THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

LIQUID PROPELLANT MISSILE LONGITUDINAL OSCILLATION

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I. INTRODUCTION

This report is concerned with the type of longitudinal oscillation of a liquid propellant missile that is caused by a regenerative coupling between the missile structure and the propulsion system. The purpose is not to give a complete detailed analysis of the problem, but rather to convey a basic understanding of the problem by discussing the role that each of the basic missile components plays in the development of an oscillation.

In addition, testing that is necessary for the proper analysis of the problem is discussed, and an analysis approach using waterhammer methods is presented to illustrate how a stability analysis can be developed using test data to substantiate the propulsion system representation.

II. DESCRIPTION OF THE OSCILLATION PROBLEM

A. Description of the Missile System.

The basic missile configuration to be considered is shown in Figure 1 with unnecessary details omitted for simplicity. Two engines are shown with identical propellant systems, and the engines are considered mounted so that their thrust acts at common points on the structure. In an analysis the engines can be considered as acting in parallel and replaced by one engine with double the thrust. This was the case in the Titan II missile oscillation where the flight data showed the pressure oscillations at respective points in each engine system to be of the same magnitude and in phase.(1) If engines are involved that are different or that have non-identical propellant systems, or are mounted differently to the structure, they should be considered separately in an analysis. The turbine gas generator, the turbine, and the pump gear drive assembly are assumed to have no affect on the oscillation. In other words, the pump speed is assumed constant. This also was the case in the Titan II missile oscillation.(1)

To get a clearer understanding of the cause of the oscillation, it is helpful to present the missile system in a block diagram representation as shown in Figure 2. In this figure the components that contribute separately to the development of the oscillation are shown as separate blocks. The oscillation is a structural oscillation so the main block of the diagram is considered the structure block. The remaining blocks then can be considered as acting as a feedback system from the output of the structure block (velocity or acceleration) to the input of the structure block (thrust). The structure vibration causes

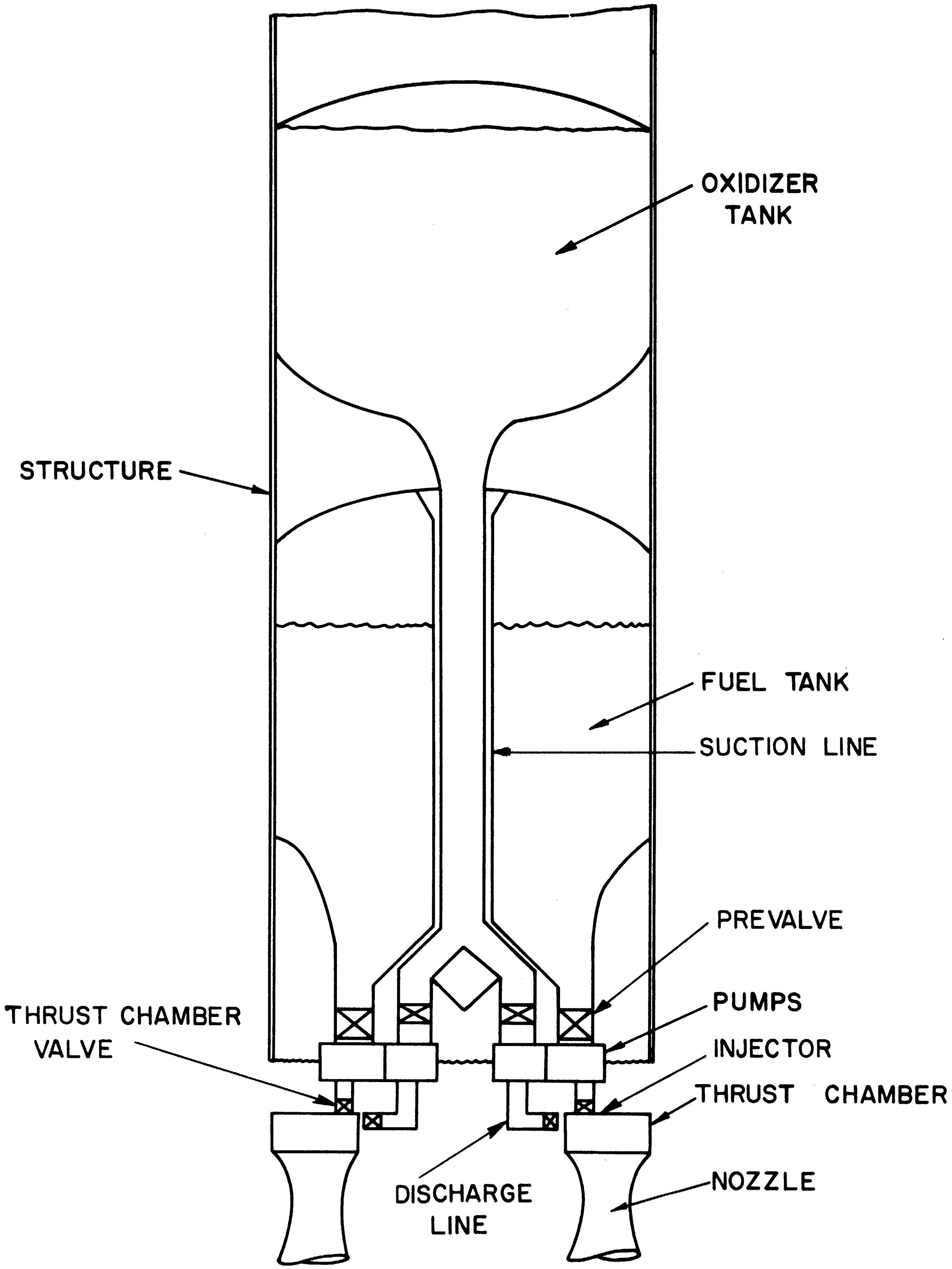

Figure 1. Missile Rocket Engine System.

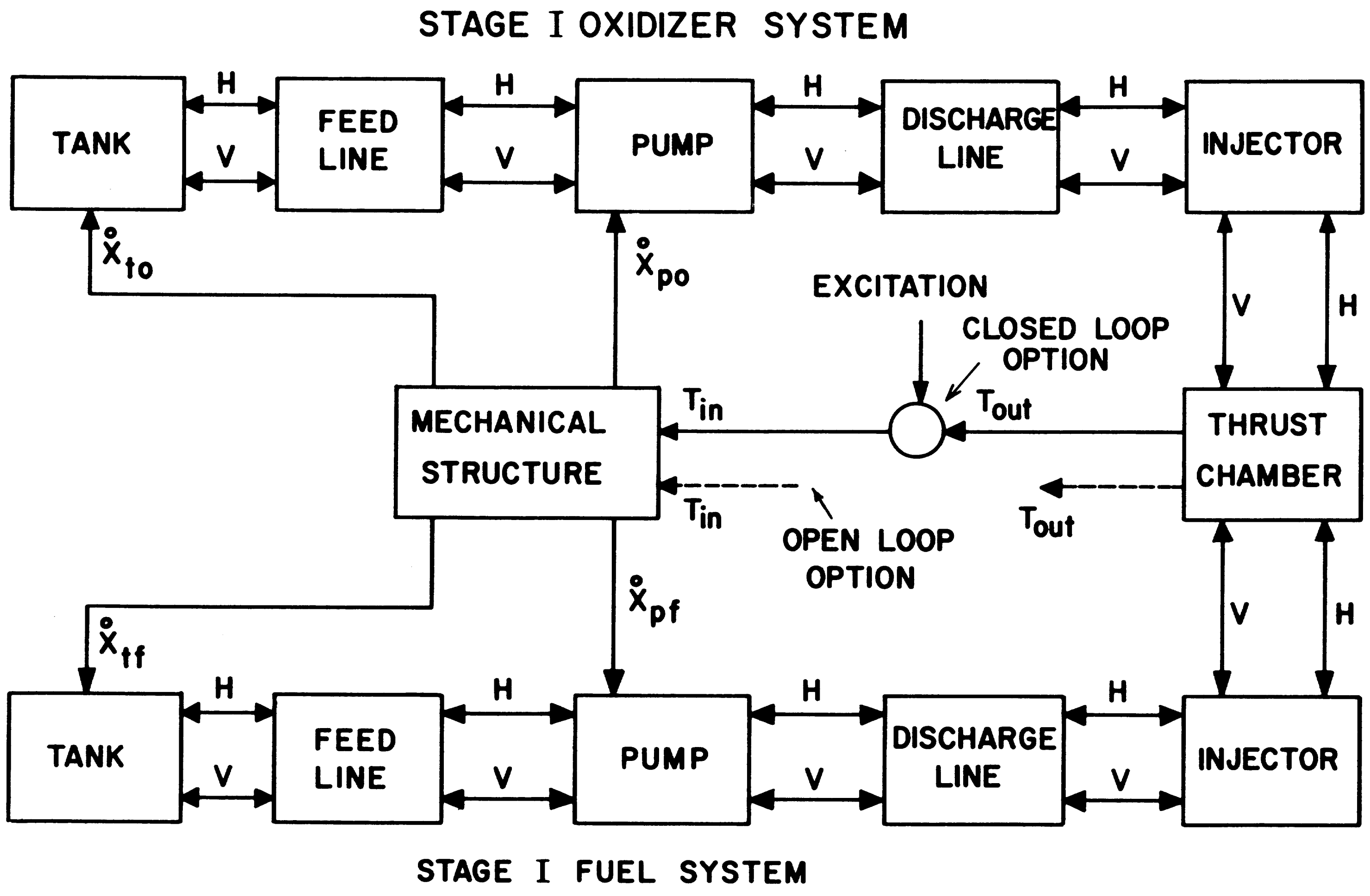

Figure 2. Diagram of Combined Propellant System, Engine, and Structure.

