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ALTERNATIVE DETECTION OF MODULATIONS IN CO-CHANNEL FREQUENCY MODULATION

Hansford W. Farris

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Doctoral Committee:

Professor Gunnar Hok, Chairman
Associate Professor Joseph A. Boyd
Associate Professor Richard K. Brown
Associate Professor Ben Dushnik
Professor Lewis N. Holland

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CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

Even a cursory examination of the literature will reveal a tremendous amount of effort invested in the study of interference problems in radio communication. In the case of frequency modulation, because of its apparent discrimination against unwanted signals and other types of interference, an appreciable amount of investigation has been carried out to show why, on the one hand, FM does tend to favor the stronger of two signals in the pass band of a receiver, or to explain, on the other hand, why interference anomalies should actually be expected. It will be found, however, that virtually all of the effort to date has been with a view to detecting a desired signal which is favored by being even slightly stronger than the noise or than another signal with which it may be competing.

In very recent years, due in part to the great demands placed on the available frequency spectrum and in part to the fact that strong signals do tend to mask or suppress the weak signals in the receiver pass band, there has arisen an interest in methods of recovering a desired weak signal in the presence of the stronger. Several schemes for accomplishing this feat in the case of FM systems have been proposed. These will be reviewed in this discussion. It is, however, the primary purpose of this paper to treat the information-bearing possibilities of two signals in the same channel, analyze the conditions under which they may be distinguished, propose and demonstrate the feasibility of a separation method and evaluate the limits of operation of the system under several conditions of interference.

1.2 Relationship to Other Studies of Interference in FM Reception

The greater interest having naturally been centered on the detection of the stronger desired signal free of interference during the period in which FM has become established, little attention has been given the problem of weak-signal detection outside of military-sponsored research with the exception of the work of Wilmotte (1). Following his paper, however, there have been at least three other proposals for detection of the weak signal in the presence of the stronger and it is appropriate to review these as they relate to this investigation. They result from the work of Meek (2) and Baghdady (3).

1.2.1 Wilmotte's Proposal. In a paper published in the British Journal of the Institute of Electrical Engineers in 1954, Wilmotte discussed a scheme by which the respective detected modulations on two FM signals might possibly be combined to yield the information of the weaker. His analysis began, as is natural for all of these studies, with the phasor diagram of Figure 1.

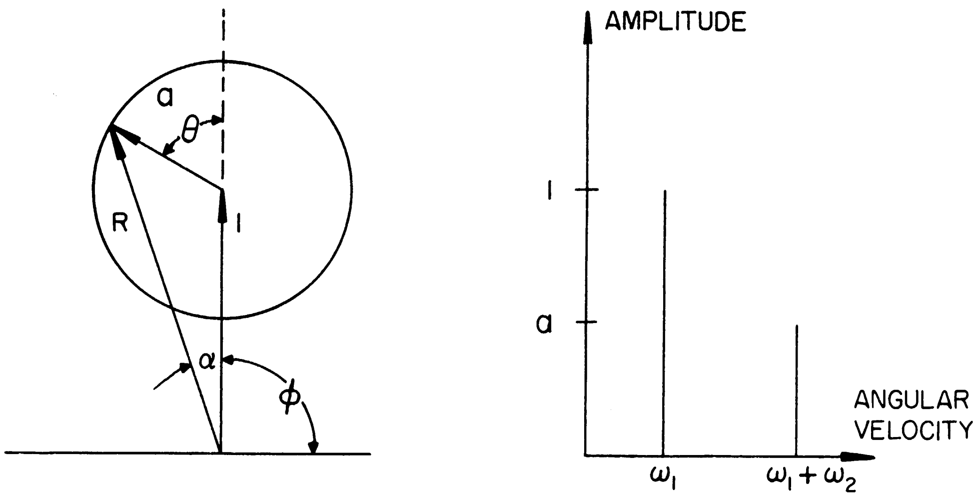

Figure 1. Two-Signal Interference (Wilmotte's Notation)

Assume a strong signal of unit amplitude, moving with angular velocity ω_1 and making an angle ϕ with an arbitrary reference line at any given instant, while the weaker signal is of relative amplitude a, is moving with angular velocity ω_2 with respect to the strong-signal phasor and at the same instant of time makes an angle θ with the strong-signal phasor. The magnitude of the resultant is R and its phasor makes an angle α with respect to the strong signal representation.

The basic parameter of the information contained in the weak signal is the angular velocity $\omega_1 + \omega_2$. R has a fundamental frequency of ω_2 and can be obtained by isolating the waveform by means of an ordinary envelope detector and then the frequency established by putting the envelope through a limiter-discriminator circuit. It is not clear whether such a limiter-discriminator arrangement would be adjustable in center frequency to accommodate all possible values of ω_2.

ω_1 is obtained in the ordinary manner by another limiter-discriminator circuit. However, it is not sufficient to add it to ω_2 to obtain $\omega_1 + \omega_2$ for this does not provide for the sense of ω_2; i.e., whether it is positive or negative at the time of combination. Wilmotte proposes that the sign of ω_2 be changed before it is added to ω_1 every time the detected envelope passes through zero. A second circuit then compares the phase of the envelope prior to the limiter and of $d\alpha/dt$ after the discriminator and a high-pass filter and "corrects the sign of ω_2 whenever it reaches a sufficiently high value outside the intelligence frequency." It is not very clear what is precisely being done here; no block diagrams were given so that particular circuits could be ascertained.

The problem of sensing the sign of ω_2 is a very real one and some additional consideration is given the post-detection scheme in a suggested method in Section 3.2. It constitutes the major difficulty in this method of weak-signal readability.

Wilmotte concluded his analysis with remarks to the effect that circuits had not been designed to realize the proposal and that no quantitative evaluation could be reported.

<u>1.2.2 A Method for the Subtraction of Strong Signal</u>. In mid-1956 at the Georgia Institute of Technology, a group (2) working on interference problems in communication suggested that the strong signal could be obtained free of the interference of the weak and then be subtracted from the original combination. In this respect the method is similar to the one proposed in this thesis. However there are differences in the manner in which the result is to be achieved.

In this method the following arrangement is used:

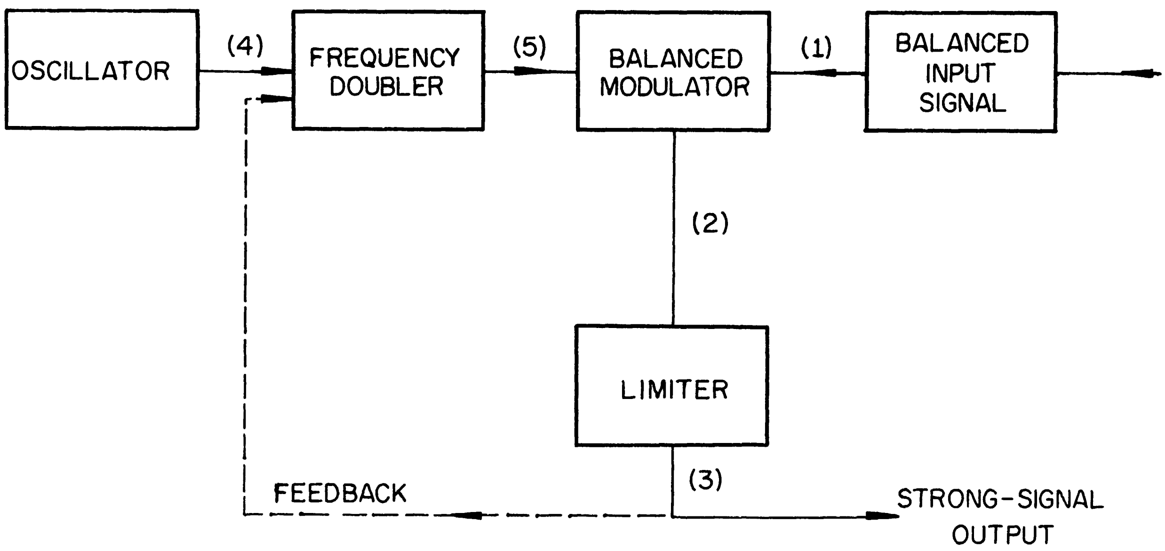

Figure 2. A Weak-Signal Detection Method

The oscillator is assumed for convenience; ultimately it is to be replaced by the limiter output. The input signal at (1) is

$$E_1 = \alpha \cos \phi_1(t) + \beta \cos \phi_2(t) \qquad \text{with } \alpha > \beta.$$

The oscillator's output is $E_4 = k_0 \cos \phi_1(t)$, so that at (5) there is obtained $E_5 = k_1 \cos 2\phi_1(t)$. Now,

$$E_2 = k_2 E_1 E_5$$

$$= k_2 \left[k_1 \cos 2\phi_1(t)\right] \left[\alpha \cos \phi_1(t) + \beta \cos \phi_2(t)\right] .$$

or

$$E_2 = k_3 \alpha \cos 3\phi_1(t) + k_3 \alpha \cos \phi_1(t) + k_3 \beta \cos \left[2\phi_1(t) \pm \phi_2(t)\right] .$$

Tuning the balanced modulator output to the strong signal frequency provides that

$$E_2 = k_3 \alpha \cos \phi_1(t) + k_3 \beta \cos \left[2\phi_1(t) - \phi_2(t)\right] .$$

Should the modulator be carefully unbalanced so as to permit the input signal to feed through and add to the output, then we obtain

$$E_2 = k_3 \beta \cos \left[2\phi_1(t) - \phi_2(t)\right] + 2k_3 \alpha \cos \phi_1(t) + k_3 \beta \cos \phi_2(t) .$$

This provides a signal output with a spectrum as shown in Figure 3 below:

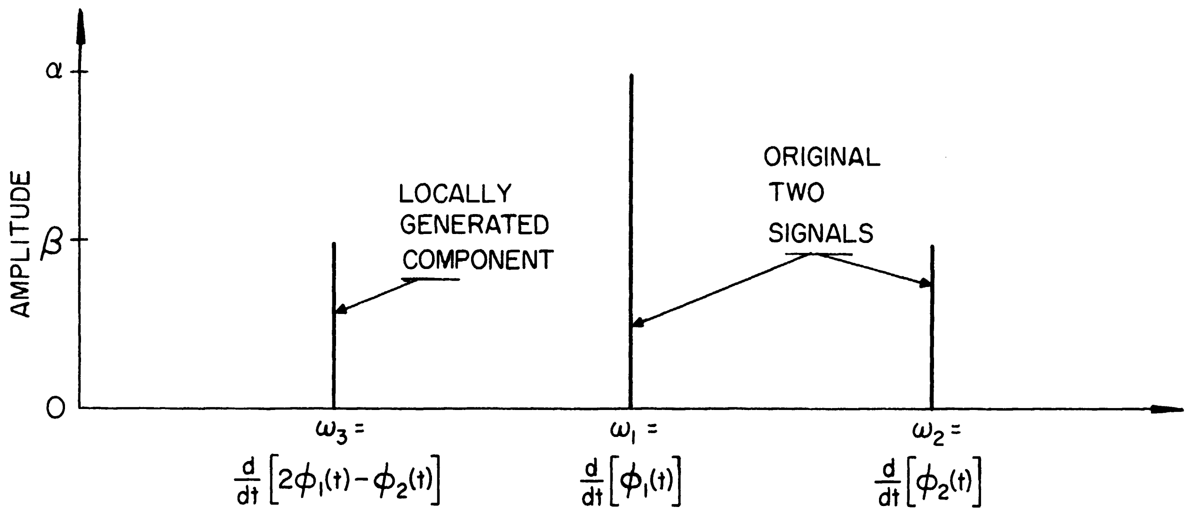

Figure 3. Spectrum of Unbalanced Modulator Output

With this symmetry about the signal, $2k_3 \alpha \cos \phi_1(t)$, regarded as a carrier, the limiter output will be

$$E_3 = k_4 \cos \phi_1(t) ,$$

the strong signal alone, assuming a narrow-band limiter which will not pass the "sideband" components. The output, E_3, is just the signal required in place of the auxiliary oscillator so points (3) and (4) are tied together. This effects a squelch circuit, since there will be no output unless there is an input of sufficient amplitude to pass the limiter. This "feedback oscillator" circuit is a form of feedback around the limiter, and the output builds up to be that of the strongest frequency component at the input.

The signal at point (3) is adjusted in amplitude and phase until cancellation of the strong signal portion of the input is effected so that only the weak signal remains. It is then detected in the usual manner.

At the time of this writing the success of the method has not been reported, although the proposal appears quite feasible.

1.2.3 Baghdady's "Feed-Forward" Method. During 1957, Baghdady (3), at Massachusetts Institute of Technology, suggested that two chains of amplifiers and limiters be excited in parallel and have their outputs connected in series so that phase cancellation of "whichever of the two signals is undesired" can be realized. His block diagram for both the feedback and feedforward connections is shown in Figure 4. The feedback arrangement resulted from his work in feedback around the limiter which is discussed in Section 2.2. He suggested the feedforward scheme as the dual of the feedback circuit. Although he states that different numbers of amplifiers and limiters are used in the two channels, it is not clear how narrow-band limiters, which have the effect of freeing the strong

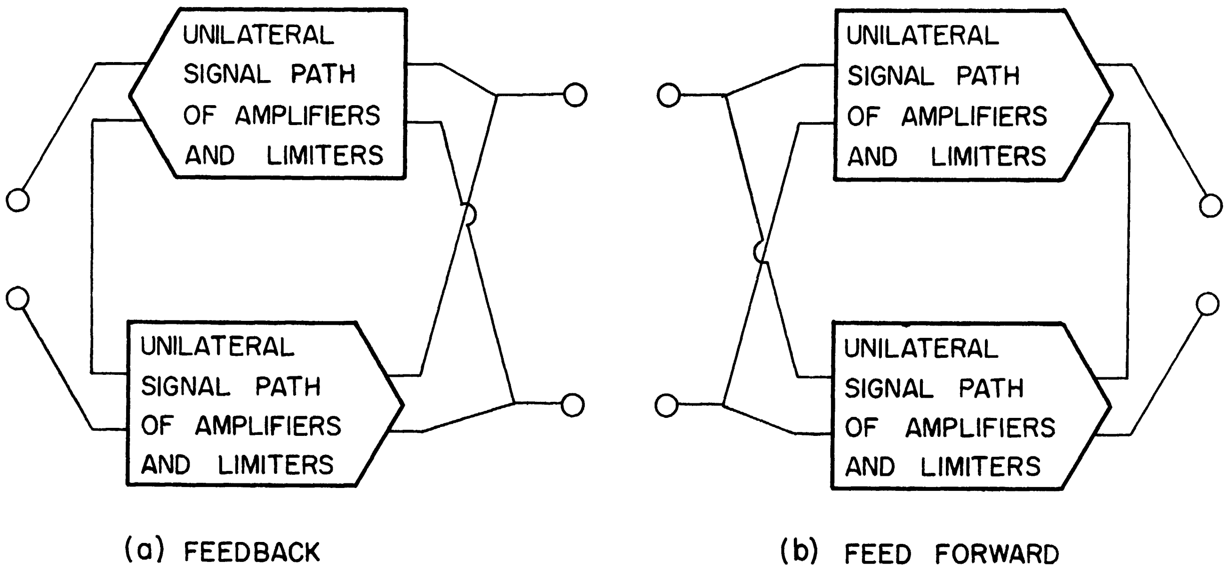

Figure 4. Baghdady Two-Channel System

signal from the weak and its influences, can be used to detect the weak signal. A linear amplifier chain, such as is proposed in this thesis, was not used in the feedforward arrangement. No experimental results were reported in late 1957.

1.2.4 Baghdady's Variable-Trap Method. At the same time as the plan discussed in Section 1.2.3 was reported, Baghdady suggested a dynamic trap approach to alter the relative amplitudes of the two co-channel FM signals. This system is shown in Figure 5.

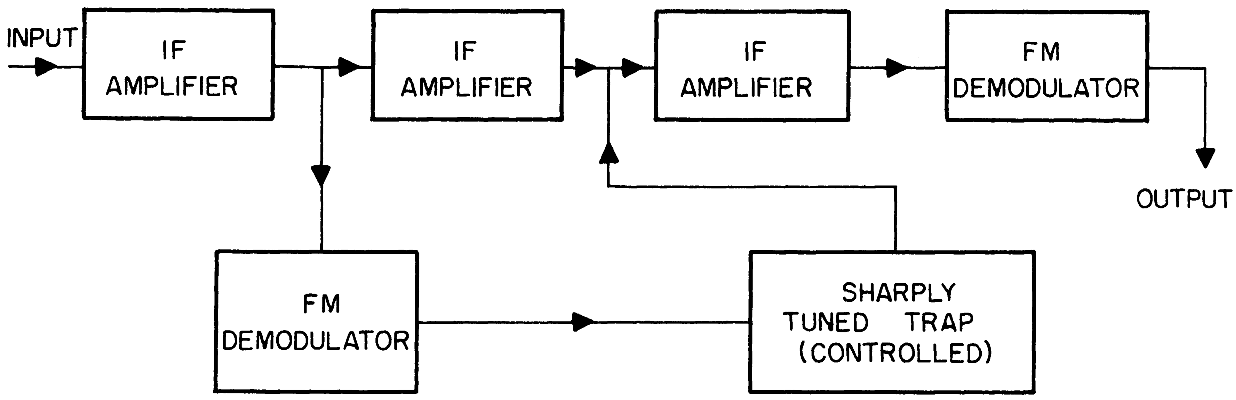

Figure 5. Baghdady Variable-Trap System

In this method the strong signal is first produced free and clear of interference, as before in other systems. It is detected in a discriminator and the low-frequency output then used to control a reactance tube circuit which is part of a sharply-tuned resonant trap at the frequency of the stronger signal, with the effect that a sharp dip is introduced into the IF pass band characteristic. It is expected that this will enable the weak signal to predominate and be subsequently detected in an FM demodulator.

Experimental work on this arrangement was in progress in late 1957. It had not been reported at the time of this writing.

1.2.5 The Status of Current Work--Summary. It will be appreciated that several of the methods for weak-signal detection which have been reviewed have elements in common. The detection of the stronger signal free of the influences of the weaker, either at the intermediate-frequency or following the discriminator, is an inherent part of every proposal. There are difficulties of no small magnitude attendant upon any method which utilizes the strong signal modulation as a control because of the need for monitoring the change in phase over hundreds of radians while maintaining correlation with the envelope variations of the composite signal.

In two cases the co-channel carrier frequencies are not permitted to approach each other arbitrarily closely, so that modulations on nearly identical frequencies could not be read in these instances.

