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Determining the actual local density of dark matter particles

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Abstract. Even if dark matter particles were unambiguously discovered in experiments, there is no clear reason to expect that this would resolve the dark matter problem. It is very easy to provide examples of dark matter scenarios (e.g. in supersymmetric models) in which nearly identical detector signals correspond to extremely different relic densities. The density of the discovered particles, therefore, must be determined before their cosmological relevance can be established. In this paper, I present a general method that utilizes both dark matter and hadron collider experimental data (once they become available) to estimate the local density of dark matter particles. These results were obtained in collaboration with Gordon Kane at the University of Michigan.

1 Introduction

We are now confident that our universe contains a large amount of cold dark matter. The most popular particle candidates for dark matter are weakly interacting massive particles (wimps). These particles are being searched for directly and indirectly by dozens of experimental groups throughout the world. If we are fortunate, wimps may soon be discovered experimentally.

Although the discovery of wimps in the galactic halo would have enormous implications for our understanding of elementary particle physics, it would infact contribute very little to our understanding of dark matter. It is unreasonable to expect that the particle discovered would represent all dark matter. This issue was first raised technically, though not resolved, in [1]. Indeed, current (and future) experiments readily provide examples of particles that in fact constitute less than 1% of all dark matter.

Even if weakly interacting massive particles were produced at colliders, it would still be very important to directly determine their local density in the galactic halo. This can only be done with experimental data.

In this paper, I describe how this can be determined in the context of supersymmetry, in which the dark matter is the lightest supersymmetric particle (LSP). Similar analyses could be done for any dark matter candidate.

I start out by illustrating why a discovery of dark matter particles in the halo would be insufficient for addressing the dark matter problem, and describing some of the uncertainties in relating the local and relic densities of dark matter. I then describe how dark matter is directly detected in experiments and present the general form of the interaction rate. This shows what is required for determining the local density of wimps. I present a very useful method for improving on these expressions by using data from different detector materials and at different recoil energies.

In order to deduce the local density of wimps, it is essential to know the wimp's mass. I present two methods for determining the mass of a wimp using direct detection data alone. The first is based on a well known relationship, and the second presents the preliminary results of the author. All of this work is then combined in the framework of the most general minimally-supersymmetric standard model, in which the neutralino is the wimp seen in dark matter experiments. A general procedure is presented for estimating the local density, and explicit bounds are given.

2 Discovering (some of?) the dark matter

Let us imagine that a weakly interacting massive particle χ has been unambiguously observed in direct detection experiments. Such a discovery would represent an enormous triumph for theoretical and experimental particle cosmology; it would have deep implications for our understanding of the universe, it may herald the existence of supersymmetry, and it would account for (at least) some of the dark matter in the universe. However, a discovery of dark matter particles is far from a solution to the dark matter problem: there is no reason to suspect that χ is *all* the dark matter.

What fraction of dark matter is represented by χ is a question that cannot be answered by experiment or theory alone. Furthermore, the answer will depend crucially on dark matter detection experiments. The purpose of this paper is to describe how this question may be answered.

It should be possible to determine the local density of χ using direct detection experiments. This is because, in a rough sense, they measure the local wimp density times its scattering cross section. Unfortunately, there are very few constraints on the scattering cross sections of most wimp candidates.

However, it is not true that the signal rate depends on the cross section and density independently, because these are somewhat related. This is because the relic density Ω_χ is related to thermal production and freeze-out in the early universe. The rate of wimp-annihilation affects the relic density and depends on the wimp annihilation cross section, which is in turn somewhat related to the scattering cross section by crossing.

As a result, there is less freedom in the observed signal rate than one may have naïvely suspected. This can be illustrated qualitatively as follows. If the cross section is large, then most wimps annihilate in the early universe and local density is small. Alternatively, if the cross section is small, then thermal freeze-out occurs very early and the density is higher. In either case, the cross section and density tend to compensate each other.

The crude arguments above suggest that even a very small component of dark matter may be detectable because it may have a higher cross section. This has been referred to as the 'no-lose theorem' in recent conferences. If even tiny fractions of dark matter may be detectable, in other words, experimentalists cannot 'lose' on making a discovery [2].

This is seen in many realistic dark matter scenarios. In Fig. 1, we have plotted the relic density against the direct detection signal for some six thousand randomly generated, constrained minimally supersymmetric standard models (without assuming any specific supersymmetry breaking scenario). By "constrained", we mean that all of the models are allowable under the current experimental constraints on supersymmetry. These models were generated and analyzed using the DarkSUSY code [3].

Notice that for any particular signal strength, the relic density fluctuates over at least two orders of magnitude.