fluid oscillations in the fuel and oxidizer systems. These fluid oscillations cause an oscillation in thrust chamber pressure which causes an oscillation in thrust. The thrust oscillation is the input to the structure which completes the loop. If this resulting thrust oscillation is of proper magnitude and phase (relative to the structure motion) to reinforce the structure oscillation, a sustaining oscillation will occur. Thus, the problem is of the common stability type. The stability analysis of the complete system is quite complex, however, due to the complicated nature of the structure and to the interaction of the component parts of the propellant systems. In Figure 2, H refers to fluid pressure, V to fluid velocity, T to thrust and $\dot{x}$ to structure velocity.

There are basically three separate systems involved, each with a natural resonant frequency: the structure, the fuel fluid system, and the oxidizer fluid system. The relation between these three resonant frequencies is of prime importance in the problem and will be discussed in the following section.

B. The Relation Between the System Resonant Frequencies.

The resonant frequencies of the structure, the oxidizer system and the fuel system all vary with the missile flight time. The structure resonant frequency increases with flight time since the propellant mass decreases. A typical variation is shown in Figure 3. Due to the complex nature of the structure, several modes of oscillation, and therefore several resonant frequencies, are possible. In the figure the two lowest frequency modes are shown.

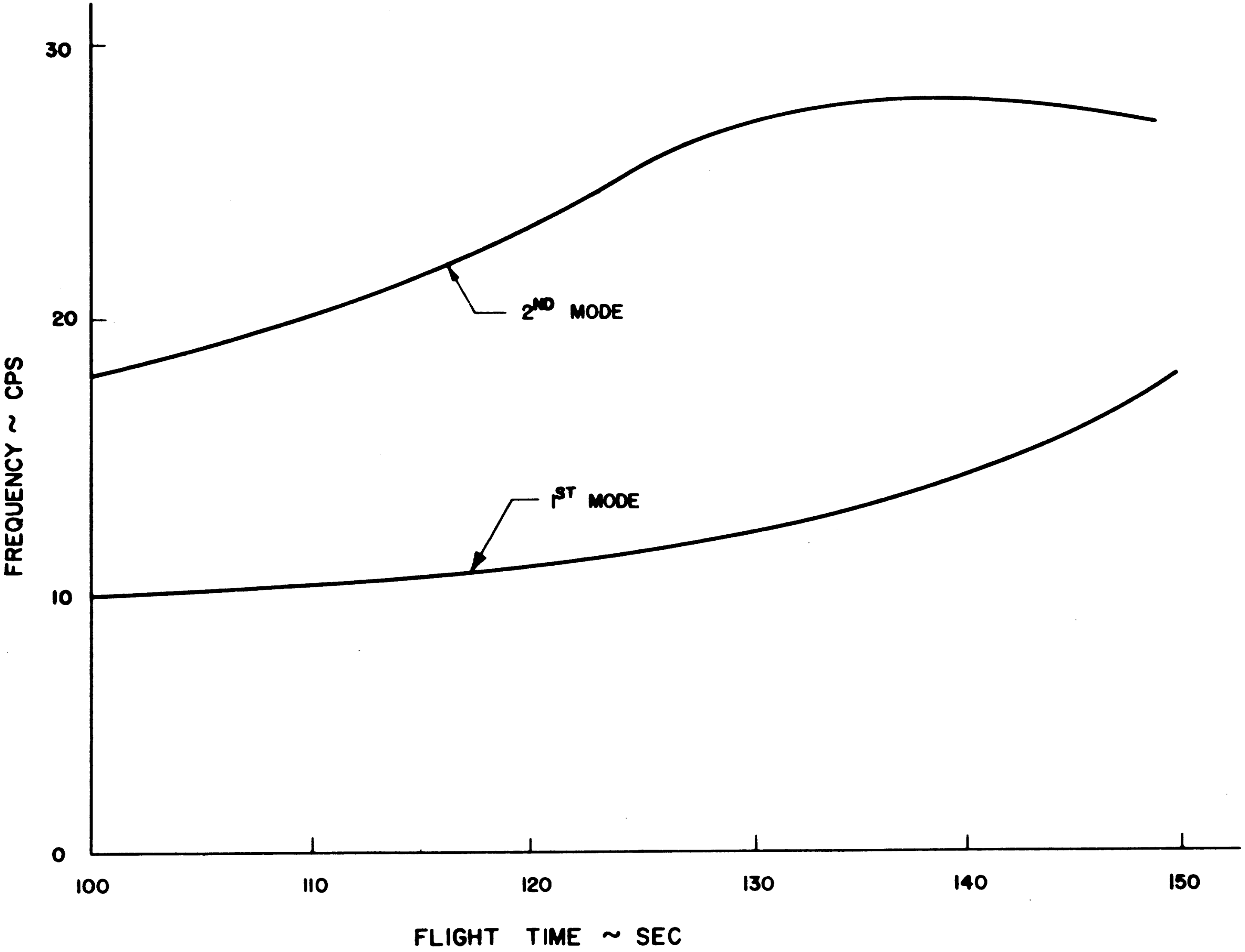

Figure 3. Structure Resonant Frequency.

The oxidizer and fuel system resonant frequencies are shown in Figures 4 and 5, respectively. In this case we are concerned with the resonance of the fluid in the tank, pump suction line, pump, and pump discharge line. The resonant frequency is not affected by the level of propellant in the tank since the hydraulic impedance at the tank-suction line junction is practically zero. This has been proven experimentally.(1) One would expect the resonant frequency to be constant, however, as Figures 4 and 5 show, this is not the case. The frequency varies with pump suction pressure because of vapor at the pump inlet caused by cavitation of the pump impeller. This cavitation vapor also lowers the frequency a considerable amount from the value one would expect with no vapor in the system. This difference is shown for a typical case in the figures. The pump suction pressure varies with the missile flight time, and therefore, the fuel and oxidizer system resonant frequencies vary with the flight time. These resonant frequency curves have to be obtained experimentally since the extent of the vapor formation by cavitation cannot be predicted.

The curves in Figures 3, 4, and 5 are plotted versus the flight time in Figure 6 to show the relationship between the resonant frequencies. This particular plot is typical of the Titan II missile (Stage I).(1) The missile oscillates at the frequency of the structure, and the oscillation begins when the oxidizer system resonant frequency becomes close to the structure frequency. The oscillation continues over to some point beyond the intersection of the fuel system frequency curve and the structure frequency curve and then stops.

