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LETTER TO THE EDITOR

Close doublet structures in ^{103}Mo, ^{109,111}Ru, and neighbours: rotation-alignment for the half-filled $h_{11/2}$ subshell?

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Abstract. Several new γ transitions are assigned to ^{103}Mo and ^{109,111}Ru in a γ–γ–γ coincidence study from the spontaneous fission of ^{252}Cf with 72 Compton suppressed Ge detectors in Gammasphere. A close doublet structure of an odd-parity band except near its bandhead is a common feature not only of the nuclei studied here but of many others with $61 \leqslant N \leqslant 67$. This doublet structure may be a general consequence of rotation alignment for configurations of half-filled j-shells, which are only weakly coupled to the deformed shapes.

The studies of beta decay and prompt fission gamma spectra in spontaneous fission [1–3] afford unique access to a strongly prolate region of the $Z = 42$–44 nuclei. The large gamma detector arrays are producing high-quality data that permit the study of odd-A nuclei. Level schemes of ^{103}Mo, ^{109}Ru and ^{111}Ru have been significantly extended. These new data provide evidence for a close doublet structure in odd-parity bands in these nuclei. The experimental details are discussed in [4].

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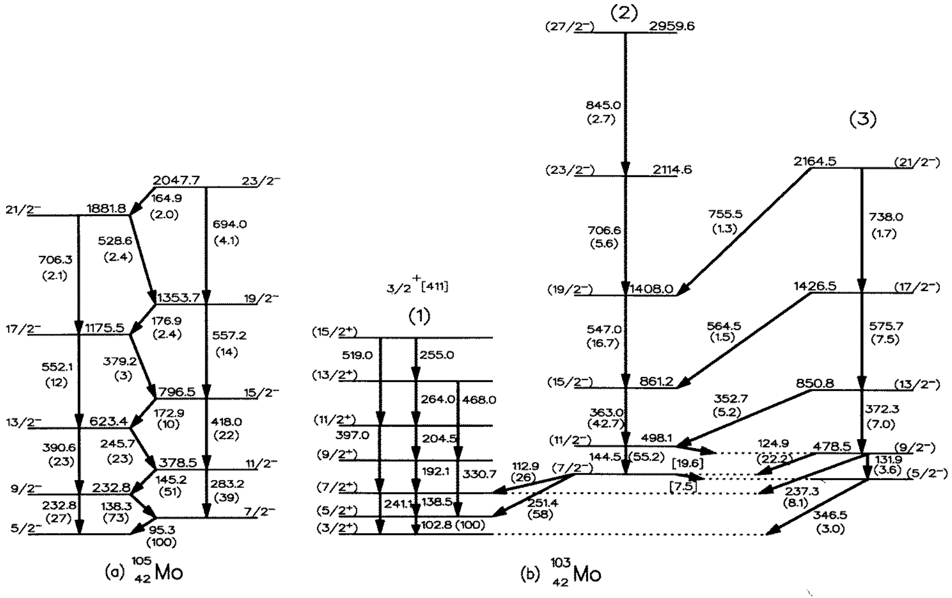

Figure 1. (a) The level scheme of ^{105}Mo. (b) The level scheme of ^{103}Mo.

