditions in this circuit. Sine-wave modulation should be used, and a sharply peaked bandpass filter, tuned to the modulation frequency should be employed in the pickup circuit. Much of the "noise" and "ringing" which causes the present trouble could be eliminated with this filter. The time and expense necessary to carry out these modifications did not seem warranted, especially as the outcome was subject to considerable doubt.

Limitations of the Wind Velocity Produced by the Revolving Plasma

The revolving discharge is a convenient experimental arrangement for studying discharges in crossed fields, and the resulting wind effects. The wind velocities which are attainable are limited by factors related to the rotational motion of the gas. The "linear flow" arrangement shown in Fig. 26, appears to be a much more promising method of producing very high wind velocities. In discussing the various factors which limit the wind velocity of a rotating gas, it is necessary to consider the situation at different ranges of gas pressure.

At very low pressures, where the mean free path has the same magnitude as the dimensions of the apparatus, the behavior is analogous to a "smooth bore" magnetron. Both ions and electrons revolve around the cathode at a velocity of approximately E/B. The percentage of ionization is very high since neutral molecules tend to be thrown out of the discharge in a tangential direction. The voltage gradient is also very high, since there are not enough collisions or plasma oscillations to permit the charged particles to move rapidly in the direction of the electric field. The large increase in voltage at the lower pressures can be seen in Fig. 15.

At intermediate pressures, where the mean free path is of the order of a centimeter, the voltage drop is less because of the increased number of collisions, and consequently the magnetic force required by Ampere's rule can be exerted more readily. The velocity is limited by the fact that the molecules are continually losing their momentum to the walls or boundaries of the discharge. In order for the neutral molecules to circulate around the cathode, they must continually receive momentum in a radial direction toward the center. A substantial fraction of the total number of molecules will, on the average, lose all of their circumferential momentum to the walls at the end of each mean free path. When these conditions are present, the wind velocity can be estimated by means of a calculation involving the electrical force on the discharge and the rate at which tangential momentum is lost to the walls by the impinging molecules. Unfortunately, this calculation requires that the gas temperature be known, and this has not yet been measured.

At the higher gas pressures, where the gas behaves as a fluid, the effects due to centrifugal force and turbulence are very important. These effects will be considered separately.

Turbulence

The rotational motion of a fluid inside a stationary cylinder is inherently an unstable type of motion. This has been demonstrated both mathematically and empirically.[11] Above a certain critical velocity, the flow breaks up into whirlpools and eddies which greatly increase the driving force necessary to maintain the rotation.

In the cylindrical geometry of the revolving discharge, this same type of turbulence is undoubtedly involved whenever the mean free path is short enough for the gas to behave as a fluid. Experimental observations have been made (at pressures of 300-500 microns), in which small flags were used to indicate the direction of the wind in various parts of the discharge. These flags do not prove conclusively that the motion is turbulent, but they do indicate a circulation of the gas due to the slower rate of rotation near the top and bottom of the cylinder. This slower rate of rotation is caused by the viscous drag of the gas against the two end surfaces. The fastest rate of rotation is in the region midway between the two end surfaces of the cylinder and the centrifugal force is also greatest in this region, so that the air moves radially outward in the middle and returns near the ends. This circulation results in a loss of rotational momentum.

Centrifugal Force Limitations

A rapidly revolving mass of gas experiences a centrifugal force which causes the density and pressure to increase at the larger radii. When a gas is revolving with uniform angular velocity, this variation is expressed by the relation

$$P/P_o = \exp\ (M\omega^2 r^2)/(2kT)$$

where M is the molecular weight;

ω is the angular velocity;

P_o is the pressure at the axis of the cylinder; and

P is the static pressure at any radius r.

For a given ω, this equation predicts a large variation of pressure with radius, since the factor r^2 appears in the exponent. Fig. 30 is a curve showing the density ratio ρ/ρ_o (or the pressure ratio P/P_o) as a function of ωr (the tangential velocity at any radius). It is to be noted that this ratio depends only on the velocity at the point where the pressure is measured.

This curve can be used as an order of magnitude estimate as to the wind velocity associated with the experimental conditions described in this report. In one typical experiment, a pressure at the center of the discharge of 30 microns was recorded simultaneously with a pressure of 60 microns at the circumference. If the gas temperature is assumed to be

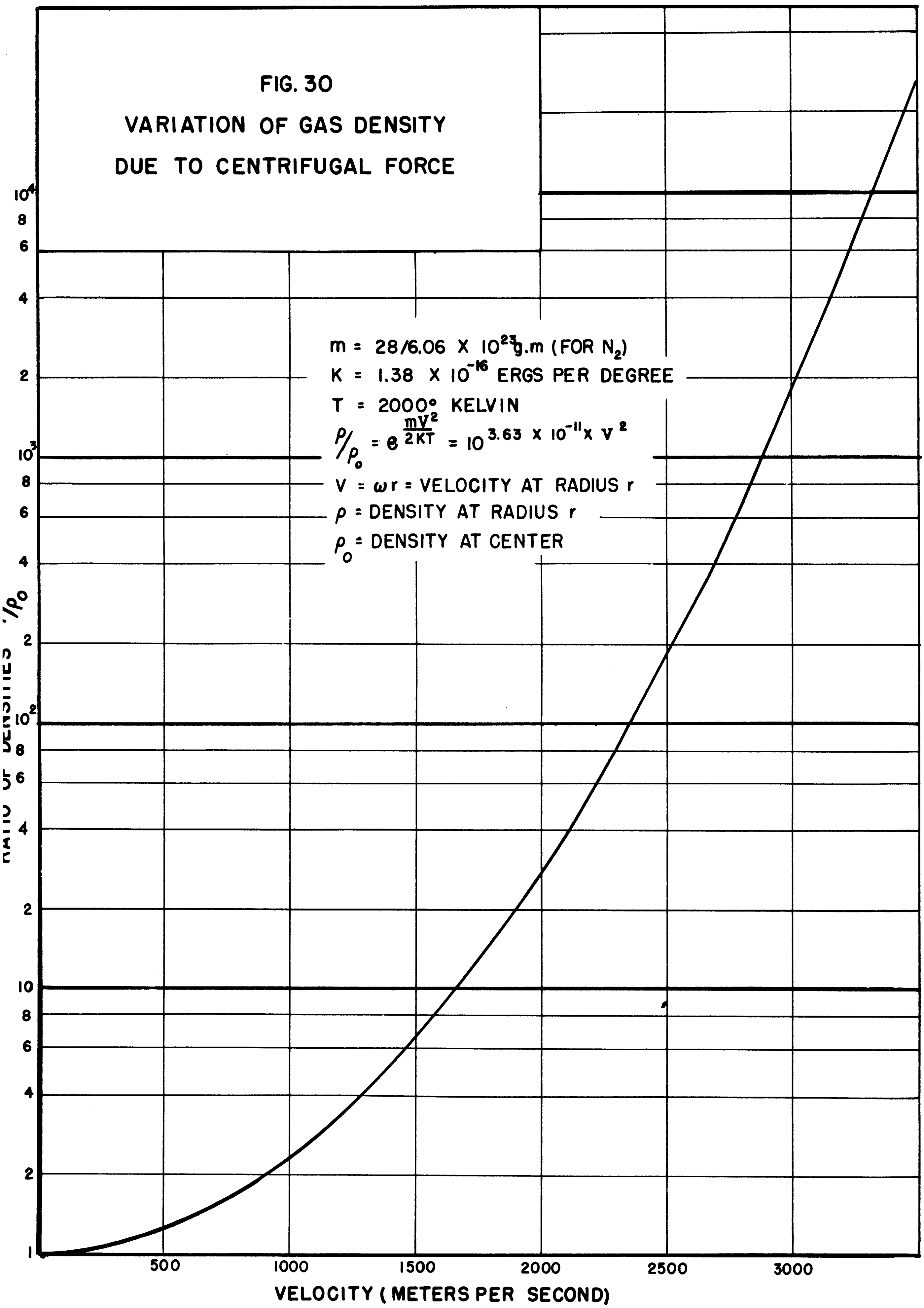


FIG. 30

VARIATION OF GAS DENSITY DUE TO CENTRIFUGAL FORCE

2000° K, and if the angular velocity is assumed to be constant with radius, the tangential velocity was 900 meters/sec. This measurement of the pressure ratio was subject to error due to temperature gradients at the end of the glass tube which measured the pressure near the cathode.

According to the curve of Fig. 30, a rotational velocity of 3600 meters/sec at the circumference of a cylinder would result in a pressure ratio of 40,000:1. A pressure of 20 microns at the axis would then correspond to atmospheric pressure at the circumference. Air of this density can not be effectively circulated by electrical forces.

