CP violation beyond the standard model*

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Abstract
In this paper a number of broad issues are raised about the origins of CP
violation and how to test the ideas.

1. Introduction

The fundamental sources of CP violation in theories of physics beyond the standard model
is an important issue which has not been sufficiently studied. In this paper, we begin by
discussing the possible origins of CP violation in string theory and the potential influence
of its phenomenology on string theory. This naturally develops into a consideration of
supersymmetry breaking, specifically the soft-breaking supersymmetric Lagrangian, $\mathcal{L}_{\text{soft}}$.
There exist two extreme scenarios which may realistically accommodate CP violation; the
first involves small soft phases and a large CKM phase, δ_{CKM}, whereas the second contains
large soft phases and small δ_{CKM}. We argue that there is reasonable motivation for δ_{CKM}
to be almost zero and that the latter scenario should be taken seriously. Consequently, it is
appropriate to consider what CP violating mechanisms could allow for large soft phases, and
hence measurements of these soft phases (which can be deduced from collider results, mixings
and decays, experiments exploring the Higgs sector or from electric dipole moment values)
provide some interesting phenomenological implications for physics beyond the standard
model.

2. Fundamental properties of CP

CP symmetry has a feature that, we believe, makes it an important tool for testing ideas
about fundamental theories: it is broken in nature and yet quite general arguments indicate
that it is an underlying symmetry of string theory. As this is a central motivation for studies
of CP beyond the standard model, it is worth first outlining these arguments and also our

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hopes for how the phenomenological study of CP violation may eventually impact on string theory.

Early studies of CP violation in string theory predominantly focused on perturbative heterotic strings. In 1985, Strominger and Witten showed [1] that, in string perturbation theory, CP existed as a good symmetry that could be spontaneously broken. They argued that a suitable extension of the four-dimensional CP operator should reverse the direction of three of the six real compactified dimensions; or, equivalently, that it should complex conjugate the three complex dimensions Z_i of the Calabi–Yau manifold. CP violation then arises if the manifold is not invariant under the transformation $Z_i \rightarrow Z_i^*$. Alternatively (or possibly equivalently) it was argued that CP violation could result from CP non-invariant compactification boundary conditions or that the breaking of CP could be understood at the field theory level through the complex vacuum expectation values (vev's) of moduli.

This observation was refined by Dine *et al* [2] who argued that CP is a gauge symmetry in string theory. Consequently, in string theory, CP cannot be explicitly broken either perturbatively or non-perturbatively. Thus string theory is a perfect example of a theory in which all CP violating quantities (such as the 'bare θ' of QCD, the CKM phase, δ_{CKM} and the phases of the soft supersymmetry-breaking Lagrangian, $\mathcal{L}_{soft}^{susy}$) are enforced to be initially zero by a gauge symmetry, and all observable CP violation arises from a spontaneous symmetry breakdown, and is therefore calculable [3]. This CP violation would then feed through to Yukawa couplings in the superpotential W and, almost inevitably, to phases in the soft supersymmetry-breaking terms $\mathcal{L}_{soft}$, which could be determined by experiment.

This has significant phenomenological implications—if the phases in $W/\mathcal{L}_{soft}$ can be determined by experiment, it may be possible to extract direct information about the patterns of the underlying theory. Some possible revelations are the moduli dependence of the Yukawas, potential mechanisms for supersymmetry breaking and its transmission to the physical world, the complex dilaton (whose imaginary part can also act as a source of CP violation) and moduli, and the geometry of the compactification manifold.

Recent progress has led to a number of alternative approaches which could significantly alter our understanding of CP violation. These include type I or type IIB theories with D-branes, CP violation in brane worlds or, possibly, theories involving warped compactifications. There has been some study of CP violation within these, for example [4], but further work is necessary.

We complete this section by making a more general remark about the relationship between string theory and experiment. It is often said that string theory is too young a subject to be applied to the real world, and that one should wait until it is fully developed. We would argue precisely the opposite; in our view the *only* way to develop string theory properly (that is in a direction that might have something to do with nature) is to deduce from experimental data how to formulate it. Indeed, if we have learned anything from recent progress in string theory, it is that heading in the direction marked 'more fundamental' usually reveals new ways to construct the standard model, and seldom eliminates any. Our hope for CP is therefore that, once the CP violating parameters of $\mathcal{L}_{soft}$ are measured and translated to the unification scale (admittedly a difficult task), they will aid string theorists in understanding how the extra dimensions are compactified and supersymmetry is broken.

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[3] An important point is that complex phases may not lead to physically observable CP violation. This is a well-known aspect of spontaneous CP violation in field theory, but it applies equally to string theories as discussed by Dent [3]. In heterotic string models, CP is preserved by any (generally complex) moduli vev's that lie on the boundary of the fundamental domain.

3. The soft-breaking supersymmetric Lagrangian

It is known that supersymmetry has to be a broken symmetry due to the fact that none of the superpartners of the standard models have as yet been discovered (if it were conserved, selectrons would have masses equal to $m_e = 0.511$ Mev, and the gluinos and photinos would be massless). Although it is known that supersymmetry must be broken in the vacuum state chosen by nature, the physics of its breaking is not yet understood. There are several potential mechanisms; it is not yet known which, if any, are correct and neither is it known how the breaking is transmitted to the superpartners. Despite this ignorance it is possible to write a general, gauge-invariant, Lorentz-invariant, effective Lagrangian (as discussed in [5]). The Lagrangian is defined to include all allowed terms that do not introduce any quadratic divergences and depends on the assumed gauge group and particle content.