The new level scheme of ^{103}Mo is shown in figure 1(b). The level at 346.5 keV was observed in the beta decay of ^{103}Nb to ^{103}Mo [5, 6]. No spin/parity assignments were made. Band-1 with the neutron configuration $3/2^+$ [411] was identified by Hotchkis *et al* [2]. They also reported a negative parity band-2 of three E2 transitions (144.6, 363.0 and 547.0 keV) based on a $7/2^-$ level at 354 keV and suggested that the 346.5 keV level is the $5/2^-$ (unfavoured) member of this band. Our results confirm and extend these findings and identify the $23/2^-$ and $27/2^-$ favoured signature as well as the $9/2^-$, $13/2^-$, $17/2^-$ and $21/2^-$ unfavoured signature members of this band. The unfavoured branch is labelled band-3 in figure 1. In figure 2(a), the coincidence spectrum obtained by double gating on the known 102.8 and 251.4 keV transitions is shown. The nuclei $^{144-147}$Ba are partners of ^{103}Mo with five to two neutrons emitted, respectively. The transitions belonging to these nuclei are also marked in figure 2(a). Based on these and several other coincidence spectra, 12 new γ-transitions of energies (7.5), (19.6), 124.9, 131.9, 237.3, 352.7, 372.3, 564.5, 575.7, 706.6, 738.0, 755.5 and 845.0 keV were identified and placed in the level scheme of ^{103}Mo including the new band-(3) as shown in figure 1(b). Even though we cannot directly observe the very low-energy transitions 7.5 and 19.6 keV, their presence can be inferred indirectly as follows. In the coincidence spectrum gated with 124.9 and 251.4 keV transitions, one clearly observes the 363.0 and 547.0 keV transitions. The only way one can explain this coincidence relationship is by assuming that there is a 19.6 keV transition as shown in figure 1(b) with a relative intensity of $\sim$30. Possible existence of a weak 7.5 keV transition between the 354.0 and 346.5 keV levels is based on the observation of the 346.5 keV transition in a coincidence spectrum double gated on the 363.0 and 144.5 keV transitions, and of the 363.0 keV transition in the coincidence spectrum with the double gates set on the 346.5 and 144.5 keV transitions. By double gating on the 372.3 and 575.7 keV transitions, the 131.9, 237.3 and 346.5 keV transitions are identified.

In the fission studies of ^{252}Cf, the $h_{11/2}5/2[532]$ rotational band with small signature splitting at the low-spin end is well established in ^{105}Mo and is similar to a band in ^{101}Zr [2]. Note the contrast between the level schemes of ^{103,105}Mo shown in figure 1, where two new levels with spins and parities of $21/2^-$ and $23/2^-$ are added to the previously established ^{105}Mo level scheme [2].

The ^{109,111}Ru nuclei were studied earlier by Hopkins *et al* [1], and by Butler-Moore *et al* [3] using the spontaneous fission source ^{252}Cf. The new level scheme of ^{109}Ru in figure 3 shows the spins and parities for each level, which were assigned on the basis of angular correlation experiments and systematics by Butler-Moore *et al* [3]. New γ-transitions of energies 314.4, 388.7, 505.3, 565.1 and 706.6 keV are identified and placed in the level scheme as shown in figure 3 in the present work. The coincidence spectrum obtained by double gating on the known 132.2 and 74.7 keV transitions in ^{109}Ru is plotted in figure 2(b).

