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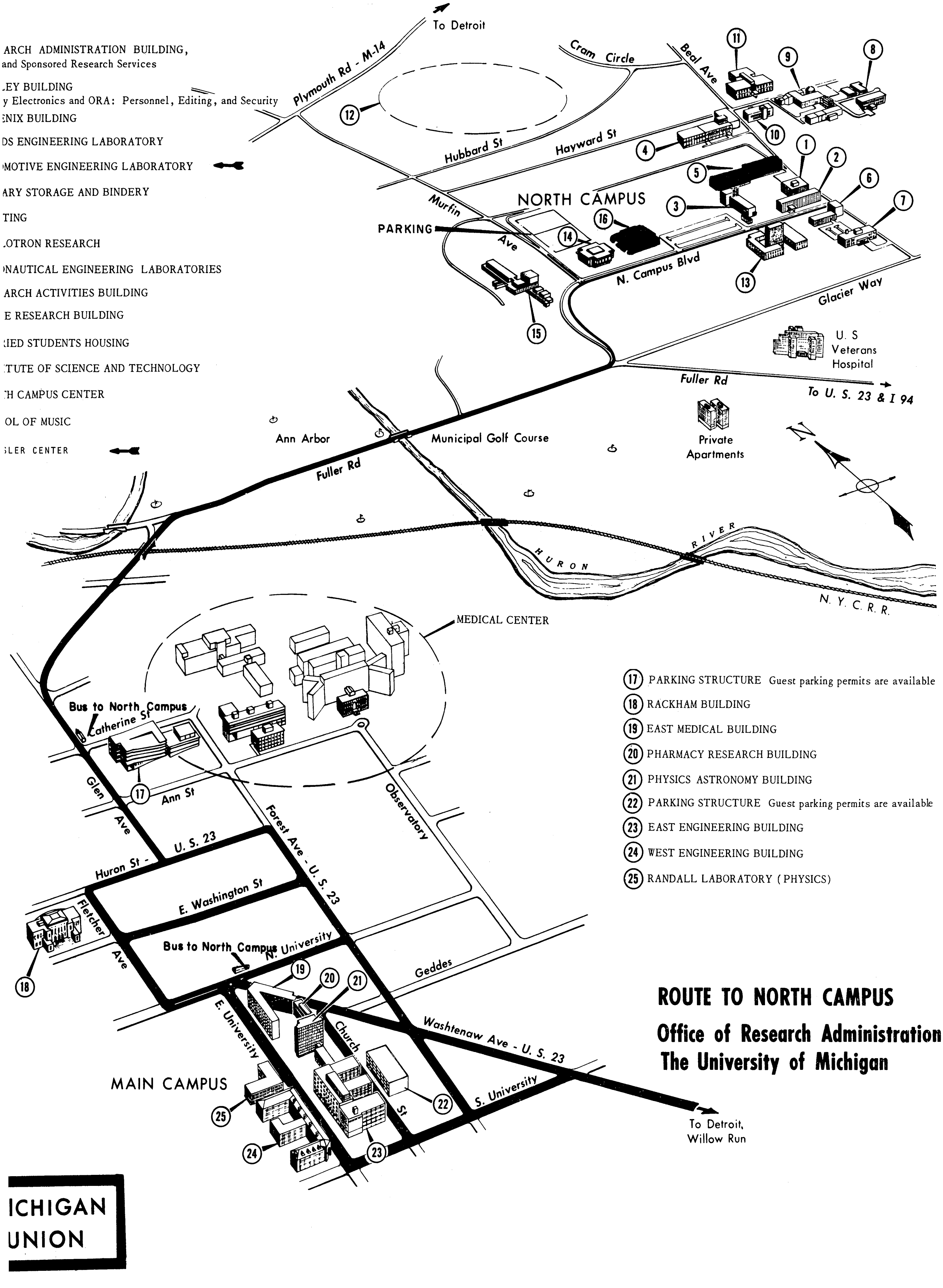

THE UNIVERSITY OF MICHIGAN

ENGINEERING SUMMER CONFERENCES

INSTRUMENTATION FOR MECHANICAL ANALYSIS

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CHAPTER 1

THE MEASUREMENT SYSTEM

1.1 INTRODUCTION

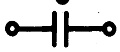
The need to measure such physical variables as displacement, velocity, force, stress, acceleration, pressure, elapsed time, etc., in operating devices or machines is a common occurrence in many technological activities. The data obtained from such measurements might be used to establish the validity of design, to predict the limit of capacity if operating requirements are increased, to determine the cause of present mal-functioning, or to provide information needed to supplement further analytical work.

It is difficult if not impossible to directly measure the above mentioned variables in any part of a machine or mechanical device while that variable is rapidly changing. To take advantage of electrical instrumentation with its ability to quickly respond to changes, a rather indirect method of measuring is used. This indirect method consists of developing an electrical voltage directly proportional to the physical variable to be measured, measuring that voltage, then converting the measured voltage back to the corresponding values of the original variable.

While this indirect method allows the use of electrical instrumentation for measurement work, it also presents considerable opportunity for errors to occur in the process of converting from the physical variable to electrical voltage and back again. This places a sizeable responsibility on the engineer or technician to be certain that the equipment and methods used are such that the measurements obtained can be accepted with confidence. To accomplish this it is necessary for the engineer or technician to have a reasonable understanding of the instrumentation and of the system in which it must be employed to utilize this indirect method of measuring. It is also necessary to employ the proper laboratory techniques, as discussed in Chapter 4, for in practice the instrumentation will not always behave as nicely as may be indicated from the discussions which follow.

1.2 CIRCUITRY

1.2.1 CIRCUIT ELEMENTS. To understand the instrumentation used in the measuring system it is necessary to have some knowledge of simple electric circuits. Figure 1.2.1.1 shows the common circuit elements, the voltage-current relations across the elements, and their impedance. In the work which follows we shall be most interested in resistance and capacitance, but for a general reference inductance has also been included in the table.

CIRCUIT ELEMENT	SYMBOL	VOLTAGE ACROSS ELEMENT	SINUSOIDAL IMPEDANCE, e/i	IMPEDANCE, OPERATOR FORM
RESISTANCE	R 	$e = iR$	R	R
CAPACITANCE	C 	$e = \frac{1}{C}\int i\,dt$	$-j\frac{1}{C\omega}$	$\frac{1}{Cp}$
INDUCTANCE	L 	$e = L\frac{di}{dt}$	$jL\omega$	Lp

R = RESISTANCE, OHMS; C = CAPACITANCE, FARADS; L = INDUCTANCE, HENRIES; e = INSTANTANEOUS VOLTAGE; i = INSTANTANEOUS CURRENT, AMPERES; $p = \frac{d(\;)}{dt}$, THE FIRST DERIVATIVE OPERATOR; $\frac{1}{p} = \int(\;)dt$, THE INTEGRAL OPERATOR

CIRCUIT ELEMENTS
FIGURE 1.2.1.1

1.2.2 KIRCHOFF'S LAWS. In addition to the voltage-current relations shown in figure 1.2.1.1, one should be familiar with Kirchoff's laws. Applied to direct current circuits they may be stated as follows:

1. The algebraic sum of all the currents flowing into and out of a junction is zero.

   In symbol form this can be written:

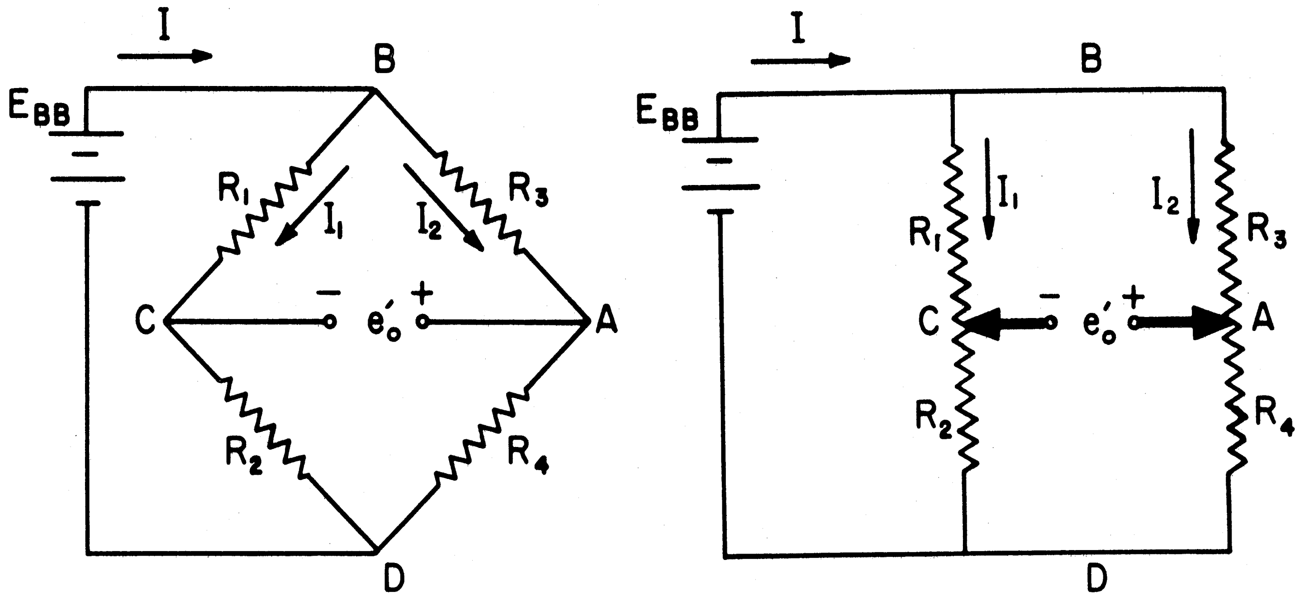
   $$\sum I = 0$$

2. The algebraic sum of all voltages or potential differences in any closed circuit is zero.

   In symbol form this can be written:

   $$\sum E = 0$$

1.2.3 BRIDGE CIRCUITS. A commonly used circuit is the well known Wheatstone bridge, most generally referred to as a bridge circuit. Figure 1.2.3.1 shows two bridge circuits, each utilizing four resistances. It will be noticed that the only difference between the two circuits is that (a) indicates the four resistances are of fixed value, while (b) shows the resistances to be variable. In each case an input voltage E_{BB}, is impressed across the bridge as shown, and the output voltage e_o is the difference between the voltage at A and the voltage at C.



(a) (b)

BRIDGE CIRCUITS
FIGURE 1.2.3.1

In the figures there are open circuits between A and C, and it is apparent that no current can flow between A and C. This condition can be described by saying there is no electrical load being driven by the output voltage. Thus the voltage e_o' is said to be the no-load output voltage of the bridge. In this book the prime notation will always be used to indicate a condition of no load.

The voltage-current relations for resistance from figure 1.2.1.1 can be utilized to obtain the following:

$$E_{cd} = I_1 \cdot R_2 \qquad (1.2.3.1)$$

$$E_{BB} = I_1 (R_1 + R_2) \qquad (1.2.3.2)$$

combining, the voltage at C with respect to D,

$$E_{cd} = E_{BB} \frac{R_2}{R_1 + R_2} \qquad (1.2.3.3)$$

similarly, the voltage at A with respect to D,

$$E_{ad} = E_{BB} \frac{R_4}{R_3 + R_4} \qquad (1.2.3.4)$$

Equations 1.2.3.3 and 1.2.3.4 show that when resistances are connected in series, the voltage drop across any one of the resistances is equal to the total voltage drop across the series multiplied by the ratio of that resistance to the total resistance of the series. This is a useful relationship to remember, especially if the general term impedance is used instead of resistance.

Continuing, the voltage output at A with respect to C,

$$e_o' = E_{ac} = E_{ad} - E_{cd} \qquad (1.2.3.5)$$

Substituting for E_{ad} and E_{cd} gives

$$e_o' = E_{BB} \cdot \frac{R_4}{R_3 + R_4} - E_{BB} \cdot \frac{R_2}{R_1 + R_2} \qquad (1.2.3.6)$$

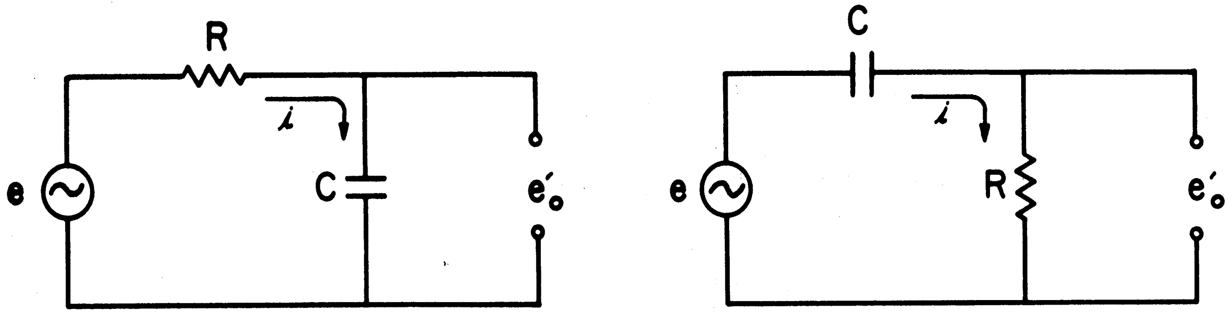
Equation 1.2.3.6 provides a general relation between the no-load output voltage, the input voltage, and the values of the four resistors in the bridge.