It is expected that a realistic phenomenological model should have a Lagrangian density which is invariant under supersymmetry, but a vacuum state which is not, i.e., supersymmetry should be an exact symmetry which is *spontaneously* broken. This enables supersymmetry to be hidden at low energies in much the same way as electroweak symmetry is concealed at low energies in the standard model. A general way to describe this is to introduce extra terms in the theory's effective Lagrangian which break supersymmetry explicitly. The extra supersymmetry-breaking couplings should be 'soft' (i.e., of positive mass dimension) in order for broken supersymmetry to provide a solution to the problem of maintaining a hierarchy between the electroweak scale and the Planck mass scale.

The effective Lagrangian for the theory can then be written in the form

$$\mathcal{L} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}} \tag{1}$$

where the first term preserves supersymmetry invariance and the second violates supersymmetry (using the notation of [6])

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}} = {} & \frac{1}{2}(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + \text{h.c.}) + M_{\tilde{Q}}^2 \tilde{Q}^* \tilde{Q} + M_{\tilde{U}}^2 \tilde{U}^{c*}\tilde{U}^c \\
& + M_{\tilde{D}}^2 \tilde{D}^{c*}\tilde{D}^c + M_{\tilde{L}}^2 \tilde{L}^*\tilde{L} + M_{\tilde{E}}^2 \tilde{E}^{c*}\tilde{E}^c + \left(a_U \tilde{U}^c \tilde{Q} H_U + a_D \tilde{D}^c \tilde{Q} H_D \right. \\
& \left. + a_E \tilde{E}^c \tilde{L} H_D + \text{h.c.}\right) + m_{H_U}^2 H_U^* H_U + m_{H_D}^2 H_D^* H_D + (B H_U H_D + \text{h.c.}) .
\end{aligned} \tag{2}
$$

Supersymmetry is broken because these terms contribute explicitly to masses and interactions of, for example, winos and squarks, but *not* to their superpartners. The mechanisms for how supersymmetry breaking is transmitted to the superpartners, and their interactions, are encoded in the parameters of (2).

Focusing on the phases of the soft parameters is justified for several reasons. If they are large they can have substantial effects on a variety of phenomena. Soft phases are a leading candidate to explain the baryon asymmetry in the universe (the inability of the CKM phase to achieve this is a primary reason to explore physics beyond the standard model) and it has been suggested [7] that all CP violation could arise from them. Such a scenario could be examined by imagining δ_{CKM} (which arises from the supersymmetry-conserving superpotential if the Yukawa couplings have a relative phase) to be very small, while allowing phases in $\mathcal{L}_{\text{soft}}$ (which arise from supersymmetry-breaking) to be significant. That is, CP violation would arise *only* in soft supersymmetry-breaking terms, with the CKM matrix being entirely real. Large soft phases could also effect the relic density and detectability of cold dark matter and rare decays. The patterns of these phases and whether they are measured to be large or small should reveal information on the mechanisms for breaking supersymmetry and string compactification.

Here, the minimal supersymmetric standard model (MSSM) is assumed to be the framework for a model of physics beyond the standard model (SM). This theory is the most

economical low-energy supersymmetric extension of the SM and consists of the SM particles and superpartners, the SM $SU(3) \times SU(2) \times U(1)$ gauge group, two Higgs doublets (necessary in supersymmetry to give masses to both the up-type quarks and to the down-type quarks and charged leptons) and has a conserved R-parity. This gives a total of 124 parameters, including masses, flavour rotation angles and phases, which all have to be measured (unless a compelling theory determines them).

Six of the parameters arise due to gaugino mass terms of the form $M_i = |M_i| e^{i\phi_i}$. The squark and slepton masses are in principle 3×3 Hermitian matrices with complex matrix elements, contributing $5 \times 6 \times 2 = 60$ parameters. Trilinear couplings between the sfermions and Higgs bosons are arbitrary 3×3 complex matrices which constitute $2 \times 9 \times 2 = 36$ parameters. Additional parameters arise due to the gravitino, which has a mass and a phase which may be observable in principle if it is the lightest supersymmetric particle (LSP), and a complex effective μ term must also be generated which is the supersymmetric version of the Higgs boson mass in the SM and has a magnitude of the order of the other soft terms. The symmetries of the theory allow some of these parameters to be absorbed or rotated away by field redefinitions; in this case, resulting in 33 mass eigenstates, 43 phases and the CKM angle.

In fact, the correct theory could be larger than the MSSM. For example, one could want to extend the theory with an extra singlet scalar or an additional $U(1)$ symmetry by adding the associated terms. To include neutrino masses in the theory, one would have to add new fields such as right-handed neutrinos and their superpartners and the associated terms in $\mathcal{L}_{\text{soft}}$. The physics of the above parameters is understood and is observable in many ways, so any extra variables could be checked experimentally.