The need for many limiters in two channels in one of the methods is probably a practical handicap because of the difficulty of controlling the limiter design to the degree necessary to permit satisfactory cancellation of the stronger signal in the output. In a single path system the displacement of the zeros of the signal is not usually audible to the ear, but the nonlinearities of the usual limiter stages can actually change the time of occurrence of the zeros when symmetrical limiting is not obtained, and this might well make the needed correlation difficult.

Developed independently, the method of this paper, described in detail in Section 3.3, is an alternate means of accomplishing weak signal detection. It is a straightforward approach to the problem and makes use of several of the aspects of those methods discussed previously. In brief, it requires the following:

(a) Two paths are necessary, one a linear path retaining the phase of the stronger signal relative to its phase in a second path composed of limiters and filters and which permits precise control of the strong signal amplitude.

(b) Good limiter characteristics in the second path which permit the strong signal to be obtained reasonably free of interference of both amplitude and phase types.

(c) Cancellation of the strong signal at intermediate-frequency so that the roles of "strong" and "weak" signals are interchanged.

The result of this method is that the two signals need not be separated in frequency, except as required by the limiter path to fulfill the (b) requirement. It is only required that the frequency be known and that the amplitudes of the two signals differ. The degree to which they will be permitted to differ is controlled by the characteristics of the limiter channel and the amount of common-mode rejection obtainable. In addition, alternative detection of the two signals may be achieved by opening or closing the linear channel, depending on whether the weaker or stronger signal's modulation is desired in the receiver output.

It is apropos to quote Granlund (4) in his conclusions after his work on successful detection of the stronger of two signals in FM reception: "...additional information about the desired signal is being neglected if the receiver, in giving an output, does not make use of both the amplitude and frequency variations of the received wave."* The receiver reported in this work does indeed make use of both types of waveform perturbations, although it is unlikely that it represents a realization as Granlund envisioned, for it appears that he intended one carrier to be both amplitude- and frequency-modulated as a means of conveying information from two sources. However, with slight modification the receiver suggested in this report should be capable of handling this situation.

*J. Granlund, Massachusetts Institute of Technology RLE Technical Report No. 42, p. 78, 1949.

CHAPTER II

HISTORICAL BACKGROUND ON THE STUDY OF INTERFERENCE IN FM RECEPTION

A single-tone amplitude modulation communication system is characterized at radio frequencies as a carrier and two sideband frequencies which are separated from the carrier by the frequency of the modulating tone. It is essentially a narrow-band system; i.e., the difference between the highest frequency present and the carrier frequency is of the order of the modulating frequency--in this case, it is exactly so. On the other hand, a single-tone frequency modulation system is characterized by a broad spectrum, the highest frequency in which may differ from the carrier frequency by an amount equal to many times the modulating frequency. If it does, the system may be referred to as a wide-band system. Should the FM case be narrow-banded, Blachman (5) has shown that it offers little advantage over the corresponding AM system insofar as signal-to-noise ratio is concerned. However, if a wide-band FM system is employed, it offers definite improvement in signal-to-noise ratio.

2.1 Recognition of the Problem and Its Early Treatment

While it may have been inevitable that frequency modulation would assume a firm place in the communications art, that Armstrong (6) succeeded in proving its advantages in the face of considered opinion to the contrary was no small achievement. Carson (7) had shown in 1922 that it was fallacious to assume that FM would permit a reduction in bandwidth over AM for comparable performance. He was joined by others, even as late as 1931, in adverse criticism of FM as a desirable means of communication.

The exchange of bandwidth occupancy for improvement in signal-to-noise ratio, however, simply meant that Armstrong saw a way to live with what had previously been regarded as the "enemy" of the time, since most schemes were of a type which suggested bandwidth reduction as a means of improved noise performance. Once the advantages of the broadband system were observed, Carson and Fry (8) wrote a classic paper explaining the reasons for the improvement and giving a detailed treatment of the frequency modulation capability.

Armstrong (6), in describing his system of frequency modulation, wrote, "The basis of the method consists in introducing into the transmitted wave a characteristic which cannot be reproduced in disturbances of natural origin and utilizing a receiving means which is substantially not responsive to the currents resulting from the ordinary types of disturbances and fully responsive only to the type of wave which has the special characteristic.

"The method to be described utilizes a new principle in radio signaling, the application of which furnishes an interesting conflict with one which has been a guide in the art for many years, i.e., the belief that the narrower the band of transmission the better the signal-to-noise ratio. That principle is not of general application. In the present method an opposite rule applies."*

He then proceeded to elaborate on his system which did indeed provide an improvement in signal-to-noise ratio over the restricted-bandwidth AM systems. As was natural, he considered interference levels of thermal noise and shot-effect noise below the signal level of interest.

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*Armstrong, Proc. IRE, Vol. 24, p. 700, May 1936.

He gave an extensive qualitative discussion of his observations for those cases and proceeded to show the industry that at last a practical method for combating static was available.

Even as Armstrong reported his results, much additional experimental work was being carried out at RCA Communications. Crosby (9) reported the results of an investigation of long-range propagation, concluding that, "where multipath transmission is encountered, frequency modulation is impracticable."* While the interest in frequency modulation was high, reports such as Crosby's and that of Eckersley (10), among others, led to the conclusion that, while it might be true that one could exchange bandwidth-occupancy for signal-to-noise ratio, there were to be other problems of interference that would require additional work for their solution. Chief among these was the multipath phenomenon which resulted in more noticeable distortion than was apparent in the AM case. The difficulty lay in the fact that the FM receiver involved a phase-sensitive system, as opposed to amplitude-sensitivity, and as the two nearly-equal carriers varied so as to cause rapid phase shifts, the receiver would detect any such shift as a frequency shift and produce a corresponding audio frequency output. The effect is enhanced under circumstances in which the signals are equal in amplitude, for then each essentially competes for control of the receiver. Corrington (11) and Granlund (4) both reported on this effect.

About a year after the appearance of Armstrong's paper and his own on FM propagation characteristics, Crosby (12) discussed the RCA work on the signal-to-noise ratio characteristics of FM, supporting his theory

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*Crosby, Proc. IRE, Vol. 24, p. 913, June 1936.

with extensive experimentation using a conventional-type FM receiver. He treated the case of two voltage quantities, one representing the cosinusoidally-modulated carrier and the other one component of the noise spectrum, combining them and passing the resultant through a limiter, after which the instantaneous frequency was obtained and studied as the signal-to-noise ratio was varied. Consideration was given to both high and low carrier-to-noise ratios and the behavior compared with the AM situation. This was apparently the first paper to show the plot of calculated waveforms of distortion produced on the instantaneous frequency deviation of the wave composed of the carrier and a single noise component as the carrier-to-noise ratio was changed. Crosby concluded that FM systems offered a S/N improvement over an equivalent AM system only when the carrier-to-noise ratio was high.

While the above discussion has mentioned both noise and distortion, it is not to be concluded that the effects wrought by an FM receiver on either are the same, but that the general method of approach to the study of these effects was the same prior to about 1942. In the present paper interest centers on the two-signal case, but the influence of noise on this normal mode of operation will be considered in Chapter V.

2.2 Recent Developments in the Study of FM Interference

Extensive broadcasting by means of frequency modulation on the 42 to 50 mc/sec band resulted in many cases of common- and adjacent-channel interference, in spite of the advantage which the FM system was able to display by providing better reception in those cases in which a high signal-to-noise ratio could be maintained. Some of the troubles reported were due to the multipath phenomenon, but, because of assignment of the same frequency to several stations, troubles due to common-channel occupancy were also found. This was also true in the 30 to 42 mc/sec band, occupied by police forces. These difficulties were simply a widespread manifestation of the fact that "capture" of an FM receiver was a much more pronounced effect than had ever been observed for the case of signal suppression in AM systems.

A comprehensive study of the problem of FM distortion due to common- and adjacent-channel interference was made by Corrington (11) and reported in 1946. With the systems commonly in use, he found that for carrier-to-noise ratios of less than two or three to one the amount of distortion increased rapidly. This limitation is one which Granlund (4) subsequently was able to improve upon appreciably. However, Corrington reported on numerous instances of long-range interference on the 42-50 mc/sec band as well as on police systems in the 30-42 mc/sec band and then analyzed three cases: (a) the fundamental case of two unmodulated carriers added together; (b) the case in which one carrier was frequency modulated, while the other was not; (c) the case in which both carriers were frequency modulated.

Corrington's analysis of the two-carrier interference situation is summarized in part by the plots in Figure 6. For two carriers, $e_1 \sin \omega t$ and $e_2 \sin (\omega + \delta)t$, having an amplitude relation of $\alpha = e_2/e_1$, the instantaneous frequency disturbance resulting from the time derivative of the argument, $d\theta/dt$, of the resultant linear combination is as depicted for one period of the difference angular frequency, δ, with particular values of α in the range of 0.2 to 1.1. The overall effect, then, of two unmodulated carriers is to create a series of frequency spikes as they alternately reinforce and cancel each other. The rapid shift in phase results in a detected equivalent frequency modulation which produces a distorted audio output whose intensity depends on the amplitude ratio of the two signals and their frequency separation.

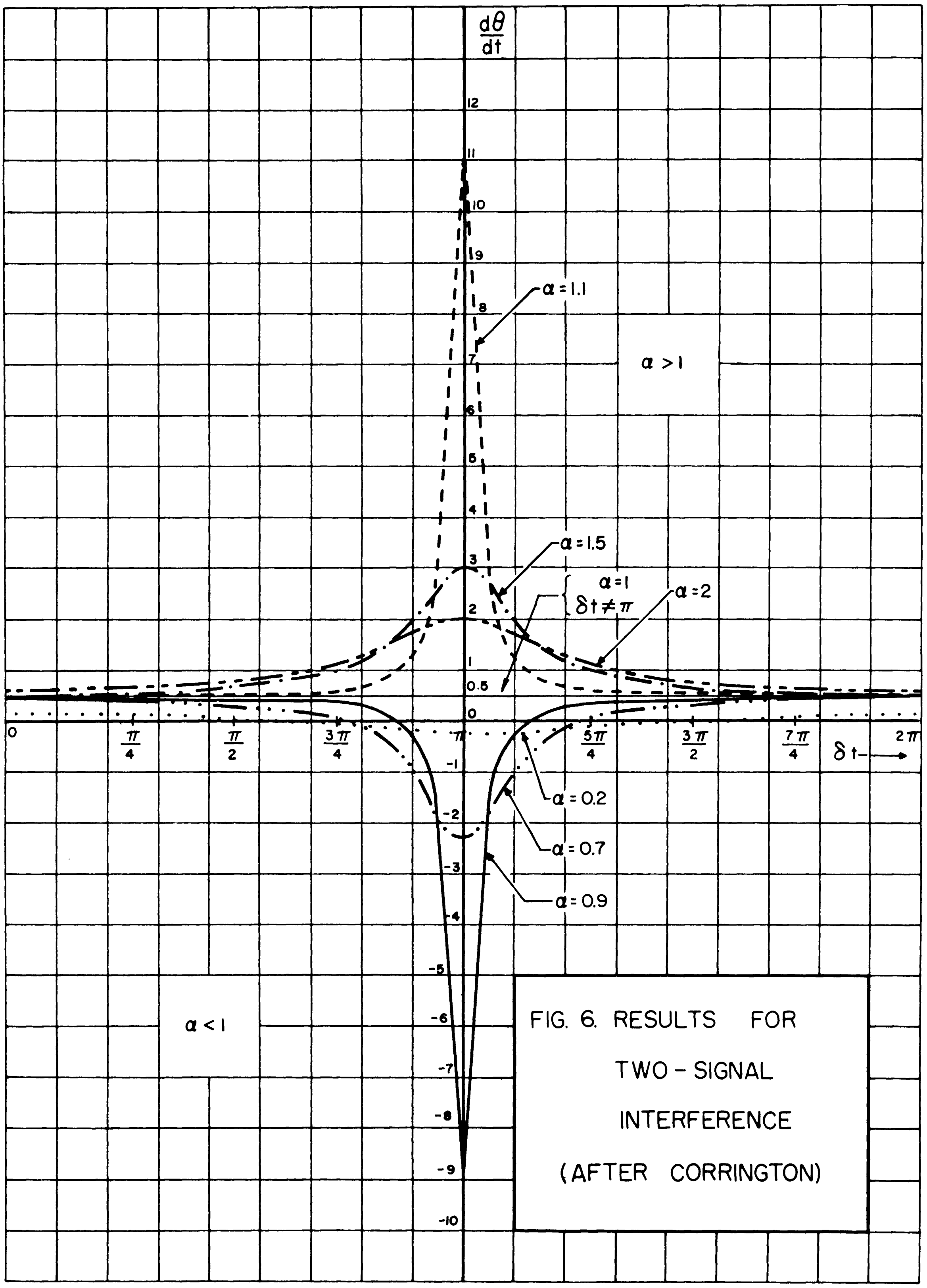

FIG. 6. RESULTS FOR TWO - SIGNAL INTERFERENCE (AFTER CORRINGTON)

In those cases involving modulation on the stronger of the two signals, the frequency spikes occur at a more rapid rate, being then a function of the modulation index, or, for a fixed frequency, varying directly with the frequency deviation.

In 1949, Granlund (4), working on the problem of receiver operation under multipath conditions, showed that a greatly improved signal-to-interference output ratio could be realized for an input consisting of an interfering signal having an rf amplitude approaching that of the desired signal. The reasoning which he followed is given below.

The interference pattern of two carriers of frequency ω and $\omega + \delta$ is shown in the figure, where the signal of frequency ω has unity amplitude and the interfering signal an amplitude of α, where $\alpha < 1$.

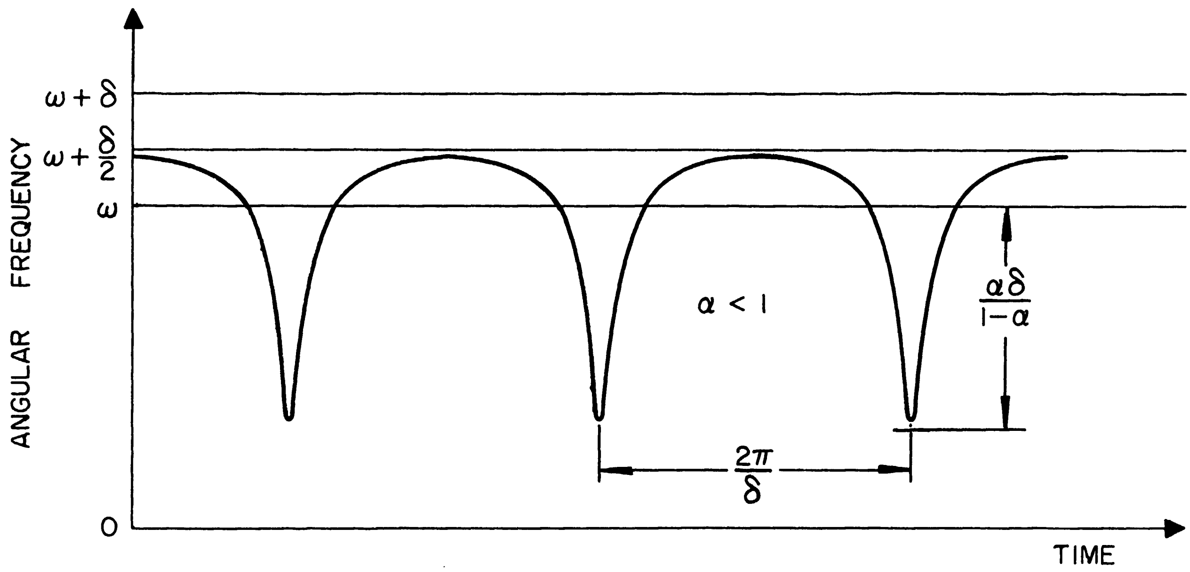

Figure 7. Instantaneous Frequency Disturbance (After Granlund)

The frequency of interest is the average frequency ω. In order to preserve the average value of ω, it is necessary to preserve the entire frequency "spike" having a peak deviation of $\frac{\alpha\delta}{1-\alpha}$ relative to ω. This large deviation is brought about by the rapid shift in phase developed as the voltage corresponding to the $\omega + \delta$ signal passes through 180° with respect to that corresponding to the ω signal.

If, as may happen in multipath situations, the roles of the two signals are interchanged at any time, the "spike" frequency range may cover as much as

$$2\left(\frac{\alpha\delta_{max}}{1-\alpha}\right) + \delta_{max} = \delta_{max}\left(\frac{1+\alpha}{1-\alpha}\right) .$$

By way of illustration, if $\alpha = 0.95$ and the instantaneous frequency separation is that permitted by normal modulation characteristics, i.e., deviations of $\pm$ 75 kc/sec, then substitution will show that the instantaneous deviation range may be as great as 6 mc/sec. This value determines the required bandwidth which must be designed into the limiter and discriminator stages.

The ratio of maximum to minimum amplitude is $\frac{1+\alpha}{1-\alpha}$ so that a limiter dynamic range must be such that it will provide constant output for signals varying by as much as 40:1, in the case of $\alpha = 0.95$. The linear stages must not introduce additional amplitude variations, so they must be flat over the pass band. However, they do not have to accommodate the 6 mc/sec bandwidth; it is undesirable that they be wider than the normal bandwidth of, say, 150 kc/sec.

Granlund built a receiver to demonstrate his theory and was indeed able to show a 30 db output signal-to-interference ratio for $\alpha = 0.95$.

As a result of his examination of the spectrum of two carriers after limiting, which spectrum is shown in Figure 8, he concluded that the components of the stronger and weaker signals lying within the receiver pass band enjoyed an effectively smaller ratio "α" at the output than had existed at the input to the limiter. For small relative interfering signal levels the improvement was as much as two to one. Repeating the process of limiting and filtering would, then, apparently reduce the interference component in the pass band to any desired value. This conclusion apparently formed the basis for Baghdady's study of narrow-band limiting in 1956.