When the no-load output voltage is zero, the bridge is said to be balanced. For this condition equation 1.2.3.6 reduces to:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (1.2.3.7)$$

The above relationship for the balanced bridge is an important one and should be remembered. It will be noted that the positions of the resistances in equation 1.2.3.7 are the same as in the diagrams of figure 1.2.3.1, which is a handy memory aid. It is apparent that the bridge is balanced if all resistances are equal, but that all need not be equal for the bridge to be balanced.

1.2.4 RESISTANCE-CAPACITANCE CIRCUITS. Two circuits combining resistance and capacitance are shown in figure 1.2.4.1 a and b. The only difference



(a) INTEGRATOR (b) DIFFERENTIATOR

R. C. CIRCUITS

FIGURE 1.2.4.1

between the two circuits is the measurement of the output voltage e_o', the output of the integrator circuit being measured across the capacitance, while that of the differentiator circuit is measured across the resistance.

R and C are in series, and according to the statement following equation 1.2.3.4 on page 3, the no-load output voltage e_o' for the RC integrator circuit will be:

$$e_o' = \frac{Z_c}{R + Z_c} e \qquad (1.2.4.1)$$

where Z_c = impedance of the capacitance.

Re-writing with the impedance in operator form as shown in figure 1.2.1.1:

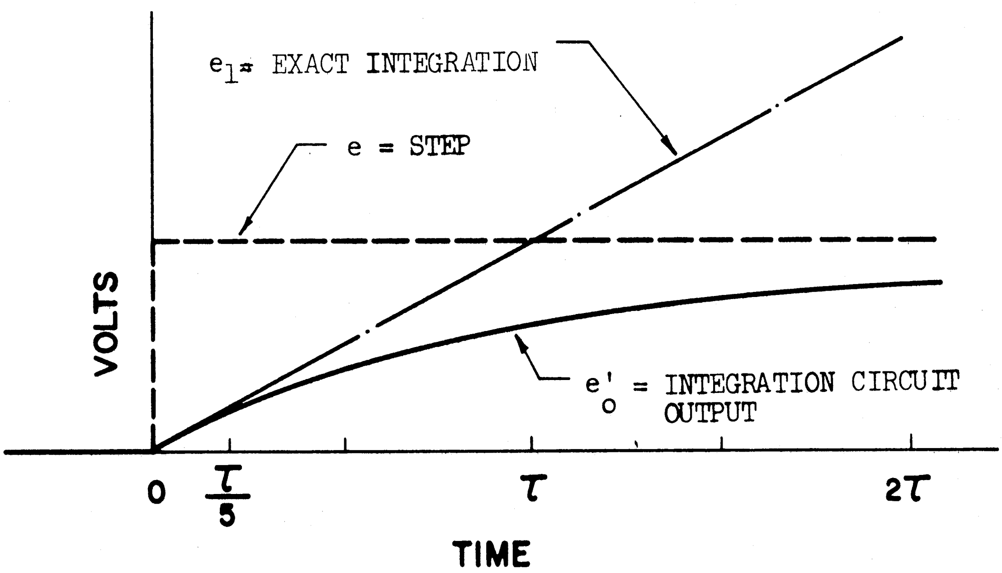
$$e_o' = \frac{1/C\,p}{R + 1/C\,p} e \qquad (1.2.4.2)$$

Multiplying by C p:

$$e_o' = \frac{1}{(R\,C\,p + 1)} e \qquad (1.2.4.3)$$

In this equation the RC product has units of time, which makes the combination RC·p unitless since the operator p has units of one divided by time. The RC product is called the time constant of the circuit, and when the effect of RC·p is large compared to unity the circuit behaves like an integrator.

A thorough analysis will not be made here, but one can at least gain some insight by considering the simple case when e(t) is a step change in the excitation as shown by the dashed line in figure 1.2.4.2, where:



INTEGRATOR CIRCUIT RESPONSE TO A STEP INPUT
FIGURE 1.2.4.2

$$RC = \tau \qquad (1.2.4.4)$$

$$e_1 = \frac{1}{\tau}\int_0^t e\,dt, \text{ for all values of } t \qquad (1.2.4.5)$$

$$e_o' \approx \frac{1}{\tau}\int_0^t e\,dt, \text{ for } t < \frac{\tau}{5} \qquad (1.2.4.6)$$

The response of the integration circuit as shown by the solid line agrees with the exact integration for a short interval of time. In fact, for a time period of one-fifth of the RC time constant, e_o' approximates the exact integral, e_1, within an error of 10%. For a time interval one-tenth of the RC time constant, the worst error will be less than 5%. If integrating a sinusoidally varying voltage is considered, it would be found that there would be errors no greater than 2% if the exciting frequency in radians per second were above 5 times the reciprical of the RC time constant.

It can be concluded that this circuit will give approximate integration if the time interval of the voltage changes is small compared to the RC time constant of the circuit, and because of this the capacitor should be as large as possible to give a large time constant for approximate integration. In such a case RC will be large compared to unity, and equation 1.2.4.3 becomes approximately:

$$e_o' = \frac{1}{R\,C\,p}\,e \qquad (1.2.4.7)$$

Substituting the integral for the operator to put the equation in more conventional form, the output voltage becomes:

$$e_o' = \frac{1}{RC}\int e\;dt \qquad (1.2.4.8)$$

By a similar analysis the no-load output of the differentiator circuit is:

$$e_o' = \frac{R}{R + \frac{1}{Cp}}\,e \qquad (1.2.4.9)$$

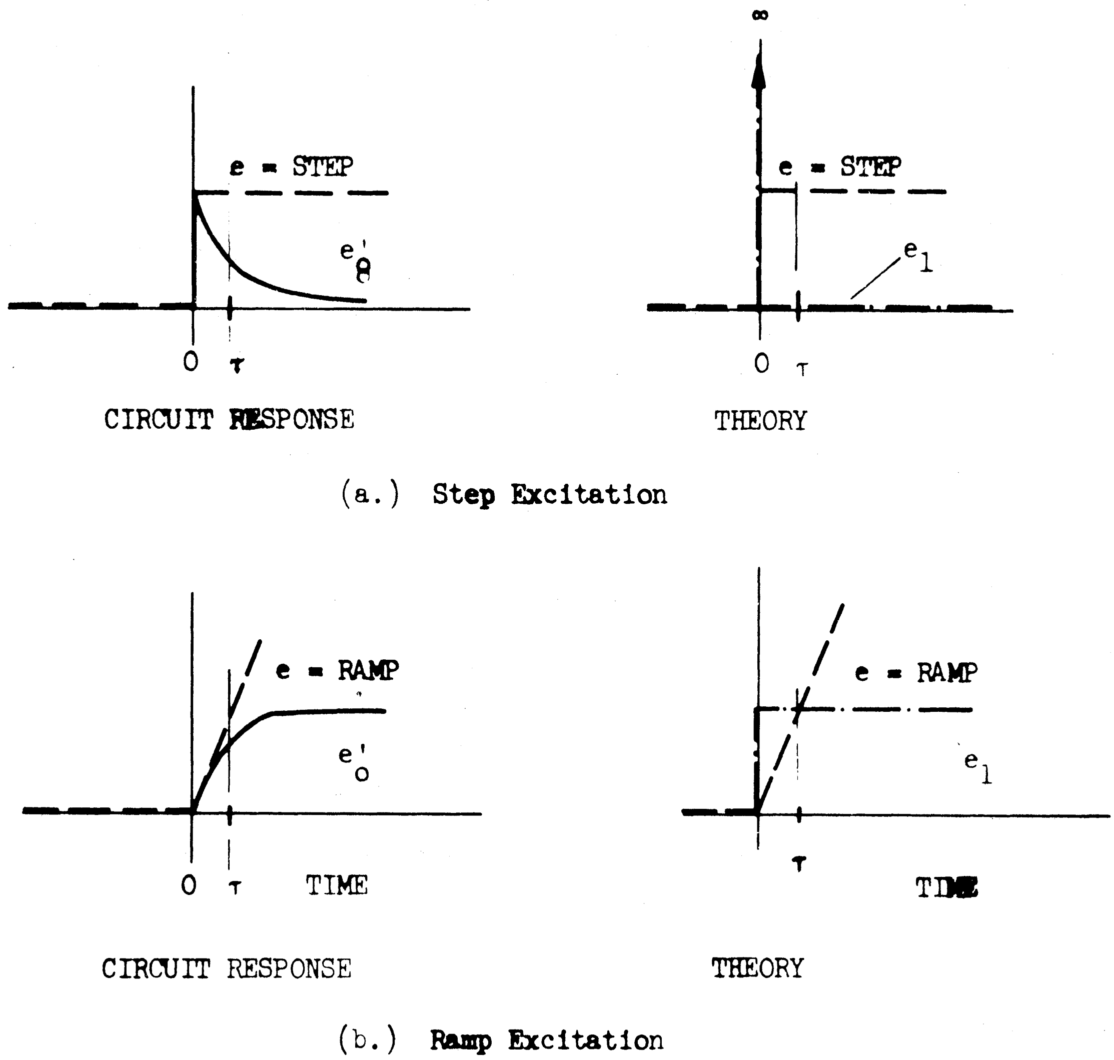
When multiplied by Cp:

$$e_o' = \frac{R\,C\,p}{(R\,C\,p + 1)}\,e \qquad (1.2.4.10)$$

From this equation it can be seen that if the effect of RCp is small compared to unity, which is the opposite effect considered previously for approximate integration, then the equation reduces to

$$e_o' = R\,C\,p\,e \qquad (1.2.4.11)$$

Again this result is an approximation depending upon a small RC time constant. This effect can be examined by comparing the approximation with exact differentiation for the case of a step and a ramp excitation as shown in figure 1.2.4.3. It can be seen by comparing e'_o with e_1 the approximation is reasonable after an initial period of several time constants. A study of the illustrations will show that if



e'_o, circuit-differentiation e_1, exact differentiation

COMPARISON OF THE DIFFERENTIATOR CIRCUIT RESPONSE
WITH EXACT DIFFERENTIATION
FIGURE 1.2.4.3

the signal to be differentiated varies slowly relative to the circuit RC time constant, the approximation will be quite accurate. It should be noted, however, that this requires a small RC time constant, and since the differentiated result is also multiplied by this value, the output voltage will be small in magnitude and difficult to measure. This is apparent from equation 1.2.4.12, where the operator has been replaced by more conventional notation.

$$e'_o = RC \frac{de}{dt} \qquad (1.2.4.12)$$

It should be mentioned that the RC integrating circuit can some times be effectively used as an integrator, but the RC differentiating circuit can seldom be effectively used to differentiate voltages. A differentiating circuit could often be very useful, but unfortunately the process of differentiating voltages rarely works because there is almost always high frequency electrical noise present. A simple example can illustrate the problem.

Example 1-1: A harmonically varying voltage signal accompanied by electrical noise is fed into an RC diffentiating circuit. The noise amplitude is 0.01 times the signal amplitude, and the noise frequency is 100 times the signal frequency. Determine the no-load output voltage of the differentiating circuit, and comment on the significance of the answer obtained.

