THE UNIVERSITY OF MICHIGAN

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# EXPLORING THE DEPTH OF SURFACE LAYERS OF THE MOON AND PLANETS FROM A RADAR SPACE OBSERVATORY


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February 1959

IP-351

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## 1. CONCEPT

For many excellent reasons, outer space observations of the moon, the stars, and the planets have been made in the visible spectrum for centuries. During this century, observations have been made of radiation from outer space at many other portions of the spectrum. These measurements include infrared, radar, and radio waves. With the beginning of the space era, space observatories which are not influenced by the earth's atmosphere or constrained by the earth's trajectory are now being proposed. It is only natural that the first such space laboratories investigate the moon. It is also to be expected that the instrumentation will be in the visible spectrum. To point out the possible value of investigating the moon by radar is the purpose of this paper, which, while not intended as an exhaustive survey of the benefits of radar for observation in outer space, is intended to give an example of how some information not yet available by other means could be obtained by radar.

Radar can be used not only to determine the heights and contours of mountains and the depths of craters but also to determine how the layer structure varies, as evidenced by a function of permeability, $\mu$, permittivity, $\epsilon$, and conductivity, s.

In Reference 1, the specular radar return at wavelengths of 0.1m to 2.5m are analyzed and the ratios $\epsilon/\mu$ and $s/\mu$ determined for the specular areas which are near the center of the moon as seen from the earth. Refined calculations yield:

$$\frac{\epsilon}{\mu} = 6.5 \times 10^{-6} \text{ mhos}^2;$$

$$\frac{s}{\mu} = 3.5 \times 10^{2} \text{ mhos/henry}.$$

A vital question in the case of the moon is where to land on its surface. This question is based on one important "if": If the moon is covered with dust--and the ratios cited above are consistent with some fine sandy particle structures--there will be a danger of sinking when landing. Radar will be able to obtain information about the depth of this outer sandy surface.

This can be done by analyzing a region on the moon at many different frequencies. The depth of electromagnetic penetration will be the order of the wavelength until a more imperfectly conducting material is reached. That is, at some discrete frequency a sharp rise in power reflection coefficient should begin, while at frequencies higher than this the power reflection coefficient should be relatively flat and of order $10^{-4}$ (measurements of the power reflection coefficient in the specular region show a variation of only a factor of two between wavelengths of 0.1m and 2.5m). By deriving the ratios of conductivity to permeability and permittivity to permeability, and knowing the wavelength--and thus depth--of occurrence, as well as the width and length of this high-reflection-coefficient region, a basis can be reached for determining the region to land in, or at least for determining a means of selection of a region for further investigation.

Now let us consider how to obtain such a measurement from our radar space laboratory. We want to measure reflection coefficients as a function of frequency for particular areas on the moon's surface. We require a space ship which will transmit at many frequencies simultaneously. In our example in Section 2 we have chosen 10 frequencies. We want to "look" at the same or almost the same area of the moon's surface at each frequency. In Section 2 we show how this can be done with off-the-shelf items.

Little or no energy will be used during data collection, as the one thing in space travel that can be obtained for almost nothing is the portion of trajectories where the ship behaves as a satellite; almost circular satellite orbits appear to be the ideal trajectories for collecting depth information by the method briefly described above and described analytically in Section 3.

The earth's ionosphere precludes us from getting all the data from a radar on the surface of the earth as long wavelengths are reflected with little or no transmission through the ionosphere. A space observatory would allow us to see beyond the moon.