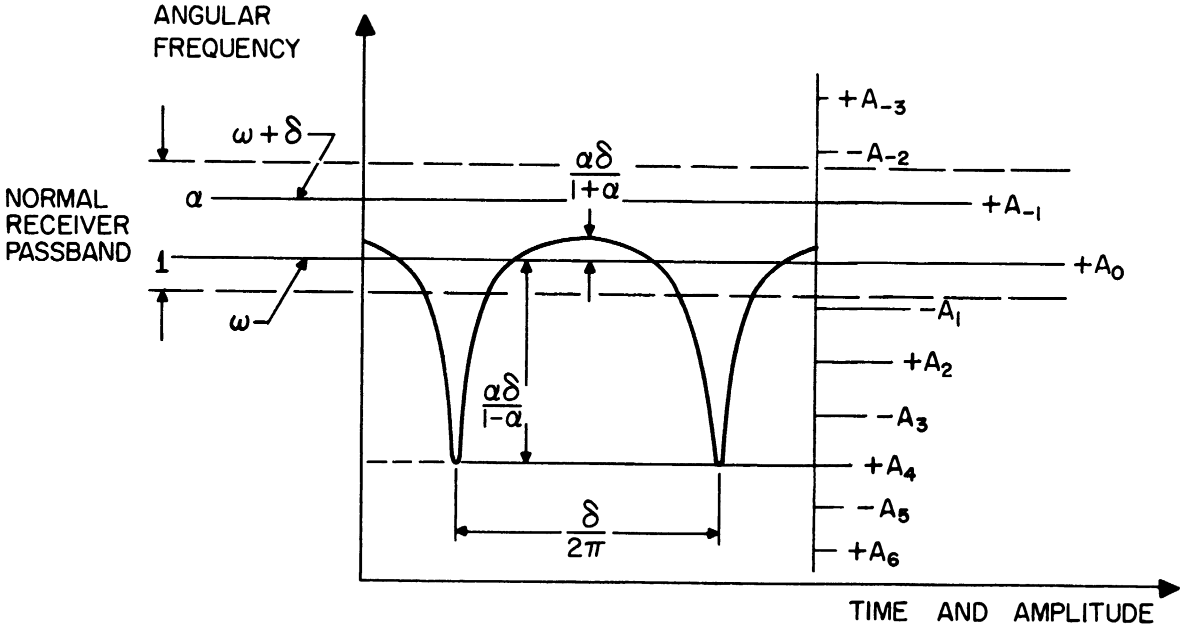

Figure 8. Input and Output Spectra for α = 0.8 (After Granlund)

Baghdady (13), following Granlund's earlier work, succeeded in showing theoretically that, for values of $\alpha < 0.84$, cascading ideal limiters and filters of one IF bandwidth would permit the reduction of the intensity of the interference component to any desired level. For values of $\alpha > 0.84$ the required bandwidth increases until at $\alpha \geq 0.95$ an ideal limiter-filter bandwidth of three times the IF bandwidth is required.

In a later paper, Baghdady (14) showed that the spectrum approach could also be exploited in another way to enhance the capture effect. By appropriate application of a narrow-band, positive feedback loop around the limiter stages, acting so as to discriminate and enhance the desired strong signal component, the value of "α" at the output of the limiter could be increased over that at the input. Auxiliary results are a lowering of the limiting threshold and an automatic inter-station noise squelch. This work was being explored experimentally in 1957 (15).

A basic requirement in both Baghdady and Granlund's works is that the frequency difference, δ, be large compared with the modulation rate on the stronger signal frequency, ω. If this is not the case, the low-frequency beat disturbance will fall inside the receiver pass band and will be detected. This restriction is discussed further in Chapter V.

Through the years since 1930 there has been a continued interest in the FM interference problem. Several methods of attack upon the two-signal case have been made, and some solutions proposed (16-21). In recent years, noise effects in FM systems have been receiving attention (22-25), due to an increased amount of effort going into the solution of noise problems in general, but also due to the use of noise-like signals as the modulation in some communication systems.

As a result of this brief survey of the literature bearing on the problem discussed in this thesis, the primary conclusion is that techniques are now available for freeing the stronger of two co-channel signals from both the amplitude and frequency perturbations imparted to it by the weaker. With this possibility available, it is feasible to obtain this signal and subtract it from the linear combination of the two signals for the purpose of detecting the modulation of the weaker carrier.

CHAPTER III

ANALYSIS OF ALTERNATIVE DETECTION OF TWO CO-CHANNEL FREQUENCY-MODULATED SIGNALS

3.1 General Analysis

Since the basis for the method of weak-signal detection under study in this paper is the subtraction of a practically undistorted version of the strong signal from the composite waveform, it should prove helpful to examine the means by which the strong signal can be freed from interference by the weak. In order to do this, consideration will first be given to the nature of the composite signal.

3.1.1 The Sum of Two Sinusoids. Although an elementary problem, consider the sum of two carriers,

$$e_1(t) = \mathrm{Re}\left[E_1 \exp j\omega t\right] \qquad (3\text{-}1)$$

and

$$e_2(t) = \mathrm{Re}\left[E_2 \exp j(\omega+\delta)t\right] , \qquad (3\text{-}2)$$

added as phasors as shown in Figure 9. The magnitude of the resultant, E, is readily obtained by the law of cosines to be

$$|E| = \sqrt{E_1{}^2 + E_2{}^2 + 2\,E_1E_2 \cos \delta t}\ . \qquad (3\text{-}3)$$

The resultant real time function is seen from the diagram to be

$$e(t) = |E| \cos (\omega t + \theta) \qquad (3\text{-}4)$$

with

$$\theta = \theta(t) = \tan^{-1} \frac{E_2 \sin \delta t}{E_1 + E_2 \cos \delta t} \qquad (3\text{-}5)$$

We introduce $\alpha = \frac{E_2}{E_1}$ and now write for Equations 3-3, 3-4, and 3-5 as follows:

$$e(t) = E_1 \sqrt{1 + \alpha^2 + 2\alpha \cos \delta t} \cos (\omega t + \theta) \tag{3-6}$$

and

$$\theta = \tan^{-1} \frac{\alpha \sin \delta t}{1 + \alpha \cos \delta t} \tag{3-7}$$

It is significant to notice in Equation 3-6 that the zeros of e(t) are not produced by the radicand when $\alpha \neq 1$. The zeros are determined, then, by the cosine factor. Figure 9 illustrates the case, since, if $E_2 < E_1$, all possible variations in δt can only cause the tip of the E phasor to describe a circle which does not enclose the origin. Thus we see that $(\omega t + \theta)$ is always within $\pm \pi/2$ of ωt, as long as $\alpha < 1$.

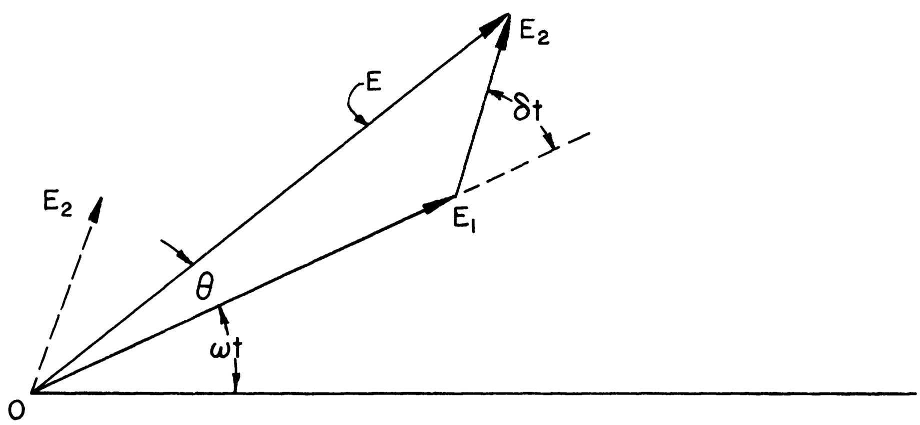

Figure 9. The Sum of Two Unequal Carriers

A transition occurs, of course, if E_2 becomes greater than E_1 and the average frequency of the system is then controlled by the frequency of E_2.

An FM receiver detects zero crossings, or, more precisely, the rate at which zero crossings occur. If the average frequency being detected is that of E_1, the larger signal, then the fluctuations of E_2 do not contribute to the average in a linear combination of the two signals. The capability of a practical receiver to detect and follow only the average number of zero crossings and their occurrence rate is a measure of its susceptibility to capture by the stronger signal.

The plot of Figure 7, from Granlund (4), shows the average frequency, ω, but also displays clearly the fact that the weaker signal is causing a disturbance insofar as the instantaneous frequency is concerned. By instantaneous frequency is meant the usual definition of the time derivative of the phase of the signal, as, for example,

$$\omega_i = \frac{d(\omega t)}{dt} = \omega \quad \text{radians/second} \tag{3-8}$$

The term, $d\theta/dt$, represents the amount by which the instantaneous frequency of the resultant phasor differs from the frequency of the stronger signal. Thus, we have

$$\frac{d\theta}{dt} = \alpha\delta \frac{\alpha + \cos \delta t}{1 + \alpha^2 + 2\alpha \cos \delta t} \tag{3-9}$$

The frequency disturbance is seen to increase with both α and δ, the relative size of the interfering signal and the frequency difference between the two carriers, respectively.

From Equation 3-9 and the diagram (Figure 9) an expression for the worst case of frequency interference--that is, largest departure from the average frequency--can be determined. The most rapid changes of phase occur as the two signals are passing through a phase difference of 180°, so from Equation 3-9,

$$\left.\frac{d\theta}{dt}\right|_{\pi} = -\left(\frac{\alpha\delta}{1-\alpha}\right) \tag{3-10}$$

This value is shown on Figure 7.

With regard to the average influence exerted on the stronger signal by the weaker, consider the average value of $d\theta/dt$ (Equation 3-9) over a half-period of the interference repetition frequency. Thus,

$$\left.\frac{d\theta}{dt}\right|_{avg} = \frac{\alpha\delta}{\pi}\left\{\int_0^{\pi}\frac{\alpha\, d(\delta t)}{1+\alpha^2+2\alpha\cos\delta t}+\int_0^{\pi}\frac{\cos\delta t\, d(\delta t)}{1+\alpha^2+2\alpha\cos\delta t}\right\} \quad (3\text{-}11)$$

Evaluation of the second integral yields, upon collection of terms,

$$\left.\frac{d\theta}{dt}\right|_{avg} = \frac{\alpha\delta}{\pi}\left\{\frac{\pi}{2\alpha}+\frac{\alpha^2-1}{2\alpha}\int_0^{\pi}\frac{d(\delta t)}{1+\alpha^2+2\alpha\cos\delta t}\right\} \quad (3\text{-}12)$$

This becomes

$$\left.\frac{d\theta}{dt}\right|_{avg} = \frac{\alpha\delta}{\pi}\left\{\frac{\pi}{2\alpha}-\frac{1-\alpha^2}{2\alpha}\left[\frac{2}{1-\alpha^2}\left(\frac{\pi}{2}\right)\right]\right\} = 0 \quad (3\text{-}13)$$

Thus the <u>average</u> value of the frequency interference is zero over the period of the difference frequency, which is to say that the presence of a co-channel weak signal does not result in altering the average frequency of the stronger signal. This determination is, of course, made from the point of view of $\alpha < 1$.

In summary, an interfering signal results in a composite waveform having both amplitude and frequency alteration when viewed with respect to a desired strong signal. However, the nature of the disturbance created is such that it suggests methods of dealing with it so that the average frequency of the desired signal may be preserved and the unwanted signal discriminated against. In Section 2.2 it was shown that Granlund's technique was to accommodate the entire frequency range resulting from the rapid phase changes so that the average frequency of the desired signal was retained and detected, while Baghdady's procedure involved the judicious discarding of portions of the spectrum after limiting so that the average value of the stronger carrier frequency was retained. This was accomplished by means of a system of cascaded narrow-band limiters and filters. The amplitude variation was handled in the usual way, by having good amplitude limiting action. Thus, the very nature of co-channel FM interference permits obtaining the stronger signal as free of both amplitude and frequency disturbances as may be desired.

3.1.2 Analysis of Two Frequency-Modulated Interfering Signals. At this point, some additional insight into the general problem may be had by examining the general case of two modulated carriers within the pass band of the receiver.

Consider two normalized angle-modulated waves as

$$e_1(t) = \exp j \left[\omega_1 t + m_1(t)\right] \tag{3-14}$$

and

$$e_2(t) = \exp j \left[\omega_2 t + m_2(t)\right] \tag{3-15}$$

where ω_1 and ω_2 are two carrier frequencies having respective phase modulations of $m_1(t)$ and $m_2(t)$ which are themselves real time functions. The time derivatives of the phase modulations would be the equivalent frequency modulations. Let $e_2(t)$ be multiplied by α, a real constant whose magnitude lies between zero and one. We can now write the summation as

$$e_1(t) + \alpha\, e_2(t) = \left[1 + m_3(t)\right] \exp j \left[\omega_1 t + m_4(t)\right] \tag{3-16}$$

where $m_3(t)$ is an apparent amplitude variation on the mean value of the sum of the two modulated waves and $m_4(t)$ is the resultant phase modulation with respect to the unmodulated angular carrier frequency, ω_1.

It is readily appreciated that the equivalent expression on the right-hand side of Equation 3-16 is a wave possessing both amplitude and phase information. That is, there are present in the resultant waveform components of varying amplitude and frequency, having the phase-altered components as shown in the exponent.

The new modulation functions, $m_3(t)$ and $m_4(t)$, can now be determined in terms of the quantities which specify the original two waves. Let

$$\beta(t) = (\omega_2 - \omega_1)t + m_2(t) - m_1(t) \ . \tag{3-17}$$

We can now write the left-hand side of Equation 3-16 as

$$e_1(t) \left[1 + \alpha \frac{e_2(t)}{e_1(t)}\right] = e_1(t) \left[1 + \alpha \exp j\beta(t)\right] \tag{3-18}$$

and determine the logarithm of the bracketed factor as

$$\log \left[1 + \alpha \frac{e_2(t)}{e_1(t)}\right] = \log \left[1 + \alpha \exp j\beta(t)\right] \tag{3-19}$$

$$= \log \left[1 + \alpha \cos \beta(t) + j\alpha \sin \beta(t)\right] \tag{3-20}$$

$$= \log \sqrt{\left[1 + \alpha \cos \beta(t)\right]^2 + \left[\alpha \sin \beta(t)\right]^2} + j\theta \tag{3-21}$$

where

$$\theta = \tan^{-1} \frac{\alpha \sin \beta(t)}{1 + \alpha \cos \beta(t)} \tag{3-22}$$

By use of the relation in Equation 3-21, we can write the logarithm of the left-hand side of Equation 3-16 as

$$\log e_1(t) + \log \left[1 + \alpha \frac{e_2(t)}{e_1(t)}\right] = j\omega_1 t + jm_1(t) + \log \left[\left[1 + \alpha \cos \beta(t)\right]^2 + \left[\alpha \sin \beta(t)\right]^2\right]^{1/2} + j\theta \tag{3-23}$$

and the right-hand side of Equation 3-16 as

$$\log \left\{ \left[1 + m_3(t)\right] \exp j \left[\omega_1 t + m_4(t)\right] \right\} = \log \left[1 + m_3(t)\right] + j\omega_1 t + jm_4(t) \qquad (3\text{-}24)$$

By equating the right-hand sides of Equations 3-23 and 3-24, we find the imaginary terms show that

$$m_4(t) = m_1(t) + \tan^{-1} \frac{\alpha \sin \beta(t)}{1 + \alpha \cos \beta(t)}, \qquad (3\text{-}25)$$

and that the real terms yield

$$m_3(t) = \sqrt{\left[1 + \alpha \cos \beta(t)\right]^2 + \left[\alpha \sin \beta(t)\right]^2} - 1 \qquad (3\text{-}26)$$

This suggests that a likely procedure for recovering the modulation of either signal could be carried out following detection. Consider a rearrangement of Equations 3-25, 3-17, and 3-26:

$$m_1(t) = m_4(t) - \tan^{-1} \frac{\alpha \sin \beta(t)}{1 + \alpha \cos \beta(t)} \qquad (3\text{-}27)$$

$$m_2(t) = m_1(t) - (\omega_2 - \omega_1)t + \beta(t) \qquad (3\text{-}28)$$

and

$$\beta(t) = \cos^{-1} \left\{ \frac{1}{2\alpha} \left[\left[1 + m_3(t)\right]^2 - 1 - \alpha^2 \right] \right\}. \qquad (3\text{-}29)$$

These forms of the expression for the quantities of interest were originally communicated to the author by Gunnar Hok. As can be seen, accurate knowledge of α, $m_3(t)$, $m_4(t)$, and the two carrier frequencies will permit the proper determination of the original modulations, $m_1(t)$ and $m_2(t)$. Two methods of approach to the problem on this basis are given in the appendix.

3.2 Detection of the Modulations

From Equation 3-25, a previously discussed situation can be recognized. The resultant output of a phase detector would yield $m_1(t)$ accompanied by a phase disturbance in the form of $\tan^{-1} \dfrac{\alpha \sin \beta(t)}{1 + \alpha \cos \beta(t)}$.

The derivatives of these functions with respect to time would give us the frequency modulation of the stronger signal, $e_1(t)$, plus a quantity which, in Equation 3-9, was $d\Theta/dt$. We have seen that the effects of this disturbance can be minimized. In ordinary FM transmission, $m_1(t)$ may easily represent a phase change of hundreds of radians, while the instantaneous disturbance of phase can be no greater than $\pi/4$ radian for α no greater than 0.5 (see Equation 3-25). Of course, the magnitude of the disturbance rises rapidly as α approaches unity, but α does have to do just that before the distortion due to the rapid changes in phase can even begin to compare with the very large phase shifts experienced in the presence of normal deviations.

The passage of the waveform represented by Equation 3-13 through an ideal limiter removes the amplitude variation and any information contained therein. If the instantaneous frequency is determined as the time derivative of the limited waveform, having zero crossings fixed by the equivalent modulation function, $m_4(t)$, there results

$$\omega_i = \omega_1 + \dot{m}_1(t) + \frac{(\omega_2 - \omega_1) + \dot{m}_2(t) - \dot{m}_1(t)}{1 + \dfrac{\cos\beta(t) + 1/\alpha}{\cos\beta(t) + \alpha}} \qquad (3\text{-}30)$$

where the dotted quantities represent time derivatives, being obtained from the original phase modulating functions in Equations 3-1 and 3-2.

The low-frequency output, after filtering, becomes

$$\omega_\ell = \dot{m}_1(t) + \frac{(\omega_2 - \omega_1) + \dot{m}_2(t) - \dot{m}_1(t)}{1 + \dfrac{\cos\beta(t) + 1/\alpha}{\cos\beta(t) + \alpha}} \qquad (3\text{-}31)$$

with the second term now representing the low-frequency disturbance of the desired strong signal modulation. As a check on the expression, consider the limiting cases of α very large (i.e., greater than unity, implying that, for the time being, the earlier restriction to values less than unity is lifted) and α very small.

(1) Very large values of α greater than unity mean that $\alpha e_2(t)$ greatly exceeds $e_1(t)$ and it should be expected that the modulation, $\dot{m}_2(t)$, should predominate in the low-frequency output. Equation 3-31 shows that, when the fraction in the denominator can be neglected, the output does consist of $\dot{m}_2(t)$ and a beat frequency component due to the difference between the two carriers.

(2) As α tends to zero, $\alpha e_2(t)$ also tends to zero and the output is dominated by the modulation of $e_1(t)$, or consists of $\dot{m}_1(t)$.