Solution: The original signal can be expressed as:

$$e = A \sin\omega t + 0.01\, A \sin 100\omega t$$

then

$$e_o' = \frac{de}{dt} = \omega A \cos\omega t + 100 \cdot 0.01\omega\, A \cos 100\omega t$$

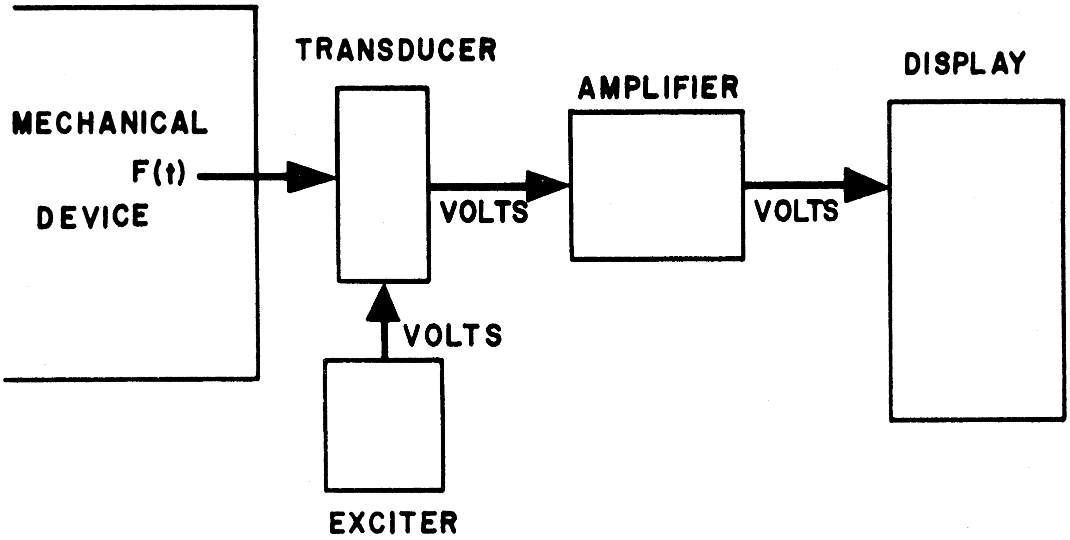
$$e_o' = \omega\, A \cos\omega t + \omega A \cos 100\omega t$$

The high frequency noise would be barely perceptible on the original signal, since it had an amplitude of only 1% of the signal. However, when differentiated the noise amplitude becomes equal to the signal amplitude, and would completely obscure the signal.

The most common use of the differentiating circuit is to produce sharp peak outputs when a step input is applied. When used in this way it is often called a "peaking circuit". This effect is illustrated by the circuit response in figure 1.2.4.3 a.

1.3 BASIC MEASURING SYSTEM AND COMPONENTS

It was pointed out in section 1.1 that the method utilized for measuring a physical variable using electrical instrumentation consisted of developing a voltage directly proportional to the physical variable, measuring that voltage, and then converting the measured voltage back to the corresponding values of the physical variable. Figure 1.3.1 shows the schematic arrangement of the basic measuring system which uses electrical instrumentation in this way to measure a physical variable on a mechanical device. It consists of the following components, each represented by a box in the figure:



BASIC MEASURING SYSTEM
FIGURE 1.3.1

1. A transducer, or sensing device, which is to develop a voltage directly proportional to the variable to be measured. The transducer is generally attached to the mechanical device.

2. An exciter, or source of electrical energy, for the transducer.

3. An amplifier, which amplifies the voltage from the transducer when this voltage is very small.

4. A display or recording instrument which displays or records the amplified voltage from the transducer.

All of the components shown in figure 1.3.1 are not always needed, and in some cases various components might be combined into a single piece of equipment. Nevertheless, the basic system remains unchanged, and one should think of the more detailed systems shown later in terms of the basic system of figure 1.3.1.

There are many different pieces of equipment which might be purchased or built and then assembled to form a measuring system such as represented by figure 1.3.1. There are so many transducers available that the next chapter is entirely devoted to their description and use. The exciter might be a battery or an audio frequency oscillator, while the amplifier might be either a separate unit or be built into a display instrument such as an oscilloscope, x-y plotter, pen recorder, or optical oscillograph. However, to make a satisfactory choice of equipment it is necessary to consider some of the important characteristics of the measuring system and its components, and to be certain that these characteristics are satisfactory for the combination of equipment to be used.

The most important characteristics of the measuring system and its components will be those characteristics which influence the accuracy with which the displayed or recorded voltage represents the physical variable to be measured. First of all, the transducer must indeed develop a voltage directly proportional to the physical variable to be measured, and the proportionality must be established by calibration. After that each component of the measuring system should faithfully maintain this proportionality of volts to physical variable. However, this proportionality can easily become distorted in the measuring system, and it is wise to consider the possible causes of such distortion, and how the distortion can be minimized.

The major part of any possible distortion which may prevent the displayed or recorded voltage from accurately representing the physical variable will probably be made up of one or more of the following:

1. Distortion due to mechanical loading of the device or system on which the measurements are to be made. The problem of mechanical loading is discussed in section 1.4

2. Distortion due to the combination of electrical impedances in the various components of the measuring system. This is commonly referred to as distortion due to impedance loading, and is considered in section 1.5

3. Distortion due to the frequency at which the voltages vary in the instrument system. The term frequency response is often used to describe the limiting ability of a measuring system or its components to accurately handle a certain type of rapidly varying voltage signal, and this is discussed in section 1.6

4. Distortion due to electrical noise, which is caused by undesirable background voltages which are not related to the physical variable of the mechanical system, but which somehow have been added to the measured voltage. This is further discussed in section 1.7

5. Distortion or inaccuracy due to improper calibration of the measuring system. This is considered in section 1.8

It will be noticed from the discussions which follow that at least a slight amount of distortion is apt to result from almost everyone of the 5 causes just listed. It is therefore necessary that the distortion due to any one of these causes be kept to a minimum.

1.4 DISTORTION DUE TO MECHANICAL LOADING

In general, electrical instrumentation is used to measure such physical variables as force, displacement, etc., only when those variables are rapidly changing. In such a case one is interested in the dynamic behavior of the mechanical device, and care must be exercised to see that the installation of the measuring system does not appreciably alter this dynamic behavior.

Mechanical loading of the device or system on which the measurements are to be made is said to exist when the installation of the measuring system alters the behavior of the mechanical device or system. Since the dynamic behavior of the mechanical device or system is governed by its mass or inertia, its spring rate, its damping, and externally applied exciting forces, it follows that mechanical loading and distortion of the physical variable will result if the installation of the measuring system changes any of these parameters. Physically attaching the transducer to the mechanical device commonly adds the inertia and friction of the transducer to that of the mechanical device, and in such a case a certain amount of mechanical loading is inevitable. However, if the inertia and friction of the transducer are small compared to those of the mechanical device, the mechanical loading and the resulting distortion will be negligible.

The addition of the inertia and friction of the transducer to that of the mechanical device is easily recognized, and this type of mechanical loading is generally prevented. Changing the spring rate of the mechanical device due to the addition of the measuring system seems to be less easily recognized, and often turns out to be the most serious type of mechanical loading. For example, in order to measure torque in the spindle of a drill press by utilizing a transducer which develops a voltage proportional to unit strain in the spindle, one is tempted to replace the original spindle with one having a smaller diameter so as to increase the unit strain for any given torque. While this increases the voltage developed by the transducer, it reduces the spring rate of the spindle, hence changing the dynamic behavior of the drill press.

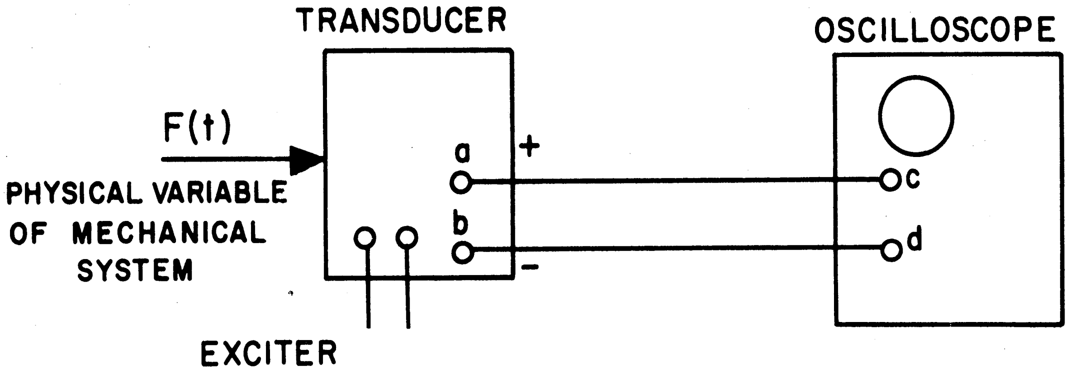
A certain amount of mechanical loading due to changing the spring rate is often necessary when measuring forces. Care must be taken to see that the amount of change is not excessive.

1.5 DISTORTION DUE TO IMPEDANCE LOADING

Impedance loading is said to exist when voltage distortion results from the combination of impedances in the various components of the measuring system. A certain amount of impedance loading is always present, but its effects can be negligible in a properly designed measuring system.

It will be seen in the next chapter that in theory the relationship between a physical variable and the output voltage of the transducer is established when the output of the transducer is not connected to anything. In such a situation there is an open circuit between the output terminals of the transducer, the transducer is said to be not loaded, and the developed voltage is called the no-load output voltage of the transducer.

In practice, however, it is necessary to connect at least a display or recording instrument to the output of the transducer, and in most cases an amplifier is needed as shown in figure 1.3.1. Connecting a component such as an amplifier to the output of the transducer causes the output voltage from the transducer to become the input voltage supplied to the amplifier. The transducer can then be considered as the voltage source, the amplifier as a load, and the transducer is said to be driving the amplifier. Under these conditions the output voltage from the transducer will no longer be the no-load voltage, but will instead be the output voltage under load. The way in which the impedance of the components affects the relation between the no-load and the load voltage is considered in the paragraphs which follow.



TRANSDUCER DRIVING OSCILLOSCOPE
FIGURE 1.5.1

Figure 1.5.1 shows an externally excited transducer driving the internal amplifier of an oscilloscope. It will be assumed that before the oscilloscope is connected to the transducer the no-load output voltage of the transducer is directly proportional to the physical variable to be measured, and under these conditions:

$$e_o' = K' \cdot F(t) \tag{1.5.1}$$

where

e_o' = no load output voltage

K' = proportionality constant at no load

$F(t)$ = the physical variable which varies with time

As mentioned on page 3, the primes indicate a condition of no-load, a convention used throughout this book.

When the output of the transducer is connected to the input of the oscilloscope as shown in figure 1.5.1, e_o' changes to e_o, the voltage output under load. One of the three following conditions then exists.

1. e_o equals e_o'; equation 1.5.1 becomes:

$$e_o = K' \cdot F(t) \qquad (1.5.2)$$

2. e_o does not equal e_o', but is directly proportional to e_o'; this is not serious so long as e_o is of a measurable magnitude, and the equation becomes:

$$e_o = K \cdot F(t) \qquad (1.5.3)$$

where K = proportionality constant with load.

3. e_o does not equal e_o', and is not directly proportional to e_o'; the transducer output voltage under load has been distorted so that it is no longer directly proportional to the physical variable, and the equation becomes:

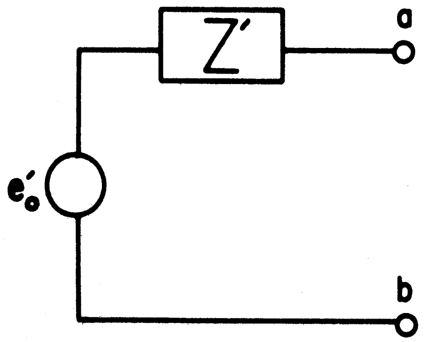
$$e_o \neq K \cdot F(t) \qquad (1.5.4)$$

To see what governs the relationship of the transducer no-load output voltage e_o' and the output voltage under load e_o, the common measuring system of figure 1.5.1 can be analysed by considering the transducer output terminals a and b as the source, and the oscilloscope input terminals c and d as the load. The equivalent circuits of the two components are shown in figure 1.5.2, where

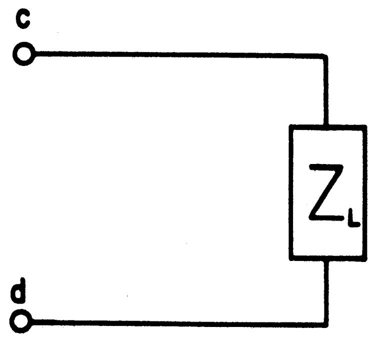
e_o' = no load output voltage of source, or transducer

Z' = source impedance, or output impedance, ohms

Z_L = load impedance, or input impedance, ohms



(a) EQUIVALENT CIRCUIT OF SOURCE



(b) EQUIVALENT CIRCUIT OF LOAD

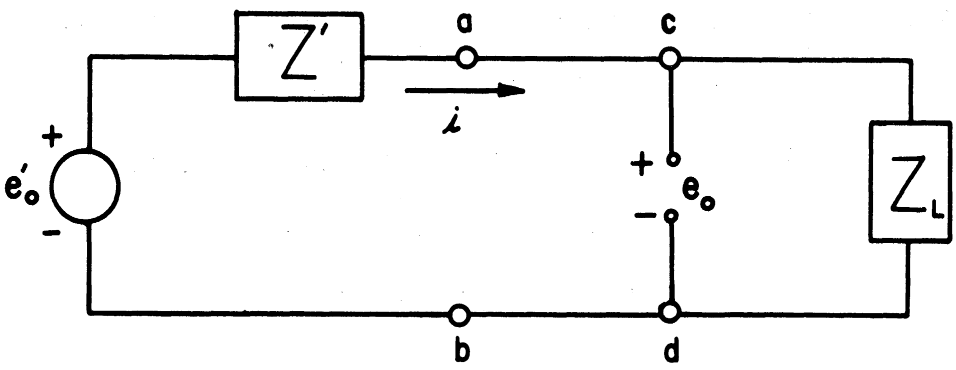
EQUIVALENT CIRCUITS OF SOURCE AND LOAD
FIGURE 1.5.2

The equivalent circuit for the source is known as the Thevenin equivalent circuit. The source impedance Z' represents the impedance of the transducer from terminal a to b when all voltage and current generators are replaced by their respective internal impedances.