Linear Flow Electrode Arrangement

Experience gained during the past year has shown that further investigation of the wind effects due to the crossed-fields discharge, can be carried out more advantageously when the electrodes are arranged so as to accelerate the air in a straight path. This arrangement not only produces higher wind velocity, but it is believed that many measurements in which the wind effects were confused by the circular symmetry of the revolving plasma can be studied more advantageously with this type of discharge.

Preliminary tests have been made to determine the problems associated with this type of electrodes. Fig. 31 is an arrangement which has been quite successful in these intial experiments. The electrodes consist of copper sheets set in the form of a dihedral angle. The tungsten filament is at the end of the cathode where the air stream enters the discharge. The emission from this filament acts as a "tickler" to produce ionization in the air stream, and prevents the magnetic field from "blowing out" the discharge in the manner of a magnetic circuit breaker.

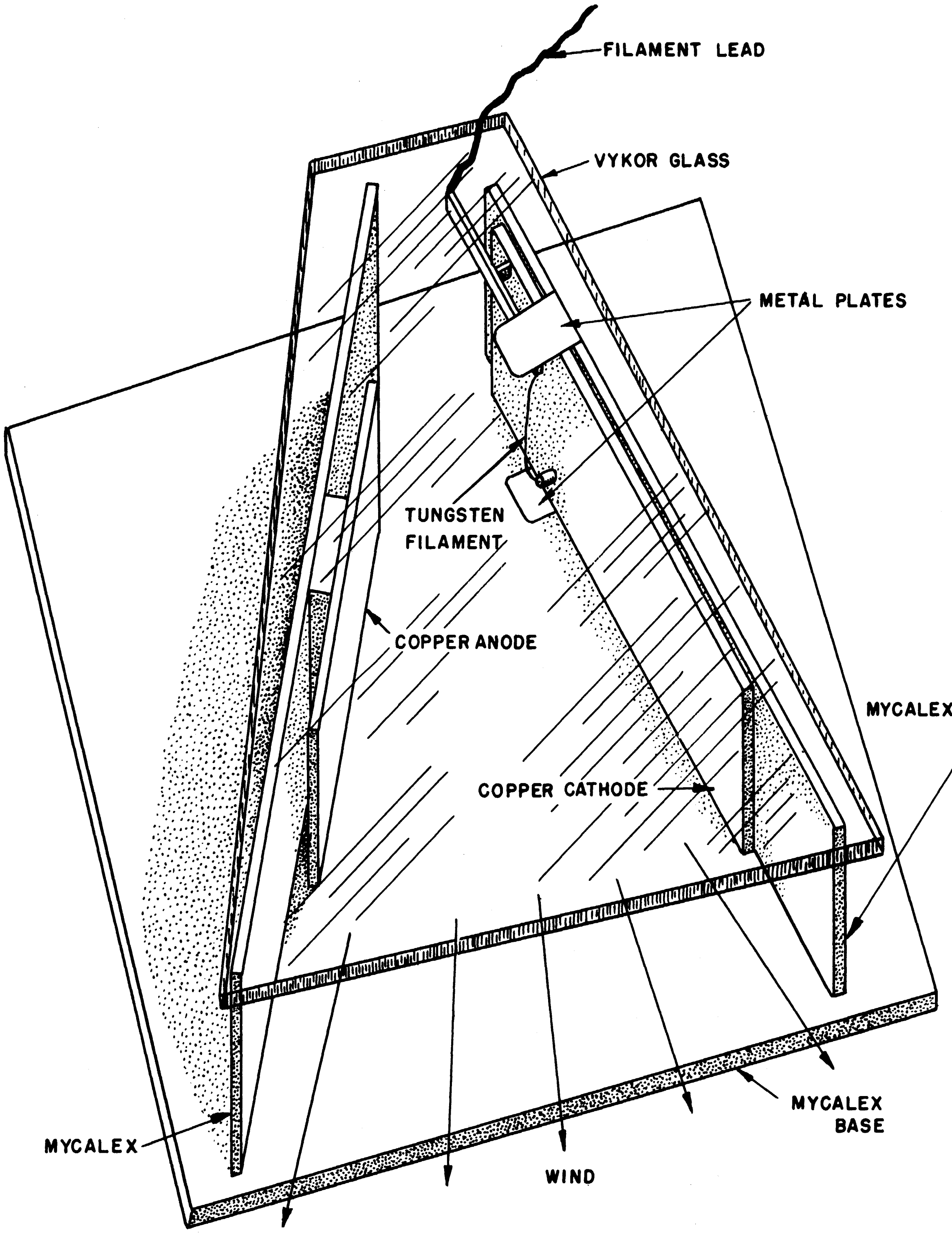


FIG. 31

ELECTRODE GEOMETRY FOR LINEAR FLOW TESTS

Fig. 32 shows the longer vacuum chamber which will be necessary in order to accelerate gas in a rectilinear manner. This vacuum box is 60 inches long and has removable glass windows both on the front end and on the top of the rear section, in order to provide accessibility and visibility. It is expected that most of the tests will involve a gas stream of small enough cross section so that the gas can be re-circulated within the box. In order to discharge the air flow into a vacuum system, as indicated in Fig. 26, a much larger diffusion pump and fore pump would be required.

As an illustration of the momentum requirements involved in accelerating an air stream with a force produced by a gas discharge, the following numerical calculation is of interest. This calculation considers only the force and momentum relations due to the action of the gas discharge. It neglects the nozzle effect tending to accelerate the stream and the viscous effect tending to retard it.

On page (12) a simple relationship was given to demonstrate that a force of 10^5 dynes could be exerted on an air stream. Let us assume that this accelerating force is independent of stream velocity. Suppose that the rate at which air enters the accelerating section is adjusted so that the mass flow is 0.2 gms/sec. Then the accelerating force is equal to the momentum imparted to the air stream per second, or

force = (mass flow/sec) x (change in velocity).

For a force of 100,000 dynes and a mass flow of 0.2 gms/sec the velocity would be 5,000 meters/sec.

If the cross-sectional area of the air stream is assumed to be 1000 cm^2, then the mass flow relationship requires that the air stream have a density of 4×10^{-10} gm/cc. This density corresponds to air at room temperature, having a pressure of 2.5×10^{-4} mm of Hg.

