THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Civil Engineering

Final Report

STRENGTH OF STEEL BEAM-COLUMNS FOR STEEL YIELD POINTS BETWEEN 33 KSI AND 100 KSI, AND INCLUDING EFFECTS OF END-RESTRAINT, BIPLANAR BENDING, AND RESIDUAL STRESS

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ABSTRACT

The purpose of this investigation is to develop simple and accurate design procedures for beam-columns composed of four corner angles closed on four sides by lacing bars. Whenever applicable, these procedures are extended to cover other cross sections. The simplified section was adopted to avoid the complications of stress redistribution across the section. Emphasis is on analysis by use of the electronic digital computer, without which the calculations would have been too time consuming. Coverage herein is limited to the cases of unrestrained and restrained beam-columns in planar bending, and unrestrained beam-columns in biplanar bending.

Tables that give the average compressive stress at the load causing initial yield for the unrestrained beam-column subjected to uniform lateral load W = kP, together with the axial load P, are provided. These tables are given for all currently used yield stress levels of structural steels. The applicability of these tables is extended by providing shape and load conversion factors to cover other cross sections under various conditions, both in planar and biplanar bending.

Tables also are given that provide the actual ultimate load of the unrestrained beam-columns of the four point section under uniform lateral loads W = kP, together with the axial load P, in planar bending. The effect of residual stresses on the individual corner element is included in the analysis. The ultimate load values are obtained by an incremental increase in load, determining the column shape by a numerical integration procedure at each step up to the load beyond which this procedure does not converge. The ultimate strength of the unrestrained four point section in biplanar bending, under uniform lateral loads $W_x = k_xP$ and $W_y = k_xP$ together with the axial load P was also investigated. A computer program is given to solve such a problem using the same procedure as that of the planar bending case, but no tables have been prepared.

Restrained columns, both in planar and biplanar bending, were investigated by procedures essentially the same as for the unrestrained column. For a given P, the restraining end moments are determined so as to be compatible with the deformed geometry of the restraining members. A computer program is given for the case of restrained beam-columns in planar bending.

INTRODUCTION

1.1 GENERAL

This final report for phase one of Contract NBy-45819 is in compliance with the contract requirements covering work done during the period June 4, 1962 to June 1, 1964, under The University of Michigan Research Contract 05154, on the subject of the ultimate strength of framed beam-columns of high strength steel.

"Beam-columns" are members subjected simultaneously to axial loads and bending moments caused either by transverse forces or by eccentricities of the axial force at one or both ends. The general analysis of a beam-column may involve end restraints to lateral displacements, twist, or rotations. The treatment of the problem to include all of these effects simultaneously is very complex. Column design may be based either on the load at which the maximum stress reaches the yield point or the ultimate load capacity of a column in a given structure.

This investigation takes up the effects on ultimate load of residual stress, end restraint, and biaxial bending. To eliminate other complications, such as the torsion of open sections, and to simplify the effects of cross-sectional shape, the column section considered in this investigation consists of four corner angles closed on the four connecting sides by lacing bars. It will be assumed that the lacing is continuously effective and that deflections due to shear have a negligible effect on column behavior. Although in some of the analyses the effect of residual stress in altering stress-strain characteristics will be included, each of the corners will be considered as a point concentration of area. The results pertaining to this simplified section may in some cases be used as a basis for estimating approximately the behavior of other cross sections.

The accurate determination of the load-deflection history of a beam-column in the inelastic range is a time consuming process, requiring an evaluation of the deflected shape of the beam column at successive increments of load. The use of an electronic computer is essential for the efficient study of this problem within reasonable limits of time and cost. The IBM 7090 computer at The University of Michigan was used and instructions to the computer were written in the MAD (Michigan Algorithm Decoder) language.

There are two approaches to the beam-column problem. If the material is steel, with no consideration given to residual stress, and with lateral-torsional buckling prevented, the design may be based conservatively on the column load at which the maximum stress due to combined axial load and bending moment reaches the yield point. When end restraint exists this procedure becomes still more conservative. If, however, a nonlinear stress-strain relationship is considered, either inherent in the material or effectively caused by the presence of residual stress, an incremental analysis involving the determination of the maximum load on a load-deflection curve is required.

1.2 PREVIOUS STUDIES

The first rational approach to the determination of the maximum load at which an eccentrically loaded column, without end restraint, passes from a stable to an unstable equilibrium configuration is due to von Karman.[1] For a given nonlinear stress-strain relationship, von Karman outlined the steps whereby the relationship between column curvature and applied moment could be established. For the eccentrically loaded column at any location, the applied moment, $M = P(e+y)$. Thus for a given value of P and e, a relationship between curvature and deflection could be established, and by numeric and/or analytic procedures a particular column curve could be approximated. A series of curves obtained by numerical integration then served as the basis from which, finally, a relationship between average column stress (P/A) and maximum deflection could be obtained. Chwalla[2] made tests confirming the correctness of von Karman's work and followed these by analytical studies regarding the effect of cross-section shape.[3] A review of the work of von Karman and Chwalla, as well as that of later investigators who introduced various simplifying modifications as to shape of stress-strain curve or column deflection curve, is presented by Bleich.[4]

In recent years emphasis has been directed toward the effects of end restraint on beam-column strength. Most columns are part of a continuously framed structure. Although Chwalla made initial rigorous but laborious studies of this problem, it has been attacked more extensively in recent years through investigations at Cambridge and Lehigh in connection with plastic design developments, and at Cornell, where the problem was studied primarily within the elastic range.

The work on columns (stanchions) at Cambridge (initiated in 1938) together with other investigations is reviewed with great descriptive detail in Chapters 13-15 of Ref. 5, by Baker, Horne and Heyman, covering both experiment and the ultimate strength elasto-plastic analysis of framed beam-columns.

The work at Cornell, initiated in 1948 under sponsorship of Column Research Council, included very extensive elastic analyses of the buckling and instability strength of framed columns and beam-columns, respectively, with approximate extension to elasto-plastic behavior, as summarized in a paper by Bijlaard, Fisher, and Winter.[6]

At Lehigh, in connection with many full scale tests, analyses in the early 1940's were initially related to the ultimate strength of unrestrained beam-columns, including for the first time the effects of residual stress caused by hot rolling and cooling of structural steel shapes, as summarized for a Column Research Council Symposium in 1955 by Ketter, Kaminsky, and Beedle.[7] Later, in a series of papers growing out of the initial Lehigh work, by means of ingenious nomographic charts, Ojalvo adapted the method of von Karman and Chwalla to procedures for calculating the ultimate strength of eccentrically loaded restrained beam-columns in the inelastic range.[8] Very complete ultimate strength tables for unrestrained beam-columns, with unequal end-eccentricities, giving critical combinations of length, end-moment and axial force have been prepared by Galambos and Prasad[9] for wide flange beam-columns bent by end-moments about their major axis. The Ojalvo charts and Galambos tables have been prepared only for structural steel in the 33-36 ksi yield point range.

Most investigations of the beam-column problem have been concerned with the case whereby bending moent is introduced only at the ends of the column, either by hypothetical brackets with a fixed eccentricity or by the angular rotation caused by the bending of restraining members framed to the column. In 1962 Ketter[10] compared the effects of end-moment and lateral load on the maximum strength of the unrestrained beam-column.

For an overall review of the beam-column problem, reference may be made to the Column Research Council "Guide to Design Criteria for Metal Compression Members."[11]

1.3 SUMMARY OF PRESENT INVESTIGATION

It has been the purpose of the present investigation to explore in a preliminary way, with the development of design aids, the laterally loaded beam-column problem, with limitations and scope outlined as follows:

1. Simplified 4 point area closed section.

2. No consideration of local buckling, torsion, or shear deformation.

3. Residual stress considered on a point area average basis.

4. Elastic stress analysis as well as ultimate load (instability) analysis.

5. Ends either unrestrained or restrained.

6. Uniform lateral load, with approximate load conversion factors to other lateral load distributions or to end-eccentricities.

7. Planar or biaxial bending.

8. Various steel strengths between 36 ksi and 100 ksi yield points.

Most of the objectives of the investigation were realized. The use of a computer comparable to the IBM 7090 at The University of Michigan was essential. However, the computer program for the final phase, biaxial bending with end restraint, was not entirely successful, with failure to achieve convergence near ultimate load. Considerable effort was expended on this portion of the program development but it is not reported upon and the work (under the authorized extension) was terminated prematurely by the early recall of the principal investigator, Dr. I. ElDarwish, to Egypt. The successful program for planar bending with end restraint, and biplanar bending without end restraint, are covered herein, and these also provide the bases for the second phase that is not reported and is now in progress (October, 1964).

2. ANALYSIS OF THE STRESSES, STRAINS, AND CURVATURES OF THE SIMPLIFIED FOUR POINT CROSS SECTION

2.1 THE SIMPLIFIED CROSS SECTION

The cross section shown in Fig. 1a, four angles boxed by lacing bars, is commonly used for very long columns. The size of the angles is assumed to be comparatively small compared to other dimensions of the member, in which case the areas of the angles may be considered as concentrated at points as shown in Fig. 1b in which:

2c = depth of the assumed cross section (distance between angle centroids).

2b = width of the assumed cross section (distance between angle centroids).

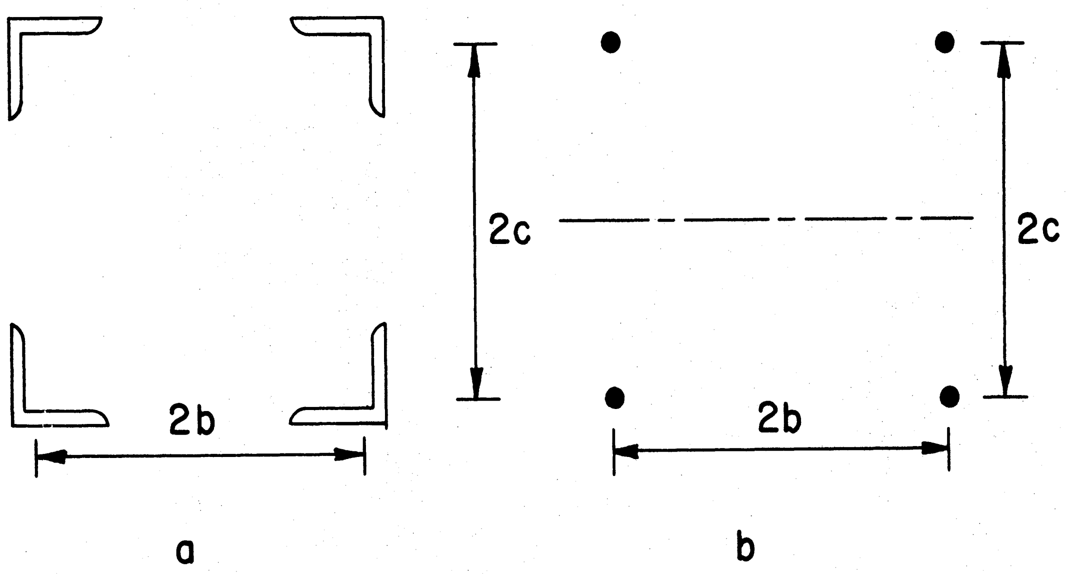


Fig. 1. The simplified cross section.

One advantage of such a simplified cross section is that its moment-curvature relationship in either the elastic or plastic range may be expressed analytically, obviating the need of curves or tables.

2.2 STRESSES AND STRAINS AT THE LOAD CAUSING INITIAL YIELD IN THE UNRESTRAINED BEAM-COLUMN UNDER PLANAR BENDING

In an unrestrained beam-column, when the yield stress is reached at the top fibers of the section shown in Fig. 1b, the resistance of that section to rotation will be at a maximum and the member cannot sustain additional column load. In the restrained beam-column, however, there may be some reserve strength left beyond the load at initial yield. (See Section 4.)

2.3 STRESS-STRAIN RELATIONSHIP MODIFIED BY RESIDUAL STRESS

A stress-strain curve to represent the average behavior of a steel angle in compression may be computed by assuming a residual stress pattern as shown in Fig. 3a. It is assumed that the stress-strain diagram in the annealed state would be as shown in Fig. 2.

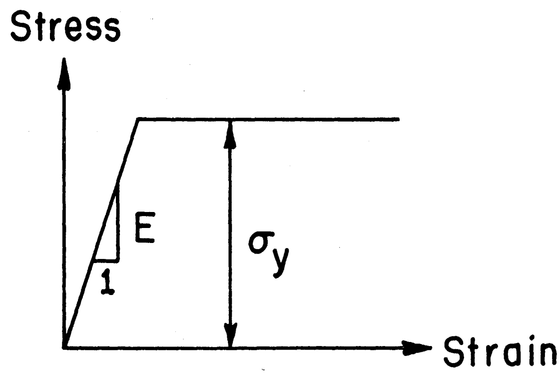


Fig. 2. Simplified stress-strain diagram for steel.

If $(\sigma_y - \sigma_R) < \sigma_a < \sigma_y$, the average stress on the section is

$$\sigma_a = \sigma_y - \sigma_R \, (x_e/c)^2 \; . \tag{1}$$

The average strain is

$$\epsilon_a = \frac{\sigma_y}{E} \left[1 + \frac{\sigma_R}{\sigma_y} - 2(\sigma_R/\sigma_y)(x_e/c) \right] . \tag{2}$$

Eliminating (X_e/c) from Eqs. (1) and (2),

$$\epsilon_a = \frac{1}{E} \left[\sigma_y + \sigma_R - 2 \sqrt{\sigma_R(\sigma_y - \sigma_a)} \right] \tag{3}$$

or

$$\sigma_a = \sigma_y - \frac{\sigma_R}{4}\left[\left(\frac{\sigma_y}{\sigma_R}+1\right) - \frac{E\epsilon_a}{\sigma_R}\right]^2 . \qquad (4a)$$

Alternatively, for $\sigma_p = \sigma_y - \sigma_R$

$$\sigma_a = -\frac{E^2(\epsilon_a-\epsilon_p)^2}{4(\sigma_y-\sigma_p)} + E\epsilon_a . \qquad (4b)$$

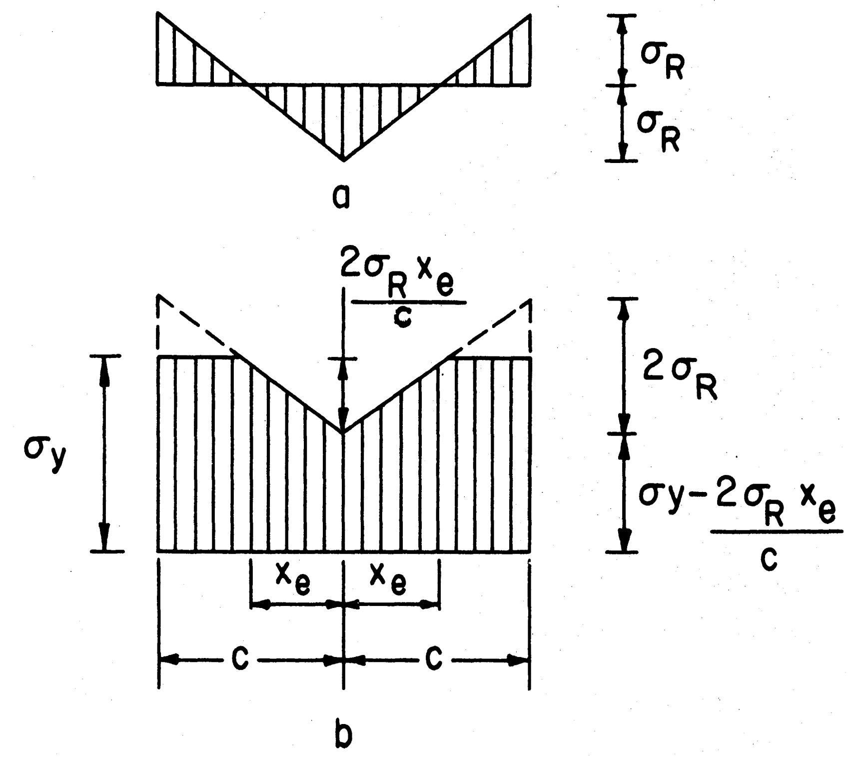


Fig. 3. a. Assumed residual stress distribution. b. Compressive stress distribution after a central compressive load initiates yielding.

From Eq. (4a), the effective tangent modulus may be determined

$$E_t = \frac{d\sigma_a}{d\epsilon_a} = \frac{E}{2\sigma_R}\left[\sigma_y+\sigma_R-E\epsilon_a\right] . \qquad (5)$$

Alternatively, E_t could have been determined directly from Eq. (2) by means of the relationship that

$$E_t = \frac{A_e}{A} E = \frac{x_e}{c} E . \qquad (6)$$

Although test information is limited, there are indications that the ratio σ_R/σ_y decreases as the yield point increases. By assuming $\sigma_R = 0.3\ \sigma_y$

for steel of yield stress of 50 ksi, and $\sigma_R = 0.2\ \sigma_y$ for steel of yield stress of 100 ksi, the following equation representing a linear interpolation will be assumed for σ_R/σ_y

$$\frac{\sigma_R}{\sigma_y} = .40 - \frac{\sigma_y}{500} . \qquad (7)$$

When the stress-strain relationship is assumed to be modified by residual stress, it will be referred to as the "general stress-strain relationship" in contradistinction to the "elasto-plastic stress-strain relationship."

2.4 REGRESSION OF STRESS

If the compressive strain at any point in the cross section decreases during a positive load increment, the stress will decrease locally at the constant elastic rate "E" as shown in Fig. 4, between points r and s. If

σ_r and ϵ_r are the stress and strain at point r after which regression of stress exists

and

σ_s and ϵ_s are the regressed stress and regressed strain at point s

then

$$\epsilon_s = \epsilon_r - (\sigma_r - \sigma_s)/E . \qquad (8)$$

The possibility of strain regression must be considered in the accurate analysis of a beam-column to obtain its ultimate loads.

2.5 STRESS-STRAIN RELATIONS IN PLANAR BENDING FOR GENERAL STRESS-STRAIN RELATIONSHIP

If the section shown in Fig. 1b is subjected to axial load P, bending moment M, and stresses at upper and lower corners, respectively, of σ_U and σ_B:

$$\begin{aligned} P &= \frac{A}{2}(\sigma_U + \sigma_B) \\ M &= \frac{A}{2} c(\sigma_U - \sigma_B) \end{aligned} \qquad (9)$$

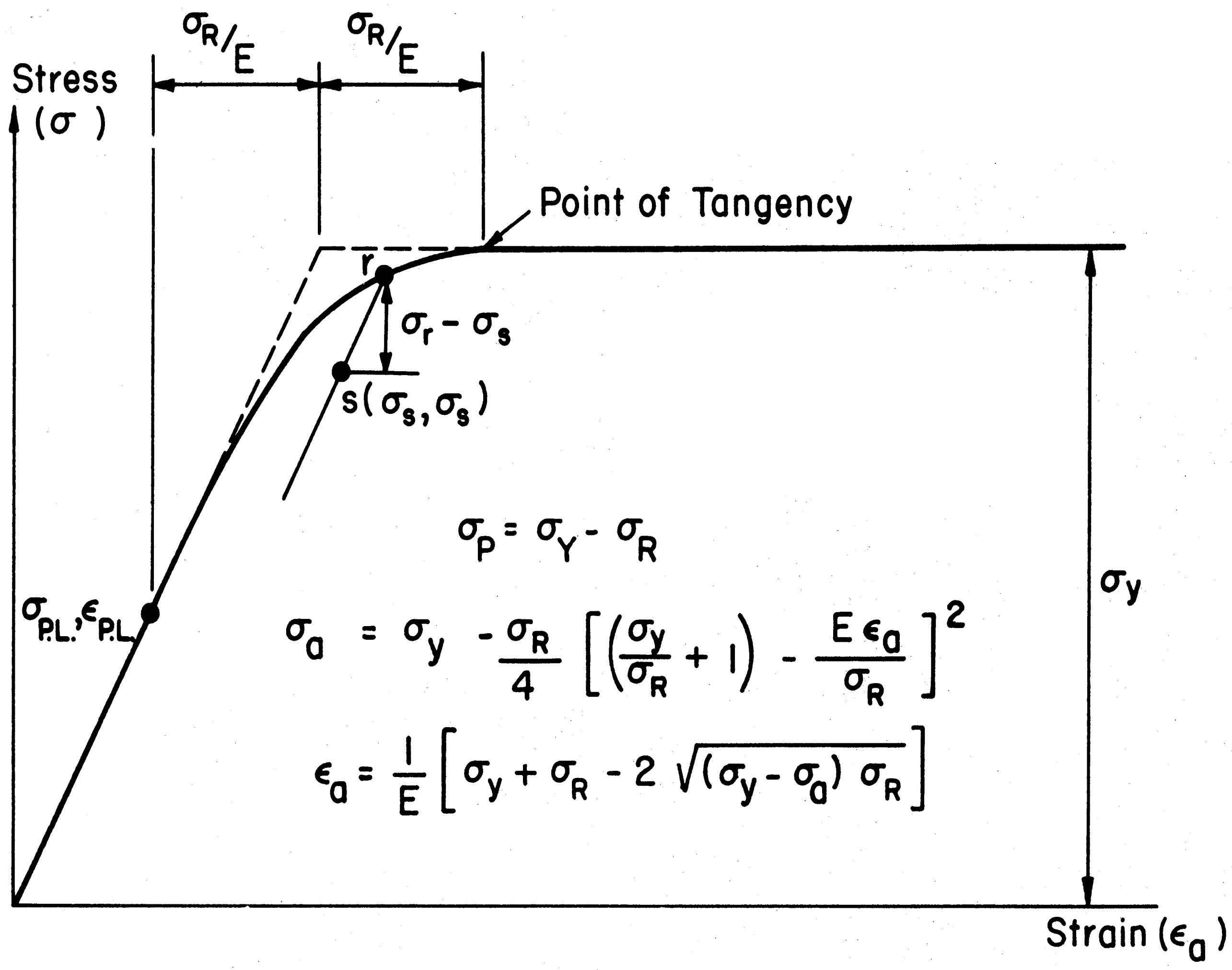


Fig. 4. Average stress-strain relation resulting from residual stress distribution in Fig. 3.

Solving

$$\sigma_U = \frac{P}{A} + \frac{M}{S}$$

$$\sigma_B = \frac{P}{A} - \frac{M}{S} \qquad (10)$$

in which

A = total area of the section

S = section modulus of the section = $Ac^2/c = Ac$.

The strain at the upper and lower fibers will be obtained by substituting in Eq. (3). Denoting these strains by ϵ_U and ϵ_B, the curvature at this particular location, assuming a linear distribution of strain, is

$$\phi = \frac{\epsilon_U - \epsilon_B}{2c} \quad . \tag{11}$$

Thus if the moment and axial load are given at a particular section, the curvature at the same section could be computed.

2.6 STRESSES AND STRAINS BEYOND THE LOAD AT INITIAL YIELD OF THE SQUARE SIMPLIFIED SECTION IN BIPLANAR BENDING FOR ELASTO-PLASTIC STRESS-STRAIN RELATIONSHIP

M_x is taken as greater than M_y.

For convenience in the computation, the diagonal axes (alternate principal axes for the square section), will be taken as reference x and y axes, as shown in Fig. 5.

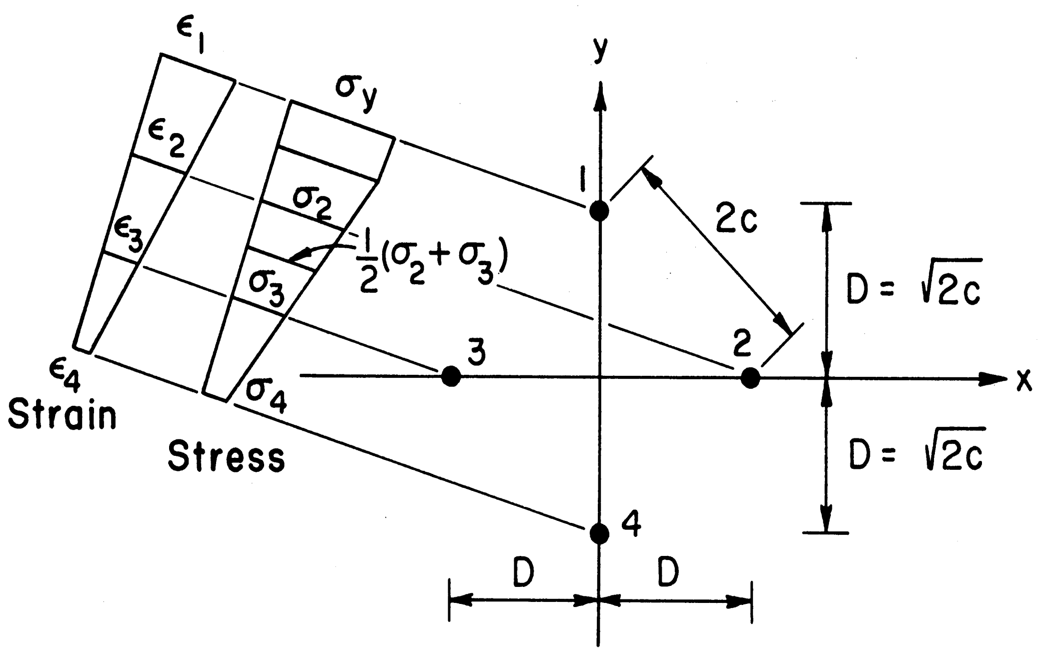


Fig. 5. Stresses and strains beyond the load at initial yield for biplanar bending and elasto-plastic stress-strain behavior.

At any point along the beam, if the initial yield strain level is exceeded at one corner, the stresses and strains will be as shown in Fig. 5. From geometry and statics, letting $D = \sqrt{2}\, c$

$$P = \frac{A}{4}(\sigma_y+\sigma_2+\sigma_3+\sigma_4)$$

$$M_x = \frac{A}{4}D(\sigma_y-\sigma_4) \qquad (12)$$

$$M_y = \frac{A}{4}D(\sigma_2-\sigma_3)$$

solving

$$\sigma_4 = \sigma_y - \frac{4M_x}{AD}$$

$$\sigma_2 = -\sigma_y + \frac{2P}{A} + \frac{2(M_x+M_y)}{AD} \qquad (13)$$

$$\sigma_3 = -\sigma_y + \frac{2P}{A} + \frac{2(M_x-M_y)}{AD}$$

The curvatures will be:

the curvature in the vertical plane

$$\phi_x = \frac{\frac{1}{2}(\sigma_2+\sigma_3)-\sigma_4}{ED} = \frac{\sigma_2+\sigma_3-2\sigma_4}{2ED} \qquad (14)$$

the curvature in the horizontal plane

$$\phi_y = \frac{\sigma_2-\sigma_3}{2ED} = \frac{M_y}{\frac{A}{4}D \times 2ED} = \frac{M_y}{EI}\,.$$

2.7 STRESSES AND STRAINS IN BIPLANAR BENDING FOR THE SQUARE SIMPLIFIED SECTION FOR THE GENERAL STRESS-STRAIN RELATIONSHIP

For convenience in the computations the diagonal principal axes are again considered as reference axes. The stresses and strains for the case of biplanar bending and with a stress-strain relation similar to that shown in Fig. 4, at any section of the beam-column, will be as shown in Fig. 6.

From statics we have

$$P = \frac{A}{4}(\sigma_1+\sigma_2+\sigma_3+\sigma_4)$$

$$M_x = \frac{A}{4}D(\sigma_1-\sigma_4) \qquad (15)$$

$$M_y = \frac{A}{4}D(\sigma_2-\sigma_3)$$

solving in terms of σ_1

$$\sigma_4 = \sigma_1 - \frac{4M_x}{AD}$$

$$\sigma_2 = -\sigma_1 + \frac{2P}{A} + \frac{2(M_x+M_y)}{AD} \qquad (16)$$

$$\sigma_3 = -\sigma_1 + \frac{2P}{A} + \frac{2(M_x-M_y)}{AD}$$

in which $D = \sqrt{2}\,c$.

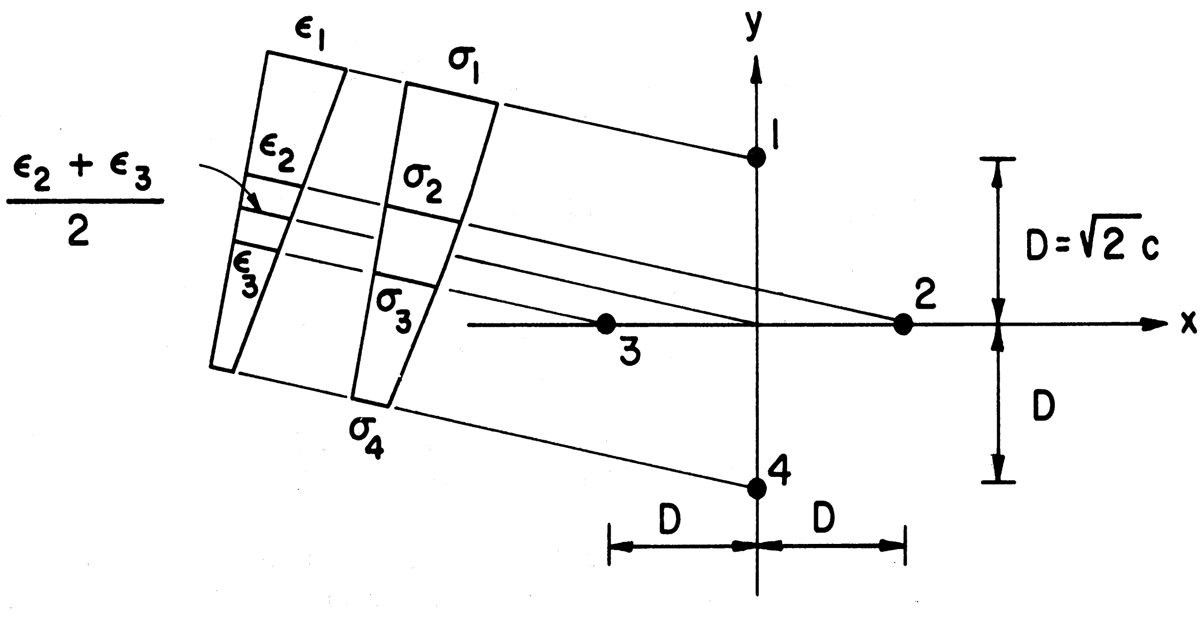


Fig. 6. Stresses and strains beyond the proportional limit stress for the biplanar bending case.

Equation (16) is not enough to determine the four stresses $\sigma_1, \sigma_2, \sigma_3$, and σ_4. The additional condition results from the linearity of the strain distribution. From Fig. 6,

$$\epsilon_1 + \epsilon_4 = \epsilon_2 + \epsilon_3 . \tag{17}$$

If the stresses and then the strains are known, the curvature will be:

$$\phi_x = (\epsilon_1 - \epsilon_4)/2D$$
$$\phi_y = (\epsilon_2 - \epsilon_3)/2D . \tag{18}$$

If both σ_2 and σ_3 are less than σ_P then $\phi_y = M_y/EI$ in which ϕ_x, ϕ_y are the curvatures in the vertical and horizontal planes respectively.

If the stresses σ_2, σ_3, and σ_4 are within the proportional limit stress, then from Eqs. (17) and (4)

$$\epsilon_2 + \epsilon_3 - \epsilon_4 = \epsilon_1$$

or

$$\frac{\sigma_2 + \sigma_3 - \sigma_4}{E} = \frac{1}{E}\{2\sigma_y - \sigma_P) - 2\sqrt{(\sigma_y - \sigma_1)(\sigma_y - \sigma_P)}\} .$$

By adding the components of Eq. (15) together and solving for σ_1 one gets

$$\sigma_1 = \frac{1}{2}\{-(\frac{4}{9}\sigma_R - 2F) + \sqrt{(\frac{4}{9}\sigma_R - 2F)^2 - 4(F^2 - \frac{4}{9}\sigma_y\sigma_R)}\} \tag{19}$$

in which

$$F = \frac{1}{3}\{\frac{4P}{A} + \frac{8M_x}{AD} - (2\sigma_y - \sigma_P)\} .$$

After assuming σ_2, σ_3, and σ_4 less than σ_P, if it turns out that the stress σ_2 exceeds the proportional limit, then the relation between σ_2 and ϵ_2 is not linear and should be obtained from Eq. (4). In this case the equation for σ_1 will take the following form:

$$\sigma_1 = \frac{P}{A} + \frac{3M_x - M_y}{AD} + \{\sqrt{(\sigma_y - \sigma_1)\sigma_R} - \sqrt{(\sigma_y + \sigma_1 - \frac{2P}{A} - \frac{2(M_x + M_y)}{AD})\sigma_R}\} . \tag{20}$$

Equation (20) may be solved by trial and error. If the assumption that only σ_1 and σ_2 exceed the proportional limit is incorrect, then Eqs. (16) and (17) may be solved directly by trial and error.

If $\sigma_1 = \sigma_y$, there is still some reserve strength, and the stresses will be obtained by putting $\sigma_1 = \sigma_y$ in Eq. (16). The resulting values of σ_2, σ_3, and σ_4 will be those given in Eq. (13). The strains ϵ_2, ϵ_3, and ϵ_4 will be obtained from Eq. (9) and the strain ϵ_1 is

$$\epsilon_1 = \epsilon_2 + \epsilon_3 - \epsilon_4 .$$

If σ_1 and σ_2 both reach the yield point there will be no reserve strength left of the section. The ultimate for unrestrained beam-columns in such a case is that load at which σ_2 just reaches the yield point.

__Example__: For a simplified section of the following properties

$$A = 40 \text{ sq in.}; \quad c = 15 \text{ in.}; \quad E = 29{,}000 \text{ ksi}$$

$$\sigma_y = 50 \text{ ksi}; \quad \sigma_P = 35 \text{ ksi}$$

and subject to an axial force of 1,200 kips and bending moments of $M_x = 8{,}485$ K-inch and $M_y = 4{,}243$ K-inch, find the stresses, strains and curvatures.

__Solution__:

$$D = 15\sqrt{2} = 21.213 \text{ in.}, \quad I = 2 \times 10 \times (21.213)^2 = 9{,}000 \text{ in.}^4$$

By the ordinary flexural formula

$$\sigma_1 = \frac{P}{A} + \frac{M_x D}{I_x} = 30 + 20 = 50.00 \text{ ksi}$$

thus $\sigma_1 > \sigma_P$. From Eq. (19)

$$\sigma_R = 15 \text{ ksi}$$

$$\sigma_P = 50\text{-}15 = 35 \text{ ksi}$$

$$F = \frac{1}{3}\left\{120 + \frac{8 \times 8.485}{40 \times 21.213} - 100 + 35\right\} = 45.0$$

$$\sigma_1 = \frac{1}{2}\left[-(\frac{4}{9} \times 15 - 90) + \sqrt{(\frac{4}{9} \times 15 - 90)^2 - 4(45^2 - \frac{4}{9} \times 50 \times 15}\right\}$$

$$= 48.3 \text{ ksi.}$$

By Eq. (16)

$$\sigma_2 = -48.3 + 60 + \frac{2 \times 12.728}{40 \times 21.213} = -48.3 + .90 = 41.7 \text{ ksi} .$$

In this case $\sigma_2 > \sigma_{PL}$, and Eq. (20) is applied:

$$\sigma_1 = 30 + \frac{3x8,485-4.243}{40x21.213} + \left\{\sqrt{50-\sigma_1} - \sqrt{50+\sigma_1-60 - \frac{2x12.728}{40x21.213}}\right\}\sqrt{15}$$

then

$$\sigma_1 = 55 + 3.873\left\{\sqrt{50-\sigma_1} - \sqrt{\sigma_1-40}\right\} .$$

Solution by trial

Assumed σ_1*	Computed σ_1	
48.30	48.89	
48.60	48.22	
48.41	48.65	
48.53	48.38	
48.46	48.55	
48.51	48.43	
48.47	48.52	
48.50	48.46	
48.48	48.50	Thus σ_1 = 48.49 ksi.

*First assumed value of σ_1 is that obtained by assuming $\sigma_2 < \sigma_{PL}$. Other assumed values of σ_1 = (last assumed σ_1+last computed σ_1)/2.