For most transducer circuits the source impedance is pure resistance. It is easily determined by short circuiting any voltage source to approximate its low impedance, opening the circuit to any current sources to approximate their high impedances, then measuring the resistance across the output terminals a and b.

The load impedance is commonly specified by the manufacturer of that piece of equipment as either the load impedance or as the input impedance.

The two equivalent circuits have been combined in figure 1.5.3. The impedances Z' and Z_L can be considered as resistances, and so by applying the voltage relation from figure 1.2.1.1 to Kirchoff's law for voltages the following equation is written:



EQUIVALENT CONNECTED CIRCUIT
FIGURE 1.5.3

$$e_o' - iZ' - iZ_L = 0 \qquad (1.5.5)$$

from which

$$e_o' = i\,(Z' + Z_L) \qquad (1.5.6)$$

but

$$iZ_L = e_o \qquad (1.5.7)$$

and so by substitution and manipulation

$$\frac{e_o}{e_o'} = \frac{1}{1 + Z'/Z_L} \qquad (1.5.8)$$

Equation 1.5.8 shows that if the ratio Z'/Z_L is very small, the output voltage under load will be approximately the same as the no-load voltage, and there will be no reduction or distortion of the transducer output due to connecting the transducer to the load.

However, the output voltage under load could be substantially reduced if the impedance ratio was not small, hence it can be concluded that the effects of impedance loading are minimized by always having a small impedance ratio.

If the impedance ratio is of significant magnitude and varies during operation of the measuring system, it follows that a linear relation between the physical variable and the displayed voltage will not be maintained. The impedance ratio can vary if the source impedance of the transducer varies during operation. The source impedance of various transducers will be given with the discussion of the transducers in chapter 2, and it will be seen that the impedance of most transducers varies with operation. Thus the impedance ratio will vary, and serious distortion will result unless the impedance ratio is very small.

Example 1-2: A strain gage bridge having an essentially constant source impedance of 120 ohms is used as a transducer, with (a) an oscilloscope, and (b) an optical oscillograph as the display instruments. Will impedance loading be a serious problem with either arrangement?

Solution: (a) Most oscilloscopes have an input or load resistance of about one megohm, in parallel with a small capacitor in the order of 5×10^{-12} farads. Because the impedance of the capacitor is very large at normal mechanical frequencies, the load impedance of the oscilloscope can be taken as approximately one megohm.

Using equation 1.5.8

$$\frac{e_o}{e_o'} = \frac{1}{1 + Z'/Z_L}$$

$$\frac{e_o}{e_o'} = \frac{1}{1 + 120/1 \times 10^6} = 1 \text{ approximately}$$

It is apparent that the output voltage under load e_o is essentially the same as the no-load voltage e_o', and impedance loading is of no consequence.

(b) The sensitive element of an optical oscillograph is a mirror galvanometer, with a resistance somewhere between 25 and 60 ohms depending upon the model used. Supposing a 26 ohm galvanometer is used:

$$\frac{e_o}{e_o'} = \frac{1}{1 + 120/26} = 0.178$$

The output voltage across the galvanometer has been reduced to about 19% of the no-load output voltage of the strain gage transducer. The voltage is not distorted, but is only reduced or attenuated. The galvonometer is sufficiently sensitive to provide ample output deflections in response to the strain gage output. A problem will arise only if the user is mislead by expecting full no-load voltage to appear across the galvanometer.

Example 1-3: A potentiometer transducer having a source impedance which varies during operation from 10 K ohms to 5 K ohms is used with the display instruments of Example 1-2. Will impedance loading be a serious problem with either arrangement?

Solution: (a) When the oscilloscope is used:

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{5 \times 10^3}{1000 \times 10^3}} = \frac{1}{1.005} \quad \text{(max.)}$$

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{10 \times 10^3}{1000 \times 10^3}} = \frac{1}{1.010} \quad \text{(min.)}$$

There is some distortion due to the change in the transducer impedance, but it is so small it can be neglected.

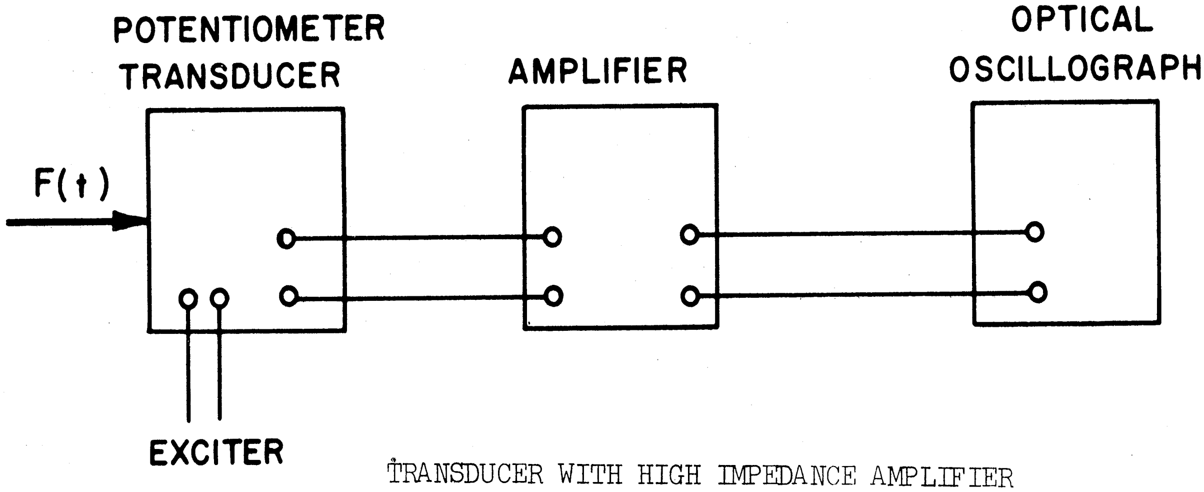
(b) When the optical oscillograph is used:

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{5 \times 10^3}{26}} = \frac{1}{193.3} \quad \text{(max.)}$$

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{10 \times 10^3}{26}} = \frac{1}{385.6} \quad \text{(min.)}$$

The output voltage is not only so greatly attenuated that it possibly could not be read, but it no longer has constant proportionality to the no-load voltage, hence is not proportional to the physical variable being measured by the potentiometer transducer.

The preceeding example does not mean that it is impossible to use the described potentiometer transducer with the optical oscillograph. A high impedance amplifier could be inserted into the circuit as shown in figure 1.5.4.



TRANSDUCER WITH HIGH IMPEDANCE AMPLIFIER
FIGURE 1.5.4

If the input impedance of the amplifier is sufficiently large the impedance ratio of the transducer and amplifier input will be small enough so that the variation in transducer impedance will have negligible effect on the transducer output voltage. This will eliminate the distortion. The output voltage of the amplifier may still be attenuated by the low impedance oscillograph, but this can be at least partially offset by the gain of the amplifier. The use of a lower impedance potentiometer would be helpful if such was available.

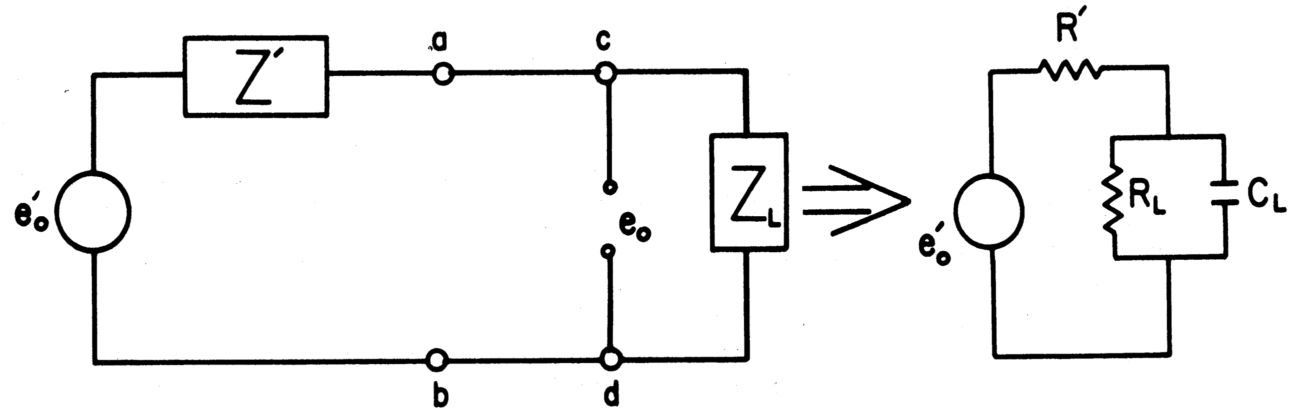
1.6 DISTORTION DUE TO SIGNAL FREQUENCY

The term frequency response is commonly used to describe the limitations of a component's output ability to reproduce sinusoidal inputs without undue distortion in either amplitude or time. For example, if an oscilloscope is described as having a frequency response of 200,000 cps at a given sensitivity, then in general, signals will be quite faithfully reproduced at lower frequencies and distorted at higher frequencies.

The sinusoidal frequency beyond which the signals become unduly distorted is also often called the "break point" frequency, for reasons which will become apparent from the analysis that follows. For most manufacturers of electrical components this corresponds to the frequency at which the amplitude of a sinusoidal input is reduced to 0.707 of its true value and there is an appreciable time lag between the signal and the output of the component.

A quantitative analysis of the distortion due to signal frequency, and the establishment of the frequency response of a measuring system, can be made by considering the source and load impedances of two inter-connected electrical components and taking into account voltage signals varying with time.

When the impedance of a circuit element varies with time, the impedance can be expressed in operator form as shown in figure 1.2.1.1 The operator notation requires that the first derivative with respect to time be taken for any variable having p as a coefficient, while any variable having 1/p as a coefficient must be integrated with respect to time. Consider a transducer as a source, and a display instrument having resistance and capacitance in parallel as a load. The circuit for analyzing the impedance loading is repeated for convenience in figure 1.6.1.



EQUIVALENT CIRCUIT
FIGURE 1.6.1

The response of the circuit can be calculated from equation 1.5.8 using the impedances in operator form:

$$\frac{e_o}{e_o'} = \frac{1}{1 + Z'/Z_L} = \frac{1}{1 + R'\left(\frac{1}{R_L} + C_L\, p\right)} \qquad (1.6.1)$$

By re-arranging:

$$\frac{e_o}{e_o'} = \frac{1}{1 + R'/R_L} \cdot \frac{1}{\frac{1}{1 + R'/R_L} \cdot R'C_L\, p + 1} \qquad (1.6.2)$$

which can be re-stated:

$$\frac{e_o}{e_o'} = \frac{1}{1 + R'/R_L} \cdot \frac{1}{\tau p + 1} \qquad (1.6.3)$$

where

$$\tau = \frac{1}{1 + R'/R_L} \cdot R'C_L \qquad (1.6.4)$$

It might be noted that in the preceeding equation:

$$\frac{1}{1 + R'/R_L} \cdot R' = \frac{R' R_L}{R' + R_L} \qquad (1.6.5)$$

The right hand term of equation of 1.6.5 is the equivalent resistance of the source and load resistances in parallel.

In most cases the ratio R'/R_L is small, and equations 1.6.3 and 1.6.4 become approximately:

$$\frac{e_o}{e_o'} = \frac{1}{\tau p + 1} \qquad (1.6.6)$$

where

$\tau = R'C_L$, the time constant of the circuit, seconds, as explained in section 1.2.4 (1.6.7)

Equation 1.6.6 can also be expressed in conventional derivative form:

$$e_o' = \tau \frac{d\, e_o}{d\, t} + e_o \qquad (1.6.8)$$

The solution of this standard differential equation will establish the relationship between the no-load output voltage e'_o and the output voltage under load e_o.