(3) If α is allowed to be unity, both signals appear with an intensity determined by their respective deviations. The output then takes the form

$$\omega_\ell = \frac{\dot{m}_1(t)}{2} + \frac{\dot{m}_2(t)}{2} + \frac{(\omega_2 - \omega_1)}{2} \tag{3-32}$$

whenever $\delta t \neq (2n+1)\pi$ for $n = 0, 1, 2, \ldots$

Perhaps some indication of the size of the beat-note to be expected should be made. If the difference frequency be regarded as a modulation frequency capable of causing a certain amount of phase modulation, it is found that the amount of modulation is a definite function of the amplitude ratio α. Consider $\alpha = 0.5$, as shown in Figure 10. The steady-state phase modulation would amount to $\pm$ 30° and would occur at the difference frequency. This phase change would cause a frequency deviation of

$$\Delta f = f_{mod} \times \phi \tag{3-33}$$

where ϕ is the change in phase and f_{mod} is the difference frequency. We have tacitly assumed carriers otherwise unmodulated. In this case, then,

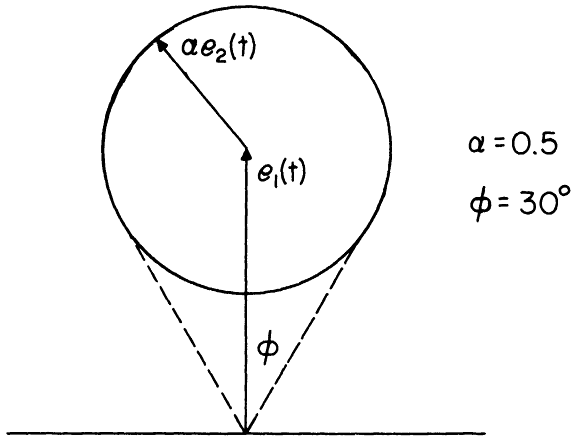

Figure 10. Phase Modulation by Difference Frequency

a 50 kc separation of carriers would result in a deviation of about 26 kc, while a 50 cycle separation would effect a deviation of 26 cycles. The output due to this type of beat-note can be seen to be of small deviation when within the audio-frequency portion of the spectrum and consequently is of small amplitude as compared with the deviations characteristic of program material. The customary de-emphasis network serves to attenuate the beat-note intensity when the frequency separation is such as to cause the interference to fall in the high end of the audio-frequency spectrum, where the larger deviations obtain. The magnitude of this type of interference as a function of the difference frequency is plotted in Chapter V.

3.3 A New Method of Weak-Signal Enhancement

In the foregoing it has been shown that the susceptibility of an FM receiver to capture by the stronger signal is facilitated by the inherent nature of two-signal interference and by proper receiver design to utilize this characteristic. It has also been demonstrated that the separation of the modulations on two unequal carriers within the pass band of the receiver should be possible. Methods of accomplishing this task subsequent to detection are faced with the problem of maintaining careful account of the relative phases of the two modulated carriers or the detected envelope variations in order to provide satisfactory correlation between the two detected signals and so permit suitable combination at the output of the receiver. This problem is alleviated to an appreciable extent if the correlation--subtraction--is accomplished at the intermediate frequency of the receiver. A method for doing this is shown in Figure 11.

The action of the circuit is as follows: Consider at point (1) the simultaneous presence of two voltages as,

$$e_{(1)} = e_1(t) + \alpha\, e_2(t)$$

which summation voltage is fed to the two paths, "A" and "B", having parallel inputs. Path "A" is a cascade of linear amplifier stages whose function is to provide, at point (2), a voltage having an instantaneous phase insofar as $e_1(t)$ is concerned, which differs from the output of Path "B" by 180° at all times. The amplitude of the output must also be correct, since it is to be used to cancel the limiter output. Path "B" is to perform the function of delivering a voltage, $e_1(t)$, to point (3), operating in the manner prescribed by Granlund (4) or Baghdady (13), if necessary, and then, of course, subject to their restrictions.

The adder stage receives inputs of $e_1(t) + \alpha e_2(t)$ and $- e_1(t)$, adds them linearly and delivers, in the absence of noise, $\alpha e_2(t)$ to the remainder of the system which comprises a conventional FM receiver. It would be expected that $\alpha e_2(t)$ would capture the receiver proper and result in a satisfactory audio frequency output representing the modulation of the weak signal carrier.

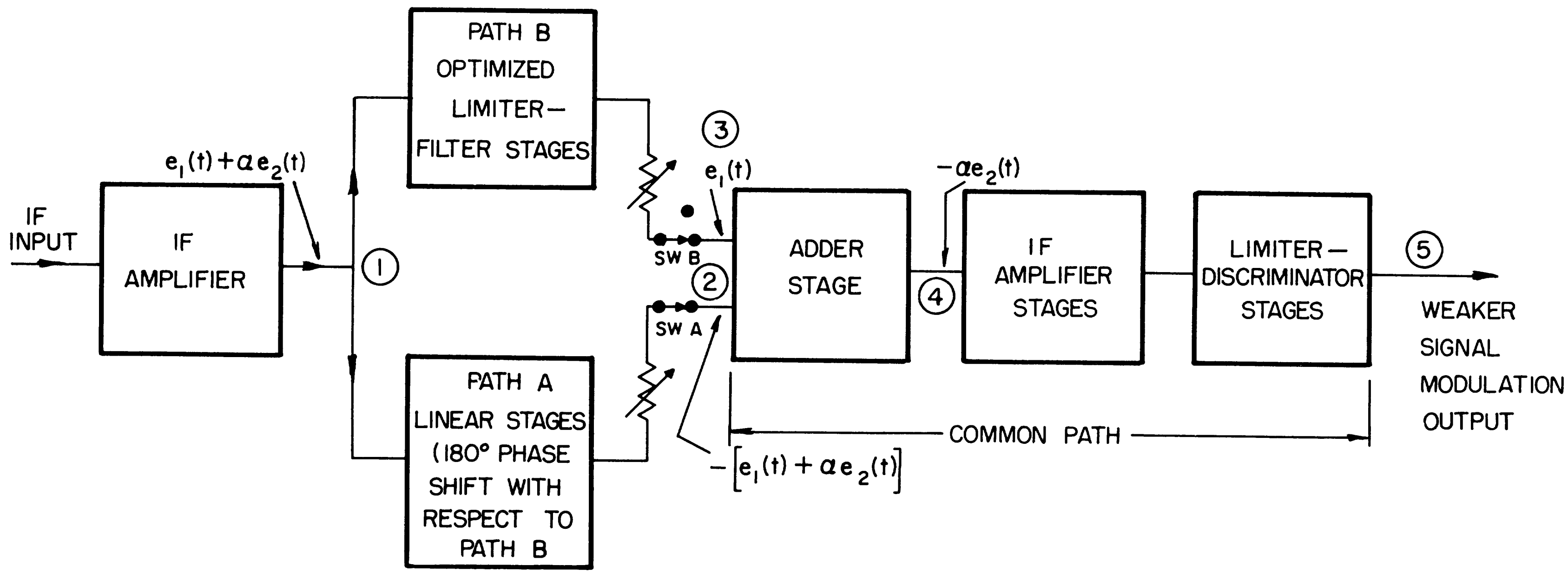

FIG.11. AMPLITUDE DISCRIMINATORY RECEIVER BLOCK DIAGRAM

There are several considerations on which the successful implementation of the configuration shown in Figure 11 depends. These may be itemized as follows:

(a) With regard to $e_1(t)$, the overall gain-frequency characteristics of the two paths must be as nearly identical as possible. Since the output of the limiter is virtually constant at the center frequency, the adjustment is most conveniently made in Path "A" by means of either an automatic- or a manual-gain-control arrangement.

(b) The IF must be chosen so that the harmonic content of the limiter output is sufficiently below any value of $\alpha e_2(t)$ expected at point (4).

(c) The gain of Path "A" must not be such as to cause harmonic or intermodulation distortion of the original input voltages; it must be a <u>linear</u> path.

(d) The second set of limiters should permit ready capture by the $\alpha e_2(t)$ signal as soon as it exceeds the residual voltage representing $e_1(t)$.

The only serious restriction on frequency separation of the two signals is that placed on the operation of the limiter path, viz., that the separation be greater than the modulation rate. Even this requirement can be relaxed somewhat as α is reduced and the common-mode rejection ratio improved (see Chapter V).

Alternative detection is realized by opening the switch, (SW A), which disconnects the linear path and permits the limited signal, $e_1(t)$, to be received through a cascade of limiters.

CHAPTER IV

EXPERIMENTAL VERIFICATION OF THE ANALYSIS

4.1 Experimental Receiver Design Considerations

An experimental receiver which embodied the arrangement shown in Figure 11 was constructed for the purpose of demonstrating the basic idea of weak-signal enhancement. In order to facilitate construction and adjustment, some compromises were made. These were as follows:

(a) The receiver would operate at the intermediate frequency on the assumption that the radio-frequency amplifier and frequency converter of any practical receiver would represent linear stages over the pass band.

(b) The intermediate frequency would be low enough to permit ready viewing of its waveform on laboratory oscilloscopes without undue loading problems, while being high enough to permit satisfactory filtering of harmonics.

(c) A low frequency IF would ease the difficulties associated with construction and trouble-shooting at conventional FM intermediate frequencies.

(d) Only a moderate degree of amplification would be required since signal generators of adequate output would be available for test purposes.

(e) Because the adjustment of phase was to be very critical, a minimum number of synchronously tuned stages would be used.

As a result of these considerations, a receiver was built having a two-megacycle intermediate-frequency. Two preamplifier stages were used to amplify the input signals to the point at which the stronger one could drive the limiter path stages effectively and still permit adequate variation of the ratio of the weaker-to-stronger signal amplitudes.

Isolation of the two parallel paths was obtained by the use of cathode followers and an additional cathode follower was used as an active adder. The lowered output impedance thus effected permitted the use of a carbon potentiometer in each path for the adjustment of the level at the input to the adder stage.

It is evident from the circuit diagram, Figure 12, that the limiter and linear paths both contain odd numbers of stages. With a symmetrical limiter there would probably be an odd number in one path and an even number in the other. However, in the receiver which was built it was found that the outputs were such that they were more nearly in phase than out of phase, and the conversion of the limiter input stage from a cathode follower to an amplifier provided the necessary flexibility to permit phase adjustment for satisfactory cancellation in the adder.

The common-path portion is straightforward, comprising the adder, an IF amplifier, two stages of limiting, a discriminator and an audio amplifier. Operation would no doubt be improved, particularly for a greater variety of input signal ratios, if more gain were provided ahead of the limiter stages. However, operation was satisfactory for the purposes of the test and no further effort was made to incorporate such additional amplification.

Provision is made for removing one or the other of the parallel paths by merely opening the supply voltage switch in the proper path. The "paths" and the balancing potentiometer can be seen in photographs of the laboratory unit, Figures 13 and 14.

The pertinent characteristics of the receiver are shown in Figures 15, 16, 17, and 18. The limiter characteristic (Figure 15) is

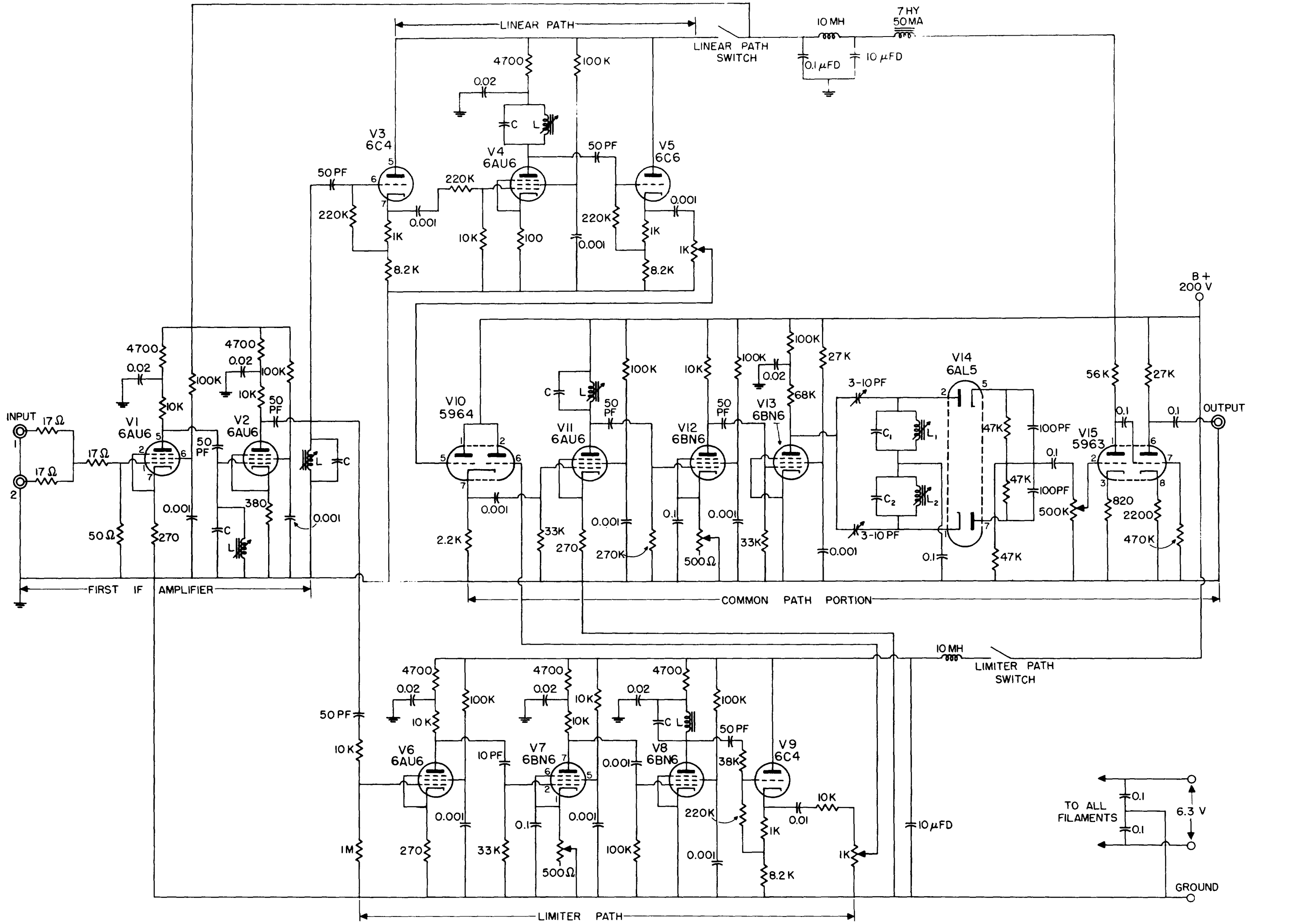

FIG. 12.

CIRCUIT OF LABORATORY MODEL AMPLITUDE-DISCRIMINATORY RECEIVER

NOTES: ALL CAPACITORS IN μFD UNLESS OTHERWISE SPECIFIED.
ALL RESISTORS 0.5 WATT.
ALL L's = 100 MH EACH, APPROXIMATELY.
ALL C's = 15 PF EACH, APPROXIMATELY.
TOTAL PLATE CURRENT DRAIN IS 80 MA.

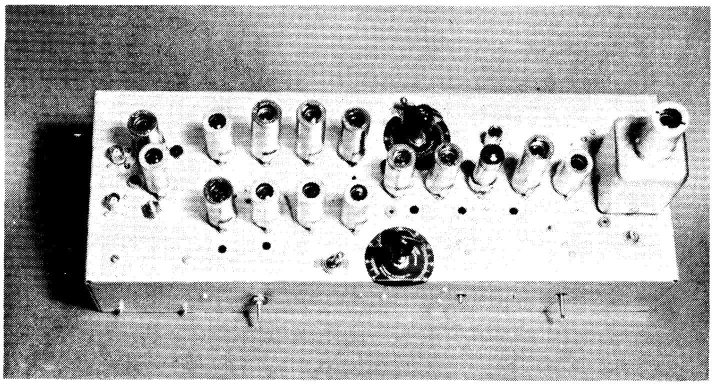

FIG. 13. TOP VIEW OF AMPLITUDE DISCRIMINATORY RECEIVER.

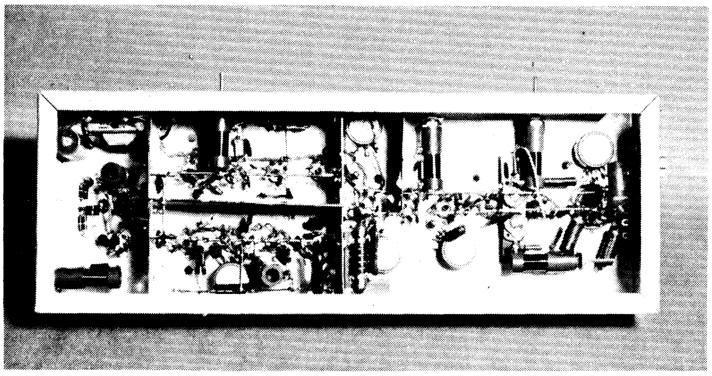

FIG. 14. UNDER-CHASSIS VIEW OF AMPLITUDE DISCRIMINATORY RECEIVER.

given for rms input voltages existing at the grid of the input IF amplifier, the output being measured at the limiter-path cathode follower. The 6BN6 gated-beam amplifier is exceptionally well suited to this application, the bias adjustment of V7 allowing precise control over the minimization of the amount of amplitude variation that can be obtained in the path's output. The usual region of operation for this receiver was about the 10- to 15-millivolt level indicated on Figure 15. The common-path portion's limiter stages were similar and provided approximately the same characteristic.

The discriminator is of the type that involves no mutual coupling to the limiter and so requires that one diode be reversed from the other. The impedances of the two tank circuits differ because of the requirement for equal Q's at different frequencies and the coupling capacitors are adjustable to allow for the necessarily differing driving voltages. The theoretical discriminator characteristic is shown in Figure 16, with a photograph of the actual characteristic given for comparison in Figure 17. The bandwidth is wide compared with the IF stages in order that the discriminator itself play no unique part in the unusual detection mode employed.