If Eqs. (15) and (16) are used to compute the other stresses, then

$$\sigma_2 = 41.51 \text{ ksi}, \quad \sigma_3 = 21.51 \text{ ksi}, \quad \text{and} \quad \sigma_4 = 8.49 \text{ ksi}$$

From Eq. (3)

$$\epsilon_1 = \frac{1}{29000} \{(50 + 15) - 2\sqrt{(50-48.49)15}\} = 1.910 \times 10^{-3}$$

$$\epsilon_2 = \frac{1}{29000} \{(50 + 15) - 2\sqrt{(50-41.54)15}\} = 1.464 \times 10^{-3}$$

and

$$\epsilon_3 = \frac{21.54}{29000} = 0.743 \times 10^{-3}; \quad \epsilon_4 = \frac{8.46}{29000} = 0.292 \times 10^{-3} .$$

As a check

$$\epsilon_2+\epsilon_3-\epsilon_4 = 10^{-3}(1.464+.743+.292) = 1.915 \times 10^{-3} = \epsilon_1$$

From Eq. (18) the curvatures are

$$\phi_x = (\frac{\epsilon_1-\epsilon_4}{2D}) = (1.91-.292)10^{-3}/42.426 = 3.8137 \times 10^{-5} \text{ rad/in.}$$

$$\phi_y = (\frac{\epsilon_2-\epsilon_3}{2D}) = (1.464-.743)10^{-3}/42.426 = 1.6994 \times 10^{-5} \text{ rad/in.}$$

<u>The same example will now be solved by trial</u>: From Eq. (16) the stresses σ_2, σ_3, and σ_4 in terms of σ_1 are

$$\sigma_4 = \sigma_1 - \frac{4M_x}{AD} = \sigma_1 - \frac{4 \times 8,485}{40 \times 21.213} = \sigma_1 - 40$$

$$\sigma_2 = -\sigma_1 + \frac{2P}{A} + \frac{2(M_x+M_y)}{AD} = -\sigma_1 + \frac{2 \times 1200}{40} + \frac{2 \times 12,728}{40 \times 21.213} = -\sigma_1 + 90$$

$$\sigma_3 = -\sigma_1 + \frac{2P}{A} + \frac{2(M_x-M_y)}{AD} = -\sigma_1 + \frac{2 \times 1200}{40} + \frac{2 \times 4,243}{40 \times 21.213} = -\sigma_1 + 70$$

First Cycle:

$$\text{Assumed } \sigma_1 = \frac{P}{A} + \frac{M_x D}{I_x} = 50 \text{ ksi} \qquad \text{Assumed } E\epsilon_1 = 65$$

Thus

$$\sigma_2 = 40 \text{ ksi} \qquad E\epsilon_2 = 40.5$$

$$\sigma_3 = 20 \text{ ksi} \qquad E\epsilon_3 = 20$$

$$\sigma_4 = 10 \text{ ksi} \qquad E\epsilon_4 = 10$$

$$\text{Computed } E\epsilon_1 = E(\epsilon_2+\epsilon_3-\epsilon_4) = 50.5$$

Second Cycle:

Assumed $E\epsilon_1$ = (Assumed $E\epsilon_1$ in the first cycle + computed $E\epsilon_1$ in the first cycle)/2

$$\sigma_1 = 49.12 \text{ ksi} \qquad \text{Assumed } E\epsilon_1 = \frac{65+50.5}{2} = 57.75$$

$$\sigma_2 = 40.88 \text{ ksi} \qquad E\epsilon_2 = 41.61$$

$$\sigma_3 = 20.88 \text{ ksi} \qquad E\epsilon_3 = 20.88$$

$$\sigma_4 = 9.12 \text{ ksi} \qquad E\epsilon_4 = 9.12$$

$$\text{Computed } E\epsilon_1 = E(\epsilon_2+\epsilon_3-\epsilon_4) = 53.37.$$

At this stage, an interpolation between two results is very helpful. Consider Fig. 7.

If $T\epsilon_1$ = assumed strain

ϵ_1 = computed strain

The linear interpolation between the two results

$$(T\epsilon_1^o, \epsilon_1^o \ \& \ T\epsilon_1', \epsilon_1')$$

is

$$T\epsilon_1 = \frac{T\epsilon_1^o - \epsilon_1^o(T\epsilon_1' - T\epsilon_1^o)/(\epsilon_1' - \epsilon_1^o)}{1 - (T\epsilon_1' - T\epsilon_1^o)/(\epsilon_1' - \epsilon_1^o)} .$$

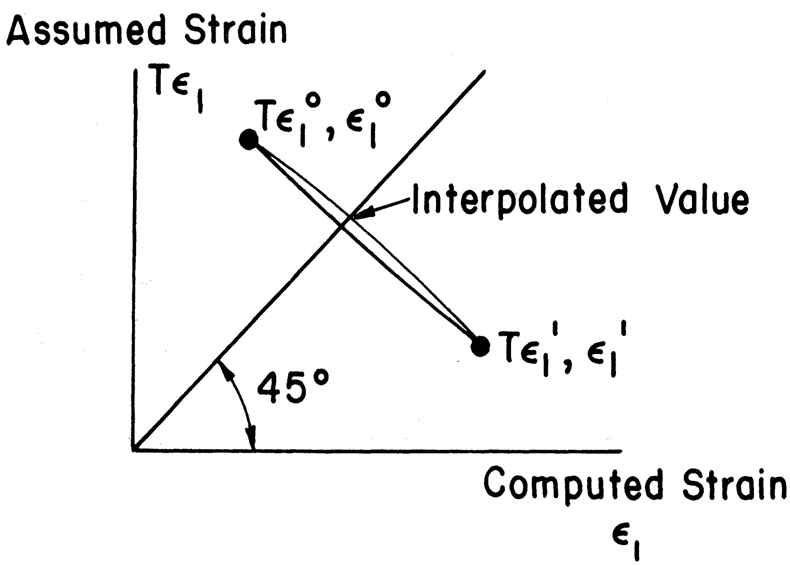


Fig. 7. Interpolation procedure.

Points of less difference between $T\epsilon_1$ and ϵ_1 should be used in the interpolation. Thus,

Third Cycle:

$$(T\epsilon_1)E = \frac{65-50.5(57.75-65)/(53.37-50.5)}{1-(57.75-65)/(53.37-50.5)} = 54.61.$$

$\sigma_1 = 48.20$ ksi $\quad E\epsilon_1 = 54.61$

$\sigma_2 = 41.80$ ksi $\quad E\epsilon_2 = 42.82$

$\sigma_3 = 21.80$ ksi $\quad E\epsilon_3 = 21.80$

$\sigma_4 = 8.20$ ksi $\quad E\epsilon_4 = 8.20$

Computed $E\epsilon_1 = E(\epsilon_2+\epsilon_3-\epsilon_4) = 56.42.$

Fourth Cycle:

$$(T\epsilon_1)E = \frac{54.61-56.42(57.75-54.61)/(53.37-56.42)}{1-(57.75-54.61)/(53.37-56.42)} = 55.53$$

$$\sigma_1 = 48.50 \text{ ksi} \qquad E\epsilon_1 = 55.53$$

$$\sigma_2 = 41.50 \text{ ksi} \qquad E\epsilon_2 = 42.42$$

$$\sigma_3 = 21.50 \text{ ksi} \qquad E\epsilon_3 = 21.50$$

$$\sigma_4 = 8.50 \text{ ksi} \qquad E\epsilon_4 = 8.50$$

$$\text{Computed } E\epsilon_1 = E(\epsilon_2+\epsilon_3-\epsilon_4) = 55.42.$$

Fifth Cycle:

$$(T\epsilon_1)E = \frac{54.61-56.42(55.53-54.61)/(55.42-56.42)}{1-(55.53-54.61)/(55.42-56.42)} = 55.48$$

$$\sigma_1 = 48.49 \text{ ksi} \qquad E\epsilon_1 = 55.48$$

$$\sigma_2 = 41.51 \text{ ksi} \qquad E\epsilon_2 = 42.43$$

$$\sigma_3 = 21.51 \text{ ksi} \qquad E\epsilon_3 = 21.51$$

$$\sigma_4 = 8.49 \text{ ksi} \qquad E\epsilon_4 = 8.49$$

$$\text{Computed } E\epsilon_1 = E(\epsilon_2+\epsilon_3-\epsilon_4) = 55.45.$$

Good agreement is obtained between the assumed and computed ϵ_1.

2.8 EFFECT OF INITIAL CURVATURE

The effect of initial curvature is of comparable importance to residual stress, especially so in the case of high strength steels. In the case of rolled sections, the specified tolerances (ASTM Specifications A6) for out-of-straightness may be used as a basis for estimating the initial curvature, as shown in Fig. 8 by the full line. On the basis of the dashed line approximation for out-of-straightness tolerance, the permissible variation δ_i is

$$\delta_i(\text{initial curvature}) = .000833L \tag{21}$$

in which δ_i and L are in inches.

If the additional bending moment due to initial curvature is assumed to take the same form as that due to a uniform lateral load on the beam, then an

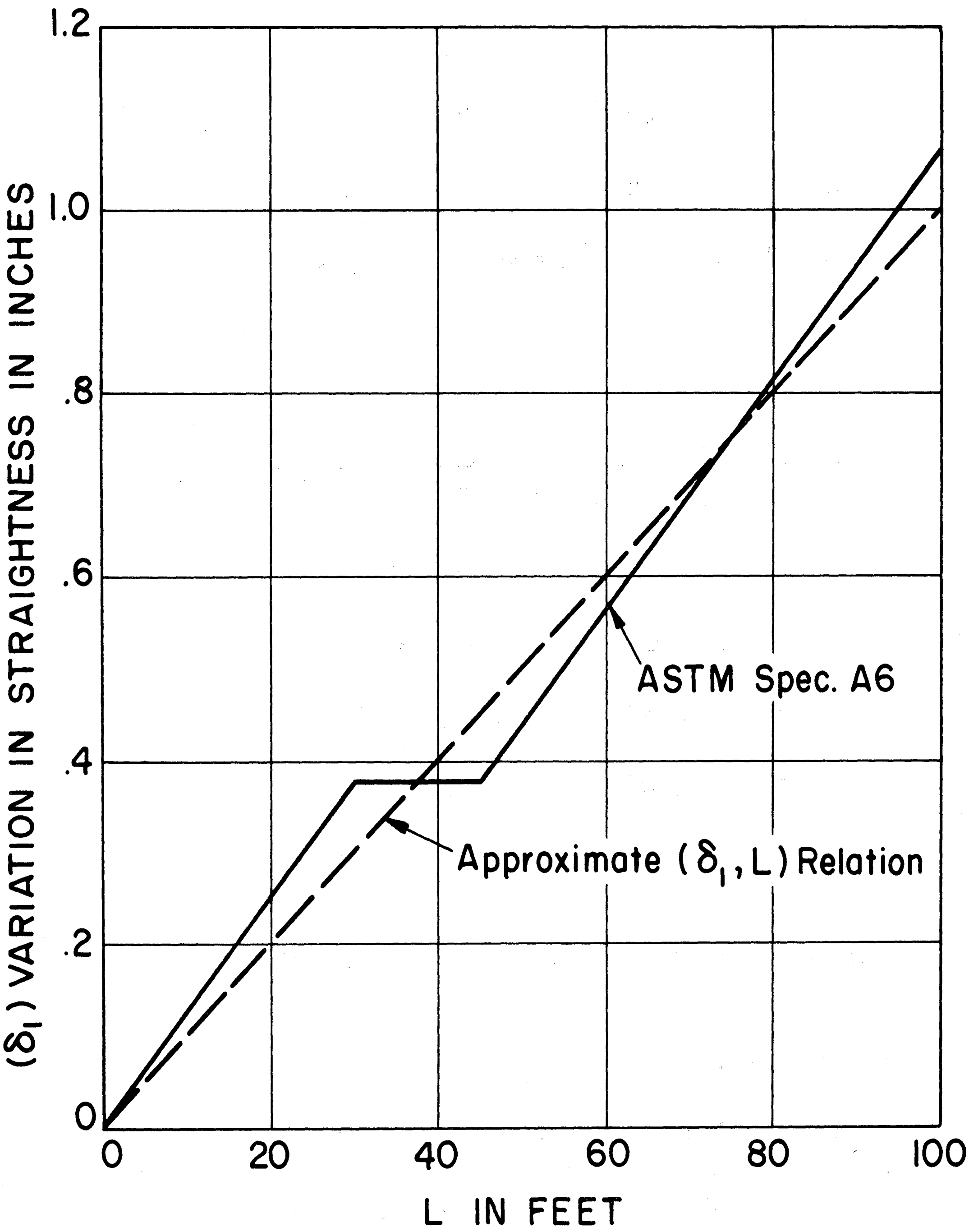


Fig. 8. Permissible variation in straightness for wide flange shapes by ASTM Spec. A6.

equivalent value (W_i) of this lateral load may be taken as

$$\frac{W_i L}{8} = P\delta_i = P \times .000833L \; .$$

If the lateral load W_i is given as a ratio of the axial load P, then

$$k_i P \frac{L}{8} = P \times .000833L$$

or

$$k_i = .00667 \; . \qquad (22)$$

In which k_i is the ratio of an equivalent uniform lateral load, introduced to allow for initial curvature, to the axial load.

If the ultimate load of a beam-column is obtained without the effect of initial curvature, a value of uniform lateral load equal to $k_i P$ should be added to the original lateral loads in both principal directions.

3. APPROXIMATE DESIGN PROCEDURE FOR BEAM-COLUMNS

3.1 UNRESTRAINED BEAM-COLUMN DESIGN BASED ON LOAD AT INITIAL YIELD

A lower bound to the ultimate load capacity of a beam-column is the load at initial yield. Such an estimate is restricted to those cases in which failure is due primarily to excessive bending. It should be noted that if the initial yield procedure is to be used to estimate the strength of columns with small eccentricities or small lateral loads, it is necessary to assume that some unintentional eccentricity exists, in order that the initial yield strength curve approaches a proper limit i.e., the strength curve for a concentrically loaded column, as the lateral load or end eccentricity diminishes to zero. The effect of all imperfections such as initial crookedness, non-homogenity, residual stresses, etc., are lumped into the equivalent initial eccentricity.

A good approximation for estimating the maximum bending moment in columns with lateral loads is given by

$$M = M_o + \frac{P\delta_o}{1 - \frac{P}{P_e}} \tag{23}$$

in which M_o and δ_o are the maximum moment and deflection respectively without regard to the added moment caused by deflection. Equation (23) can be written as

$$M = M_o \left[\frac{1 + \psi \frac{P}{P_e}}{1 - \frac{P}{P_e}} \right] \tag{24}$$

where

$$\psi = \left[\frac{\pi^2 \delta_o EI}{M_o L^2} - 1 \right] . \tag{25}$$

The value of ψ is dependent on δ_o and M_o, which can be obtained easily from a structural handbook. Some values of ψ for different cases of loading are given in Table II. The maximum combined stress is therefore

$$\sigma_m = \frac{P}{A} + \left[\frac{1+\psi\alpha}{1-\alpha} \right] \frac{M_o c}{I} \tag{26}$$

in which

$$\alpha = P/P_e .$$

The load at initial yield (P_p) can be obtained simply by putting $\sigma_m = \sigma_y$ in Eq. (26), then solving for P_p. Equation (26) gives estimates that are in error by less than 1% for all usual cases of load distributions at all levels of P.

As mentioned in Section 2.2, the unrestrained four point cross section in planar bending reserves no resistance to rotation beyond the load at initial yield. Thus, the ultimate load for unrestrained beam-columns of the simplified cross section in planar bending is the load at initial yield.

3.2 LOAD AT INITIAL YIELD FOR THE UNRESTRAINED BEAM-COLUMN (SIMPLIFIED FOUR POINT CROSS SECTION) IN PLANAR BENDING UNDER UNIFORM LATERAL LOAD (kP)

For the case of the unrestrained beam-columns in planar bending under uniform load kP Eq. (26) may be written:

$$\sigma_m = \frac{P}{A} + \left[\frac{1+.028\alpha}{1-\alpha}\right] \frac{\frac{kPL}{8} c}{Ar^2} \tag{27}$$

Putting $\sigma_m = \sigma_y$ and solving for P/A,

$$\frac{P}{A} = \sigma_e \left\{ \frac{\frac{\sigma_y}{\sigma_e} + \frac{kLc}{8r^2} + 1 - \sqrt{\left(\frac{\sigma_y}{\sigma_e} + 1 + \frac{kLc}{8r^2}\right)^2 - 4 \frac{\sigma_y}{\sigma_e}\left(1 - \frac{.028kLc}{8r^2}\right)}}{2(1-.028\ kLc/8r^2)} \right\} \tag{28}$$

in which

σ_e = Euler stress equal to $\pi^2 E/(L/r)^2$

r = radius of gyration.

For the simplified section in which

$$I = Ac^2 \text{ or } r = c$$

Eq. (28) reduces to

$$\frac{P}{A} = \sigma_e \left\{ \frac{\left(\frac{\sigma_y}{\sigma_e} + 1 + \frac{kL}{8r}\right) - \sqrt{\left(\frac{\sigma_y}{\sigma_e} + 1 + \frac{kL}{8r}\right)^2 - 4\,\frac{\sigma_y}{\sigma_e}\left(1 - \frac{.028kL}{8r}\right)}}{2(1-.028\ kL/8r)} \right\} \qquad (29)$$

Equation (29) is a function of L/r, E, σ_y, and k. The modulus of elasticity of steel is usually taken as 29,000,000 psi. Column slenderness ratios L/r vary between 0 and 200. The various yield stresses, that cover the current constructional steels, are σ_y = 33, 36, 42, 46, 50, 60, 70, and 100 ksi. In Table I, Eq. (29) has been solved to cover this wide range of parameters, together with variation of the load ratio k between .01 to 0.3, in increments of .01.

3.3 BEAM-COLUMN DESIGN BASED ON THE LOAD AT INITIAL YIELD FOR OTHER CROSS SECTIONS AND LOADING CONDITIONS FOR PLANAR BENDING

The solution, as tabulated in Table I is for c/r = 1, applicable to a column with the area concentrated at the full depth, as in the case of four corner angles or tubes connected on all four sides by lacing bars. Modification to other values of c/r may be made by simple multiplication.

In using Table I, a value of k = 0.01 should be added to take care of column imperfections. This allows for an initial curvature of 20% more than allowed by usual mill tolerances for camber or sweep and is based on an assumed maximum deviation from straightness of L/800. It is 50% more than the value of 0.0067 arrived at by ASTM tolerance specifications. The additional amount will roughly compensate for other imperfections, such as residual stress, that are not explicitly included. Thus, the equivalent lateral load moment is

$$\frac{kPL}{8} = \frac{PL}{800} \text{ and } k = 0.01 .$$

In addition, for nonuniform lateral loads, an equivalent uniform load may be established by multiplying k by

(1) A load distribution factor, and

(2) A deflection factor.

The load distribution factor is the same as that tabulated in the beam tables of the AISC Manual and is

$$LF = 8M_o/WL . \qquad (30)$$

The deflection factor is

$$DF = \frac{1 + \psi \frac{P}{P_e}}{1+0.0281 \frac{P}{P_e}} \tag{31}$$

but for design purposes may be approximated simply by

$$DF = 1 + \psi \frac{P}{P_e} \, . \tag{32}$$

Some values of both the load and deflection factors are tabulated in Table II.

For end eccentricities e_1 and e_2, where $e_1 < e_2$, the load factor is

$$k_{ecc} = \left[\frac{3.2e_1+4.8e_2}{L}\right] \left[1 + 0.23 \frac{P}{P_e}\right] \tag{33}$$

but is not to be less than

$$k_{ecc} = \frac{3.2e_2}{L} \left[1 + 0.23 \frac{P}{P_e}\right] \, . \tag{34}$$

Finally, the combined equivalent load factor must be multiplied by the ratio of c/r appropriate to the cross section being used.

Equation (33) in different form corresponds to CRC and AISC recommendations.

In summary, then for any beam-column situation,

1. Calculate $k_W = W/P$.

2. Add $k_I = 0.01$ for column imperfection.

3. Add k_{ecc} by Eq. (33) if there are applied end eccentricities.

4. Estimate approximate P/P_e (or σ_a/σ_e).

5. Determine load distribution factor (Eq. (30) or Table II), deflection factor (Eq. (32) or Table II), and c/r for section type to be used.

6. Determine equivalent uniform load ratio for use in Table I, as follows:

$$k_e = \frac{c}{r}\,[(k_w)(LF)(DF) + k_{ecc} + (0.01)] \,. \qquad (35)$$

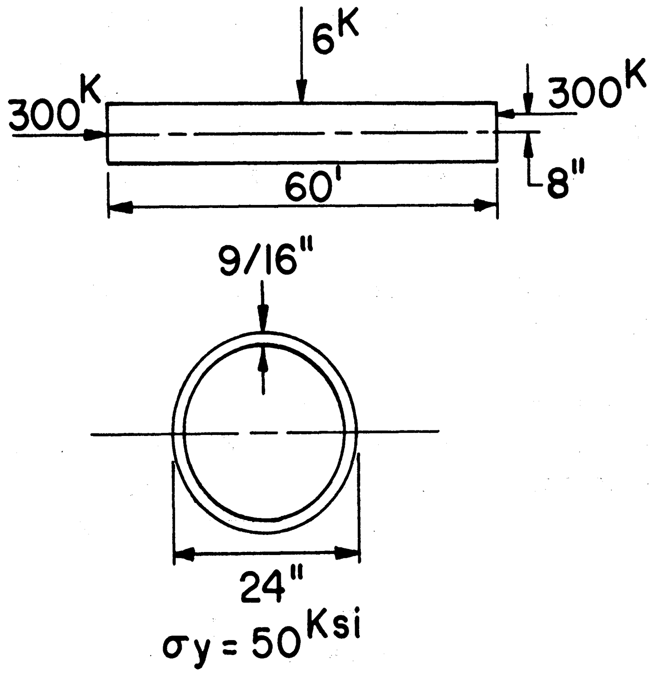
7. Enter Table I with the chosen σ_y, the computed k_e, and the estimated L/r.

8. Determine P/A at yield and divide by the desired safety factor.

9. Select column and redesign if necessary as may be indicated.

For uniform lateral load with no eccentricity of column load, the procedure is simplified because the load and deflection factors are unity, and $k_{ecc} = 0$. In numerous other cases the load factor is close to unity and may be taken as such. (See Table II.)

Example 1. Design by Use of Table I a Beam-Column for Load Conditions as Illustrated Below:



After preliminary trial design the above cross-section is arrived at for check analysis.

$A = 42.41 \text{ in.}^2$

$I = 3054 \text{ in.}^4$

$S_{xx} = 3054/12.28 = 248.7 \text{ in.}^3$

The average stress P/A (load factor = 1.75) = 300 x 1.75/42.41 = 12.38 ksi. L/r = 84.8.

$$P_e = \frac{\pi^2(29000)}{(84.8)^2} = 39.80 \text{ ksi.}$$

Moment Factors for Use in Table I

1. Load distribution factor = 2.

2. $c/r = \sqrt{2} = 1.414.$

3. Deflection factor $= 1 - .018 \times \dfrac{12.38}{39.80} = .944.$

4. Eccentricity $k_{ecc} = \dfrac{4.8 \times 8}{720}(1 + 0.23(11.67/39.80)) = 0.057.$

The dead weight of the tube is 7.68/37.7 x 42.41 = 8.64 K for which $k_w = 8.64/300 = 0.029$; crookedness allowance k = 0.01.

$$k_e = (.029 \times 1 \times 1 + .02 \times 2 \times .944 + .057 + .01)1.414 = 0.190$$

Referring to Table I, and interpolating between L/r of 80 and 90, for $\sigma_y = 50$ ksi and k = 0.190, the maximum stress reaches $\sigma_y = 50$ ksi at $\sigma_a = 12.59$. This being greater than the trial $\sigma_a = 12.38$ ksi, the design is confirmed.

3.4 UNRESTRAINED BEAM-COLUMN DESIGN BASED ON THE LOAD AT INITIAL YIELD FOR BIPLANAR BENDING CASE

Table I may be adapted to the case of biplanar bending. Under the assumption that the maximum moments in each of the principal planes of bending occur at the same location, Eq. (27) may be adapted to biplanar bending (modified for the simplified section by putting c = r):

$$\sigma_m = \frac{P}{A} + \left\{\left(\frac{1+.028\alpha_x}{1-\alpha_x}\right) k_{xe} \frac{P}{8A}\frac{L}{r_x}\right\} + \left\{\left(\frac{1+.028\alpha_y}{1-\alpha_y}\right) k_{ye} \frac{P}{8A}\frac{L}{r_y}\right\} \qquad (36)$$

in which k_{xe} and k_{ye} are the equivalent uniform load ratios, for bending about the principal axis x and y respectively, α_x and α_y are the load ratios P/P_{ex} and P/P_{ey}, and r_x and r_y are the radii of gyration about the principal axes x and y respectively. Where P_{ex} and P_{ey} are Euler's loads about the principal axis x and y respectively. If the following two factors are introduced:

1. The (slenderness-ratio) ratio factor

$$SFR = \frac{L}{r_y} \Big/ \frac{L}{r_x} = \frac{r_x}{r_y} \tag{37}$$

2. The amplification ratio factor

$$AFR = \frac{1+.028\alpha_y}{1-\alpha_y} \Big/ \frac{1+.028\alpha_x}{1-\alpha_x} \tag{38}$$

which can be approximated by:

$$AFR = \frac{1-\alpha_x}{1-\alpha_y} .$$

Then, Eq. (36) may be rewritten as

$$\sigma_m = \frac{P}{A} + \left\{\left(\frac{1+.028\alpha_x}{1-\alpha_x}\right) k_{xe} \frac{P}{8A} \frac{L}{r_x}\right\} \left\{1 + \frac{k_{ye}}{k_{xe}} (SFR)(AFR)\right\} . \tag{39}$$

If the biplanar bending equivalent uniform load factor k_{Be} is introduced as

$$k_{Be} = k_{xe} \left\{1 + (SFR)(AFR) \frac{k_{ye}}{k_{xe}}\right\} . \tag{40}$$

Then Table I, can be used to find the load at initial yield for unrestrained biplanar bending. The procedure will be as follows:

1. Assume P.

2. Find the equivalent uniform load ratios for bending about both principal axis from Eq. (35).

3. Find (the slenderness-ratio) factor ratio (SFR), from Eq. (37).

4. Find the amplification factor ratio (AFR), from Eq. (38).

5. Find the biplanar equivalent uniform load ratio from Eq. (40).

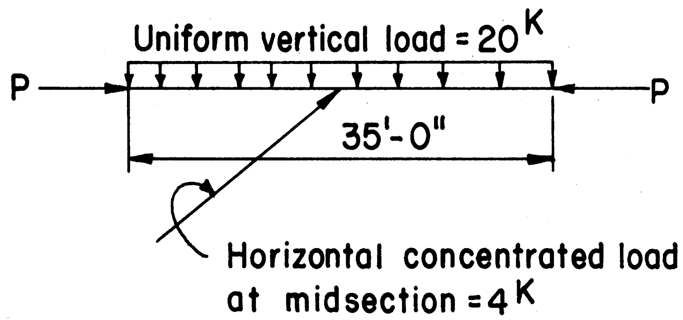
6. Enter Table I with the chosen σ_y, the computed k_{Be}, and the estimated (L/r_x).

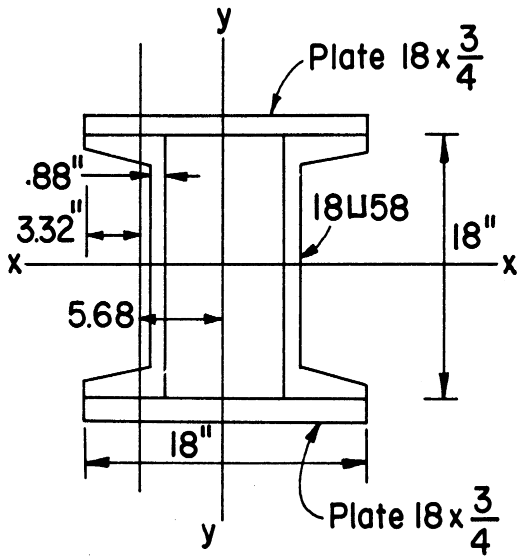
7. Determine P/A at yield and divide by the desired safety factor.

8. Select column and redesign if necessary.

Example 2.

The beam shown is subjected to a uniform vertical load of 20 Kips, together with a horizontal concentrated load of 4 Kips acting at mid section. Calculate the maximum load P if this yield point is 33 ksi.





$A = 2 \times 16.98 + 2 \times 18 \times 3/4 = 60.96 \text{ in.}^2$

Weight of beam $= 7.29$ Kips

$I_x = 2 \times 670.7 + 2 \times 18 \times .75 \times (9.375)^2 = 1341.4 + 2373.05 = 3714.5 \text{ in.}^4$

$I_y = 2 \times 18.5 + 2 \times 16.98 \times (5.68)^2 + 2 \times 3/4 \times 18^3/12 = 37 + 1095.6 + 729$
$= 1861.6 \text{ in.}^4$

$r_x = \sqrt{3714.5/60.96} = 7.81$ in. $\qquad c_x = 9.75$ in.

$r_y = \sqrt{1861.6/60.96} = 5.52$ in. $\qquad c_y = 9.0$ in.

$L/r_x = 53.78 \qquad S_x = 380.97 \text{ in.}^3$

$L/r_y = 76.10 \qquad S_x = 206.84 \text{ in.}^3$

$c_x/r_x = 1.25 \qquad c_y/r_y = 1.63$

$$\sigma_{ex} = \pi^2 E/(L/r_x)^2 = 98.96 \text{ ksi}$$

$$\sigma_{ey} = \pi^2 E/(L/r_y)^2 = 49.42 \text{ ksi}$$

$$SFR = r_x/r_y = 1.415.$$

Assume P = 600 Kips, with assumed factor of safety = 1.65

$$\sigma_a = 600 x 1.65/60.96 = 16.24 \text{ ksi}$$

$$\alpha_x = 16.24/98.96 = .164$$

$$\alpha_y = 16.24/49.52 = .329$$

	Bending About xx	Bending About yy
Load factor	1	2
c/r	1.25	1.63
Deflection factor	1	1-.18x329 = .9407
k_w	$\frac{27.29}{600}$ = .0455	$\frac{4}{600}$ = .0067
k_e	(.0455x1x1+.01) 1.25 = .0694	(.0067x2x.9407+.01) 1.63 = .0368

$$AFR = \frac{1-.164}{1-.329} = \frac{.836}{.671} = 1.246$$

$$k_{Be} = .0694(1+1.415 x 1.246 x \frac{.0368}{.0694}) = .1343$$

By interpolation from Table I for L/r = 53.78 and k = .1343, gives σ_a = 15.9 ksi.

Assume P = 580 Kips with assumed factor of safety = 1.65

$$\sigma_a = 580 x 1.65/60.96 = 15.70 \text{ ksi}$$

$$\alpha_x = 15.70/98.96 = .159$$

$$\alpha_y = 15.70/49.42 = .318$$

	Bending About xx	Bending About yy
Deflection factor	1	1-.18x.138 = .9428
k_w	$\frac{27.29}{580}$ = .0471	$\frac{4}{580}$ = .0069
k_e	(.0471x1x1+.01) 1.25 = .0714	(.0069x2x.9428+.01) 1.63 = .0375

AF = 1-.159/1-.318 = .841/.682 = 1.233

k_{Be} = .0714(1+1.415x1.233 x .0375/.0714) = .1368

By interpolation from Table I for L/r = 53.78 and k = .1368, gives σ_a = 15.77 ksi > 15.70.

3.5 LOAD AT INITIAL YIELD FOR THE RESTRAINED BEAM-COLUMN OF SIMPLIFIED SECTION IN PLANAR BENDING

Usually there is no direct solution for the load at initial yield for a restrained column in planar bending under uniform lateral loads. The solution may be obtained indirectly, using the computer, by increasing the loads on the beam-column until the maximum stress, at either the end or middle section, equals the yield stress. In Table III, Appendix A, the average stress at the load at initial yield for the simplified section for the symmetrical case under uniform lateral load is given only for a yield stress of 50 ksi and E = 29000 ksi.* This average stress is given as a function of the following dimensionless parameters:

1. Slenderness ratio L/r,

2. Ratio of lateral load to axial load k,

3. $\eta = \beta L/10EI$,

in which β is the rotational stiffness summed for all the members, except the beam-column itself, rigidly connected at one end of the beam-column. L/r ratio ranges between 10 and 200 with increments of 10. k ranges between .02 and .3 with increment of .02 and η = 0.2, 0.4, 0.6, 0.8, and 1.0. The case η = 0 is that of the unrestrained column and is given in Table I. Reference should be made to Section 5.1 for equations used in this solution.

*This particular program was developed near the end of the contract period, in 1964, and funds were not available to tabulate results for all yield points. This work will be completed under the Phase 2 continuation of the project.

4. ULTIMATE STRENGTH OF UNRESTRAINED BEAM-COLUMNS

4.1 ULTIMATE STRENGTH OF UNRESTRAINED BEAM-COLUMN OF THE FOUR POINT SECTION IN PLANAR BENDING, FOR THE GENERAL STRESS-STRAIN RELATION

Incremental procedure will be used to determine the ultimate strength of the unrestrained beam-column of the simplified section in planar bending. The procedure is as follows:

1. Find the load at the proportional limit stress from Eq. (28), by putting $\sigma_m = \sigma_p$ and solving for P. For a beam-column of the simplified cross section subject to a uniform lateral load W = kP,

$$P_p = A\sigma_e \left\{ \frac{\left(\frac{\sigma_P}{\sigma_e} + 1 + \frac{k}{8}\frac{L}{r}\right) - \sqrt{\left(\frac{\sigma_P}{\sigma_e} + 1 + \frac{k}{8}\frac{L}{r}\right)^2 - 4\frac{\sigma_P}{\sigma_e}\left(1 - .028\frac{L}{r}\frac{k}{8}\right)}}{2 \times \left(1 - .028\frac{L}{r}\frac{k}{8}\right)} \right\} \qquad (41)$$

2. Increase the load P by an arbitrary increment ΔP, say $P_y/200$, and use Newmark's numerical procedure[12] to find the beam deflection. A good assumption for the initially assumed deflection curve is the deflection curve due to the lateral loads multiplied by the amplification factor $1/(1-P/P_e)$.

3. Repeat Step 2, until the numerical procedure method does not converge, in which case the nonconvergent load is greater than the ultimate load. Convergence in this report is considered to occur when the ratio between the assumed deflection and the calculated deflection curve at any nodal point is less than or equal to $1 \pm .005$. Divergence will be considered when this condition is not satisfied within 200 cycles. If the maximum stress at any stage equals σ_y, the ultimate load will have been reached.

Table IV in the Appendix gives the ultimate average stress (P/A) for unrestrained beam-columns of the four point cross section subject to uniform lateral load W = kP. A comparison between the ultimate strength as given in Table IV (with the effect of residual stress included) and the strength based on the load at initial yield (ultimate without the effect of residual stress in case of the simplified section), shows that the ratio between the two strength's is much closer to unity than the ratio between proportional limit determined by the residual stress and the yield stress. This average stress at ultimate is given as a function of the slenderness ratio, which ranges from 10 to 120 with increments of 10, and the k ratio, which ranges from 0.02 to 0.30, with increments of .02. Each table corresponds to a certain yield stress. The yield stresses include 33, 36, 42, 46, 50, 60, 70, and 100 ksi which cover the current constructional steels. E is assumed to be 29,000 ksi. The effect of residual stress is included and is assumed to be present as discussed in Section 2.3 and in the proportion given by Eq. (7).

Illustrative Example 3 is given to show how the ultimate strength may be computed.

The numerical calculations are made by using the IBM 7090 digital computer. In Appendix B, Flow Diagram No. 1, is given for finding the ultimate strength of the unrestrained beam-column under uniform lateral load $W = kP$, for the simplified section with the effect of residual stresses. This flow diagram was the basis for a computer program that requires as data the following, with computer names shown in parentheses:

L (L) = length of beam in inches

c (C) = half depth of section in inches

k (K) = the (lateral load)/(axial load) ratio

A (A) = area of section in square inches

E (E) = modulus of elasticity of steel in ksi

σ_P (SIGPL) = proportional limit stress $(\sigma_y - \sigma_R)$ in ksi

σ_y (SIGY) = yield stress level in ksi

N (N) = half number of beam divisions.

This program (No. 1) is given also in the Appendix.

<u>Example 3</u>. For an Unrestrained Beam-Column of the Simplified Section, of the Following Properties:

$L = 900$ in.

$c = 15$ in.

$A = 40$ sq in.

$E = 29{,}000$ ksi

$\sigma_P = 35$ ksi

$\sigma_y = 50$ ksi

and subjected to axial loads and uniform lateral load of ratio .04 to that of the axial load, find the utlimate load.

1. With $L/r = L/c = 60$, $I = 9000$ in.^{4} and $S = 600$ in.^{3}, compute P_p by means of Eq. (41).

$$\sigma_e = \pi^2 E/(L/r)^2 = 79.505 \text{ ksi.}$$

By substituting in Eq. (41) $P_p = 974.57$ K.

2. Take $\Delta P = A\sigma_y/200 = 40x50/200 = 10$ Kips. For an increment of P of 10 Kips one cycle of the integration to determine deflection will be illustrated. The beam is divided into 8 divisions. For each load, the deflection at nodal points are computed, the assumed deflection having been arrived at as the iterative result of many earlier applications of the same procedure, as follows:

	W = .04 P								
P = 1124.6	←λ=112.5"→		←112.5"→		←112.5"→		←112.5"→		Common Factor
Moment due to lateral load, M_o(K-in.)		2,214		3,796		4,745		5,061	
Assumed deflection, in.		1.0069		1.855		2.419		2.617	
P_{xy}(K-in.)		1,132		2,086		2,720		2,943	
$M = M_o + P_y$		3,306		5,882		7,465		8,004	
M/S (ksi)		5.58		9.80		12.44		13.34	
σ_U = P/A + M/S (ksi)		33.70		37.92		40.56		41.46	
σ_B = P/A - U/S (ksi)		22.54		18.32		15.68		14.78	
ϵ_U		1.162		1.315		1.422		1.465	10^{-3}
ϵ_B		.777		.632		.5407		.5097	10^{-3}
$\epsilon_U - \epsilon_B$		.385		.683		.875		.955	10^{-3}
Curvature ϕ		.128		.227		.292		.318	10^{-4}
Concentrated angle change		1.413		2.52		3.248		3.528	10^{-3}
Average slope	8.945		7.532		5.012		1.764		10^{-3}
Deflection		8.945		16.477		21.489		23.253	$\lambda x10^{-3}$
Deflection (in.)		1.0063		1.854		2.417		2.616	
Assumed deflection/ computed deflection		1.0006		1.0005		1.0008		1.0004	

The above Newmark numerical calculation of deflection and moments for the case of an unrestrained beam-column under uniform lateral load in planar bending is shown only for the final cycle. Whenever $\sigma < \sigma_P$, then $\epsilon = \sigma/E$, otherwise Eq. (3) may be used to calculate the strain.