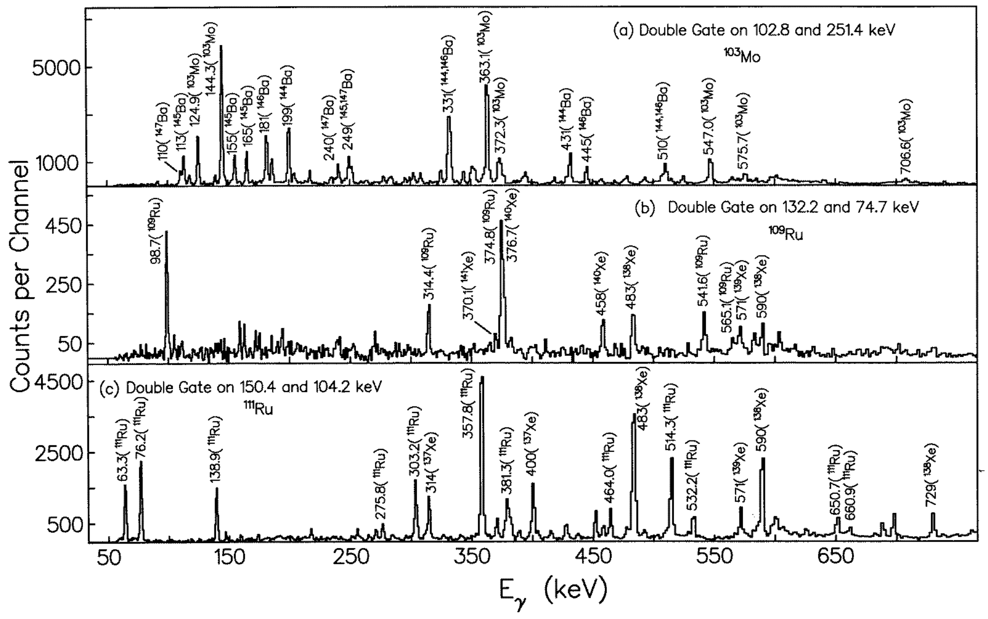

Figure 2. The partial coincidence spectra obtained by double gating on (a) the 102.8 and 251.4 keV transitions in ^{103}Mo, (b) the 132.2 and 74.7 keV transitions in ^{109}Ru, (c) the 150.4 and 104.2 keV transitions in ^{111}Ru.

The new level scheme of ^{111}Ru is shown in figure 4. Previously, several members of band-2 were identified. However, the 138.9 keV transition was missing. Therefore, the 167.0 and 104.2 keV transitions fed the ground state in ^{111}Ru, but with the observation of the 138.9 keV transition in the present work, the level energies change systematically as shown in figure 4. The 138.9 keV transition is seen in the coincidence spectrum obtained by double gating on the 357.8 and 514.3 keV transitions in ^{111}Ru, and was also confirmed in the x ray–γ coincidence spectrum. 13 new γ-transitions with the energies 138.9, 275.8, 303.2, 378.8, 381.3, 464.0, 477.0, 492.0, 532.2, 625.3, 660.9, 747.4 and 765.0 keV were identified in the present work. Several of these new transitions are shown in figure 2(c) obtained by double gating on the 150.4 and 104.2 keV transitions. The spins of the levels in ^{111}Ru (figure 4) are tentatively assigned by comparing with the levels in ^{109}Ru (figure 3). A new band-3 identified in the present work is also shown in figure 4. No parity assignment

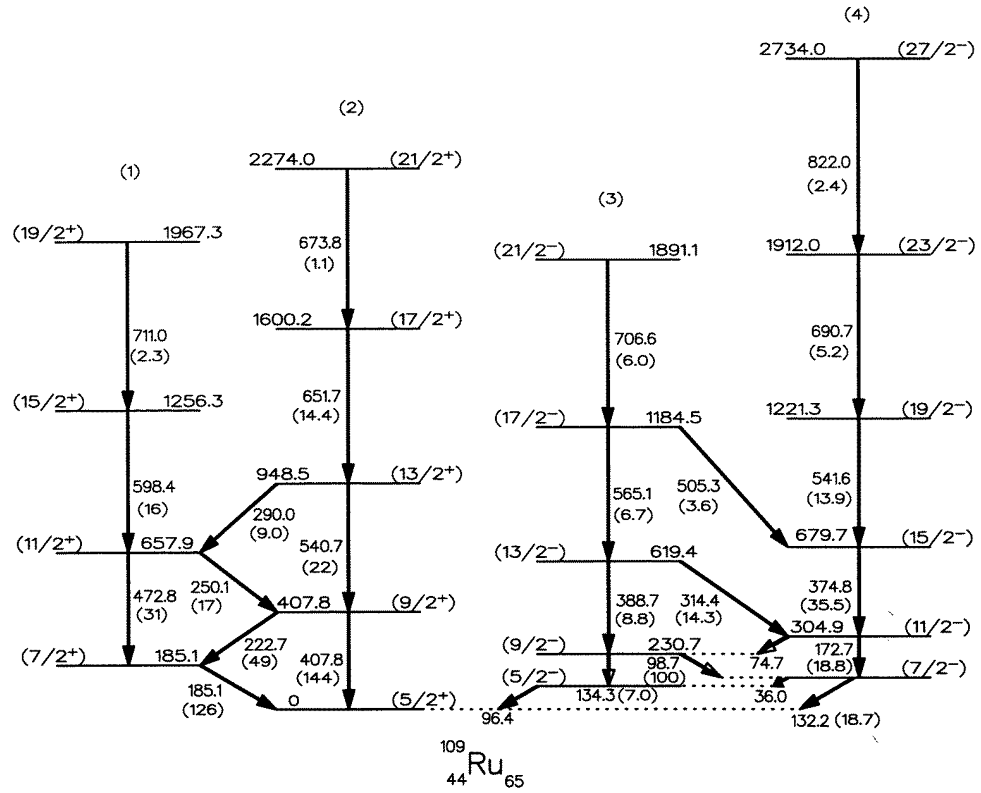

Figure 3. The level scheme of ^{109}Ru.