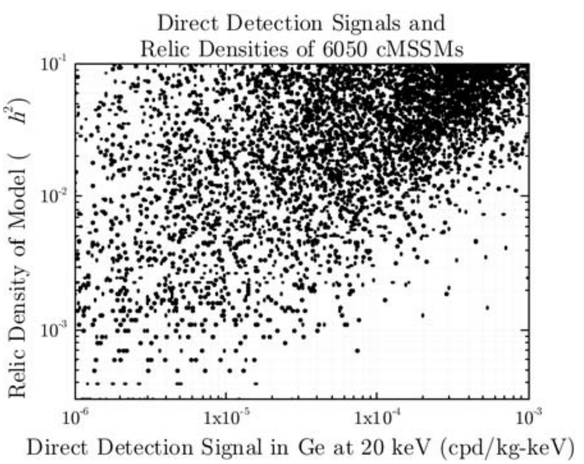

Fig. 1. The relic densities of 6050 constrained MSSMs as a function of direct detection signal strength in germanium. Experiments currently in planning or under construction may be able to observe signals of the order 10^{-4} cpd/kg·keV

In accordance with the 'no-lose theorem,' experiments in the near future may detect even 1% of the dark matter or less.

However, the no-lose theorem also implies, unfortunately, that the discovery of wimps in the galactic halo tells us very little about how much of the dark matter they compose. A wimp discovery could easily represent a negligible fraction of the dark matter.

Therefore, although wimps may be discovered in the near future, the dark matter problem will not be addressed until the density of wimps has been directly determined.

3 Local and relic densities

From studies of the cosmic microwave background, large scale structure formation, and big bang nucleosynthesis, we know the cosmic-scale relic density of cold dark matter to be approximately $\Omega_{\rm cdm}h^2 \sim 0.11$ [4]. From our knowledge of the rotation of the Milky Way galaxy, the local dark matter halo density is known to be approximately $\rho_{\rm cdm} \sim 0.3$ GeV/cm^3 [5]. It is obvious that any dark matter experiment on Earth is only sensitive to the local density and not the relic density.

Unfortunately, the relationship between local and relic densities involves many details of galaxy formation and structure that are still not understood. Even if we were able to demonstrate that χ has a local density of precisely 0.3 GeV/cm^3, there remain important subtleties about our understanding of dark matter in the universe as a whole.

Acknowledging these shortcomings, it nevertheless remains extremely important to determine the local density of any wimp discovered in direct detection experiments. It would be very promising if the entire local dark matter halo could be accounted for by wimps discovered in these experiments.

Because direct detection experiments are sensitive to small-scale structure in the local halo density, a knowledge of the ambient halo density, $\rho_{\rm cdm} \sim 0.3$ GeV/cm^3 may not be sufficient. Our knowledge of the local halo density is based on large scale surveys of star velocities in the Milky Way, and these measurements are not very sensitive to small-scale structure in the halo.

There are several types of small-scale halo structure that may affect direct detection experiments. For example, the earth may be within a stream of dark matter. This situation has been suggested in studies of the Sgr A stream and it has been estimated that our local halo density could be $0.3 - 23\%$ higher than the ambient density [6]. Alternatively, some authors have proposed that the halo may be clumpy or contain caustic structures [7]. These small-scale perturbations in dark matter density could have significant effects on direct detection rates.

Most of the small-scale structure considered in the literature involves local, high-density regions of dark matter within the halo. Although these structures may make it easier to discover dark matter, they make it nearly impossible to assess what fraction of the halo is composed of χ.

Fortunately, there exist ways to check the smoothness of the halo profile. For example, dark matter streams or caustics may be identified or excluded using directional dark matter experiments like DRIFT [8]. Also, it may be possible to identify a clumpy dark matter halo by studying the time-dependence of a wimp signal. These questions, therefore, may find answers in the foreseeable future.

These ambiguities must be addressed before the dark matter problem can be put to rest. For the purposes of this paper, however, we assume that the halo is locally smooth and that if $\rho_\chi = \rho_{\rm cdm} = 0.3$ GeV/cm^3, then χ represents all of the dark matter.

4 Dark matter direct detection

Dark matter particles in the halo can be observed directly through their interaction with ordinary matter[1]. Although wimps interact only weakly, occasionally they will scatter off matter in detectors, depositing a small amount of energy. Because wimps typically have masses on the order of a hundred GeV and move relatively slowly in the halo (on the order of a few hundred km/s) they typically deposit recoil energies from $\sim 1 - 200$ keV. Using very sensitive detectors, experiments can observe signals as low as a few keV. Using sophisticated coincidence algorithms, most experiments can remove virtually all background noise except scattering from neutrons.

In essence, direct detection experiments measure the χ-nucleus scattering rate as a function of recoil energy and time. In general, the signal rate is a function of the cross section for χ-nucleon scattering, the nuclear physics describing the nuclei in a particular detector, and the local velocity profile of the wimp fraction of the dark matter halo.