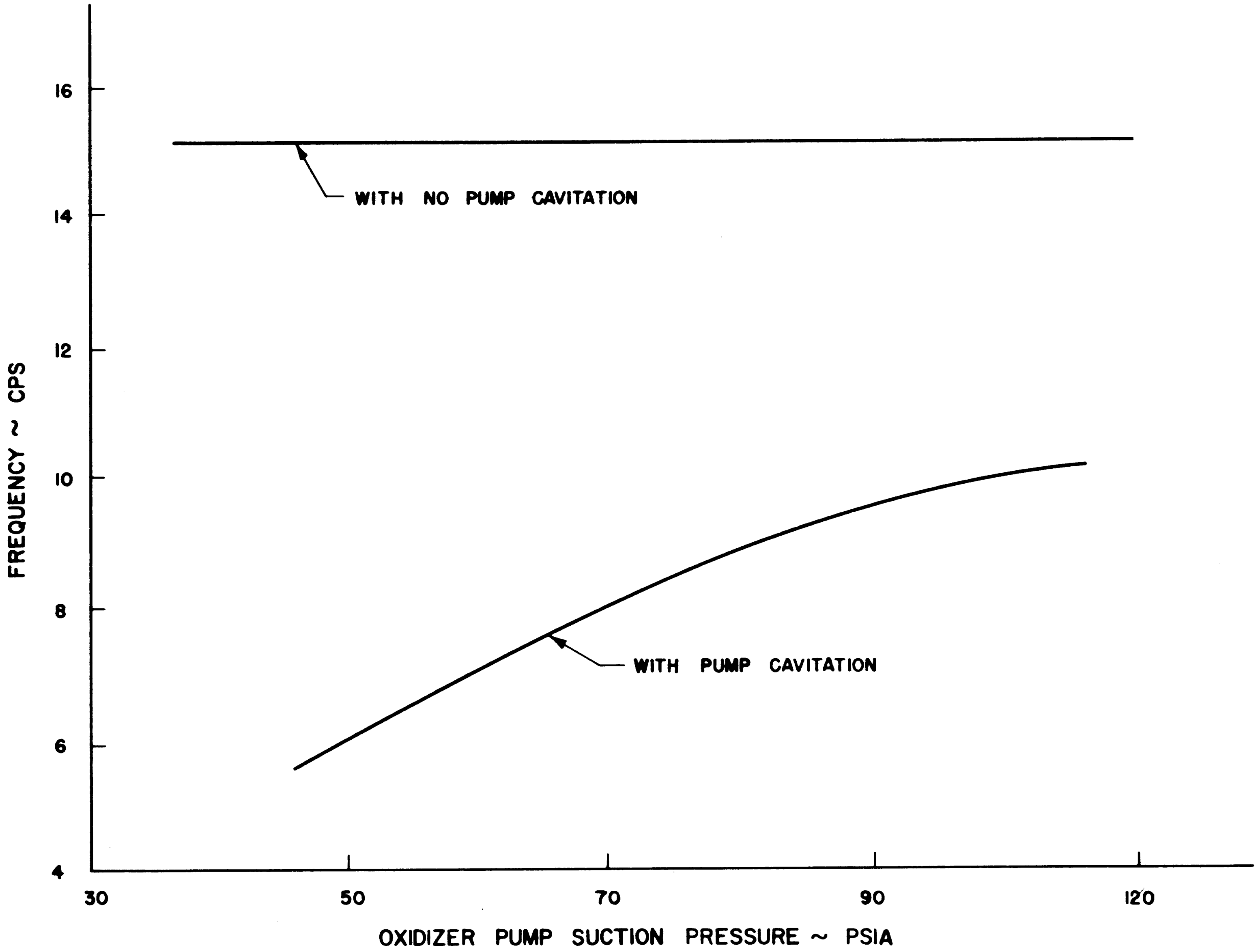

Figure 4. Oxidizer System Resonant Frequency.

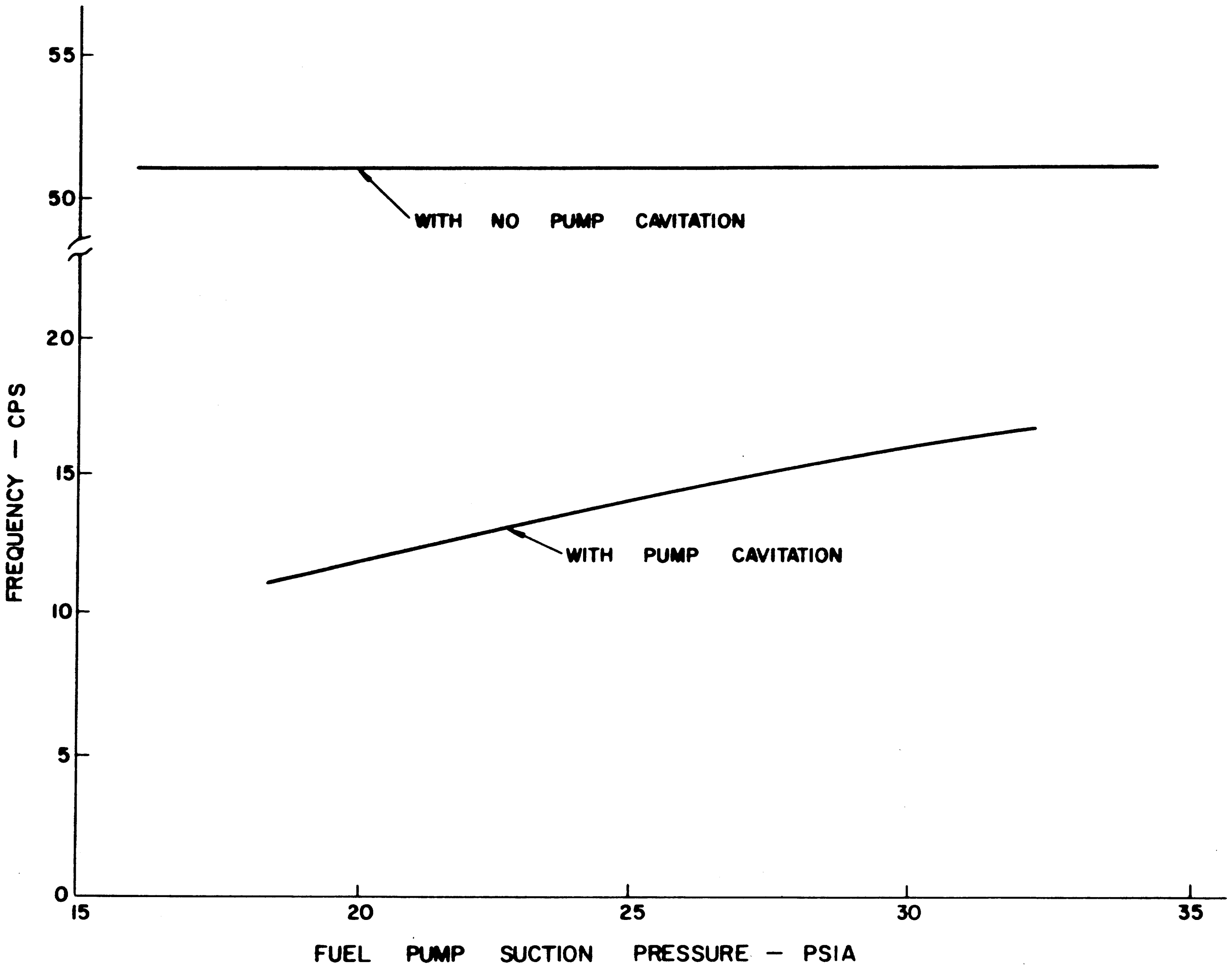

Figure 5. Fuel System Resonant Frequency.