The average stress, σ_a, at the foregoing load of 1124.6K is 28.11 ksi, the foregoing being a manual computation of one stage of the computer print out as marked by arrows in the following tabulation:

P/A	DEFLECTION IN INCHES AT NODAL POINTS			
24.61	.8032	1.4767	1.9217	2.0769
24.86	.8151	1.4986	1.9503	2.1079
25.11	.8272	1.5210	1.9794	2.1394
25.36	.8396	1.5438	2.0092	2.1717
25.61	.8522	1.5671	2.0397	2.2048
25.86	.8651	1.5908	2.0709	2.2386
26.11	.8782	1.6151	2.1028	2.2732
26.36	.8928	1.6423	2.1387	2.3123
26.61	.9070	1.6687	2.1735	2.3500
26.86	.9218	1.6961	2.2095	2.3892
27.11	.9370	1.7245	2.2469	2.4298
27.36	.9528	1.7539	2.2857	2.4720
27.61	.9711	1.7880	2.3307	2.5208
27.86	.9886	1.8207	2.3739	2.5678
→ 28.11	1.0069	1.8549	2.4191	2.6169 ←
28.36	1.0261	1.8907	2.4664	2.6685
28.61	1.0462	1.9283	2.5162	2.7226
28.86	1.0696	1.9722	2.5744	2.7859
29.11	1.0924	2.0150	2.6311	2.8477
29.36	1.1166	2.0605	2.6915	2.9135
29.61	1.1425	2.1092	2.7562	2.9840
29.86	1.1703	2.1615	2.8257	3.0599
30.11	1.2037	2.2244	2.9095	3.1513
30.36	1.2374	2.2880	2.9943	3.2440
30.61	1.2748	2.3586	3.0887	3.3472
30.86	1.3214	2.4466	3.2066	3.4764
31.11	1.3763	2.5506	3.3465	3.6301
31.36	1.4473	2.6858	3.5295	3.8321

3. For the next increment of load P beyond 1254.4 Kips (31.36 ksi), at which the maximum stress is

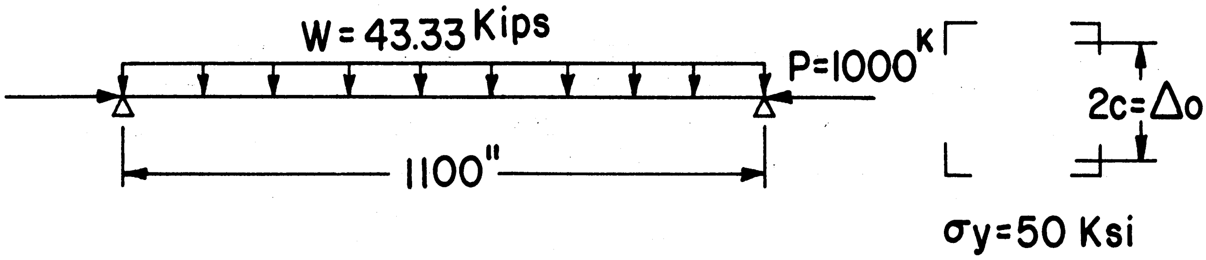
$$\sigma_{max} = \frac{P}{A} + \frac{kPL}{8S} + \frac{Py_{max}}{S} = \frac{1254.4}{40} + \frac{.04x1254.4x900}{8x600} + \frac{1254.4x3.8321}{600}$$

$$= 31.36 + 9.408 + 8.01 = 48.778 \text{ ksi}$$

the maximum stress was found to be equal to σ_y = 50 ksi. Thus the ultimate load is closely estimated as 1255 Kips.

The use of Table IV will now be illustrated by Example 4.

Example 4. By use of Table III, find the required area of cross section for proportion as indicated based on a required ultimate load capacity as shown.



k = 43.33/1000 + .00667 = .050

(.00667: Initial Curvature Allowance)

L/r = L/c = 1100/20 = 55.

By interpolation from Table III (Appendix)

P/A = 31.57 ksi or A = 31.68 sq in.

4L 3-1/2 x 2-1/2 x 7/16 of A = 8.3 x 4 = 33.2 sq in. will be chosen.

If the load P is not known, find the ultimate load this member can carry, for an area A = 40 sq in., and c = 20 in.

Assumed P/A	k	Computed P/A From Table III
30	.0361+.00667 = .0428	32.76
33	.0328+.00667 = .0395	33.36
33.4	.0324+.00667 = .0391	33.43 (thus P_{ult} = 1336K)

If P = 1000K, W = 43.33K, A = 40.00 sq in., find the smallest c

k = .05

P/A = 25 ksi.

From Table IV by interpolation between L/r = 70 and 80 for k = .05, L/r = 73.6, (c=r)

or c = 14.95 in. ≃ 15 in.

4.2 ULTIMATE STRENGTH OF THE UNRESTRAINED BEAM-COLUMN OF THE FOUR POINT SECTION IN BIPLANAR BENDING FOR THE GENERAL STRESS-STRAIN RELATION

The procedure for finding the ultimate strength of the unrestrained beam-column in biplanar bending is essentially the same as that for planar bending. The procedure may be summarized as follows:

1. Find the load at the proportional limit stress from Eq. (28), by putting $\sigma_m = \sigma_y$ and $c = D$ (see Fig. 5). This value will be dependent on the largest lateral load in one of the diagonal directions compared to the other direction. For a beam-column subject to uniform lateral loads:

$$\begin{aligned} W_y^* &= k_y^* P \\ W_x^* &= k_x^* P \quad . \end{aligned} \tag{42}$$

In which k_y^* and k_x^* are the ratios of the lateral loads to the axial load in the diagonal directions and $k_y > k_x$. The load at the proportional limit stress is

$$P_P = A\sigma_e \left\{ \frac{\left(\frac{\sigma_P}{\sigma_e} + 1 + k_y LD/8r^2\right) - \sqrt{\left(\frac{\sigma_P}{\sigma_e} + 1 + k_y LD/8r^2\right)^2 - \frac{4\sigma_P}{\sigma_e}\left(\frac{1.028 k_y LD}{8r^2}\right)}}{2(1-.028k_y LD/8r^2)} \right\} \tag{43}$$

2. Increase the load P by an arbitrary increment ΔP, say $P_y/200$, and use the numerical procedure to find the deflections in both directions. This can be accomplished by assuming deflection curves in both directions equal the deflection curve corresponding to the lateral loads in the same direction multiplied by the amplification factor $1/(1-P/P_e)$ or $1/1-\frac{P}{P_e}$. After the deflection curves in both directions have been assumed, the bending moments M_x and M_y will be tentatively evaluated, and thus the curvatures at any nodal point may be obtained as outlined in Section 2.7. The new deflection line will be computed in both directions. The procedure will be repeated until a desired degree of correspondance is achieved between the assumed and computed deflection curves in both directions.

3. Repeat Step 2, till the numerical procedure method does not converge, this means that this load is greater than the ultimate load. Convergence in this report is assumed when the ratio between the assumed deflection and the calculated deflection is less or equal to $1 \pm .005$. Divergence will be assumed when this condition is not satisfied within 200 cycles. If it happens that both the stresses σ_1 and σ_2 at any location equal the yield point, then no reserve strength of the beam-column will be left. In such a case the ultimate load is that load at which σ_1 and σ_2 equal at any location the yield point.

In Appendix B, Flow Diagram No. 2 is given for finding the ultimate strength of the unrestrained beam-column under uniform lateral load $W_y = k_y P$, and $W_x = k_x P$, for the simplified section with the effect of residual stresses. This flow diagram was the basis for a computer program that requires the following data:

L = length of beam in inches

c = half depth of section in inches

k_y = ratio: (vertical uniform lateral load)/(axial load)

Q = ratio: (horizontal lateral load)/(vertical lateral load) i.e., k_x/k_y

A = area of section in sq in.

E = modulus of elasticity of steel in ksi

σ_P = proportional limit stress $(\sigma_y - \sigma_R)$ in ksi

N = half number of beam divisions.

It may be noted that both k_y and k_x/k_y should be related to the x and y axis not to the diagonals. The program converts these ratios as follows:

$$
\begin{aligned}
k_y^* &= k_y(1+Q)/\sqrt{2} \\
k_x^* &= ((1-Q)/(1+Q))k_y^*
\end{aligned}
\qquad (44)
$$

in which

$$Q = k_x/k_y$$

and k_y^* and k_x^* are related to the diagonal axis x^* and y^* as shown in Fig. 9.

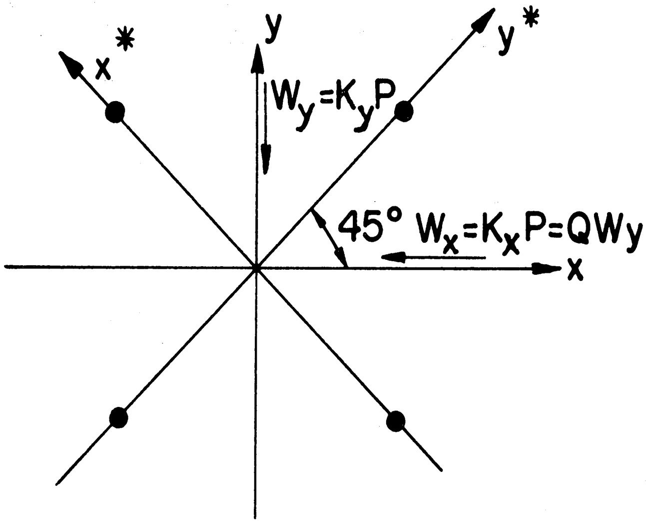
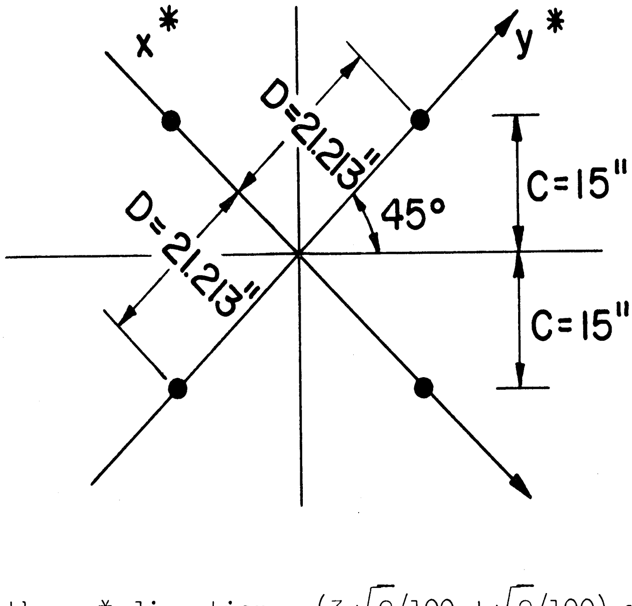


Fig. 9. Diagonal principal axes used for biplanar bending.

Example 5. An unrestrained beam-column has the following properties:

$L = 900$ in., $c = 15$ in., $A = 40$ sq. in., $E = 29{,}000$ ksi, $\sigma_R = 15$ ksi, $\sigma_{PL} = 35$ ksi, $\sigma_y = 50$ ksi.

It is subjected to vertical lateral load of $W_y = 3\sqrt{2}/100\ P$, and to horizontal lateral load $W_x = \sqrt{2}/200\ P$. Find the ultimate load.



1. Loads in the -y* direction = $(3\sqrt{2}/100 + \sqrt{2}/100) \times P/\sqrt{2} = .04P$.
   Loads in the -x* direction = $(3\sqrt{2}/100 - \sqrt{2}/100) \times P/\sqrt{2} = .02P$.

2. Load at proportional limit from Eq. (42). $I = 9{,}000$ in., $r = 15$ in., $D = 21.213$ in., $S = 424.26$ in.^{3}, $k_y^* = 0.4$, $\sigma_e = \pi^2 E/(L/r)^2 = 79.5$ ksi, $P_{PL} = 879.74$ Kips.

3. Increase the load P by increments $\Delta P = A\sigma_y/200 = 10$ Kips and perform numerical procedures similar to that illustrated in the next page. Stop computation when the procedure does not converge, or when $\sigma_1 = \sigma_2 = \sigma_y$. Typical deflection calculations at nodal points by the Newmark[12] procedure shown only for the last cycle, meeting the convergence criterion, at one of the intermediate load stages, are as follows:

$W_y = .04P = 40.39K$, $W_x = .02P = 20.19K$

P = 1009.74

P/A = 25.2435 ksi

$\lambda = 112.5''$

L = 900 in.

Moments and Deflections in the x Direction									Common Factor
M_{ox} (Kips x in.)		1,988		3,408		4,260		4,544	
Assumed y(in.)		.846		1.558		2.030		2.196	
P_y (Kips x in.)		.854		1,573		2,050		2,217	
$M_x = M_{ox}+P_y$		2,842		4,981		6,310		6,761	
$P/A + M_x/S$ (ksi)		31.942		36.984		40.116		41.179	
$\phi_x EI$		2,842		5,004		6,419		6,934	
Concentrated ϕ_x		33,424		59,301		76,128		2x41,089	$\lambda/12EI$
Average slope	209,942		176,518		117,217		41,089		$\lambda/12EI$
Deflection		209,942		386,460		503,677		544,766	$\lambda^2/12EI$
y (in.)		.848		1.562		2.035		2.201	
$y_{assumed}/y_{computed}$		.9976		.9974		.9975		.9977	
Moments and Deflections in the y Direction									
M_{oy} (Kips x in.)		.994		1,704		2,130		2,272	
Assumed x (in.)		.417		.766		.997		1.077	
P_x (Kips x in.)		.421		.773		1,007		1,087	
$M_y = M_{oy}+P_x$		1,415		2,477		3,137		3,359	
$\phi_y EI$		1,415		2,477		3,137		3,359	
Concentrated $\phi_y EI$		16,627		29.322		37,206		2x19,932	$\lambda/12EI$
Average slope	103,087		86,460		57,206		19,932		$\lambda/12EI$
Deflection		103,087		189,547		246,685		266,617	$\lambda^2/12EI$
x (in.)		.417		.766		.997		1.077	
$x_{assumed}/x_{computed}$									

Details of computations for ϕ_x EI and ϕ_y in the foregoing table are as follows:

<u>At 1/8 L</u>

$$F = \frac{1}{3}\{\frac{4P}{A} + \frac{8M_x}{AD} - (2\sigma_y - \sigma_{PL})\} = \frac{1}{3}\{100.974 + \frac{8x4,981}{40x21.213} - 65\} = 27.65$$

$$\sigma_1 = \frac{1}{2}\left[-(\frac{4}{9}\sigma_R - 2F) + \sqrt{(\frac{4}{9}\sigma_R - 2F)^2 - 4(F^2 - \frac{4}{9}\sigma_y\sigma_R)}\right] = \frac{1}{2}\left[-(\frac{60}{9} - 55.30) + \sqrt{(48.63)^2 - 4(764.62 - 333.35)}\right]$$

$$= 36.96 \text{ ksi}$$

$$\sigma_4 = \sigma_1 - 4M_x/AD = 36.96 - \frac{4x4,981}{40x21.213} = 13.48 \text{ ksi}$$

$$\sigma_2 = -\sigma_1 + 2P/A + \frac{2(M_x + M_y)}{AD} = -36.96 + 50.49 + \frac{2(4,981 + 2,477)}{40x21.213} = 31.11 \text{ ksi}$$

$$\sigma_3 = -\sigma_1 + 2P/A + \frac{2(M_x - M_y)}{AD} = -36.96 + 50.49 + \frac{2(4,981 - 2,477)}{40x21.213} = 19.43 \text{ ksi}$$

$$\epsilon_1 E = \{(2\sigma_y - \sigma_{PL}) - 2\sqrt{(\sigma_y - \sigma_1)\sigma_R}\} = \{65 - 7.746\sqrt{50 - 36.96}\} = 37.07$$

$$\phi_x EI = \{(\epsilon_1 - \epsilon_4)/2D\}EI = \frac{9,000}{2x21.213} E(\epsilon_1 - \epsilon_4) = 5004.1$$

$$\phi_y EI = M_y$$

If σ_2 and $\sigma_3 < \sigma_{PL}$, $\phi_y EI = M_y$.

At 3/8 L

$$F = \frac{1}{3}\{100.974 + \frac{8x6,310}{40x21.213} - 65\} = 31.82$$

$$\sigma_1 = \frac{1}{2}\{-(\frac{60}{9} - 63.64) + \sqrt{(56.97)^2-4(1012.51-333.33)}\} = 39.99 \text{ ksi}$$

$$\sigma_2 = -39.99 + 50.49 + \frac{2(6,310+3,137)}{21.213x40} = 32.77 \text{ ksi}$$

$$\sigma_3 = -39.99 + 50.49 + \frac{2(6,310-3,137)}{40x21.213} = 17.98 \text{ ksi}$$

$$\sigma_4 = 39.99 - \frac{4x6,310}{40x21.213} = 10.24 \text{ ksi}$$

$$\epsilon_1 E = \{65 - 7.746\sqrt{50-39.99} = 40.50$$

$$\phi_x EI = \frac{9,000}{2x21.213} E(\epsilon_1-\epsilon_4) = 6,419$$

$$\phi_y EI = M_y$$

At L/2

$$F = \frac{1}{3}\{100.974 + \frac{8x6,761}{40x21.213} - 65\} = 33.57$$

$$\sigma_1 = \frac{1}{2}\{-(+\frac{60}{9} - 67.14\} + \sqrt{(60.47)^2-4(1126.94-333.33)}\} = 41.21 \text{ ksi}$$

$$\sigma_2 = -41.21 + 50.49 + 2(6,761+3,359/40x21.213) = 33.13 \text{ ksi}$$

$$\sigma_3 = -41.21 + 50.49 + 2(6,761-3,359/40x21.213) = 17.30 \text{ ksi}$$

$$\sigma_4 = 41.21 - 4x6,761/40x21.213 = 9.34 \text{ ksi}$$

$$\epsilon_1 E = \{65 - 7.746\sqrt{50-41.21}\} = 42.03$$

$$\phi_x EI = 9,000/2x21.213\ E(\epsilon_1-\epsilon_4) = 6,934$$

$$\phi_y EI = M_y$$

The load deflection curves in the x and y directions, plotted from the computer output for this particular problem are shown in Fig. 10

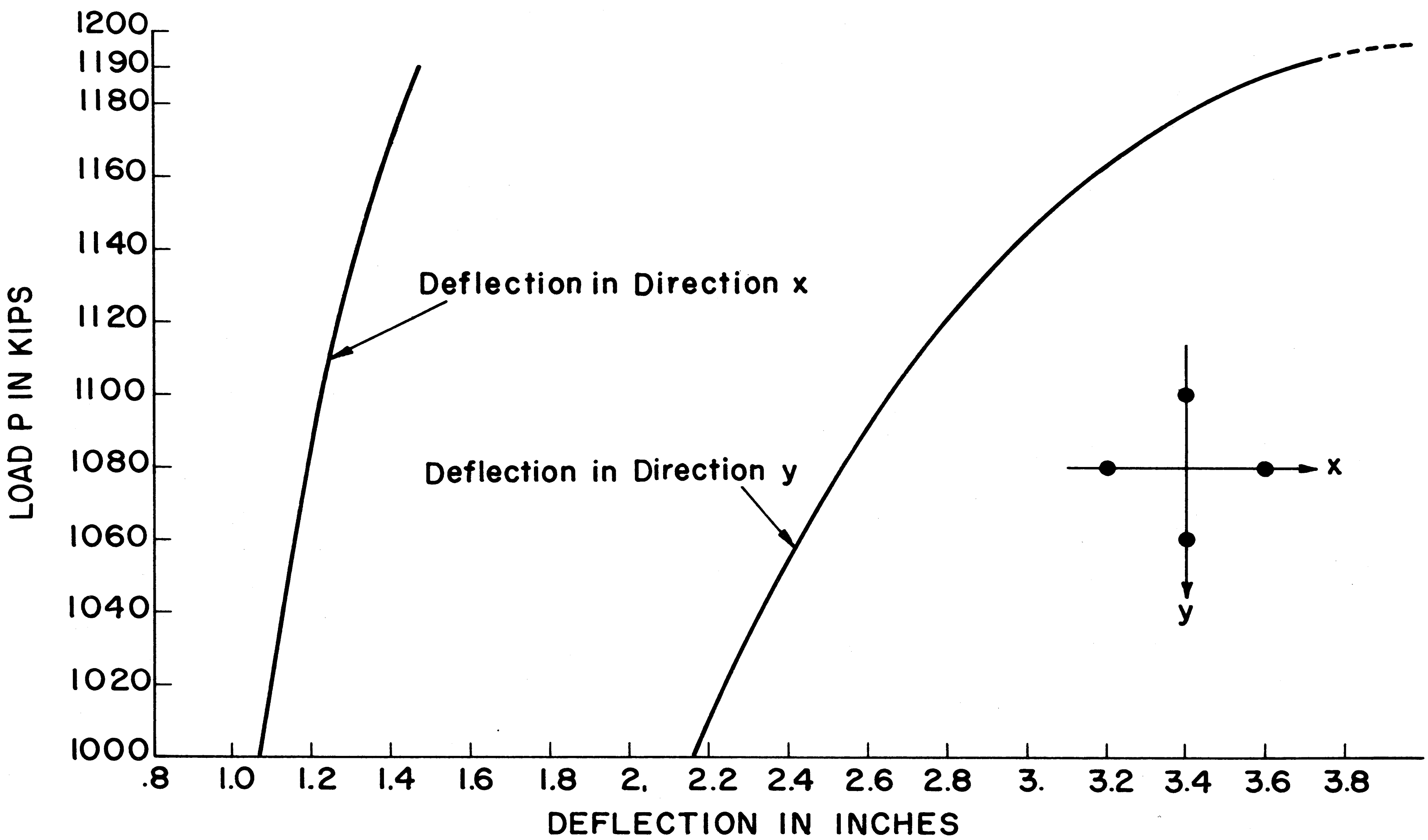


Fig. 10. Deflection of an unrestrained beam-column in planar bending.

5. ULTIMATE STRENGTH OF RESTRAINED BEAM-COLUMN IN PLANAR BENDING

5.1 MODIFIED SLOPE DEFLECTION EQUATIONS FOR COMPRESSION MEMBERS WITH TRANSVERSE LOADS AND END MOMENTS WITHIN THE ELASTIC LIMIT

The following is limited to cases where lateral relative movement of the ends of the beam-column is prevented. (Fig. 11)

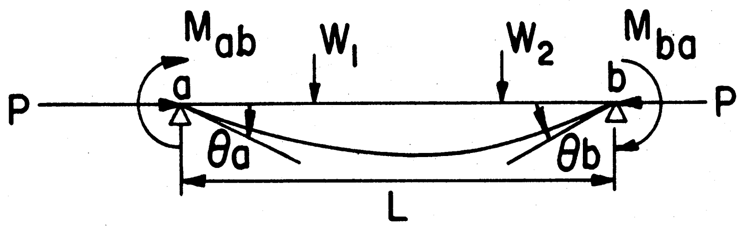


Fig. 11. Beam-column with end moments.

Within the elastic range of behavior, the usual form of the slope deflection equation (modified for axial load) are applicable to case of compression members with transverse loads.[4,13,14] The slope-deflection equations (with no lateral movement of ends) are:

$$M_{ab} = \frac{EI}{L}(C_1\theta_a + C_2\theta_b) + M_{Fab}$$

$$M_{ba} = \frac{EI}{L}(C_2\theta_a + C_1\theta_b) + M_{Fba} \qquad (45)$$

where

θ_a, θ_b = end slopes at a and b respectively, considered positive when clockwise.

M_{Fab}, M_{Fba} = fixed end moments at a and b respectively, considered positve when acting on a member in a clockwise direction.

$$C_1 = \frac{(1-2u \cos 2u)}{\left(\frac{\tan u}{u} - 1\right)} \qquad (46)$$

$$C_2 = \frac{2u \csc 2u - 1}{\left(\frac{\tan u}{u} - 1\right)} \tag{47}$$

$$r = \text{carry over factor} = S_2/S_1 = \frac{2u \operatorname{cosec} 2u - 1}{1 - 2u \cot 2u} \tag{48}$$

where

$$2u = \frac{PL^2}{ES} .$$

Tables (Ref. 4, 13) and graphs (Ref. 14) give values of C_1 and C_2 and factor r, for different values of 2u.

For a uniform load w pounds per unit length, the fixed end moments are:

$$M_{Fab} = -M_{Fba} = \frac{1}{4a^2}\left[\frac{u}{\tan u} - 1\right] wL^2 . \tag{49}$$

Reference 14, Chapter 5, gives a chart for fixed end moments in beam-columns for uniform and other load distributions.

For the symmetrical case of loading u and end restraint, Eq. (45) reduces to

$$M_{ab} = \frac{EI}{L}(C_1 - C_2)\theta_a + M_{Fab} . \tag{50}$$

5.2 SYMMETRICALLY RESTRAINED BEAM-COLUMNS ABOVE THE ELASTIC LIMIT

If the maximum stress in the beam shown in Fig. 11 exceeds the proportional limit, Eq. (45) is not applicable and the solution may be obtained by a numerical integration procedure. If the beam shown in the same figure is framed continuously to other members at its ends, the end moments M_{ab} are not known, adding to the complexity of the problem. The analysis of the beam-column shown in Fig. 12 will be divided into two parts. Initially the load

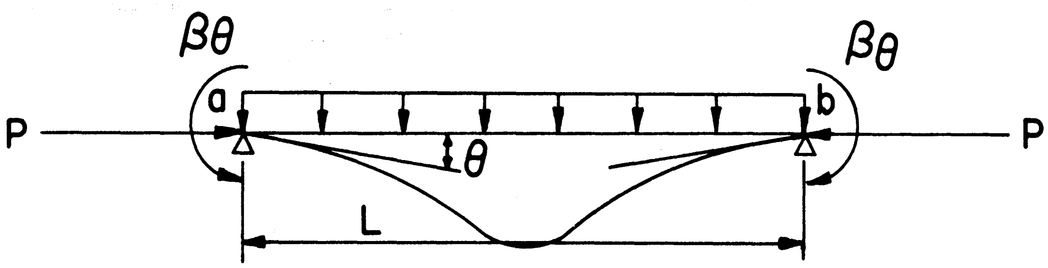


Fig. 12. Restrained beam-column under uniform lateral load.

at which the maximum stress almost equals the proportional limit stress will be determined. Assume a load P let "β" equal the summed stiffnesses of all members at joint a or b except ab. Θ = rotation at end a. Then the slope deflection equations at a will be:

$$(C_1-C_2)\,\frac{EI}{L}\,\Theta + M_{Fab} + \beta\Theta = 0 \tag{51}$$

and

$$M_{ab} = -\beta\Theta\ . \tag{52}$$

The deflection and bending moments along the beam will be

$$y = \frac{1}{P}\left(A' \cos\frac{2xu}{L} + B' \sin\frac{2xu}{L} - M_o + wL^2/4u^2\right) \tag{53}$$

$$M_x = A' \cos\frac{2xu}{L} + B' \sin\frac{2xu}{L} - wL^2/4u^2 \tag{54}$$

where

x = distance from the end of beam

$A' = M_{ab} + wL^2/4u^2$

$B' = -(A' \cos 2u + M_{ba} - wL^2/4u^2)/\sin 2u$.

The maximum deflection and maximum positive moment at the center of the beam are obtained by substituting $x/L = 0.5$ in Eqs. (53) and (54), respectively. Then, by trial, the load is increased until the maximum stress $\frac{P}{A} + \frac{M_{max}}{S}$ is almost equal to the proportional limit stress.

Secondly, the deflection curve of the restrained column is determined after the stresses exceed the proportional limit. The analysis would be essentially the same as for the case of the unrestrained column, using the numerical integration procedure, if the end moments $M_{ab} = -\beta\Theta$ were known. Since these cannot be known in advance, they also must be assumed and then modified by trial until the condition $M_{ab} = -\beta\Theta$ is satisfied.

To obtain the ultimate strength of the restrained beam in planar bending, under uniform lateral load $W = kP$, the procedure for computer analysis was as follows:

1. Divide the beam into a reasonable number of divisions (2N).

2. Assume an initial value P and corresponding w. A good start for P is the load at the proportional limit stress assuming no end restraint, as given by Eq. (41).

3. For this P find 2u, C_1, C_2 from Eqs. (46) and (47) and the fixed end moments M_{Fab} as given in Eq. (49).

4. Find the end slope θ from Eq. (51), then M_{ab} follows from Eq. (52).

5. Find the maximum positive moment from Eq. (54), by putting x/L = 0.5.

6. Calculate the maximum stresses from the equation

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S}$$

   M_{max} is the larger of end moments M_{ab} and the maximum positive moment obtained in Step 5.

7. If the maximum stresses do not exceed the proportional limit stress, increase P by any arbitrary amount say $\Delta P = P_y/200$ and repeat from Step 3 to Step 7, otherwise proceed to Step 8.

8. With the last value of M_{ab} as given in Step 4 and the deflection corresponding to the last value of P and w as given by Eq. (53) calculate at all nodal points the bending moment by the equation

$$M = M_o + Py - M_{ab} \qquad (55)$$

9. For the end moment M_{ab} and the assumed deflection curve, find by the numerical procedure the deflection at all nodal points.

10. Calculate the end slope θ, from the equation: (See Fig. 13).

$$\theta = (96y_1 - 72y_2 + 32y_3 - 6y_4)/24\lambda \ . \qquad (56)$$

11. If the ratio between the assumed M_{ab} and $\beta\theta$ is within a reasonable limit (a reasonable limit in this report is assumed when this ratio is within limits 1 ± .005), then the boundary conditions are satisfied, and one proceeds to Step 12. If not, assume a new value of

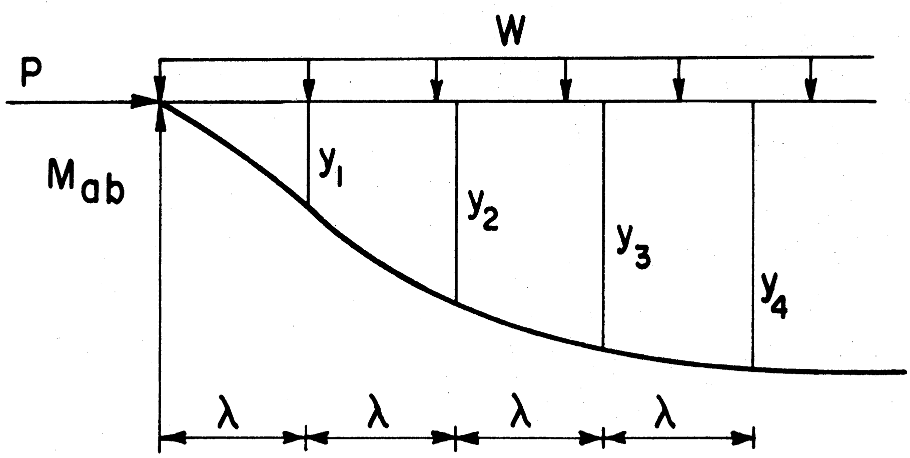


Fig. 13. Numerical evaluation of end slope.

M_{ab} and repeat from Step 8 to Step 11. As a second assumption of the end moment it may be taken as

$$M_{ab} = (\text{first assumed } M_{ab} + \beta\theta/2) \ . \tag{57}$$

A linear interpolation in the other trials is efficient in treating this problem, referring to Fig. 14, the linear interpolation between two values is as follows:

$$M_{ab} = \frac{[\beta^o\theta^o - M^o_{ab}(\beta'\theta' - \beta^o\theta^o)/(M'_{ab} - M^o_{ab})]}{[1-(\beta'\theta' - \beta^o\theta^o)/(M'_{ab} - M^o_{ab})]} \tag{58}$$

Points of the least difference between M_{ab} and $\beta\theta$ should be used in the interpolation.

12. Increase the load P by ΔP and assume new deflection curves and end moments as follows:

$$\begin{aligned} P_{N+1} &= P_N + \Delta P & w &= k \times P_{N+1} \\ M_{ab(N+1)} &= M_{ab}P_{N+1}/P_N & y_{N+1} &= y_N \times P_{N+1}/P_N \ . \end{aligned} \tag{59}$$

13. Repeat from Step 8 to 12 with the new value of P.

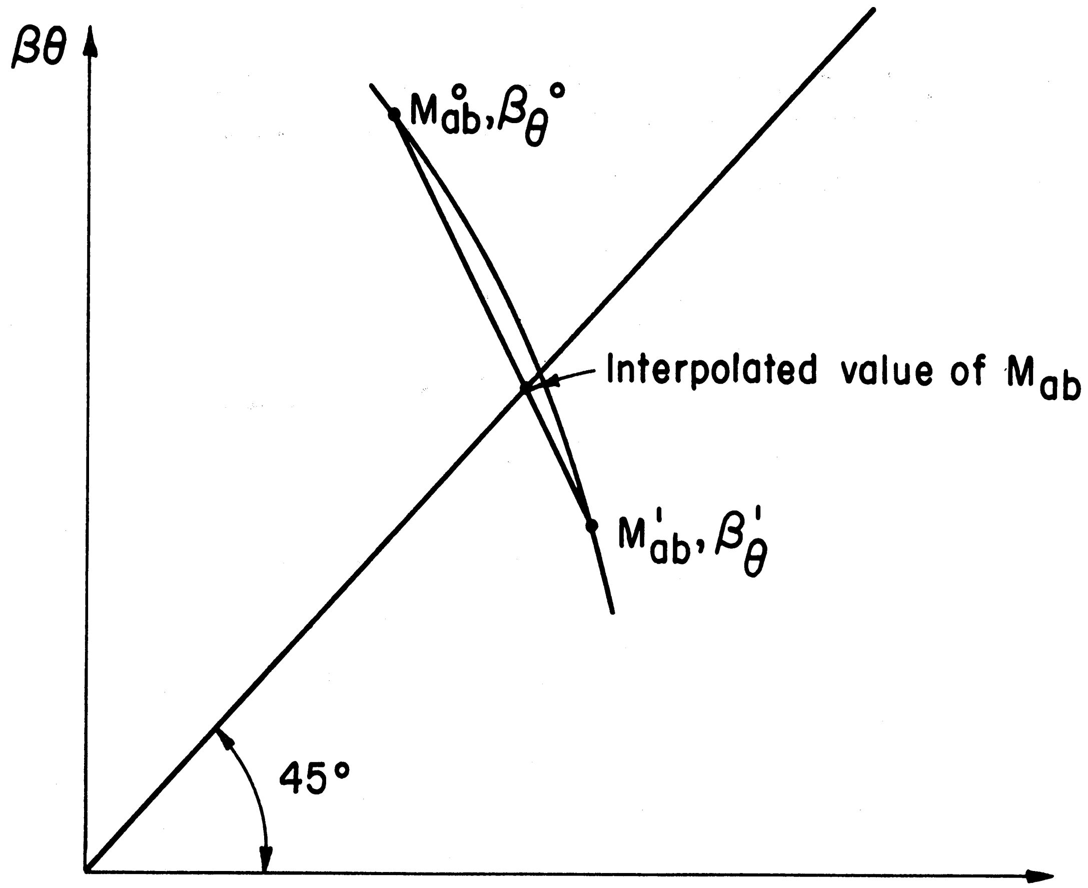


Fig. 14. Linear interpolation for satisfying the end condition of a restrained beam-column.

14. Ultimate Strength: Upon increasing loads P on the beam by increments of say, $\Delta P = P_y/200$, the ultimate load is finally reached when either of the following two conditions is satisfied:

A. The numerical calculation procedure does not converge (convergence is assumed when the ratio between the trial deflection and the computed deflection is equal to $1 \pm .005$. Divergence is assumed when this condition is not satisfied within 200 cycles.)

B. When both the end section and middle section of the beam yield. After one section yields, there is still reserve strength left in the restrained beam-column whereas in the case of the unrestrained beam-column, there is none left. The strain which corresponds to the yield stress at the section which yields first is indefinite. Since yielding exists only in a very small portion of the beam, the strain at this location initially may be assumed as that strain corresponding to σ_y as given by the equation:

$$\epsilon_y = (2\sigma_y - \sigma_{PL})/E \tag{60}$$

If the end section yields first, which will happen when the beam is subject to large restrains, the end moments will be independent of the restraint $\beta\theta$ and will be given by: (See Fig. 15)

$$M_{ab} = (P_y - P)c \tag{61}$$

in which M_{ab} is the end moment.