4.1 Elastic scattering rate

It is helpful here to state the explicit form of the differential interaction rate for a particular detector at the recoil energy q. Let the detector in question be composed of nuclei labeled with index j, each with mass fraction c_j. Thus the differential rate of wimp scattering at recoil energy q is,

$$\left.\frac{dR}{dQ}\right|_{Q=q} = \frac{2\rho_\chi}{\pi m_\chi}\sum_j c_j \int_{v_{\min_j}(q)}^{\infty} \frac{f(v,t)}{v} dv \times \left\{ F_j^2(q)[Z_j f_p + (A_j - Z_j) f_n]^2 + \frac{4\pi}{(2J_j+1)} \left[a_1^2 S_{j_{00}}(q) + a_0^2 S_{j_{11}}(q) + a_1 a_0 S_{j_{01}}(q)\right] \right\}, \quad (1)$$

where $v_{\min_j}(q)$ is the minimum velocity kinematically capable of depositing energy q into the $j^{\rm th}$ nucleus, $f(v,t)$ is the local velocity distribution function for the galactic halo, $F_j^2(q)$ and $S_{j_{mn}}(q)$ are nuclear form factors for coherent and incoherent scattering, respectively, Z_j and A_j are atomic and mass numbers, J_j is the nuclear spin, $a_1 \equiv a_p + a_n$ and $a_0 \equiv a_p - a_n$, and the constant parameters $f_{p,n}$ and $a_{p,n}$ describe the coherent and incoherent wimp-nucleon scattering cross sections, respectively[2]. For a more detailed discussion of equation 1, please refer to any modern review of dark matter (see, e.g., [5]).

It is important to note that (1) depends on several unknown parameters: (1) the wimp's mass, m_χ; (2) the particle physics of χ which determines the interaction parameters $f_{p,n}$ and $a_{p,n}$; (3) the velocity distribution of the halo, $f(v,t)$; (4) the local density of wimps, ρ_χ. These details will not be known when wimps are first discovered and may take many years to resolve.

4.2 Prerequisites to determine ρ_χ

From the discussion above, it is clear that to determine the density ρ_χ, one must first: (1) identify the particle χ; determine m_χ; estimate the halo profile; calculate the interaction parameters from the theory describing χ. Each of these will require enormous efforts from both dark matter and collider physics experiments.

The most important and perhaps most difficult requirement is the identification of χ. This is not possible through dark matter experiments alone. This is because these experiments observe only a few of χ's quantum numbers. For example, it is unlikely that any amount of direct detection data would enable us to differentiate between the lightest supersymmetric particle and the lightest Kaluza-Klein particle; if possible at all, this would probably require very precise data from several different nuclei. Therefore, while direct detection experiments may unequivocally discover dark matter wimps, they would not be able to explain the dark matter itself.

Perhaps the most important parameter describing χ is its mass. This determines all of its kinematics and is crucially linked to the local density. Fortunately, m_χ may be calculable from direct detection experiments alone. The known methods of calculating m_χ from dark matter experiments are described below.

Because the mass may be observable, it may prove to be the key to the identification of χ. If a neutral, stable particle is observed at colliders with the same mass as that observed in dark matter experiments, then we might suspect that they are the same particle. Although this association is imprecise, it appears to be one of the best methods of identification.

We should note, however, that determining the mass of χ may be very difficult at hadron colliders. For example, if χ is the LSP, it could take several years and an enormous effort to determine m_χ in a model-independent way. Most of the known techniques for determining the mass of the LSP rely on either the framework of mSUGRA or

[1] This was first proposed by Goodman and Witten in [12].

[2] Informally, coherent scattering is sometimes called 'spin-independent' and incoherent scattering 'spin-dependent.'

$$\left.\frac{dR}{dQ}\right|_{Q=q} = \frac{2\rho_\chi}{\pi m_\chi}\left\{f_p^2\left(\sum_j c_j\int_{v_{\min_j}(q)}^\infty \frac{f(v,t)}{v}dvF_j^2(q)Z_j^2\right) + a_p^2\left(4\pi\sum_j c_j\int_{v_{\min_j}(q)}^\infty\frac{f(v,t)}{v}dv\frac{[S_{j00}(q)+S_{j11}(q)+S_{j01}(q)]}{2J_j+1}\right)\right.$$
$$+f_n^2\left(\sum_j c_j\int_{v_{\min_j}(q)}^\infty \frac{f(v,t)}{v}dvF_j^2(q)(A_j-Z_j)^2\right) + a_n^2\left(4\pi\sum_j c_j\int_{v_{\min_j}(q)}^\infty\frac{f(v,t)}{v}dv\frac{[S_{j00}(q)+S_{j11}(q)-S_{j01}(q)]}{2J_j+1}\right)$$
$$\left.+f_pf_n\left(2\sum_j c_j\int_{v_{\min_j}(q)}^\infty \frac{f(v,t)}{v}dvF_j^2(q)Z_j(A_j-Z_j)\right) + a_pa_n\left(8\pi\sum_j c_j\int_{v_{\min_j}(q)}^\infty\frac{f(v,t)}{v}dv\frac{[S_{j00}(q)-S_{j11}(q)]}{2J_j+1}\right)\right\}. \tag{2}$$

specific assumptions about the relative masses of squarks and sleptons. Therefore, it should be stressed that χ may not be identified until long after it is discovered.