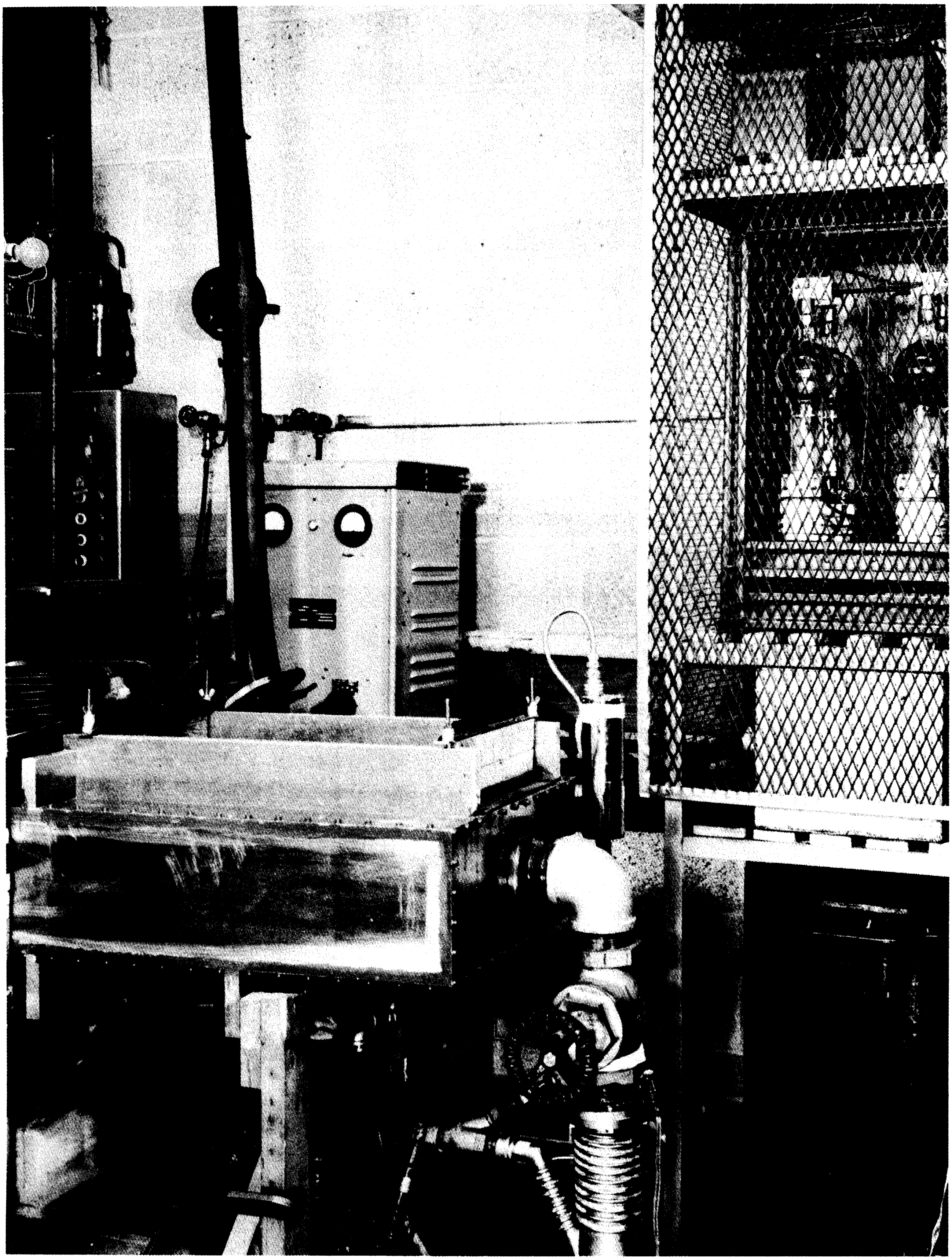


FIG. 32

SIDE VIEW OF APPARATUS SHOWING THE 60 INCH LONG VACUUM BOX FOR LINEAR FLOW EXPERIMENTS

INSTRUMENTATION FOR MEASURING PROPERTIES OF AIR STREAM

The experimental investigation of electrical wind effects involves a number of very unusual problems in instrumentation. An essential requirement for studying wind phenomena is a method of determining the velocity, temperature, density and pressure of the stream of gas. Of these four quantities, only the static pressure can be measured in a conventional manner. Because of the very low air density and partial ionization, none of the ordinary wind tunnel techniques are applicable to this problem. Neither are the techniques ordinarily used with gas discharges, because of the high wind velocity and the lack of thermal equilibrium.

This instrumentation problem is in some respects analogous to the problem of developing rocket-borne apparatus to measure the pressure, density and temperature of the upper atmosphere. The authors of this report have been involved for a number of years in this type of rocket instrumentation for use at high altitudes, where the pressure is of the same order of magnitude as in the wind generator. However, in the rocket case the velocity is known with fair accuracy, which is a great advantage. Some of the successful principles which have been used in rocket work are:

1. Simultaneous measurement of ram pressure and cone wall pressure
2. Velocity of sound data obtained by firing grenades
3. Anomalous sound propagation methods
4. Light scattering methods
5. Measurement of the Mach angle of a shock wave
6. Paschen law spark breakdown

None of these principles could be adapted to the wind generator problem.

A number of attempts have been made to measure velocity directly without determining density. These tests have not been encouraging; hence recent work has all been directed toward the development of the molecular effusion method of measuring density and temperature. It is now believed that this measurement can be made in a manner that will be essentially independent of the stream velocity. Before describing the molecular effusion method, some of the other measurement methods will be reported.

Static Pressure Measurement

The static pressure of the gas stream can be measured by means of a vacuum gage, provided care is taken to avoid errors due to thermal transpiration. This type of error results when a temperature gradient occurs near the opening of a pressure measuring tube, unless the diameter of the tube is sufficiently large compared to a mean free path. Data for estimating the magnitude of this type of error is given in Reference (7), page 65. In most experimental situations, it is possible to measure static stream pressure at locations where large tube opening are no problem. Since measurement of impact pressures requires small tube openings and results in errors due to large and unknown thermal gradients, no particular significance has been attached to the results of such measurements.

Mica-Disk Experiments

If a small sphere or disk is mounted in the center of a whirling gas, and if the friction of the supports is sufficiently low, the viscous drag of the gas tends to cause the object to revolve at approximately the same speed as the gas. The velocity of rotation can easily be measured with a strobotac.

This approach to the problem was investigated experimentally and a variety of mica disks and metal spheres were tested. Needle point bearings and magnetic suspension were both tried and found to have sufficiently low friction at low speeds, but at high speeds vibration or other types of dynamic instability always developed.

The most successful tests involved a 2-inch diameter mica disk with a tiny metal hub which was supported by a needle bearing. This was mounted on top of a shortened cathode "post" in the center of the cylindrical discharge region. This disk would accelerate from 0 to 20,000 rpm in 30 seconds and would require 3 or 4 minutes to decelerate after the power was turned off. Evidently the gas was revolving at a velocity considerably higher than the maximum disk velocity. If the gas were assumed to rotate at a constant angular velocity, and if the rate of gas rotation were 40,000 rpm, this would correspond to a circumferential velocity of 2100 feet per second. However, the assumptions which were required in order to interpret the data could not be justified, and for this reason these tests were not continued.

A type of measurement somewhat related to the revolving disk experiments has recently been suggested by Dr. Newman A. Hall of the Office of Ordnance Research. He suggests that the wind velocity might be determined by a device resembling an axial flow type of gas turbine. If the pitch of the blades of such a device is properly adjusted, a relatively slow rate of spin of the turbine corresponds to a very high stream velocity. Such an arrangement might avoid the mechanical difficulties associated with high rates of rotation, and permit the velocity of the gas stream to be approximately determined.

This proposed method will be investigated further.

Nitrogen Afterglow

Wind tunnels operating at static gas pressures of 2-3 mm of H_g have successfully used nitrogen afterglow as a method of visualizing air flow and shock waves.[10] The use of active nitrogen as a method of determining air velocity looked very attractive since it offered the possibility of intermittently "tagging" the gas as it passed through a small spark gap and then stroboscopically observing the time lag required for it to move downstream.

Tests in a bell jar indicated that afterglow, of significant intensity, could be produced at pressures as low as 20 microns, by using high current pulses from capacitor discharges between tungsten electrodes. However, when the same spark-gap and circuitry for exciting the afterglow were tried in connection with the wind generator, the afterglow was too weak to be useful. It is presumed that the decrease in the afterglow under these conditions may be related to the higher gas temperature, or to contamination from the copper electrodes.

Another experiment was made to see if the excitation processes in the revolving plasma discharge were capable of exciting sufficient afterglow in the gas. (In the photograph of Fig. 8, a luminosity due to active nitrogen can be seen scattered throughout the vacuum box.) The electric current to the discharge was "chopped" or continuously interrupted at regular intervals, and during these intervals the discharge region was viewed through slots in a synchronized revolving disk. It was hoped that the afterglow, visible during these intervals, would be of sufficient intensity to permit visual observation of the flow of the gas; but the maximum intensity which could be obtained was too weak for this purpose. If the afterglow had been of sufficient intensity, a method of measuring velocity could undoubtedly have been devised.

A trip was made to Langley Field, Virginia, to discuss these experiments with J. M. Benson, who has had considerable experience with flow-visualization techniques utilizing nitrogen afterglow. His work, however, involved gas pressures, and was concerned with density variations which did not relate directly to the velocity measurement problem.

The work with nitrogen afterglow was discontinued when it became evident that the intensity of the afterglow was too weak for adequate observation during the very short time intervals when an optical shutter would be open. Also, a shutter was required which would have an exposure time of less than 100 microseconds. The development of such a shutter of either the mechanical or the Kerr Cell type would involve a considerable amount of time and expense.

Wind Pressure on Wire Vane

One method of instrumentation which was investigated, involves the measurement of the wind pressure on a wire vane mounted in the air stream. This investigation so far has been mainly theoretical. The experimental work has been carried only far enough to demonstrate that drag forces of sufficient magnitude for quantitative measurement, are exerted on the wire. In one experiment, a force of the order of 8 dynes was developed on a 4 cm length of 0.004" tungsten wire.

Theoretical analysis indicates that information obtained from drag force measurements can be combined with independent pressure measurements, and the density, temperature and velocity of the gas stream can be calculated. Such a calculation is subject to a possible error due to a lack of available information regarding the magnitude of the accommodation coefficient, or even the exact meaning of the term.

The proposed method of interpreting the drag force data can be outlined in a few paragraphs. The diameter of the wire vane is assumed to be small compared to the mean free path, and the forces acting on the wire will be analyzed on the basis of free molecule flow theory. The pressure on the back side of the vane need not be considered in this discussion, since this effect is not large and a correction can be made in a complete analysis.

The momentum imparted to the front surface can be conveniently separated into two components:

1) a contribution due to the stream of molecules striking the surface;

2) a contribution due to the stream "re-emitted" from it.

The first contribution can be calculated by summing over all the molecules striking the surface. To calculate the momentum due to the re-emitted molecules, some assumptions must be made regarding the manner in which the molecules are scattered. In general, it is reasonable to assume diffuse scattering: all the molecules upon striking the surface lose their previous history of motion, and on re-emission, emerge with a Maxwellian velocity distribution characteristic of a temperature T_r. This temperature may or may not be the temperature of the surface. It is closely related to the accommodation coefficient of the momentum and energy exchange that takes place at the surface.