If the transducer in the equivalent circuit of figure 1.6.1 produces a no-load output voltage which varies sinusoidally with time, it could be expressed as:

$$e'_o = E'_o \sin \omega t \qquad (1.6.9)$$

Substituting this into equation 1.6.8 and allowing the transients to die out, the steady state solution will be:

$$e_o = E_o \sin (\omega t - \phi) \qquad (1.6.10)$$

where ϕ = phase angle, and

$$E_o = \frac{E'_o}{\sqrt{1 + (\tau\omega)^2}} \qquad (1.6.11)$$

Equation 1.6.10 shows that the output voltage under load e_o is sinusoidal and has the same frequency as the no-load voltage e'_o. However, the amplitude is attenuated, and the ratio of the amplitudes is:

$$\frac{E_o}{E'_o} = \frac{1}{\sqrt{1 + (\tau\omega)^2}} \qquad (1.6.12)$$

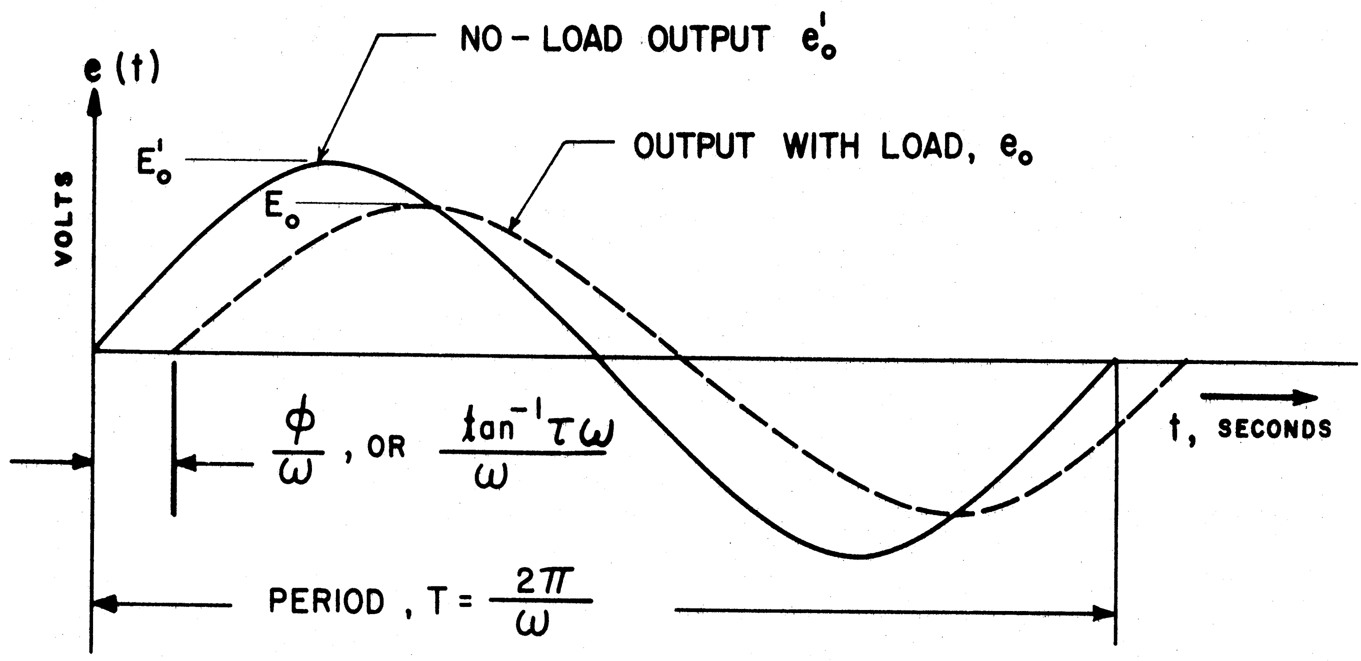
The output voltage e_o is also delayed, lagging the no-load sinusoid by the phase angle ϕ, where:

$$\phi = \tan^{-1} \tau\omega \quad , 0^\circ \leq \phi \leq 90^\circ \qquad (1.6.13)$$

The relation between e_o and e'_o is illustrated in figure 1.6.2, where e_o is delayed and attenuated.

The amplitude ratio of equation 1.6.12 is dimensionless, and by taking the logarithm of both sides:

$$\log_{10} \frac{E_o}{E'_o} = - \log_{10} \sqrt{1 + (\tau\omega)^2} \qquad (1.6.14)$$



DISTORTION OF A SINUSOIDAL OUTPUT DUE TO LOAD
FIGURE 1.6.2

A logarithmic plot of amplitude ratio vs. ω will show the relationship of amplitude ratio and frequency. A simple approximation of this relationship can be made if one separately considers the logarithm of the amplitude ratios for very low frequencies and for very high frequencies. For these extremes the log of the amplitude ratio is asymptotic to straight lines. The low frequency end of the curve will be approaching the middle frequencies along a straight horizontal line. From the high frequency end the curve will be moving towards the middle frequencies along a sloped straight line. These two straight line asymptotes will give a good approximation of the logarithm of the amplitude ratio versus the logarithm of excitation frequency.

The low frequency asymptote is in the region where ω is small enough so that $\tau\omega$ is very much less than unity, in which case equation 1.6.14 becomes approximately

$$\log_{10} \frac{E_o}{E_o'} = \log_{10} 1 = 0 \qquad (1.6.15)$$

This shows that the low frequency asymptote is a horizontal line at an amplitude ratio of one.

The high frequency asymptote is in the region where ω is very large, and $\tau\omega$ is much greater than unity. In this situation:

$$1 + (\tau\omega)^2 \approx (\tau\omega)^2 \qquad (1.6.16)$$

Solving equation 1.6.14 for the high frequency asymptote:

$$\log_{10} \frac{E_o}{E_o'} = -\log_{10} \tau\omega \qquad (1.6.17)$$

or

$$\log_{10} \frac{E_o}{E_o'} = -\log_{10}\omega + \log_{10} \frac{1}{\tau} \qquad (1.6.18)$$

Frequency ω_o is now defined as:

$$\omega_o = \frac{1}{\tau} \qquad (1.6.19)$$

When this is substituted for ω , equation 1.6.14 for the high frequency asymptote becomes:

$$\log_{10} \frac{E_o}{E_o'} = 0 \qquad (1.6.20)$$

This is the same as equation 1.6.15 for the low frequency asymptote, hence the asymptotes intersect at ω_o.

Equation 1.6.18 for the high frequency asymptote is in the form of the equation for a straight line, $y = m x + b$, where:

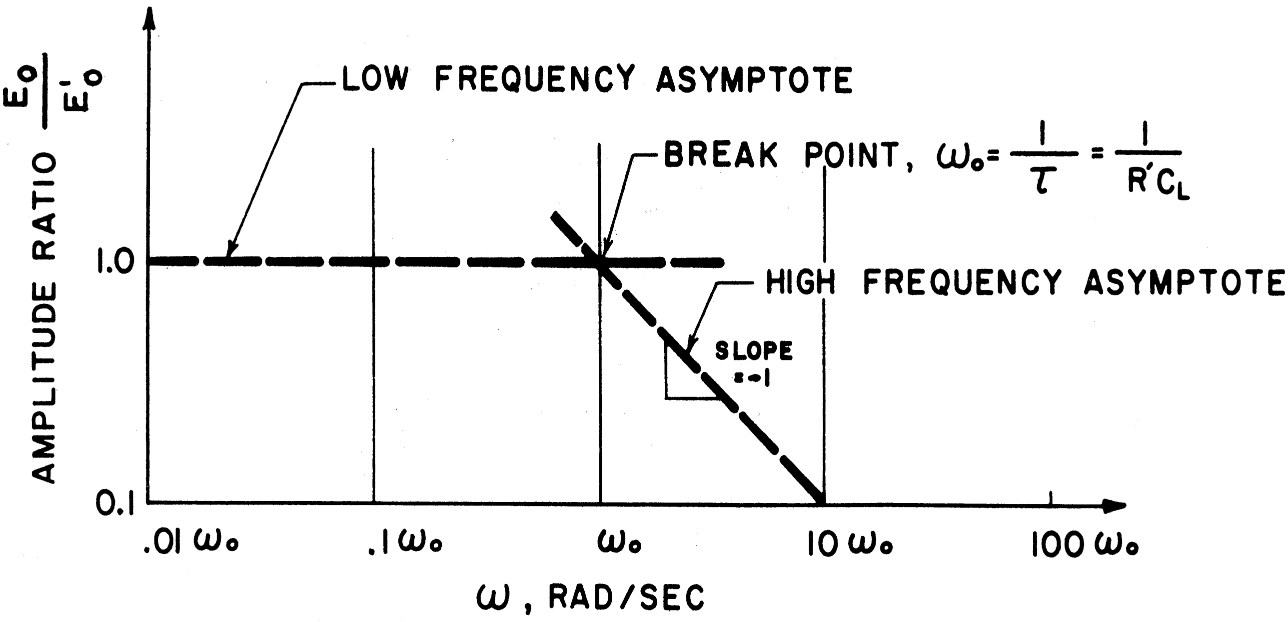
$\log_{10} \frac{E_o}{E_o'}$ corresponds to y

-1 corresponds to m

$\log_{10} \omega$ corresponds to x

$\log_{10} \frac{1}{\tau}$ corresponds to b

Figure 1.6.3 shows the asymptotic approximation of equations 1.6.15 and 1.6.18, plotted on logarithmic scales and in terms of ω_0.



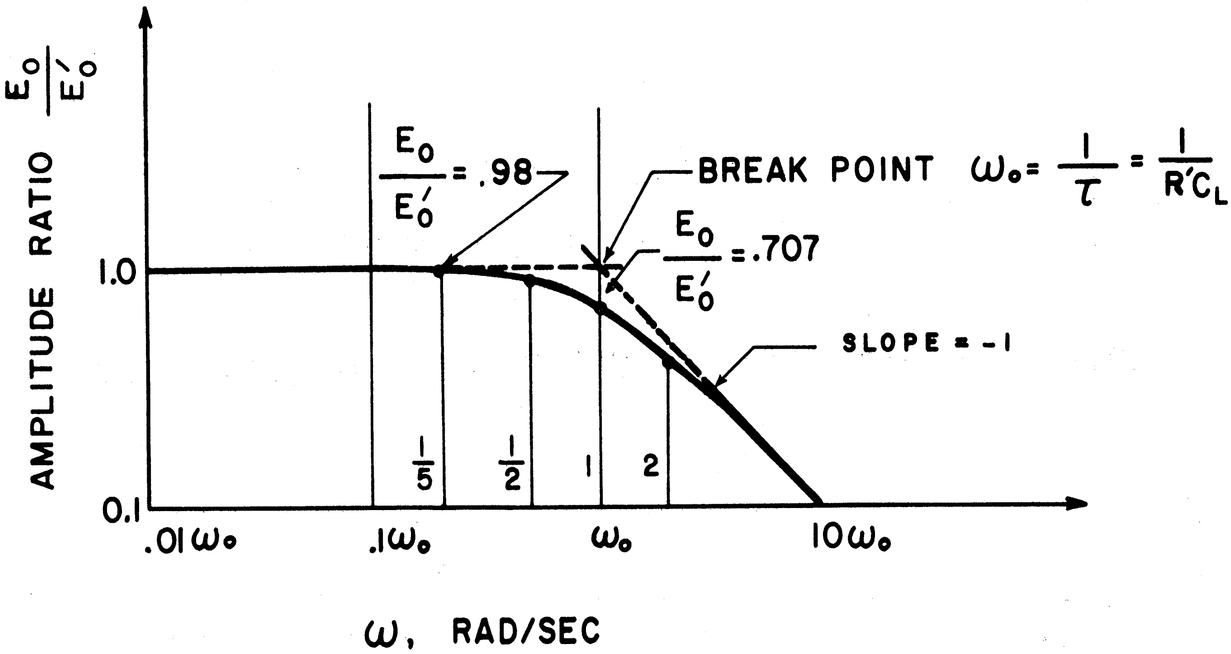
APPROXIMATION OF LOG VS. LOG PLOT OF AMPLITUDE RATIO VS. FREQUENCY
FIGURE 1.6.3

As previously mentioned in this section, $\tau = R'C_L$, the time constant of the circuit of figure 1.6.1. Then as defined by equation 1.6.19 and shown on figure 1.6.3:

$$\omega_0 = \frac{1}{\tau} = \frac{1}{R'C_L} \qquad (1.6.21)$$

This is the "break point" frequency referred to at the beginning of this section, and the reason for this name becomes apparent from figure 1.6.3. It is also apparent that at frequencies beyond ω_0 the amplitude of the output voltage with load e_0 will be attenuated.

The actual curve of amplitude vs. frequency is shown in figure 1.6.4. The maximum deviation from the approximation shown in figure 1.6.3 occurs at the break point frequency ω_0, at which the actual amplitude ratio is 0.707. At twice the break point frequency and at one-half the break point frequency the actual curve varies from the approximation by 10.8%, and at one-fifth the break point frequency the amplitude is reduced 2%.