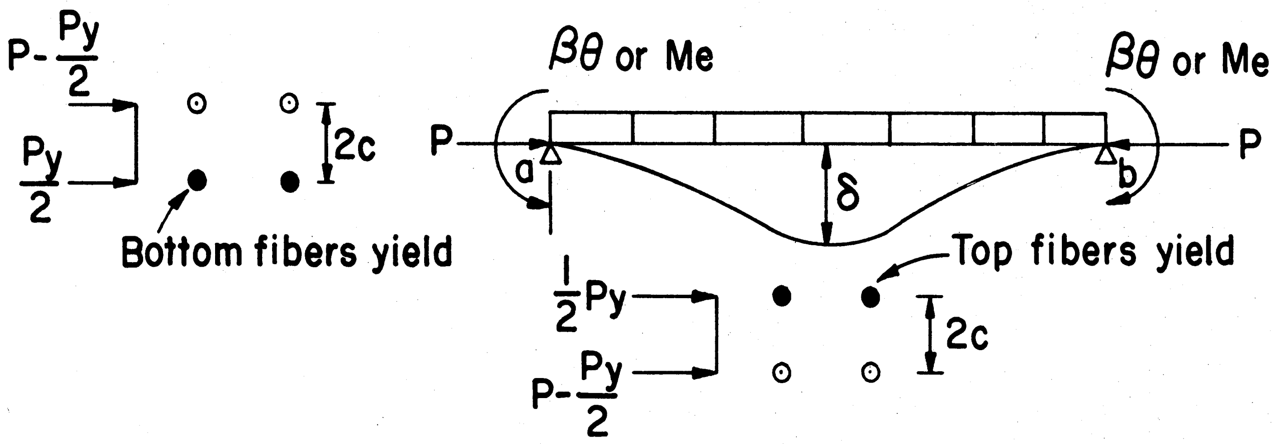


Fig. 15. Yield condition of beam-column with equal end restraints.

If the ultimate load is reached after the end section yields, then this ultimate load will be the same for all larger end restraints. If the middle section yields first, which will occur for small end restraints, the corresponding bending moment at the middle section will be known. For a given end moment M_{ab} the deflection at the centerline δ will be:

$$\delta = \{(P_y - P)c + M_{ab} - M_o\}/P \; . \tag{62}$$

If both the middle and end section yield simultaneously, the moment at both locations will be given by Eq. (61). The maximum deflection in such a case will be

$$\delta = 2c\left(\frac{P_y}{P} - 1\right) - \frac{M_o}{P}. \tag{63}$$

From Eq. (63), it may be seen that as P decreases δ increases. In such a case the ultimate load is that load at which both end and middle section yields.

Illustrative Example 6 is given to show how the ultimate strength has been computed.

In Appendix B, Flow Diagram No. 3 is given for determining the ultimate strength of the restrained beam-column under uniform load W = kP, for the case of the simplified section in planar bending. This flow diagram should be translated to a computer program, and that program requires as data the following:

L = length of beam in inches

c = half depth of section in inches

N = half number of beam divisions

k = ratio of the lateral load/axial load

A = area of section in sqaure inches

E = modulus of elasticity in ksi

β = stiffness of all members at end joint except the beam in K in./rad

σ_P = proportional limit stress in ksi

σ_y = yield stress level in ksi

<u>Example 6</u>. A restrained column of the simplified section has the following properties: L = 300 in., c = 15 in., A = 40 sq. in., E = 29,000 ksi, σ_P = ksi, σ_y = 50 ksi.

If subjected to a vertical load W = .15 P, and restrained by members having β = 4,350,000 Kips in./rad, find its ultimate strength.

1. The load at the proportional limit stress, without effect of end restraint is about 1000 Kips.

2. $\Delta P = P_y/200$ = 5Kips.

3. At the load P = 1167.85, the stresses computed by the usual elastic theory as outlined in Section 3 were found to be greater than σ_P at the end section.

4. For increment of P of 5 Kips, the numerical integration procedure will be illustrated. The beam is divided into 8 divisions. For each load, the deflection at nodal points are computed.

5. Beyond the load P = 1,672.85 K, no convergence can be obtained, thus this load is the ultimate load.

β = 4,350,000, K in./rad, c= 15 in., A = 40 sq in., E = 29,000 ksi, σ_P = 35 ksi, σ_y = 50 ksi, L = 300 in.

W = .15 P = 249.43

P = 1662.85 K						
P/A = 41.57 ksi	37.5"	37.5"	λ=37.5"	37.5"		
	M_{ab} = 5057.27		L=300"			Common factor
M_o (Kips x in.)		4.092	7,015	8,769	9,354	
Assumed y(in.)		.0577	.1236	.1741	.1933	
P_y (Kips x in.)		.096	.206	.290	.321	
$M = M_o + P_y$	-5,057	-.869	2,164	4,002	4,618	
M/S = ksi	-8.43	-1.45	3.61	6.67	7.70	
σ_U = P/A + M/S	33.14	40.12	45.18	48.24	49.27	
σ_B = P/A - M/S	50.	43.02	37.96	34.90	33.87	
ϵ_U	1.143	1.402	1.655	1.887	2.013	10^{-3}
ϵ_B	2.241	1.535	1.316	1.203	1.168	10^{-3}
Curvature ϕ	-3.66	-.443	1.130	2.28	2.817	10^{-5}
Concentrated ϕ		-6.96	13.137	26.747	2x16.365	$\frac{\lambda}{12} 10^{-5}$
Average slope	49.289	56.249	43.112	16.365		$\frac{\lambda}{12} 10^{-5}$
Deflection		49.289	105.538	148.650	165.015	$\frac{\lambda}{12} 10^{-5}$
Deflection (in.)		.0578	.1237	.1742	.1934	
$y_{assumed}/y_{calculated}$		.9983	.9992	.9994	.9995	

$\theta = (96x.0578-72x.1237+32x.1742-6x.1934)/(24x37.5) = 1.1738\ 10^{-3}$
$\beta\theta = 5.106$, $M/\beta\theta = .9904$.

The foregoing illustrates for a specific case, just short of maximum load, the numerical calculation of deflection and moments for the case of a restrained column under uniform load in planar bending.

The following tabulation shows the computer printout for loads above those causing initial yielding for the foregoing example, together with a graph of the results in Fig. 16.

It is of interest also to compare these results with the load at initial yield of the same column, assuming no residual stress, as discussed in Section 3.5 with solution tabulated in Table III.

$$\eta = \frac{\beta L}{10EI} = \frac{4,350,000x300}{10x29000x9000} = 0.50$$

$$L/r = 20.$$

By interpolation, from Table III, $\sigma_a = 41.52$ ksi, or P = 40x41.52 = 1661 Kips, which is to be compared with the 1673 Kips ultimate strength with residual stress included. It should not be assumed that these two different procedures will always agree so well.

$N = 4$, $k = .15$, $\beta = 4{,}350{,}000$ K in./rad, $L = 3000.0$ in.
$c = 15.0$ in., $A = 40.0$ in.^{2}, $E = 29{,}000$ ksi
$\sigma_{PL} = 35.0$ ksi, $\sigma_y = 50.0$ ksi

P(Kips)	w(Kips)	Deflection in Inches at Nodal Points				End Moments (K in.)
1167.85	175.18	.0332	.0682	.0937	.1030	-3180.06
1172.85	175.93	.0333	.0685	.0941	.1034	-3193.68
1177.85	176.68	.0335	.0688	.0945	.1039	-3207.29
1182.85	177.43	.0336	.0691	.0950	.1044	-3220.91
1187.85	178.18	.0338	.0694	.0954	.1048	-3234.52
1192.85	178.93	.0339	.0697	.0958	.1053	-3248.14
1197.85	179.68	.0341	.0700	.0963	.1058	-3261.75
1202.85	180.43	.0342	.0704	.0967	.1063	-3275.37
1207.85	181.18	.0344	.0707	.0971	.1068	-3288.98
1212.85	181.93	.0346	.0710	.0976	.1073	-3302.60
1217.85	182.68	.0347	.0713	.0980	.1078	-3316.21
1222.85	183.43	.0349	.0717	.0985	.1083	-3329.83
1227.85	184.18	.0349	.0717	.0986	.1084	-3352.79
1232.85	184.93	.0350	.0720	.0991	.1089	-3366.44
1237.85	185.68	.0352	.0724	.0995	.1094	-3380.10
1242.85	186.43	.0354	.0727	.1000	.1100	-3393.75
1247.85	187.18	.0356	.0731	.1005	.1105	-3407.40
1252.85	187.93	.0357	.0735	.1010	.1111	-3421.06
1257.85	188.68	.0359	.0738	.1015	.1116	-3434.71
1262.85	189.43	.0361	.0742	.1020	.1122	-3448.36
1267.85	190.18	.0363	.0746	.1026	.1128	-3462.02
1272.85	190.93	.0363	.0747	.1028	.1131	-3481.60
1277.85	191.68	.0365	.0751	.1034	.1137	-3495.27
1282.85	192.43	.0367	.0755	.1039	.1143	-3508.95
1287.85	193.18	.0369	.0759	.1045	.1149	-3522.63
1292.85	193.93	.0370	.0761	.1048	.1152	-3542.87
1297.85	194.68	.0372	.0765	.1053	.1158	-3556.58
1302.85	195.43	.0374	.0769	.1059	.1165	-3570.28
1307.85	196.18	.0374	.0770	.1061	.1167	-3594.74
1312.85	196.93	.0376	.0774	.1067	.1173	-3608.48
1317.85	197.68	.0378	.0779	.1073	.1180	-3622.22
1322.85	198.43	.0380	.0783	.1079	.1187	-3635.97
1327.85	199.18	.0383	.0788	.1085	.1194	-3649.71
1332.85	199.93	.0384	.0790	.1089	.1198	-3670.30
1337.85	200.68	.0386	.0795	.1095	.1205	-3684.07
1342.85	201.43	.0386	.0796	.1098	.1208	-3707.76
1347.85	202.18	.0389	.0801	.1105	.1215	-3721.56
1352.85	202.93	.0391	.0806	.1112	.1223	-3735.37
1357.85	203.68	.0394	.0811	.1119	.1231	-3749.17
1362.85	204.43	.0395	.0814	.1122	.1235	-3770.53
1367.85	205.18	.0397	.0819	.1130	.1243	-3784.36
1372.85	205.93	.0399	.0822	.1134	.1248	-3804.91
1377.85	206.68	.0401	.0828	.1142	.1257	-3818.76
1382.85	207.43	.0402	.0831	.1146	.1262	-3839.73
1387.85	208.18	.0405	.0836	.1154	.1270	-3853.61
1392.85	208.93	.0406	.0839	.1159	.1275	-3874.88
1397.85	209.68	.0409	.0845	.1167	.1284	-3888.79
1402.85	210.43	.0411	.0848	.1171	.1290	-3910.36
1407.85	211.18	.0414	.0854	.1180	.1299	-3924.30
1412.85	211.93	.0415	.0857	.1185	.1304	-3946.19
1417.85	212.68	.0418	.0864	.1193	.1314	-3960.15
1422.85	213.43	.0419	.0867	.1198	.1319	-3982.38
1427.85	214.18	.0422	.0873	.1207	.1329	-3996.37

1432.85	214.93	.0424	.0877	.1212	.1335	-4018.97
1437.85	215.68	.0427	.0883	.1221	.1345	-4032.99
1442.85	216.43	.0428	.0887	.1226	.1351	-4056.09
1447.85	217.18	.0432	.0894	.1236	.1362	-4070.15
1452.85	217.93	.0433	.0897	.1241	.1367	-4093.78
1457.85	218.68	.0436	.0904	.1251	.1379	-4107.87
1462.85	219.43	.0438	.0908	.1256	.1384	-4132.07
1467.85	220.18	.0441	.0915	.1267	.1396	-4146.20
1472.85	220.93	.0443	.0919	.1272	.1402	-4171.21
1477.85	221.68	.0446	.0926	.1283	.1414	-4185.37
1482.85	222.43	.0448	.0930	.1289	.1420	-4210.78
1487.85	223.18	.0450	.0936	.1297	.1429	-4230.80
1492.85	223.93	.0453	.0942	.1305	.1439	-4251.00
1497.85	224.68	.0455	.0948	.1314	.1448	-4271.38
1502.85	225.43	.0458	.0954	.1322	.1458	-4291.94
1507.85	226.18	.0461	.0960	.1331	.1468	-4312.68
1512.85	226.93	.0463	.0966	.1340	.1478	-4333.61
1517.85	227.68	.0466	.0972	.1349	.1488	-4354.74
1522.85	228.43	.0469	.0979	.1359	.1499	-4376.05
1527.85	229.18	.0472	.0985	.1368	.1509	-4397.57
1532.85	229.93	.0475	.0992	.1378	.1520	-4419.30
1537.85	230.68	.0478	.0999	.1388	.1531	-4441.24
1542.85	231.43	.0481	.1006	.1398	.1542	-4463.39
1547.85	232.18	.0484	.1013	.1408	.1554	-4485.77
1552.85	232.93	.0487	.1020	.1418	.1565	-4508.37
1557.85	233.68	.0490	.1027	.1429	.1578	-4531.19
1562.85	234.43	.0494	.1035	.1440	.1590	-4554.25
1567.85	235.18	.0497	.1042	.1451	.1602	-4577.55
1572.85	235.93	.0500	.1050	.1462	.1615	-4601.10
1577.85	236.68	.0504	.1058	.1474	.1628	-4624.90
1582.85	237.43	.0507	.1066	.1485	.1641	-4648.96
1587.85	238.18	.0510	.1074	.1497	.1655	-4673.29
1592.85	238.93	.0514	.1082	.1510	.1668	-4697.91
1597.85	239.68	.0518	.1091	.1522	.1683	-4722.82
1602.85	240.43	.0521	.1099	.1535	.1697	-4748.02
1607.85	241.18	.0525	.1108	.1548	.1712	-4773.54
1612.85	241.93	.0529	.1117	.1562	.1728	-4799.37
1617.85	242.68	.0533	.1127	.1576	.1743	-4825.53
1622.85	243.43	.0537	.1136	.1590	.1760	-4852.02
1627.85	244.18	.0541	.1146	.1605	.1777	-4878.84
1632.85	244.93	.0545	.1157	.1621	.1794	-4905.99
1637.85	245.68	.0549	.1167	.1637	.1813	-4933.44
1642.85	246.43	.0554	.1178	.1653	.1832	-4961.15
1647.85	247.18	.0559	.1190	.1671	.1852	-4989.00
1652.85	247.93	.0564	.1202	.1690	.1874	-5016.72
1657.85	248.68	.0569	.1216	.1710	.1897	-5043.44
1662.85	249.43	.0577	.1236	.1741	.1933	-5057.27 —
1667.85	250.18	.0626	.1327	.1864	.2069	-4982.27
1672.85	250.93	.0690	.1449	.2033	.2262	-4907.27

NO CONVERGENCE

33.64	50.00	.11601E-02	.22411E-02
40.70	42.95	.14266E-02	.15332E-02
45.81	37.83	.16945E-02	.13103E-02
48.91	34.73	.19626E-02	.11977E-02
49.95	33.69	.21842E-02	.11617E-02

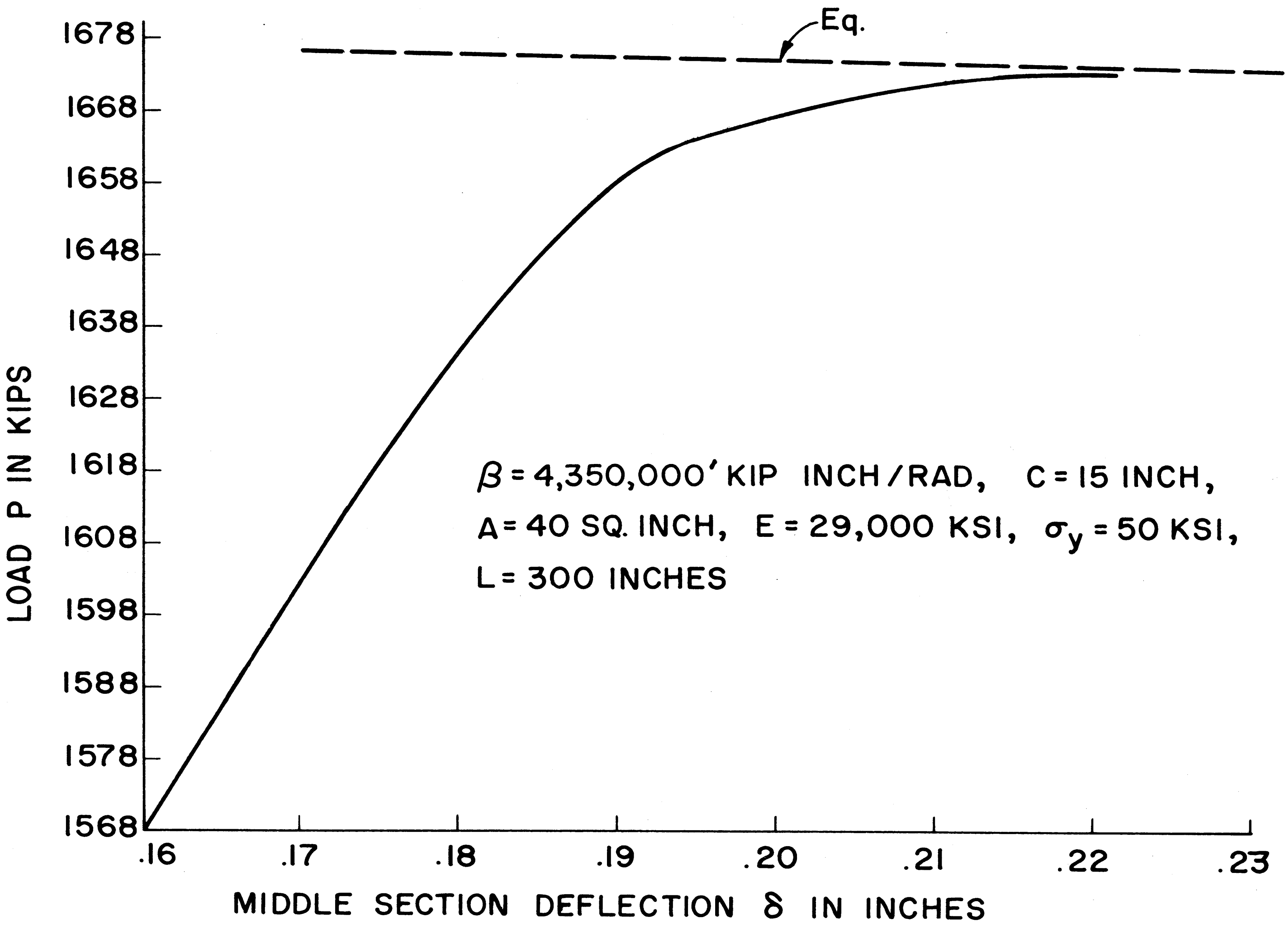


Fig. 16. Load deflection curve of a restrained beam-column in planar bending.

6. SUMMARY

By means of a simplified "four point area closed section" a conservative estimate of column strength has been provided. The problems of stress redistribution as well as local buckling and torsion have not been considered. Computer programs have been developed both to determine the beam-column load at which the maximum stress reaches the yield point and, alternatively, the maximum column load that the beam-column can carry as it reaches a condition of instability. Residual stresses are considered on a point area basis. Complete design tables for each of these approaches are furnished for the unrestrained case in planar bending for various yield points of steel from 33 to 100 ksi and various ratios of total uniform lateral beam load to column load up to 0.30. A partial table for design use (for yield point of 50 ksi) is provided to give the load at which the maximum stress reaches yield point for the end-restrained symmetrically loaded beam-column. It is shown that the unrestrained beam-column in biplanar bending can be designed by means of the tables prepared for planar bending. A program is developed for the determination of the ultimate strength in biplanar unrestrained bending including effects of residual stress. The latter part of the report is somewhat exploratory in nature and does not provide complete design tables as does the initial part of the report but it should point the way to further research in this important area of work.

Computer facilities of The University of Michigan were used in obtaining the numerical results. The work was initiated through the interest of Mr. A. Amirikian, Special Design Consultant, Bureau of Yards and Docks, U.S. Navy. The results on Phase I as here reported include some preliminary partial results for Phase II. A wide range of design information is provided, including a rather detailed comparison of the two entirely different procedures of analysis, one based on initial yield and the other based on ultimate strength. For the simplified section that was considered herein, the results in most cases have shown remarkably close agreement between the two procedures.

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APPENDIX A

DESIGN TABLES

1. Average column loads for which combined stress in unrestrained beam-columns with uniform lateral load just reaches yield point.

2. Load and deflection coefficients to give equivalent lateral loads for use in Table I.

3. Average column load for which combined stress in restrained beam-columns with uniform lateral load just reaches the yield point. (This table limited to yield point of 50 ksi.)

4. Ultimate strength of unrestrained beam-columns under uniform lateral load, in planar bending, including the effect of residual stress.

TABLE I

AVERAGE COLUMN LOADS FOR WHICH COMBINED STRESS IN UNRESTRAINED BEAM-COLUMNS WITH UNIFORM LATERAL LOAD JUST REACHES YIELD POINT

(YIELD STRESS = 33 KSI)

L/R	VALUES OF P/A IN KSI FOR k =														
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	32.59	32.19	31.79	31.41	31.04	30.67	30.32	29.97	29.63	29.30	28.97	28.66	28.35	28.04	27.75
20	32.16	31.36	30.60	29.88	29.19	28.54	27.91	27.31	26.74	26.20	25.67	25.17	24.68	24.22	23.77
30	31.68	30.47	29.35	28.32	27.37	26.48	25.66	24.88	24.16	23.48	22.83	22.23	21.65	21.11	20.59
40	31.11	29.46	28.00	26.70	25.53	24.47	23.50	22.61	21.79	21.04	20.34	19.69	19.08	18.51	17.97
50	30.39	28.27	26.49	24.97	23.64	22.46	21.41	20.47	19.62	18.84	18.13	17.47	16.86	16.30	15.78
60	29.45	26.86	24.81	23.13	21.71	20.49	19.42	18.47	17.62	16.86	16.16	15.53	14.95	14.42	13.92
70	28.17	25.17	22.96	21.22	19.79	18.57	17.53	16.62	15.81	15.08	14.43	13.84	13.30	12.80	12.34
80	26.43	23.21	20.98	19.28	17.90	16.76	15.77	14.92	14.17	13.50	12.90	12.36	11.86	11.41	10.99
90	24.20	21.07	18.97	17.39	16.12	15.06	14.17	13.39	12.71	12.10	11.56	11.06	10.62	10.21	9.83
100	21.65	18.88	17.01	15.60	14.46	13.52	12.72	12.02	11.41	10.87	10.38	9.94	9.54	9.17	8.83
110	19.09	16.79	15.19	13.96	12.97	12.14	11.43	10.81	10.27	9.79	9.35	8.96	8.60	8.27	7.97
120	16.74	14.88	13.54	12.49	11.63	10.91	10.29	9.74	9.26	8.84	8.45	8.10	7.78	7.49	7.22
130	14.68	13.19	12.08	11.19	10.45	9.82	9.28	8.81	8.38	8.01	7.66	7.35	7.07	6.81	6.57
140	12.92	11.72	10.80	10.04	9.41	8.87	8.40	7.98	7.61	7.27	6.97	6.70	6.44	6.21	6.00
150	11.43	10.46	9.68	9.05	8.50	8.03	7.62	7.26	6.93	6.63	6.36	6.12	5.89	5.68	5.49
160	10.16	9.37	8.72	8.17	7.71	7.30	6.94	6.62	6.33	6.07	5.83	5.61	5.41	5.22	5.05
170	9.08	8.43	7.88	7.41	7.01	6.65	6.34	6.06	5.80	5.57	5.35	5.16	4.98	4.81	4.65
180	8.16	7.61	7.15	6.75	6.40	6.09	5.81	5.56	5.33	5.12	4.93	4.76	4.59	4.44	4.30
190	7.37	6.91	6.51	6.16	5.86	5.58	5.34	5.12	4.91	4.73	4.56	4.40	4.25	4.12	3.99
200	6.69	6.29	5.95	5.65	5.38	5.14	4.92	4.72	4.54	4.37	4.22	4.08	3.95	3.82	3.71

L/R	VALUES OF P/A IN KSI FOR k =														
	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	27.45	27.17	26.89	26.62	26.35	26.09	25.83	25.58	25.33	25.09	24.85	24.62	24.39	24.16	23.94
20	23.34	22.93	22.53	22.14	21.77	21.41	21.06	20.72	20.39	20.08	19.77	19.48	19.19	18.91	18.64
30	20.10	19.64	19.19	18.77	18.36	17.97	17.60	17.25	16.91	16.58	16.26	15.96	15.67	15.39	15.12
40	17.47	17.00	16.55	16.12	15.72	15.34	14.98	14.63	14.30	13.99	13.69	13.40	13.13	12.86	12.61
50	15.29	14.83	14.40	14.00	13.62	13.26	12.92	12.59	12.29	12.00	11.72	11.46	11.20	10.96	10.73
60	13.46	13.03	12.63	12.26	11.91	11.58	11.26	10.97	10.69	10.42	10.17	9.93	9.71	9.49	9.28
70	11.92	11.52	11.16	10.82	10.50	10.19	9.91	9.64	9.39	9.15	8.93	8.71	8.51	8.31	8.13
80	10.61	10.25	9.92	9.61	9.32	9.05	8.79	8.55	8.32	8.11	7.91	7.71	7.53	7.35	7.19
90	9.49	9.16	8.87	8.59	8.33	8.08	7.85	7.64	7.43	7.24	7.06	6.88	6.72	6.56	6.41
100	8.52	8.23	7.97	7.72	7.48	7.26	7.06	6.86	6.68	6.51	6.34	6.19	6.04	5.90	5.77
110	7.69	7.43	7.19	6.97	6.76	6.56	6.38	6.20	6.04	5.88	5.74	5.60	5.46	5.34	5.22
120	6.97	6.74	6.53	6.32	6.14	5.96	5.79	5.64	5.49	5.35	5.22	5.09	4.97	4.86	4.75
130	6.34	6.14	5.94	5.76	5.59	5.44	5.29	5.14	5.01	4.88	4.76	4.65	4.54	4.44	4.34
140	5.80	5.61	5.44	5.27	5.12	4.98	4.84	4.72	4.59	4.48	4.37	4.27	4.17	4.08	3.99
150	5.31	5.15	4.99	4.84	4.71	4.58	4.45	4.34	4.23	4.12	4.03	3.93	3.84	3.76	3.68
160	4.89	4.74	4.59	4.46	4.34	4.22	4.11	4.01	3.91	3.81	3.72	3.64	3.55	3.48	3.40
170	4.51	4.37	4.24	4.12	4.01	3.91	3.80	3.71	3.62	3.53	3.45	3.37	3.30	3.23	3.16
180	4.17	4.05	3.93	3.82	3.72	3.62	3.53	3.45	3.36	3.28	3.21	3.14	3.07	3.00	2.94
190	3.87	3.76	3.65	3.55	3.46	3.37	3.29	3.21	3.13	3.06	2.99	2.93	2.86	2.81	2.75
200	3.60	3.50	3.40	3.31	3.23	3.15	3.07	3.00	2.93	2.86	2.80	2.74	2.68	2.63	2.57

TABLE I (Continued)

(YIELD STRESS = 36 KSI)

VALUES OF P/A IN KSI FOR k =

L/R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	35.55	35.11	34.68	34.27	33.86	33.46	33.07	32.69	32.32	31.96	31.60	31.26	30.92	30.59	30.26
20	35.08	34.20	33.37	32.58	31.83	31.11	30.43	29.78	29.15	28.56	27.98	27.43	26.90	26.40	25.91
30	34.54	33.21	31.99	30.86	29.82	28.84	27.94	27.09	26.30	25.56	24.85	24.19	23.57	22.97	22.41
40	33.90	32.07	30.47	29.04	27.75	26.59	25.53	24.57	23.68	22.85	22.09	21.38	20.72	20.10	19.52
50	33.07	30.71	28.75	27.08	25.62	24.34	23.20	22.17	21.25	20.40	19.63	18.92	18.26	17.66	17.09
60	31.95	29.06	26.81	24.98	23.44	22.11	20.95	19.93	19.02	18.19	17.45	16.77	16.14	15.57	15.03
70	30.38	27.06	24.66	22.78	21.24	19.95	18.83	17.85	16.99	16.21	15.51	14.88	14.30	13.77	13.29
80	28.22	24.75	22.38	20.57	19.11	17.90	16.86	15.96	15.16	14.45	13.82	13.24	12.72	12.24	11.79
90	25.49	22.25	20.07	18.42	17.10	16.00	15.07	14.25	13.54	12.90	12.33	11.81	11.34	10.91	10.52
100	22.52	19.75	17.86	16.42	15.26	14.29	13.46	12.74	12.11	11.54	11.04	10.58	10.16	9.77	9.42
110	19.65	17.43	15.84	14.61	13.61	12.77	12.04	11.41	10.85	10.36	9.91	9.50	9.13	8.79	8.48
120	17.11	15.35	14.04	13.01	12.15	11.43	10.80	10.25	9.76	9.32	8.93	8.57	8.24	7.94	7.66
130	14.94	13.54	12.47	11.60	10.87	10.25	9.71	9.23	8.80	8.42	8.07	7.75	7.46	7.20	6.95
140	13.10	11.99	11.11	10.38	9.76	9.23	8.76	8.34	7.97	7.63	7.32	7.04	6.79	6.55	6.33
150	11.56	10.67	9.93	9.32	8.79	8.33	7.93	7.56	7.24	6.94	6.67	6.42	6.19	5.98	5.78
160	10.27	9.53	8.92	8.40	7.95	7.55	7.20	6.88	6.59	6.33	6.09	5.87	5.67	5.48	5.31
170	9.17	8.56	8.05	7.60	7.21	6.87	6.56	6.28	6.03	5.80	5.59	5.39	5.21	5.04	4.88
180	8.23	7.72	7.29	6.91	6.57	6.27	6.00	5.75	5.53	5.33	5.14	4.96	4.80	4.65	4.51
190	7.43	7.00	6.62	6.30	6.00	5.74	5.50	5.29	5.09	4.91	4.74	4.58	4.44	4.30	4.17
200	6.73	6.37	6.05	5.76	5.51	5.27	5.06	4.87	4.70	4.53	4.38	4.24	4.11	3.99	3.87

VALUES OF P/A IN KSI FOR k =

L/R	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	29.95	29.63	29.33	29.03	28.74	28.45	28.17	27.90	27.63	27.36	27.10	26.85	26.60	26.36	26.11
20	25.44	24.99	24.55	24.13	23.72	23.33	22.95	22.58	22.23	21.88	21.55	21.22	20.91	20.61	20.31
30	21.88	21.37	20.88	20.42	19.98	19.56	19.16	18.77	18.40	18.04	17.70	17.37	17.05	16.75	16.45
40	18.97	18.46	17.97	17.51	17.07	16.66	16.27	15.89	15.54	15.19	14.87	14.56	14.26	13.97	13.70
50	16.56	16.07	15.60	15.16	14.75	14.36	14.00	13.65	13.32	13.00	12.70	12.42	12.15	11.89	11.64
60	14.54	14.08	13.65	13.25	12.87	12.51	12.18	11.86	11.56	11.27	11.00	10.75	10.50	10.27	10.04
70	12.83	12.41	12.02	11.66	11.31	10.99	10.69	10.40	10.13	9.88	9.63	9.40	9.18	8.98	8.78
80	11.39	11.01	10.65	10.33	10.02	9.73	9.46	9.20	8.96	8.73	8.51	8.31	8.11	7.93	7.75
90	10.15	9.81	9.50	9.20	8.93	8.67	8.43	8.20	7.98	7.78	7.58	7.40	7.23	7.06	6.90
100	9.10	8.79	8.51	8.25	8.00	7.77	7.56	7.35	7.16	6.98	6.80	6.64	6.48	6.33	6.19
110	8.19	7.92	7.67	7.43	7.21	7.01	6.81	6.63	6.46	6.29	6.14	5.99	5.85	5.72	5.59
120	7.40	7.16	6.94	6.73	6.53	6.35	6.18	6.01	5.86	5.71	5.57	5.44	5.31	5.19	5.08
130	6.72	6.51	6.31	6.12	5.94	5.78	5.62	5.48	5.34	5.21	5.08	4.96	4.85	4.74	4.64
140	6.12	5.93	5.75	5.59	5.43	5.28	5.14	5.01	4.88	4.77	4.65	4.55	4.44	4.35	4.25
150	5.60	5.43	5.27	5.12	4.98	4.85	4.72	4.60	4.49	4.38	4.28	4.18	4.09	4.00	3.91
160	5.14	4.99	4.84	4.71	4.58	4.46	4.35	4.24	4.14	4.04	3.95	3.86	3.78	3.69	3.62
170	4.74	4.60	4.47	4.35	4.23	4.12	4.02	3.92	3.83	3.74	3.66	3.58	3.50	3.42	3.35
180	4.37	4.25	4.13	4.02	3.92	3.82	3.73	3.64	3.55	3.47	3.39	3.32	3.25	3.18	3.12
190	4.05	3.94	3.83	3.73	3.64	3.55	3.46	3.38	3.31	3.23	3.16	3.09	3.03	2.97	2.91
200	3.76	3.66	3.57	3.47	3.39	3.31	3.23	3.15	3.08	3.02	2.95	2.89	2.83	2.78	2.72

TABLE I (Continued)

(YIELD STRESS = 42 KSI)

VALUES OF P/A IN KSI FOR k =

L/R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	41.47	40.96	40.46	39.97	39.50	39.03	38.58	38.13	37.70	37.28	36.86	36.46	36.06	35.68	35.30
20	40.91	39.88	38.91	37.98	37.10	36.26	35.46	34.70	33.97	33.27	32.60	31.95	31.34	30.74	30.17
30	40.27	38.69	37.24	35.91	34.68	33.54	32.48	31.49	30.56	29.69	28.87	28.10	27.37	26.68	26.02
40	39.45	37.27	35.35	33.66	32.15	30.79	29.55	28.42	27.39	26.43	25.54	24.72	23.95	23.24	22.56
50	38.36	35.51	33.17	31.20	29.49	28.00	26.68	25.50	24.43	23.46	22.57	21.75	21.00	20.30	19.65
60	36.80	33.30	30.65	28.52	26.74	25.23	23.90	22.74	21.70	20.77	19.92	19.15	18.44	17.79	17.19
70	34.50	30.59	27.85	25.72	23.99	22.53	21.28	20.19	19.22	18.36	17.58	16.88	16.23	15.64	15.09
80	31.33	27.49	24.89	22.92	21.33	20.01	18.87	17.89	17.02	16.24	15.54	14.91	14.33	13.80	13.32
90	27.57	24.26	21.99	20.27	18.87	17.71	16.71	15.84	15.07	14.39	13.77	13.21	12.70	12.24	11.80
100	23.82	21.19	19.31	17.85	16.66	15.66	14.80	14.04	13.38	12.78	12.24	11.75	11.30	10.89	10.51
110	20.48	18.45	16.93	15.72	14.72	13.87	13.13	12.48	11.91	11.39	10.92	10.49	10.10	9.74	9.41
120	17.66	16.09	14.87	13.88	13.04	12.32	11.69	11.14	10.64	10.19	9.78	9.41	9.07	8.75	8.46
130	15.32	14.09	13.11	12.29	11.59	10.98	10.45	9.97	9.54	9.15	8.80	8.47	8.17	7.90	7.64
140	13.38	12.41	11.61	10.93	10.34	9.83	9.37	8.96	8.59	8.25	7.94	7.66	7.40	7.16	6.93
150	11.77	10.99	10.33	9.77	9.27	8.83	8.44	8.09	7.77	7.47	7.20	6.95	6.72	6.51	6.31
160	10.42	9.79	9.24	8.77	8.35	7.97	7.64	7.33	7.05	6.79	6.56	6.34	6.13	5.94	5.77
170	9.29	8.77	8.31	7.91	7.55	7.23	6.93	6.67	6.42	6.20	5.99	5.80	5.61	5.45	5.29
180	8.33	7.89	7.51	7.16	6.85	6.57	6.32	6.09	5.87	5.67	5.49	5.32	5.16	5.01	4.87
190	7.51	7.14	6.81	6.51	6.25	6.00	5.78	5.58	5.39	5.21	5.05	4.90	4.75	4.62	4.49
200	6.80	6.49	6.20	5.95	5.72	5.50	5.31	5.13	4.96	4.80	4.66	4.52	4.39	4.27	4.16