It is interesting to examine how phases will enter the theory. The physics is embedded in the soft-breaking Lagrangian and the superpotential, which is of the form

$$W \sim Y_{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \tag{3}$$

where the $Y_{\alpha\beta\gamma}$ are Yukawa couplings in the scalar field basis. The trilinear soft terms are of the form

$$A_{\alpha\beta\gamma} = F^m [\hat{K}_m + \partial_m \log Y_{\alpha\beta\gamma} - \partial_m \log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)]. \tag{4}$$

Here the Latin indices refer to the hidden sector fields while the Greek indices refer to the observable fields; F^m is the hidden auxiliary fields. The Kähler potential is expanded in observable fields as $K = \hat{K} + \tilde{K}_\alpha |C^\alpha|^2 + \cdots$ and $\hat{K}_m \equiv \partial_m \hat{K}$. The A's depend on linear combinations of Yukawas and their derivatives. The CKM phase arises as the relative phases of the Yukawas. Note that if the Yukawas have large phases it is very likely that the trilinears also have large phases, but the converse is not necessarily true.

Thus the theory suggests that if δ_{CKM} is large then so are the soft phases, but soft phases could be large even if δ_{CKM} is small. Since the baryon asymmetry cannot be described by the SM alone, some other phases are needed; presumably the soft phases. Consequently it is very interesting to consider the possibility that δ_{CKM} is small [7]. As we shall see, all CP violation today can be described by the soft phases—there is no phenomenological evidence [18] that δ_{CKM} is large. (Of course, this does not necessarily exclude the possibility that *both* δ_{CKM} and the soft phases are large.)

Let us now discuss how most of these phases affect observables and hence how the couplings in $\mathcal{L}_{\text{soft}}$ can be measured.

4. Measuring the phases of $\mathcal{L}_{\text{soft}}$

Experiments measure kinematical masses of superpartners, and cross-sections × branching ratios, electric dipole moments and so forth, whilst phenomenologists need to examine how these measurements can be expressed in terms of the soft parameters. At most, two soft parameters could potentially be measured directly, the gluino mass and the gravitino mass, and the latter only if it is the LSP and then only approximately.

Forty three of the parameters in $\mathcal{L}_{\text{soft}}$ are phases and these soft phases have effects on many observables, not just CP violation. The measurements which are the focus of discussion here are the EDMs of the electron, neutron and mercury; the K, D, B systems, observables being $\triangle m_{B_d}$, $\triangle m_{B_s}$, ϵ, ϵ', $\sin 2\beta$; baryon asymmetry; the Higgs sector, parameters being m_h, σ_h, branching ratios and $h - A$ mixing; the relation of superpartner masses to Lagrangian masses and the cross-sections and branching ratios of the superpartners. Other sectors include the decay $b \rightarrow s\gamma$, observables being its branching ratio and CP asymmetry; rare decays such as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, $K_L \rightarrow \pi^0 \nu\bar{\nu}$; and the relic density and detectability of the lightest supersymmetric particle.

4.1. Baryon asymmetry

It is appropriate to start by looking at the problem of baryon asymmetry as the standard model cannot explain it whatever the value of δ_{CKM}. There are a variety of reasonable approaches [8] that seek to achieve this, but in all cases the analysis is very complicated and the resulting values are still uncertain.

One appealing mechanism is that of Affleck–Dine baryogenesis, where supersymmetry-breaking gives rise to a potential in the so-called 'flat directions' (the many-parameter set of vacuum states in supersymmetric unified theories) with a curvature of the order of m^2 (where m is the scale at which explicit soft breaking occurs and is comparable to M_W). This small curvature allows scalar fields to be pushed to large vev's, resulting in the universe developing a substantial baryon number. In this case the origin of the CP violating phase is most likely to be supersymmetric soft phases.

A different idea involves leptogenesis from the decay of heavy Majorana neutrinos, or their superpartners, which have masses of the order of 10^{11} GeV. In $B - L$ conserving theories, sphaleron interactions will generate a baryon asymmetry from the lepton number-violating Majorana neutrino decay. The origin of the CP violating phase here is rarely considered. Some possibilities are that it occurs in the couplings of these heavy neutrinos in the superpotential, Yukawa couplings or higher dimension operators, or in soft terms involving Majorana fields. The mass matrix would be given by

$$\begin{pmatrix} m & m_D \\ m_D & M \end{pmatrix} \tag{5}$$

where M is related to the soft phases and m_D contains the lepton Yukawa phase which needs to be of a sufficient magnitude to accommodate the level of baryon asymmetry. There are other possible mechanisms which have been explored, such as grand unified theory (GUT) baryogenesis, which is preserved by $B - L$ conservation and involves GUT Yukawa phases contributing to the asymmetry.

A particularly attractive mechanism involves the electroweak phase transition. It is known that $B + L$ violating transitions would wash out any net $B + L$ at temperatures much higher than the weak scale. However various processes can generate a baryon asymmetry at the electroweak phase transition itself and, provided it is strongly first order, this asymmetry will not be washed out by sphalerons and arises due to soft phases, not δ_{CKM}.