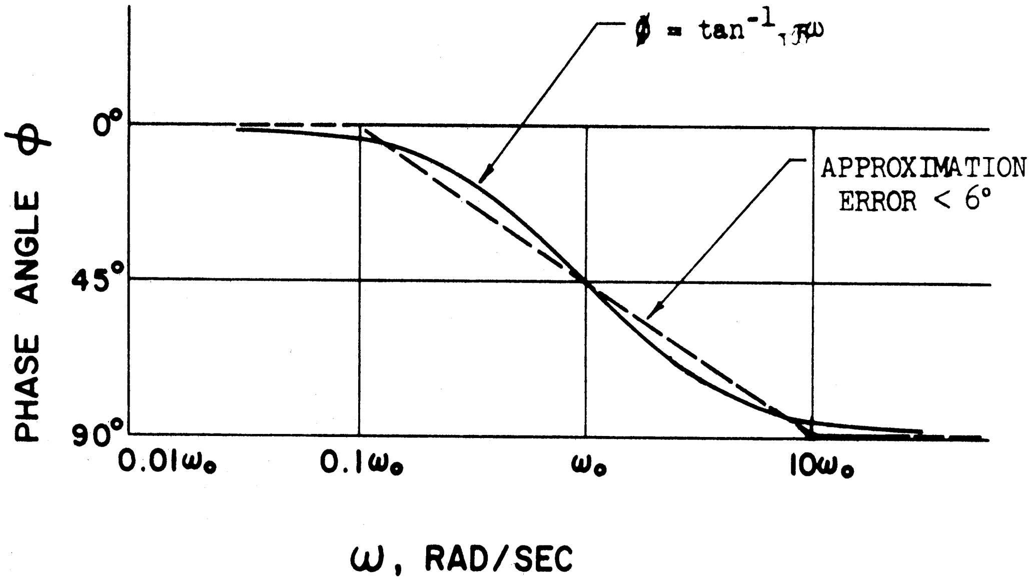


LOG$_{10}$ AMPLITUDE VS. LOG$_{10}$ FREQUENCY
FIGURE 1.6.4

The phase angle ϕ by which e_o lags behind e_o' is shown in figure 1.6.5. At the break point frequency ω_o the output sinusoid lags the no-load sinusoid by 45^o.

An approximate curve shown by the dashed lines is also drawn. The approximation assumes no phase lag at a frequency one-tenth the break point frequency and a maximum phase lag of 90^o at a frequency ten times the break point frequency. At the break point the approximation is exact. It is never off by more than 6^o. The approximation is simply drawn and is fairly accurate.

It must be remembered that the preceeding analysis is based on a circuit where the input or load resistance R_L was very large compared to the source or transducer resistance R_T, the resulting simplification being used to obtain equation 1.6.6.



PHASE ANGLE ø VS. $LOG_{10}\omega$
FIGURE 1.6.5

Example 1-4: A strain gage bridge having a source impedance of 120 ohms is used as a transducer, and an oscilloscope having a 1 megohm resistance in parallel with a 0.05 μfd capacitor is the display instrument. For what maximum sinusoidal signal frequency might this circuit be considered satisfactory?

Solution: Using equation 1.6.21 the break point frequency ω_o will be:

$$\omega_o = \frac{1}{\tau} = \frac{1}{R^T C_L} = \frac{1}{120 \times 0.05 \times 10^{-6}}$$

$$\omega_o = .167 \times 10^6 \text{ radians/second}$$

In cycles/second this frequency will be:

$$f_o = \frac{1}{2\pi}\omega_o = 26{,}600 \text{ cycles/second}$$

From the curve of figure 1.6.4 it can be seen that at the break point frequency the output of the strain gage transducer will be attenuated to 0.707 of its no-load amplitude. The output will also lag 45^o, or will be delayed 0.005 milliseconds.

As a rule of thumb it might be said that this circuit would be satisfactory for a maximum signal frequency of one-fifth the break point frequency, or about 5,000 cycles/second, where the amplitude is reduced only 2%.

1.7 DISTORTION DUE TO ELECTRICAL NOISE

Electrical noise was defined in section 1.3. The small voltage signals which are generated in a measuring system are easily distorted or obscured by electrical noise, and one of the most troublesome characteristics of the electrical measuring system is its tendency to generate or gather noise. This noise may vary in a random fashion, or it may be of a particular frequency such as 60 cycles/second.

The noise can be reduced and in some cases eliminated by keeping the wires of the measuring system away from noise generating machinery, by carefully grounding all of the electrical and mechanical equipment being used, and when necessary, by shielding all of the connecting wires and grounding the shields. Further discussion on grounding problems will be found in chapter 4 on laboratory techniques.

Capacitors can be shunted across the circuit to filter out high frequency noise by reducing the break point frequency of the measuring system as explained in the preceeding section. However, care must be taken not to distort the main signal by this procedure.

1.8 DISTORTION DUE TO IMPROPER CALIBRATION

The relationship between the physical variable being measured and the output voltage displayed or recorded by the measuring system is established by a suitable calibration. To a large extent it is the accuracy of this calibration which establishes the accuracy and hence the validity of the measurements made by the system. Conversely, improper or careless calibration can distort or reduce the accuracy of the system.

Applicable calibration methods and techniques are discussed with the various transducers in chapter 2. At this point it can only be emphasized that the calibration must be carefully and accurately carried out, for calibration is by far the greatest potential source of error in the entire measuring system.

1.9 CONSIDERATIONS FOR COMPONENT SELECTION

The preceeding discussions should make one aware of the importance of considering mechanical loading, impedance, and frequency response when selecting components for a measuring system. There are many other considerations such as sensitivity, resolution, size, cost, and reliability. While each of these are important, it would seem that no special discussion of these considerations is needed here. However, there is one other consideration which is worth some emphasis, and that is simplicity. The more simple the measuring system and its circuits, the less difficulty one is apt to encounter. Each refinement of the system tends to substantially contribute to the complexity, hence it is generally a good idea to start off with as simple a measuring system as seems reasonably possible. This simple measuring system will almost always allow the measuring problem to be defined, and the range of the parameters established. This in turn should give some indication of whether or not a more involved measuring system is needed.

CHAPTER 2

TRANSDUCERS

2.1 INTRODUCTION

For the engineer or technician who wishes to measure some of the mechanical variables mentioned in section 1.1, the transducer is the most important part of the measuring system. A transducer has been previously defined as a sensing device which develops a voltage proportional to the variable to be measured, and it was mentioned that the transducer is generally attached to the mechanical device or machine on which the measurements are to be made. To complete the system it is generally possible to purchase standard components in the form of amplifiers and recorders, but the choice or construction of a suitable transducer which will develop a reasonable voltage, can be suitably mounted and calibrated, and have the proper combination of satisfactory characteristics often requires ingenuity and imagination as well as a good understanding of how transducers operate.

Many different devices find use as transducers, depending upon what is to be measured. The following list contains some devices used as transducers for measuring common variables in mechanical devices:

1. Displacement Transducers
   a. wire wound potentiometer
   b. differential transformer
   c. solar cell
   d. photo cell
   e. resistance strain gages

2. Velocity Transducers
   a. coil and magnet
   b. a.c. or d.c. tachometer generators

3. Acceleration Transducers
   a. any displacement or force transducer sensing an inertial reaction

4. Force or Pressure Transducers
   a. resistance strain gages
   b. piezo-electric crystal

In the sections which follow a number of the transducers listed are discussed in detail, but this write-up is not intended to be a complete coverage of transducers. The transducers to be discussed have been selected because they are useful, and because they utilize a number of different principles for measuring. It is hoped that these will serve as a good introduction, and will provide enough background so that those who are interested may be able to use these transducers and proceed to learn about others.

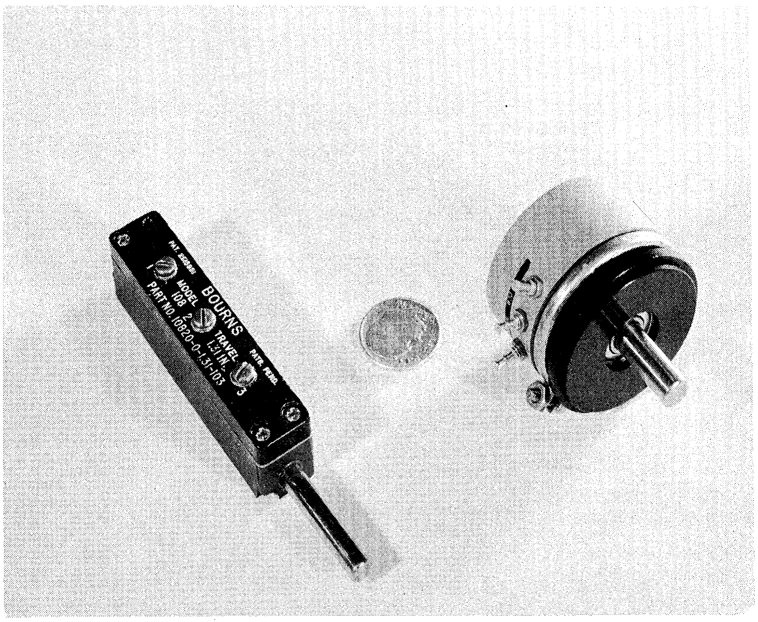
Up to this time a transducer has been referred to as a device. It will soon become apparent that a number of circuits with various components are often needed to go with the device, in which case the transducer becomes a small system of its own.

2.2 WIRE WOUND POTENTIOMETER

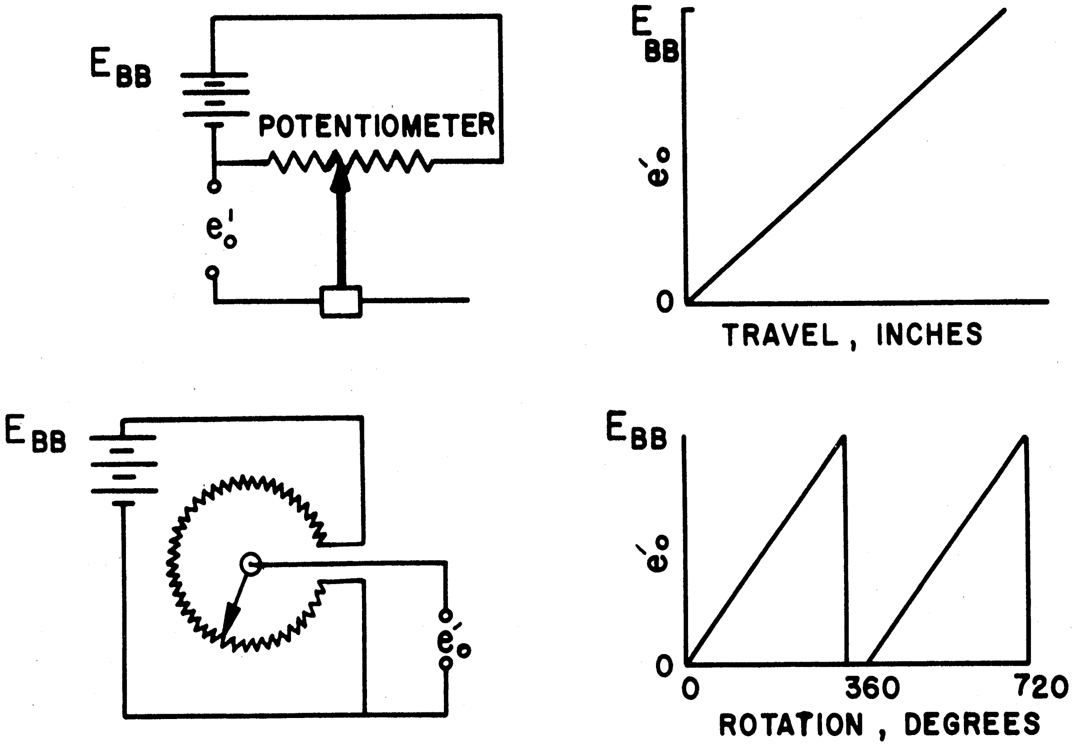
2.2.1 PRINCIPLE AND CIRCUITS OF THE WIRE WOUND POTENTIOMETER. The wire wound potentiometer can be used as a linear displacement transducer or as an angular displacement transducer, depending upon its construction. It consists of a very uniform resistance element formed in a straight line or in a circle, and a movable contact driven by the mechanical device the displacement of which is to be measured.

The most common potentiometer is the wire wound type discussed here. However, a number of manufacturers provide other types of resistance elements such as a thin metallic film strip, or a carbon strip. They can be considered equivalent to the wire wound potentiometer in all but a few minor details.