VALUES OF P/A IN KSI FOR k =

L/R	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	34.93	34.56	34.21	33.86	33.52	33.18	32.86	32.54	32.22	31.91	31.61	31.31	31.02	30.73	30.45
20	29.62	29.10	28.59	28.09	27.62	27.16	26.72	26.29	25.88	25.47	25.09	24.71	24.34	23.99	23.64
30	25.40	24.81	24.25	23.71	23.20	22.71	22.24	21.79	21.36	20.94	20.55	20.16	19.80	19.44	19.10
40	21.93	21.34	20.78	20.24	19.74	19.26	18.81	18.38	17.97	17.57	17.20	16.84	16.50	16.17	15.85
50	19.05	18.48	17.95	17.45	16.98	16.53	16.11	15.71	15.34	14.98	14.63	14.31	14.00	13.70	13.41
60	16.63	16.10	15.62	15.16	14.73	14.33	13.95	13.59	13.25	12.93	12.62	12.33	12.05	11.78	11.53
70	14.59	14.12	13.68	13.27	12.89	12.53	12.19	11.86	11.56	11.27	11.00	10.74	10.50	10.26	10.04
80	12.87	12.45	12.06	11.69	11.35	11.03	10.73	10.45	10.18	9.92	9.68	9.45	9.23	9.02	8.83
90	11.41	11.04	10.69	10.37	10.07	9.78	9.52	9.27	9.03	8.80	8.59	8.38	8.19	8.01	7.83
100	10.16	9.84	9.53	9.25	8.98	8.73	8.50	8.27	8.06	7.86	7.67	7.49	7.32	7.16	7.00
110	9.10	8.81	8.55	8.30	8.06	7.84	7.63	7.43	7.24	7.07	6.90	6.74	6.59	6.44	6.30
120	8.19	7.94	7.70	7.48	7.27	7.07	6.89	6.71	6.55	6.39	6.24	6.09	5.96	5.83	5.70
130	7.40	7.18	6.97	6.77	6.59	6.41	6.25	6.09	5.94	5.80	5.67	5.54	5.42	5.30	5.19
140	6.72	6.52	6.33	6.16	5.99	5.84	5.69	5.55	5.42	5.29	5.17	5.06	4.95	4.85	4.75
150	6.12	5.95	5.78	5.63	5.48	5.34	5.21	5.08	4.96	4.85	4.74	4.64	4.54	4.45	4.36
160	5.60	5.44	5.30	5.16	5.03	4.90	4.78	4.67	4.56	4.46	4.36	4.27	4.18	4.10	4.02
170	5.14	5.00	4.87	4.74	4.63	4.51	4.41	4.31	4.21	4.12	4.03	3.95	3.87	3.79	3.71
180	4.73	4.61	4.49	4.38	4.27	4.17	4.07	3.98	3.90	3.81	3.73	3.66	3.58	3.51	3.45
190	4.37	4.26	4.15	4.05	3.96	3.87	3.78	3.70	3.62	3.54	3.47	3.40	3.33	3.27	3.21
200	4.05	3.95	3.85	3.76	3.68	3.59	3.51	3.44	3.37	3.30	3.23	3.17	3.11	3.05	2.99

TABLE I (Continued)

(YIELD STRESS = 46 KSI)

VALUES OF P/A IN KSI FOR k =

L/R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	45.42	44.86	44.31	43.78	43.25	42.74	42.25	41.76	41.29	40.82	40.37	39.92	39.49	39.07	38.65
20	44.80	43.67	42.60	41.58	40.61	39.69	38.81	37.97	37.17	36.40	35.66	34.96	34.28	33.63	33.01
30	44.07	42.32	40.73	39.26	37.90	36.65	35.48	34.40	33.38	32.42	31.52	30.68	29.88	29.12	28.41
40	43.14	40.70	38.58	36.71	35.04	33.55	32.19	30.95	29.82	28.78	27.81	26.91	26.08	25.30	24.56
50	41.84	38.64	36.05	33.87	32.01	30.38	28.94	27.65	26.49	25.44	24.47	23.59	22.77	22.02	21.32
60	39.91	36.00	33.09	30.77	28.85	27.21	25.78	24.53	23.41	22.41	21.50	20.67	19.91	19.21	18.57
70	37.01	32.75	29.80	27.53	25.69	24.14	22.82	21.66	20.63	19.71	18.89	18.14	17.45	16.82	16.24
80	33.06	29.08	26.38	24.33	22.67	21.29	20.11	19.07	18.16	17.35	16.62	15.95	15.34	14.79	14.27
90	28.63	25.37	23.09	21.34	19.92	18.72	17.69	16.80	16.00	15.29	14.65	14.07	13.54	13.05	12.60
100	24.46	21.95	20.12	18.67	17.48	16.46	15.59	14.82	14.14	13.52	12.97	12.47	12.00	11.58	11.19
110	20.88	18.98	17.52	16.34	15.36	14.51	13.77	13.12	12.53	12.00	11.53	11.09	10.69	10.32	9.98
120	17.92	16.47	15.32	14.36	13.54	12.83	12.21	11.65	11.15	10.70	10.29	9.91	9.56	9.24	8.94
130	15.50	14.37	13.45	12.67	11.99	11.40	10.87	10.40	9.97	9.58	9.23	8.90	8.60	8.32	8.06
140	13.51	12.62	11.87	11.23	10.67	10.17	9.72	9.32	8.95	8.62	8.31	8.02	7.76	7.52	7.29
150	11.87	11.16	10.54	10.01	9.54	9.11	8.73	8.39	8.07	7.78	7.51	7.27	7.04	6.82	6.62
160	10.50	9.92	9.41	8.97	8.57	8.21	7.88	7.58	7.31	7.06	6.82	6.61	6.40	6.21	6.04
170	9.35	8.87	8.45	8.07	7.73	7.42	7.14	6.88	6.65	6.42	6.22	6.03	5.85	5.68	5.53
180	8.38	7.98	7.62	7.30	7.01	6.74	6.50	6.27	6.06	5.87	5.69	5.52	5.36	5.22	5.08
190	7.55	7.21	6.91	6.63	6.38	6.15	5.93	5.74	5.56	5.38	5.23	5.08	4.94	4.80	4.68
200	6.83	6.55	6.29	6.05	5.83	5.63	5.44	5.27	5.11	4.95	4.81	4.68	4.55	4.44	4.32

VALUES OF P/A IN KSI FOR k =

L/R	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	38.24	37.85	37.46	37.08	36.70	36.34	35.98	35.63	35.28	34.94	34.61	34.28	33.97	33.65	33.34
20	32.41	31.83	31.27	30.73	30.21	29.71	29.22	28.75	28.30	27.86	27.44	27.02	26.62	26.24	25.86
30	27.73	27.08	26.47	25.88	25.32	24.79	24.27	23.78	23.31	22.86	22.43	22.01	21.61	21.22	20.85
40	23.88	23.23	22.62	22.04	21.49	20.97	20.48	20.01	19.56	19.14	18.73	18.34	17.96	17.61	17.26
50	20.66	20.05	19.47	18.93	18.43	17.95	17.49	17.06	16.65	16.26	15.89	15.54	15.20	14.88	14.57
60	17.97	17.41	16.89	16.40	15.94	15.50	15.09	14.71	14.34	14.00	13.67	13.35	13.05	12.77	12.50
70	15.70	15.20	14.74	14.30	13.89	13.51	13.14	12.80	12.48	12.17	11.88	11.60	11.34	11.09	10.85
80	13.80	13.36	12.94	12.56	12.20	11.86	11.54	11.24	10.95	10.68	10.42	10.18	9.95	9.73	9.52
90	12.19	11.80	11.44	11.10	10.78	10.48	10.20	9.94	9.69	9.45	9.22	9.01	8.80	8.61	8.42
100	10.82	10.48	10.17	9.87	9.59	9.33	9.08	8.85	8.63	8.42	8.22	8.03	7.85	7.68	7.51
110	9.66	9.36	9.09	8.83	8.58	8.35	8.13	7.93	7.73	7.55	7.37	7.21	7.05	6.89	6.75
120	8.67	8.41	8.16	7.94	7.72	7.52	7.32	7.14	6.97	6.81	6.65	6.50	6.36	6.23	6.10
130	7.81	7.58	7.37	7.17	6.98	6.80	6.63	6.47	6.32	6.17	6.03	5.90	5.77	5.65	5.54
140	7.07	6.87	6.68	6.51	6.34	6.18	6.03	5.88	5.75	5.62	5.49	5.38	5.26	5.16	5.05
150	6.43	6.25	6.09	5.93	5.78	5.64	5.50	5.38	5.25	5.14	5.03	4.92	4.82	4.72	4.63
160	5.87	5.71	5.56	5.42	5.29	5.16	5.04	4.93	4.82	4.72	4.62	4.52	4.43	4.34	4.26
170	5.38	5.24	5.11	4.98	4.86	4.75	4.64	4.54	4.44	4.35	4.26	4.17	4.09	4.01	3.93
180	4.94	4.82	4.70	4.59	4.48	4.38	4.28	4.19	4.10	4.02	3.94	3.86	3.78	3.71	3.64
190	4.56	4.45	4.34	4.24	4.14	4.05	3.97	3.88	3.80	3.73	3.65	3.58	3.51	3.45	3.39
200	4.22	4.12	4.02	3.93	3.84	3.76	3.68	3.61	3.53	3.46	3.40	3.33	3.27	3.21	3.15

TABLE I (Continued)

(YIELD STRESS = 50 KSI)

L/R	VALUES OF P/A IN KSI FOR k =														
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	49.37	48.76	48.16	47.58	47.01	46.46	45.92	45.39	44.87	44.36	43.87	43.39	42.92	42.46	42.00
20	48.69	47.45	46.28	45.17	44.11	43.11	42.15	41.23	40.36	39.52	38.72	37.96	37.22	36.51	35.83
30	47.88	45.95	44.20	42.59	41.11	39.74	38.47	37.29	36.18	35.14	34.16	33.24	32.38	31.56	30.78
40	46.81	44.11	41.77	39.72	37.90	36.27	34.80	33.45	32.22	31.09	30.05	29.07	28.17	27.33	26.54
50	45.27	41.71	38.86	36.49	34.46	32.70	31.14	29.75	28.50	27.37	26.33	25.38	24.51	23.70	22.95
60	42.90	38.60	35.44	32.94	30.87	29.12	27.60	26.26	25.07	24.00	23.03	22.15	21.34	20.60	19.91
70	39.30	34.74	31.62	29.23	27.29	25.67	24.27	23.05	21.97	21.01	20.14	19.35	18.62	17.96	17.35
80	34.53	30.48	27.73	25.62	23.91	22.48	21.26	20.19	19.24	18.39	17.63	16.94	16.30	15.72	15.18
90	29.49	26.32	24.06	22.30	20.87	19.65	18.60	17.69	16.87	16.14	15.48	14.88	14.33	13.83	13.36
100	24.97	22.59	20.81	19.39	18.20	17.19	16.31	15.53	14.84	14.21	13.65	13.13	12.66	12.22	11.82
110	21.20	19.43	18.03	16.89	15.92	15.08	14.34	13.69	13.10	12.57	12.09	11.64	11.24	10.86	10.51
120	18.14	16.79	15.69	14.77	13.98	13.28	12.67	12.12	11.62	11.17	10.75	10.37	10.02	9.70	9.39
130	15.65	14.61	13.74	12.99	12.34	11.76	11.24	10.78	10.36	9.97	9.61	9.29	8.98	8.70	8.44
140	13.62	12.80	12.10	11.49	10.95	10.46	10.03	9.63	9.27	8.94	8.64	8.35	8.09	7.84	7.62
150	11.95	11.29	10.72	10.22	9.76	9.36	8.99	8.65	8.34	8.06	7.79	7.55	7.32	7.10	6.90
160	10.57	10.03	9.56	9.14	8.75	8.41	8.10	7.81	7.54	7.29	7.06	6.85	6.65	6.46	6.28
170	9.40	8.96	8.57	8.21	7.89	7.59	7.32	7.07	6.84	6.63	6.43	6.24	6.06	5.90	5.74
180	8.42	8.05	7.72	7.42	7.14	6.89	6.65	6.44	6.23	6.05	5.87	5.71	5.55	5.40	5.27
190	7.58	7.27	6.99	6.73	6.49	6.27	6.07	5.88	5.70	5.54	5.38	5.24	5.10	4.97	4.85
200	6.86	6.60	6.35	6.13	5.93	5.73	5.56	5.39	5.23	5.09	4.95	4.82	4.70	4.58	4.47

L/R	VALUES OF P/A IN KSI FOR k =														
	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	41.56	41.13	40.71	40.29	39.88	39.49	39.10	38.71	38.34	37.97	37.61	37.26	36.91	36.57	36.23
20	35.18	34.55	33.94	33.36	32.79	32.25	31.72	31.21	30.72	30.24	29.78	29.33	28.90	28.48	28.07
30	30.04	29.34	28.67	28.04	27.43	26.85	26.30	25.76	25.25	24.77	24.30	23.85	23.41	22.99	22.59
40	25.79	25.09	24.43	23.81	23.22	22.66	22.13	21.62	21.14	20.68	20.24	19.82	19.42	19.03	18.66
50	22.24	21.59	20.97	20.39	19.85	19.33	18.84	18.38	17.94	17.53	17.13	16.75	16.39	16.05	15.71
60	19.27	18.67	18.12	17.60	17.11	16.65	16.21	15.80	15.41	15.04	14.69	14.35	14.03	13.73	13.44
70	16.78	16.25	15.76	15.30	14.86	14.46	14.07	13.71	13.37	13.04	12.73	12.44	12.16	11.89	11.64
80	14.69	14.22	13.79	13.39	13.01	12.65	12.32	12.00	11.70	11.41	11.14	10.89	10.64	10.41	10.18
90	12.93	12.52	12.15	11.80	11.47	11.15	10.86	10.58	10.32	10.07	9.83	9.61	9.39	9.19	8.99
100	11.44	11.09	10.77	10.46	10.17	9.90	9.64	9.40	9.17	8.95	8.74	8.54	8.36	8.18	8.00
110	10.18	9.88	9.59	9.33	9.07	8.84	8.61	8.40	8.20	8.01	7.82	7.65	7.48	7.32	7.17
120	9.11	8.85	8.60	8.36	8.14	7.93	7.74	7.55	7.37	7.20	7.04	6.89	6.74	6.60	6.47
130	8.19	7.96	7.74	7.54	7.34	7.16	6.99	6.82	6.67	6.52	6.37	6.24	6.11	5.98	5.86
140	7.40	7.20	7.01	6.83	6.66	6.49	6.34	6.19	6.05	5.92	5.80	5.67	5.56	5.45	5.34
150	6.71	6.54	6.37	6.21	6.06	5.91	5.78	5.65	5.52	5.41	5.29	5.18	5.08	4.98	4.89
160	6.12	5.96	5.81	5.67	5.54	5.41	5.29	5.17	5.06	4.95	4.85	4.76	4.66	4.57	4.49
170	5.59	5.45	5.32	5.20	5.08	4.96	4.86	4.75	4.65	4.56	4.47	4.38	4.30	4.22	4.14
180	5.13	5.01	4.89	4.78	4.67	4.57	4.47	4.38	4.29	4.21	4.13	4.05	3.97	3.90	3.83
190	4.73	4.62	4.51	4.41	4.32	4.22	4.14	4.05	3.97	3.90	3.82	3.75	3.68	3.62	3.55
200	4.37	4.27	4.17	4.08	4.00	3.92	3.84	3.76	3.69	3.62	3.55	3.49	3.42	3.36	3.31

TABLE I (Continued)

(YIELD STRESS = 60 KSI)

L/R	VALUES OF P/A IN KSI FOR k =														
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	59.24	58.51	57.79	57.09	56.40	55.73	55.08	54.45	53.83	53.22	52.62	52.04	51.48	50.92	50.38
20	58.41	56.90	55.48	54.13	52.85	51.64	50.48	49.37	48.32	47.31	46.35	45.42	44.54	43.69	42.87
30	57.36	54.99	52.84	50.87	49.07	47.41	45.87	44.44	43.11	41.86	40.69	39.58	38.54	37.56	36.63
40	55.90	52.51	49.62	47.11	44.91	42.94	41.16	39.56	38.09	36.74	35.50	34.35	33.28	32.28	31.35
50	53.61	49.12	45.62	42.76	40.34	38.25	36.42	34.79	33.33	32.01	30.81	29.70	28.68	27.74	26.87
60	49.82	44.56	40.84	37.94	35.57	33.56	31.83	30.30	28.95	27.74	26.64	25.64	24.72	23.87	23.09
70	44.03	39.03	35.62	33.01	30.89	29.11	27.58	26.24	25.05	23.98	23.02	22.15	21.35	20.61	19.93
80	37.24	33.32	30.54	28.38	26.61	25.11	23.81	22.68	21.67	20.76	19.94	19.19	18.50	17.87	17.28
90	30.99	28.15	26.02	24.30	22.87	21.64	20.58	19.63	18.79	18.02	17.33	16.70	16.12	15.58	15.08
100	25.85	23.81	22.19	20.85	19.70	18.71	17.84	17.06	16.36	15.72	15.14	14.61	14.12	13.66	13.24
110	21.76	20.26	19.02	17.97	17.06	16.26	15.55	14.90	14.32	13.79	13.30	12.85	12.44	12.05	11.69
120	18.51	17.38	16.43	15.60	14.86	14.21	13.63	13.09	12.61	12.16	11.75	11.37	11.02	10.69	10.38
130	15.92	15.05	14.29	13.63	13.03	12.50	12.02	11.57	11.17	10.79	10.44	10.12	9.82	9.54	9.27
140	13.82	13.13	12.53	11.99	11.50	11.06	10.66	10.29	9.95	9.63	9.33	9.06	8.80	8.56	8.33
150	12.10	11.55	11.06	10.62	10.22	9.85	9.51	9.20	8.91	8.63	8.38	8.14	7.92	7.71	7.51
160	10.68	10.24	9.83	9.47	9.13	8.82	8.53	8.26	8.02	7.78	7.56	7.36	7.17	6.98	6.81
170	9.50	9.13	8.79	8.48	8.20	7.94	7.69	7.46	7.25	7.05	6.86	6.68	6.51	6.35	6.20
180	8.50	8.19	7.91	7.64	7.40	7.18	6.97	6.77	6.58	6.41	6.24	6.09	5.94	5.80	5.67
190	7.65	7.39	7.15	6.92	6.71	6.52	6.34	6.17	6.00	5.85	5.71	5.57	5.44	5.32	5.20
200	6.92	6.69	6.49	6.30	6.12	5.95	5.79	5.64	5.50	5.36	5.24	5.11	5.00	4.89	4.79

L/R	VALUES OF P/A IN KSI FOR k =														
	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	49.85	49.33	48.82	48.32	47.83	47.35	46.89	46.43	45.98	45.53	45.10	44.68	44.26	43.85	43.45
20	42.08	41.33	40.60	39.90	39.22	38.57	37.93	37.32	36.73	36.16	35.61	35.07	34.55	34.05	33.56
30	35.75	34.92	34.12	33.36	32.64	31.95	31.29	30.66	30.05	29.47	28.91	28.38	27.86	27.36	26.88
40	30.47	29.65	28.87	28.14	27.44	26.78	26.16	25.56	25.00	24.45	23.94	23.44	22.97	22.52	22.08
50	26.05	25.29	24.58	23.91	23.27	22.68	22.11	21.58	21.07	20.59	20.13	19.69	19.27	18.87	18.48
60	22.36	21.69	21.05	20.46	19.90	19.38	18.88	18.41	17.96	17.54	17.14	16.76	16.39	16.04	15.71
70	19.29	18.70	18.15	17.64	17.15	16.70	16.27	15.86	15.47	15.11	14.76	14.43	14.11	13.81	13.52
80	16.74	16.23	15.76	15.32	14.90	14.51	14.13	13.78	13.45	13.13	12.83	12.55	12.27	12.01	11.76
90	14.62	14.18	13.78	13.40	13.04	12.70	12.38	12.08	11.79	11.52	11.26	11.01	10.77	10.55	10.33
100	12.84	12.47	12.12	11.80	11.49	11.20	10.92	10.66	10.41	10.17	9.95	9.73	9.53	9.33	9.15
110	11.35	11.03	10.73	10.45	10.19	9.94	9.70	9.47	9.26	9.05	8.86	8.67	8.49	8.32	8.16
120	10.09	9.82	9.56	9.32	9.09	8.87	8.67	8.47	8.28	8.10	7.93	7.77	7.61	7.46	7.32
130	9.02	8.79	8.57	8.36	8.16	7.97	7.79	7.62	7.45	7.30	7.15	7.00	6.87	6.73	6.61
140	8.11	7.91	7.71	7.53	7.36	7.19	7.03	6.88	6.74	6.60	6.47	6.34	6.22	6.11	5.99
150	7.33	7.15	6.98	6.82	6.67	6.52	6.38	6.25	6.12	6.00	5.89	5.77	5.67	5.56	5.46
160	6.65	6.49	6.34	6.20	6.07	5.94	5.82	5.70	5.59	5.48	5.38	5.28	5.18	5.09	5.00
170	6.06	5.92	5.79	5.67	5.55	5.43	5.33	5.22	5.12	5.03	4.93	4.84	4.76	4.68	4.60
180	5.54	5.42	5.31	5.20	5.09	4.99	4.89	4.80	4.71	4.62	4.54	4.46	4.38	4.31	4.24
190	5.09	4.98	4.88	4.78	4.69	4.60	4.51	4.43	4.35	4.27	4.19	4.12	4.05	3.99	3.92
200	4.69	4.59	4.50	4.41	4.33	4.25	4.17	4.09	4.02	3.95	3.89	3.82	3.76	3.70	3.64

TABLE I (Continued)

(YIELD STRESS = 70 KSI)

VALUES OF P/A IN KSI FOR k =

L/R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	69.11	68.25	67.41	66.59	65.79	65.01	64.25	63.50	62.78	62.07	61.37	60.69	60.03	59.38	58.75
20	68.11	66.33	64.66	63.07	61.56	60.13	58.77	57.47	56.24	55.06	53.93	52.85	51.81	50.82	49.86
30	66.81	63.96	61.39	59.06	56.93	54.97	53.16	51.49	49.92	48.47	47.10	45.82	44.61	43.47	42.39
40	64.86	60.72	57.26	54.28	51.68	49.38	47.32	45.45	43.76	42.20	40.77	39.45	38.22	37.07	36.00
50	61.55	56.07	51.94	48.62	45.84	43.46	41.37	39.52	37.87	36.38	35.02	33.77	32.63	31.57	30.58
60	55.74	49.73	45.58	42.37	39.76	37.55	35.65	33.98	32.49	31.15	29.95	28.84	27.83	26.90	26.04
70	47.43	42.39	38.89	36.17	33.95	32.08	30.46	29.03	27.77	26.63	25.60	24.66	23.80	23.00	22.27
80	38.97	35.38	32.71	30.58	28.80	27.29	25.96	24.79	23.75	22.80	21.95	21.16	20.44	19.77	19.15
90	31.92	29.43	27.47	25.84	24.46	23.25	22.19	21.24	20.39	19.62	18.91	18.26	17.66	17.10	16.58
100	26.40	24.64	23.19	21.95	20.87	19.92	19.07	18.30	17.61	16.98	16.39	15.85	15.36	14.89	14.46
110	22.11	20.83	19.73	18.78	17.93	17.18	16.50	15.88	15.31	14.79	14.31	13.86	13.44	13.05	12.69
120	18.76	17.79	16.95	16.20	15.53	14.93	14.37	13.87	13.40	12.97	12.57	12.20	11.85	11.52	11.21
130	16.09	15.35	14.69	14.10	13.56	13.06	12.61	12.20	11.81	11.45	11.12	10.80	10.51	10.23	9.97
140	13.95	13.37	12.84	12.36	11.92	11.52	11.14	10.80	10.47	10.17	9.89	9.62	9.37	9.14	8.91
150	12.20	11.74	11.31	10.92	10.55	10.22	9.91	9.62	9.35	9.09	8.85	8.62	8.41	8.21	8.01
160	10.76	10.38	10.03	9.71	9.40	9.12	8.86	8.62	8.38	8.17	7.96	7.77	7.58	7.41	7.24
170	9.56	9.25	8.95	8.68	8.43	8.19	7.97	7.76	7.56	7.37	7.20	7.03	6.87	6.72	6.57
180	8.55	8.29	8.04	7.81	7.60	7.39	7.20	7.02	6.85	6.69	6.54	6.39	6.25	6.12	5.99
190	7.69	7.47	7.26	7.06	6.88	6.70	6.54	6.38	6.23	6.09	5.96	5.83	5.71	5.59	5.48
200	6.95	6.76	6.58	6.42	6.26	6.11	5.96	5.83	5.70	5.57	5.46	5.34	5.24	5.13	5.03

VALUES OF P/A IN KSI FOR k =

L/R	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	58.13	57.52	56.92	56.34	55.77	55.21	54.67	54.13	53.60	53.09	52.58	52.09	51.60	51.12	50.65
20	48.94	48.06	47.21	46.39	45.60	44.84	44.10	43.39	42.71	42.04	41.39	40.77	40.17	39.58	39.01
30	41.37	40.40	39.47	38.60	37.76	36.96	36.20	35.46	34.76	34.09	33.45	32.83	32.23	31.66	31.11
40	35.00	34.06	33.17	32.33	31.53	30.78	30.06	29.38	28.73	28.12	27.53	26.96	26.42	25.90	25.41
50	29.67	28.81	28.01	27.25	26.54	25.87	25.23	24.63	24.05	23.51	22.99	22.50	22.02	21.57	21.14
60	25.24	24.49	23.79	23.13	22.51	21.93	21.38	20.86	20.37	19.90	19.45	19.02	18.62	18.23	17.86
70	21.58	20.94	20.35	19.79	19.26	18.76	18.29	17.85	17.42	17.02	16.64	16.28	15.93	15.60	15.28
80	18.57	18.04	17.53	17.06	16.61	16.19	15.79	15.41	15.05	14.71	14.38	14.07	13.78	13.49	13.22
90	16.10	15.64	15.22	14.82	14.44	14.08	13.74	13.42	13.11	12.82	12.54	12.28	12.03	11.78	11.55
100	14.05	13.67	13.31	12.97	12.65	12.34	12.05	11.78	11.52	11.27	11.03	10.80	10.59	10.38	10.18
110	12.35	12.02	11.72	11.43	11.16	10.90	10.65	10.42	10.19	9.98	9.77	9.58	9.39	9.21	9.04
120	10.92	10.65	10.39	10.14	9.91	9.69	9.47	9.27	9.08	8.89	8.72	8.55	8.38	8.23	8.08
130	9.72	9.49	9.26	9.05	8.85	8.66	8.48	8.30	8.13	7.97	7.82	7.67	7.53	7.39	7.26
140	8.70	8.50	8.31	8.13	7.95	7.79	7.63	7.48	7.33	7.19	7.05	6.93	6.80	6.68	6.57
150	7.83	7.66	7.49	7.33	7.18	7.04	6.90	6.76	6.64	6.51	6.40	6.28	6.17	6.07	5.97
160	7.08	6.93	6.79	6.65	6.52	6.39	6.27	6.15	6.04	5.93	5.83	5.73	5.63	5.54	5.44
170	6.43	6.30	6.17	6.05	5.94	5.83	5.72	5.62	5.52	5.42	5.33	5.24	5.15	5.07	4.99
180	5.87	5.75	5.64	5.53	5.43	5.33	5.24	5.15	5.06	4.97	4.89	4.81	4.74	4.66	4.59
190	5.37	5.27	5.17	5.08	4.99	4.90	4.82	4.73	4.66	4.58	4.51	4.44	4.37	4.30	4.24
200	4.94	4.85	4.76	4.68	4.60	4.52	4.44	4.37	4.30	4.23	4.16	4.10	4.04	3.98	3.92

TABLE I (Concluded)

(YIELD STRESS = 100 KSI)

VALUES OF P/A IN KSI FOR k =

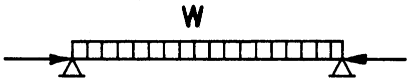
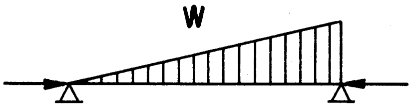
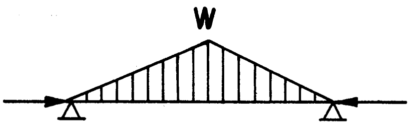
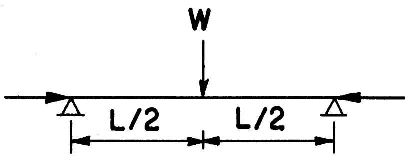
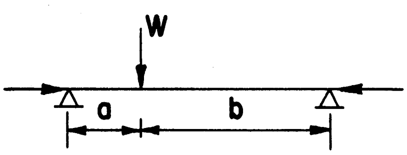
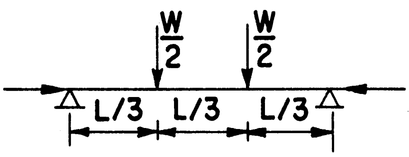
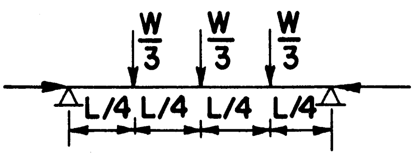
L/R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15
10	98.72	97.47	96.26	95.08	93.92	92.80	91.70	90.63	89.59	88.57	87.57	86.59	85.64	84.71	83.80
20	97.18	94.53	92.05	89.71	87.50	85.40	83.42	81.54	79.74	78.04	76.41	74.85	73.36	71.93	70.56
30	94.89	90.45	86.53	83.02	79.86	76.99	74.36	71.93	69.69	67.61	65.67	63.85	62.14	60.54	59.02
40	90.68	83.97	78.67	74.29	70.55	67.30	64.41	61.83	59.50	57.38	55.44	53.64	51.98	50.44	49.00
50	81.76	73.69	68.04	63.63	60.01	56.93	54.27	51.91	49.81	47.91	46.18	44.60	43.15	41.80	40.55
60	67.07	60.69	56.15	52.57	49.62	47.11	44.92	42.99	41.26	39.70	38.28	36.98	35.78	34.67	33.64
70	52.60	48.54	45.39	42.80	40.60	38.70	37.01	35.51	34.16	32.93	31.80	30.76	29.80	28.91	28.08
80	41.47	38.89	36.75	34.91	33.32	31.90	30.63	29.48	28.44	27.48	26.60	25.78	25.02	24.31	23.64
90	33.30	31.57	30.08	28.77	27.59	26.54	25.58	24.70	23.89	23.14	22.45	21.80	21.20	20.63	20.10
100	27.25	26.04	24.96	24.00	23.12	22.32	21.58	20.90	20.27	19.68	19.13	18.62	18.14	17.68	17.25
110	22.67	21.79	21.00	20.27	19.60	18.98	18.41	17.87	17.37	16.90	16.46	16.05	15.66	15.29	14.94
120	19.15	18.49	17.88	17.32	16.80	16.31	15.86	15.43	15.03	14.65	14.30	13.96	13.64	13.33	13.04
130	16.38	15.87	15.40	14.95	14.54	14.15	13.79	13.44	13.12	12.81	12.52	12.24	11.97	11.72	11.48
140	14.17	13.77	13.39	13.03	12.70	12.38	12.09	11.80	11.54	11.28	11.04	10.81	10.59	10.38	10.18
150	12.37	12.05	11.74	11.46	11.18	10.92	10.68	10.44	10.22	10.01	9.81	9.61	9.43	9.25	9.08
160	10.90	10.63	10.38	10.14	9.92	9.70	9.50	9.30	9.12	8.94	8.77	8.60	8.44	8.29	8.15
170	9.67	9.45	9.24	9.05	8.86	8.67	8.50	8.34	8.18	8.03	7.88	7.74	7.60	7.47	7.35
180	8.64	8.46	8.28	8.11	7.95	7.80	7.65	7.51	7.38	7.25	7.12	7.00	6.88	6.77	6.66
190	7.77	7.61	7.46	7.32	7.18	7.05	6.92	6.80	6.69	6.57	6.46	6.36	6.26	6.16	6.07
200	7.02	6.88	6.76	6.63	6.52	6.40	6.29	6.19	6.09	5.99	5.89	5.80	5.71	5.63	5.55

VALUES OF P/A IN KSI FOR k =

L/R	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
10	82.91	82.04	81.19	80.35	79.53	78.73	77.95	77.18	76.43	75.69	74.96	74.25	73.56	72.87	72.20
20	69.25	67.99	66.77	65.60	64.48	63.39	62.35	61.33	60.36	59.41	58.50	57.61	56.75	55.92	55.12
30	57.59	56.23	54.95	53.72	52.56	51.45	50.39	49.37	48.40	47.47	46.58	45.72	44.89	44.10	43.34
40	47.65	46.38	45.19	44.07	43.00	42.00	41.04	40.13	39.26	38.44	37.65	36.89	36.17	35.48	34.81
50	39.38	38.29	37.27	36.31	35.40	34.54	33.72	32.95	32.22	31.52	30.85	30.22	29.61	29.02	28.47
60	32.67	31.77	30.93	30.13	29.38	28.67	28.00	27.36	26.75	26.17	25.62	25.09	24.59	24.11	23.65
70	27.30	26.57	25.88	25.24	24.62	24.04	23.49	22.97	22.47	22.00	21.54	21.11	20.70	20.30	19.92
80	23.02	22.43	21.87	21.35	20.85	20.38	19.93	19.50	19.09	18.70	18.33	17.97	17.63	17.30	16.98
90	19.59	19.12	18.67	18.24	17.83	17.45	17.08	16.73	16.39	16.07	15.76	15.46	15.18	14.90	14.64
100	16.84	16.45	16.08	15.73	15.40	15.08	14.78	14.49	14.21	13.94	13.68	13.43	13.20	12.97	12.75
110	14.60	14.28	13.98	13.69	13.42	13.15	12.90	12.65	12.42	12.20	11.98	11.77	11.57	11.38	11.19
120	12.77	12.50	12.25	12.01	11.78	11.56	11.35	11.14	10.95	10.76	10.57	10.40	10.23	10.06	9.91
130	11.25	11.03	10.82	10.62	10.42	10.23	10.05	9.88	9.71	9.55	9.40	9.25	9.10	8.96	8.83
140	9.98	9.80	9.62	9.45	9.28	9.12	8.97	8.82	8.68	8.54	8.41	8.28	8.15	8.03	7.91
150	8.91	8.76	8.60	8.46	8.32	8.18	8.05	7.92	7.80	7.68	7.56	7.45	7.34	7.24	7.14
160	8.00	7.87	7.74	7.61	7.49	7.37	7.26	7.15	7.04	6.94	6.84	6.74	6.65	6.56	6.47
170	7.23	7.11	7.00	6.89	6.78	6.68	6.58	6.49	6.39	6.30	6.21	6.13	6.05	5.97	5.89
180	6.56	6.45	6.36	6.26	6.17	6.08	5.99	5.91	5.83	5.75	5.67	5.60	5.52	5.45	5.38
190	5.97	5.89	5.80	5.72	5.63	5.56	5.48	5.41	5.33	5.26	5.20	5.13	5.07	5.00	4.94
200	5.47	5.39	5.31	5.24	5.17	5.10	5.03	4.96	4.90	4.84	4.78	4.72	4.66	4.61	4.55

TABLE II

LOAD AND DEFLECTION COEFFICIENTS TO GIVE EQUIVALENT LATERAL LOAD FOR USE IN TABLE I

CASE NO.	LOADING CONDITION	EQUIVALENT LOAD FACTOR	Ψ IN DEFLECTION FACTOR
1		1	+ .028
2		1.026	0.003
3		1.333	-0.013
4		2	- 0.178
5		$8\frac{ab}{L^2}$	$\left(\frac{\pi^2 ab}{3L^2}-1\right)$ *
6		1.333	+ 0.051 **
7		1.333	- 0.023 **

* At load point

** At Center

Note: For design purposes it would be satisfactory to assume $\psi = 0$, with resulting deflection factor of 1.0, for cases 1, 2, 3, 6, and 7.