4.2. An example—the Chargino mass matrix

An important case and the simplest example of phenomenology is the chargino sector. The chargino mass matrix can be derived from $\mathcal{L}_{susy}$ and is given by (in the basis shown)

$$M_{\tilde{C}} = \begin{pmatrix} M_2 e^{i\phi_2} & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu e^{i\phi_\mu} \end{pmatrix} \begin{matrix} \tilde{W} \\ \tilde{H} \end{matrix} \tag{6}$$
$$\qquad\qquad\quad \tilde{W} \qquad\qquad\quad \tilde{H}$$

When electroweak symmetry is broken and the neutral Higgs field gets vev's, the spin $\frac{1}{2}$ fermion superpartners of the $W^\pm$ bosons mix with those of the charged Higgs bosons, $H^\pm$, producing the above matrix. The physical mass eigenstates $M_{\tilde{C}_1}$, $M_{\tilde{C}_2}$ are

$$M_{\tilde{C}_1}^2 + M_{\tilde{C}_2}^2 = \operatorname{Tr} M_{\tilde{C}}^\dagger M_{\tilde{C}} = M_2^2 + \mu^2 + 2M_W^2 \tag{7}$$

$$M_{\tilde{C}_1}^2 M_{\tilde{C}_2}^2 = \operatorname{Det} M_{\tilde{C}}^\dagger M_{\tilde{C}} = M_2^2 \mu^2 + 2M_W^4 \sin^2 2\beta - 2M_W^2 M_2 \mu \sin 2\beta \cos(\phi_2 + \phi_\mu) \tag{8}$$

with

$$\tan\beta = \frac{\langle H_U \rangle}{\langle H_D \rangle}. \tag{9}$$

In order to relate experiment and theory, we need to measure $\tan\beta$, M_2, μ, $\phi_2 + \phi_\mu$. Ultimately there are only two mass eigenstates and four unknowns. If more observables such as cross-sections are added, more parameters such as sneutrino or squark masses enter. Thus, it is not possible, in general, to measure $\tan\beta$ and μ and the Lagrangian parameters that include phases at hadron colliders. Claims that $\tan\beta$ can be ascertained at hadron colliders are based on assumptions about soft parameters, so these would not be direct measurements. On the other hand, at lepton colliders with polarized beams, and energies above the threshold for some superpartners, it is possible to measure $\tan\beta$, μ, etc.

The phases enter the masses $M_{\tilde{C}_1}^2$, $M_{\tilde{C}_2}^2$ in the last term of (8). Hence it is important to note that, in general, masses (which are not CP violating) also depend strongly on the phases.

4.3. Phases at colliders

The phase of the gluino is a prime example of the subtleties of including and measuring phases. The gluino part of the Lagrangian is given by

$$\mathcal{L} \sim M_3 e^{i\phi_3} \lambda_{\tilde{g}} \lambda_{\tilde{g}} + \text{h.c.} \tag{10}$$

where $\lambda_{\tilde{g}}$ is the gluino field. This can be redefined so that the masses are real and the vertices pick up the phases

$$\psi_{\tilde{g}} = e^{i\frac{\phi_3}{2}} \lambda_{\tilde{g}}. \tag{11}$$

The Feynman rules introduce factors of $e^{i\phi_3/2}$ or its complex conjugate at each of the vertices. If the gluino then decays via a quark to (for example) $q\bar{q}\tilde{\gamma}$, as illustrated in figure 1, a factor of $e^{i\phi_3/2}$ enters at the gluino vertex and a factor of $e^{-i\phi_1/2}$ at the photino vertex. This results in a differential cross-section of

$$\frac{d\sigma}{dx} \sim m_{\tilde{g}}^4 \left(\frac{1}{\tilde{m}_L^4} + \frac{1}{\tilde{m}_R^4} \right) \left[x - \frac{4x^2}{3} - \frac{2y^2}{3} + y(1 - 2x + y^2)\cos(\phi_3 - \phi_1) \right] \tag{12}$$

where $x = E_{\tilde{\gamma}}/m_{\tilde{g}}$ and $y = m_{\tilde{\gamma}}/m_{\tilde{g}}$. ϕ_3 also enters ϵ, ϵ' in the kaon system. See [10] for detailed discussions of how various distributions depend on this phase and other soft parameters, enabling measurements at Tevatron and LHC.
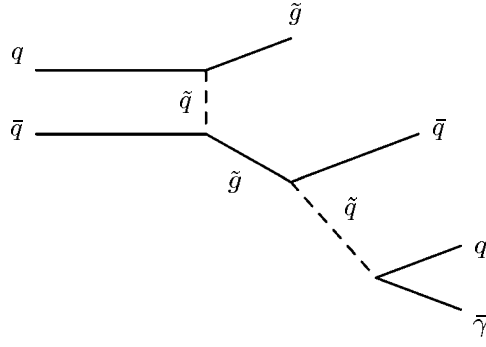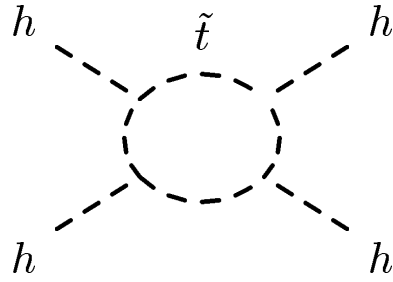
Figure 1. Gluino decay.

Figure 2. Higgs corrections.

4.4. The Higgs sector

We now consider how the phases affect the physics of the Higgs sector. See [20] for details and further references. The Higgs potential includes radiative corrections of the form shown in figure 2.