Figure 2.2.1.1 shows the physical appearance and size of a linear and a rotary potentiometer, while figure 2.2.1.2 shows the schematic circuits and no-load output voltage characteristics. With both the linear and rotary displacement potentiometers the movable contact, or brush, is physically attached to the mechanical part on which the displacement measurements are to be made.



WIRE WOUND POTENTIOMETERS
FIGURE 2.2.1.1

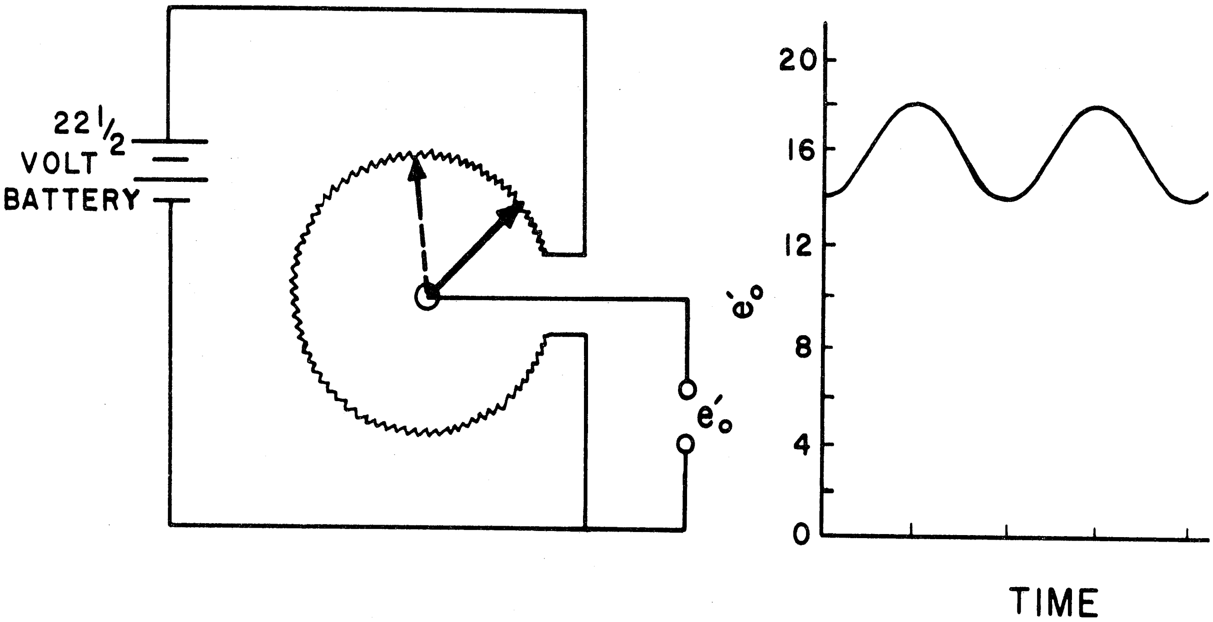


POTENTIOMETER SCHEMATICS
FIGURE 2.2.1.2

It can be seen from figure 2.2.1.2 that the no-load output voltage e_o' from the potentiometer is directly proportional to the displacement of the movable contact from any given starting point, thus is also directly proportional to the displacement of the mechanical device to which the contact is attached.

The potentiometer as shown in figure 2.2.1.2 could be used to measure displacement, since it develops a voltage proportional to displacement. In practice, however, the potentiometer must be used in a bridge circuit similar to that shown in figure 1.2.3.1(b). The reason for this is best shown by the example which follows.

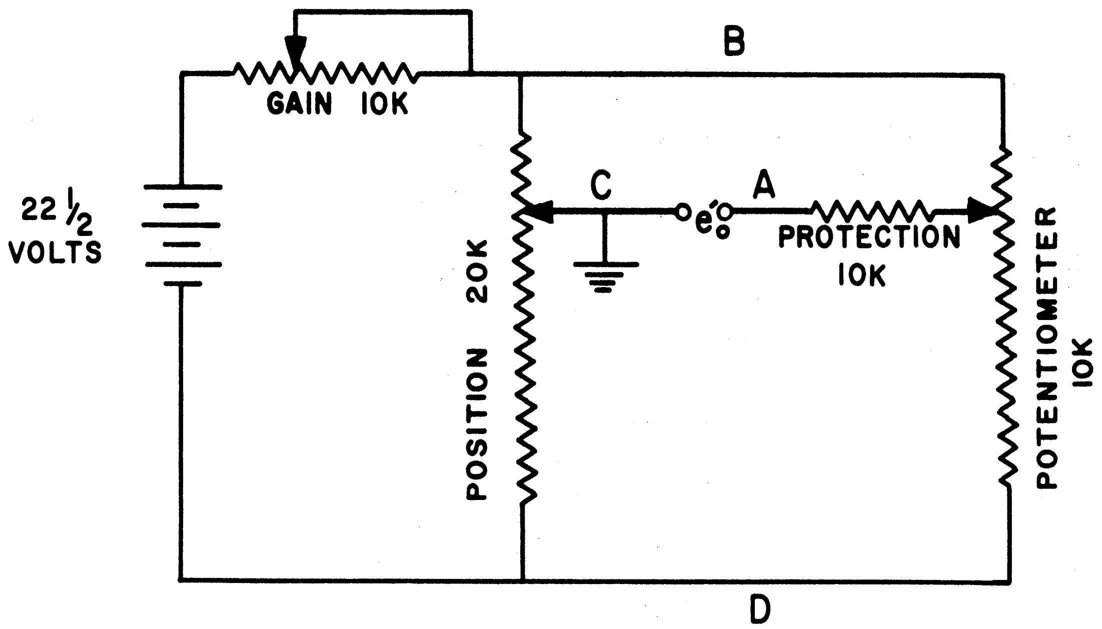
Consider a situation in which the movable contact of the rotary potentiometer is attached to a machine shaft which oscillates through a rather small arc so that the no-load output voltage varies a total of say 4 volts. Consider further that the potentiometer contact happens to do this oscillating in such a range that the actual no-load output voltage varies from say 14 volts to 18 volts, approximately as shown in figure 2.2.1.3. The same figure shows how the output voltage might vary with time as the shaft oscillates. For this illustration it is assumed that the output is to be displayed on a cathode ray oscilloscope. Being interested in the shaft displacement as indicated



OSCILLATING POTENTIOMETER
FIGURE 2.2.1.3

by the voltage variation, one would logically wish to amplify the output voltage so that the 4-volt variation covered approximately the full screen of the oscilloscope which is being used as a display instrument. The oscilloscope has a 10 cm high display screen, and it might be decided to use 8 cm of this to display the 4-volt variation, at 0.5 volts per cm. Remembering, however, that the 4-volt variation was from 14 to 18 volts, it can be concluded that if the sensitivity of the oscilloscope is set at 0.5 volts per cm, the variation will be from 28 cm to 36 cm ---an obvious impossibility on the oscilloscope having a maximum display range of only 10 cm! The result is that it would not be possible to make the display appear on the screen of the oscilloscope, hence no measurement could be obtained.

From this example it can be concluded that one could easily amplify the 4-volt variation to the size wished on the oscilloscope if the 4-volt variation was closer to the zero level, say from 0 volts to 4 volts, or from -2 volts to +2 volts. This can be done by using the potentiometer in a bridge circuit as shown in figure 2.2.1.4. This figure shows a typical potentiometer transducer circuit, and although the potentiometer is shown as formed in a straight line, the circuit applies to the rotary potentiometer as well.



POTENTIOMETER TRANSDUCER CIRCUIT
FIGURE 2.2.1.4

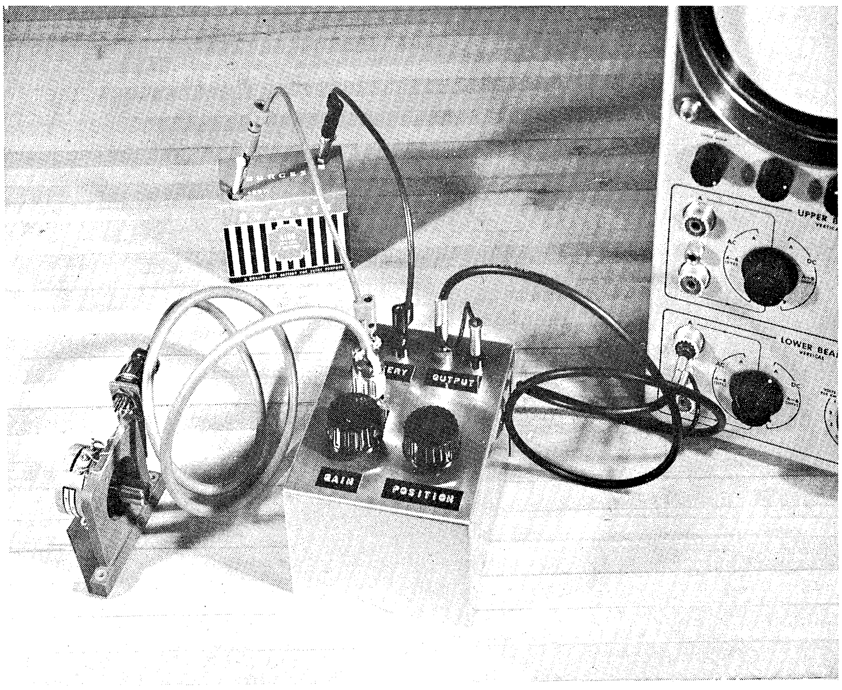
Referring to figure 2.2.1.4 it will be noticed that a variable resistor, labelled "position resistor", has been added to form the bridge circuit, the adjustable contact of the position resistor is grounded to establish this as a point of zero voltage, and that the no-load output voltage e_o' is read between the movable contact of the potentiometer and the adjustable contact of the position resistor. The adjustable contact of the position resistor is varied by hand, and does not move with the potentiometer contact, hence the position resistor can be manually adjusted until its contact is in a position which approximately corresponds with the mid-point of the potentiometer motion on the circuit diagram. Thus when the potentiometer moves with the mechanical device the bridge will be operating near the balanced position as explained in section 1.2.3, and the output voltage will be close to zero.

If in the preceding example it is assumed that 16 volts is the mean of the 14 to 18 volt variation shown in figure 2.2.1.3, then the position resistor should be set at approximately 16/22.5 x (total resistance of position resistor) to have the output voltage vary from -2 to +2 volts.

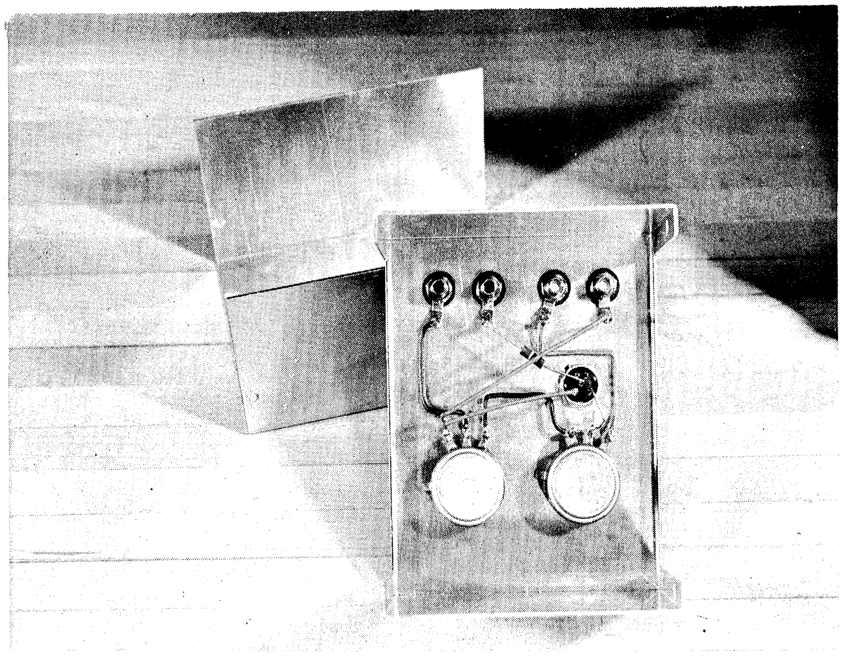
In actual operation it is seldom necessary to establish the setting of the position resistor as has just been illustrated. Instead, the position resistor is varied until the transducer output voltage can be displayed or recorded on the instrument being used for this purpose, and no attention need be paid to the exact setting which results from procedure. Since adjusting the position resistor changes the point where zero volts occurs in the circuit, it follows that this adjusts the position of the output trace on the display or recording instrument, and it is because of this that it is called a "position" resistor.