According to the analysis given in Reference 1, there are approximately 11 major specular regions on the surface of the moon facing the earth. Our space ship, of course, will be looking at many aspects that are different from those seen from the earth. The specular region under the space ship, when it exists, will occasionally be the same as one seen from the earth. There will be other specular regions not near the point on the moon nearest to the satellite, but in some cases even on the periphery on the line of sight from the satellite to the moon, which could produce other specular regions. However, these regions are easy to resolve by range gating. The key question concerning resolution, as far as the important specular returns are concerned, is associated with the beam width of a specular region. There are many types of specular reflections possible. Let us consider only two types here, namely, those in which we get definite focusing, or those which involve the radii of curvature of the moon itself. The radar cross section due to the radii of curvature of the moon itself will give us an answer in the order of magnitude of 300 sq. mi. Focusing effects, i.e., flat plate, as well as all other specular effects which are significant in amplitude (such as multiple bounce - corner reflector effects), will vary from pulse to pulse, and on integration we will lose them. This variation is due to the narrow beam width of the forward return for all reflectors of significant amplitude. This is precisely what we want, as then the only specular region moving with the space ship itself is the one involving the radii of curvature of the moon. This is what we want to occur to determine depth, as we shall see in Section 3.

The space vehicle and radar described in Section 2 represent a compromise, in that they are described within current state-of-the-art; in general, one of several possible choices has been made to suggest how the design can be accomplished.

## 2. THE VEHICLE

The requirements placed on the radar and vehicle are quite stringent. Indeed, they can only be met conceptually, for we would require the scanning of a few square feet of area simultaneously by a continuum of frequencies, which is not practically possible, since we are limited in size, weight, complexity, and by a vehicle speed of orbital magnitude.

We have chosen an orbital vehicle of practicable dimensions (not much larger and no heavier than Sputnik III) and thus have placed further limitations on our design. The length is 150 ft-- 1/2-wavelength at the longest wavelength chosen--and the diameter is 6 feet; the total weight of the orbiting vehicle will be less than 4000 pounds.

Ten frequencies between 3 and 300 Mc, are employed, 1 oscillator, and suitable doublers furnishing the reference frequencies for the final amplifiers. Each transmitter feeds a 1 or 1/2-wavelength radiator, which are spaced around the periphery of the vehicle. To provide scanning (as well as stabilization), the vehicle is spun prior to launching at 300 rpm. At an orbital altitude of 100 nautical miles above the moon's surface, and with a pulse repetition rate of 500 pps, each frequency will be pulsed 10 times per cycle--a cycle being a 360° rotation of the vehicle. Since orbital velocity at 100 nautical miles is about 1 nautical mile per second, the vehicle track on the moon's surface will be about 0.2 nautical mile per cycle.

Although the antennas will provide little resolution, resolution will be provided by the nature of specular reflection as described in

Section 1 of this paper. Because of this specular reflection and the extremely broad antenna pattern, small deviations in vehicle attitude about the yaw and pitch axes can be tolerated.

A means of vehicle stabilization in addition to spinning is required because of the shape of the moon, (i.e., deviation from spherical) and other perturbation forces. However, very small reaction forces can be used to achieve this stabilization.

The vehicle will carry a nuclear power source to supply 3 or 4 Kw of electric power. Since this power source will operate at approximately 1000°, a considerable amount of energy will be available to operate small jet reactors, a high-density fluid being carried to become ejected mass. Four such jets will be spaced at 90° around the periphery of the vehicle at each end.

Since only at the highest frequency employed is it practicable to carry antennas of optimum design for the configuration, two 300 Mc 1-wavelength radiators in corner reflectors will be mounted, in line, at opposite ends of the surface of the vehicle, thus separated by about 150 ft. Returns from these antennas will synchronize operation of the vehicle system.

The 10 transmitters must be pulsed sequentially at a time when the associated antenna is pointed generally along the gravity vector. To accomplish this, a synchronizer will be activated by minimum-range signals from the 300-Mc antennas. The reaction jets will be activated by error signals resulting from comparison of returns received by these 300-Mc antennas; error signals will result from both range change, indicating pitch, and doppler content dissimilarities, indicating yaw.

In operation, the outputs of the receivers will be integrated over 10 pulses, stored, read out on demand from earth and transmitted via the 300-Mc transmitter. In order to compare transmitted and received power, i.e., to obtain reflection coefficient, it will be necessary to calibrate the antennas carefully prior to the operation. Power level of the transmitters will be stored and telemetered, along with the associated returns.