The phases enter at the one loop order with stop loops being dominant for low to medium values of $\tan \beta$. Much like the chargino mass matrix, the stop mass matrix is in general complex, so the phases enter into the scalar effective potential. One can write

$$H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} V_d + h_d + ia_d \\ h_d^- \end{pmatrix} \tag{13}$$

$$H_u = \frac{e^{i\phi}}{\sqrt{2}} \begin{pmatrix} h_u^+ \\ V_u + h_u + ia_u \end{pmatrix} \tag{14}$$

with the vev's taken to be real and using (9). A phase $\theta = \theta(\phi_{A_t}, \phi_\mu)$ allows a relative phase between the two vev's at the minimum of the Higgs potential and cannot be rotated away. The stop mass matrix is

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{L}}^2 + m_t^2 & Y_t \left(A_t H_u^0 - \mu^* H_d^0 \right) \\ * & m_{\tilde{R}}^2 + m_t^2 \end{pmatrix} \tag{15}$$

The Higgs mass matrix is derived by minimizing the scalar potential, setting $\partial V / \partial h_d$, $\partial V / \partial h_u$, $\partial V / \partial a_d$ and $\partial V / \partial a_u$ to be equal to zero. Of the resulting four equations, only two are independent, so three conditions remain. The Higgs sector has 12 parameters; $V_u, V_d, \phi_{A_t}, \phi_\mu, \theta, |A_t|, |\mu|, Q$ and the $\mathcal{L}_{\text{soft}}$ parameters $m_{\tilde{Q}}^2, m_{\tilde{u}}^2, b, m_{H_u}^2, m_{H_d}^2$. Four of these can be eliminated using the above three conditions and the fact that the renormalization scale,
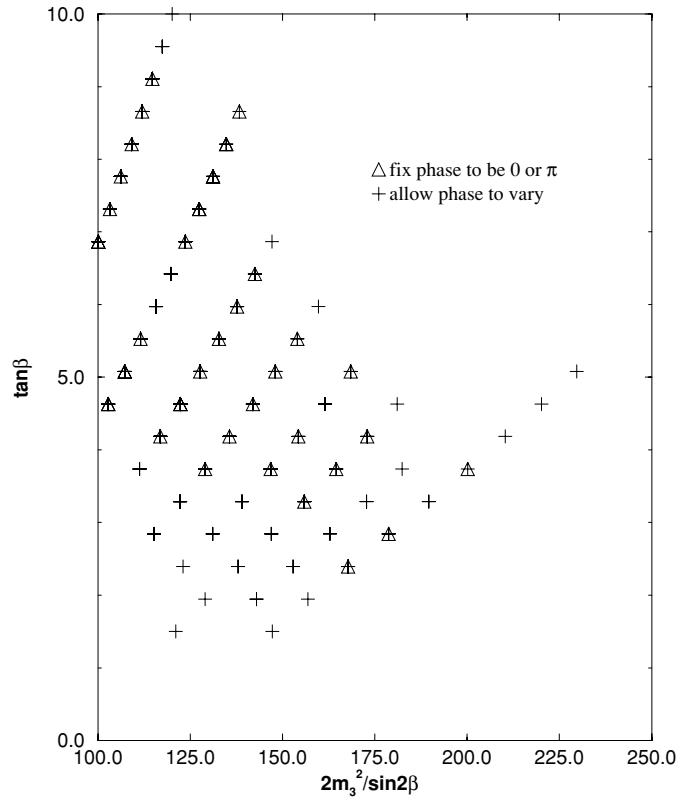
Figure 3. Higgs found at LEP.

Q, is chosen so as to minimize any higher order corrections. Applicaton of the conditions for electroweak symmetry breaking (replacing V_u, V_d with M_Z, $\tan \beta$) further reduces the number of parameters to seven so that any descriptions of the Higgs sector are now based on $\tan \beta$, $\phi_{A_t} + \phi_\mu$, $|A_t|$, $|\mu|$, $m_{\tilde{L}}^2$, $m_{\tilde{R}}^2$, b. This number cannot be reduced without some further theoretical or experimental information.

If $\tan \beta$ is large, bottom loops could be large and also enter the scalar potential, and additional parameters due to, for example, $\tilde{C}$, $\tilde{N}$ loops could be significant, (see [11]). If the phase $\phi_{A_t} + \phi_\mu$ is non-zero, it is not possible to separate the pseudo-scalar A from h, H and it is necessary to diagonalize a 3×3 matrix for neutral scalars. (In the limit of no CP-violating phase, the three mass eigenstates H_i are $H_1 \rightarrow h$, $H_2 \rightarrow A$, $H_3 \rightarrow H$). These can then decay into any given final state or could be produced in any channel, producing three mass ($b\tilde{b}$) peaks in a decay channel resulting in, for example, $Z +$ Higgs. All of the branching ratios and cross-sections depend on the phase and so can change significantly. There are two interesting phenomenological situations to consider, depending on whether a Higgs is found at LEP or not.