is possible without angular distribution studies, but a positive parity would imply octupole collectivity, which seems less likely.

The $11/2 \rightarrow 7/2$ and $9/2 \rightarrow 5/2$ transition energies decrease from 298.7 keV in ^{101}Zr to 144.5 keV in ^{103}Mo and from 250.5 keV in ^{101}Zr to 131.9 keV in ^{103}Mo, respectively. The gap between the band heads of favoured and unfavoured bands decreases to 7.5 keV in ^{103}Mo from 104.2 keV in ^{101}Zr. Above the $17/2^-$ level the band-2 members are lower in energy than the band-3 members. The intensities of the 144.5 and 363.0 keV transitions are much stronger than those of the 131.9 and 372.3 keV transitions. The close doublet nature (average gap of $\sim$5.8 keV) can be seen clearly in figures 1(b) and 5(a).

In the $N = 60$–70 nuclear region, we find bands in ^{109,111}Ru similar that observed in ^{103}Mo. In figures 5(a)–(d) the level energies of negative parity bands are plotted for the $N = 61$, 63, 65 and 67 nuclei, respectively. Close doublets are observed in many of these nuclei especially at the higher spins.

By the ordinary Bohr–Mottelson strong coupling model, one would expect mid-shell bands of odd parity in the $N = 50$–82 shell to show regular structures with $I(I+1)$ spacing. In figure 5, only ^{101}Zr and ^{105}Mo approach this behaviour. In most of the other nuclei shown, the bands establish a doublet structure between the signature partners as the general pattern. What can be the reason for such a widespread occurrence of doublet structure? Examination of a Nilsson level diagram for the region of $61 \leqslant N \leqslant 67$ shows that for prolate deformations of β between 0.3 and 0.5 there are usually three orbitals of