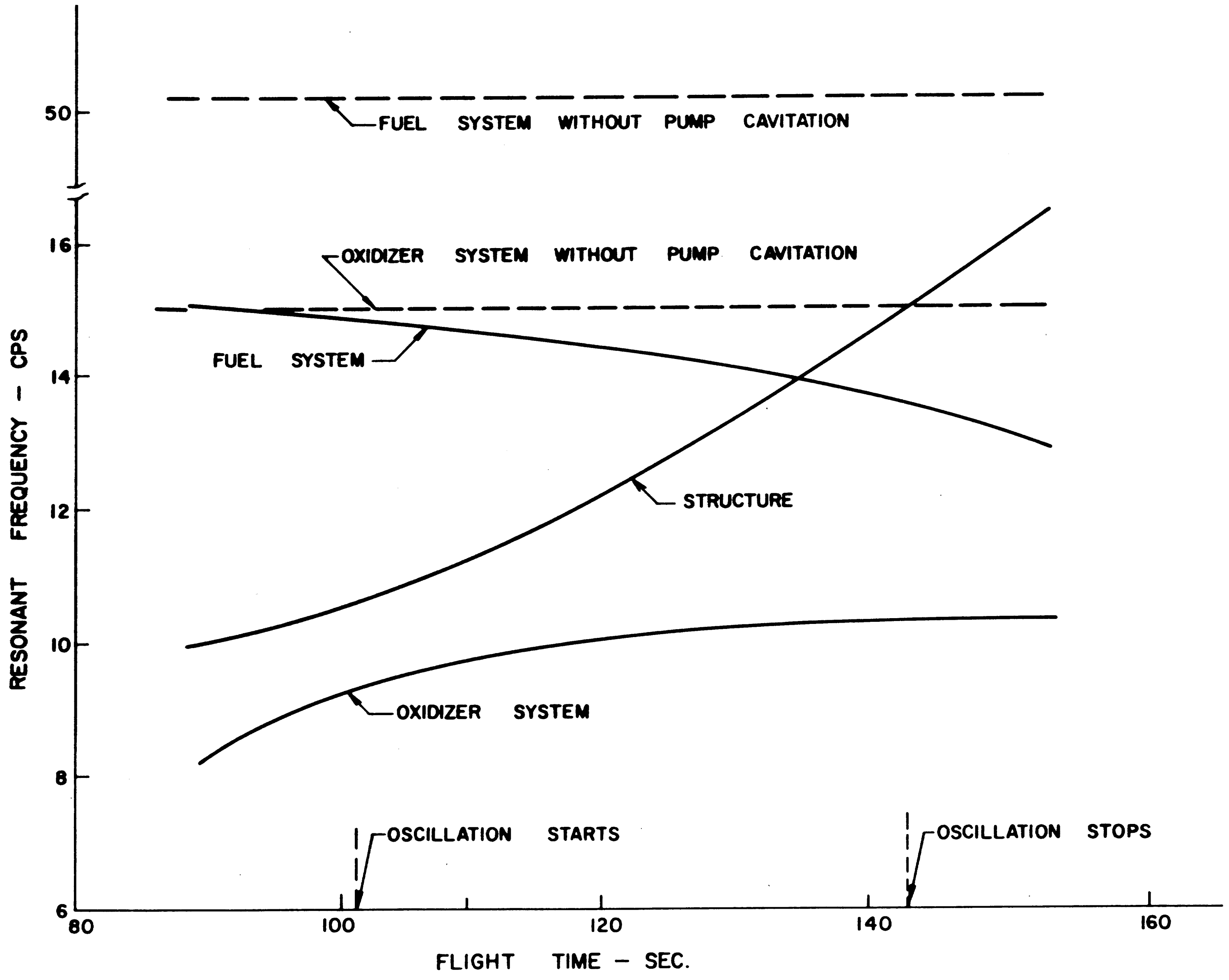

Figure 6. Comparison of System Frequencies.

The predicted fuel and oxidizer system resonant frequencies without the effect of pump cavitation are shown on Figure 6 to illustrate the importance of accurately knowing the effect of cavitation on the frequency. As seen, the pump cavitation completely changes the problem. Also, any gas that is trapped in the pump suction system (such as in a PVC) can have a similar effect.

C. Factors Affecting the Oscillation Other Than Frequency.

The possibility of the development of an oscillation in a system depends upon many factors, in addition to the natural frequency of the component parts. The main factors are: the structure gain, the pressure losses in the fuel and oxidizer systems, the dynamic gain of the fuel and oxidizer pumps, the pressure oscillation generated at the bottom of the fuel and oxidizer tanks due to tank motion, the pressure oscillation generated at the fuel and oxidizer pump inlets due to pump motion, the thrust oscillation per unit amplitude of thrust chamber pressure oscillation, and the combustion time lag.

The structure gain at a particular point on the structure is the oscillation amplitude at that point per unit amplitude of thrust oscillation. It depends upon the mode shape of the oscillation, and for a given mode of oscillation varies from point to point along the missile. A vibration analysis of the missile structure will yield the structure gain as well as the structure frequencies. In the case of the Titan II missile, the structure gains and frequencies agreed adequately with values measured in flight.(1)

The fuel and oxidizer system pressure losses are important in the overall gain of the propulsion system. The pump discharge lines typically have a much higher pressure loss than the pump suction lines. As a consequence, the resonant frequency of the system is essentially determined by the fluid in the pump suction line (including the cavitation vapor), and the pump discharge line (including injector) provides the predominant damping which limits the amplitude of the resonant curve. It should be mentioned that in representing the long suction lines analytically, a distributed representation should be used, the most accurate being a waterhammer type analysis. When the resonant frequencies are known the fuel and oxidizer systems can be represented very well by a waterhammer analysis. The measured steady state pressure losses can be used to determine the pump discharge system friction factors.(2,3)

The dynamic gain of the propellant pumps is the oscillation amplitude of the pump discharge pressure per unit oscillation amplitude of the pump suction pressure. This ratio depends upon the steady state characteristic curves of the pump, and can be less than or greater than one. In the case of the Titan II missile, the oxidizer pump gain was about 0.8, whereas the fuel pump gain was in the neighborhood of 4.0.(1) This will be discussed in more detail in section D.

In setting up an analysis of the oscillation, it is difficult to properly represent the effect of the motion of the structure on the fluid systems. The main points where the fluid system is excited are at the tank bottom, the pump, and the thrust chamber injector. It is important that the phase angle between the generated pressure oscillations and the structure oscillation be correct as well as the relation between the oscillation amplitudes, since the system stability is sensitive to this phase angle. One method used is simply to apply the acceleration of the tank directly to the fluid in the tank and the acceleration of the pump directly to the fluid in the suction line, with the fluid system represented by a lumped compliance and lumped inertance. It is felt that the tank excitation is adequately represented in this manner, but that the excitation at the pump is questionable because the change in pump pressure rise due to the change in the fluid velocity relative to the pump caused by the motion of the structure is neglected. Also, because the flow can change through the pump as the pump moves, the fluid in the suction line does not experience the same acceleration as the pump structure. Another approach is to represent the system with the waterhammer analysis,(2,3) and excite the system by varying the pump pressure rise with flow relative to the moving pump, and move the pump boundary with the motion. The waterhammer equations will yield the proper transients developed. A test is currently being conducted at the University of Michigan to establish the latter method of representing the pump motion.

The thrust chamber injector motion causes an oscillation of the pressure drop across the injector. This effect is probably small but can easily be introduced into a waterhammer analysis of the system.

The thrust oscillation depends directly upon the thrust chamber pressure oscillation, and is given by,(4)

$$T = n \ A_t \ C_f \ P_c$$

where n is the number of engines considered in parallel, A_t is the nozzle throat area, C_f is the thrust coefficient, and P_c is the thrust chamber pressure. The thrust coefficient varies slightly with the thrust chamber pressure and the mixture ratio (flow rate of oxidizer over flow rate of fuel), but very little error is made by treating it as constant. The thrust chamber pressure is given by,(4)

$$P_C = \frac{C^* \; (\dot{w}_o + \dot{w}_f)}{A_t \; g}$$

where, C^* is the characteristic velocity which is a function of the mixture ratio, $\dot{w}_O$ and $\dot{w}_f$ are the oxidizer and fuel flow rates respectively, and g is the gravitational constant. To illustrate the variation of the characteristic velocity with mixture ratio, a typical curve is given in Figure 7 with a typical operating point shown. Since the slope at the operating point can be quite steep, small variations in the mixture ratio can cause substantial variations in the characteristic velocity. Therefore, this effect should be included in an analysis.

The combustion time lag is small compared to a period of oscillation at the frequencies concerned with herein (11 cps in the Titan II case), and therefore causes only a small phase shift. This time lag is due to the time required for the combustion gas to accelerate from zero velocity at the inlet end of the combustion chamber to sonic velocity at the nozzle throat. It can be estimated from,(4)

$$\tau_c = \frac{\bar{\rho}_c \; C^* \; L_c}{P_c}$$

where, $\bar{\rho}_c$ is the average mass density of the gas in the combustion chamber, and L_c is the average combustion chamber length. To include this time lag in a waterhammer digital computer type analysis, it is only necessary to store the P_c values for the time τ_c and then compute the thrust. In an analogue analysis the Laplace operator S can be