Also, the halo profile must be known sufficiently well. As described above, any small-scale structure in the halo will dramatically alter the analysis of the local density. It is imperative that these issues be sufficiently resolved.

Lastly, to compute the local density ρ_χ, one must know the interaction parameters $f_{p,n}$ and $a_{p,n}$. To compute these parameters, one must know a great deal about the theory describing χ. It is clear that these parameters cannot be obtained from direct detection data alone: they depend on the many parameters of whichever extended standard model χ is a part of. For example, if χ is the LSP, then these parameters will be functions of the squark masses, mixing angles, gauge-content of the LSP, and higgs parameters. It is extremely unlikely that all of these will be known when χ is discovered in direct detection experiments.

4.3 Combining data

All of the required analysis can be strengthened and empowered by combining data from different detectors over a range of recoil energies. There are many important insights and results that derive from the following framework.

In general, the expression for the scattering rate (1) is a second order polynomial in the four unknown interaction parameters $f_{p,n}$ and $a_{p,n}$. To highlight this, it can be recast in the suggestive form shown in (2) on top of the page.

It is clear from the expressions above that by using data from

1. different detector materials (varying the mass fractions, nuclear form factors, nuclear spins, and minimum velocities),
2. different recoil energies (varying the nuclear form factors and minimum velocities),

one could invert (2) to solve for $\sqrt{\rho_\chi}f_{p,n}$ and $\sqrt{\rho_\chi}a_{p,n}$ if the halo velocity distribution and m_χ were known. Given a halo model and the wimp mass, that is, the data from different detector materials and different recoil energies are sufficient for determining $\sqrt{\rho_\chi}f_{p,n}$ and $\sqrt{\rho_\chi}a_{p,n}$ (up to quadratic ambiguities).

We should mention that there are many important situations in which the above analysis can be simplified. For example, because $a_{p,n}$ are already scaled by linearly independent combinations of the incoherent nuclear form factors, $S_{j_{mn}}(q)$, it is not necessary to use different detector materials to solve for $\sqrt{\rho_\chi}a_{p,n}$. However, this will only work if there is data available from a detector with nuclei that have non-zero spin, which is sufficiently sensitive to incoherent scatteri ng.

Although knowing the scaled interaction parameters will not directly determine the local density, it can give enormous insight into the particle physics of χ. For example, if χ is the LSP, then the ratios a_p/a_n or a_p/f_n could possibly lead to important insights on $\tan\beta$, the degeneracy of the squark masses, mixing, and perhaps other information as well. This could be very important for collider physics and disentangling the MSSM.

5 Determining the Wimp mass

Of all the factors required to interpret the observed signal rate, perhaps the most important is m_χ. Not only is the mass required to compute the density, but it also plays a critical role in the identification of χ as described above. Fortunately, it may be possible to determine m_χ from direct detection experiments alone.

There are roughly two ways to determine m_χ from direct detection data. One method, that using the annual modulation crossing energy, was described in the dark matter review article by Primack et. al. in 1988 [9]. Although it seems unlikely to have originated in a review article, we have been unable to find any previous work mentioning this effect. The other method has been developed by the author in collaboration with Gordon Kane and represents work still in preparation. Both of these methods rely solely on the kinematics of the halo, although neither are particularly sensitive to the precise halo model [3] (although, see caveats in [10]).

5.1 Annual modulation crossing energy

As the earth orbits the sun, its velocity through the galactic dark matter halo varies between roughly 250 and 190

[3] Specifically, these calculations are insensitive to which isothermal halo model is assumed if the halo is in fact locally isothermal. Any extra structure in the halo (e.g. streams, caustics, etc.), however, can disturb these calculations significantly.

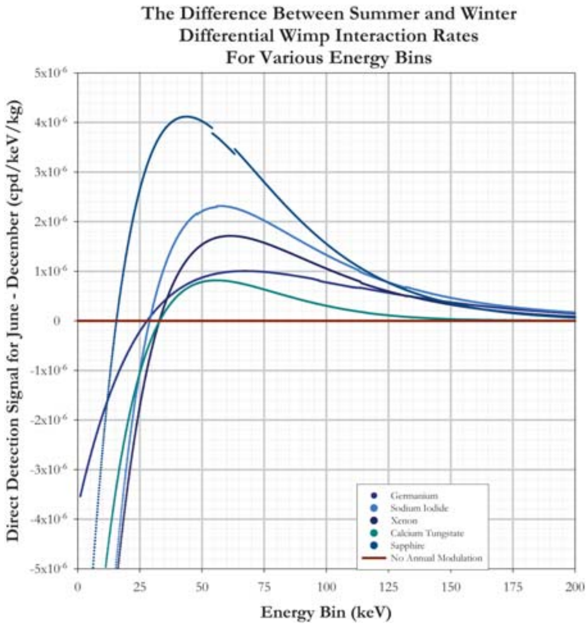

Fig. 2. The difference between direct detection signals in germanium in June and December as a function of recoil energy. This plot was generated for an MSSM with $m_\chi \sim 161$ GeV

km/s [8]. This in turn results in annual modulation in the scattering rate. However, the amplitude of this modulation varies as a function of recoil energy and changes sign. For a more detailed description, see, for example, [10].