The gross weight of the rocket will be about 500,000 lbs; this assumes an over-all payload ratio of 125; i.e., for each pound on orbit 125 lbs of "on-the-ground" weight is required.(2)

To keep over-all length within reason, clustered (parallel) boosters will be used in the first and second stages. With a take-off acceleration of 1/2g, 750,000 lbs of thrust are required in the first stage. If materials permitting a structural factor of about 0.20 are available, and with fuels of I-specific of the order of 300, it may be feasible to use only 3 stages; however, 4 stages would present no unsolvable problems. A burnout velocity of approximately 6 nautical miles per second is required; firing eastward from near the equator would, of course, add the earth's rotation speed--about 0.2 nautical miles per second.

A retro-rocket will be used to slow the vehicle to the desired orbital speed upon its arrival at the moon.

## 3. DISCUSSION

When a homogeneous sphere of arbitrary electromagnetic properties is illuminated by plane-wave radiation whose wavelength is small in respect to the dimensions of the sphere, the radar cross section can be given to the leading term of its asymptotic expansion by the equation

$$\sigma = |R|^2 \, 4\pi a^2, \tag{1}$$

where $|R|^2$ is the power reflection coefficient and a is the radius of the sphere.

Although Equation (1) is simple in form, it is an essentially new result and a word about its derivation is in order. Using the exact expression for the field scattered back from a perfectly conducting sphere when a plane wave is incident upon it, the amplitude of the received signal is[3]

$$\left(\frac{a}{2r-a}\right) \left\{ 1 + O\left(\frac{\lambda}{a}\right) \right\}, \tag{2}$$

where a is the radius of the sphere and r is the distance from the receiver to the center of the sphere. This should be compared with

$$\left(\frac{a}{2r}\right) \left\{ 1 + O\left(\frac{\lambda}{a}\right) \right\}, \tag{3}$$

which is the limiting form of (2) when $r \gg a$, and (3) leads to the usual value $\pi a^2$ for the far-zone cross section of a perfectly conducting sphere.

For finite conductivity, the expressions (2) and (3) must be multiplied by $|R|^2$ and this value of R is precisely the one obtained by considering a uniform slab whose electromagnetic constants are the same as for the sphere.

In order to determine the depth of the outer layer of the moon, the procedure is as follows. We first measure the cross section $|R|^2 4\pi a^2$, and for sufficiently small wavelength the reflection coefficient R will depend only on the outer layer. As the wavelength is increased, the effect of the inner layer will be manifested by the presence of oscillations in the curve of reflection coefficient against wavelength; as the wavelength is increased still further, the fact that the constants of the inner layer are, by assumption, large compared with those of the outer, means that the inner layer will eventually dominate the return.

The reflection coefficient is, therefore, measured at a variety of different wavelengths and from these values the depth and properties of the inner layer are calculated. The experiment will be a complete success when we have determined not only the depth as a function of position on the moon's surface but also the appropriate values (or bounds) for the electromagnetic constants. It should be observed, however, that the constants obtained in Reference 1 and the validity of equations such as (1) above, are dependent on using data obtained from specular regions alone.

The theoretical problem to be considered is the reflection coefficient of a two-layered structure. For a "coated" sphere whose outer layer is of depth d, the voltage reflection coefficient R is given by

$$R = \frac{R_1 + R_2\, e^{2ik_2 d}}{1 + R_1 R_2\, e^{2ik_2 d}}, \qquad (4)$$

where $R_1$ and $R_2$ are the voltage reflection coefficients of the outer and inner layers respectively, and $k_2$ is the propagation constant in the outer layer. Another form of the same result, but in terms of the electromagnetic constants, is

$$R = \frac{1-n}{1+n} + \frac{n-m}{n+m} e^{i2nd\left(\frac{2\pi}{\lambda_o}\right)} , \qquad (5)$$