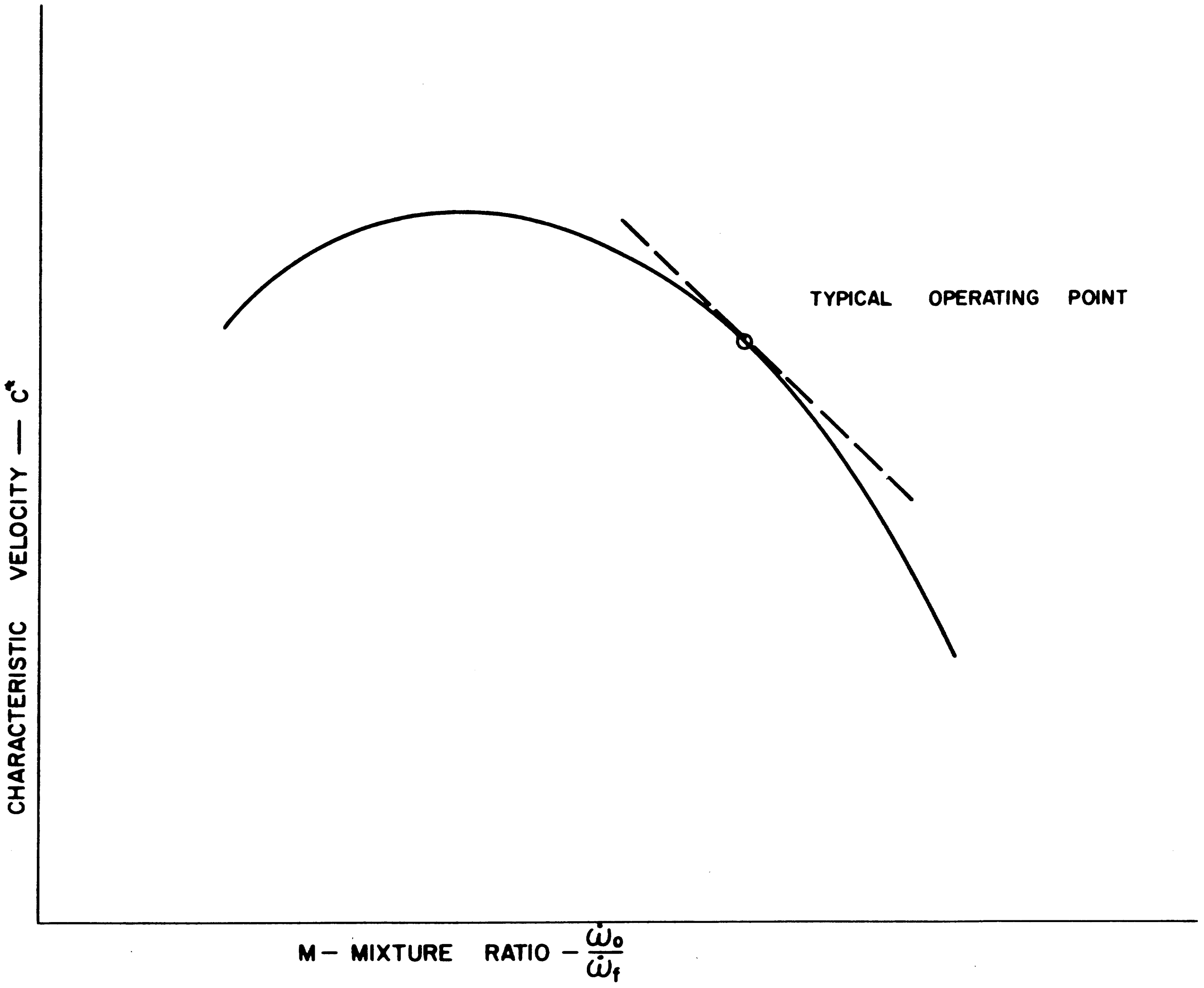

Figure 7. Typical C* versus Mixture Ratio Curve.

used and the chamber pressure computed from,

$$P_c = \frac{1}{\tau_c \; S + 1} \left[\frac{C^* (\dot{w}_o + \dot{w}_f)}{A_t \; g} \right]$$

D. <u>The Dynamic Gain of the Propellant Pumps</u>.

The propellant pump pressure rise is a function of the pump suction pressure, the flow rate and the pump speed. A typical set of steady state pump characteristic curves is shown in Figure 8. This set of curves will be used to obtain the dynamic pump pressure rise. Using the steady state curves to obtain the dynamic or transient pump pressures has been varified.(2) The pump speed will be considered constant since the mass of the pump rotor, gear train, and turbine combined is large. (Titan II experience varifies this).(1) The pump discharge pressure, P_d, is given by,

$$P_d = P_p + P_s$$

where P_p is the pump pressure rise and P_s is the suction pressure. Since the pump pressure rise is a function of the suction pressure and flow rate, so is the discharge pressure, and we can write the change in the discharge pressure as,

$$dP_d = \frac{\partial P_d}{\partial P_s} \, dP_s + \frac{\partial P_d}{\partial \dot{w}} \, d\dot{w}$$

where $\dot{w}$ is the pump flow rate. Also, from above,

$$\frac{\partial P_d}{\partial P_s} = \frac{\partial P_p}{\partial P_s} + 1$$

$$= m + 1$$

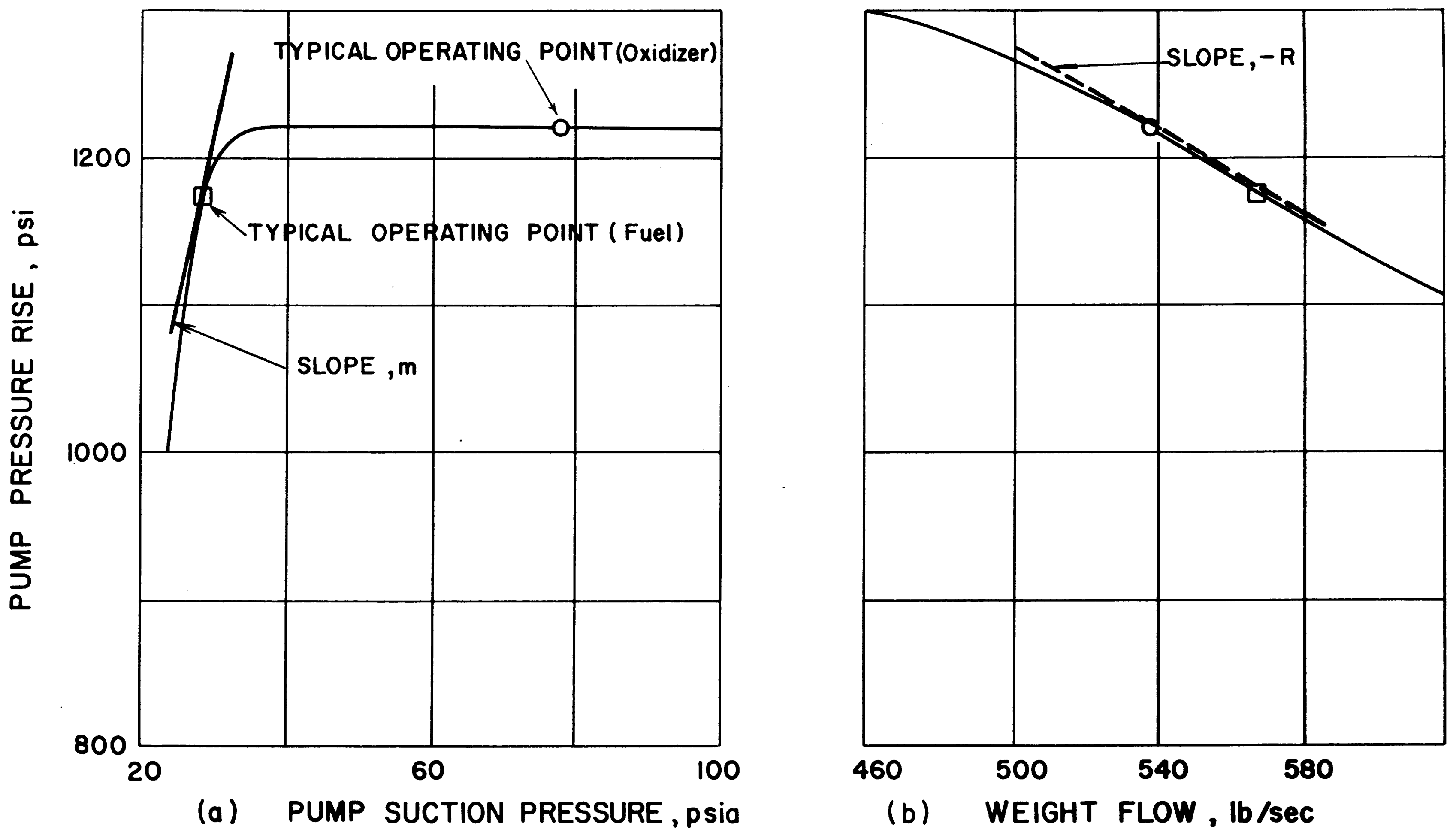

Figure 8. Typical Pump Steady State Characteristic Curves.

where m is the slope of the pressure rise versus suction pressure curve, Figure 8a. (Note: NPSH is usually the parameter used, but suction pressure is used here since the fluid temperature and thus the vapor pressure is constant). Similarly,

$$\frac{\partial P_d}{\partial \dot{w}} = \frac{\partial P_p}{\partial \dot{w}} = - R$$

where - R is the slope of the pressure rise versus flow rate curve, Figure 8b. Since the slope is negative, the minus sign is put in so that R is a positive number. Combining we obtain,

$$dP_d = (m + 1) dP_s - Rd\dot{w}$$

and the pump dynamic gain is therefore given by,

$$\frac{dP_d}{dP_s} = (m + 1) - R \frac{d\dot{w}}{dP_s}$$

The first term on the right side represents the effect of the pump pressure rise - suction pressure characteristic, and the second term represents a gain loss due to the pressure - rise - flow rate characteristic. R can be thought of as the pump resistance. The second term also reflects the nature of the fluid impedance of the pump piping system. This shows that the pump gain depends upon the dynamic characteristics of the pump suction and discharge systems as well as the pump characteristics, a fact that should be considered when measuring pump gain in a test.