4.4.1. Higgs found at LEP or the Tevatron. If a Higgs were found and m_{H_1} and its $\sigma \times BR$ were measured, what region of the full seven-dimensional parameter space would be allowed? There are different answers depending on whether the phase is set to zero or π or whether a
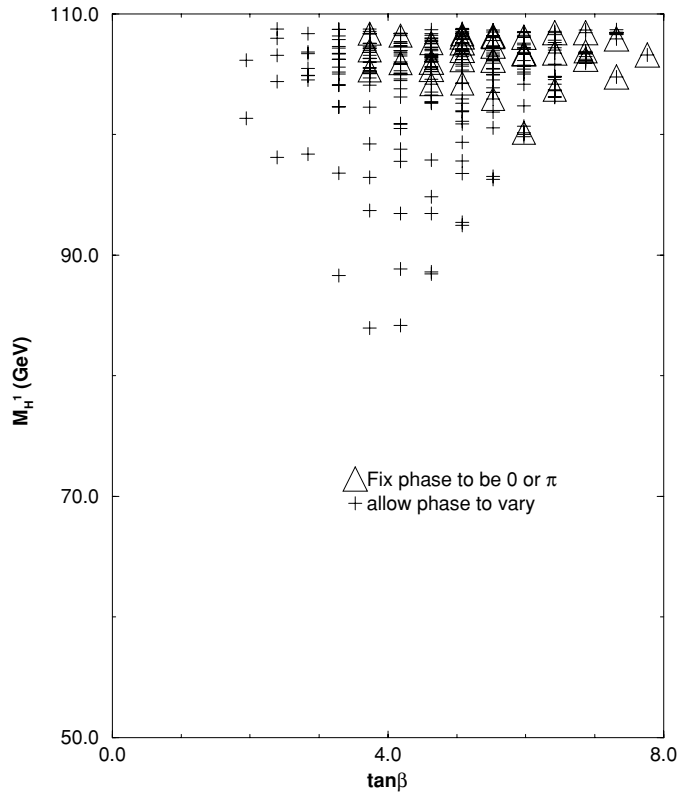
Figure 4. No Higgs found at LEP.

general phase is allowed. Consequently, it would be extremely misleading not to include a phase in the data analysis if there were a discovery.

To illustrate this, the allowed parameter region is shown in figure 3 [20]. The factor $2m_3^2/\sin 2\beta$ on the horizontal axis would be m_A^2 if there were no phase (if it was set to zero or π). The diagram only illustrates the effect of the phase; the full range of other parameters is not included and experimental aspects are not taken into consideration, except for crude estimates of the efficiencies. If the heavier Higgs were heavy (the decoupling limit) the effect of the phase decreases for the lower limit on the mass of the lightest eigenstate, but the effect of the phase on the lower limit of $\tan \beta$ is still significant.

4.4.2. No Higgs found at LEP. If no Higgs were found at LEP, this would produce an experimental limit on $\sigma(H_1) \times BR(H_1 \to b\bar{b})$. So, what would be the lower limits on m_{H_1} and $\tan \beta$ in the full seven-parameter theory? It is clear from the diagram (figure 4) that the mass of the lightest Higgs, H_1, is allowed to be significantly lighter if a phase is present, i.e., the lower limit for the MSSM without a phase is approximately 10% below the SM limit, whereas the lower limit when the phase is allowed to vary is a further reduction of 10%. Here $\tan \beta$ also has lower values allowed if the phase is non-zero.

With seven parameters, at least seven observables would be required to determine any of the soft parameters from the Higgs sector alone. Potential observables include the three neutral mass eigenstates, the charged Higgs mass, the three $\sigma \times BR$ for $Z +$ Higgs and the three

$\sigma \times BR$ for the mass eigenstate channels, and finally the two stop mass eigenstate masses. It could also be possible to measure the ratio

$$R = \frac{\sigma(gg \rightarrow H^2 \rightarrow b\bar{b})}{\sigma(gg \rightarrow H^1 \rightarrow b\bar{b})}. \tag{16}$$

If all of these measurements could be made $\tan \beta$ and $\phi_{A_t} + \phi_\mu$ could be measured directly in the Higgs sector and the former could be compared with results from the gaugino sector. Such analysis could be enabled with LHC data, but it is unlikely.

4.5. Electric dipole moments

The most stringent constraints on models for sources of CP violation come from the experimental upper limits on the absolute values for the electric dipole moments (EDMs) of the electron, neutron and mercury atom. The standard model predicts very small values (an upper limit of order $10^{-32} e$ cm), which are consistent with experimental constraints (upper limits of 4.3×10^{-27} [12], 6.3×10^{-26} (90% CL) [13] and 2.1×10^{-28} [14] e cm). However generic supersymmetry models predict much higher values, for example, $d_n \sim 10^{-23} e$ cm. As a result, it is necessary to suppress the CP violating phases responsible for EDMs. There are a variety of models suggested to achieve this [15], including models with small CP phases, models with heavy sfermions, cancellation scenarios and models with flavour off-diagonal CP violation.

It has long been known that supersymmetry predicts values for the neutron and electron EDMs that are approximately 50 times the experimental limits if all the soft masses and phases are independent. From this, it could be naively concluded that in a supersymmetric world EDMs should have already been observed. However, in any theory there can be relations among the soft phases that lead to large cancellations which can occur over a large region of parameter space even if all the soft phases are present and large.

There are in fact two possible methods of avoiding the constraints the dipole moment measurements place on supersymmetric models. One is to assume that all the supersymmetry phases are zero or unnaturally small ($\lesssim 0.01$). The second possibility is that the phases may be large while certain approximate relations hold among the mass parameters and phases, resulting in cancellations of the order of 5–10 in the EDM calculations. (Note that some cancellation effects were neglected in earlier analyses, such as $\tilde{C} - \tilde{N}$ from the Lagrangian.) However, both these possibilities seem 'unnatural' without some deeper understanding of what is going on. No symmetry or dynamics is known that would imply phases are small. One can think of arguments such as dilaton dominance, but that is an *ad hoc* and not well-motivated choice unless it is determined by some deeper argument. The second scenario could only become acceptable if the cancellations were due to some symmetry of high scale theory, a condition that looks like fine tuning if we can only see the low energy theory.