The most critical adjustment of the receiver is that of the overall gain-versus-frequency characteristic of each path. This adjustment determines the degree of common-mode rejection obtainable at the adder output, since the phase characteristic is inseparable from the amplitude response. It is apparent that the limiter tends toward broad-band behavior, amplifying off-frequency weak signals more than does the linear path. However, by suitably adjusting the loading on the several stages the characteristics of Figure 18 were obtained. These represent steady-state response and resulted in a common-mode rejection value of greater

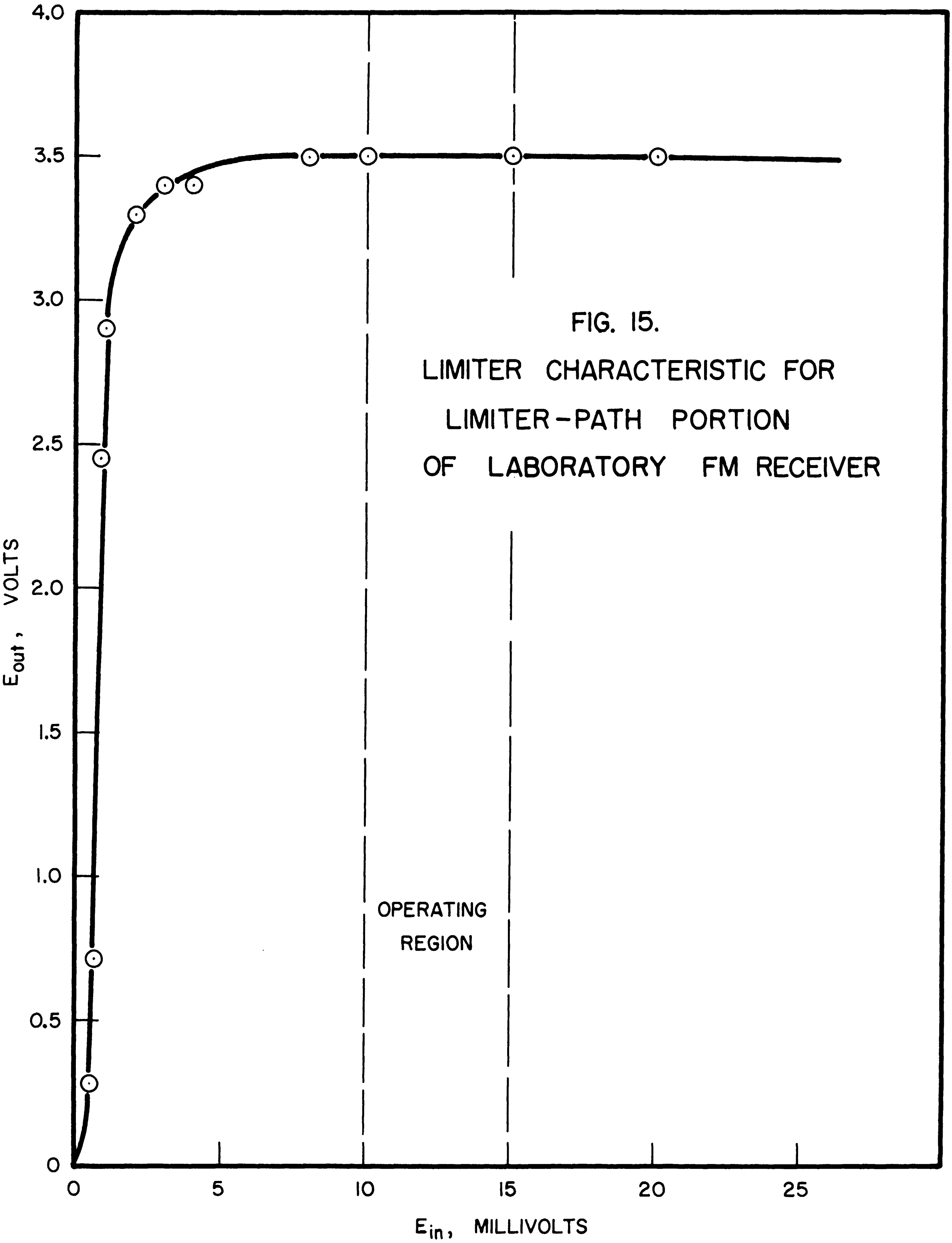

FIG. 15.
LIMITER CHARACTERISTIC FOR LIMITER-PATH PORTION OF LABORATORY FM RECEIVER

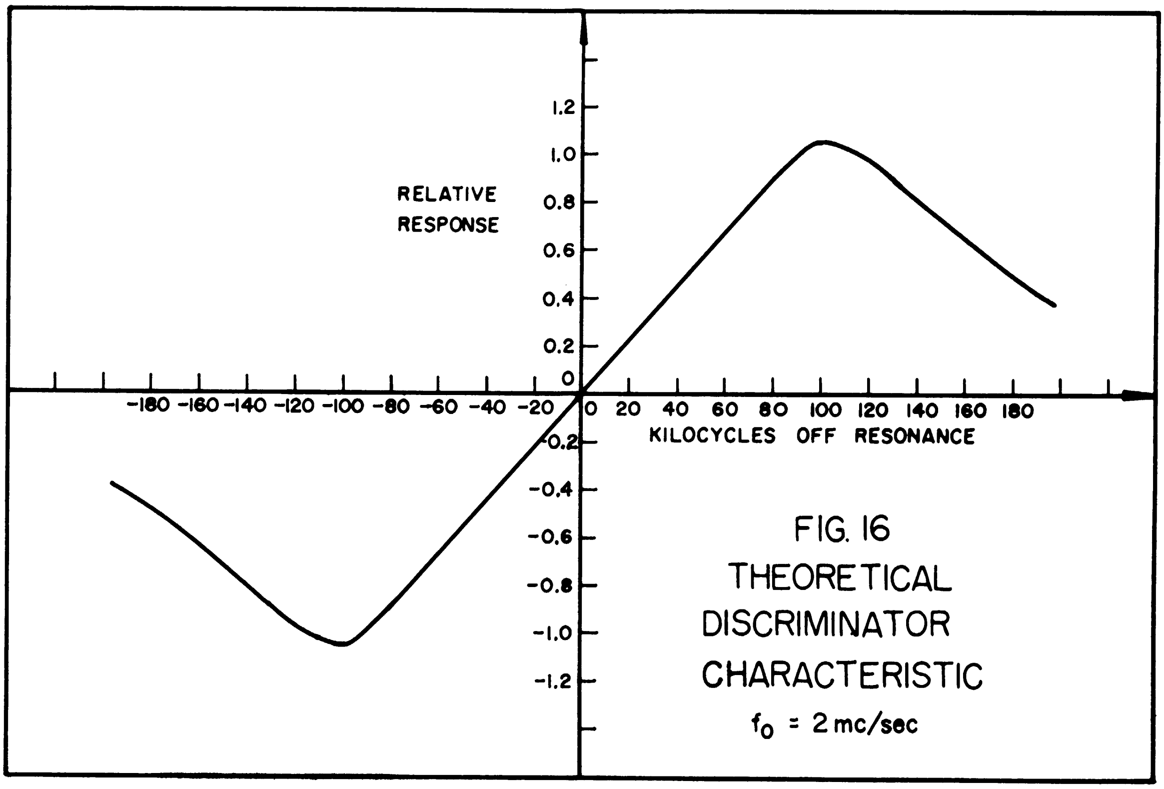

FIG. 16
THEORETICAL
DISCRIMINATOR
CHARACTERISTIC
f_0 = 2 mc/sec

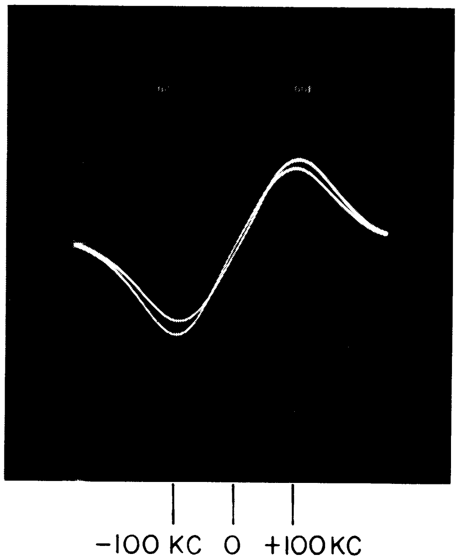

FIG. 17. ACTUAL DISCRIMINATOR CHARACTERISTIC.

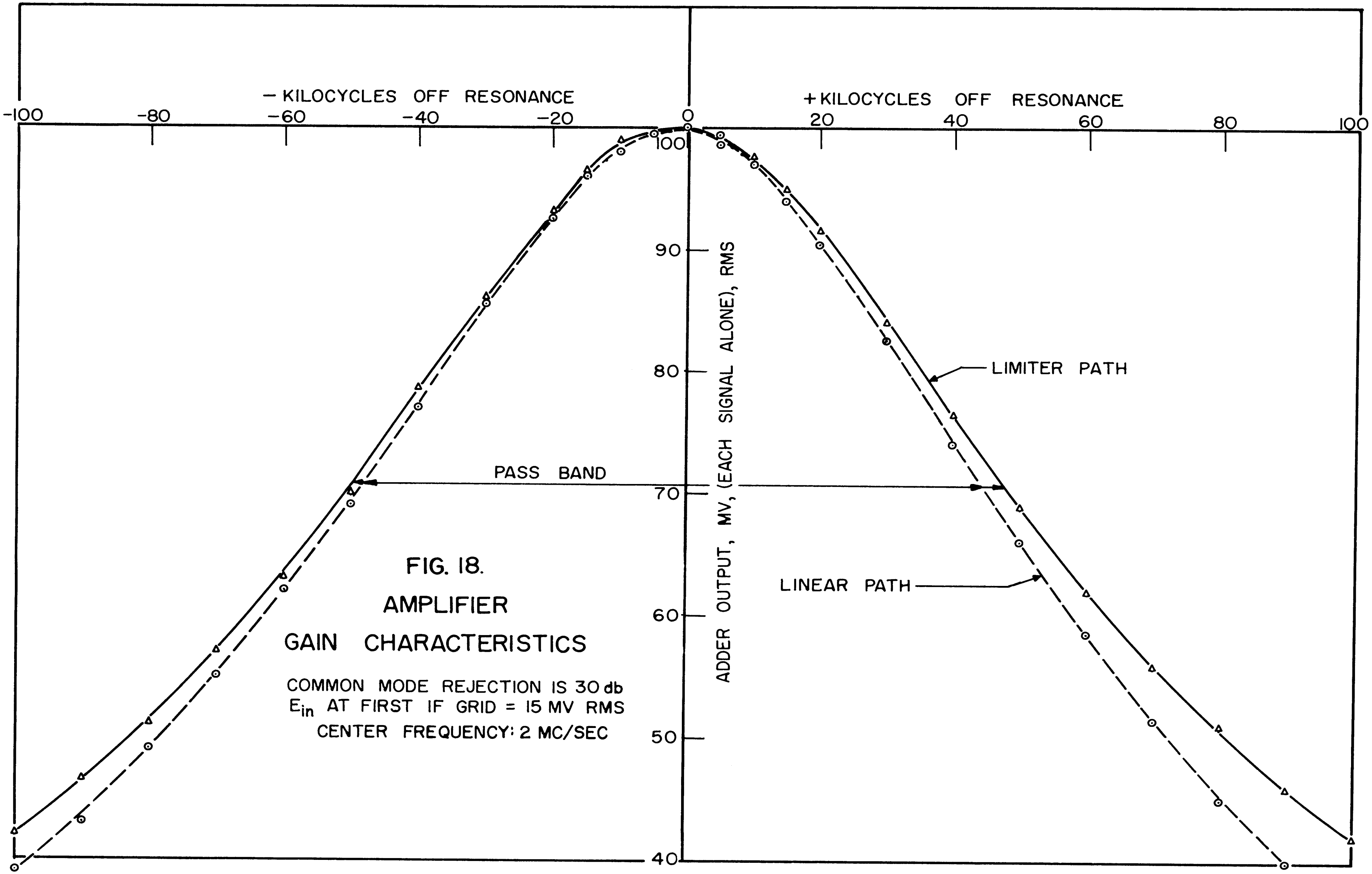

FIG. 18.
AMPLIFIER
GAIN CHARACTERISTICS

COMMON MODE REJECTION IS 30 db
E_{in} AT FIRST IF GRID = 15 MV RMS
CENTER FREQUENCY: 2 MC/SEC

than thirty decibels. The same amount of rejection was obtained under modulated conditions for deviations of up to twelve kilocycles for sinusoidal modulation rates of up to about five kilocycles/second.

The "linear" path was linear for amplitudes varying over a range of $\pm$ 5 db about the mean input level of 15 millivolts, while the "limiter" path limited effectively for signals varying by $\pm$ 10 db about the same mean.

4.2 Test Procedures

A block diagram of the laboratory test arrangement is shown in Figure 19 and a photograph of the bench arrangement in Figure 20. Tests were run with CW signals, separate tone-modulated signals, separate speech-modulated signals, and with speech on the weaker carrier while AF noise was applied to the stronger carrier.

The receiver adjustments were made as follows:

(a) With a strong-signal unmodulated carrier of from ten to fifteen millivolts at the input IF amplifier grid and the limiter path off, the tuning of the signal generator to the linear path was checked for maximum voltage at the adder's output.

(b) The limiter path having been previously aligned to match the linear path, it was energized and the reduction in adder output maximized by adjustment of the two potentiometers in the separate path outputs. A single path output was approximately fifteen millivolts rms.

(c) The reduction in combined signal output was further enhanced by manipulation of the limiter path bias adjustment.

(d) Operations (b) and (c) were repeated until maximum common-mode rejection was obtained.

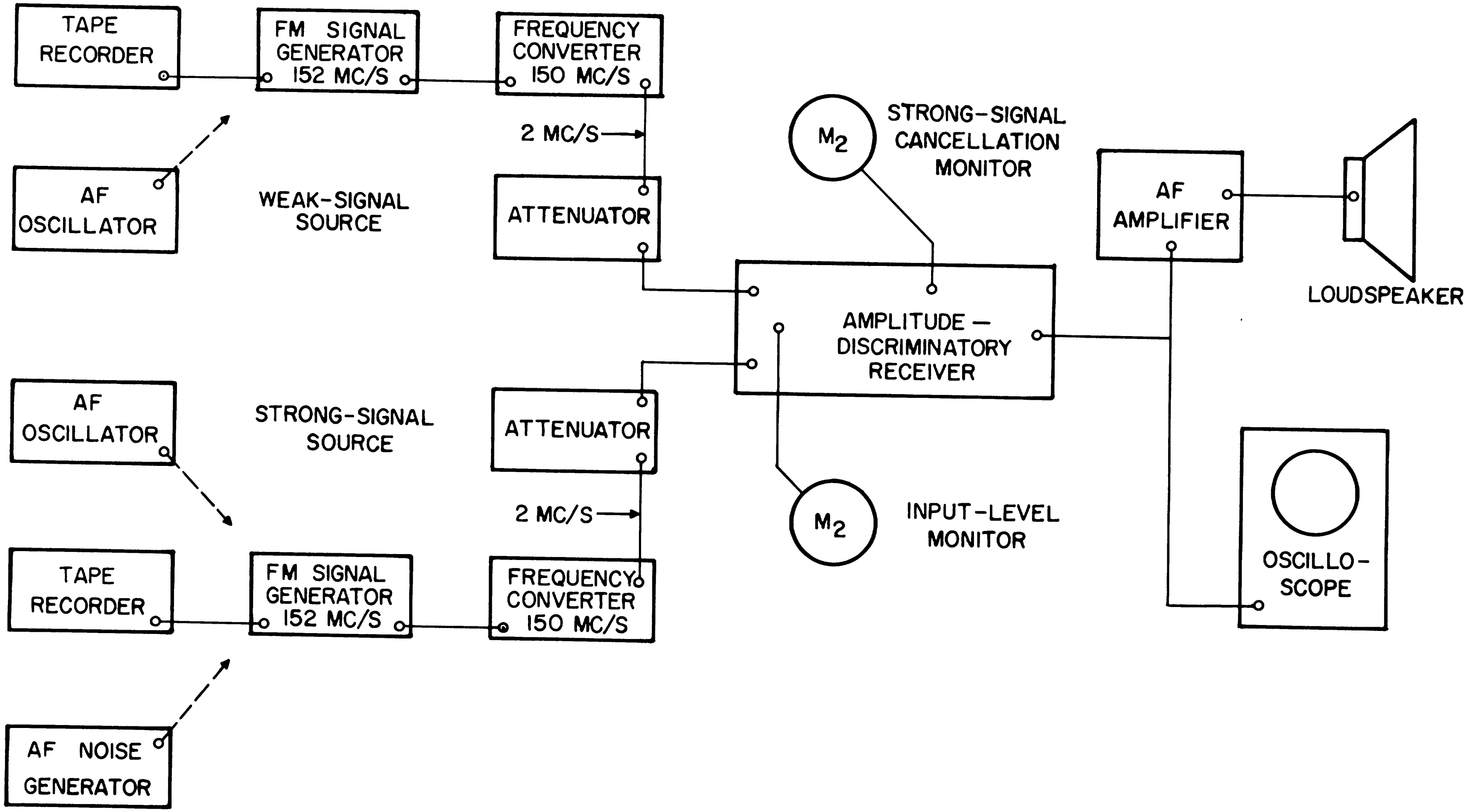

Fig. 19
LABORATORY TEST ARRANGEMENT

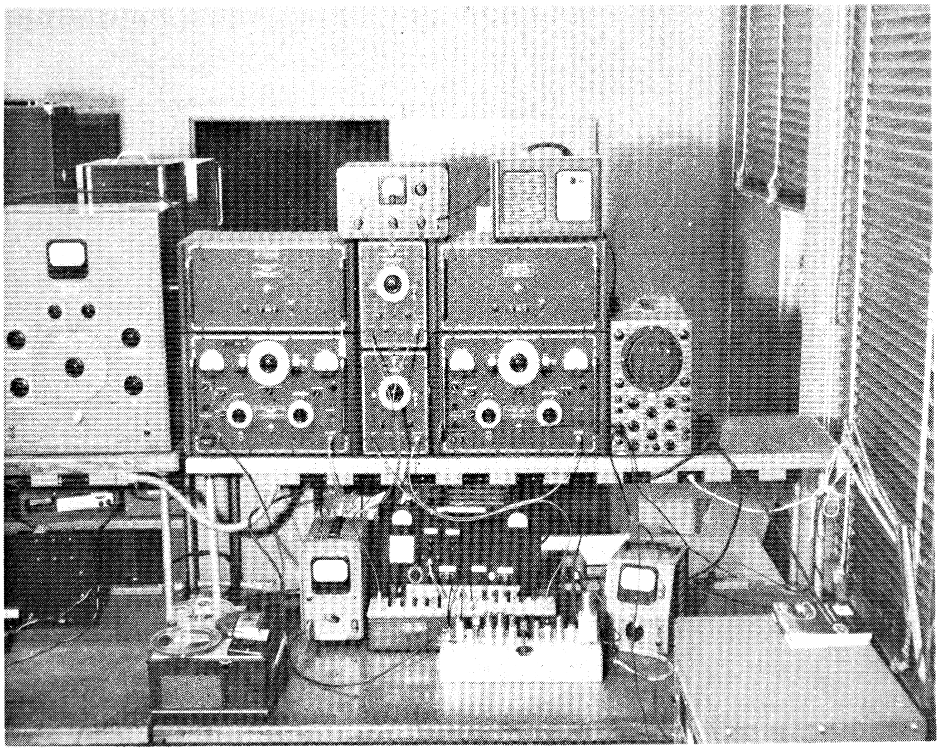

Figure 20. Photograph of Laboratory Arrangement

The adjustments were often made with the strong signal modulated by means of a one kilocycle/second sine wave, a deviation of ten kilocycles being used. Whether modulation was used or not, the above procedure realized a common-mode rejection figure of better than thirty decibels.