TABLE III

AVERAGE COLUMN LOAD FOR WHICH COMBINED STRESS IN RESTRAINED BEAM-COLUMNS WITH UNIFORM LATERAL LOAD JUST REACHES YIELD POINT

σ_y = 50.000000

η = .200000

VALUES OF P/A IN KSI FOR k_ =

L/R	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30
10	49.13	48.33	47.58	46.82	46.11	45.39	44.71	44.06	43.39	42.76	42.16	41.59	40.98	40.47	39.92
20	48.27	46.73	45.23	43.86	42.59	41.39	40.20	39.12	38.13	37.17	36.22	35.34	34.53	33.71	32.94
30	47.33	45.03	42.93	41.03	39.33	37.74	36.24	34.91	33.67	32.55	31.48	30.50	29.54	28.66	27.84
40	46.29	43.16	40.52	38.20	36.15	34.32	32.70	31.23	29.87	28.65	27.56	26.51	25.55	24.66	23.84
50	44.96	41.11	37.94	35.31	33.11	31.13	29.44	27.93	26.59	25.34	24.28	23.25	22.31	21.45	20.71
60	43.40	38.80	35.30	32.44	30.12	28.14	26.47	25.02	23.68	22.54	21.46	20.53	19.62	18.84	18.12
70	41.36	36.16	32.47	29.67	27.31	25.40	23.77	22.40	21.13	20.05	19.07	18.18	17.42	16.66	16.01
80	38.79	33.36	29.66	26.9	24.70	22.93	21.40	20.06	18.91	17.94	17.00	16.26	15.51	14.83	14.24
90	35.75	30.42	26.89	24.37	22.32	20.63	19.26	18.05	17.02	16.09	15.29	14.57	13.89	13.27	12.74
100	32.34	27.44	24.31	21.96	20.08	18.63	17.34	16.25	15.32	14.48	13.76	13.10	12.49	11.98	11.50
110	28.92	24.69	21.88	19.81	18.13	16.76	15.67	14.68	13.84	13.07	12.42	11.82	11.32	10.86	10.36
120	25.66	22.08	19.65	17.80	16.35	15.18	14.19	13.29	12.53	11.89	11.30	10.74	10.29	9.86	9.46
130	22.79	19.80	17.69	16.09	14.84	13.78	12.88	12.06	11.43	10.84	10.30	9.79	9.37	8.98	8.61
140	20.24	17.73	15.96	14.57	13.44	12.47	11.72	11.02	10.44	9.90	9.40	8.99	8.61	8.24	7.90
150	17.98	15.96	14.42	13.21	12.24	11.42	10.66	10.09	9.55	9.05	8.65	8.21	7.92	7.58	7.26
160	16.09	14.38	13.03	11.99	11.16	10.41	9.77	9.24	8.75	8.35	7.91	7.56	7.29	6.97	6.74
170	14.46	13.02	11.84	10.95	10.19	9.55	9.02	8.53	8.07	7.70	7.35	7.03	6.72	6.48	6.20
180	13.05	11.79	10.82	10.06	9.36	8.77	8.28	7.82	7.45	7.11	6.79	6.48	6.25	5.97	5.76
190	11.83	10.73	9.90	9.19	8.60	8.11	7.65	7.23	6.89	6.56	6.32	6.03	5.82	5.56	5.36
200	10.72	9.82	9.11	8.45	7.96	7.51	7.08	6.74	6.42	6.12	5.83	5.62	5.42	5.17	4.99

TABLE III (Continued)

σ_y = 50.0C0000

η = .400000

VALUES OF P/A IN KSI FOR k =

L/R	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30
10	49.26	48.58	47.96	47.32	46.68	46.08	45.52	44.94	44.39	43.82	43.28	42.78	42.30	41.78	41.30
20	48.58	47.23	45.98	44.80	43.71	42.64	41.64	40.68	39.75	38.86	38.03	37.22	36.47	35.71	35.00
30	47.83	45.84	44.06	42.41	40.83	39.43	38.12	36.91	35.73	34.68	33.67	32.69	31.79	30.97	30.15
40	46.98	44.35	42.02	39.95	38.09	36.38	34.89	33.48	32.24	31.03	29.94	28.95	27.99	27.10	26.28
50	45.96	42.67	39.88	37.50	35.36	33.51	31.88	30.43	29.09	27.84	26.78	25.75	24.81	23.89	23.09
60	44.78	40.80	37.61	34.94	32.74	30.77	29.09	27.58	26.24	25.04	23.96	22.97	22.06	21.22	20.50
70	43.30	38.72	35.22	32.42	30.13	28.21	26.52	25.02	23.75	22.61	21.57	20.61	19.79	18.97	18.26
80	41.41	36.36	32.73	29.93	27.64	25.74	24.15	22.74	21.54	20.44	19.44	18.57	17.76	17.08	16.37
90	39.18	33.86	30.21	27.49	25.32	23.50	21.95	20.67	19.52	18.52	17.61	16.75	16.02	15.39	14.74
100	36.46	31.19	27.74	25.15	23.08	21.44	20.03	18.81	17.70	16.79	15.95	15.23	14.55	13.92	13.38
110	33.48	28.57	25.32	22.93	21.06	19.51	18.23	17.12	16.15	15.32	14.55	13.88	13.26	12.67	12.17
120	30.41	25.95	23.09	20.92	19.23	17.81	16.63	15.61	14.72	13.95	13.30	12.68	12.10	11.61	11.15
130	27.42	23.55	21.01	19.02	17.53	16.28	15.20	14.31	13.49	12.78	12.17	11.60	11.06	10.60	10.17
140	24.61	21.36	19.08	17.38	16.00	14.85	13.90	13.08	12.38	11.71	11.15	10.68	10.17	9.74	9.40
150	22.10	19.34	17.35	15.84	14.62	13.61	12.79	12.02	11.36	10.80	10.28	9.83	9.42	9.02	8.63
160	19.90	17.57	15.84	14.49	13.41	12.53	11.77	11.11	10.50	9.97	9.47	9.06	8.66	8.35	8.05
170	17.96	15.96	14.47	13.32	12.31	11.55	10.83	10.22	9.70	9.20	8.79	8.40	8.09	7.73	7.45
180	16.24	14.54	13.26	12.25	11.36	10.64	10.03	9.45	9.02	8.55	8.16	7.79	7.50	7.22	6.95
190	14.77	13.29	12.15	11.25	10.47	9.86	9.28	8.79	8.32	7.94	7.57	7.28	7.01	6.74	6.49
200	13.47	12.19	11.23	10.39	9.71	9.13	8.64	8.18	7.80	7.43	7.08	6.81	6.55	6.30	6.05

TABLE III (Continued)

σ_y 50.000000

η = .600000

L/R	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30
	VALUES OF P/A IN KSI FOR k =														
10	49.32	48.77	48.14	47.57	46.99	46.45	45.89	45.37	44.89	44.38	43.91	43.40	42.92	42.47	42.05
20	48.70	47.54	46.36	45.30	44.27	43.27	42.33	41.43	40.57	39.79	38.97	38.22	37.47	36.77	36.13
30	48.02	46.28	44.62	43.10	41.64	40.30	39.12	37.91	36.86	35.80	34.86	33.94	33.04	32.22	31.46
40	47.29	44.91	42.77	40.83	39.09	37.51	36.07	34.73	33.49	32.34	31.25	30.26	29.36	28.48	27.66
50	46.40	43.42	40.82	38.62	36.61	34.82	33.26	31.80	30.47	29.28	28.15	27.13	26.19	25.32	24.52
60	45.40	41.80	38.80	36.31	34.12	32.21	30.59	29.08	27.74	26.54	25.46	24.41	23.50	22.66	21.87
70	44.17	39.97	36.66	33.92	31.69	29.78	28.08	26.59	25.25	24.11	23.01	22.05	21.17	20.34	19.63
80	42.66	37.86	34.35	31.56	29.33	27.37	25.72	24.31	23.04	21.88	20.88	20.01	19.14	18.39	17.68
90	40.75	35.61	32.02	29.24	27.01	25.19	23.57	22.24	21.02	19.96	18.98	18.13	17.39	16.64	15.99
100	38.52	33.19	29.68	26.96	24.83	23.06	21.59	20.31	19.20	18.23	17.33	16.54	15.80	15.17	14.56
110	35.92	30.69	27.32	24.81	22.81	21.14	19.79	18.62	17.59	16.63	15.86	15.13	14.45	13.86	13.30
120	33.04	28.20	25.09	22.74	20.91	19.43	18.13	17.04	16.09	15.26	14.55	13.87	13.23	12.68	12.21
130	30.17	25.80	23.01	20.84	19.21	17.85	16.63	15.69	14.80	14.03	13.36	12.72	12.18	11.67	11.24
140	27.36	23.55	21.02	19.13	17.63	16.35	15.34	14.40	13.63	12.90	12.27	11.74	11.23	10.74	10.34
150	24.73	21.46	19.23	17.52	16.18	15.04	14.10	13.27	12.55	11.93	11.34	10.83	10.35	9.95	9.57
160	22.40	19.57	17.59	16.12	14.91	13.91	13.02	12.24	11.62	11.03	10.53	10.06	9.60	9.22	8.86
170	20.27	17.90	16.15	14.82	13.75	12.80	12.02	11.34	10.76	10.20	9.73	9.34	8.90	8.61	8.26
180	18.43	16.35	14.82	13.62	12.67	11.83	11.15	10.51	10.02	9.48	9.10	8.67	8.31	7.97	7.70
190	16.77	14.98	13.65	12.63	11.72	10.98	10.34	9.79	9.32	8.88	8.45	8.10	7.76	7.49	7.18
200	15.28	13.75	12.61	11.64	10.90	10.19	9.64	9.12	8.67	8.25	7.90	7.56	7.24	6.98	6.74

TABLE III (Continued)

σ_y = 50.0C0C00

η = .800000

VALUES OF P/A IN KSI FOR k =

L/R	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30
10	49.32	48.64	48.02	47.45	46.80	46.26	45.71	45.12	44.58	44.07	43.53	43.03	42.55	42.10	41.61
20	48.64	47.36	46.17	45.05	43.96	42.96	41.95	41.05	40.13	39.29	38.47	37.66	36.90	36.21	35.50
30	47.95	46.09	44.37	42.78	41.33	39.93	38.68	37.48	36.36	35.30	34.29	33.38	32.48	31.66	30.84
40	47.23	44.78	42.58	40.58	38.84	37.20	35.70	34.29	33.06	31.90	30.81	29.76	28.80	27.91	27.09
50	46.40	43.36	40.69	38.44	36.36	34.57	32.94	31.49	30.15	28.90	27.78	26.75	25.81	24.89	24.09
60	45.46	41.80	38.80	36.25	34.06	32.14	30.41	28.89	27.55	26.29	25.14	24.16	23.18	22.34	21.56
70	44.36	40.16	36.85	34.11	31.81	29.78	28.08	26.52	25.19	23.98	22.88	21.93	20.98	20.16	19.38
80	43.10	38.36	34.79	31.99	29.64	27.62	25.90	24.43	23.10	21.94	20.88	19.94	19.08	18.26	17.56
90	41.56	36.42	32.71	29.87	27.51	25.56	23.88	22.49	21.21	20.09	19.11	18.19	17.39	16.64	15.99
100	39.65	34.32	30.62	27.77	25.52	23.63	22.03	20.69	19.51	18.48	17.51	16.67	15.93	15.23	14.63
110	37.30	32.07	28.51	25.74	23.63	21.83	20.35	19.05	17.97	17.01	16.11	15.32	14.63	13.98	13.42
120	34.66	29.70	26.40	23.86	21.85	20.18	18.82	17.61	16.59	15.64	14.86	14.12	13.48	12.86	12.34
130	31.92	27.30	24.32	22.02	20.21	18.66	17.38	16.25	15.30	14.47	13.73	13.04	12.43	11.92	11.42
140	29.17	25.05	22.33	20.32	18.63	17.22	16.09	15.08	14.19	13.40	12.71	12.11	11.54	11.06	10.59
150	26.54	22.96	20.54	18.71	17.24	15.98	14.91	13.96	13.18	12.43	11.84	11.21	10.73	10.27	9.82
160	24.09	21.01	18.84	17.18	15.91	14.78	13.83	12.99	12.25	11.60	10.97	10.43	9.98	9.54	9.17
170	21.90	19.21	17.34	15.88	14.69	13.67	12.83	12.03	11.38	10.76	10.23	9.78	9.34	8.92	8.57
180	19.93	17.60	15.95	14.62	13.61	12.70	11.96	11.26	10.64	10.05	9.54	9.10	8.69	8.34	8.01
190	18.15	16.17	14.71	13.56	12.60	11.80	11.09	10.48	9.89	9.38	8.95	8.53	8.20	7.81	7.49
200	16.60	14.88	13.61	12.58	11.71	10.94	10.33	9.80	9.30	8.81	8.40	8.00	7.67	7.36	7.05

TABLE III (Concluded)

σ_y = 50.000000

η = 1.000000

VALUES OF P/A IN KSI FOR k =

L/R	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30
10	49.26	48.58	47.96	47.32	46.68	46.08	45.52	44.94	44.39	43.82	43.35	42.78	42.30	41.78	41.30
20	48.58	47.29	46.04	44.86	43.77	42.71	41.70	40.74	39.82	38.98	38.10	37.34	36.53	35.83	35.13
30	47.89	45.97	44.18	42.60	41.08	39.68	38.37	37.10	35.98	34.93	33.92	32.94	32.04	31.22	30.40
40	47.10	44.60	42.33	40.33	38.53	36.82	35.32	33.91	32.68	31.47	30.37	29.32	28.42	27.48	26.66
50	46.27	43.17	40.51	38.19	36.11	34.26	32.57	31.11	29.78	28.53	27.40	26.31	25.38	24.45	23.65
60	45.40	41.68	38.61	36.00	33.74	31.83	30.09	28.58	27.18	25.92	24.83	23.78	22.81	21.97	21.12
70	44.30	40.04	36.66	33.92	31.56	29.53	27.77	26.27	24.88	23.67	22.57	21.55	20.67	19.84	19.07
80	43.10	38.30	34.73	31.81	29.39	27.43	25.65	24.18	22.85	21.63	20.63	19.63	18.76	18.01	17.24
90	41.62	36.42	32.71	29.74	27.39	25.38	23.70	22.24	21.02	19.84	18.86	17.94	17.14	16.39	15.74
100	39.90	34.44	30.62	27.77	25.46	23.56	21.91	20.56	19.32	18.29	17.33	16.48	15.74	15.04	14.38
110	37.92	32.32	28.63	25.81	23.63	21.83	20.29	18.99	17.84	16.82	15.98	15.19	14.45	13.79	13.23
120	35.60	30.20	26.65	23.99	21.91	20.18	18.75	17.54	16.47	15.58	14.73	13.99	13.35	12.74	12.21
130	33.04	28.05	24.69	22.21	20.28	18.72	17.38	16.25	15.30	14.40	13.67	12.97	12.37	11.79	11.30
140	30.42	25.92	22.83	20.63	18.81	17.35	16.15	15.08	14.19	13.40	12.65	12.05	11.48	10.93	10.46
150	27.79	23.90	21.10	19.09	17.43	16.11	14.98	14.02	13.18	12.43	11.78	11.21	10.67	10.20	9.76
160	25.28	21.95	19.53	17.68	16.16	14.97	13.96	13.05	12.25	11.60	10.97	10.43	9.98	9.54	9.11
170	23.02	20.21	18.03	16.38	15.06	13.92	12.96	12.15	11.45	10.83	10.23	9.78	9.28	8.92	8.51
180	20.99	18.54	16.70	15.18	13.98	12.95	12.09	11.32	10.70	10.11	9.60	9.10	8.69	8.34	7.95
190	19.21	17.04	15.46	14.13	13.04	12.11	11.28	10.60	10.01	9.50	9.01	8.60	8.20	7.81	7.49
200	17.60	15.69	14.29	13.14	12.15	11.26	10.58	9.93	9.36	8.87	8.46	8.06	7.67	7.36	7.05

TIME ESTIMATE EXCEEDED
AT LOC 20076

TABLE IV

ULTIMATE STRENGTH OF THE UNRESTRAINED BEAM COLUMNS OF THE SIMPLIFIED SECTION IN PLANAR BENDING UNDER UNIFORM LATERAL LOAD (W = kP)

Yield Stress = 33 ksi, Residual Stress = 11.02 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	32.00	31.32	30.50	29.87	29.26	28.50	27.93	27.37	26.83	26.31	25.79	25.30	24.81	24.34	23.88
20	31.13	29.66	28.28	27.14	26.07	25.06	24.10	23.19	22.31	21.64	20.84	20.23	19.65	19.10	18.57
30	29.91	27.86	26.16	24.61	23.18	22.02	20.94	19.94	19.00	18.11	17.44	16.81	16.05	15.48	14.94
40	28.66	26.06	23.93	22.21	20.66	19.43	18.30	17.27	16.32	15.59	14.76	14.13	13.55	13.00	12.49
50	27.18	24.23	21.92	19.92	18.49	17.06	15.93	15.07	14.13	13.42	12.78	12.18	11.62	11.10	10.61
60	25.46	22.22	19.78	17.90	16.46	15.20	14.09	13.26	12.35	11.67	11.06	10.49	9.96	9.47	9.17
70	23.65	20.19	17.86	16.14	14.72	13.50	12.42	11.63	10.92	10.28	9.69	9.16	8.66	8.36	7.93
80	21.58	18.32	16.15	14.31	13.09	11.92	11.06	10.30	9.63	9.03	8.64	8.14	7.67	7.40	6.99
90	19.57	16.45	14.34	12.89	11.74	10.78	9.97	9.26	8.63	8.06	7.71	7.23	6.79	6.54	6.32
100	17.48	14.78	12.92	11.55	10.47	9.57	8.81	8.31	7.72	7.18	6.86	6.41	6.16	5.93	5.56
110	15.68	13.14	11.58	10.30	9.45	8.61	7.90	7.44	6.88	6.55	6.25	5.83	5.59	5.22	5.03
120	13.89	11.73	10.30	9.28	8.51	7.73	7.22	6.64	6.28	5.97	5.53	5.30	5.08	4.73	4.56

Yield Stress = 36 ksi, Residual Stress = 11.81 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	34.94	34.19	33.29	32.60	31.75	31.10	30.47	29.86	29.26	28.69	28.13	27.58	27.05	26.53	26.02
20	33.98	32.37	30.85	29.60	28.24	27.13	26.26	25.26	24.30	23.56	22.86	22.02	21.38	20.77	20.19
30	32.64	30.38	28.51	26.80	25.23	23.96	22.78	21.68	20.65	19.86	18.94	18.24	17.59	16.97	16.20
40	31.25	28.38	26.04	24.15	22.46	21.10	19.87	18.74	17.69	16.90	15.98	15.30	14.66	14.06	13.50
50	29.43	26.17	23.62	21.62	20.04	18.48	17.24	16.30	15.27	14.50	13.80	13.14	12.53	11.96	11.43
60	27.50	23.92	21.43	19.37	17.79	16.42	15.21	14.30	13.31	12.57	11.90	11.28	10.71	10.35	9.85
70	25.28	21.65	19.10	17.23	15.68	14.53	13.36	12.50	11.73	11.03	10.40	10.00	9.45	8.95	8.66
80	23.10	19.54	17.19	15.37	14.05	12.78	11.85	11.03	10.31	9.83	9.24	8.69	8.36	7.89	7.62
90	20.63	17.44	15.34	13.60	12.37	11.34	10.64	9.88	9.20	8.59	8.21	7.69	7.39	6.94	6.70
100	18.43	15.55	13.58	12.12	11.14	10.19	9.37	8.83	8.19	7.80	7.27	6.97	6.52	6.27	6.05
110	16.38	13.72	12.08	10.91	10.02	9.13	8.54	7.87	7.46	6.92	6.60	6.32	5.90	5.67	5.47
120	14.35	12.33	10.85	9.78	8.97	8.15	7.62	7.17	6.61	6.28	6.00	5.56	5.34	5.13	4.95

TABLE IV (Continued)

Yield Stress = 42 ksi, Residual Stress = 13.27 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	40.83	39.74	38.89	38.06	37.06	36.29	35.55	34.83	34.12	33.44	32.78	32.14	31.51	30.90	30.30
20	39.49	37.57	35.99	34.30	32.91	31.59	30.56	29.37	28.46	27.37	26.55	25.76	24.80	24.08	23.39
30	37.89	35.20	32.98	30.96	29.32	27.81	26.42	25.12	23.90	22.97	22.10	21.07	20.30	19.57	18.88
40	36.00	32.79	30.02	27.78	25.99	24.38	22.93	21.60	20.38	19.44	18.58	17.78	17.03	16.33	15.66
50	33.99	30.10	27.09	24.93	22.87	21.24	19.80	18.69	17.70	16.59	15.76	15.00	14.29	13.83	13.20
60	31.41	27.35	24.41	22.00	20.15	18.55	17.35	16.29	15.35	14.49	13.71	12.99	12.32	11.91	11.32
70	28.63	24.54	21.57	19.40	17.81	16.28	15.14	14.14	13.25	12.65	11.92	11.25	10.83	10.25	9.91
80	25.65	21.72	19.22	17.14	15.64	14.39	13.32	12.39	11.76	11.01	10.54	9.91	9.53	8.98	8.68
90	22.52	19.12	16.97	15.22	13.82	12.66	11.87	11.00	10.44	9.74	9.31	8.72	8.38	7.86	7.59
100	19.93	16.80	14.83	13.43	12.35	11.27	10.56	9.76	9.24	8.59	8.20	7.86	7.34	7.06	6.82
110	17.36	14.99	13.23	11.97	11.00	10.01	9.37	8.83	8.16	7.76	7.40	7.09	6.60	6.35	6.13
120	15.31	13.10	11.75	10.63	9.76	9.06	8.48	7.78	7.37	7.00	6.68	6.40	5.94	5.71	5.51

Yield Stress = 46 ksi, Residual Stress = 14.17 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	44.62	43.64	42.47	41.56	40.68	39.83	38.78	37.98	37.21	36.46	35.73	35.02	34.55	33.88	33.22
20	43.36	41.24	39.26	37.62	36.08	34.63	33.26	32.18	30.94	29.97	29.06	28.19	27.13	26.34	25.58
30	41.59	38.60	36.14	33.91	31.86	30.20	28.66	27.46	26.12	25.09	23.90	22.99	22.14	21.34	20.58
40	39.24	35.67	32.83	30.35	28.15	26.38	24.78	23.55	22.20	21.17	20.22	19.34	18.52	17.74	17.01
50	36.75	32.65	29.54	26.93	24.89	23.11	21.52	20.31	18.99	18.00	17.09	16.25	15.70	14.97	14.29
60	33.81	29.30	26.28	23.64	21.85	20.10	18.79	17.63	16.60	15.67	14.81	14.02	13.29	12.84	12.20
70	30.61	26.11	23.08	20.95	19.00	17.56	16.32	15.23	14.26	13.62	12.82	12.09	11.64	11.00	10.63
80	27.18	23.12	20.43	18.41	16.79	15.45	14.29	13.28	12.61	11.80	11.29	10.60	10.19	9.60	9.27
90	23.79	20.17	17.90	16.03	14.76	13.52	12.68	11.75	11.14	10.38	9.92	9.28	8.92	8.59	8.29
100	20.57	17.78	15.73	14.26	12.89	11.98	11.22	10.36	9.82	9.34	8.70	8.33	8.00	7.70	7.21
110	17.88	15.52	13.71	12.63	11.39	10.57	9.90	9.33	8.84	8.19	7.81	7.48	7.18	6.69	6.45
120	15.59	13.64	12.30	11.14	10.24	9.52	8.92	8.41	7.74	7.35	7.02	6.72	6.45	6.21	5.76

TABLE IV (Continued)

Yield Stress = 50 ksi, Residual Stress = 15 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	22	.24	.26	.28	.30
10	48.64	47.31	46.28	45.28	44.07	43.14	42.24	41.37	40.52	39.69	38.89	38.11	37.35	36.61	36.14
20	47.00	44.68	42.76	40.96	39.27	37.68	36.17	34.99	33.62	32.57	31.57	30.36	29.46	28.59	27.75
30	45.05	41.76	39.06	36.61	34.61	32.79	31.10	29.79	28.32	27.19	25.89	24.90	23.97	23.09	22.26
40	42.45	38.51	35.38	32.66	30.50	28.57	26.82	25.47	24.00	22.87	21.84	20.87	19.98	19.13	18.34
50	39.65	35.12	31.71	28.86	26.63	24.69	23.20	21.89	20.45	19.37	18.39	17.73	16.88	16.08	15.34
60	36.33	31.36	28.07	25.44	23.25	21.60	20.18	18.68	17.81	16.80	15.87	15.02	14.48	13.74	13.04
70	32.43	27.76	24.49	22.19	20.34	18.78	17.45	16.28	15.23	14.54	13.68	13.14	12.40	11.96	11.31
80	28.49	24.39	21.52	19.36	17.64	16.45	15.21	14.13	13.41	12.53	11.99	11.50	10.81	10.42	9.81
90	24.61	21.08	18.69	16.97	15.64	14.31	13.43	12.43	11.79	11.23	10.49	10.05	9.66	9.06	8.74
100	21.47	18.40	16.29	14.75	13.57	12.61	11.82	11.15	10.33	9.83	9.39	8.75	8.40	8.09	7.80
110	18.47	16.15	14.31	12.96	11.92	11.07	10.37	9.77	9.26	8.81	8.42	7.82	7.51	7.22	6.97
120	15.95	14.09	12.75	11.58	10.66	9.91	9.29	8.76	8.30	7.90	7.55	6.98	6.70	6.45	6.22

Yield Stress = 60 ksi, Residual Stress = 16.8 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	58.33	56.71	55.45	54.22	53.04	51.90	50.79	49.72	48.68	47.67	46.69	45.74	44.81	43.90	43.32
20	56.32	53.46	51.09	48.88	46.81	44.85	43.31	41.86	40.19	38.90	37.67	36.50	35.39	34.33	33.31
30	53.58	49.81	46.47	43.76	41.31	39.07	37.01	35.40	33.61	32.23	30.95	29.74	28.61	27.54	26.53
40	50.30	45.68	41.82	38.78	35.83	33.78	31.65	30.02	28.53	26.87	25.61	24.75	23.66	22.64	21.67
50	46.35	41.00	37.12	33.64	31.25	28.90	27.12	25.53	24.11	22.81	21.63	20.53	19.81	18.86	18.26
60	41.62	36.10	32.12	29.28	26.96	25.01	23.32	21.83	20.50	19.29	18.49	17.48	16.83	15.95	15.42
70	36.65	31.37	27.82	25.13	23.27	21.46	19.90	18.82	17.60	16.79	15.77	15.14	14.27	13.75	13.28
80	31.28	26.95	24.00	21.84	19.87	18.52	17.40	16.15	15.32	14.61	13.67	13.11	12.60	11.85	11.43
90	26.70	23.00	20.71	18.83	17.36	16.17	15.18	14.04	13.32	12.68	12.12	11.33	10.88	10.48	10.11
100	22.62	19.90	18.00	16.38	15.10	14.07	13.20	12.46	11.82	10.97	10.47	10.03	9.64	9.28	8.96
110	19.51	17.11	15.57	14.18	13.08	12.47	11.72	10.77	10.21	9.72	9.28	8.89	8.54	8.22	7.94
120	16.81	14.92	13.68	12.49	11.54	10.76	10.10	9.53	9.04	8.60	8.22	7.87	7.56	7.28	7.02

TABLE IV (Concluded)

Yield Stress = 70 ksi, Residual Stress = 18.2 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	68.01	66.44	64.58	63.12	61.71	60.35	59.03	57.75	56.52	55.67	54.50	53.36	52.26	51.18	50.48
20	65.61	62.18	59.34	56.70	54.58	52.25	50.41	48.68	46.69	45.15	43.69	42.30	40.98	39.72	38.86
30	62.28	57.74	54.08	50.48	47.55	45.24	42.79	40.88	39.11	37.12	35.60	34.17	33.18	31.92	30.72
40	58.23	52.62	47.98	44.36	41.22	38.78	36.28	34.34	32.59	30.99	29.51	28.14	27.21	26.01	24.88
50	52.87	46.74	42.12	38.38	35.57	32.83	30.74	28.89	27.58	26.07	24.69	23.41	22.58	21.46	20.77
60	46.72	40.52	36.24	32.96	30.29	28.04	26.10	24.74	23.21	21.82	20.90	19.73	18.99	18.32	17.35
70	39.78	34.50	30.87	27.85	25.76	23.72	22.30	21.09	19.70	18.79	17.98	16.90	16.25	15.67	14.78
80	33.50	29.31	26.14	23.81	21.99	20.51	19.27	17.87	16.96	16.16	15.45	14.47	13.90	13.39	12.92
90	28.09	24.80	22.14	20.51	18.95	17.67	16.61	15.69	14.55	13.86	13.24	12.69	12.20	11.75	11.34
100	23.89	21.08	19.23	17.57	16.25	15.16	14.24	13.46	12.77	12.17	11.63	11.15	10.72	10.32	9.97
110	19.95	18.15	16.36	15.32	14.20	13.27	12.48	11.80	11.21	10.68	10.22	9.80	9.42	9.07	8.76
120	17.26	15.64	14.17	13.36	12.41	11.61	10.94	10.35	9.83	9.37	8.96	8.59	8.26	7.95	7.68

Yield Stress = 100 ksi, Residual Stress = 20 ksi

Values of k:

L/r	.02	.04	.06	.08	.10	.12	.14	.16	.18	.2	.22	.24	.26	.28	.30
10	96.99	94.59	92.28	90.05	87.90	86.33	84.33	82.40	81.02	79.20	77.44	76.22	74.55	73.43	71.85
20	93.25	88.47	84.59	80.53	77.26	74.22	71.40	68.76	66.28	63.94	61.73	59.63	58.14	56.23	54.42
30	87.86	81.15	75.42	70.91	66.46	62.93	59.76	56.87	54.22	51.78	50.01	47.89	45.90	44.52	42.75
40	80.21	71.70	65.58	60.62	55.95	52.35	49.69	46.86	44.31	42.48	40.34	38.85	37.00	35.76	34.13
50	70.01	61.55	55.29	50.44	46.97	43.60	40.67	38.58	36.75	34.63	33.19	31.40	30.22	29.16	28.18
60	58.09	50.75	45.61	41.78	38.76	36.28	33.69	31.89	30.33	28.95	27.22	26.12	25.12	24.21	23.39
70	46.98	41.29	37.61	34.47	31.97	29.90	28.16	26.66	25.35	24.19	23.15	21.73	20.89	20.12	19.42
80	37.88	33.83	31.09	28.59	26.57	24.89	23.45	22.21	21.12	20.15	19.29	18.51	17.80	17.16	16.57
90	30.85	28.03	25.55	24.09	22.47	21.10	19.92	18.90	17.99	17.19	16.46	15.81	15.21	14.67	14.16
100	25.24	23.29	21.88	20.32	19.02	17.91	16.95	16.10	15.34	14.67	14.06	13.50	13.00	12.54	12.11
110	21.24	19.89	18.40	17.16	16.11	15.70	14.91	14.20	13.57	13.00	12.49	12.02	11.59	11.19	10.82
120	18.09	16.66	15.98	14.99	14.13	13.38	12.72	12.12	11.59	11.11	10.67	10.27	10.40	10.06	9.75

APPENDIX B

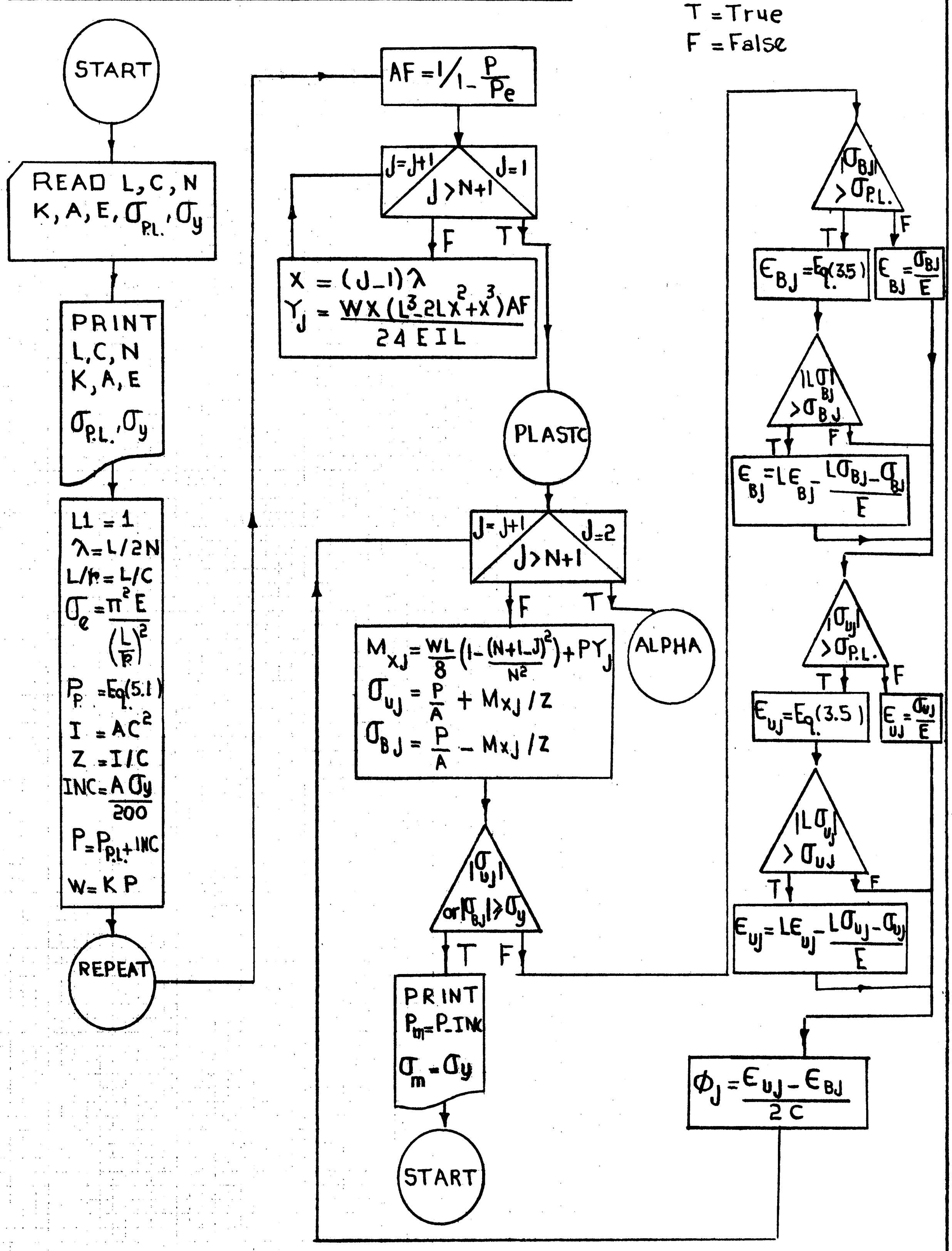
FLOW DIAGRAMS AND RELATED MAD PROGRAMS FOR COMPUTER ANALYSIS OF VARIOUS BEAM-COLUMN PROBLEMS

1. Ultimate strength of unrestrained beam-columns of simplified section under uniform lateral load. (Used for the printout of Table IV, Appendix A.)

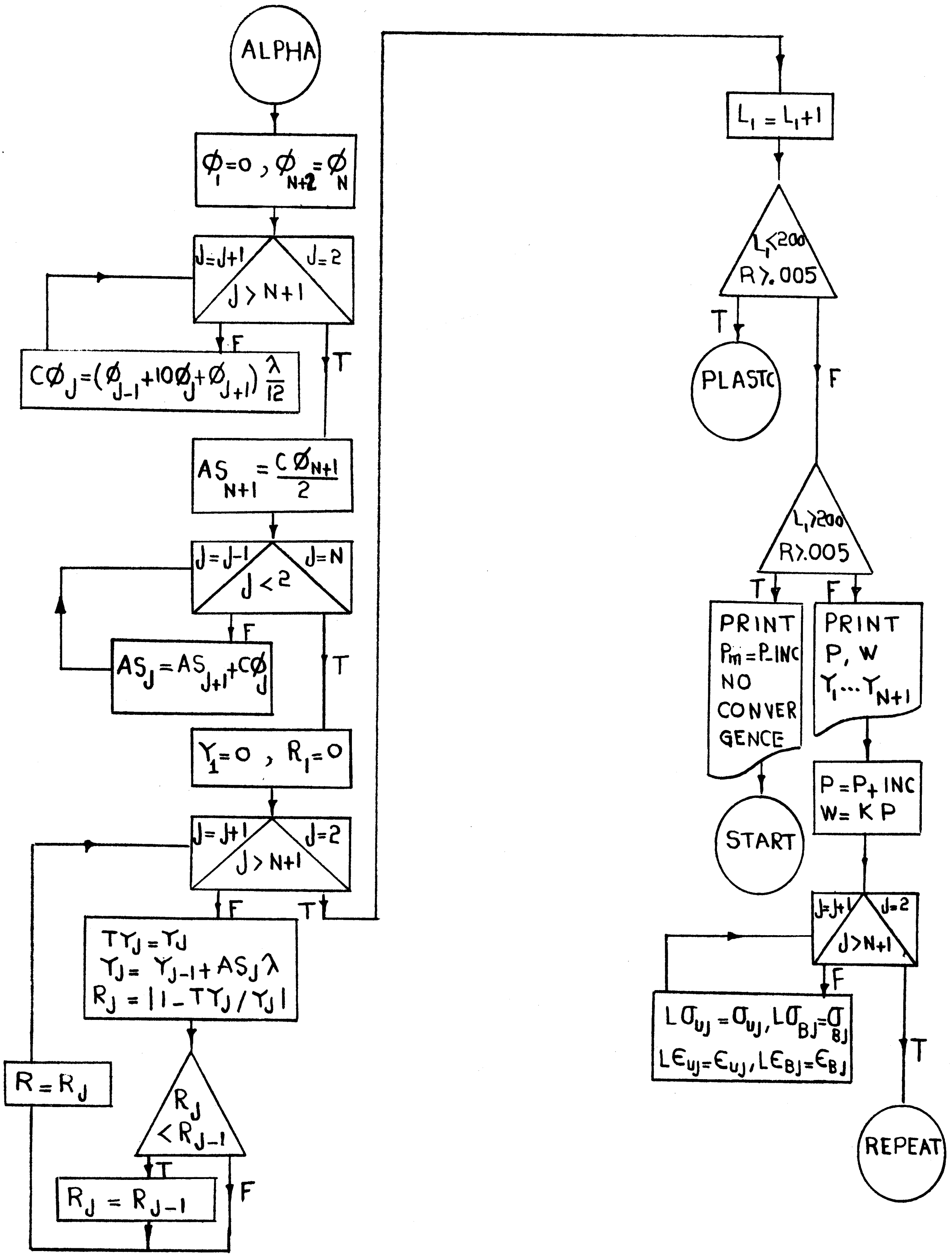
2. Ultimate strength of unrestrained beam-columns of simplified section in biplanar bending under uniform lateral load.

3. Ultimate strength of the restrained beam-column of the simplified section in planar bending under uniform lateral load.

FLOW DIAGRAM NO.1 – ULTIMATE STRENGTH OF UNRESTRAINED BEAM COLUMNS OF THE SIMPLIFIED SECTION IN PLANAR BENDING UNDER UNIFORM LATERAL LOAD W = KP.