In Fig. 2, we plot the difference between the scattering rates in June and December as a function of recoil energy for several detector materials. Notice that there is a particular energy, called the 'crossing energy,' at which no annual modulation is observed. As pointed out by Primack et al. [9], the crossing energy is an explicit function of the masses of the wimp and detector nuclei that can be derived easily from kinematics. Therefore, if crossing is observed, one can determine the mass of the wimp explicitly.

This method is moderately robust. Specifically, if there is an energy at which the annual modulation amplitude changes sign, then one can confidently determine the wimp mass to within approximately 10%. There are, however, some important subtleties and caveats to this analysis as described in, for example, [10]. These include the effects of bin-sizes and small-scale halo structure. Furthermore, if the wimp is very light, then the crossing energy may be well below the threshold of the detector and therefore not observed at all.

5.2 Kinematical consistency

Recall that if the halo velocity profile and m_χ are known, then direct detection data from different detector materials and different energies can be used to solve for $\sqrt{\rho_\chi} f_{p,n}$ and $\sqrt{\rho_\chi} a_{p,n}$. If the halo velocity distribution is known, then only the wimp mass is required to determine these.

Let us assume that the local halo velocity profile can be adequately approximated and that there exists enough data to solve for $\sqrt{\rho_\chi} f_{p,n}$ and $\sqrt{\rho_\chi} a_{p,n}$ as long as the mass is known. (If, for example, there was enough data to solve for $a_{p,n}$, it would be clear how to proceed along similar lines). Because many direct detection experiments observe scattering rates in a large number of recoil energy bins, we can generally expect to have many more measurements than the minimum required to solve the system of equations.

Because the interaction parameters are absolute constants, all minimal, linearly independent combination of measurements used to solve for the scaled interaction parameters will agree as long as the correct mass is used in the derivation. However, if an arbitrary m'_χ was used to solve for these parameters, different calculations will tend to disagree.

This motivates us to define a 'kinematical consistency' function, $\zeta(m'_\chi)$, which compares the values of $\sqrt{\rho_\chi} f_{p,n}$,$\sqrt{\rho_\chi} a_{p,n}$ obtained using different independent subsets of the data as a function of the m'_χ used. Specifically, let $\zeta(m'_\chi)$ be given by

$$\zeta(m'_\chi) \equiv \sum_{i \neq j} \sqrt{\rho_\chi} \left\{ \left(a_p(i) - a_p(j)\right)^2 + \text{similar terms} \right\},$$

where the indices i, j represent a minimal set of data used to compute the constants given the particular m'_χ. It is necessary that $\zeta(m'_\chi) = 0$ when $m'_\chi = m_\chi$, but this is not a sufficient condition. Specifically, we have not found any way to demonstrate that m_χ is the unique root of $\zeta(m'_\chi)$, although we have found no example where it has multiple roots.

To determine the wimp mass, one varies m'_χ until $\zeta(m'_\chi) = 0$. To test how useful this technique is, we applied it to some six thousand random, constrained MSSMs. In every single model tested, the correct mass was determined to near-arbitrary precision. Figure 3 illustrates a typical plot of $\zeta(m'_\chi)$. Notice that ζ has an extremely sharp minimum, decreasing many orders of magnitude within a few GeV of the true mass of the LSP. It should be stressed, however, that experimental uncertainties and resolutions were not considered in these calculations.

Although this method appears quite promising, these results should be considered preliminary. Several questions remain regarding how robust this calculation would be upon the introduction of experimental uncertainties and halo model ambiguities. Without these uncertainties, the algorithm yields the correct mass to seemingly arbitrary precision. It will be interesting to see how this changes in more realistic circumstances.