5. A string-motivated model

It is now important to return to the general structure of $\mathcal{L}_{\text{soft}}$ and consider potential models which will reduce the number of soft parameters. String models can provide some motivation for large phases in the soft breaking parameters, suggesting that low-energy data revealing the patterns of cancellations could reveal clues about high (Planck) scale theory. A particularly interesting class results from embedding the SM on D-branes within simple type IIB models [16]. Open type I strings on D-branes that intersect at some non-vanishing angle give rise to chiral fermions and explicit compactifications with intersecting branes exist. An
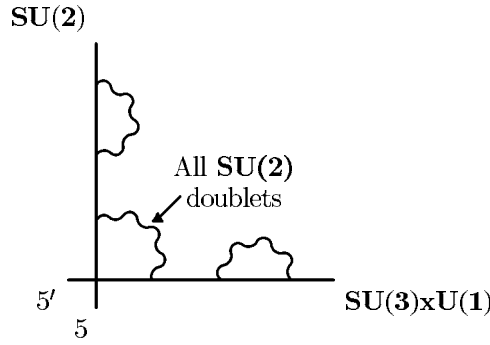
Figure 5. Embedding the SM on two intersecting 5-branes.

acceptable phenomenological example arises by assuming that the supersymmetry breaking effects are communicated dominantly via the F-component vev's of the dilaton and moduli [17]. Consider, for example, the case of two intersecting 5-branes shown in figure 5.

The SM gauge group is chosen to be embedded in such a way that $SU(3)$ and $U(1)$ are associated with one of the intersecting branes, and $SU(2)$ is associated with the other. We only note here the resulting phase structure

$$m_1 = m_3 = -A_t \sim \mathrm{e}^{-i\alpha_1} \qquad m_2 \sim \mathrm{e}^{-i\alpha_2} \tag{17}$$

and all the other soft terms are real. Then using symmetries and rotations, the previous 105 parameters can be reduced to just 8, $\alpha_2 - \alpha_1$, the mass scale $m_{3/2}$, $\tan\beta$, $|\mu|$ (from electroweak symmetry breaking), ϕ_μ, and the relative amounts of dilaton and moduli, X_1, X_2 and X_3. It is not yet understood fully how to include a reliable mechanism that provides the effective μ parameter in models such as these; so, for now, we treat it as an arbitrary complex parameter. With this model it is then possible, qualitatively, to describe all CP violation with no contributions from the CKM phase δ_{CKM}, by using a 'different' flavour structure [18].

6. Fine tuning

The gluino sector of the above model provides an example of the fact that although some relations may appear to be fine tuned in a low-energy theory, they can originate in the structure of the high energy theory. If the gluino–squark box diagram, which would probably be one of the dominant contributions to CP violation in the K system, is to be consistent with observed values of ϵ and ϵ', the argument of the gluino phase must satisfy (see [19])

$$\arg\left((\delta_{12})^d_{LR} M_3^*\right) \approx 10^{-2} \tag{18}$$

where $(\delta_{12})^d_{LR} \sim A^d_{12}$. This value can be achieved in models with large flavour violation in the A-terms. It seems like fine tuning, but in a D-brane model, assuming that

$$\phi_{(\delta_{12})^d_{LR}} = \phi_{A_{sd}} = \phi_{M_3} \tag{19}$$

would force $\arg\left((\delta_{12})^d_{LR} M_3^*\right)$ to be zero at high energies. However, the phases run differently and can generate the factor of 10^{-2}.

7. The K and B systems

The angle β is one of the angles of the CKM unitarity triangle and is an important test for physics beyond the SM. The SM ratio $|V_{ub}/V_{cb}|$ is generally thought of to be unaffected by BSM
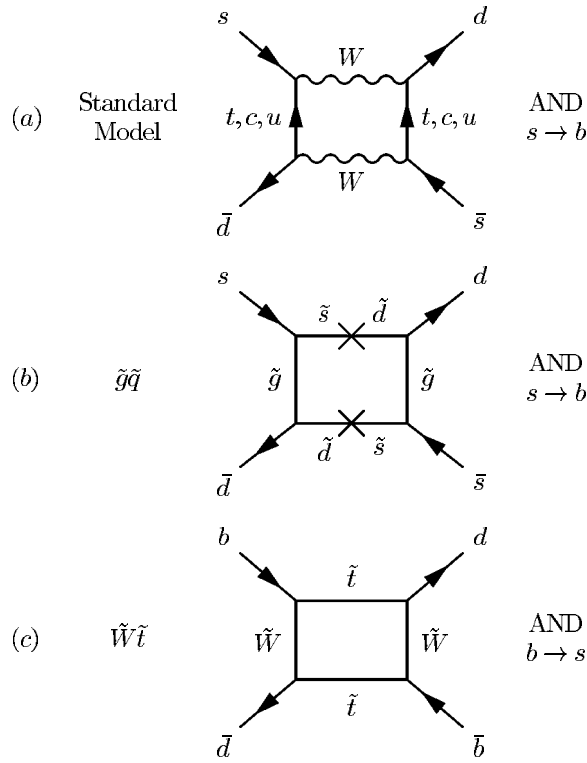
Figure 6. Loop contributions in the K and B systems.

physics as it arises from tree level decays, but all the other constraints $\left(\epsilon_K, \triangle m_{B_d} \text{ and } \triangle m_{B_s}\right)$ can be affected. It is possible that the unitarity triangle could be flat, giving a β and hence $\sin 2\beta$ of zero for the SM where $\sin 2\beta$ is a measure of the CP asymmetry for the decay $B^0 \bar{B}^0 \rightarrow \psi K_{K,L}$; a large $\sin 2\beta$ could arise from soft phases.