in which

$$n = \sqrt{\frac{\mu_o \epsilon_1}{\mu_1 \epsilon_o} \left(1+i \frac{s_1}{\omega \epsilon_1}\right)} ,$$

$$m = \sqrt{\frac{\mu_o \epsilon_2}{\mu_2 \epsilon_o} \left(1+i \frac{s_2}{\omega \epsilon_2}\right)} ,$$

$$R_1 = \frac{1-n}{1+n} \text{ and } R_2 = \frac{n-m}{n+m} .$$

In going from Equation (4) to Equation (5), the second term in the denominator has been omitted in the belief that this is small compared to unity in the case of the moon, and in addition, the equation has been expressed in terms of the free-space propagation constant. The symbols n and m in (5) denote the indices of refraction of the outer and inner layers respectively, and

$\mu$ = permeability

$\epsilon$ = permittivity

s = conductivity

$\omega$ = $2\pi \cdot$ frequency

subscript o = free-space

subscript 1 = outer layer

subscript 2 = inner layer.

When $\lambda_o \gg 4\pi nd$, it is seen from Equation (5) that if the properties of the inner layer are such that $s_2 \gg s_1$, and

$$\frac{\epsilon_2}{\mu_2} \gg \frac{\epsilon_1}{\mu_1} \quad ,$$

this layer will govern the return. On the other hand, when $\lambda_o \ll 4\pi nd$, but with the same electromagnetic constants, the very large phase factor will cause the second term (corresponding to the inner layer) to be negligible over any band of frequencies.

As given in Equation (5), the voltage reflection coefficient depends only on

$$\omega,\ \epsilon_o,\ \mu_o,\ \frac{\epsilon_1}{\mu_1},\ \frac{s_1}{\mu_1},\ \frac{\epsilon_2}{\mu_1},\ \frac{s_2}{\mu_1},\ \text{and } d,$$

and since the first three are, of course, known, we are left with 5 quantities to determine. A possible procedure for the data analysis is to measure the return at 5 frequencies, and hence calculate d. In practice it may be best to use additional frequencies and smooth out the minor wiggles by rounding off the measured values. We now have more equations than there are unknowns, and the equations should be tested for consistency.

The depth d need only be determined when certain conditions are simultaneously satisfied. In the first place, only data which is indicative of a two-layer structure is considered and in cases of doubt it is natural to throw away the data. In addition, the fact that we are interested in selecting places to land on the moon implies that the only cases to be analyzed are those in which d is small, with an inner layer having electromagnetic constants characteristic of a hard surface (based on our experiences on mother earth).

For an inner layer of given properties, the smallest d will occur when the smoothed curve of $|R|^2$ against $\lambda$ has its maximum at the smallest wavelength. If, at a particular wavelength, the value of $|R|^2$ is within a factor 10 of unity, the corresponding d can be regarded as minimal, or alternatively, the wavelength which has been used is too large. To decide which of these is correct, a comparison with the data obtained for the same location at a higher frequency should be sufficient.

## 4. CONCLUSIONS

In this paper a method is outlined for determining the depth of the surface layer of the moon by radar. Hardware now under development in the United States, could be used.

Concerning the exploration of outer space, it is envisioned that space observatories operating in the visible spectrum will work very closely with radar space observatories (possibly using the same vehicle) to obtain the maximum amount of information per space flight.

## REFERENCES


1. Senior, T.B.A. and Siegel, K. M., "Reflection Characteristics of the Moon", Presented at the Symposium on Radio Astronomy; Paris, France; July 30 - August 6, 1958. To be published in a book to be edited by R. N. Bracewell.

2. The Air Force stated recently that in the "near future" manned flight around the moon would be feasible with an over-all payload ratio of 75. General B. A. Schriever, USAF, quoted in Aviation Week, November 17, 1958.

3. Weston, V. H., "Near Zone Scattering from Large Spheres", Radiation Laboratory Memorandum 2778-509-M, December 18, 1958.