A kinetic theory analysis of the momentum given to the surface by the incident molecules is presented in Appendix II. This analysis assumes that the surface of the vane is flat instead of cylindrical and the mathematical expressions are thereby considerably simplified. A flat surface could be achieved in practice by means of a thin ribbon. The more involved solution of the cylindrical case is given in Reference (4).

For normal incidence on a flat surface

$$\text{Force due to incident molecules} = mU \frac{N\alpha}{2\sqrt{\pi}}\left[e^{-\frac{U^2}{\alpha^2}} + \sqrt{\pi}\,\frac{U}{\alpha}\left(1 + \operatorname{erf}\frac{U}{\alpha}\right)\right] + \frac{Nm\,\alpha^2}{4}\left(1 + \operatorname{erf}\frac{U}{\alpha}\right)$$

This expression contains several unknown quantities which would have to be evaluated, such as: α, the characteristic or most probable molecular velocity, U, the stream velocity and N, the number of molecules in the stream per cm^3. In order to determine these quantities, two types of drag measurements would be required, as follows:

(a) a measurement with the vane transverse to the stream;

(b) a measurement with the vane parallel to the stream.

This information would then be combined with a static pressure measurement to obtain a complete solution.

Wire Vane Transverse to Stream

The wind pressure on the transverse wire would be measured as a function of the temperature of the wire. This could be accomplished by heating the wire electrically to a temperature T_V, and at the same time using it as a resistance thermometer to determine T_V. Information would be obtained as to the value of T_V which makes the average energy of the re-emitted molecules equal to the average energy of the incident molecules. This condition exists when sufficient electrical heat is being generated in the vane to compensate exactly for the heat lost by radiation. Under this condition, T_V would be expected to remain the same, whether the wire was surrounded by a vacuum or by the air stream. Calculations indicate that this equilibrium T_V would be approximately 2250°K, if the static stream temperature were 1000°K and if the molecular velocity ratio, $\frac{U}{\alpha} = \sqrt{2}$. Measurements taken at this value of T_V would be less affected by uncertainties regarding the accommodation coefficient than those taken at lower temperatures.

An expression for the predicted variation of drag force with T_V can readily be derived. Preliminary to this derivation, it should be noted that the rate of arrival of incident molecules, and the momentum which they deliver, are independent of T_V. The rate of re-emission is also independent of T_V, but the velocity and momentum of re-emission do depend on T_V. Hence the variation of drag force due to a change of T_V is due only to changes in the momentum given to the re-emitted molecules.

Let us assume that: (1) The wire is near the equilibrium temperature, for which the incident energy input per second equals the re-emission energy output per second. (2) The velocity distribution of the re-emitted molecules is Maxwellian and characteristic of the same temperature as the wire. These two assumptions are equivalent to saying that T_v and T_r are the same.

It is shown in Eq 7, Appendix I, also Reference (1), page 377, that in a stationary Maxwellian gas, the molecules which strike a surface have an average velocity normal to the surface equal to $\frac{\sqrt{\pi}}{2}\alpha$, where α is the characteristic or most probable velocity. The same expression applies for a Maxwellian type of emission from a surface. If there are Γ_N molecules emitted per second per cm^2, each of mass m, the force F_r due to re-emission is:

$$F_r = \frac{\sqrt{\pi}}{2}\Gamma_N m\alpha = \frac{\sqrt{\pi}}{2}\Gamma_N m\sqrt{\frac{2kT}{m}} = \sqrt{\pi}\,\Gamma_N\sqrt{\frac{mkT}{2}} \text{ dynes/cm}^2$$

and

$$\frac{\partial F_r}{\partial T_v} = \frac{\sqrt{\pi}\,\Gamma_N}{2}\sqrt{\frac{mk}{2T}} .$$

If the experimental value for $\frac{\partial F_r}{\partial T_v}$ were substituted in this expression, it could be solved for Γ_N, the number of incident molecules per cm^2 per second. A cross check on this value of Γ_N might also be obtained by using energy, instead of momentum relations.

The mass of the incident molecules per second, $m\Gamma_N$, is approximately equal to ρU, where ρ is the stream density, provided that the molecular velocity ratio $\frac{U}{\alpha}$ is 2 or more. (c.f. Appendix I.) When this ratio is less a correction can be applied.

Thus the measurement of the force on the transverse wire provides a numerical value for the product ρU.

Wire Vane Parallel to Stream

Additional data can be obtained by measuring the viscous drag force when the wire is oriented parallel to the air stream. In this position, the rate at which molecules strike the surface is determined by the transverse components of thermal velocity, and is independent of stream velocity. The mass striking unit area per second is $1/4 \rho \bar{v}$, where $\bar{v}$ is the average molecular velocity. The stream momentum delivered to a unit area per second is $1/4 \rho U \bar{v}$. The net force due to re-emission is zero. (This assumes complete absorption before re-emission and hence no persistence of tangential momentum at the surface.)

If the value of ρU from the previous transverse vane experiment is substituted in the above expression, the value of $\bar{v}$ can be obtained, and the static stream temperature, density and velocity can then be calculated, provided the value of static pressure $p = nkT$ is known from an independent measurement.

At the present time, the "molecular effusion" method of instrumentation appears to involve less elaborate experimental apparatus than the "wind pressure on vane" method and is therefore more attractive. However, if the molecular effusion experiments should run into serious difficulties, it might be advantageous to revive the "force on vane" investigation.

Molecular Effusion

One method of determining the state of the gas in a very low density air stream, which is currently being investigated, utilizes the rate of molecular effusion through a small hole. At atmospheric pressure such a method of measurement is not feasible because of the short mean free path. At much lower pressures, where the mean free path is a centimeter or more in length, a much more precise measurement is possible. This method is based on the kinetic theory relationship that the rate at which the molecules of a stationary gas arrive at a surface is given by the relation:

$$\Gamma_n = 1/4\, N\bar{v}$$

where N is the number of molecules per unit volume, and $\bar{v}$ is the average molecular velocity. The mass of gas arriving at a surface per second is

$$\Gamma_m = 1/4\,\rho\bar{v} = \rho\sqrt{\frac{RT}{2\pi}} = \frac{p}{\sqrt{2\pi RT}}$$

where ρ is the density, T is the absolute temperature, p is the pressure, and R is the gas constant per gram.

These relations also define the rate at which gas effuses through a small aperture, on the tip of a hollow probe, provided certain conditions are maintained.

In utilizing the effusion rate relationship to determine the state of a stream of gas, two separate measurements are required as follows:

(a) a measurement with the aperture parallel to the air stream;

(b) a measurement with the aperture perpendicular to the air stream.

This information would then be combined with an independent determination of static pressure, and the temperature, density, and velocity could be calculated.

Aperture Parallel to Stream

Consider the situation shown in Fig. 33. The aperture consists of one (or more) holes in the tip of a probe which projects into the gas stream. In this position, the number of molecules entering the aperture presumably can be made essentially independent of stream velocity, and the effusion rate is determined only by the static temperature and density of the gas. In order to achieve this condition, a number of requirements must be met. These may be listed as follows:

(1) The diameter of the aperture must be small enough so that the drain of molecules through the hole, does not appreciably affect the conditions in the region where the molecules that enter the hole, have had their last collision.