Following the initial adjustments on the receiver, the weaker-signal level was established and both signals independently modulated, first with tones and subsequently with speech from recorded tapes. The tapes provided words from lists prepared for articulation score tests, the strong-signal modulation being PB word lists read by a young woman and the weak-signal modulation comprising similar words read by a young man, but with each word embedded in the carrier sentence, "You will write _______."

On frequency, some difficulty was experienced with an excessive amount of beat noise due to incidental modulation of the signal generators at their power supply frequencies. A conversion process was necessary to obtain the two 2-megacycle signals and each of the oscillators beat with the other, resulting in components in the output which were comparable with the weak-signal modulation after the adder. The effect was minimized by detuning one or the other of the oscillators by a few kilocycles. This noise should not be confused with that which is ordinarily due to the clean frequency spike which results from the two carriers being on slightly different frequencies, for in this latter case the amount of interference increases with the separation frequency as well as with the ratio of weaker-to-stronger signal amplitudes. The on-frequency case, however, does place an added burden on the limiter-filter path and results in uncancelled components of the modulated strong signal (see Chapter V).

It should be remarked that in the laboratory receiver and in this test no pre-emphasis or de-emphasis was incorporated. The advantages of such differentiation and integration procedures are well known, but their use in this experiment was not deemed necessary for demonstration of the techniques involved.

4.3 Results of Tests

The performance of the receiver was best under conditions of speech modulation. Articulation scores of seventy-five to eighty-five per cent for an untrained observer were obtained on the detection of the modulation of the weaker signal in the presence of the stronger for a ten- to twenty-decibel difference in level ($\alpha = 0.3$ to $\alpha = 0.1$). Articulation score for the strong signal alone, on an alternative detection basis, was about ninety per cent and was virtually the same as that taken from the tape recorder directly at the audio level. On the other hand, the strong-signal modulation under conditions for weak-signal detection resulted in articulation scores of near zero, for the words were unintelligible and extremely weak. Articulation score conditions and results are summarized in Table 1.

With tones the modulations were separately detectable, and the weaker signal modulation could be easily read aurally in the presence of the stronger. However, the fact that the sinusoidally modulated signals spent most of their time out on the slopes of the gain-versus-frequency curves resulted in a severe test of the phase cancellation scheme, for it is in those frequency ranges that divergence between the two path characteristics begins to appear. On frequency, the two signals competed with the incidental beating of the oscillators and better results were obtained with one carrier slightly detuned. The results are shown in Figure 21 for two values of amplitude ratio, α, and as a function of the detuning. Although detuned with respect to one another, the two signals were both within the receiver pass band. The results of Figure 21 were obtained with the aid of a sharply-tuned audio-frequency filter and demonstrate the possibilities for alternative or simultaneous detection of two tone-modulated carriers.

The photograph of Figure 22 shows examples of reception under two conditions of tone modulation. In the upper trace the predominant audio signal is that of the 400-cycle modulation of the weak signal when no modulation is applied to the strong signal carrier. The lower trace is the same except that the strong signal is modulated by a one-kilocycle tone. The deviation in both cases was 10 kilocycles, and the weak-to-strong signal ratio was -10 decibels. Co-channel operation was used, the separation between unmodulated carriers being about 10 kilocycles.

Modulations	Receiver Connection	Percentage of Words Correctly Identified
(a) Speech on both carriers	Linear path inoperative, S-signal modulation read	90
(b) Speech on both carriers	Both paths operative, S-signal modulation read	~ 0
(c) Speech on both carriers	Linear path inoperative, W-signal modulation read	~ 0
(d) Speech on both carriers	Both paths operative, W-signal modulation read	85
(e) Speech on W-carrier, AF white gaussian noise of 20-kc/s bandwidth giving 3 kc rms deviation on S-carrier	Both paths operative, W-signal modulation read	78
(f) Speech on W-carrier, S-carrier unmodulated	Both paths operative, W-signal modulation read	86
General conditions of operation: Speech signals provided approximately 10 kc deviation; the two carriers were co-channel but separated by 8 kc; the amplitude of the weak (W) signal was 10 db below that of the strong (S) signal.		

TABLE 1

RESULTS OF ARTICULATION TESTS MADE WITH THE LABORATORY AMPLITUDE-DISCRIMINATORY RECEIVER

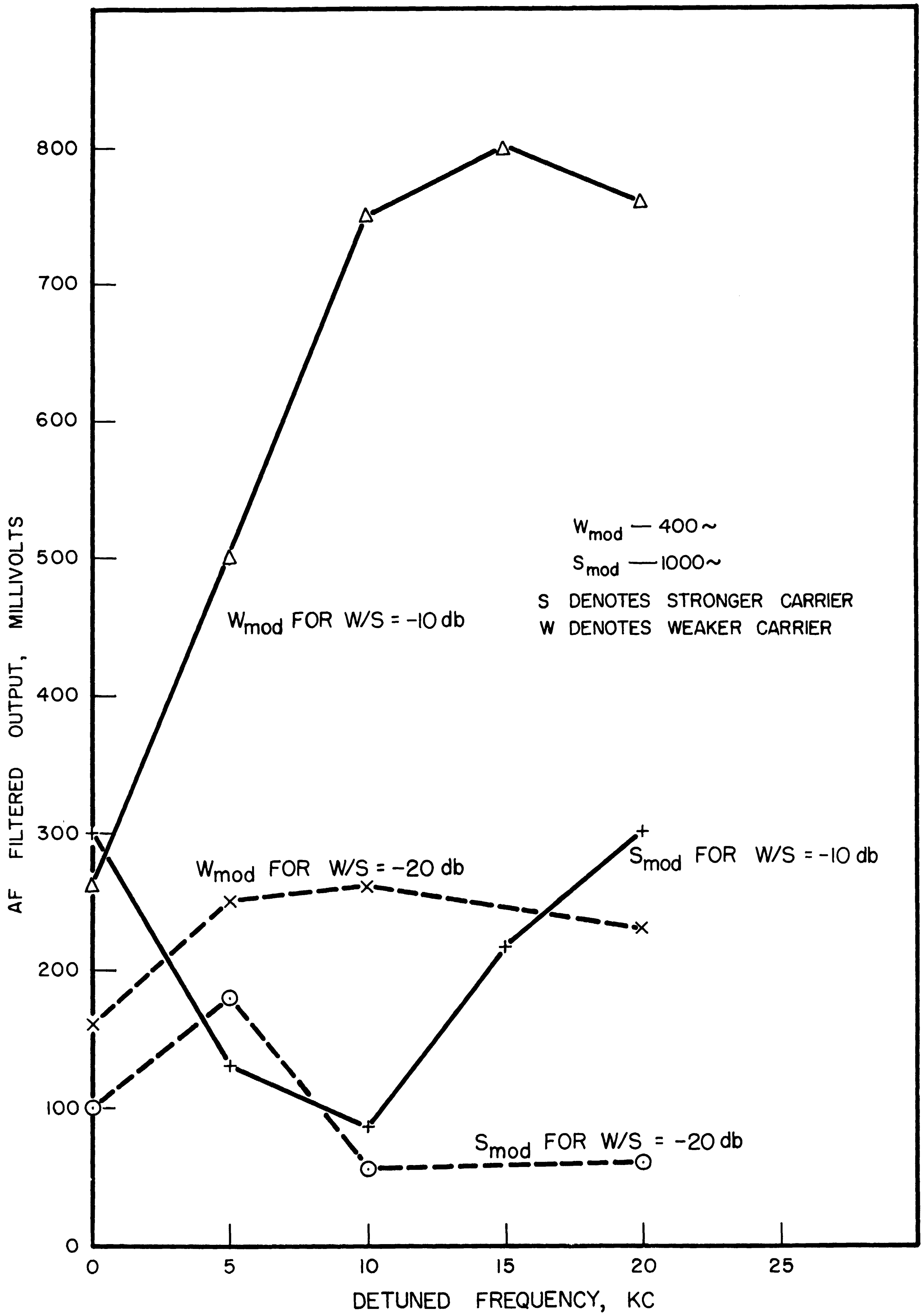

Figure 21. Tone Modulation Test for Amplitude Discriminatory Receiver.

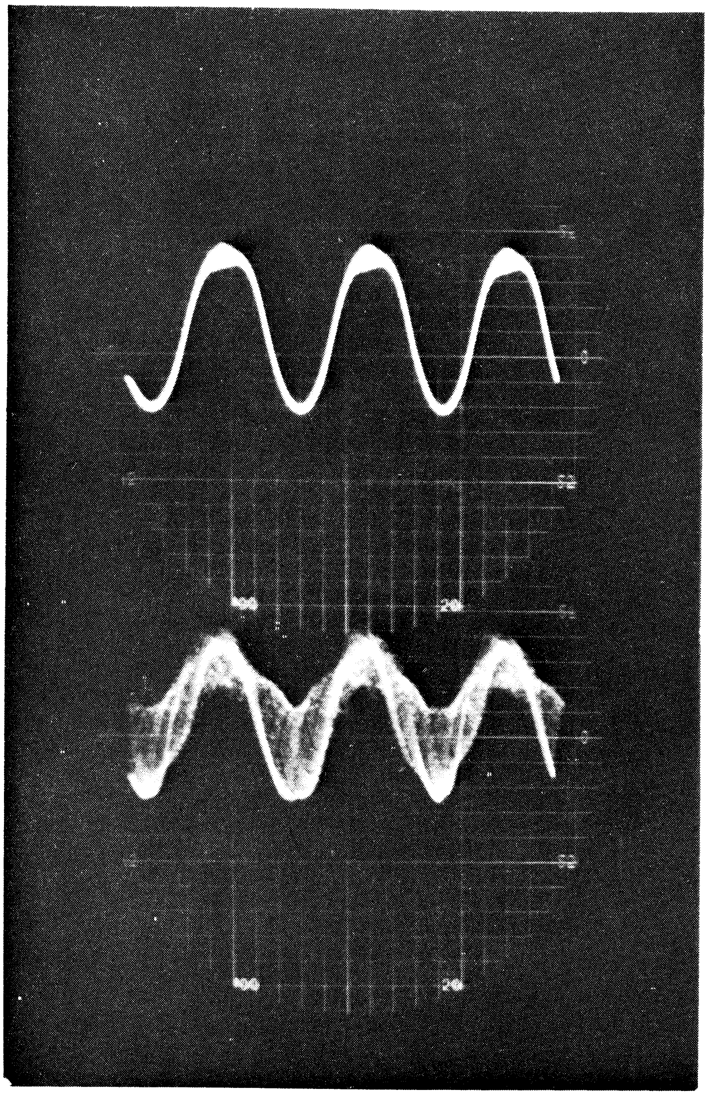

Figure 22. Output Waveforms with Sinusoidal Modulation

4.4 Discussion of Results of Experimental Work

The experimental work has demonstrated the feasibility of the strong-signal cancellation method as a means of weak signal enhancement. Without an excessive amount of effort sufficient cancellation was obtained to permit reading the modulation of the weaker carrier for strong-to-weak signal ratios of from ten to twenty decibels, the performance being audibly better for speech and noise modulations than for tones, although tone modulations as received through very narrow-band filters were of large amplitude and could be useful in an application requiring the separation of two modulations in the same channel, as for the operation of frequency-sensitive devices.

The satisfactory performance of the receiver in those instances involving speech and noise modulations of the carriers was probably aided by the fact that small modulating amplitudes are more probable than large ones for modulating functions of these types, with the result that small deviations predominate in the receiver most of the time. Cancellation is easier to achieve in the region close to the center frequency when both signals are present. The articulation scores show the readability of the signals in the alternative type of detection to be of a high order.

The inability of the receiver to cope with the noise from the incidental modulation of the generators used, thus requiring a slight separation in the frequency of the two carriers, was at first disappointing. However, the limiter path ordinarily requires that the signal separation be greater than the highest audio frequency that modulates the strong carrier in order that the interference be inaudible (13). As most of the difficulty arises for values of α approaching unity, the situation is eased by utilizing a small value of α while maintaining a large degree of common-mode rejection. Experimentally, the rapid improvement in intelligibility as the carrier separation was effected was notable.

Optimization of the receiver could be undertaken in several of its sections, the most important being in the effective action of the limiter path in obtaining the strong signal alone. While not necessary for all conceivable applications of co-channel reception, the path, in order to deal with values of α approaching unity, should be of either the Granlund type (broadband) or the Baghdady type (narrowband). Special attention should be given to the problem of symmetrical limiting in order to assist the phase control of the strong signal fundamental components. A second section worthy of further attention is the common-path portion of the receiver. Sufficient gain must be provided ahead of the limiters in this path to assure adequate capture of the receiver by the predominating weak signal.

In a practical design, several other considerations deserve attention. The most important of these is the incorporation of an automatic gain control loop around the linear path and the earlier common intermediate frequency amplifier stages for the purpose of helping the receiver to accommodate changes in the value of α. In the laboratory model a manual adjustment was satisfactory but a dynamic arrangement is desirable.

The necessity for a de-emphasis network in the output of the discriminator stage is obvious because of its use in current practice in conjunction with pre-emphasis of the transmitter.

Design and operation may very well be facilitated at a higher intermediate frequency, such as the more conventional value of 10.7 megacycles. The filtering problem should be made easier and the control of phase over the relatively narrow pass band made more tractable.

CHAPTER V

EFFECTS OF INTERFERENCE AND NOISE ON RECEIVER OPERATION

5.1 Idealization of the Receiver

The amplitude-discriminatory receiver which has been discussed in the earlier portion of this report can be considered idealized to a certain extent in order that attention be given the problems of two-signal interference and noise as they affect the operation of the device. The idealization which is required comprises the following:

(a) The linear path operates in a linear manner over all ranges of the amplitude ratio, α.

(b) The output of the limiter path is the strong signal only.

(c) The pass band characteristics of all filters are rectangular and the phase characteristics are linear with frequency.

We do not stipulate that the receiver be noiseless but do wish to assume the presence of only white Gaussian noise. It is not required that the common-mode rejection be infinite, inasmuch as this is one type of interference which we will wish to consider.

5.2 Interference Conditions in the Common Path

Neglecting, for the time being, the effects of noise in the channel, we investigate the type of interference signal which results from the simultaneous occurrence of two carriers which differ in both amplitude and frequency but fall within the pass band of the receiver.

The situation which is of interest here was illustrated in Figure 7, where it was evident that the repetition rate of the frequency spike was the angular difference frequency, δ, and the maximum amplitude of the spike (in radians) was given by

$$A = \frac{\alpha\delta}{1 - \alpha} . \qquad (5\text{-}1)$$

If it is assumed that this instantaneous frequency deviation is applied to a linear discriminator-detector circuit, proportional output voltages result. Under these conditions, the plot of Figure 23 can be made, where the ordinate is presented in terms of the per cent relative amplitude caused by the maximum deviation, the amplitude due to a normal 75 kilocycle deviation being taken as the reference. Curves are shown for values of α corresponding to weak-to-strong signal ratios of -3, -10, and -20 decibels. The increase in the magnitude of the detected interference with both frequency difference and α is readily seen. If the standard de-emphasis (τ = 75 microseconds) is employed, the curves in the region above about 2100 cycles/second flatten out and a maximum value of the interference amplitude is reached in the vicinity of the upper frequency limit for human hearing.

With particular reference to the amplitude discriminatory receiver, the existence of the frequency spike will not be apparent through the linear path because there is no $d\theta/dt$ detector present. The idealized limiter path has the characteristic that it produces only the strong signal and its modulation. Thus, the only signals present in the linear path are $e_1(t)$ and $\alpha e_2(t)$ and the availability of $-e_1(t)$ alone from the limiter path permits proper operation to the extent that a residual value for the strong signal is obtained and the weak signal allowed to predominate in the common-path portion of the receiver. That is to say, the fact that the phase may change rapidly in the linear path does not introduce any difficulties as far as the point of subtraction.