The missile oscillation is sinusoidal so we can represent the pressures and flows as vectors and,

$$\vec{P_d} = (m + 1) \vec{P_s} - R \vec{\dot{w}_s}$$

which is a linearlized form where the slopes m and - R are considered constant for a small oscillation around the operating point.

The above is useful in gaining an understanding of the pump dynamics and in developing an analogue analysis of the pump. If the waterhammer characteristic method(3) is used the steady state pump curves are used directly in the program. This yields the proper pump gain with the non-linear aspects of the curves present.(2)

E. Suppressing the Missile Oscillation.

It is of interest to consider some methods of suppressing the missile oscillation. The methods to be briefly discussed are: varying the propellant tank gas pressure, changing the pump suction line, and the addition of an accumulator (surge chamber) to the suction line at the inlet to the pump.

Changing the gas pressure in the top of the propellant tanks has two effects on the system. One effect is to change the operating point of the pump on the pressure rise - inlet pressure curve, Figure 8a, and the second is to change the resonant frequency of the propellant system by changing the amount of cavitation and the effective compliance of the cavitation vapor, Figure 4. Changing the operating point changes the pump gain by changing the slope, m. The gain is reduced by increasing the pressure. If the pump is normally operating on the zero slope part of the curve, the pump gain cannot, of course, be reduced by increasing the pressure. In the Titan II case the fuel pump was operating well to the left of the zero slope region, and the oxidizer pump was operating on the zero slope part,(1) so only the fuel tank pressure was changed.

An alteration of the propellant system resonant frequency by changing gas pressure has two effects. This frequency can be moved farther from the structure frequency which reduces the gain of the propellant system, and also changes the phase angle between the structure motion and the thrust oscillation output of the propellant system. The phase angle change might or might not help, depending on whether the phase angle between input thrust and output thrust is increased or decreased. With the frequencies shown in Figure 6, increasing the fuel tank pressure will move the fuel resonance away from the structure resonance and reduce the fuel pump gain. A decrease of the oxidizer tank pressure will move the oxidizer resonance away from the structure resonance, but will not appreciably change the oxidizer pump gain.

In the case of the long pump suction lines changing the line diameter, wall thickness or material can change the resonant frequency of the system. The resonant frequency is decreased if the pressure wave velocity in the suction line is decreased, and the pressure wave velocity is decreased if the diameter of the line is increased, the wall thickness decreased, and/or the line material modulus of elasticity decreased. In the example of Figure 6, the oxidizer frequency can be moved away from the structure frequency for instance by changing from a stainless steel line to an aluminum line.

The above methods are limited by missile design restrictions and might only succeed in reducing the amplitude of the missile oscillation. A more effective way of suppressing the oscillation is the addition of accumulators at the pump inlets. By properly designing the accumulators, the fundamental resonance of both propellant systems can be moved considerably below the structure resonance which will stabilize the missile. However, the accumulators introduce two additional resonances to each propellant system, an anti-resonance (zero) which is of no concern (except in phase considerations), and a higher resonance (pole) which needs to be above any prominent structure resonances. Fortunately, at the frequency of the high resonance, the propellant system gain is much lower than at the low resonance. The suction system response curve is shown in Figure 9 with and without an accumulator. The curve is a log - log plot of the ratio of the amplitude of suction pressure oscillation over the amplitude of the fluid acceleration oscillation versus frequency.

The accumulators can be either of a piston - spring type design or a compressed gas type (standpipe). The latter is preferred because the mass (inertance) in the accumulator can be low which allows the high resonance to be higher than with the piston type. Also, since a piston has a static breaking force, the piston type accumulator is not effective until an oscillation starts, or unless the noise present (or random vibration) causes large enough pressure fluctuations to overcome the static friction. In designing an accumulator it is advantageous to get the low resonance as far below the structure resonance as possible without having the high resonance too low. The pump cavitation vapor can extend three or four feet into the suction line, and the accumulator will most likely be right in this region since it should be as close to pump as possible to get the low resonance low and the high resonance high. This vapor causes difficulty in analyzing the system with an accumulator. A waterhammer analysis is probably the best since it can approximate the effect of the vapor distribution.

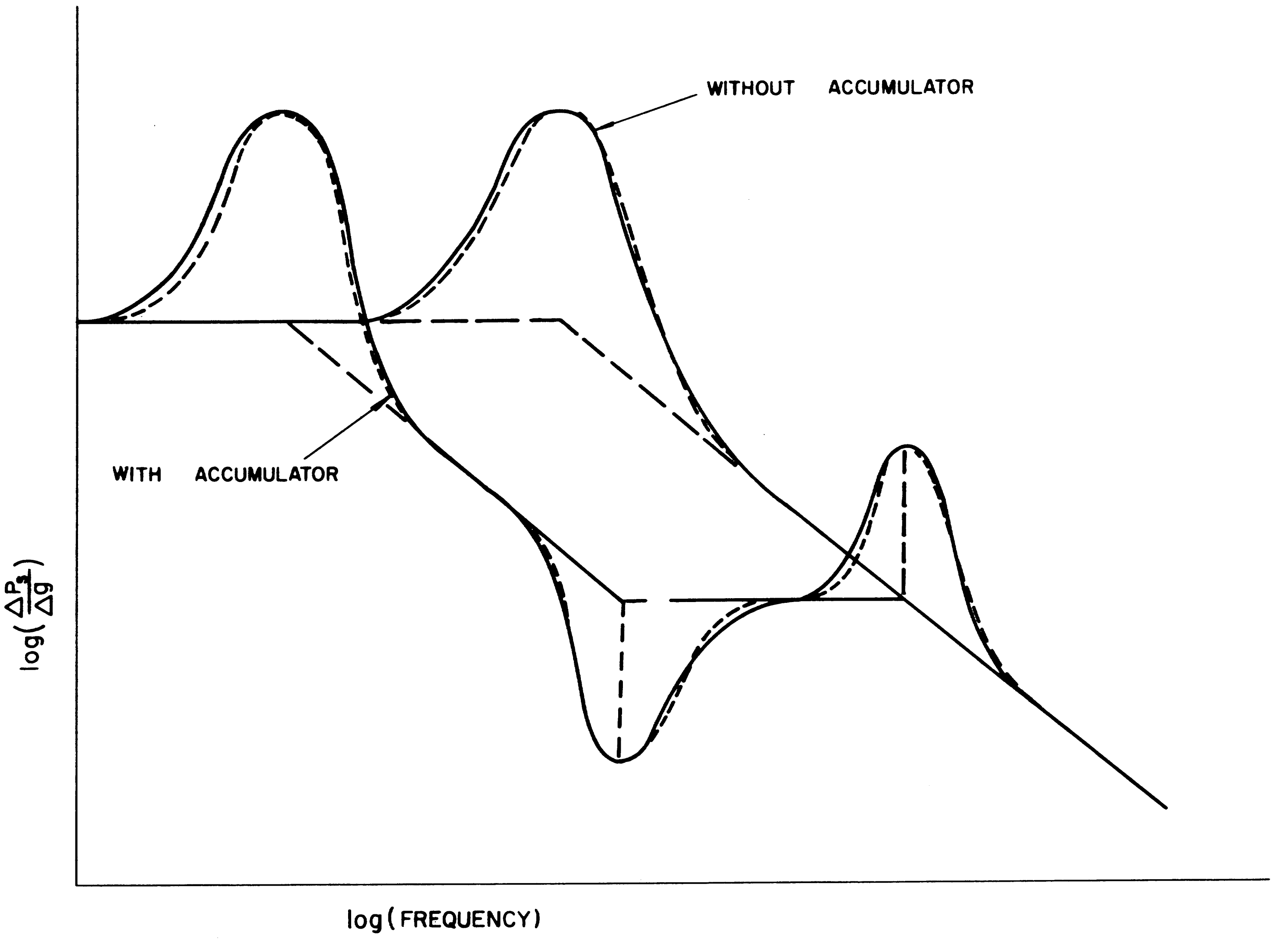

Figure 9. Suction System Response with Accumulator.

As might be expected, putting an accumulator on one propellant system and not the other might cause the missile oscillation to become worse. It is possible for the propellant systems to be out of phase such that one is cancelling the effects of the other, and the removal of one by an accumulator can increase the overall propulsion system gain causing an increased amplitude of oscillation.

III. TESTING TO DETERMINE THE PROPELLANT SYSTEM RESONANT FREQUENCY AND PUMP GAIN.

As already pointed out, the oxidizer and fuel system resonant frequencies can only be determined by testing because of the effect of pump cavitation. In fact, each particular pump has to be tested since a correlation between pumps for this factor does not presently exist. It is possible that a correlation might exist through the pump cavitation index parameter (fluid velocity head over equivalent impeller tip speed head), but there is not enough data available on missile pumps to establish any correlation.