FLOW DIAGRAM NO. 1 - CONT.



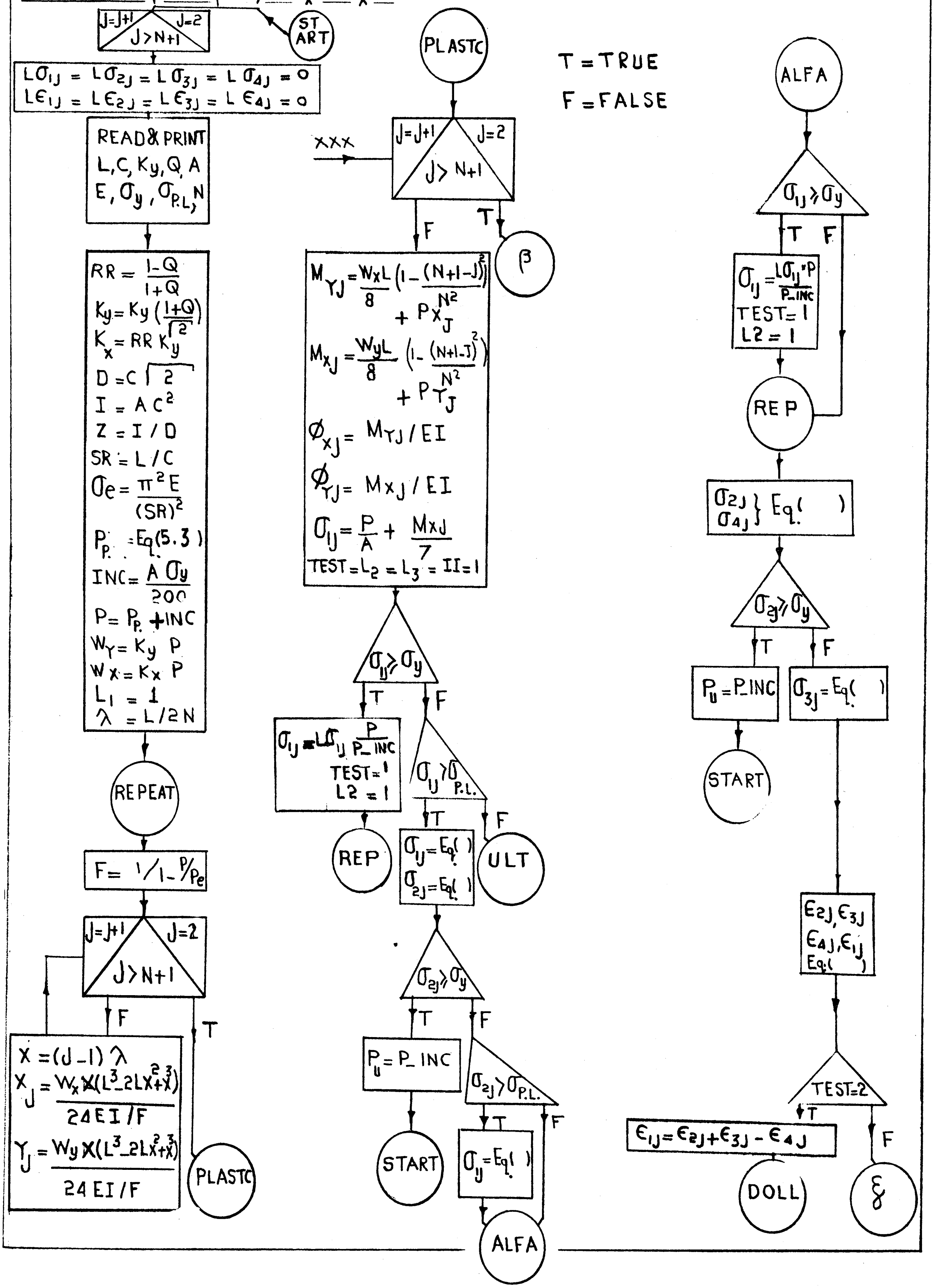
MAD PROGRAM NO. 1

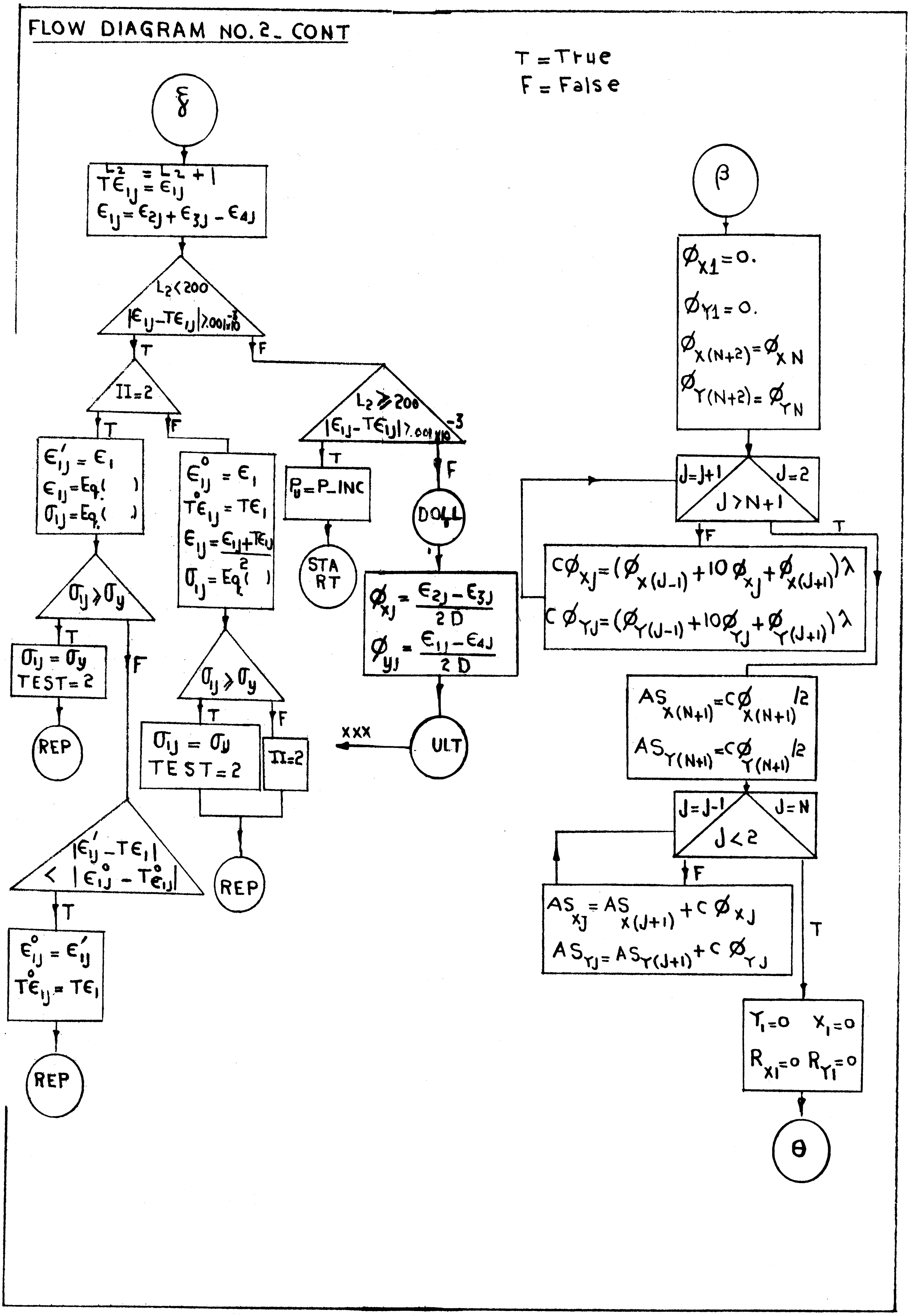
```
$COMPILE MAD ,EXECUTE,DUMP
          RPROGRAM FOR FINDING THE ULTIMATE STRENGTH OF UNRESTRAINED
          RBEAM COLUMNS IN PLANAR BENDING UNDER UNIFORM LATERAL LOAD,
          RFOR THE 4 POINT SECTION
          RTHIS PROGRAM REQUIRES AS A DATA-
          RL=LENGTH OF BEAM IN INCHES
          RC=HALF DEPTH OF SECTION IN INCHES
          RN=HALF NUMBER OF THE BEAM DIVISIONS
          RK=RATIO OF THE LATERAL LOAD TO THE AXIAL LOAD
          RA=AREA OF SECTION IN SQ.INCHES
          RE=MODULUS OF ELASTICITY IN KSI
          RSIGPL=PROPOTIONAL LIMIT SRESS IN KSI
          RSIGY=YIELD STRESSS IN KSI
START      READ DATA C,N,A,E,SIGY ,L,SIGPL,K
           PRINT COMMENT$1$
           PRINT RESULTS L,C,N,K,A,E,SIGPL,SIGY
           L1=1
           LAM=L/(2.*N)
           SR=L/C
           SIGE=286218.5276/(SR*SR)
           PE=A*SIGE
           H=SIGPL/SIGE
           G=-(H+1.+K*SR/8.)
           F=(1.-.03*SR*K/8.)
           R=(-G-SQRT.(G*G-4.*H*F))/(2.*F)
           SIGA=R*SIGE
           PPL=A*SIGA
           I=A*C*C
           WPL=(5.*K*PPL*L.P.3)/(384.*E*I)*(1./(1.-R))
           PRINT RESULTS SR,SIGE,PPL,WPL
           INC=A*SIGY/200.
           P=PPL+INC
           W=K*P
           PRINT FORMAT TYTLE
           VECTOR VALUES TYTLE=$44H  P/A   DEFLECTION IN INCHES AT NODAL
          1POINTS*$
REPEAT     AFAC=1./(1.-P/PE)
           THROUGH INIT,FOR J=1,1,J.G.(N+1)
           X=(J-1)*LAM
INIT       Y(J)=(W*X*(L.P.3-2.*L*X*X+X.P.3)/(24.*E*I*L))*AFAC
PLAST C    THROUGH ULT,FOR J=2,1,J.G.(N+1)
           MX(J)=W*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.+P*Y(J)
           SIGU=P/A+MX(J)*C/I
           SIGB=P/A-MX(J)*C/I
           WHENEVER .ABS.SIGU.GE.SIGY.OR..ABS.SIGB.GE.SIGY
           PRINT RESULTS P,W
           PRINT FORMAT EXCEED
           VECTOR VALUES EXCEED=$11H SIGM=SIGY *$
           TRANSFER TO START
           END OF CONDITIONAL
           WHENEVER .ABS.SIGU.G.SIGPL
           AA=SIGU/.ABS.SIGU
           SIGU=.ABS.SIGU
           EPSU=(1.-SQRT.(1.-(SIGU-SIGPL)/(SIGY-SIGPL)))/(E/(2.*(SIGY-
          1SIGPL)))+SIGPL/E
           EPSU=EPSU*AA
           OTHERWISE
           EPSU=SIGU/E
           END OF CONDITIONAL
```

```
           WHENEVER .ABS.SIGB.G.SIGPL
           BB=SIGB/.ABS.SIGB
           SIGB=.ABS.SIGB
           EPSB=(1.-SQRT.(1.-(SIGB-SIGPL)/(SIGY-SIGPL)))/(E/(2.*(SIGY-
          1SIGPL)))+SIGPL/E
           EPSB=EPSB*BB
           OTHERWISE
           EPSB=SIGB/E
           END OF CONDITIONAL
           WHENEVER .ABS.(LSIGU(J)).G.SIGU,EPSU=LEPSU(J)-(LSIGU(J)-SIGU)
          1/E
           WHENEVER .ABS.(LSIGB(J)).G.SIGB,EPSB=LEPSB(J)-(LSIGB(J)-SIGB)
          1/E
           CSIGU(J)=SIGU
           CSIGB(J)=SIGB
           CEPSU(J)=EPSU
           CEPSB(J)=EPSB
'ULT       PHI(J)=(EPSU-EPSB)/(2.*C)
           PHI(1)=0.0
           PHI(N+2)=PHI(N)
           THROUGH LOAP,FOR J=2,1,J.G.(N+1)
LOAP       CPHI(J)=(PHI(J-1)+10.*PHI(J)+PHI(J+1))*LAM/12.
           ASL(N+1)=CPHI(N+1)/2.
           THROUGH LOOP1,FOR J=N,-1,J.L.2
LOOP1      ASL(J)=ASL(J+1)+CPHI(J)
           Y(1)=0.0
           R(1)=0.0
           THROUGH LOOP2,FOR J=2,1,J.G.(N+1 )
           TRY(J)=Y(J)
           Y(J)=Y(J-1)+ASL(J)*LAM
           R(J)=.ABS.(1.-(TRY(J)/Y(J)))
           WHENEVER R(J).L.R(J-1),R(J)=R(J-1)
LOOP2      R=R(J)
           L1=L1+1
           WHENEVER R.G.(.005).AND.L1.L.200,TRANSFER TO PLAST C
           WHENEVER R.G.(.005).AND.L1.G.200
           PRINT RESULTS P,W
           PRINT FORMAT NOCON
           VECTOR VALUES NOCON=$39H NO CONVERGENCE BEYOND THE LAST P GIV
          1EN*$
           END OF CONDITIONAL
           WHENEVER R.LE.(.005).AND.L1.LE.200
           SIG=P/A
           PRINT FORMAT RESULT,SIG,Y(2)...Y(N+1)
           VECTOR VALUES RESULT=$F8.2,S2,4F8.4*$
           THROUGH REVERS ,FOR J=2,1,J.G.(N+1)
           LSIGU(J)=CSIGU(J)
           LSIGB(J)=CSIGB(J)
           LEPSU(J)=CEPSU(J)
REVERS     LEPSB(J)=CEPSB(J)
           P=P+INC
           W=K*P
           TRANSFER TO REPEAT
           END OF CONDITIONAL
           DIMENSION          Y(50),MX(50),PHI(50),CPHI(50),ASL(50),TRY
          1(50),R(50),
           DIMENSION LSIGU(50),LSIGB(50),LEPSU(50),LEPSB(50)
           DIMENSION CSIGU(50),CSIGB(50),CEPSU(50),CEPSB(50)
           INTEGER J,N,L1
```

```
             TRANSFER TO START
             END OF PROGRAM
$DATA
L=900.,C=15.,N=4,K=.04,A=40.,E=29000.,SIGPL=35.,SIGY=50.*
```

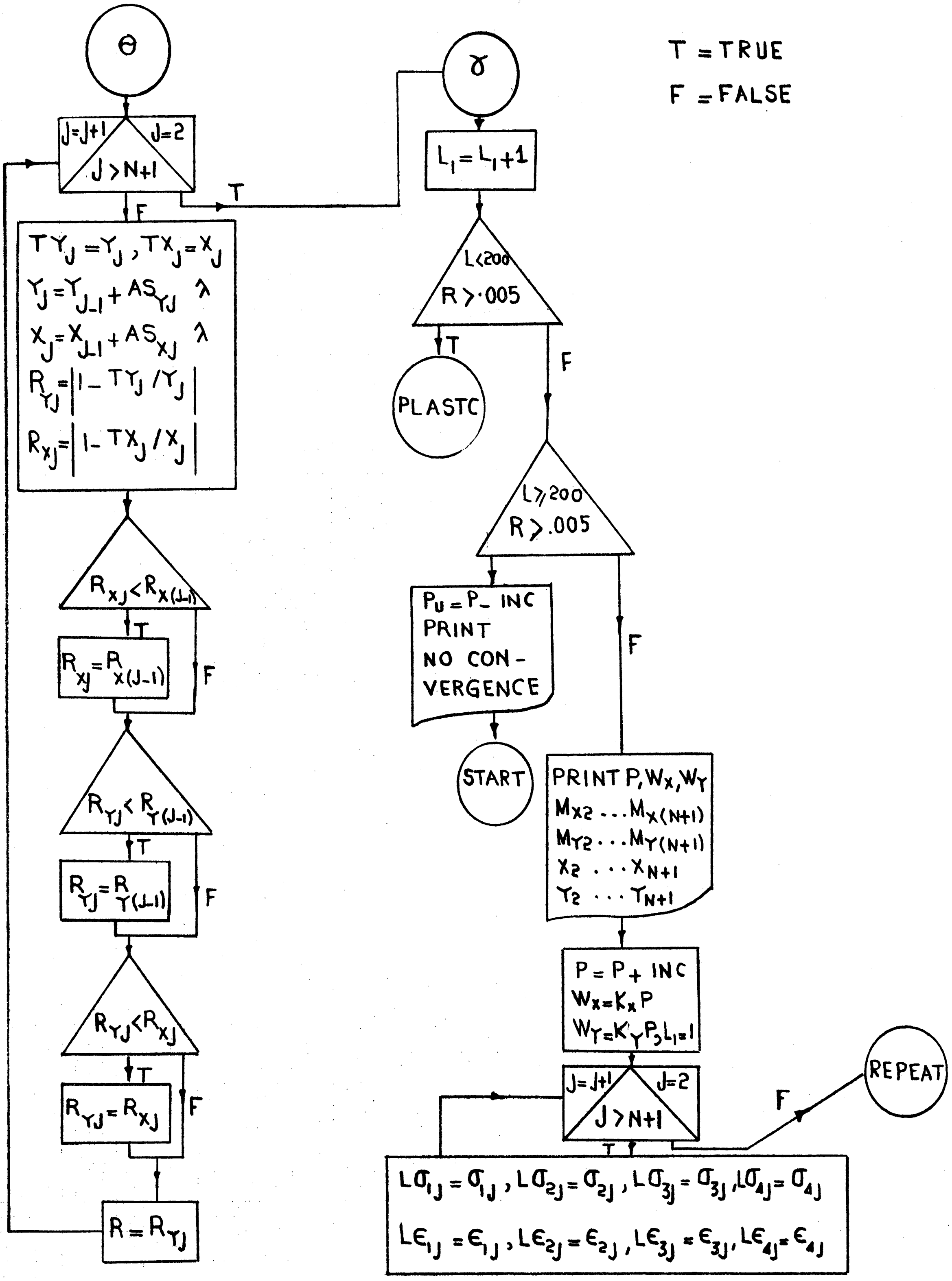
FLOW DIAGRAM NO.2 _ ULTIMATE STRENGTH OF UNRESTRAINED BEAM COLUMNS OF THE SIMPLIFIED SECTION IN BIBLANAR BENDING UNDER UNIFORM LATERAL LOADS $W_Y = K_Y P$, $W_X = K_X P$.





FLOW DIAGRAM NO. 2 - CONT
T = True
F = False
REP
STA RT
DOLL
ULT
XXX
β
θ

FLOW DIAGRAM NO. 2 – CONT.



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$COMPILE MAD ,EXECUTE                                                        002644    10/22/63      2 41 33.3 PM
$PUNCH LIBRARY,PUNCH OBJECT,FULL DUMP

MAD (21 OCT 1963 VERSION) PROGRAM LISTING ... ... ...

                PROGRAM FOR FINDING THE ULTIMATE STRENGTH OF UNRESTRAINED
                BEAM COLUMN IN BIPLANAR BENDING UNDER UNIFORM LOAD ,FOR
                THE 4 POINT SECTION
                THIS PROGRAM REQUIRES AS A DATA
                L=LENGTH OF BEAM IN INCHES
                C= HALF DEPTH OF SECTION IN INCHES
                N=HALF NUMBER OF BEAM DIVISIONS
                KY=RATIO OF VERTICAL LATERAL LOAD /AXIAL LOAD
                Q=RATIO OF VERTICAL LATERAL LOADS/HORIZONTAL LATERAL LOADS
                A=AREA OF SECTION IN SQ.INCHES
                E=MODULUS OF ELASTICITY IN KSI
                SIGPL=PROPOTIONAL LIMIT STRESS IN KSI
                SIGY=YIELD STRESS IN KSI
BEGIN            READ DATA   L,C,N,KY,Q,A,E,SIGY,SIGPL                                   *001
                 PRINT COMMENT$1$                                                        *002
                 PRINT RESULTS L,C,N,KY,Q,A,E,SIGY,SIGPL                                 *003
                 TEST=4                                                                  *004
                 THROUGH REVER1,FOR J=2,1,J.G.N+1                                        *005
                 CSIG1(J)=0.0                                                            *006
                 CSIG2(J)=0.0                                                            *007
                 CSIG3(J)=0.0                                                            *008
                 CSIG4(J)=0.0                                                            *009
                 CEPS1(J)=0.0                                                            *010
                 CEPS2(J)=0.0                                                            *011
                 CEPS3(J)=0.0                                                            *012
                 CEPS4(J)=0.0                                                            *013
                 LSIG1(J)=0.0                                                            *014
                 LSIG2(J)=0.0                                                            *015
                 LSIG3(J)=0.0                                                            *016
                 LSIG4(J)=0.0                                                            *017
                 LEPS1(J)=0.0                                                            *018
                 LEPS2(J)=0.0                                                            *019
                 LEPS3(J)=0.0                                                            *020
REVER1           LEPS4(J)=0.0                                                            *021
                 INTEGER L3                                                              *022
                 RR=(1.-Q)/(1.+Q)                                                        *023
                 KY=KY*(1.+Q)/(SQRT.(2.))                                                *024
                 KX=RR*KY                                                                *025
                 PRINT RESULTS RR,KY,KX                                                  *026
                 D=C*SQRT.(2.)                                                           *027
                 LAM=L/(2.*N)                                                            *028
                 SR=L*SQRT.(2.)/D                                                        *029
                 I=A*D*D/2.                                                              *030
                 Z=I/D                                                                   *031
                 PE=A*SIGE                                                               *033

                 ALL=(SIGPL+SIGE+PE*KY*L/(8.*Z)-SQRT.((SIGPL+SIGE+PE*KY*L/(8.*           *034
               1 Z)).P.2-4.*SIGE*SIGPL*(1.-.03*KY*L*A/(8.*Z))))/(2.*SIGE*(1.-            *034
               2 .03*KY*L*A /(8.*Z)))                                                    *034
                 PPL=ALL*PL                                                              *035
                 PRINT RESULTS D,LAM,SR,I,PE,PPL                                         *036
                 INC=A*SIGY/200.                                                         *037
```

```
         P=PPL+INC                                                          *038
         WY=KY*P                                                            *039
         WX=KX*P                                                            *040
         L1=1                                                               *041
REPEAT   AFAC=1./(1.-P/PE)                                                  *042
         THROUGH INIT,FOR J=1,1,J.G.(N+1)                                   *043
         X=(J-1)*LAM                                                        *044
         X(J)=(WX*X*(L.P.3-2.*L*X*X+X.P.3)/(24.*E*I*L))*AFAC                *045
INIT     Y(J)=(WY*X*(L.P.3-2.*L*X*X+X.P.3)/(24.*E*I*L))*AFAC                *046
PLASTC   THROUGH ULT,FOR J=2,1,J.G.(N+1)                                    *047
         II=1                                                               *048
         INTEGER II                                                         *049
         TEST=1                                                             *050
         L2=1                                                               *051
         L3=1                                                               *052
         MY(J)=WX*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.+P*X(J)              *053
         MX(J)=WY*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.+P*Y(J)              *054
         PHIX(J)=MY(J)/(E*I)                                                *055
         PHIY(J)=MX(J)/(E*I)                                                *056
         SIG1=P/A+MX(J)/Z                                                   *057
         WHENEVER SIG1.LE.SIGPL                                             *058
         SIG4=P/A-MX(J)/Z                                                   *059
         SIG2=P/A+MY(J)/Z                                                   *060
         SIG3=P/A-MY(J)/Z                                                   *061
         EPS1=SIG1/E                                                        *062
         EPS2=SIG2/E                                                        *063
         EPS3=SIG3/E                                                        *064
         EPS4=SIG4/E                                                        *065
         TRANSFER TO DOLL                                                   *066
         END OF CONDITIONAL                                                 *067
         WHENEVER SIG1.GE.SIGY                                              *068
         SIG1=LSIG1(J)*P/(P-INC)                                            *069
         WHENEVER SIG1.G.SIGY,SIG1=SIGY                                     *070
         L2=1                                                               *071
         TEST=1                                                             *072
         TRANSFER TO REP                                                    *073
         END OF CONDITIONAL                                                 *074
         WHENEVER SIG1.G.SIGPL                                              *075
         O=(4.*P/A+8.*MX(J)/(A*D)-2.*SIGY+SIGPL)/3.                         *076
         SIG1=(-4.*(SIGY-SIGPL)/9.+2.*O +SQRT.((-4.*(SIGY-SIGPL)/9.+2.      *077
       1 *O).P.2-4.*(O*O-4.*SIGY*(SIGY-SIGPL)/9.)))/2.                      *077
         SIG2=-SIG1+2.*P/A+(MX(J)+MY(J))/(.5*A*D)                           *078
         WHENEVER SIG2.GE.SIGY                                              *079
         PRINT FORMAT END                                                   *080
         VECTOR VALUES END=$20H SIGM = SIG2 = SIGY *$                       *081
         THROUGH KAP,FOR J=2,1,J.G.N+1                                      *082
KAP      PRINT FORMAT ABCD,LSIG1(J),LSIG2(J),LSIG3(J),LSIG4(J),LEPS1        *083
       1 (J),LEPS2(J),LEPS3(J),LEPS4(J)                                     *083
         TRANSFER TO START                                                  *084
         END OF CONDITIONAL                                                 *085
         WHENEVER SIG2.G.SIGPL                                              *086
         S=P/A+(3.*MX(J)-MY(J))/(A*D)                                       *087
TOP      TSIG1=SIG1                                                         *088
       1 (SIGY-TSIG1))*SQRT.(SIGY-SIGPL)                                    *089

         WHENEVER SIG1.GE.SIGY                                              *090
         SIG1=LSIG1(J)*P/(P-INC)                                            *091
         WHENEVER SIG1.G.SIGY,SIG1=SIGY                                     *092
         L2=1                                                               *093
         TEST=1                                                             *094
```

```
               TRANSFER TO REP                                                   *095
               END OF CONDITIONAL                                                *096
               L3=L3+1                                                           *097
               WHENEVER .ABS.(SIG1-TSIG1).G..005.AND.L3.G.200                    *098
               TRANSFER TO DONE                                                  *099
               END OF CONDITIONAL                                                *100
               WHENEVER .ABS.(SIG1-TSIG1).G.0.005.AND.L3.LE.200                  *101
               SIG1=(SIG1+TSIG1)/2.                                              *102
               WHENEVER SIG1.GE.SIGY                                             *103
               SIG1=LSIG1(J)*P/(P-INC)                                           *104
               WHENEVER SIG1.G.SIGY,SIG1=SIGY                                    *105
               L2=1                                                              *106
               TEST=1                                                            *107
               TRANSFER TO REP                                                   *108
               END OF CONDITIONAL                                                *109
               TRANSFER TO TOP                                                   *110
               END OF CONDITIONAL                                                *111
               END OF CONDITIONAL                                                *112
 REP           SIG4=SIG1-4.*MX(J)/(A*D)                                          *113
               SIG2=-SIG1+2.*P/A+(MX(J)+MY(J))/(0.5*A*D)                         *114
               WHENEVER SIG2.GE.SIGY                                             *115
               PRINT FORMAT END                                                  *116
               VECTOR VALUES END=$20H SIGM = SIG2 = SIGY *$                      *117
               THROUGH KBP,FOR J=2,1,J.G.N+1                                     *118
 KBP           PRINT FORMAT ABCD,LSIG1(J),LSIG2(J),LSIG3(J),LSIG4(J),LEPS1       *119
             1 (J),LEPS2(J),LEPS3(J),LEPS4(J)                                    *119
               TRANSFER TO START                                                 *120
               END OF CONDITIONAL                                                *121
               SIG3=-SIG1+2.*P/A+(MX(J)-MY(J))/(0.5*A*D)                         *122
               EPS2=F.(SIGY,SIGPL,SIG2,E,LEPS2(J),LSIG2(J))                      *123
               EPS3=F.(SIGY,SIGPL,SIG3,E,LEPS3(J),LSIG3(J))                      *124
               EPS4=F.(SIGY,SIGPL,SIG4,E,LEPS4(J),LSIG4(J))                      *125
               EPS1=F.(SIGY,SIGPL,SIG1,E,LEPS1(J),LSIG1(J))                      *126
               WHENEVER TEST.E.2                                                 *127
               EPS1=EPS2+EPS3-EPS4                                               *128
               TRANSFER TO DOLL                                                  *129
               END OF CONDITIONAL                                                *130
               TEPS1=EPS1                                                        *131
               EPS1=EPS2+EPS3-EPS4                                               *132
               L2=L2+1                                                           *133
               WHENEVER .ABS.((EPS1-TEPS1)*1000).G..0010.AND.L2.L.800            *134
               WHENEVER II.E.2,TRANSFER TO LOOP13                                *135
               OEPS1=EPS1                                                        *136
               OTEPS1=TEPS1                                                      *137
               EPS1=(OEPS1+OTEPS1)/2.                                            *138
               SIG1=E*EPS1-((E*EPS1-SIGPL).P.2)/(4.*(SIGY-SIGPL))                *139
               WHENEVER SIG1.L.LSIG1(J)                                          *140
               SIG1=LSIG1(J)-E*(LEPS1(J)-EPS1)                                   *141
               END OF CONDITIONAL                                                *142
               WHENEVER SIG1.GE.SIGY                                             *143
               SIG1=SIGY                                                         *144
               TEST=2                                                            *145
               TRANSFER TO REP                                                   *146
               END OF CONDITIONAL                                                *147
               II=2                                                              *148
               TRANSFER TO REP                                                   *149
 LOOP13        EPS11=EPS1                                                        *150
               EPS1=(OTEPS1-OEPS1*(TEPS1-OTEPS1)/(EPS1-OEPS1))/(1.-(TEPS1-       *151
             1 OTEPS1)/(EPS1-OEPS1))                                             *151
               SIG1=E*EPS1-((E*EPS1-SIGPL).P.2)/(4.*(SIGY-SIGPL))                *152
```

```
          WHENEVER SIG1.L.LSIG1(J)                                          *153
          SIG1=LSIG1(J)-E*(LEPS1(J)-EPS1)                                   *154
          END OF CONDITIONAL                                                *155
          WHENEVER SIG1.GE.SIGY                                             *156
          SIG1=SIGY                                                         *157
          TEST=2                                                            *158
          TRANSFER TO REP                                                   *159
          END OF CONDITIONAL                                                *160
          WHENEVER .ABS.(EPS11-TEPS1).L..ABS.(OEPS1-OTEPS1)                 *161
          OEPS1=EPS11                                                       *162
          OTEPS1=TEPS1                                                      *163
          END OF CONDITIONAL                                                *164
          TRANSFER TO REP                                                   *165
          END OF CONDITIONAL                                                *166
          WHENEVER .ABS.((EPS1-TEPS1)*1000).G..0010.AND.L2.GE.800           *167
          PRINT RESULTS P,WX,WY,J,L2                                        *168
          PRINT FORMAT ABC                                                  *169
          VECTOR VALUES ABC=$37H NO EQUILIBRUIM OF STRESSES POSSIBLE *$     *170
          THROUGH KCP,FOR J=2,1,J.G.N+1                                     *171
KCP       PRINT FORMAT ABCD,LSIG1(J),LSIG2(J),LSIG3(J),LSIG4(J),LEPS1       *172
         1 (J),LEPS2(J),LEPS3(J),LEPS4(J)                                   *172
          TRANSFER TO START                                                 *173
          END OF CONDITIONAL                                                *174
DOLL      CSIG1(J)=SIG1                                                     *175
          CSIG2(J)=SIG2                                                     *176
          CSIG3(J)=SIG3                                                     *177
          CSIG4(J)=SIG4                                                     *178
          CEPS1(J)=EPS1                                                     *179
          CEPS2(J)=EPS2                                                     *180
          CEPS3(J)=EPS3                                                     *181
          CEPS4(J)=EPS4                                                     *182
          PHIX(J)=(EPS2-EPS3)/(2.*D)                                        *183
          PHIY(J)=(EPS1-EPS4)/(2.*D)                                        *184
ULT       END OF CONDITIONAL                                                *185
          PHIX(1)=0.0                                                       *186
          PHIY(1)=0.0                                                       *187
          PHIX(N+2)=PHIX(N)                                                 *188
          PHIY(N+2)=PHIY(N)                                                 *189
          THROUGH LOAP,FOR J=2,1,J.G.(N+1)                                  *190
          CPHIX(J)=(PHIX(J-1)+10.*PHIX(J)+PHIX(J+1))*LAM/12.                *191
LOAP      CPHIY(J)=(PHIY(J-1)+10.*PHIY(J)+PHIY(J+1))*LAM/12.                *192
          ASLX(N+1)=CPHIX(N+1)/2.                                           *193
          ASLY(N+1)=CPHIY(N+1)/2.                                           *194
          THROUGH LOOP1,FOR J=N,-1,J.L.2                                    *195
          ASLX(J)=ASLX(J+1)+CPHIX(J)                                        *196
LOOP1     ASLY(J)=ASLY(J+1)+CPHIY(J)                                        *197
          Y(1)=0.0                                                          *198
          X(1)=0.0                                                          *199
          RX(1)=0.0                                                         *200
          RY(1)=0.0                                                         *201
          THROUGH LOOP2,FOR J=2,1,J.G.(N+1)                                 *202
          TRYY(J)=Y(J)                                                      *203
          TRYX(J)=X(J)                                                      *204
          Y(J)=Y(J-1)+ASLY(J)*LAM                                           *205
          X(J)=X(J-1)+ASLX(J)*LAM                                           *206
          RX(J)=.ABS.(1.-(TRYX(J)/X(J)))                                    *207
          RY(J)=.ABS.(1.-(TRYY(J)/Y(J)))                                    *208
          WHENEVER RX(J).L.RX(J-1),RX(J)=RX(J-1)                            *209
          WHENEVER RY(J).L.RX(J),RY(J)=RX(J)                                *211
```

```
LOOP2      R=RY(J)                                                      *212
           L1=L1+1                                                      *213
           WHENEVER R.G.(.005).AND.L1.L.200,TRANSFER TO PLASTC          *214
           WHENEVER R.G.(.005).AND.L1.GE.200,TRANSFER TO DONE           *215
           PRINT FORMAT RESULT,P,X(2)...X(N+1),Y(2)...Y(N+1)            *216
           VECTOR VALUES RESULT=$1F8.2,8F10.4*$                         *217
           P=P+INC                                                      *218
           WX=KX*P                                                      *219
           WY=KY*P                                                      *220
           THROUGH REVERS,FOR J=2,1,J.G.N+1                             *221
           LSIG1(J)=CSIG1(J)                                            *222
           LSIG2(J)=CSIG2(J)                                            *223
           LSIG3(J)=CSIG3(J)                                            *224
           LSIG4(J)=CSIG4(J)                                            *225
           LEPS1(J)=CEPS1(J)                                            *226
           LEPS2(J)=CEPS2(J)                                            *227
           LEPS3(J)=CEPS3(J)                                            *228
REVERS     LEPS4(J)=CEPS4(J)                                            *229
           L1=1                                                         *230
           TRANSFER TO REPEAT                                           *231
DONE       PRINT RESULTS P,WX,WY                                        *232
           PRINT FORMAT CON                                             *233
           VECTOR VALUES CON=$16H NO CONVERGENCE *$                     *234
           THROUGH KDP,FOR J=2,1,J.G.N+1                                *235
KDP        PRINT FORMAT ABCD,LSIG1(J),LSIG2(J),LSIG3(J),LSIG4(J),LEPS1  *236
         1 (J),LEPS2(J),LEPS3(J),LEPS4(J)                               *236
START      TEST=4                                                       *237
           DIMENSION X(100),MY(100),PHIX(100),CPHIX(100),ASLX(100),TRYX *238
         1 (100),RX(100),Y(100),MX(100),PHIY(100),CPHIY(100),ASLY(100), *238
         2 TRYY(100),RY(100)                                            *238
           VECTOR VALUES ABCD=$4F10.2,4E12.5*$                          *239
           DIMENSION LEPS1(50),LEPS2(50),LEPS3(50),LEPS4(50),LSIG1(50), *240
         1 LSIG2(50),LSIG3(50),LSIG4(50)                                *240
           DIMENSION CEPS1(50),CEPS2(50),CEPS3(50),CEPS4(50),CSIG1(50), *241
         1 CSIG2(50),CSIG3(50),CSIG4(50)                                *241
           INTEGER L1,L2,J,N                                            *242
           INTEGER TEST                                                 *243
           TRANSFER TO BEGIN                                            *244
           END OF PROGRAM                                               *245
```

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      $COMPILE MAD ,PUNCH OBJECT                                           002844   10/22/63      2 41 47.9 PM

MAD (21 OCT 1963 VERSION) PROGRAM LISTING ... ... ...

           EXTERNAL FUNCTION (SIGY,SIGPL,SIG,E,LEPS,LSIG)                  *001
           ENTRY TO F.                                                     *002
           WHENEVER SIG.LE.SIGPL                                           *003
           EPS=SIG/E                                                       *004
           OR WHENEVER .ABS.LSIG.G..ABS.SIG                                *005
           AA=SIG/.ABS.SIG                                                 *006
           EPS=(.ABS.LEPS-(.ABS.LSIG-.ABS.SIG)/E)*AA                       *007
           OTHERWISE                                                       *008
           AA=SIG/.ABS.SIG                                                 *009
           SIG=.ABS.SIG                                                    *010
           EPS=(1.-SQRT.(1.-(SIG-SIGPL)/(SIGY-SIGPL)))/(E/(2.*(SIGY-       *011
         1 SIGPL)))+SIGPL/E                                                *011
           EPS=EPS*AA                                                      *012
           END OF CONDITIONAL                                              *013
           FUNCTION RETURN EPS                                             *014
           END OF FUNCTION                                                 *015
```