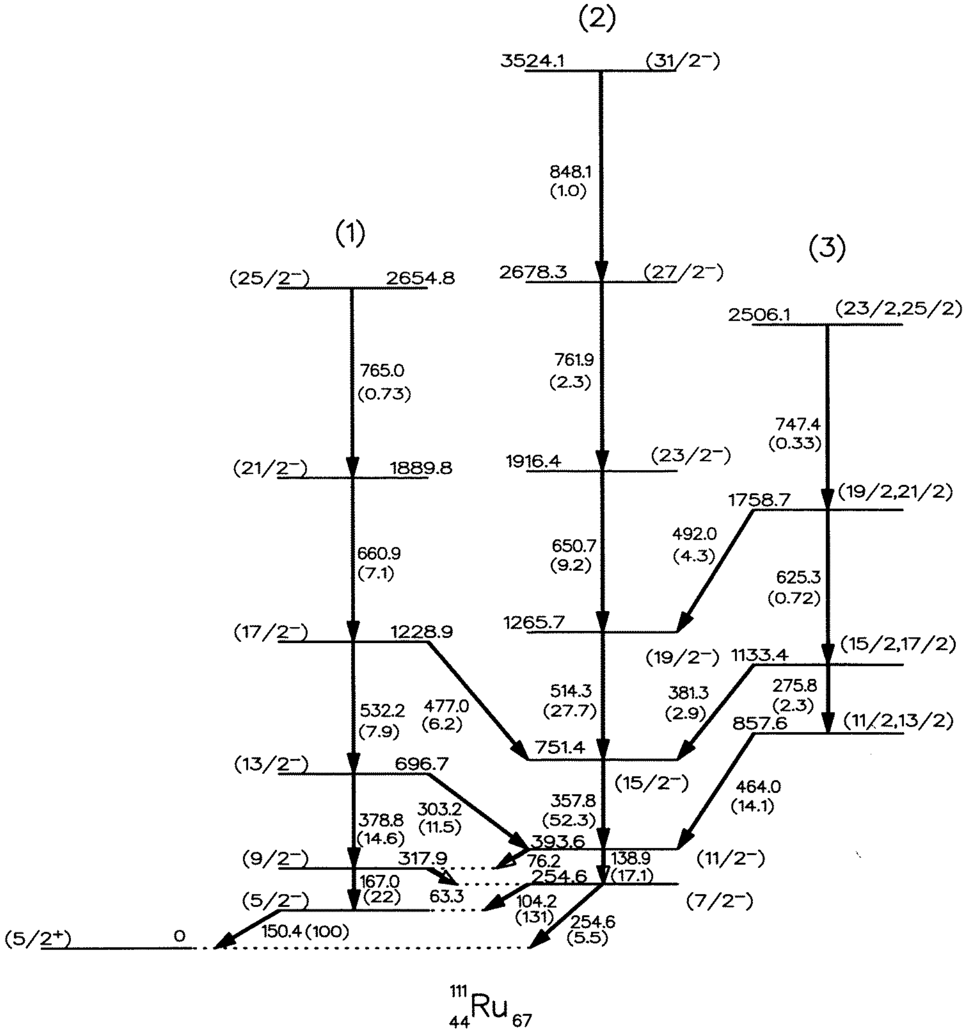

Figure 4. The level scheme of ^{111}Ru.

other types between the $\mathrm{h}_{11/2}5/2[413]$ and $\mathrm{h}_{11/2}7/2[523]$ orbitals. That is to say the $\mathrm{h}_{11/2}$ family will remain half-filled over this region of neutron numbers. The half-filled j-shell will have about the same total energy in oblate shapes as in prolate or spherical shapes. Meyer-ter-Vehn [7] studied the problem of one odd nucleon in a triaxial potential well, but we are not aware of theoretical work on odd systems near the half-filled shell. From his single-particle energy diagram for a $j = 11/2$ shell in a triaxial potential (figure 4 in [7]) we can infer that half-filled shells are rather independent of deformation parameters β and γ. Removing or adding one nucleon from or to the half-filled shell will leave $\mathrm{h}_{11/2}$ configurations that are still only weakly coupled to the nuclear shape. Thus, we would expect the yrast bands for odd-parity states to be rotation-aligned bands on $5/2^-$ and $7/2^-$ bandheads, because the parallel alignment of rotational and odd-nucleon angular momentum minimizes the rotational energy for a given I value. That is, the Coriolis coupling dominates. The quasiparticle energies of the $5/2^-$ (hole) and $7/2^-$ (particle) states will be similar for the