In the common-path part of the receiver, a different situation exists. Here, the presence of the spike due to the linear combination of

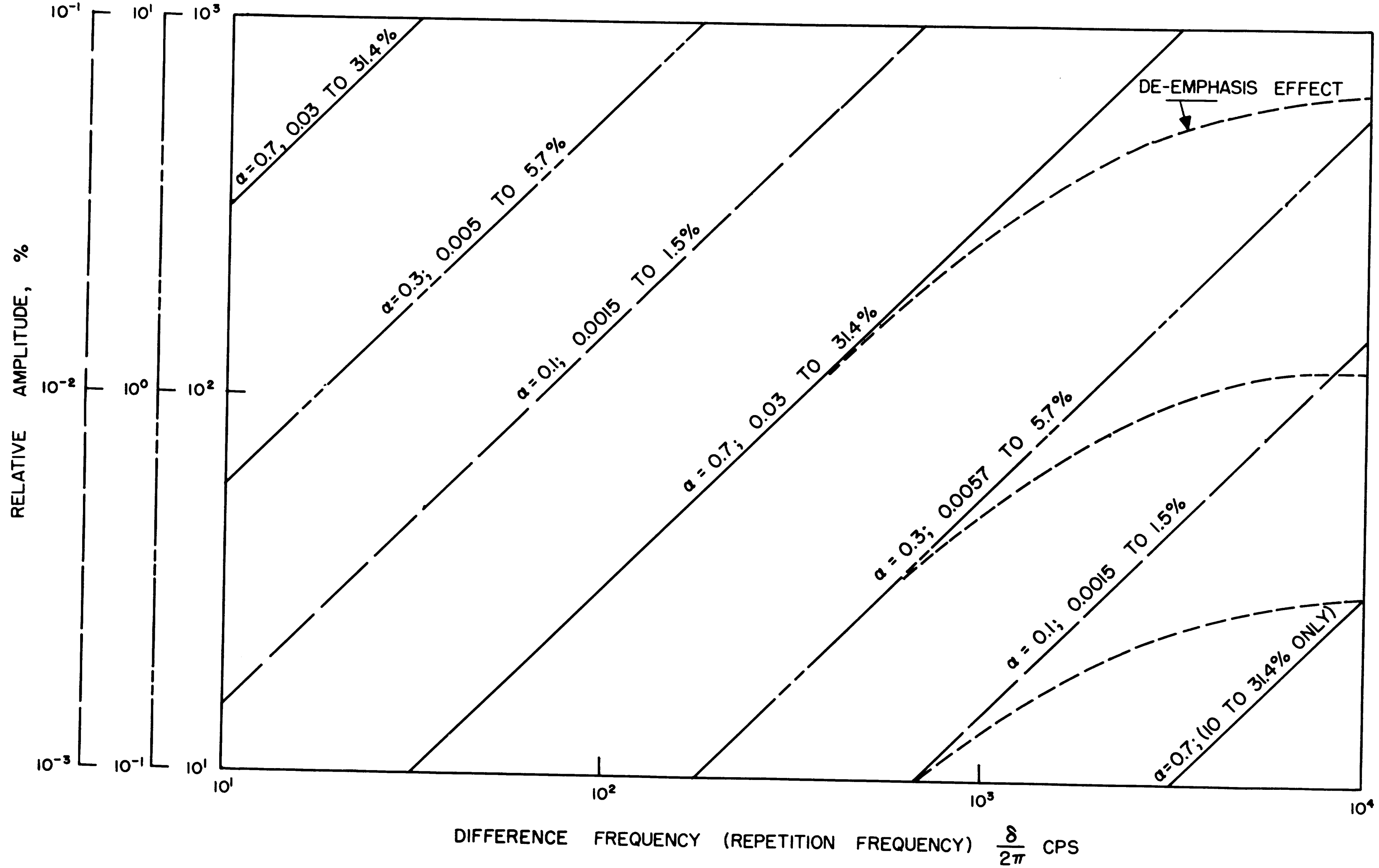

FIG. 23. EFFECT OF THE FREQUENCY SPIKE $\frac{\delta}{2\pi}\left(\frac{\alpha}{1-\alpha}\right)$

the residual value of $e_1(t)$ and the larger amplitude, $\alpha e_2(t)$, must be dealt with as a frequency deviation which takes on the values shown in Figure 23. The distinction is that the combination signal, assuming imperfect cancellation, is to be applied to a conventional limiter-discriminator combination. Let us define the ratio of signals in the common-path portion as

$$\alpha' = \frac{\text{Residual strong-signal amplitude}}{\text{Linear-path weak-signal amplitude}} \tag{5-2}$$

where it is assumed that the linear amplitude value of the weak signal was not affected by the strong-signal subtraction process. It is then feasible to plot the ratio of signal amplitudes following subtraction against the ratio that existed in the linear path, both ratios being taken as quantities less than unity. Figure 24 shows the resultant curves, with various values of common-mode rejection as a parameter.

The turnabout nature of the ratio relations as they are affected by a specified degree of common-mode rejection is interesting, the more so because the effect was very pronounced in experimental observations. For example, the laboratory receiver realized about thirty decibels of common-mode rejection and, for a value of $\alpha = 0.3$ in the linear path, the common path should show a value of $\alpha' = 0.1$. This is, of course, the same as saying that for a 10-db difference in strong-to-weak signal amplitudes in the linear path, a 30-db common-mode rejection ratio results in the weak signal being 20 db greater than the residual strong signal in the common path. The advantage of obtaining as large a value as possible for the common-mode rejection ratio can be appreciated from the plot, although it is inherently obvious as well.

To return to the effect of the spike amplitude disturbance, it is found that the size of the disturbance is lessened by having a small

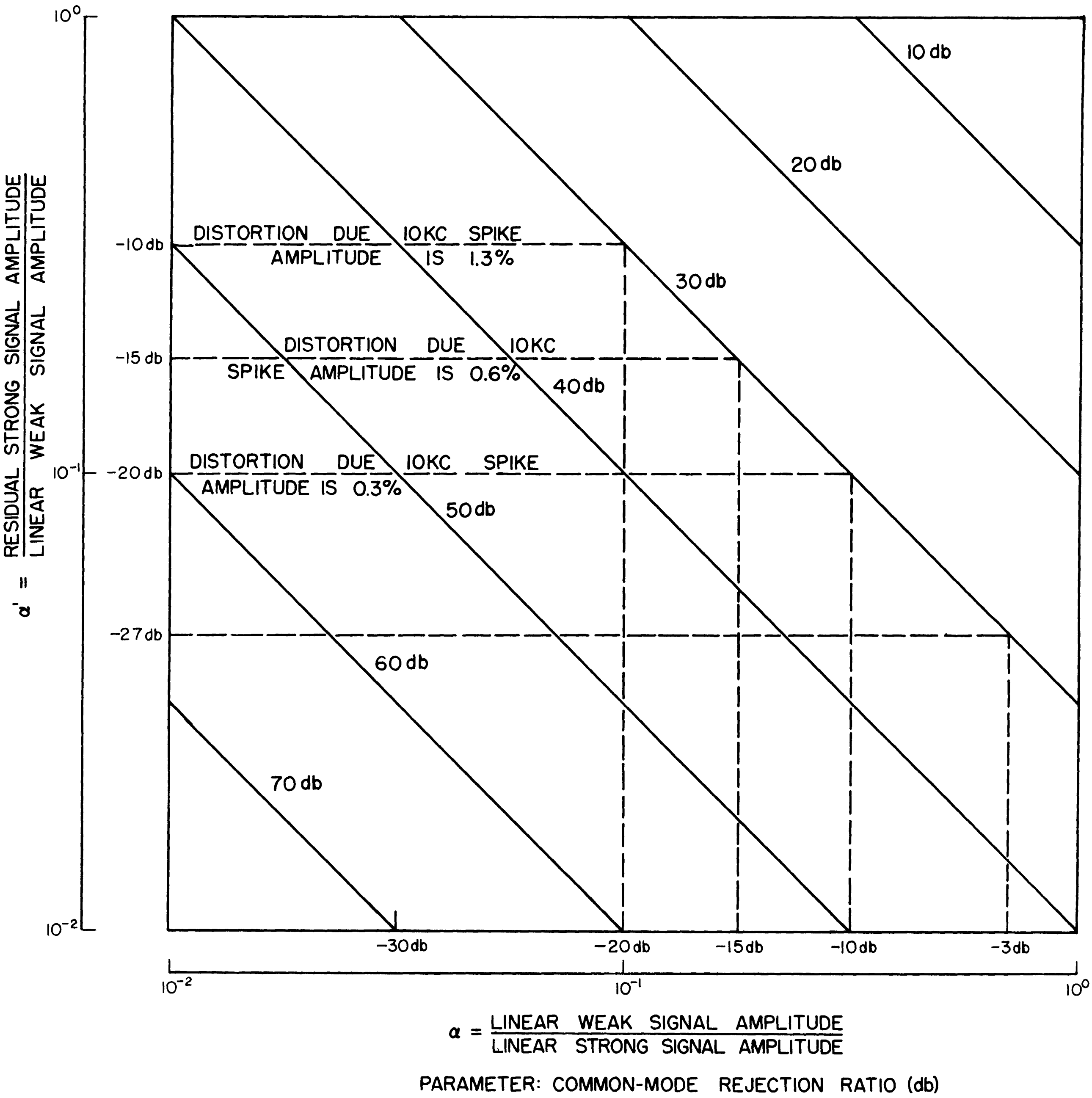

FIG. 24.

α VS α' FOR SEVERAL VALUES OF COMMON-MODE REJECTION difference frequency and a small value of α' in the common-path. In the worst case plotted for $\alpha' = 0.7$, assuming de-emphasis were to be used, the peak amplitude is about seven per cent of that caused by a modulating signal which causes full output to appear with a 75 kilocycle deviation. This value would very likely be intolerable if it recurred often during low-level modulating signals. Appreciable improvement results from a smaller value of α', the maximum amplitude becoming only 1.3% for $\alpha' = 0.3$ and only about 0.3% for $\alpha' = 0.1$, both cases being taken for a difference frequency of ten kilocycles. The actual difference frequency involved is, of course, that resulting from the beating of the two modulated carriers, and it may well cover the range from zero to about fifteen kilocycles per second on an instant-by-instant basis to produce small amplitude but audible components in the receiver unless α' is of the order of 0.1.

For two-signal interference, then, it is reasonable to expect satisfactory operation of the amplitude discriminatory receiver if the value of α' in the common path can be kept small and a high degree of common-mode rejection achieved. A small value of α in the linear stages eases the task of the limiter-filter path markedly, permitting ready capture of that section. However, a small value of α makes more difficult the problem of suppression, since the value of α' is determined thereby.

5.3 Rates of Transmission of Information for Co-Channel FM Reception

Because of the apparent simultaneous occupancy of the communication channel by two FM signals and the inherent noise of the system, the question naturally arises as to the effect that the unwanted interferences may have on the rate of transmission of information which can be achieved with the desired signal. With the exception of treatments of several types of pulse modulation, the determination of such rates of transmission for various types of modulating schemes has not received very much attention during the rapid growth of the literature in the area of communication theory. Goldman (26), Jelonek (27) and Blachman (28), have made calculated comparisons of several transmission systems employing either continuous or discrete modulation signals. Their methods differ in the particulars because of the initial assumptions made, but results are generally compatible. The development for the case used here is somewhat different in approach.

In a receiving situation such as indicated in the present discussion, the desirable condition of extremely large signal-to-noise ratio may not be realized in the case of the weaker of the two signals which are to be distinguished. It is therefore of interest to determine the amount of uncertainty imposed upon the weak signal by the inherent noise level of the channel employed, with an upper bound for the rate of transmission of information established as a consequence.

Maximum entropy for the received signal results from a uniform distribution of frequency of the transmitted signal over the radio-frequency band. Such a distribution will be assumed to exist and the frequency-modulated signal will be taken to have a peak deviation of Ω_d, resulting from a modulating function covering the audio-frequency band ω_1 to ω_2. Such a signal, for large modulation indices, is in very nearly the same category as those having a peak power limitation and as such has an expressible entropy of

$$H_S = \log\sqrt{4\Omega_d^2} , \tag{5-3}$$

using the relation shown by Shannon (26).

We assume the noise to be characterized as a band-limited white Gaussian process over the radio-frequency bandwidth, ω_a to ω_b, where

$$\Omega_d = \frac{\omega_b - \omega_a}{2} \tag{5-4}$$

Figure 25 shows the relationship between the two spectra.

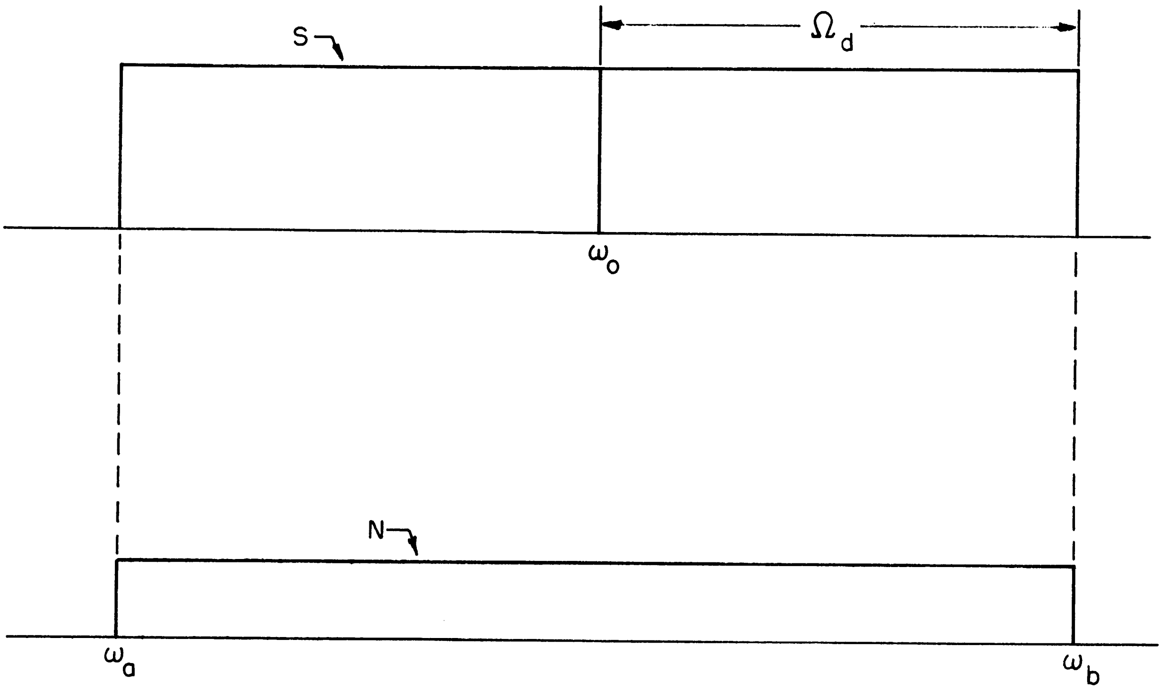

Figure 25. Spectra of FM Signal and Interfering Noise

Rice (30) has demonstrated that the noise current in the output of a narrow band-pass filter can be viewed as a wave expressible as

$$I_N = I_{N_c} \cos \omega_o t - I_{N_s} \sin \omega_o t \tag{5-5}$$

where ω_o is the center frequency of the band-limiting device and both I_{N_c} and I_{N_s} are normally distributed random variables having white low-pass spectra. Only those components of the noise which are in quadrature with the components of the signal are effective in creating uncertainty in the phase of the received signal. The variance of the quadrature components is equal to the noise power, N. However, only the fraction of the power which is evident as interference over the modulation bandwidth of the signal is of interest, since all other components will be filtered out. This fraction is

$$N_Q = N \frac{2(\omega_2 - \omega_1)}{\omega_b - \omega_a} \tag{5-6}$$

If we denote the troublesome noise current component as I_Q and the signal current amplitude as I_S, then, under the assumption that the former is very much smaller than the latter, the phase disturbance is very nearly equal to the ratio of these two envelope values, as

$$\Delta\varphi \cong \frac{I_Q}{I_S} \tag{5-7}$$

From our previous definition of instantaneous frequency, the fluctuations in the radian frequency due to the noise are given by

$$\frac{d\Delta\varphi}{dt} = \frac{1}{I_S}\frac{dI_Q}{dt} \tag{5-8}$$

The derivative of I_Q is also a random variable having a Gaussian distribution.

With the above development, we are in a position to determine the variance for the frequency fluctuation due to the noise. First, however, we note that the spectrum, $w(\omega)$, for the noise power, N_Q, can be specified in terms of the correlation function, $\psi(\tau)$, as

$$\psi(0) = N_Q = \int_0^{\infty} w(\omega)\, d\omega \tag{5-9}$$

where $\psi(0)$ is the correlation function evaluated at $\tau = 0$. The noise components, I_Q, which comprise $w(\omega)$, appear in Equation 5-8 in derivative form. Rice has shown that the second moment for such a derivative process can be found to be

$$\psi''(0) = \int_0^\infty \omega^2 \, w(\omega) \, d\omega \qquad (5\text{-}10)$$

where $\psi''(0)$ denotes the second derivative of $\psi(\tau)$ with respect to time, evaluated at $\tau = 0$. With this notation the variance for the frequency fluctuation can be written as

$$\sigma_{N\omega}^{2} = \frac{-\psi''(0)}{I_S^2} \qquad (5\text{-}11)$$

Substitution for the quantities previously determined yields

$$\sigma_{N\omega}^{2} = \frac{N}{2S} \cdot \frac{2(\omega_2 - \omega_1)}{\omega_b - \omega_a} \cdot \left[\frac{-\psi''(0)}{\psi(0)}\right] \qquad (5\text{-}12)$$

where, in the ratio, I_S^2 has been replaced by the equivalent quantity, 2S, where S is the signal power.

The ratio $-\psi''(0)/\psi(0)$ must be evaluated over the audio frequency range of interest, that is, from ω_1 to ω_2. When this is done,

$$\frac{-\psi''(0)}{\psi(0)} = \frac{1}{3} \cdot \frac{\omega_2^3 - \omega_1^3}{\omega_2 - \omega_1} \quad . \qquad (5\text{-}13)$$

Substitution of this ratio in Equation 5-12 gives

$$\sigma_{N\omega}^{2} = \frac{N}{3S} \cdot \frac{\omega_2^3 - \omega_1^3}{\omega_b - \omega_a} \qquad (5\text{-}14)$$

which is equivalent to

$$\sigma_{N\omega}^{2} = \frac{N}{6S} \cdot \frac{\omega_2^3 - \omega_1^3}{\Omega_d} \qquad (5\text{-}15)$$

The variance obtained, which characterizes the one-dimensional Gaussian process involved, permits us to utilize another of Shannon's results, namely, that the entropy of such a continuous process is

$$H_N = \log \sqrt{2\pi e \, \sigma_{N\omega}{}^2} \tag{5-16}$$

The rate of transmission of information is specified as being proportional to the number of independent values of the signal where these are determined by the frequencies of the modulating function. Specifically, the rate which corresponds to the distribution selected and having a maximum value is

$$R = 2 \, (f_2 - f_1) \, (H_S - H_N) \, . \tag{5-17}$$

For the frequency-modulated weak signal having a uniform distribution of frequency and perturbed by band-limited, white Gaussian noise in the channel, the rate of transmission becomes

$$R_U = (f_2 - f_1) \log \frac{12S}{\pi e N} \frac{\Omega_d{}^3}{\omega_2{}^3 - \omega_1{}^3} \tag{5-18}$$

where the subscript, U, indicates the uniform-distribution case.

If ω_1 is allowed to be zero, the ready dependence of the rate of transmission of information on the modulation index is apparent, since the ratio Ω_d/ω_2 defines the index.

Equation 5-18 gives an upper bound for the rate of transmission of information for FM signals and applies to the system discussed in this report insofar as the effects of band-limited, white Gaussian noise on the weak-signal reception are concerned, assuming ideal receiver operation. The more general treatment would include the effect of noise on the strong signal and then would determine the rate of transmission for the weak signal when the added uncertainties resulting from failure of the limiter path to deliver a signal which could be ideally cancelled, the influence of noise in the linear and nonlinear portions of the receiver being different. An attack on this problem was begun but the work involved rapidly assumed proportions which necessitated a postponement of solution. It is recommended that this determination be made, however, for the case of two FM signals in a channel disturbed by white Gaussian noise.

With regard to the simultaneous presence of the two frequency-modulated signals in the channel, it might well be expected that their mutual interference would result in a loss of information, even assuming linear processes wherever necessary. However, it is clear that, in the absence of noise, granting the amplitude-discriminatory receiver the ability to separate the two signals is the same as concluding that the existence of the two signals in the same channel has no effect on limiting the rate of transmission of information.