The Feynman diagrams in figure 6 show the loop contributions to mixing (there are analogous penguin diagrams for decays contributing to direct CP violation). For both the K and B systems, the supersymmetry effects arise from loops. As mentioned in the previous section, within the K system the dominant contributions are most likely to be gluino–squark boxes (and penguins). The first diagram is the usual SM box diagram for calculating the value of the neutral kaon mixing parameter $\triangle m_K$, but supersymmetric boxes would also contribute. With the postulate that $\delta_{\text{CKM}} \approx 0$, the SM box would be entirely real and would not contribute to ϵ_K, the indirect CP violating parameter. The supersymmetric box shown in the second diagram is the gluino–squark box discussed in the previous section (the '$\times$' refers to a $L - R$ chirality flip). The magnitude of A_{sd}, the triscalar coupling, must be of the right size to describe ϵ_K and the supersymmetric approach is only able to describe it, not explain it. In this case, in fact, ϵ_K is described more naturally by the SM.

Within the B system, all decays (except $b \rightarrow s\gamma$) have a tree level contribution, implying that the B system with $\delta_{\text{CKM}} \approx 0$ is superweak and all CP violating effects arise due to mixing, with $\epsilon'_B \approx 0$. The dominant mixing is usually assumed to be caused by the chargino-stop box shown in the third diagram. It is predicted from this that because the decay phase is zero

$$\sin 2\alpha = -\sin 2\beta \tag{20}$$

where $\sin 2\alpha$ and $\sin 2\beta$ are defined to be the CP asymmetries measured in the decays $B_d \to \pi^+\pi^-$ and ψK_s. It is also known that if $\delta_{\mathrm{CKM}} \approx 0$, then we can take

$$V_{td} = |V_{cb} \sin \theta_c| - |V_{ub}| \approx 0.005. \tag{21}$$

These results do not depend on the soft parameters and so act as an independent test of the approach. Studies show that there are regions of parameter space where the values for recent measurements and the neutral B mixing parameter can be achieved, using the estimate for $\sin 2\beta$ calculated by this model.

8. Some further notes

The recent measurement of $g_\mu - 2$ and the LEP Higgs lower limit suggest that $\tan \beta \gtrsim 5$. The natural value is of the order $\tan \beta \sim 1$, the supersymmetric limit, and a naive estimate using Yukawa unification is ~ 35.

CP violation in the lepton sector is very interesting. It will also arise via the superpotential Yukawa matrix (for leptons) and soft phases. We will not discuss it further in this paper.

The decay $b \to s\gamma$ has a CP asymmetry which does not involve any tree level contribution. The SM predicts a value of the order of half a percent, whereas supersymmetric estimates range up to 15% or more. This difference is interesting as the decay is a relatively clean one and could potentially be the first place that physics beyond the SM is found.

An important consideration is whether the CPT theorem is in fact only approximate and CPT symmetry could be minimally violated. As yet there is no good theoretical motivation for this to occur.

In section 3 it was shown that the MSSM had 43 soft phases, but a question remains as to which phases are constrained by which experiments. For example, the measurement of $g_\mu - 2$ constrains $\phi_2 + \phi_\mu$ if $\tan \beta$ is large and it constrains $\phi_2 + \phi_\mu$, $\phi_1 + \phi_\mu$ and $\phi_{A_\mu} + \phi_\mu$ in general. The decay $b \to s\gamma$ constrains $\phi_{A_t} + \phi_\mu$, $\phi_2 + \phi_\mu$ and $\phi_3 + \phi_\mu$, whereas the Higgs sector only constrains $\phi_{A_t} + \phi_\mu$. Sometimes insufficient care is taken to ensure that one is examining the relevant reparametrization invariant phases.

9. Outlook

It has been argued that a softly broken supersymmetric Lagrangian can provide a suitable framework to accommodate CP violation effects. This involved the requirement that some soft phases were large, which would need to be supported experimentally. Experimental results that could achieve this include the observation of an electron EDM, the ratio of neutron to mercury EDMs being different from what would occur with only a strong CP phase, a value of $\sin 2\beta$ which is not equal to that predicted by the SM, Higgs sector observations, measurements of superpartner masses and $\sigma \times BR$ at colliders and the decay $K_L \to \pi^0 \nu\bar\nu$ not being equal to that predicted by the SM. On the other hand evidence for non-zero δ_{CKM} could possibly be observed at b-factories or in the decay $K_L \to \pi^0 \nu\bar\nu$. Optimistically these issues could be clarified over the next years at BaBar, BELLE and CDF, by ever more stringent limits on the values (and perhaps even discovery) of EDMs and confirmation of the result for $g_\mu - 2$.

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