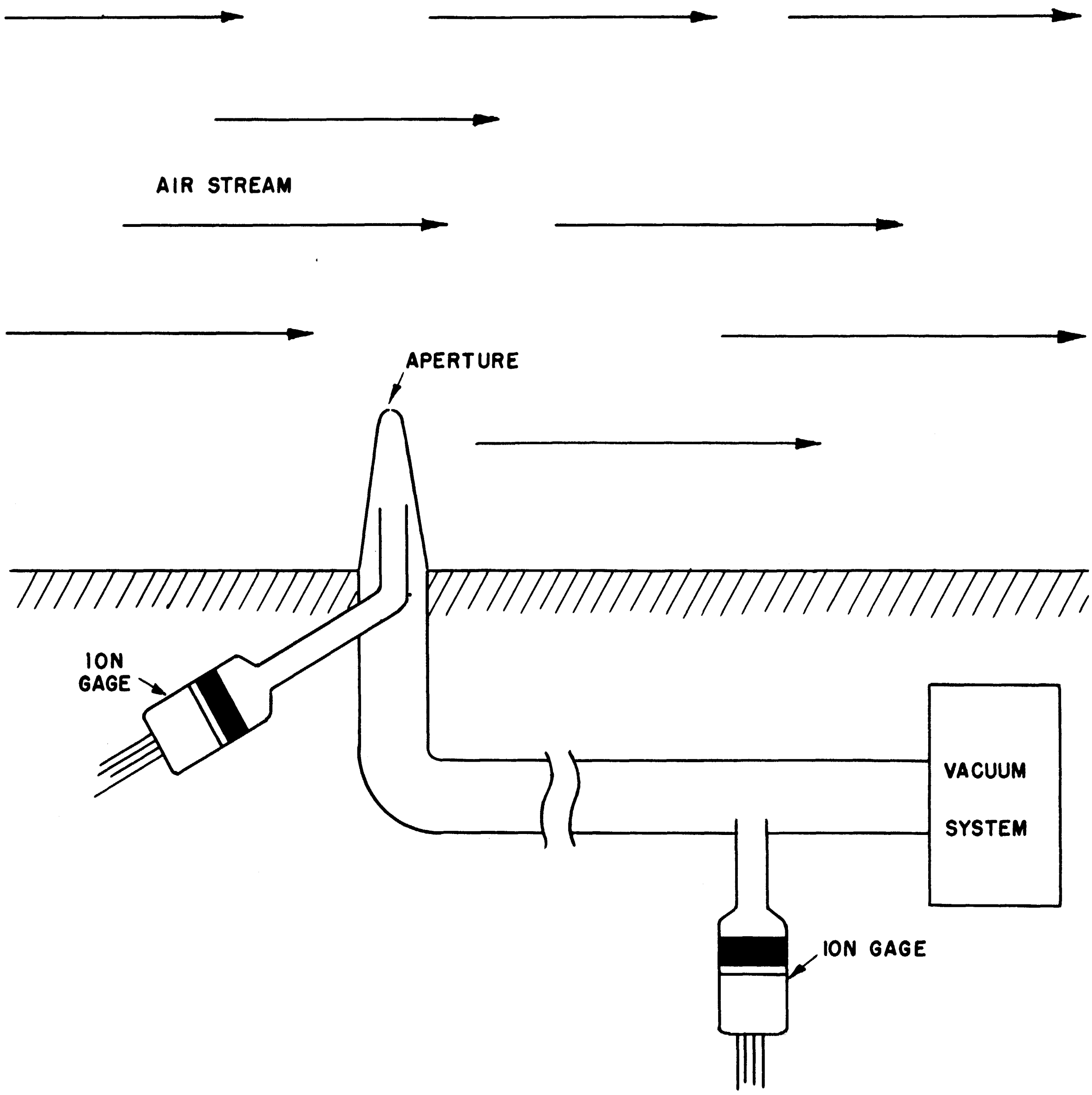


FIG. 33

PROBE IN AIR STREAM

(2) If the number of molecules entering the aperture is to be independent of stream velocity, the plane of the aperture must be parallel to the direction of flow, so that only the components of thermal velocity which are transverse to the stream, will determine the number of molecules entering the aperture.

(3) The molecular velocity distribution must consist of a Maxwellian distribution superimposed on a stream velocity.

(4) The thickness of the metal wall must be small compared to the diameter of the hole, so that the proportion of the entering molecules striking the edges of the hole is minimized.

(5) The surface surrounding the aperture will have a different temperature than the gas being measured. This effect is reduced by locating the aperture on the end of the probe, as in Fig. 33, so that the presence of the probe creates a minimum disturbance, in the region of the gas where the entering molecules had their last collision.

(6) If the molecules entering the aperture are not removed rapidly enough to provide unidirectional flow, then a proper correction for the molecules returning to the air stream requires information as to the temperature, density and pressure of the air inside the probe. In many cases, this will be the same as the wall temperature. However, if the stream velocity is large compared to the thermal velocity, there is some uncertainty as to how much effect the entering molecules might have, in raising the gas temperature inside the enclosure above wall temperature.

If the above requirements are met, then a numerical value for $\rho\bar{v}$ can be obtained which, combined with the value of $\rho\bar{v}^2$ obtained from a static pressure measurement, enables the state of the gas to be calculated. Additional information is necessary in order to determine the velocity.

Aperture Perpendicular to Stream

The rate at which molecules arrive at a surface (which is small compared to a mean free path), is derived in Appendix II, Eq. (3). The mass of arriving molecules per unit area per second, Γ_m, is shown to be approximately equal to ρU, provided $\frac{U}{\alpha} \geq 2$. Since ρ is known from the previous measurement, the measurement of ρU determines the stream velocity.

Method of Measuring Rate of Flow Through Aperture

There are two methods by which the rate of effusion can be measured:

(1) One method requires that the pressure inside the probe be kept low enough (by means of a vacuum pump), so that the flow through the aperture is unidirectional.

(2) The other method allows the molecular density inside the probe to reach an equilibrium, so that the rates of flow through the aperture in both directions are equal.

In the case of the unidirectional flow, it is necessary to determine the rate of flow from the probe into the vacuum system. The pressure gradient in the connecting pipe between the probe and the vacuum pump, would be measured by means of ion gages. The relation between this pressure gradient and the rate of flow, can be determined by means of a separate calibration experiment.

It is anticipated that the experimental procedure will involve the following approximate pressure ranges: Suppose the gas being measured has a pressure of 10^{-2} mm of H_g. If the pressure inside the probe were of the order of 2×10^{-4}mm, the unidirectional flow condition would be approximately met. The pressure at the entrance to the vacuum pump could be maintained at 10^{-5} mm or less, and a pressure drop of the order of 2×10^{-4} mm would thus be present in the connecting line. This pressure drop would be calibrated in terms of mass flow. Apparatus for carrying out this differential pressure measurement has been assembled and tests are in progress.

The other method of determining the mass flow utilizes the equilibrium condition, where the flow through the aperture is the same in both directions. Suppose the probe in Fig. 33 is connected only to a pressure gage (without the vacuum pump), and the flow through the aperture is allowed to continue until an equilibrium condition is reached. Let us assume that the gas inside of the probe has the same temperature as the wall of the probe, and this can be measured with a thermocouple. Let p_1, T_1, N_1 represent the state of the gas in the stream and p_2, T_2, N_2 represent the state of the gas inside the probe. Then, in the steady state,

$$\frac{p_1}{\sqrt{2\pi RT_1}} = \frac{p_2}{\sqrt{2\pi RT_2}}$$

hence,

$$\frac{p_1}{p_2} = \sqrt{\frac{T_1}{T_2}}$$

and

$$T_1 = \left(\frac{p_1}{p_2}\right)^2 T_2$$

An independent measurement of the static stream pressure p_1 will thus determine T_1.

When the probe is in a high velocity stream, the mass motion of the gas past the end of the probe tends to minimize any effect on the gas due to the presence of the probe. If the gas is stationary or is moving at low velocity, the lowering of the gas temperature caused by the lower temperature of the probe, can be minimized by heating the probe electrically until $p_1 = p_2$; which means that $T_1 = T_2$ and the probe is in thermal equilibrium with the gas.

This heating procedure would introduce temperature gradients, and possible pressure gradients, in the pipe connecting the probe to the pressure gage. These temperature gradients need not, however, introduce errors in the measurement of probe pressure, because these effects can be accurately determined by auxiliary experiments, so that the correction factors for various probe temperatures will be known.

Experimental Tests of Molecular Effusion

Experimental data regarding molecular flow through an aperture is given in the literature.[5] In order to gain familiarity with the experimental problems involved and the sources of error, a series of measurements were made with a stationary gas in which all variables were known with reasonable accuracy. These tests were primarily concerned with the requirements as to aperture size and the permissible thickness of the metal surrounding the hole. It was more convenient to measure the mass flow before the air passed through the holes in the diaphram instead of afterwards, as would be required when measuring an air stream of unknown density. The apparatus is shown in Fig. 34. The pressure was measured with an Alphatron which was calibrated against a McLeod gage. An oil-filled manometer was used to measure the volume of atmospheric air which was bled into the system during a 5-minute interval. The reverse flow through the diaphram was assumed to be zero, since the back pressure on the diaphram was less than 1% of the forward pressure.

Tests were made with different pressures, and sizes and numbers of holes. Fig. 35 is a plot of some of the experimental data taken during the 5-minute runs, and Fig. 36 is a tabulation of the comparison between observed values and calculated values. The only serious discrepancy is in the case of the test with the smaller holes which were 0.0101" in diameter in a diaphram

0.001" thick. This error is probably related to the much lower flow rate and a greater error due to gas liberated from the walls of the apparatus.