6 Bounds on the local density

Because m_χ can be determined, in principle, using the methods described above and because we are working under the assumption that the local halo velocity profile can be adequately approximated, we can determine $\sqrt{\rho_\chi} f_{p,n}$

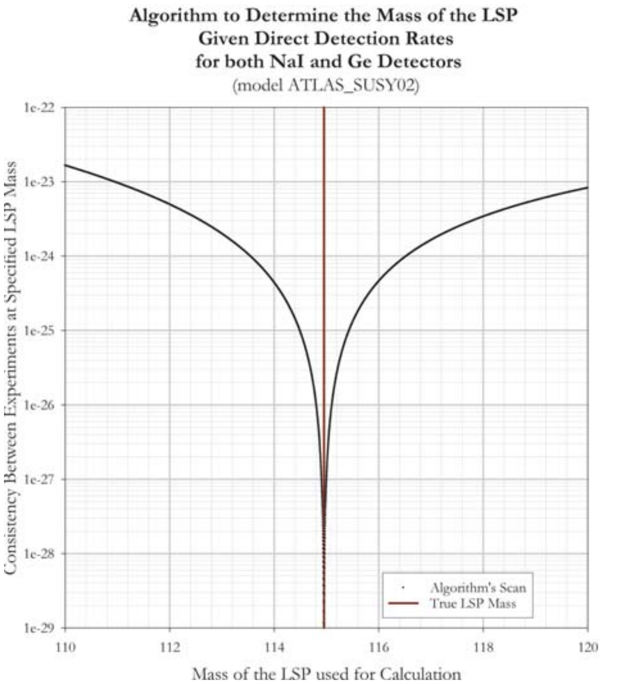

Fig. 3. The function $\zeta(m'_\chi)$, where the wimp corresponds to the neutralino in the MSSM specified by ATLAS SUSY point 2 [11]. The models and data were generated within the framework of the DarkSUSY package, [3]

and/or $\sqrt{\rho_\chi}a_{p,n}$ independently of the identification of χ. Therefore, to determine ρ_χ it is sufficient to know any one of the interaction parameters. This is an enormous improvement over the general case, for which all of the interaction parameters were required.

Therefore, any bounds on the interaction parameters will translate into bounds on the local density. Unfortunately, these parameters can only be computed in the framework of a very explicit model for the wimp. Furthermore, these parameters are typically very poorly constrained.

In order to address the question of the local density ρ_χ, one must specialize on a particular candidate particle in a specific extension of the standard model. In other words, we cannot proceed further without losing some generality.

7 Neutralino dark matter

The most popular and perhaps best-justified candidate for cold dark matter is the lightest supersymmetric particle (LSP), as predicted by supersymmetric extensions of the standard model that conserve R-parity. Indeed, the existence of supersymmetric dark matter was predicted before it was known that non-baryonic dark matter would be needed. In most MSSMs allowed by experimental constraints, the LSP is the neutralino, χ, which is the supersymmetric partner of the neutral gauge and higgs bosons. For an extensive and authoritative review of supersymmetric dark matter, see Jungman et al. [5].

7.1 Interaction parameters

Given a completely specified supersymmetric standard model, it is rather straightforward to calculate the interaction parameters. It should be noted, however, that one does not compute $f_{p,n}$ or $a_{p,n}$ directly. Rather, χ-quark (an χ-gluon) interaction parameters are calculated and these are used to determine the χ-nucleon parameters.

Even at tree level, it is clear that the χ-quark interaction parameters will depend on many of the parameters in the model. Specifically, they are functions of the

1. gauge content of the lightest neutralino,
2. most of the squark masses and mixing angles,
3. $\tan\beta$, the ratio of the vacuum expectation value of the two higgs bosons,
4. higgs mass parameters (only for the coherent interactions).

It should be emphasized that most of these parameters will be extraordinarily difficult to measure in practice (especially at hadron colliders). Currently, no general, model-independent methods exist for determining most of these parameters.

To illustrate the dependence on each of these parameters (although we do not derive them here), the incoherent scattering of χ with a u-quark is given by (see equation on bottom of the page).

In the expression above, the matrices $\Pi_{L,R}$ are 3×6 projection matrices given in the basis $(\tilde{u}_L,\tilde{c}_L,\tilde{t}_L,\tilde{u}_R,\tilde{c}_R,\tilde{t}_R)$; Θ_u is a unitary matrix which

$$
\begin{aligned}
a_u = -\frac{g^2}{16m_W^2}(N_{\tilde{H}_1}^2 - N_{\tilde{H}_2}^2) + \frac{g^2}{8}\sum_{\tilde{q}_j}\frac{1}{m_{\tilde{q}_j}^2-(m_\chi+m_u)^2}\Bigg\{\; & 2\left(\frac{1}{2}N_{\tilde{W}}^* + \frac{1}{6}\tan\theta_W N_{\tilde{B}}^*\right)^2 (\Pi_L\Theta_u)_{1j}^2 \\
&+\frac{m_u}{m_W\sin\beta}\mathfrak{Re}\left[\left(N_{\tilde{H}_2}N_{\tilde{W}}^* + \frac{1}{3}\tan\theta_W N_{\tilde{H}_2}N_{\tilde{B}}^*\right)(\Pi_R\Theta_u)_{1j}^*(\Pi_L\Theta_u)_{1j}\right] \\
&+\frac{m_u^2}{2m_W^2\sin^2\beta}N_{\tilde{H}_2}(\Pi_R\Theta_u)_{1j}^2 + \frac{8}{9}\tan^2\theta_W N_{\tilde{B}}^2(\Pi_R\Theta_u)_{1j}^2 \\
&-\frac{4m_u\tan\theta_W}{3m_W\sin\beta}\mathfrak{Re}\left[N_{\tilde{H}_2}N_{\tilde{B}}^*(\Pi_L\Theta_u)_{1j}^*(\Pi_R\Theta_u)_{1j}\right] + \frac{m_u^2}{2m_W^2\sin^2\beta}N_{\tilde{H}_2}^2(\Pi_L\Theta_u)_{1j}^2 \;\Bigg\}.
\end{aligned}
$$