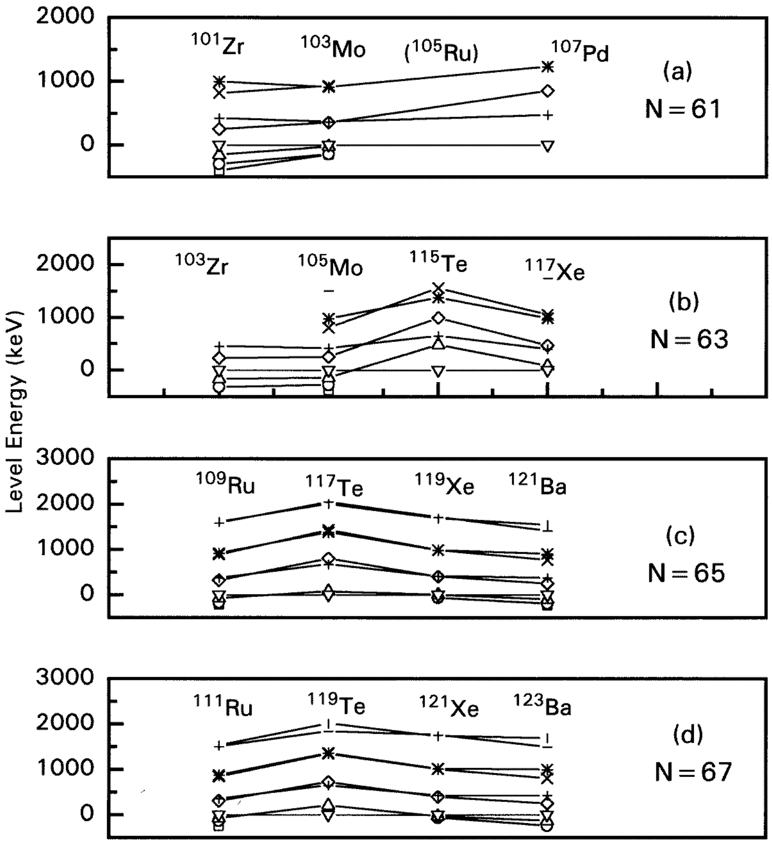

Figure 5. Comparisons of odd-parity level energies in nuclei with (a) $N = 61$, (b) $N = 63$, (c) $N = 65$ and (d) $N = 67$. □, 5/2; ○, 7/2; △, 9/2; ∇, 11/2; ◇, 13/2; +, 15/2; ×, 17/2; ∗, 19/2; –, 21/2; and |, 3/2.

$61 \leqslant N \leqslant 67$ region, where the Fermi energy lies between $h_{11/2}5/2[413]$ and $h_{11/2}7/2[523]$ levels. Here, one will get a doublet structure for these nearly half-filled j-shells, regardless of the deformation or shape of the core. We should note the 1982 calculations of $h_{11/2}$ bands by Barci *et al* [8] in their paper on some odd-A Xe isotopes. They considered both the triaxial-rotor-plus-particle model and interacting boson fermion model (IBFM), the latter including the interactions among $h_{11/2}$, $f_{7/2}$ and $h_{9/2}$ j-shells. The models Barci *et al* [8] used have prompted us to look for effects of triaxiality in the neutron-excess Mo–Ru region, where theoretical studies have indicated possible strongly deformed triaxial shapes.

Even–even nuclei in the region have low-lying second excited 2^+ states (gamma-bands), an indication of softness or instability in the triaxiality shape parameter, γ. Numerous theoretical studies have explored the shape coexistence possibilities in this deformed region, accessible almost exclusively through fission gamma studies. The recent nuclear theory paper of Skalski *et al* [9] cited 51 references on theoretical calculations of ‘equilibrium deformations and moments, potential energy surfaces, microscopic structure of coexisting configurations and shape transitions in the heavy-Zr region.’ Many concur that triaxial shapes should be a general feature of this region of nuclei. We shall not cite here all these studies but refer to the Skalski *et al* [9] paper’s listing.

A sufficiently strong triaxial deformation should quench the orbital angular momentum of the odd nucleon(s) and manifest itself as a spectrum of the even–even core with spin values of core angular momentum $\pm 1/2$. This quenching is analogous to that experienced by an unpaired electron in a field of triaxial symmetry in a molecule or solid. Obviously, this quenching, if it occurs in nuclei, is an uncommon and special phenomenon. The even-parity bands of the nuclei examined here appear to be normal strong-coupling rotational bands, and the lower ends of the odd-parity bands in several cases show regular spacing. To gain any insight we must examine the microscopic nature of the odd nucleon's response to a rotating triaxial shape field. The triaxial (Y_{22}) field induces a mixing of nucleonic states (Nilsson basis) differing in long-axis projection Ω by ± 2 and j and ℓ by 0, ± 2. To decouple the intrinsic spin and quench orbital angular momentum the Y_{22} mixing must be sufficient to strongly admix other j values, and the energy separation of the high-j shell from like-parity orbitals across a major shell is usually sufficient to prevent this strong mixing. Note, however, in the single-neutron Nilsson level diagram of figure 1 of Skalski *et al* [9] for ^{100}Zr that the 1/2[541] orbital, which is largely a mix of $f_{7/2}$ and $h_{9/2}$ from the major shell above 82 neutrons dives steeply with increasing deformation β_2 and comes close to the $h_{11/2}$5/2[532] orbital at large deformation. The Nilsson diagrams of Möller and Nix [10] show an even closer approach of the above levels, which would extrapolate to an actual crossing around deformation ε_2 of 0.45 (i.e. $\beta_2 = 0.48$).