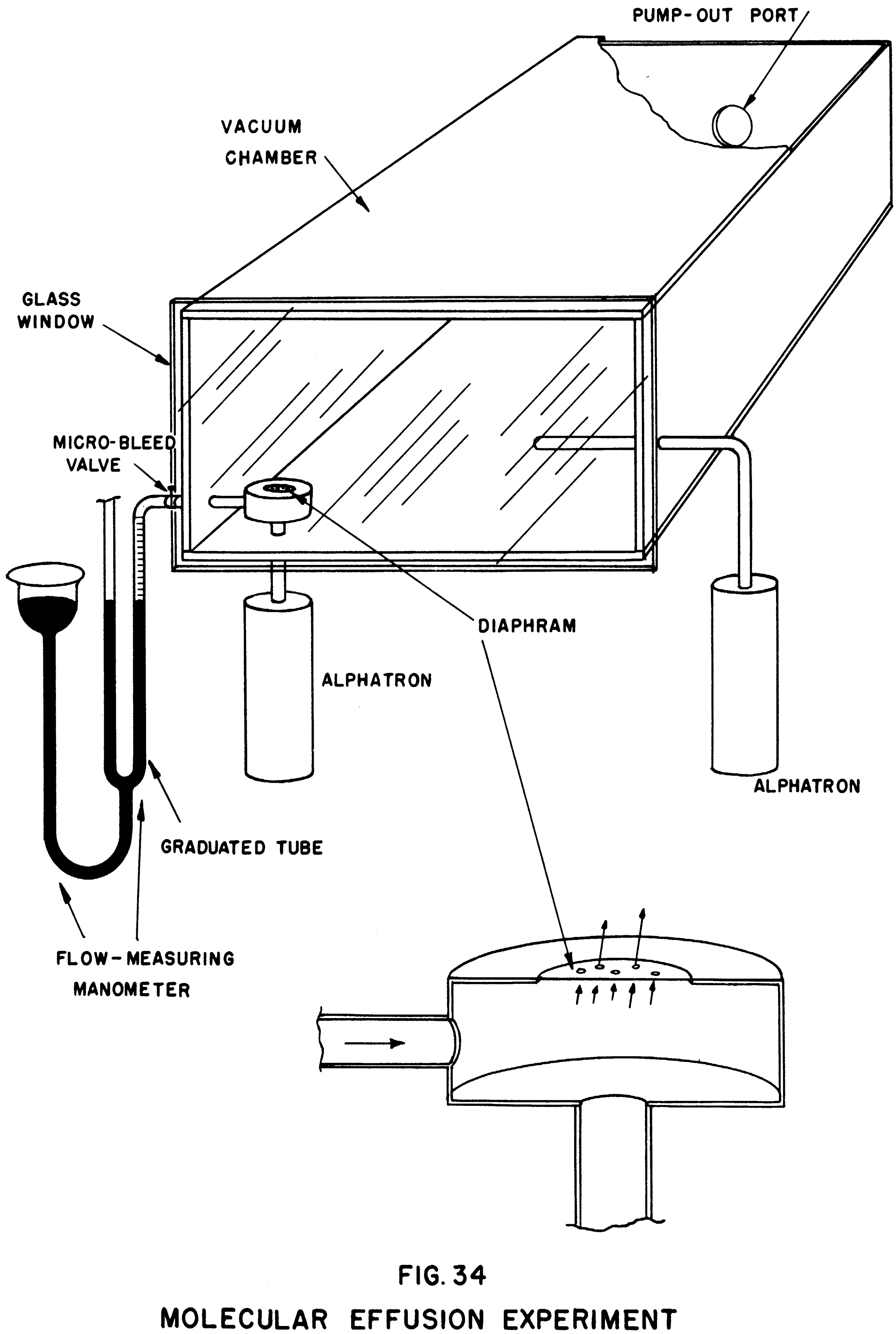


FIG. 34

MOLECULAR EFFUSION EXPERIMENT

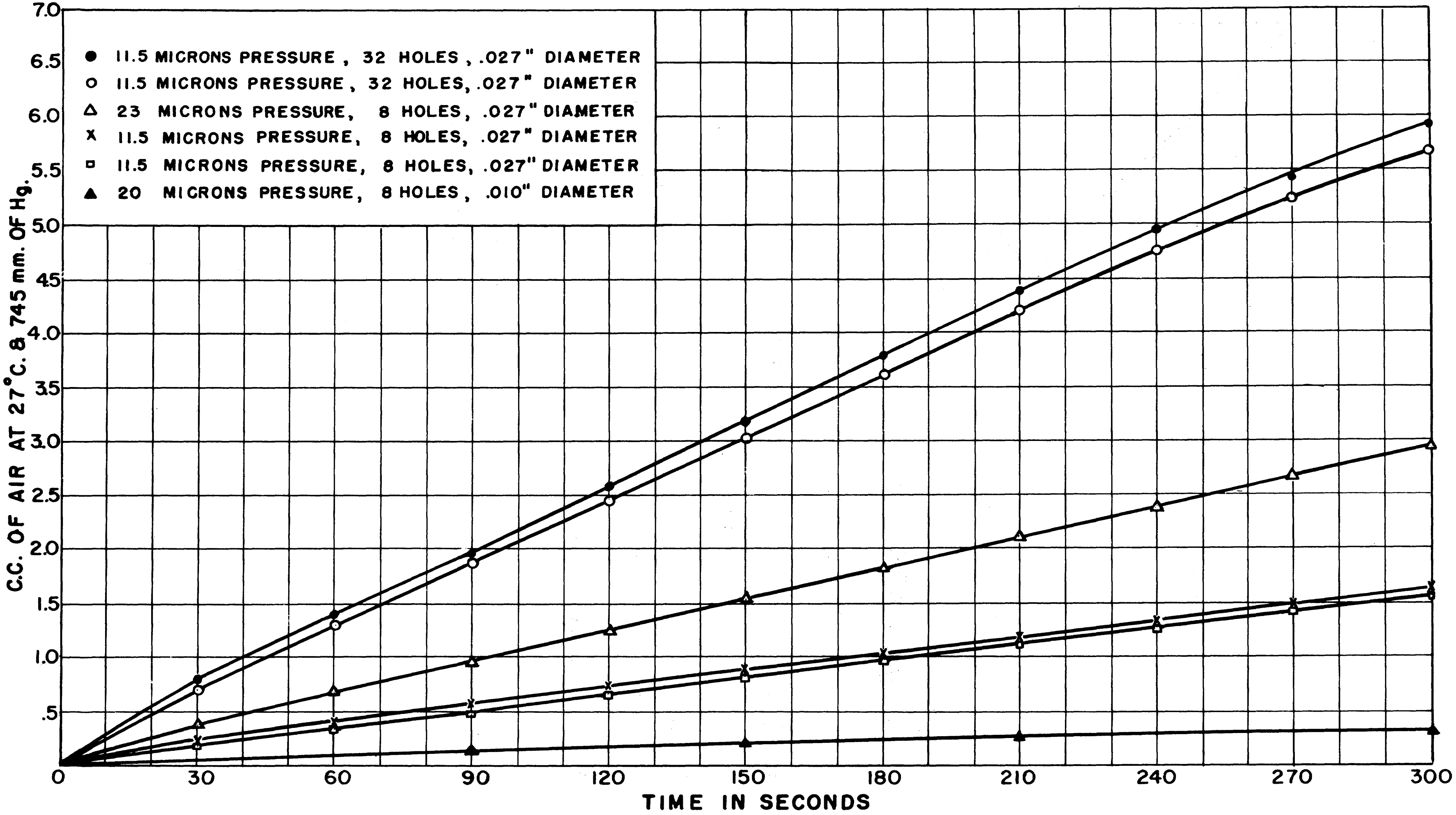


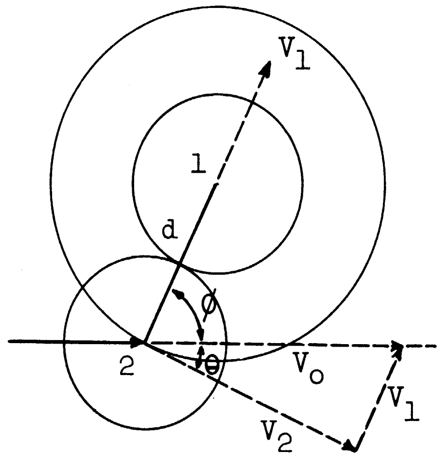
FIG. 35 EXPERIMENTAL DATA USED IN DETERMINING THE RATE OF MOLECULAR EFFUSION THRU HOLES IN THIN DIAPHRAM

AIR PRESSURE (MICRONS)	MEAN FREE PATH (CM)	TYPE OF DIAPHRAM	OBSERVED FLOW THRU DIAPHRAM gms/sec.	OBSERVED FLOW PER CM^2 OF AREA OF APERTURE gms/sec	CALCULATED FLOW PER CM^2 OF AREA OF APERTURE gms/sec.
11.5	0.50	8 HOLES .027" DIA.	$.613 \times 10^{-5}$ (AV. OF 2 RUNS)	20.8×10^{-5}	20.8×10^{-5}
11.5	0.50	32 HOLES .027" DIA.	2.23×10^{-5} (AV. OF 2 RUNS)	18.8×10^{-5}	20.8×10^{-5}
23	0.25	8 HOLES .027" DIA.	1.135×10^{-5}	38.5×10^{-5}	41.6×10^{-5}
20	0.29	8 HOLES .0102" DIA.	1.15×10^{-6}	27.2×10^{-5}	36.2×10^{-5}

FIG. 36 COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES FOR RATE OF MOLECULAR EFFUSION

APPENDIX I

The statistical average value of energy and momentum transferred from a high speed ion to a stationary molecule, through a collision, can be calculated in the following simplified case. Let us assume that the ion and the molecule have the same mass, and that the collision is of the same nature as between two force-free elastic spheres. And let us write,



V_o = velocity of ion before collision

V_1 = velocity of molecule after collision

V_2 = velocity of ion after collision.