$$a_u \leq -\frac{g^2}{16m_W^2}(N_{\tilde{H}_1}^2 - N_{\tilde{H}_2}^2) + \frac{g^2}{8}\frac{1}{(m_{\tilde{q}_\ell}^2 - (m_\chi + m_u)^2}\left\{\frac{17}{18}\tan^2\theta_W N_{\tilde{B}}^2 + \frac{1}{2}N_{\tilde{W}}^2 + \frac{m_u^2}{m_W^2\sin^2\beta}N_{\tilde{H}_2}^2 \right.$$
$$\left. +\frac{1}{3}\tan\theta_W|N_{\tilde{B}}||N_{\tilde{W}}|\cos(\alpha_{\tilde{W}}) + \frac{m_u}{m_W\sin\beta}|N_{\tilde{W}}||N_{\tilde{H}_2}|\cos(\alpha_{\tilde{H}_2}-\alpha_{\tilde{W}}) - \frac{m_u}{m_W\sin\beta}\tan\theta_W|N_{\tilde{B}}||N_{\tilde{H}_2}|\cos(\alpha_{\tilde{H}_2})\right\},$$

diagonalizes $\tilde{M}_u^2$ so that $\tilde{M}_u^{2\ \mathrm{diag}} = \Theta_u^\dagger \tilde{M}_u^2 \Theta_u$ [4]; the subscript j on $\tilde{q}_j$ corresponds to the flavor and handedness of the quarks so that $j = 1, \ldots, 6$ corresponds to $(\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)$; and gauge content of χ is given by $\chi = N_{\tilde{B}}|\tilde{B}\rangle + N_{\tilde{W}}|\tilde{W}\rangle + N_{\tilde{H}_1}|\tilde{H}_1\rangle + N_{\tilde{H}_2}|\tilde{H}_2\rangle$. A similar expression describes scattering with d, s-quarks.

The coherent parameters are similar to the incoherent ones except that they also contain higgs exchange at tree-level. This implies that in addition to squark masses, mixing angles, $\tan\beta$, and the gauge content of χ, one must also know the higgs masses. Therefore, in general, less knowledge of the MSSM is required to compute $a_{p,n}$ than $f_{p,n}$.

7.2 Limits on scattering parameters

As we have shown, the interaction parameters depend on a very detailed knowledge of the MSSM. Unfortunately, these may not be known until well after dark matter particles have been directly observed in experiments. We should try to estimate them somehow using partial information and any bounds on the MSSM that are available.

It should be clear that even if all of the squark masses and mixing angles are unknown, we can still place limits on the interaction parameters by using exclusion bounds. In general, one can typically find a way to make use of what is known and constrain what is not known in order to estimate and limit the interaction parameters.

Beginning with almost none of the MSSM parameters determined, we find that we can still place rather strong limits on $a_{p,n}$. For example, we have found that given only upper and lower bounds on $\tan\beta$ and a lower bound on the lightest squark mass, there is a strict upper bound for the incoherent χ-quark scattering parameters. If there is a strict lower bound on the lightest squark mass, such as $m_{\tilde{q}_\ell}$, and $\tan\beta$ is bounded so that $\sin\beta \geq \sin\beta_\ell$ [5], then there is a strict upper bound on $a_{p,n}$. It should be mentioned that these types of bounds already exist today, at least in the framework of particular supersymmetry breaking scenarios. In this case, it can be shown that the magnitude of a_u is strictly bounded by (see equation on top of the page), where $\alpha_{\tilde{H}_2}$, and $\alpha_{\tilde{W}}$ are the relative phases between $N_{\tilde{H}_2}, N_{\tilde{W}}$ and $N_{\tilde{B}}$, respectively.

[4] We have chosen the basis for the squark-mass matrices so that M_u is diagonal.

[5] Both the upper and lower bounds of $\tan\beta$ are important and it is not sufficient to have only a lower bound on $\sin\beta$. This is because the nucleon scattering parameters $a_{p,n}$ will depend on u- d- and s-quark scattering. In particular, a lower bound on $\cos\beta$ is needed to place bounds on $a_{d,s}$.