A typical test configuration is shown in Figure 10 for an oxidizer system. A pulser is used to introduce sinusoidal flow oscillations although other methods could be used such as oscillating a valve or vibrating the pump. It is important that the pump suction and discharge systems be the same from a fluid dynamic standpoint as the actual missile system. The length, the pressure wave velocity, the inertance, and friction losses in each line should be the same as in the missile system. The tank can be other than a missile tank since it does not affect the frequency. The test system should be isolated from the facility return lines by a cavitating venturi or cavitating orifice placed at the engine nozzle location in the missile system. The thrust chamber injector can be simulated by an orifice. Proper simulation of the missile system is important in order to obtain the correct pump gain as well as the resonant frequency.

The resonant frequency can be obtained by pulsing over a frequency range to obtain the frequency at maximum pump suction pressure

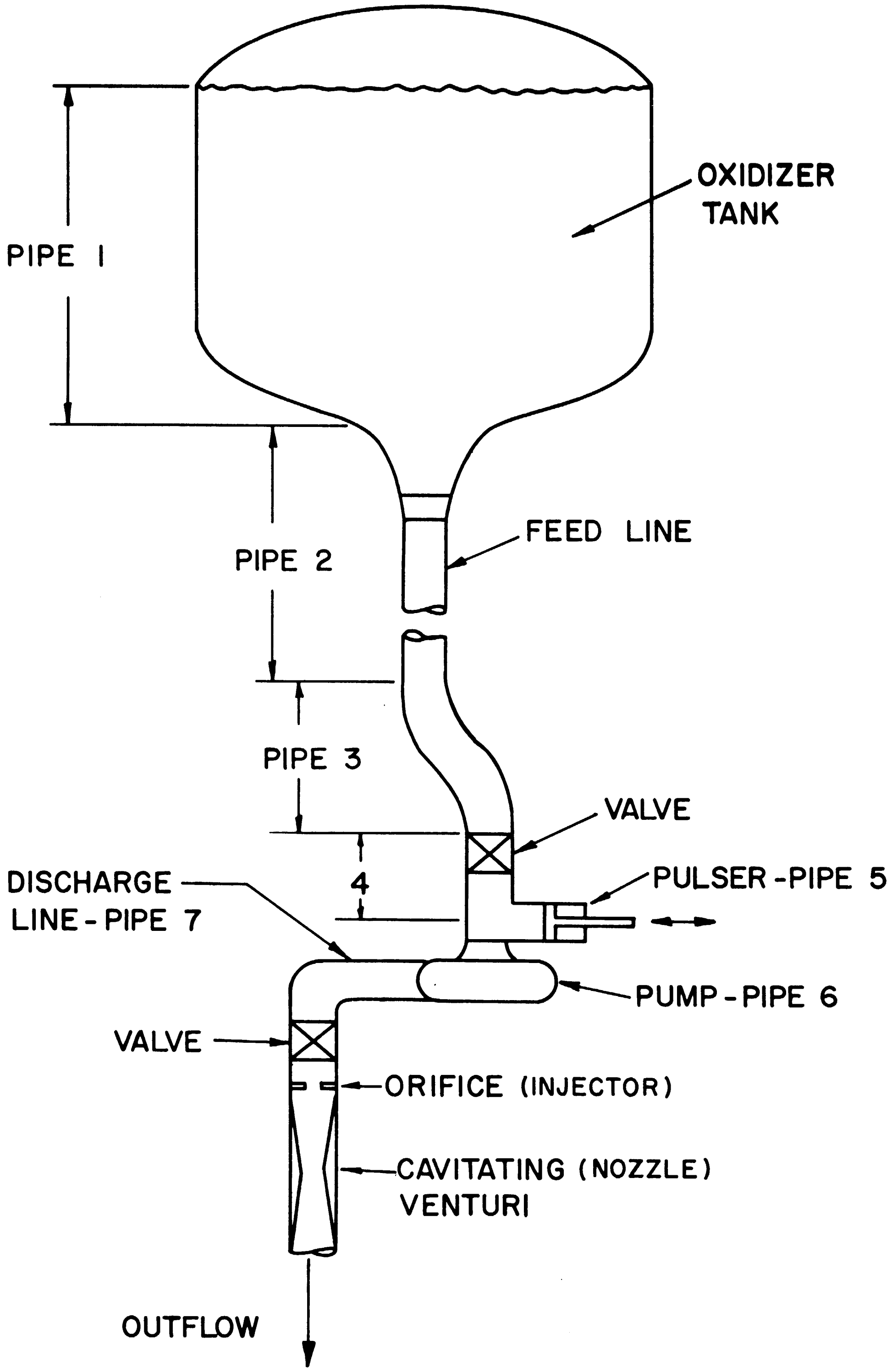

Figure 10. Oxidizer Propellant System Test Configuration.

oscillations. A typical plot is shown in Figure 11 (Titan II stage I oxidizer system).(2) A typical plot of the pump gain is shown in Figure 12. Figure 12 also shows a curve of the pump discharge pressure oscillation amplitude over the orifice outlet pressure oscillation amplitude versus frequency which is helpful in substantiating an analysis of the system.

The pulsing should be done at more than one amplitude to seek any dependence of frequency on amplitude that might be present.

Other information can be obtained from the test such as the extent of penetration of the cavitation region into the pump suction line. Measuring the standing pressure wave pattern along the suction line and/or pulsing the discharge line with a square pulse and measuring the pulse time delay up the system can accomplish this. It was found that the cavitation effect extended approximately three to four feet into the oxidizer pump suction line in the Titan II stage I system.(1,2)

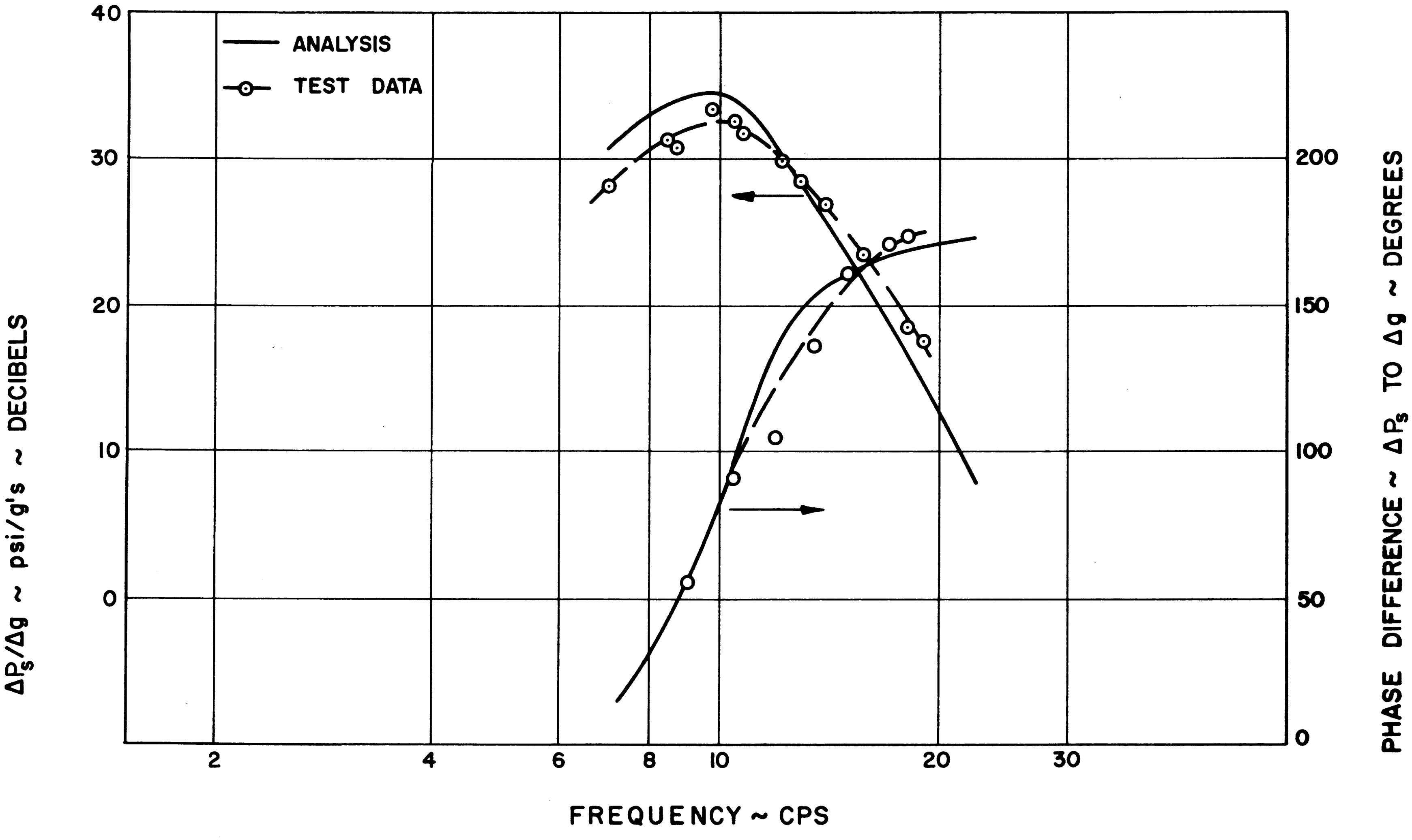

Figure 11. Oxidizer Pump Transient Suction Pressure.