L = 900.000000, C = 15.000000, N = 4, KY = .042426

Q = .333333, A = 40.000000, E = 2.900000E 04, SIGY = 50.000000

SIGPL = 35.000000

RR = .500000, KY = .040000, KX = .020000

D = 21.213203, LAM = 112.500000, SR = 60.000000, I = 8999.999878

PE = 3180.205841, PPL = 879.743546

889.74	.3480	.6397	.8323	.8994	.6960	1.2793	1.6646	1.7990
899.74	.3534	.6497	.8453	.9136	.7069	1.2995	1.6909	1.8274
909.74	.3589	.6598	.8585	.9278	.7180	1.3200	1.7176	1.8563
919.74	.3645	.6700	.8718	.9421	.7294	1.3408	1.7449	1.8859
929.74	.3700	.6803	.8852	.9566	.7409	1.3621	1.7727	1.9161
939.74	.3757	.6906	.8987	.9712	.7527	1.3839	1.8012	1.9470
949.74	.3814	.7011	.9123	.9860	.7646	1.4060	1.8303	1.9785
959.74	.3871	.7117	.9261	1.0009	.7778	1.4305	1.8625	2.0136
969.74	.3929	.7223	.9400	1.0159	.7906	1.4543	1.8938	2.0476
979.74	.3987	.7331	.9540	1.0311	.8039	1.4788	1.9262	2.0827
989.74	.4046	.7439	.9682	1.0464	.8175	1.5042	1.9596	2.1190
999.74	.4106	.7549	.9824	1.0618	.8316	1.5304	1.9942	2.1566
1009.74	.4166	.7659	.9968	1.0774	.8462	1.5576	2.0300	2.1955
1019.74	.4226	.7771	1.0114	1.0931	.8630	1.5888	2.0712	2.2403
1029.74	.4287	.7884	1.0260	1.1090	.8790	1.6187	2.1107	2.2832
1039.74	.4349	.7997	1.0409	1.1250	.8956	1.6498	2.1518	2.3279
1049.74	.4411	.8112	1.0558	1.1412	.9130	1.6822	2.1947	2.3746
1059.74	.4474	.8228	1.0710	1.1576	.9311	1.7161	2.2395	2.4233
1069.74	.4539	.8347	1.0864	1.1743	.9500	1.7516	2.2865	2.4746
1079.74	.4605	.8469	1.1024	1.1917	.9720	1.7928	2.3412	2.5340
1089.74	.4674	.8596	1.1190	1.2097	.9934	1.8331	2.3946	2.5922
1099.74	.4745	.8728	1.1364	1.2285	1.0160	1.8755	2.4510	2.6537
1109.74	.4819	.8865	1.1544	1.2481	1.0397	1.9202	2.5104	2.7185
1119.74	.4896	.9008	1.1733	1.2687	1.0654	1.9687	2.5749	2.7889
1129.74	.4978	.9161	1.1936	1.2907	1.0958	2.0259	2.6512	2.8723
1139.74	.5065	.9323	1.2150	1.3141	1.1267	2.0843	2.7291	2.9572
1149.74	.5158	.9497	1.2381	1.3392	1.1606	2.1483	2.8146	3.0509
1159.74	.5259	.9685	1.2631	1.3665	1.1980	2.2190	2.9094	3.1546
1169.74	.5372	.9898	1.2914	1.3974	1.2451	2.3083	3.0296	3.2866
1179.74	.5500	1.0139	1.3237	1.4329	1.3013	2.4151	3.1737	3.4452
1189.74	.5658	1.0437	1.3640	1.4773	1.3774	2.5602	3.3710	3.6636

P = 1199.743546, WX = 23.994652, WY = 47.989268, J = 5

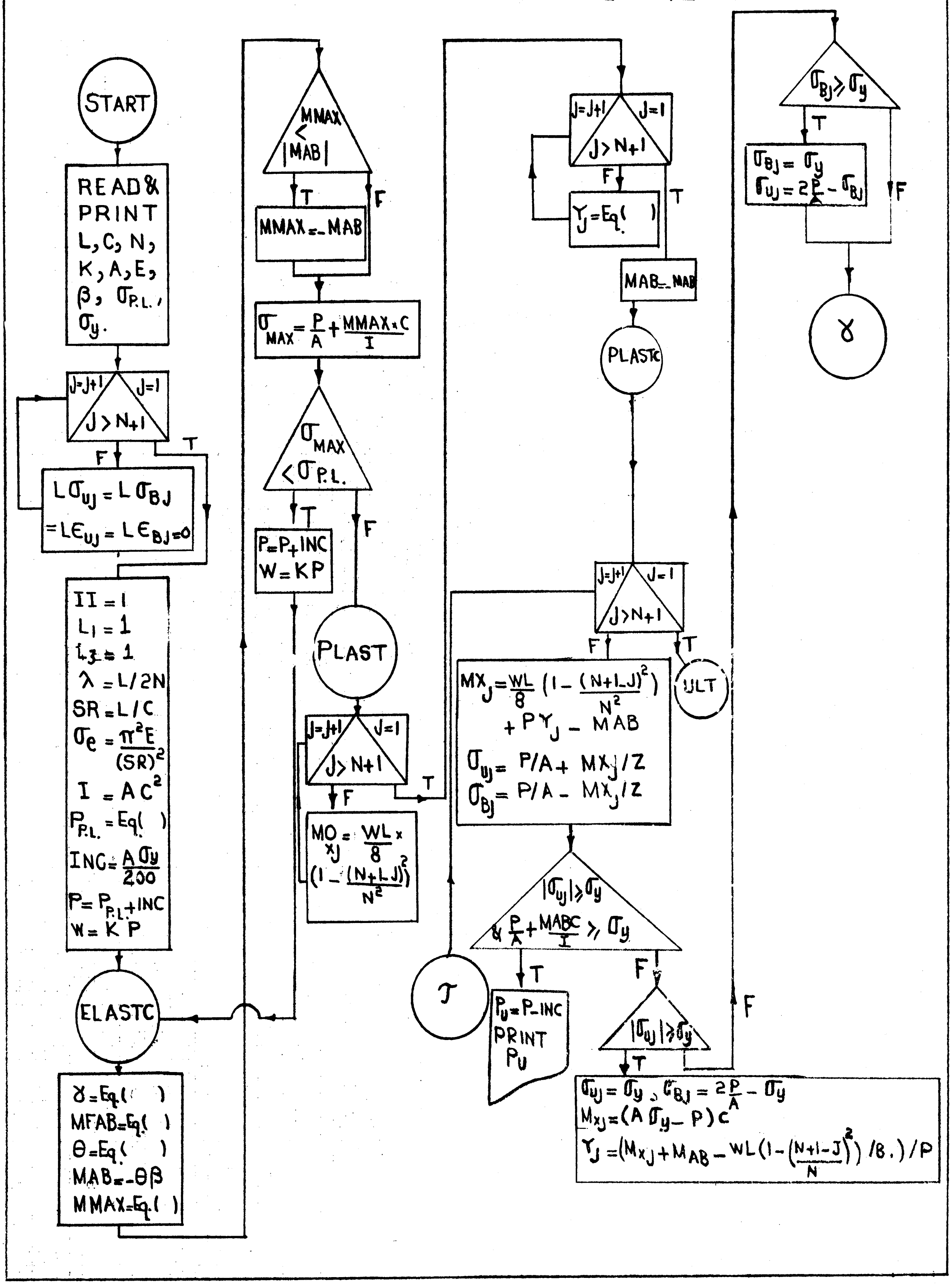
L2 = 800

NO EQUILIBRUIM OF STRESSES POSSIBLE

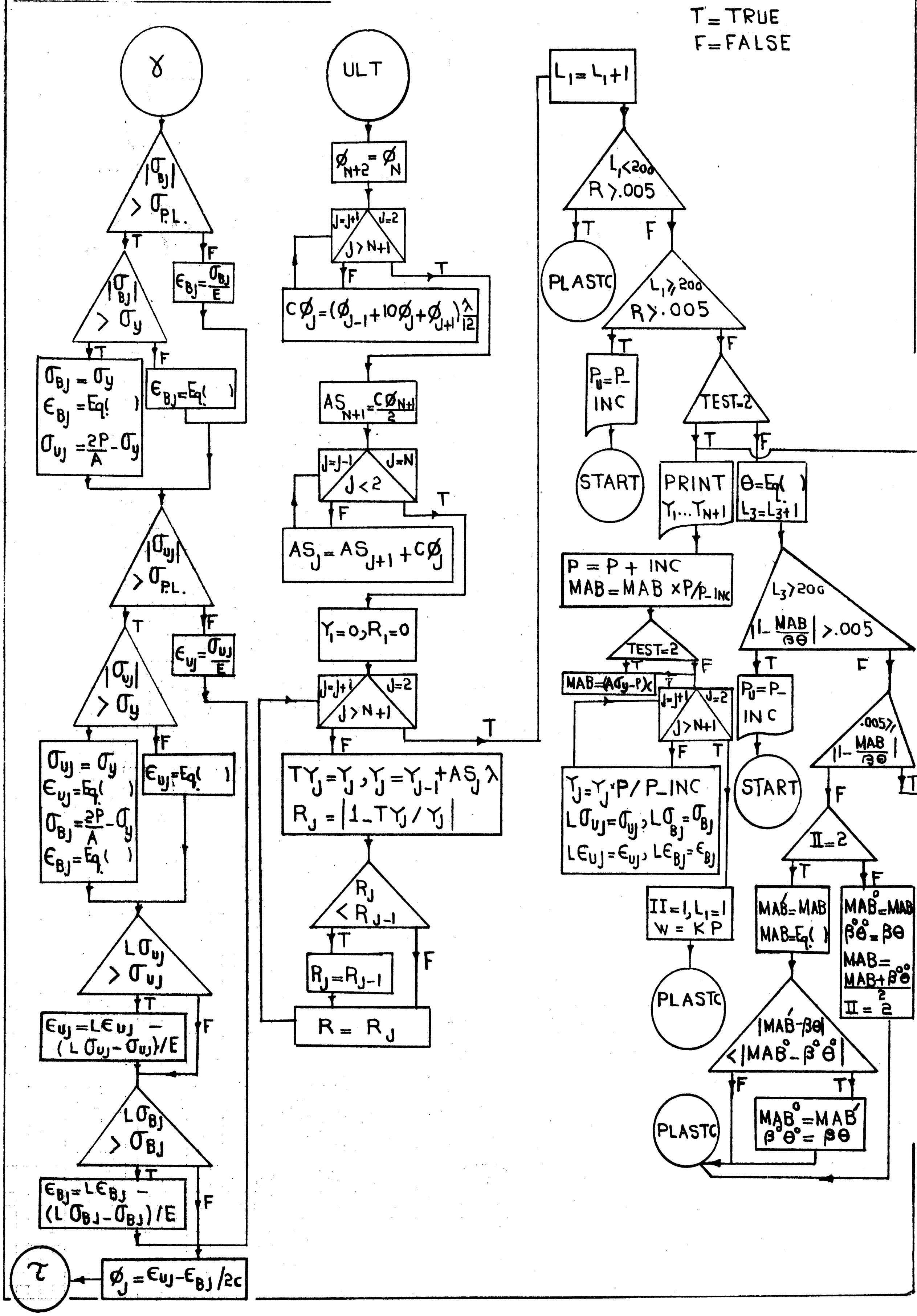
39.03	34.17	25.48	20.30	.13569E-02	.11782E-02	.87851E-03	.69984E-03
45.61	38.15	22.83	12.38	.16822E-02	.13218E-02	.78734E-03	.42692E-03
49.08	41.39	21.91	6.59	.19858E-02	.14575E-02	.75563E-03	.22725E-03
49.87	42.92	22.02	4.17	.21460E-02	.15304E-02	.75928E-03	.14372E-03

FLOW DIAGRAM NO. 3_ ULTIMATE STRENGTH OF THE RESTRAINED BEAM COLUMN OF THE SIMPLIFIED SECTION IN PLANAR BENDING UNDER UNIFORM LATERAL LOAD W=KP.

T = TRUE F = FALSE



FLOW DIAGRAM NO. 3_ CONT.



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    $COMPILE MAD ,EXECUTE,FULL DUMP  ,PRINT OBJECT                          004986   11/19/63     8 51 26.6 PM
    $PUNCH LIBRARY,PUNCH OBJECT,FULL DUMP

MAD (23 OCT 1963 VERSION) PROGRAM LISTING ... ... ...


                PROGRAM FOR FINDING THE ULTIMATE STRENGTH OF RESTRAINED
                BEAM COLUMN OF THE FOUR POINT SECTION IN PLANAR BENDING.
                THIS PROGRAM REQUIRES AS A DATA -
                L=LENGTH OF BEAM IN INCHES
                C=HALF DEPTH OF SECTION IN INCHES
                N=HALF NUMBER OF BEAM DIVISIONS
                K=RATIO OF THE LATERAL LOAD /AXIAL LOAD
                A=AREA OF SECTION IN SQ.INCHES
                E=MODULUS OF ELASTICITY IN KSI
                BETA=STIFFNESS OF ALL MEMBERS AT END JOINT EXCEPT THE BEAM
                IN KIPS INCH /RADIAN
                SIGPL=PROPORTIONAL LIMIT STRESS IN KSI
                SIGY=YIELD STRESS IN KSI
    BEGIN        READ DATA L,C,N,K,A,E,BETA,SIGPL,SIGY                              *001
                 PRINT COMMENT $1$                                                  *002
                 PRINT RESULTS L,C,N,K,A,E,BETA,SIGPL,SIGY                          *003
                 TEST=1                                                             *004
                 I=A*C*C                                                            *005
                 THROUGH REVER1,FOR J=1,1,J.G.(N+1)                                 *006
                 CSIGU(J)=0.0                                                       *007
                 CSIGB(J)=0.0                                                       *008
                 CEPSU(J)=0.0                                                       *009
                 CEPSB(J)=0.0                                                       *010
                 LSIGU(J)=0.0                                                       *011
                 LSIGB(J)=0.0                                                       *012
                 LEPSU(J)=0.0                                                       *013
    REVER1       LEPSB(J)=0.0                                                       *014
                 II=1                                                               *015
                 L1=1                                                               *016
                 L3=1                                                               *017
                 INTEGER L3                                                         *018
                 LAM=L/(2.*N)                                                       *019
                 SR=L/C                                                             *020
                 SIGE=286218.5276/(SR*SR)                                           *021
                 H=SIGPL/SIGE                                                       *022
                 G=-(H+1.+K*SR/8.)                                                  *023
                 F=(1.-.03*SR*K/8.)                                                 *024
                 R=(-G-SQRT.(G*G-4.*H*F))/(2.*F)                                    *025
                 PPL=A*R*SIGE                                                       *026
                 INC=A*SIGY/200.                                                    *027
                 P=PPL+INC                                                          *028
                 W=K*P                                                              *029
    ELAST C      ALFA=SQRT.(P*L*L/(E*I))                                            *030
                 F=ALFA/2.                                                          *031
                 F2=ALFA                                                            *032
                 MFAB=-W*L*(1.-2.*SIN.(F)/(COS.(F)*ALFA))/(2.*ALFA*SIN.(F)/         *033
               1 COS.(F))                                                           *033
                 S1=(1.-ALFA*COS.(F2)/SIN.(F2))/(2.*SIN.(F)/(COS.(F)*ALFA)-1.)      *034
                 S2=S1*(ALFA/SIN.(F2)-1.)/(1.-ALFA*COS.(F2)/SIN.(F2))               *035
                 T=E*I/L                                                            *036
                 CITA=MFAB/((S1-S2)*T+BETA)                                         *037
                 MAB=-CITA*BETA                                                     *038
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```
          M=MAB+W*L/(ALFA*ALFA)                                          *039
          B=(-M*COS.(F2)+MAB+W*L/(ALFA*ALFA))/SIN.(F2)                   *040
          MS=W*L/8.+MAB                                                  *041
          O=F2/2                                                         *042
          MMAX=M*COS.(O)+B*SIN.(O)-W*L/(ALFA*ALFA)                       *043
          WHENEVER .ABS.MAB.G.MMAX,MMAX=-MAB                             *044
          SIGX=MMAX*C/I+P/A                                              *045
          WHENEVER SIGX.L.SIGPL                                          *046
          P=P+INC                                                        *047
          W=K*P                                                          *048
          TRANSFER TO ELAST C                                            *049
          OTHERWISE                                                      *050
          TRANSFER TO PLAST                                              *051
          END OF CONDITIONAL                                             *052
PLAST     THROUGH MOMENT ,FOR J=1,1,J.G.(N+1)                            *053
MOMENT    MOX(J)=W*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.                 *054
          THROUGH LOOP,FOR J=1,1,J.G.(N+1)                               *055
          MS=MOX(J)+MAB                                                  *056
          O=(J-1)*F2/(2.*N)                                              *057
LOOP      Y(J)=(M*COS.(O)+B*SIN.(O)-MS-W*L/(ALFA*ALFA))/P                *058
          MAB=-MAB                                                       *059
PLAST C   THROUGH ULT,FOR J=1,1,J.G.(N+1)                                *060
          MX(J)=W*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.+P*Y(J)-MAB       *061
          SIGU=P/A+MX(J)*C/I                                             *062
          SIGB=P/A-MX(J)*C/I                                             *063
          WHENEVER .ABS.SIGU.GE.SIGY.AND.(P/A+MAB*C/I).GE.SIGY           *064
          PRINT RESULTS P,W                                              *065
          PRINT FORMAT EXCEED                                            *066
          VECTOR VALUES EXCEED =$18H SIGY =SIGM          *$              *067
          THROUGH KAP,FOR J=1,1,J.G.N+1                                  *068
KAP       PRINT FORMAT ABCD,LSIGU(J),LSIGB(J),LEPSU(J),LEPSB(J)          *069
          VECTOR VALUES ABCD=$2F10.2,2E12.5*$                            *070
          TRANSFER TO START                                              *071
          END OF CONDITIONAL                                             *072
          WHENEVER .ABS.SIGU.G.SIGY                                      *073
          SIGU=SIGY                                                      *074
          SIGB=2.*P/A-SIGY                                               *075
          MX(J)=(A*SIGY-P)*C                                             *076
          Y(J)=(MX(J)+MAB-W*L*(1.-(N+1-J)*(N+1-J)*1./(N*N*1.))/8.)/P     *077
          END OF CONDITIONAL                                             *078
          WHENEVER .ABS.SIGB.G.SIGY                                      *079
          SIGB=SIGY                                                      *080
          SIGU=2.*P/A-SIGB                                               *081
          END OF CONDITIONAL                                             *082
          WHENEVER .ABS.SIGU.G.SIGPL                                     *083
          AA=SIGU/.ABS.SIGU                                              *084
          SIGU=.ABS.SIGU                                                 *085
          EPSU=(1.-SQRT.(1.-(SIGU-SIGPL)/(SIGY-SIGPL)))/(E/(2.*(SIGY-    *086
        1 SIGPL)))+SIGPL/E                                               *086
          EPSU=EPSU*AA                                                   *087
          OTHERWISE                                                      *088
          EPSU=SIGU/E                                                    *089
          END OF CONDITIONAL                                             *090
          WHENEVER .ABS.SIGB.G.SIGPL                                     *091
          BB=SIGB/.ABS.SIGB                                              *092
          SIGB= .ABS.SIGB                                                *093
          EPSB=(1.-SQRT.(1.-(SIGB-SIGPL)/(SIGY-SIGPL)))/(E/(2.*(SIGY-    *094
        1 SIGPL)))+SIGPL/E                                               *094
          EPSB=EPSB*BB                                                   *095
          OTHERWISE                                                      *096
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```
           EPSB=SIGB/E                                                      *097
           END OF CONDITIONAL                                               *098
           WHENEVER .ABS.(LSIGU(J)).G.SIGU,EPSU=LEPSU(J)-(LSIGU(J)-SIGU)    *099
         1 /E                                                               *099
           WHENEVER .ABS.(LSIGB(J)).G.SIGB,EPSB=LEPSB(J)-(LSIGB(J)-SIGB)    *100
         1 /E                                                               *100
           CSIGU(J)=SIGU                                                    *101
           CSIGB(J)=SIGB                                                    *102
           CEPSU(J)=EPSU                                                    *103
           CEPSB(J)=EPSB                                                    *104
ULT        PHI(J)=(EPSU-EPSB)/(2.*C)                                        *105
           PHI(N+2)=PHI(N)                                                  *106
           THROUGH LOAP,FOR J=2,1,J.G.(N+1)                                 *107
LOAP       CPHI(J)=(PHI(J-1)+10.*PHI(J)+PHI(J+1))*LAM/12.                   *108
           ASL(N+1)=CPHI(N+1)/2.                                            *109
           THROUGH LOOP1,FOR J=N,-1,J.L.2                                   *110
LOOP1      ASL(J)=ASL(J+1)+CPHI(J)                                          *111
           Y(1)=0.0                                                         *112
           R(1)=0.0                                                         *113
           THROUGH LOOP2,FOR J=2,1,J.G.(N+1 )                               *114
           TRY(J)=Y(J)                                                      *115
           Y(J)=Y(J-1)+ASL(J)*LAM                                           *116
           R(J)=.ABS.(1.-(TRY(J)/Y(J)))                                     *117
           WHENEVER R(J).L.R(J-1),R(J)=R(J-1)                               *118
LOOP2      R=R(J)                                                           *119
           L1=L1+1                                                          *120
           WHENEVER R.G.(.005).AND.L1.L.200,TRANSFER TO PLAST C             *121
           WHENEVER R.G.(.005).AND.L1.GE.200 ,TRANSFER TO LOOP5             *122
           WHENEVER TEST.E.2,TRANSFER TO ENDY                               *123
           CITA=(96.*Y(2)-72.*Y(3)+32.*Y(4)-6.*Y(5)) /(24.*LAM)             *124
           L3=L3+1                                                          *125
           WHENEVER .ABS.(1.-MAB/(CITA*BETA)).G.005.AND.L3.G.200            *126
           PRINT RESULTS P,W                                                *127
           PRINT FORMAT ENDMT                                               *128
           VECTOR VALUES ENDMT=$36H END CONDITIONS CANNOT BE SATISFIED *    *129
         1 $                                                                *129
           THROUGH KUP,FOR J=1,1,J.G.N+1                                    *130
KUP        PRINT FORMAT ABCD,LSIGU(J),LSIGB(J),LEPSU(J),LEPSB(J)            *131
           TRANSFER TO START                                                *132
           END OF CONDITIONAL                                               *133
           WHENEVER .ABS.(1.-MAB/(CITA*BETA)).LE..005                       *134
ENDY       PRINT FORMAT TWO,P,W,Y(2)...Y(N+1) ,MX(1)                        *135
           VECTOR VALUES TWO=$2F10.2, 4F8.4,F10.2*$                         *136
           P=P+INC                                                          *137
           MAB=MAB*P/(P-INC)                                                *138
           WHENEVER (P/A+MAB*C/I).GE.SIGY                                   *139
           TEST=2                                                           *140
           MAB=(A*SIGY-P)*C                                                 *141
           END OF CONDITIONAL                                               *142
           THROUGH LOOP4,FOR J=2,1,J.G.(N+1)                                *143
LOOP4      Y(J)=Y(J)*P/(P-INC)                                              *144
           II=1                                                             *145
           L1=1                                                             *146
           W=K*P                                                            *147
           THROUGH REVERS ,FOR J=1,1,J.G.(N+1)                              *148
           LSIGU(J)=CSIGU(J)                                                *149
           LSIGB(J)=CSIGB(J)                                                *150
           LEPSU(J)=CEPSU(J)                                                *151
REVERS     LEPSB(J)=CEPSB(J)                                                *152
           TRANSFER TO PLAST C                                              *153
```

```
         END OF CONDITIONAL                                            *154
         WHENEVER II.E.2,TRANSFER TO LOOP3                             *155
         WHENEVER .ABS.(1.-MAB/(CITA*BETA)).G..005                     *156
         OMAB=MAB                                                      *157
         OBC=BETA *CITA                                                *158
         MAB=(MAB+OBC)/2.                                              *159
         II=2                                                          *160
         TRANSFER TO PLAST C                                           *161
         END OF CONDITIONAL                                            *162
LOOP3    MAB1=MAB                                                      *163
         MAB=(OBC-OMAB*(CITA*BETA-OBC)/(MAB-OMAB))/(1.-(CITA*BETA-OBC) *164
       1 /(MAB-OMAB))                                                  *164
         WHENEVER.ABS.(MAB1-BETA*CITA).L..ABS.(OMAB-OBC)               *165
         OMAB=MAB1                                                     *166
         OBC=BETA*CITA                                                 *167
         END OF CONDITIONAL                                            *168
         TRANSFER TO PLAST C                                           *169
LOOP5    PRINT RESULTS P,W                                             *170
         PRINT FORMAT CON                                              *171
         VECTOR VALUES CON=$16H NO CONVERGENCE *$                      *172
         THROUGH KOP,FOR J=1,1,J.G.N+1                                 *173
KOP      PRINT FORMAT ABCD,LSIGU(J),LSIGB(J),LEPSU(J),LEPSB(J)         *174
START    TEST=1                                                        *175
         DIMENSION  MOX(50),Y(50),MX(50),PHI(50),CPHI(50),ASL(50),TRY  *176
       1 (50),R(50)                                                    *176
         DIMENSION LSIGU(50),LSIGB(50),LEPSU(50),LEPSB(50)             *177
         DIMENSION CSIGU(50),CSIGB(50),CEPSU(50),CEPSB(50)             *178
         INTEGER J,N,JJ,L1,II ,TEST                                    *179
         TRANSFER TO BEGIN                                             *180
         END OF PROGRAM                                                *181
```

L = 900.000000, C = 15.000000, N = 4, K = .040000

A = 40.000000, E = 2.900000E 04, BETA = 4.350000E 06, SIGPL = 35.000000

SIGY = 50.000000

1174.57	46.98	.1526	.3538	.5124	.5718	-3444.36
1184.57	47.38	.1536	.3564	.5164	.5762	-3476.35
1194.57	47.78	.1552	.3600	.5216	.5820	-3509.30
1204.57	48.18	.1567	.3635	.5267	.5877	-3541.36
1214.57	48.58	.1581	.3668	.5316	.5932	-3574.62
1224.57	48.98	.1597	.3705	.5368	.5990	-3606.66
1234.57	49.38	.1611	.3738	.5417	.6045	-3640.08
1244.57	49.78	.1626	.3774	.5469	.6103	-3672.98
1254.57	50.18	.1641	.3809	.5521	.6161	-3706.10
1264.57	50.58	.1656	.3845	.5574	.6221	-3739.55
1274.57	50.98	.1672	.3882	.5627	.6281	-3773.40
1284.57	51.38	.1687	.3918	.5682	.6343	-3807.65
1294.57	51.78	.1703	.3956	.5738	.6406	-3842.33
1304.57	52.18	.1719	.3995	.5795	.6471	-3877.25
1314.57	52.58	.1735	.4033	.5853	.6536	-3912.77
1324.57	52.98	.1755	.4080	.5922	.6614	-3948.74
1334.57	53.38	.1771	.4120	.5983	.6682	-3985.31
1344.57	53.78	.1789	.4163	.6048	.6756	-4022.05
1354.57	54.18	.1807	.4208	.6115	.6832	-4059.18
1364.57	54.58	.1826	.4255	.6185	.6912	-4096.70
1374.57	54.98	.1842	.4296	.6249	.6984	-4135.57
1384.57	55.38	.1863	.4348	.6327	.7072	-4173.97
1394.57	55.78	.1884	.4400	.6405	.7161	-4212.88
1404.57	56.18	.1910	.4463	.6499	.7266	-4252.25
1414.57	56.58	.1933	.4520	.6584	.7362	-4292.23
1424.57	56.98	.1956	.4580	.6674	.7464	-4332.62
1434.57	57.38	.1981	.4642	.6768	.7570	-4373.56
1444.57	57.78	.2007	.4708	.6867	.7682	-4414.92
1454.57	58.18	.2034	.4775	.6968	.7796	-4457.00
1464.57	58.58	.2067	.4856	.7087	.7930	-4499.75
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1484.57	59.38	.2125	.5007	.7315	.8187	-4587.61
1494.57	59.78	.2157	.5087	.7436	.8325	-4632.26
1504.57	60.18	.2189	.5170	.7562	.8466	-4677.63
1514.57	60.58	.2223	.5259	.7696	.8618	-4725.01
1524.57	60.98	.2258	.5349	.7833	.8774	-4772.25
1534.57	61.38	.2293	.5442	.7974	.8934	-4820.30
1544.57	61.78	.2333	.5545	.8129	.9110	-4870.45
1554.57	62.18	.2371	.5646	.8285	.9287	-4920.65
1564.57	62.58	.2411	.5752	.8447	.9470	-4971.82
1574.57	62.98	.2461	.5878	.8637	.9686	-5024.06
1584.57	63.38	.2503	.5992	.8814	.9888	-5078.55
1594.57	63.78	.2553	.6124	.9016	1.0117	-5133.66
1604.57	64.18	.2599	.6252	.9215	1.0345	-5190.58
1614.57	64.58	.2656	.6402	.9446	1.0608	-5248.27
1624.57	64.98	.2709	.6552	.9679	1.0875	-5307.25
1634.57	65.38	.2782	.6747	.9978	1.1216	-5366.01
1644.57	65.78	.3983	.9048	1.3082	1.4630	-5331.47

P = 1654.568802, W = 66.182751

NO CONVERGENCE

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38.71	43.52	.13440E-02	.15644E-02

43.95	38.28	.15844E-02	.13294E-02
47.37	34.86	.18078E-02	.12024E-02
48.56	33.67	.19208E-02	.11610E-02

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AIIM SCANNER TEST CHART #2

Spectra

4 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:",./?$0123456789

6 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:",./?$0123456789

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Times Roman

4 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:'',./?$0123456789

6 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:'',./?$0123456789

8 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:'',./?$0123456789

10 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:'',./?$0123456789

Century Schoolbook Bold

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6 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:",./?$0123456789

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News Gothic Bold Reversed

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Bodoni Italic

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10 PT ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz;:",./?$0123456789

Greek and Math Symbols

4 PT ΑΒΓΔΕΞΘΗΙΚΛΜΝΟΠΦΡΣΤΥΩΞΨΖαβγδεξθηιφκλμνοπφρστυωχψζ≥±",./≤=≠°><≡

6 PT ΑΒΓΔΕΞΘΗΙΚΛΜΝΟΠΦΡΣΤΥΩΞΨΖαβγδεξθηιφκλμνοπφρστυωχψζ≥±",./≤=≠°><≡

8 PT ΑΒΓΔΕΞΘΗΙΚΛΜΝΟΠΦΡΣΤΥΩΞΨΖαβγδεξθηιφκλμνοπφρστυωχψζ≥±",./≤=≠°><≡

10 PT ΑΒΓΔΕΞΘΗΙΚΛΜΝΟΠΦΡΣΤΥΩΞΨΖαβγδεξθηιφκλμνοπφρστυωχψζ≥±",./≤=≠°><≡

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Black

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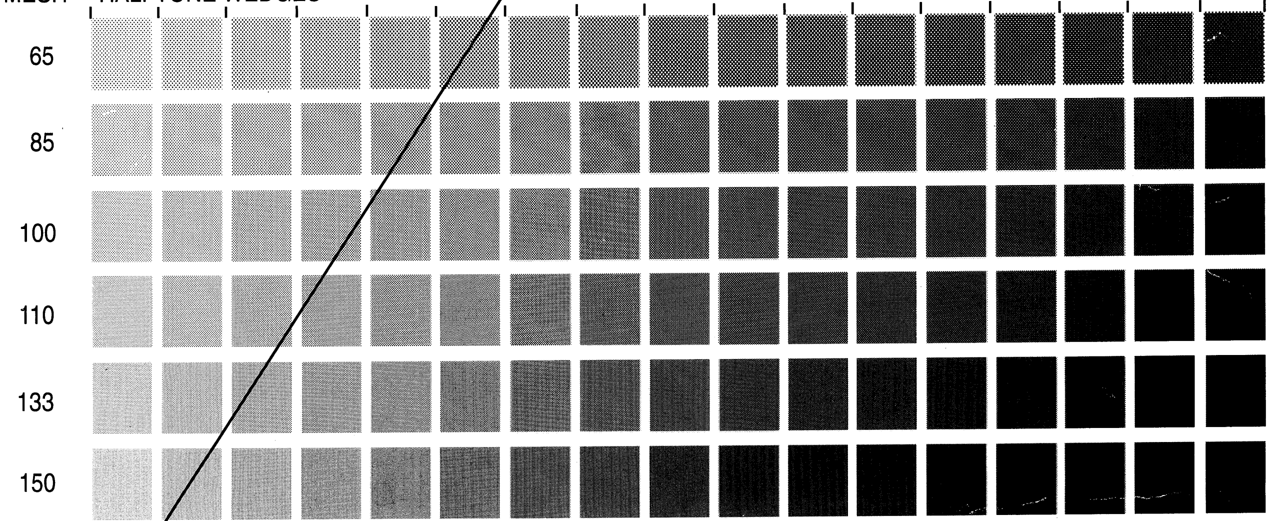

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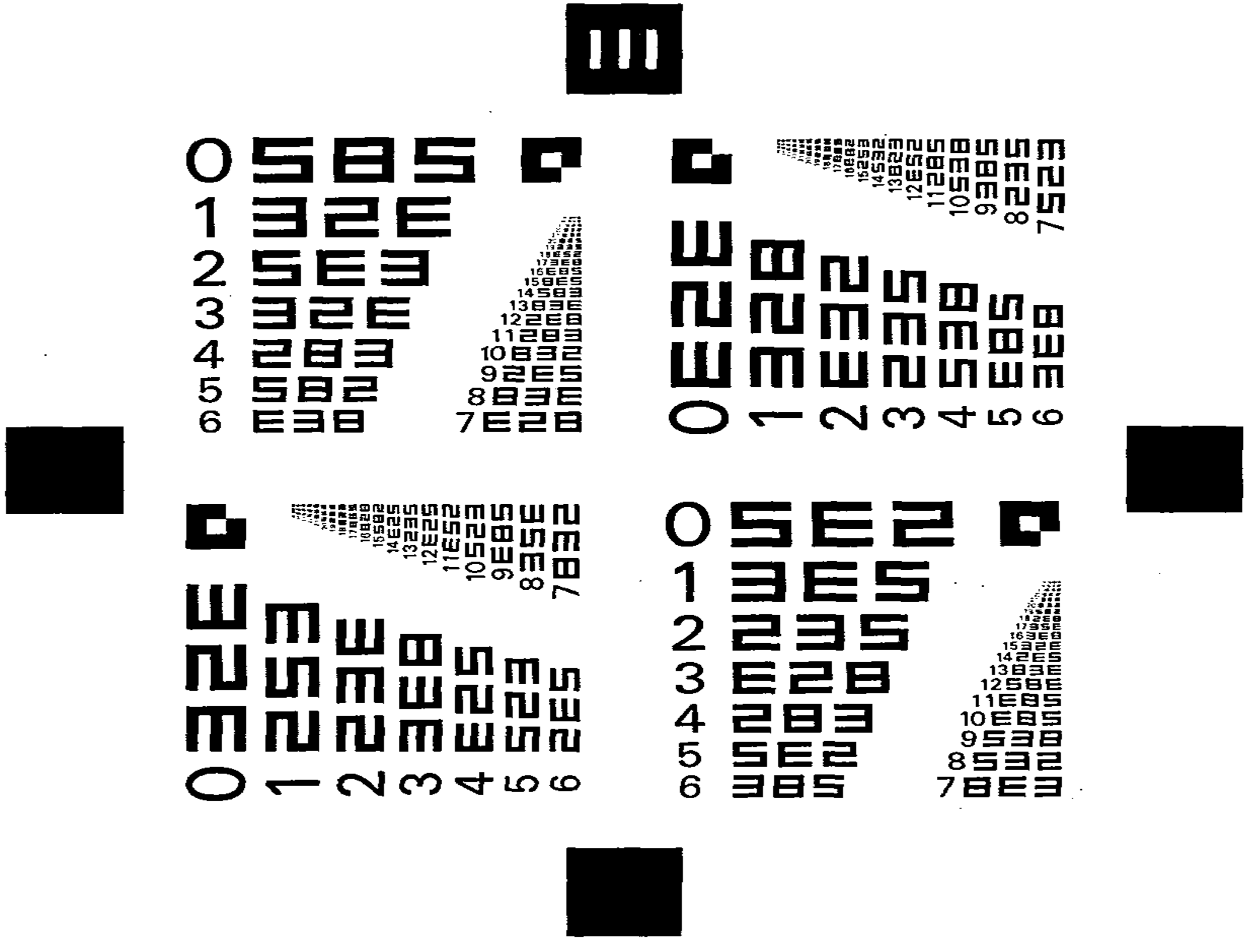

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