This expression has six real unknowns. Notice that by the normalization of the neutralino wave function, $|N_{\tilde{B}}|^2 + |N_{\tilde{W}}|^2 + |N_{\tilde{H}_1}|^2 + |N_{\tilde{H}_2}|^2 = 1$, the parameter space is compact. Therefore, a_u can be absolutely maximized with respect to all six unknowns. Specifically, although all of the gauge content of the neutralino may be unknown, one can limit the χ-quark and hence the χ-nucleon interaction parameters absolutely.

It should be emphasized that the analysis used to derive the above bound was for the most general softly-broken supersymmetric standard model; no *ad hoc* supersymmetry breaking scenarios such as mSUGRA were assumed. It is obvious that if a particular supersymmetry breaking scenario were assumed, the above expressions would be enormously simplified. However, these types of assumptions are very difficult to justify (theoretically or experimentally) and therefore greatly limit the generality of the work.

It is important to note the flexibility of the derivation involved in computing these bounds. If, for example, the masses of several light squarks were known, one could greatly improve the above bounds by including these in the explicit expression for a_q and then maximizing it relative to the parameters that remain unknown. In this manner, almost any additional knowledge can be incorporated to arrive at stronger statements. Therefore, not only do these bounds grow more restrictive with increasing knowledge of the MSSM, but they continue to approach a realistic estimate of the interaction parameters.

7.3 Strong lower bound on ρ_χ

From the work above it is clear that, given adequate bounds on $\tan\beta$ and a lower bound on the lightest squark mass, there exist strong, model-independent upper bounds on $a_{p,n}$. These in turn can be used to place a very strong lower bound on the neutralino relic density, because we know $\sqrt{\rho_\chi}a_{p,n}$.

To test this method, we considered some six thousand randomly generated MSSMs that are consistent with all known bounds on supersymmetry. For each of these models, we calculated the interaction rates for a NaI detector in twelve recoil energy bins. This (idealized) data was used to compute the mass of the LSP – using the kinematical consistency function – and to solve for $\sqrt{\rho_\chi}a_{p,n}$. Upper bounds were calculated for $a_{p,n}$ assuming 10% uncertainty in $\tan\beta$ and a lower bound on the lowest squark mass of either 200 GeV or the actual mass of the lightest squark, whichever was less. For the sake of computational simplicity, however, the specific gauge content of the neutralino

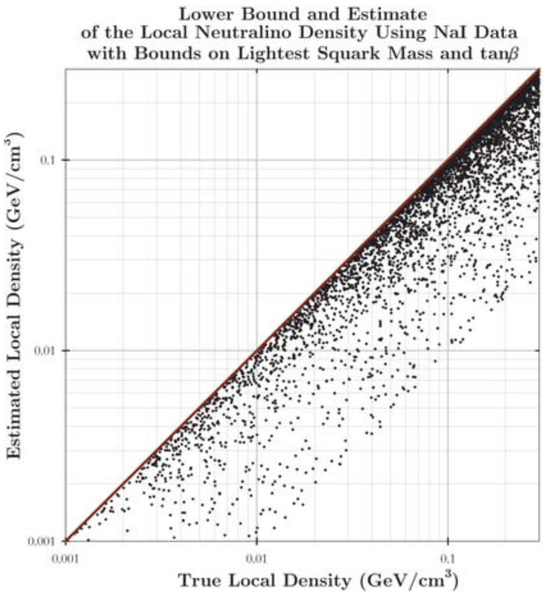

Fig. 4. This plot compares the lower bound and estimate of the local density computed using the strong upper bound for $a_{p,n}$ to the true local density for each model. The red line indicates perfect agreement. Notice that the procedure correctly determined a lower bound for the local density for every model.

was taken to be known for each model[6]. Using the upper bounds for $a_{p,n}$, we obtained a lower bound on the local density ρ_χ.

Figure 4 illustrates the results of this algorithm for each of the randomly generated MSSMs. Notice that the estimated local density is always strictly less than the true local density—as required for a lower bound. Notice also that for many models the lower bound was not such a poor estimate. This will be the case, for example, when the lightest squark mass is near or below the 200 GeV bound.

8 Conclusions

We have seen that, by itself, a discovery of dark matter particles in our galactic halo cannot address directly the dark matter problem of the universe. However, when combined with data from colliders in order to identify the particle and limit its interaction parameters, we can give a general estimate of its local density.

In the framework of supersymmetry, we have presented a robust and iteratively improvable method for estimating, using any information available about the MSSM, the local density of neutralino LSPs observed in direct detection experiments .

Thus we can conclude that even though the observation of wimps is unlikely to resolve the dark matter problem immediately, there do exist clear and general methods for addressing their cosmological significance.

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[6] If the gauge content of the neutralino was unknown, the interaction parameters could have been maximized with respect to these parameters as described earlier. In general, therefore, the upper bounds obtained were more restrictive than they would be in practice.