The 1996 theoretical shape studies of Troltenier *et al* [11] for ^{108,110,112}Ru show all three nuclei triaxial with β values slightly greater than 0.4, where table 2 of Skalski *et al* [9] for Mo and Ru shows rather smaller β_2 values. Obviously, the level diagrams and the equilibrium deformations are sensitive to some of the many input parameters, so that different calculations will have somewhat different results. Certainly, it is likely that the 1/2[541] and 5/2[532] orbitals are both near the Fermi energy and close enough to be mixed by the Y_{22} deformation term for the Mo and Ru nuclei of central interest here.

The decoupling parameter required by the experimental doublet pattern is near -1, not the $+1$ value for complete decoupling of a $j = 1/2$ orbital. The 1/2[541] orbital has a mix of j values giving a decoupling parameter near -1, as observed. We now have a surfeit of coupling schemes that could give rise to the observed doublet structure. First, the nearly half-filled $h_{11/2}$ neutron family might readily decouple from the nuclear shape to give a stretched rotation-aligned band. Second, the 1/2[541] orbital or others from the shell above 82 may admix at large deformation with the $h_{11/2}$5/2[532] orbital to exploit and contribute to the triaxial potential and decouple the intrinsic spin of the neutron to couple with the triaxial core rotation. Third, the 1/2[541] band itself has about the right decoupling parameter to produce the doublet structure, although it should show $1/2^-$ or $3/2^-$ levels, not observed. Distinguishing among these models may need M1–E2 mixing and branching ratios. The new band-(3) in ^{111}Ru might possibly fit the first picture as the $h_{11/2}$7/2[523] neutron coupled to the second 2^+ band (gamma band) of the triaxial rotor. It would be desirable to carry out a band-mixing model calculation in a triaxial field. A troublesome problem with the two band model was accounting for the lack of experimentally observed $1/2^-$ and $3/2^-$ levels. The quenching picture, however, appears to handle that problem naturally. We infer from experimental doublet spins that symmetry conditions evidently restrict coupling of intrinsic spin-1/2 only to odd-value asymmetric rotor states 3^+, 5^+, 7^+, etc. Thus the bandhead spins are either $5/2^-$ or $7/2^-$. From a microscopic perspective a minimum basis for a band-mixing study should include eight orbitals, not only the two above-mentioned Nilsson orbitals, but also the in-shell $h_{11/2}$7/2[523], 1/2[550], 3/2[541] and the higher-shell $f_{7/2}h_{9/2}$ 3/2[532], 1/2[530] and 3/2[521]. These basis states are interconnected by Coriolis and Y_{22} matrix elements. Liang *et al* [12] in 1993 published a study of ^{103}Mo including a particle-rotor model calculation. They did not have the benefit of the more recent experimental level scheme, since at that time only a few of the odd-parity levels were known. Their calculation was made assuming axial symmetry, and their figure 4 shows that with their Nilsson parameters and modest deformation the 1/2[541] configuration, which we regard as important, lies too high to mix significantly. Calculations of this kind should be repeated for triaxial shapes and with some variation of the parameters affecting the proximity of next-shell orbitals to the Fermi surface.

In conclusion we have found a number of new γ-transitions and new band structure in ^{103,105}Mo and ^{109,111}Ru. These data show that the band structure of ^{103}Mo$_{61}$ is different from that of ^{101}Zr$_{61}$ in that ^{103}Mo shows a more pronounced close doublet structure. The odd-parity bands of ^{109,111}Ru exhibit similar intruder band character at higher spins, but ^{111}Ru shows more regular spacing at low spins. Further experimental work and theoretical modelling studies are needed.

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