It is evident that V_1 must be in the direction along the line joining the centers of the two particles at the instant of impact, since the only effective force during the impact is along that direction. The conservation of energy and momentum requires the following relations:

$$V_o^2 = V_1^2 + V_2^2 \tag{1}$$

$$V_o = V_1 \cos \phi + V_2 \cos \theta \tag{2}$$

$$0 = V_1 \sin \phi - V_2 \sin \theta \tag{3}$$

Squaring both sides of eqs. (2) and (3), adding and then subtracting eq. (1), we get

$$\cot \theta = \tan \phi, \; \theta + \phi = \pi/2 \tag{4}$$

Substituting this relation into eqs. (2) and (3), we see that

$$\left. \begin{aligned} V_1 &= V_o \sin \theta = V_o \cos \phi \\ V_2 &= V_o \cos \theta = V_o \sin \phi \end{aligned} \right\} \tag{5}$$

If the molecule is in the path of an ion beam of uniform density, the total collision cross-section of the molecule for the beam is evidently πd^2, where d is the distance between the centers of the particles at impact, i.e., the diameter of the molecule (or the ion). The differential cross-section for collisions of such a nature that the line joining the centers of the particles makes an angle between ϕ to $\phi + d\phi$ with V_o, is equal to the cross-section of the ring-shaped area defined by the limits ϕ and $\phi + d\phi$, on a sphere of radius d, (as shown in the figure, also p. 124, Ref. 8) i.e., $2\pi d^2 \sin \phi \, d\phi \cdot \cos \phi$. The ratio of the differential to the total cross-section represents the fraction of the total number of collisions which are characterized by the angle ϕ, i.e., the probability for such collisions.

$$P(\phi) \, d\phi = \frac{2\pi d^2 \sin \phi \cos \phi \, d\phi}{\pi d^2} = \sin 2\phi \, d\phi \tag{6}$$

The momentum acquired by one molecule in a collision is, according to eq. (5), mV cos ϕ in the direction ϕ. To calculate the average value of this momentum, we multiply this expression by the probability given by eq. (6) and sum vectorally over all possible collisions. Because of the geometrical symmetry, however, it is evident that the resultant will be in the forward direction, i.e., in the direction of V_o.

Thus it is only the forward component of the motion V_1 that will contribute to the resultant average forward momentum. This component of the momentum can be written as

$$M(\phi) \cos \phi = mV_o \cos^2 \phi$$

and the average momentum gained per collision is

$$\left[\begin{array}{l}\text{average momentum transfer} \\ \text{per collision in the for-} \\ \text{ward direction.}\end{array}\right] = \int_0^{\pi/2} \vec{M}(\phi)\, P(\phi)\, d\phi$$

$$= \int_0^{\pi/2} mV_o \cos^2 \phi \sin 2\phi \, d\phi \qquad (7)$$

$$= \frac{1}{2} mV_o$$

i.e., the average momentum transfer is half of the maximum transfer when the particles collide head on. To calculate the average energy transfer, we noticed, from eq. (5), that the energy acquired by a molecule during a collision is given by:

$$E(\phi) = \frac{1}{2} mV_1^2 = \frac{1}{2} mV_o^2 \cos^2 \phi$$

The average value is therefore:

$$\left[\begin{array}{l}\text{average energy transfer} \\ \text{per collision}\end{array}\right] = \overline{E} = \int_0^{\pi/2} E(\phi)\, P(\phi)\, d\phi$$

$$= \int_0^{\pi/2} \frac{m}{2} V_o^2 \cos^2 \phi \sin 2\phi \, d\phi \qquad (8)$$

$$= \frac{1}{2}\left(\frac{m}{2} V_o^2\right)$$

On the average an ion transfers half of its energy to a molecule per collision. Of the energy transferred, we can associate a certain fraction with the forward motion of the molecule. This we shall denote by

$$E_f = \frac{1}{2} m(V_o \cos^2 \phi)^2$$

and the average value is

$$\begin{bmatrix}\text{average transfer of} \\ \text{forward energy}\end{bmatrix} = \overline{E}_f = \int_0^{\pi/2} E_f \, P(\phi) \, d\phi$$

$$= \int_0^{\pi/2} \frac{m}{2} V_o^{\,2} \cos^4 \phi \sin 2\phi \, d\phi \tag{9}$$

$$= \frac{1}{3} \cdot \left(\frac{1}{2} mV_o^{\,2}\right)$$

APPENDIX II

The wind pressure on a flat vane, having a width small compared to a mean free path, can be calculated as follows:

Consider a "free molecule" gas of N molecules per cm^3 moving with a mass velocity U in the positive "x" direction. The velocity distribution function for such a gas is:

$$f(C)\, dC_x\, dC_y\, dC_z = \left(\frac{1}{\alpha\sqrt{\pi}}\right)^3 e^{-\frac{1}{\alpha^2}\left[(C_x - U)^2 + C_y^{\,2} + C_z^{\,2}\right]} dC_x\, dC_y\, dC_z \qquad (1)$$

where C_x, C_y, C_z are the components of C, the total velocity of a molecule;

$\alpha = \sqrt{\frac{2kT}{m}}$ = most probable thermal velocity of a molecule,

m = mass of a molecule (in grams),

T = temperature of the gas (in degrees K),

k = 1.38×10^{-16} ergs per degree .

Let this gas be incident normally upon a surface which is parallel to the YZ plane. The number of molecules striking a surface element dS, per second, with velocities between the limits C_x, C_y, C_z and $C_x + dC_x$ $C_y + dC_y$ $C_z + dC_z$ is equal to the number of such molecules lying in a column of volume C_x dS. Utilizing Eq 1 and dividing by dS, we find

$$\left[\begin{array}{l}\text{number of molecules}\\ \text{arriving per sec. per}\\ \text{unit area with veloc-}\\ \text{ities between } C_x\ C_y\ C_z\\ \text{and } C_x + dC_x,\ C_y + dC_y,\\ C_z + dC_z\end{array}\right] = d\Gamma_N(C_x, C_y, C_z) = C_x N f(C)\, dC_x\, dC_y\, dC_z$$

$$= N\left(\frac{1}{\alpha\sqrt{\pi}}\right)^3 C_x e^{-\frac{1}{\alpha^2}\left[(C_x - U)^2 + C_y^{\,2} + C_z^{\,2}\right]} dC_x\, dC_y dC_z \qquad (2)$$

Integrating this expression over all velocities between the limits

$$C_x = 0 \rightarrow +\infty, \quad C_y = -\infty \rightarrow +\infty, \quad C_z = -\infty \rightarrow +\infty, \quad \text{we get}$$

$$\begin{bmatrix}\text{number of molecules} \\ \text{arriving per sec.} \\ \text{per unit area with} \\ \text{all velocities}\end{bmatrix} = \Gamma_N = N\left(\frac{1}{\alpha\sqrt{\pi}}\right)^3 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} C_x \, e^{-\frac{1}{\alpha^2}\left[(C_x-U)^2 + C_y^2 + C_z^2\right]}$$

$$\bullet \; dC_x \, dC_y \, dC_z = \frac{N\alpha}{2\sqrt{\pi}}\left[e^{-\frac{U^2}{\alpha^2}} + \sqrt{\pi}\,\frac{U}{\alpha}\left((1 + \operatorname{erf}\frac{U}{\alpha}\right)\right] \tag{3}$$

The total momentum carried by all molecules to the surface per second is equivalent to the force imparted to the surface. Because of the geometrical symmetry, this force has a direction normal to the surface. The Y and Z components of the total momentum are zero. Consider the group of molecules specified by Eq 2. The X - component of the momentum carried by this group is evidently

$$dP_i = mC_x d\Gamma_N = mC_x^2 \, Nf(C) \, dC_x \, dC_y \, dC_z \quad . \tag{4}$$

Integrating this over all molecules striking the surface per unit area per second, we obtain

$$\begin{bmatrix}\text{total force on} \\ \text{unit area due to} \\ \text{all incident} \\ \text{molecules}\end{bmatrix} = P_i = Nm\left(\frac{1}{\alpha\sqrt{\pi}}\right)^3 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} C_x^2 \, e^{\frac{1}{\alpha^2}\left[(C_x-U)^2 + C_y^2 + C_z^2\right]}$$

$$\bullet \; dC_x \, dC_y \, dC_z = mU\Gamma_N + \frac{Nm\alpha^2}{4}\left(1 + \operatorname{erf}\frac{U}{\alpha}\right), \tag{5}$$

where Γ_N is the total number of molecules striking the surface per second per unit area, given by Eq 3.