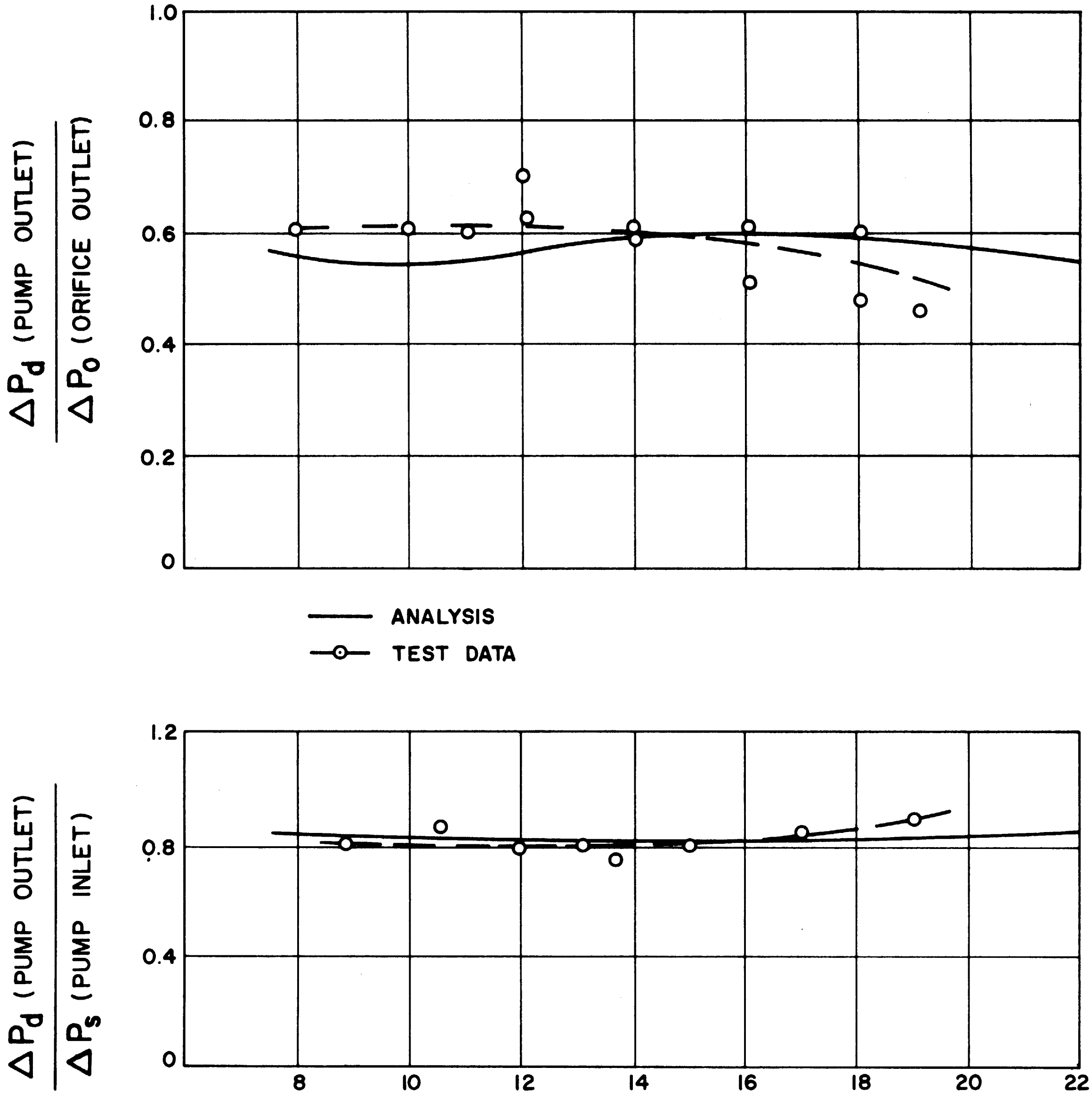

Figure 12. Oxidizer Pump Gain and Discharge System Pressure Ratio.

IV. AN ANALYSIS OF THE PROPELLANT SYSTEM USING THE WATERHAMMER - CHARACTERISTIC METHOD.

In order to establish a method of analyzing the fluid transients in a missile propellant system a waterhammer analysis of the Titan II stage I test was made, and the results of the analysis were compared with the test data. A detailed description of the analysis will not be given here since it is presented in Reference 2, and it will be assumed here that the reader is familiar with the material in References 2 and 3. The main difficulties in analyzing the propellant system are representing the flow transients through the pump and the pump cavitation. The other parts of the system are represented as explained in Reference 3.

In this analysis the pump is considered as a pipe with a length equal to the mean flow path length through the pump. The same fluid characteristic equations are used as are used in a constant area pipe except the friction term is replaced by a pump pressure rise term with a change in sign. This pressure rise term can be derived in the same manner as the friction term is derived in Reference 3 or as was done in Reference 2. For reference, the pump flow characteristic equations are,

$$V_p = 1/2 \left[(V_R + V_S) + g\,(H_R - H_S)/a_C + 2g\,\left(\frac{\Delta H_p}{L_p}\right) \Delta t \right]$$

$$H_p = 1/2 \left[(H_R + H_S) + a_C\,(V_R - V_S)/g \right]$$

where the notation follows that used in Reference 2. The comparison of the test data and computed data are presented in Figures 11 and 12 for the oxidizer system. These data show that for a stationary pump the analysis predicts the pump gain quite well. The ΔH_p used in the above equations is the pump pressure rise predicted from the steady state curves of Figure 8.

The pump cavitation is accounted for in the analysis by selecting a short length of the suction line at the pump inlet end in which there is assumed cavitation vapor. The pressure wave velocity is reduced in this section by the vapor. The value of this pressure wave velocity is computed such that the system resonant frequency will agree with the measured value of Figure 4. This is explained in Reference 2. The analysis is not sensitive to the length selected for the cavitation region since the resonant frequency will be computed the same regardless of the length, and the propellant system damping which determines the shape of the resonance curve is primarily in the discharge line. It should be mentioned, however, that if higher order resonances of the suction system are important the cavitation region length could have an effect. In the Titan II analysis a length was used that was determined approximately from the test.

V. ANALYSIS OF MISSILE IN FLIGHT

The propellant system analysis presented in Section IV, which deals with the static test stand, forms the basis of a complete missile stability study. The pulser is removed and fluid system excitation caused by the pump motion, tank motion, and injector motion is added. The pump movement affects the relative velocity to the pump which affects the pump head rise, ΔH_p, in the above equation. Flow continuity at the pump boundaries is also affected by the motion. The tank motion is inserted by maintaining flow continuity in a region containing the tank bottom to suction line junction with the tank bottom moving. The effects of the injector motion can be inserted by writing the injector pressure drop in terms of the flow velocity relative to the moving injector.

The cavitating venturi in the test analysis is replaced by the engine thrust chamber and nozzle equations given in Section I to convert to the missile system.

The structure is represented by a linear 2nd order differential equation with constant coefficients, with the constants determined from the structure vibration analysis, such as,

$$\ddot{x} + a\dot{x} + bx = cT$$

where x is displacement of a point on the structure such as the pump, a, b, c are constants from the vibration analysis, and T is the thrust inputed to the structure.

VI. SUMMARY AND CONCLUSIONS.

The problem of longitudinal oscillation in a liquid propellant missile is reviewed in this study. Some of the causes are outlined, remedies are suggested, and a complete analysis is presented. The role played by each of the important component parts of the system on the development of an oscillation is outlined in an effort to convey a broader understanding of the entire problem.

A comparison is given between analytical and test stand data. The favorable comparison of these results lead to the conclusion that an accurate stability analysis of the entire missile system can be made.

Many of the uncertainties associated with the analysis have been resolved through the experience gained on the Titan II analysis. The use of the steady state pump characteristic curves in the dynamic situation is shown to be satisfactory. A structure vibration analysis can be adequately made (Titan II, Reference 1). The tests underway at the University of Michigan will lead to an improved understanding of the boundary condition at an oscillating pump. With these results in hand, the response of the entire system should be predicted reasonably well.

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