Consider the situation in the common path portion of the receiver, where imperfect cancellation may exist. Along with the noise that is present, the residual strong signal may be supposed to influence the phase or frequency of the desired weak signal with appropriate regard for the desired-signal-to-interference ratio. The problem of ascertaining the composite behavior under conditions such as those previously described becomes difficult because of the complexity of the resultant expressions for the interfering current or voltage. However, if an ideal receiver is assumed, the presence of the residual strong signal does not potentially cause any difficulty because it is known to a reasonably high degree of accuracy at each sampling point due to its complete dominance at the output of the limiter path. Thus the ideal receiver will be able to use this information to remove the uncertainties attendant upon the weak signal being sought in the receiver output.

5.4 Effect of Incidental Amplitude Modulation and Other Signal Variations

The presence in the linear path of any components of signal or noise other than those associated with the FM signals alone will lead to common-path interference levels which may easily be comparable with the desired signal level at the output of the adder circuit. If there be incidental amplitude modulation on the weak signal the second set of limiters can be expected to remove it. However, incidental AM on the strong signal may contribute to the variation of the combined envelope in the linear path to an appreciable extent. Referring to Equation 3-26, we find the amplitude variation due to the combination of the two ideal signals alone to be

$$1 + m_3(t) = \sqrt{\left[1 + \alpha \cos \beta(t)\right]^2 + \left[\alpha \sin \beta(t)\right]^2} \quad . \qquad (3\text{-}26)$$

If $\alpha^2 << 1$, then the right-hand side can be simplified and expanded to become approximately

$$1 + m_3(t) \cong 1 + \alpha \cos \beta(t) \; . \qquad (5\text{-}22)$$

Now for small values of α it can be seen that the variation in amplitude of the composite signal is very small, being $\pm \alpha$ at most, while recurring at the rate of the instantaneous difference frequency which is a function of the difference between the two unmodulated carriers as well as their respective modulations. For such a small variation in amplitude, which is actually the variation carrying the information that is desired from the weak signal, incidental AM on the strong signal may easily be of a comparable and troublesome size. As a criterion value, it appears reasonable to require that the incidental AM be no larger than about ten per cent of the value of α.

Variations in the overall level of the output of the linear path are a potential source of trouble, since the output of the limiter path is substantially constant over a wide range of input variation. The solution to this problem lies in the incorporation of an automatic gain control loop around the linear path. This loop may encompass part or all of the IF amplifier stages prior to the parallel excitation of the limiter and linear paths, if this is necessary to establish sufficient gain.

To the extent that the nonlinear circuitry can cope with the multipath problem in single signal transmission and reception, it will also be able to handle the multipath situation in the two signal case. The burden is placed on the limiter-filter combinations and will be most severe for multipath single signal amplitude ratios approaching unity.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

It has been demonstrated that effective use can be made of the familiar capture effect, which is characteristic of frequency modulation reception, to permit the satisfactory reception of a weak FM signal which is co-channel with a stronger FM signal. The additional circuitry required for weaker signal detection comprises a linear path to achieve phase and gain control to match the limiter path, a subtraction circuit, and another IF-limiter amplifier preceding the detector stage. Even with a moderate amount of circuitry and reasonably careful adjustment, sufficient cancellation of the strong signal can be realized to permit excellent readability of the modulation of the weaker signal.

While it is not necessary that complete cancellation of the strong signal be established, since the capture effect should be manifested in the final limiter stages once the weaker signal is brought up to a level above the residual strong signal, the extent to which differing signal levels can be accommodated is a function of the degree of common-mode rejection attained. Down to the point, at the adder output, at which the weak signal competes ineffectively with the noise level in the receiver, the greater the amount of rejection, the smaller the ratio of weak-to-strong signal amplitudes that can be handled. On the other hand, ratios approaching unity place a severe burden on the limiter path, requiring some of the more recent techniques for freeing the strong signal from the phase disturbances brought about by the presence of the weak one.

Under the assumption that the limiter path has as its output the strong signal only, the ideal receiver can always make use of such potential information to remove any uncertainty which the strong signal would tend to impose upon the weak signal, the latter always being subject to the influence of noise in the channel. In the actual situation, however, noise also causes uncertainty in the instantaneous value of the strong signal and as a consequence effects a further reduction in the rate of transmission of information for the weak signal. The quantitative solution of this problem is worthy of further investigation.

Although the receiver in this discussion has emphasized the reception of the modulation of the weaker of two signals, it is obvious that the stronger signal is readily available from the limiter path and that all that is required to detect its modulation is a conventional discriminator circuit suitably isolated from the adder circuitry. Consequently, both signals are simultaneously receivable should this be desirable.

The availability of a means of receiving two signals in the same assigned channel without undue interference suggests possibilities for further work on and exploitation of the amplitude-discriminatory receiver. Because of its sensitivity to incidental amplitude-modulation, fading and the multipath phenomenon, the incorporation of a means of automatically controlling the gain of the linear path to optimize the cancellation ratio at all times is essential. Upon development of such control and with the availability of suitable transmitter power flexibility and frequency control, the uses of the receiver in the areas of stereophonic broadcasting, frequency-shift telegraphy, and tone-operated relay circuitry appear potentially widespread. The possible increase in the number of available channels in a crowded spectrum is also an application which should not be overlooked.

The cost of the added flexibility in potential applications is found to be in the greater receiver complexity and the associated balance and phasing control circuitry, in the substantial duplication of transmitter facilities for co-channel operation, and in the sacrifice of performance based on a given signal-to-noise ratio for that resulting from a smaller ratio. The restriction inherent in the last phrase limits the likely service area to that in which the smaller carrier power is sufficient to provide normal receiver operation, for it is the smaller signal level which competes with the noise of the channel for control of the receiver. Within the framework of assigned frequencies and transmitter powers, it may very well be that the greatest application of the amplitude-discriminatory receiver will be in service areas of high population density in which larger-than-minimal field strengths are found for all signals.

APPENDIX A

WEAK-SIGNAL DETECTION BY MEANS OF AN ANALOGUE COMPUTER TECHNIQUE

In Section 3.1 the suggestion was made that the Equations 3-25 through 3-29 provided sufficient information to permit the detection of the weak signal modulation, $m_2(t)$. One proposal for accomplishing this involves a special-purpose analogue computer technique which is discussed here for its interest.

The pertinent equations are:

$$e_1(t) + \alpha\, e_2(t) = \left[1 + m_3(t)\right] \exp j \left[\omega_1 t + m_4(t)\right] \tag{3-16}$$

$$m_3(t) = \sqrt{\left[1 + \alpha \cos \beta(t)\right]^2 + \left[\alpha \sin \beta(t)\right]^2} - 1 \tag{3-26}$$

$$m_1(t) = m_4(t) - \tan^{-1} \frac{\alpha \sin \beta(t)}{1 + \alpha \cos \beta(t)} \tag{3-27}$$

$$m_2(t) = m_1(t) - (\omega_2 - \omega_1)t + \beta(t) \tag{3-28}$$

$$\beta(t) = \cos^{-1} \left\{ \frac{1}{2\alpha} \left[\left[1 + m_3(t)\right]^2 - 1 - \alpha^2 \right] \right\} \tag{3-29}$$

To simplify the present work, let us take the case for the frequencies of the two signals to be the same ($\omega_1 = \omega_2$) and for values of $\alpha^2 << 1$. Under these conditions we obtain

$$1 + m_3(t) \cong 1 + \alpha \cos \beta(t) \tag{A-1}$$

$$m_1(t) \cong m_4(t) \tag{A-2}$$

$$m_2(t) = m_1(t) + \beta(t) \tag{A-3}$$

$$\beta(t) \cong \cos^{-1} \frac{1}{2\alpha} \left\{ \left[1 + m_3(t)\right]^2 - 1 \right\} \tag{A-4}$$

It is apparent that the small phase disturbance accompanying $m_4(t)$ has been assumed disposed of by the usual limiter-discriminator circuitry so that only the modulation of the strong signal is contained in Equation A-2. From the above set of equations, then, assuming that we obtain $m_1(t)$ free of interference, the determination of $\beta(t)$ will, in combination with $m_1(t)$, yield $m_2(t)$, the phase modulation on the weaker carrier. Thus, the computer is required to implement Equation A-4.

A block diagram to accomplish the determination of $\beta(t)$ is shown in Figure A-1. Most of the components are straightforward. However, the arc-cosine function generator requires a diode-resistor type of circuit suitable for operation at high frequencies.

Since $m_1(t)$ and $m_2(t)$ will be varying over hundreds of radians during normal modulation processes, the problem of determining the sign of $\beta(t)$ on a continuous basis must be solved. It is conceivable that a circuit which differentiates the output of the envelope detector and drives a clipper circuit could act as a control on a sensing circuit which changes the sign of $\beta(t)$ at the right instant by taking the voltage value representing $\beta(t)$ from the function generator directly or from a sign-reversing amplifier. A more elegant method, first suggested by Professor Gunnar Hok, would be to develop an analogue to the two-phase synchro-type resolver, suitable for operation at high frequencies. The lack of a solution to the sensing problem, together with the realization that $m_1(t)$ had to be obtained free of the effects of $m_2(t)$ led to the method discussed in the body of the paper.

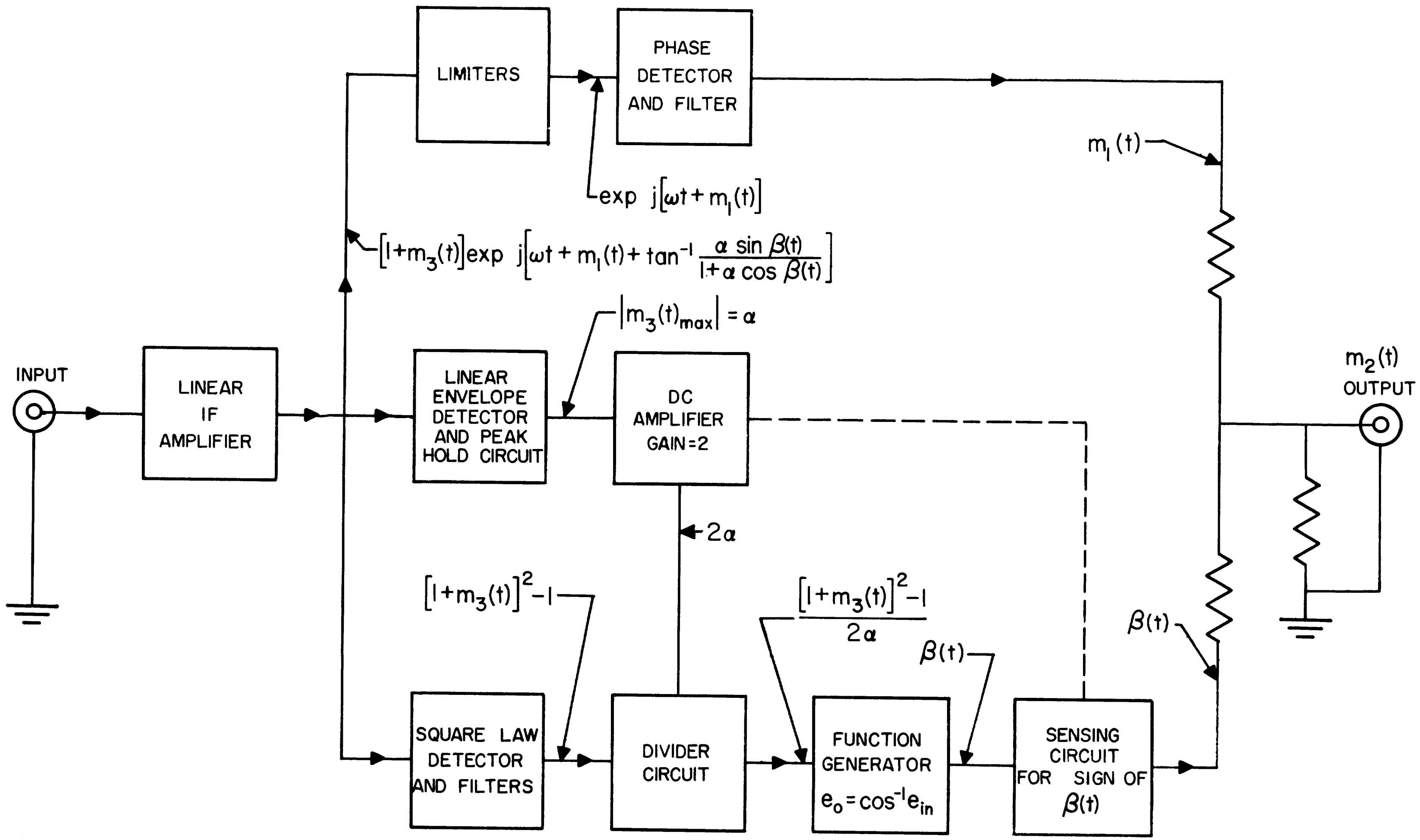

FIG. A-1. BLOCK DIAGRAM FOR ANALOGUE COMPUTER METHOD OF WEAK-SIGNAL DETECTION
(PROPORTIONALITY CONSTANTS OMITTED)

APPENDIX B

SIDEBAND METHOD OF WEAK SIGNAL DETECTION

The difficulties attendant upon the development of a proper sensing device for use in the method outlined in Appendix A led to the consideration of other solutions to the problem, still with regard for the same mathematical relations. That is to say, it was desired to obtain the phase angle, $\beta(t)$, in order to combine it with the strong signal modulation appropriately.

Professor J. A. Boyd suggested that the detected envelope might be used to amplitude modulate an oscillator and the result applied to a phase detector which would produce a DC output voltage which would be continuously proportional to the instantaneous phase quantity desired. That is, the detected quantity

$$e_{\beta(t)} = \alpha \cos \beta(t) \qquad \text{(B-1)}$$

should be used to modulate

$$e_{osc} = E_o \sin \omega_o t \qquad \text{(B-2)}$$

with the result

$$e_R = E_o \sin \omega_o t + \frac{\alpha E_o k}{2} \sin \left[\omega_o t + \beta(t)\right] + \frac{\alpha E_o k}{2} \sin \left[\omega_o t - \beta(t)\right] \qquad \text{(B-3)}$$

Since all that is required is one sideband or the other, more satisfactory means of obtaining the single sideband could be resorted to. With only the one sideband available, a conventional type of phase detector or discriminator can be employed to derive the output which is proportional to $\beta(t)$. The voltage representative of $\beta(t)$ can then be appropriately added to the strong signal modulation, $m_1(t)$, in the manner expressed in Equation A-3.

Because of the realization of the desirability of the method of the body of the report, this sideband method was not pursued further. However, a block diagram of a circuit to utilize the method is shown in Figure B-1.

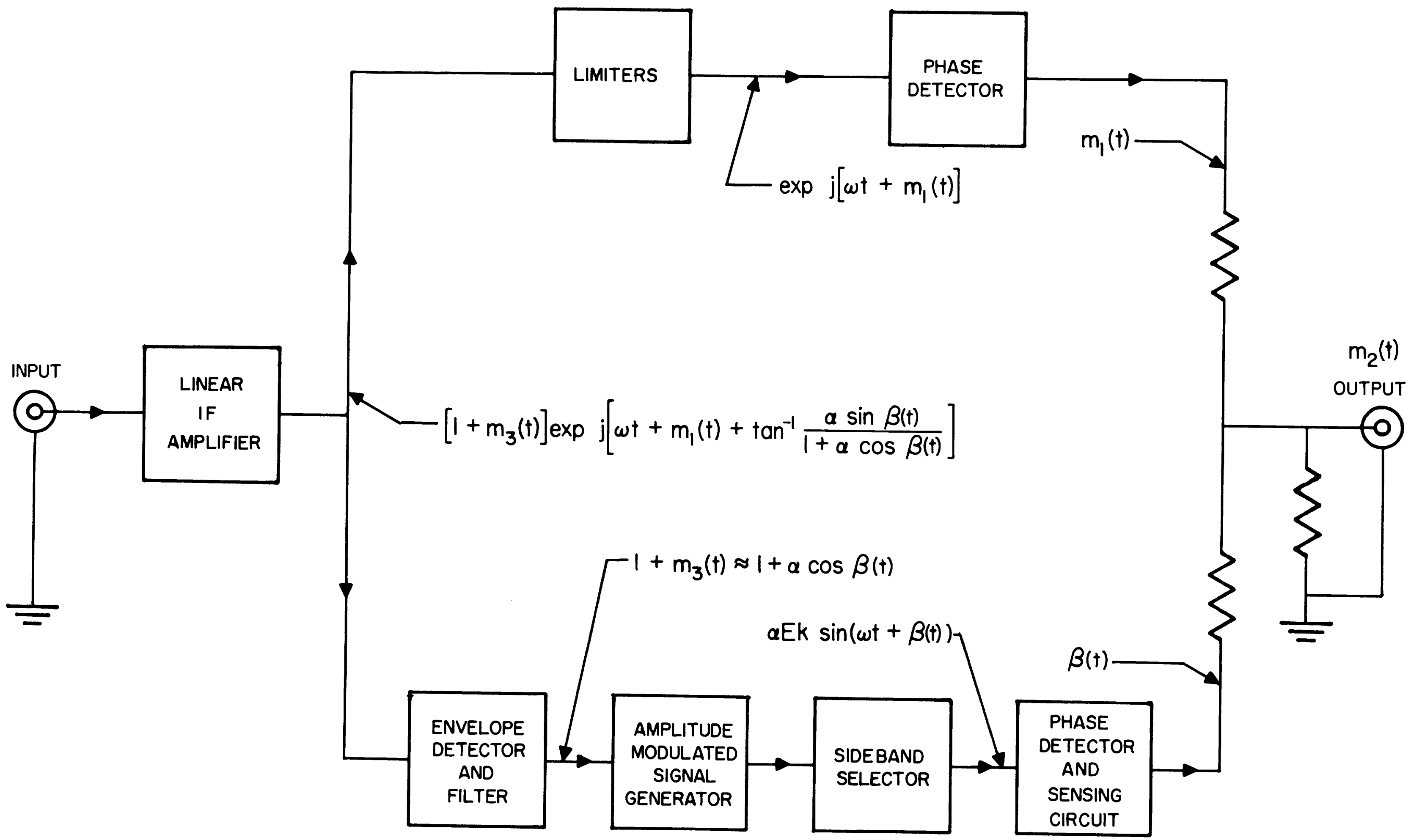

FIG. B-1. BLOCK DIAGRAM OF SIDEBAND METHOD OF WEAK-SIGNAL DETECTION
(PROPORTIONALITY CONSTANTS OMITTED)

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