As special cases of interest, we see that,

1) when U = 0, Eq 3 becomes

$$\Gamma_N = \frac{N\alpha}{2\sqrt{\pi}} = \frac{1}{4} N\overline{C} \quad , \tag{6}$$

where $\overline{C}$ is the average total velocity of a stationary Maxwellian gas, defined as:

$$\overline{C} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C \cdot f(C)\, dC_x\, dC_y\, dC_z = \left(\frac{1}{\alpha\sqrt{\pi}}\right)^3 \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} C^3 e^{-\frac{C^2}{\alpha^2}}\, dC \sin\theta\, d\theta\, d\phi$$

$$= \frac{2}{\sqrt{\pi}}\alpha \tag{6'}$$

and Eq 5 reduces to:

$$P_i = \frac{1}{4} Nm\alpha^2 = \frac{1}{2} NkT \quad , \tag{7}$$

i.e., half of the static pressure. The factor 1/2 arises from the fact that we have not included the pressure contribution from the re-emitted molecules. Dividing Eq 7 by Eq 6, we obtain

$$\frac{P_i}{\Gamma_N} = m\frac{\sqrt{\pi}}{2}\alpha \quad .$$

This is equal to the average value of the x-directed momentum carried to a surface per particle striking the surface. Dividing this by the mass of each particle, we obtain an expression for the mean value of the x-directed velocity of each particle, striking the surface. This we shall denote by $\overline{V}_x$,

$$\begin{bmatrix}\text{average x-directed velocity}\\ \text{striking a surface}\end{bmatrix} = \overline{V}_x = \frac{\sqrt{\pi}}{2}\alpha \tag{7'}$$

2) when $\frac{U}{\alpha} >> 1$, Eq 3 reduces approximately to

$$\Gamma_N \cong NU \quad , \tag{8}$$

which is to be expected from a rough consideration neglecting the thermal straggling. And Eq 5 becomes,

$$P_i \cong mU\Gamma_N \cong NmU^2 \quad . \tag{9}$$

It is interesting to see how good these approximations are at $\frac{U}{\alpha} = 2$. Substituting the numerical values: $e^{-4} \cong 0.018$, erf (2) $\cong 0.9546$, into Eq 3, we see that Eq 8 is accurate to within 2.5%; Eq 9 is accurate only to within 10% in comparison with Eq 5.

APPENDIX III

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Supersonic Wind at Low Pressures Produced by Arc in Magnetic Field

H. C. Early and W. G. Dow
Department of Electrical Engineering, University of Michigan, Ann Arbor, Michigan
January 20, 1950

The properties of a low pressure discharge are greatly modified by the presence of a transverse magnetic field of several thousand gauss. The over-all behavior, here described, is believed to be unique to values of gas concentration for which the mean free time of the electrons is much greater than their cyclotron periodicity, and the mean free time of the ions much less than their cyclotron periodicity. The experiments were performed inside a large vacuum chamber having a volume of approximately one cubic meter. When the arc is in a large unconfined region, wind effects are observed which are not present when the discharge is in a small glass tube.

Figure 1 illustrates the general appearance of an arc at 0.5 millimeter pressure, transverse to a magnetic field of 6000 gauss. The voltage gradient is approximately 100 volts/cm, and the power input and the current density are many times larger than when the magnetic field is not present. The arc column, in air or nitrogen, is pale blue and quite transparent. Although the power dissipated in the positive column of the arc is more than one-half kw/cm of arc length, the gas temperature is surprisingly low because of the cooling effect of the wind. This temperature is greatly dependent on the manner in which the wind is circulating inside the chamber, and to what extent it is cooled during the recirculating process. Observations based on the melting point of chemical salts indicate that in most cases the arc temperature is less than 600 degrees centigrade.

When the air flow is blocked by placing a ceramic sheet on the "downwind" side of the discharge, the arc flattens out against the sheet and forms a white hot surface layer. If the sheet is placed on the "upwind" side of the arc, so as to block the air from entering the region, the arc then spreads out in all directions. Under such conditions, manometer pressure measurements show that the discharge is acting like an air pump. The air pressure on the side of the ceramic sheet which is adjacent to the arc is about 20 percent of the pressure on the opposite side of the sheet.

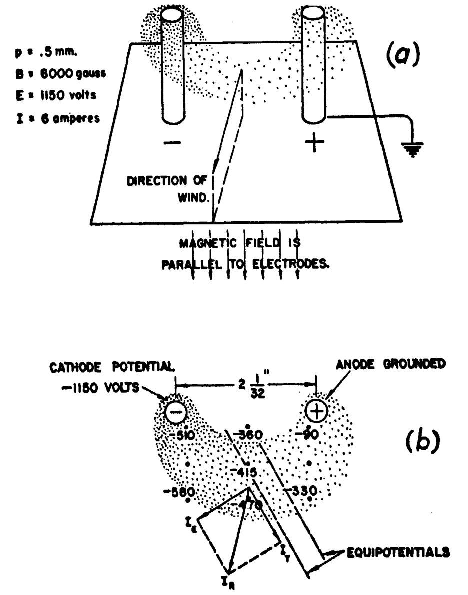

Fig. 1. (a) Arc between two copper rods. (b) Section of arc taken perpendicular to electrodes and probe measurements of potentials with respect to anode. I_E = component of ion drift velocity in direction of electric field. I_T = component of ion drift velocity transverse to electric field. I_R = resultant velocity in direction of wind.

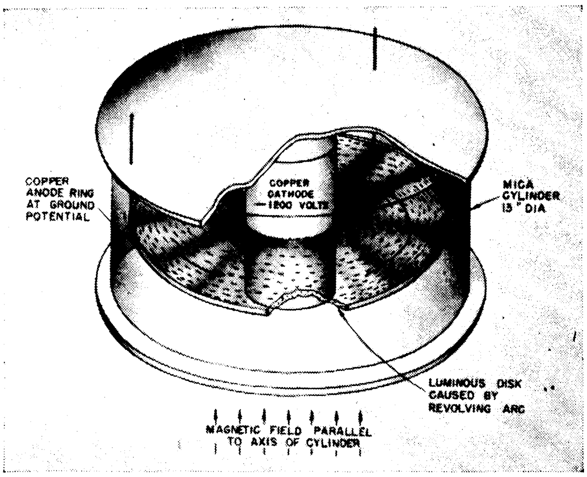

Fig. 2. Revolving arc inside a transparent mica cylinder.

A significant factor in this type of discharge is the low mobility of the electrons. It is believed that during each mean free path any electron makes many rotations about the magnetic flux lines and that its drift movement is almost entirely at right angles to the electric gradient. In contrast, an ion apparently does not follow a rotational or cycloidal path, because a collision will always occur before a fraction of one rotation is completed. As a result the ion mobility is not greatly reduced by the magnetic field. Available information as to the ion mobility to be expected under these conditions indicates that an assumption of nearly 100 percent ion conduction is quite consistent with the observed values of current density and electric gradient. For the conditions in Fig. 1, calculations predict that the component of the ion drift velocity, transverse to the electric gradient, has approximately the same magnitude as the component in the direction of the gradient. Because of the skewed position of the equipotentials, Fig. 1(b), the resultant ion current is believed to be perpendicular to the plasma boundary and in the same direction as the wind. This wind is presumably caused by the ion drift movement and the resultant interchange of momentum with neutral molecules. Since a substantial part of the energy which the ions receive from the field is utilized in producing directed momentum of gas molecules, this device appears to be a fairly efficient wind generator.

The electrons which are formed by the ionization processes throughout the discharge also move to the downwind edge of the plasma where they recombine with the ions. This electron drift is probably nearly parallel to the equipotential lines. A more detailed study of these experiments, now in progress, promises to provide a satisfactory explanation for the skewed position of these equipotentials.

Figure 2 is a sketch of an arc between a copper cylinder and a surrounding ring. An axial magnetic field causes this arc to revolve like a spoke in a wheel. The rate of rotation can be measured by means of a plasma probe connected to an oxcilloscope. When the air pressure and other variables are similar to those shown in Fig. 1, the rate of rotation is about 17,000 r.p.s. The air inside the cylinder also revolves, but its velocity is less than that of the arc. Measurement of the wind velocity is difficult because the air density is too low for the usual techniques. One method of measurement involved the sudden injection into the cylinder of a chemical vapor which colored the discharge. Stroboscopic observations of this experiment indicated that the color, caused by this vapor, spread downstream at a rate of at least 4500 miles/hr. Another experiment involved the measurement of the force on a small tungsten vane which was deflected by the air stream. On the basis of this latter data, the air speed was estimated to be approximately 3000 miles/hr.

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