The circuit diagram of figure 2.2.1.4 also shows a gain resistor and a protective resistor. The purpose of the adjustable gain resistor is to allow the voltage across the bridge to be varied, thus permitting the establishment of a convenient and easily used proportionality factor between displacement of the potentiometer contact and output voltage. The protective resistor prevents the movable contact of the potentiometer from being burned off in case the movable contacts of the potentiometer, position resistor, and gain resistor all happen to be placed so as to allow what would otherwise be a short circuit across the battery.

A potentiometer transducer with a laboratory constructed circuit box is shown in figure 2.2.1.5, and the interior of the circuit box is shown in figure 2.2.1.6. The potentiometer has been mounted onto a hanger bracket which also supports a three-pin socket connected to the two ends of the resistor and to the movable brush. A three-wire cable connects the socket to a similar socket in the circuit box. The variable resistors used for gain and position are one half watt radio volume control resistors. Banana jacks, an aluminum chassis box, and patch cords complete the assembly.

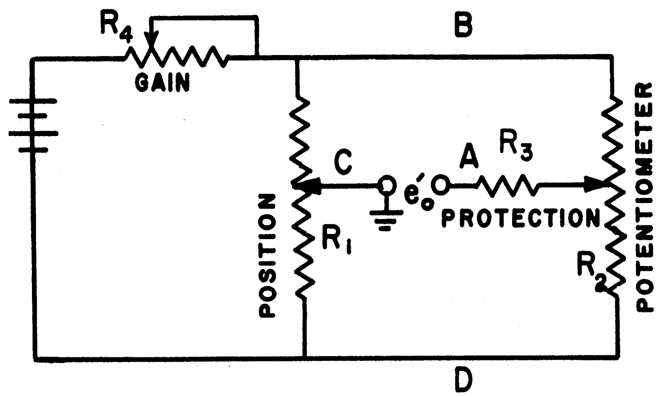


POTENTIOMETER TRANSDUCER
FIGURE 2.2.1.5



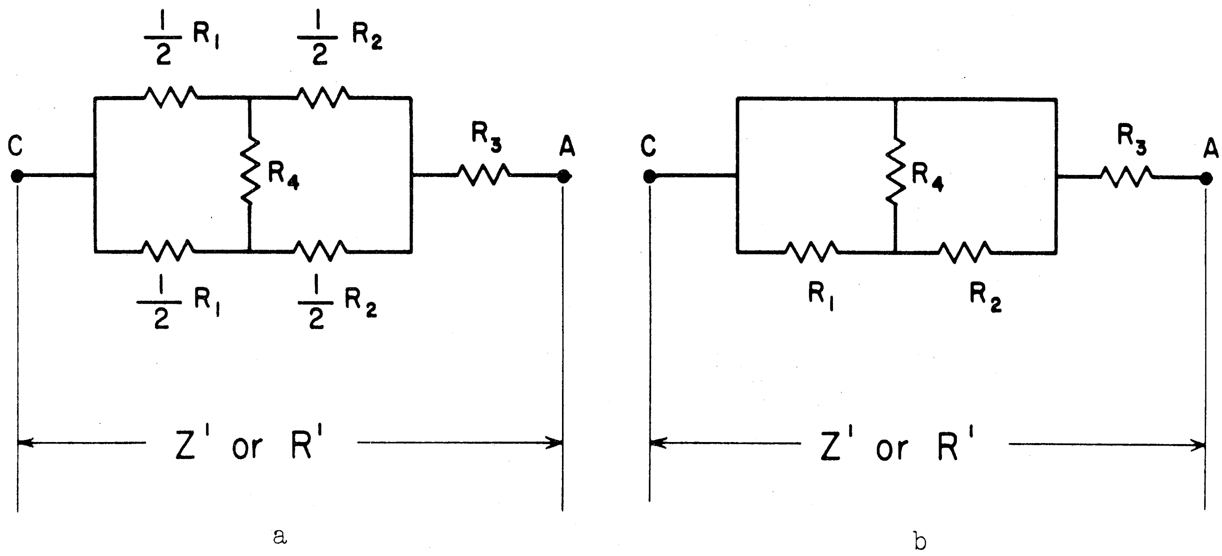
POTENTIOMETER CIRCUIT BOX
FIGURE 2.2.1.6

2.2.2 <u>IMPEDANCE OF THE POTENTIOMETER TRANSDUCER.</u> The source impedance of the potentiometer transducer circuit is the impedance from A to C with voltage source short circuited as shown in figure 2.2.2.1.



CIRCUIT TO DETERMINE THEVENIN SOURCE RESISTANCE
FOR POTENTIOMETER TRANSDUCER
FIGURE 2.2.2.1

The impedance is pure resistance, and will vary as the position resistor is adjusted, and as the potentiometer contact moves during operation. The equivalent circuits to determine the maximum and minimum values of the source impedance are shown in figure 2.2.2.2 a and b. Maximum impedance occurs when both the position resistor and



MAXIMUM IMPEDANCE

MINIMUM IMPEDANCE

EQUIVALENT CIRCUITS FOR SOURCE RESISTANCE
OF THE POTENTIOMETER TRANSDUCER
FIGURE 2.2.2.2

the potentiometer are in their mid-positions, and from figure 2.2.2.2 a this will be:

$$Z'_{max} = R'_{max} = \frac{1}{4}(R_1 + R_2) + R_3 \qquad (2.2.2.1)$$

where:

R_1 = position resistance, ohms

R_2 = potentiometer resistance, ohms

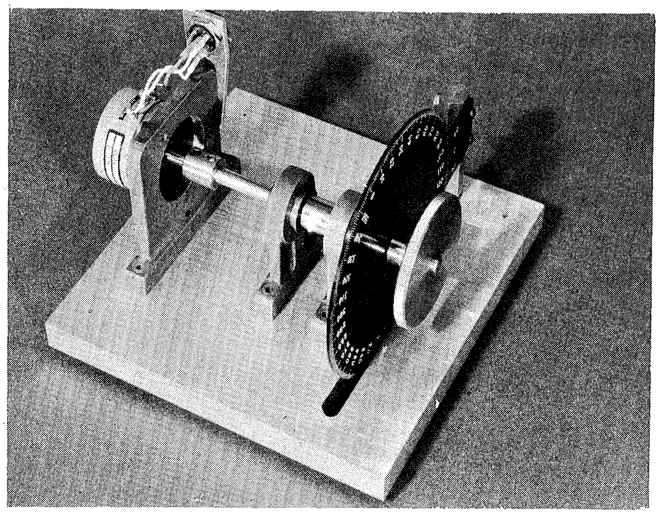
R_3 = protection resistance, ohms

Minimum impedance occurs when both the position resistor and the potentiometer are at one extreme, and from figure 2.2.2.2 b:

$$Z'_{min} = R'_{min} = R_3 \qquad (2.2.2.2)$$

The source impedance of the typical potentiometer circuit of figure 2.2.1.4 could vary from a maximum of 17.5 K ohms to a minimum of 10 K ohms. The effects of this impedance variation on the output voltage was discussed in section 1.5, and Example 1-3 on page 16 illustrated the distortion problem encountered when the potentiometer transducer is used with a low impedance display instrument. However, potentiometers having a lower resistance than that shown in figure 2.2.1.4 are available, and could be used with a lower position resistor and with the protection resistor eliminated when a low impedance display or recording instrument must be used.

2.2.3 CALIBRATION OF THE POTENTIOMETER TRANSDUCER. The purpose of the calibration is to accurately establish the relationship, or proportionality constant, between the displacement of the movable contact and the output voltage. The potentiometer can be calibrated before it is attached to the mechanical device on which the measurements are to be made. The calibration can be carried out by mounting the potentiometer on some sort of calibrating stand which would accurately indicate degrees of rotation for the rotary potentiometers, or inches of travel for the linear potentiometers. A rotary potentiometer is shown on a calibrating stand in figure 2.2.3.1.



ROTARY POTENTIOMETER ON CALIBRATION STAND
FIGURE 2.2.3.1

However, since the potentiometer in a circuit with components having proper impedances always produces a constant or linear relationship between displacement and output voltage, it is generally only necessary to establish this relationship for one given displacement and the corresponding change in voltage. For example, if we assume for a moment that the circular resistance element of the rotary potentiometer covers a full 360 degrees, calibration can be accomplished by simply rotating the movable contact through one complete revolution and noting the corresponding change in voltage. Simple division provides the volts per degree, and this can be adjusted by the gain resistor to obtain a convenient value to use. The voltage for a full revolution can also be determined without rotating the shaft by merely measuring the voltage drop across the total potentiometer resistance. In figure 2.2.1.4 this would correspond to the voltage from B to D.

Actual construction of the rotary potentiometer requires a small gap between the ends of the circular resistance element. This gap and its effect on the output voltage are shown in figure 2.2.1.2. Most potentiometers have a gap of 4 to 8 degrees. It follows that if the gap is measured, or otherwise known, the above procedure can still be used, taking 360 degrees minus the gap as the displacement corresponding to the noted change in voltage.

If the rotary potentiometer is connected to a rotating shaft it can be easily calibrated during operation, for the total voltage variation always corresponds to 360 degrees minus the gap.

A method similar to that described can be used for linear potentiometers once the total length of the resistance element is accurately determined, since the voltage variation across the potentiometer will always correspond to the length of the resistance element.

In practice it is convenient to mark the total displacement of the potentiometer on its exterior for future calibration, whether it be a rotary potentiometer or a linear potentiometer.

2.2.4 <u>ADVANTAGES AND DISADVANTAGES OF THE POTENTIOMETER TRANSDUCER</u>. The potentiometer is a simple and rugged transducer which can be used for a variety of applications. It can measure large displacements, the linear type with up to 6 inches of travel being readily available, while the rotary type can measure almost a full 360 degrees. The output voltage is well above the general level of most electric noise, and is large enough to drive almost any type of display or recording instrument.

Since the resistance wire is wound in a coil, displacement of the contact changes the resistance in steps corresponding to the length of one loop of the coil. Because of this the potentiometer is not satisfactory for measuring small displacements, for the output then appears as a series of steps instead of as a smooth line. The smallest wire sizes will result in incremental steps of between 0.001 and 0.002 inches. Incidentally, if the displacement per step is accurately measured, the potentiometer can be used to indicate velocity by displaying the steps in such a way that the time between steps can be noted. Average velocity between the steps can then be calculated.

Life of the potentiometer is limited by wear of the contact and the wire, and a badly worn potentiometer becomes noisy. The wear rate is considerably increased, and the life expectancy therefore decreased, by oscillatory or reciprocating motion as compared to rotary motion. Under good conditions the life expectancy of a typical rotary potentiometer is apt to be about one million revolutions.

The source impedance varies with the motion of the potentiometer contact, hence it is essential that the potentiometer be used with a display or recording instrument having a large enough input or load impedance so that distortion due to impedance loading is kept within reasonable limits.

The frequency response for reciprocating or oscillating motion is somewhat limited by the physical ability to transmit enough force through the coupling to overcome the friction and inertia of the potentiometer. High sliding velocities of the contact on the wires tend to generate noise, and this also tends to limit the useful frequency response. Experience shows that the noise problem tends to become troublesome when the sliding velocity exceeds about 10 ft/sec.

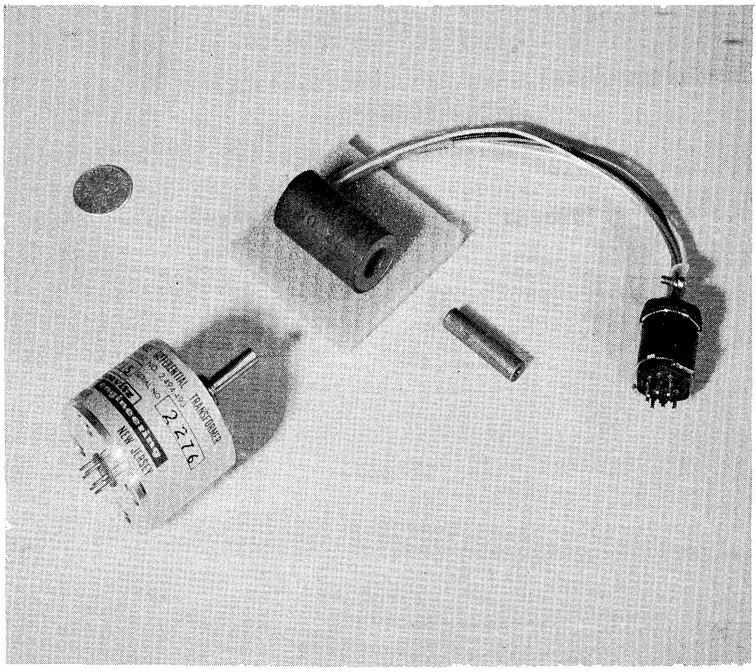
Finally, the potentiometer must be rigidly attached to the moving part on which the measurements are to be made, with its own inertia and friction added to that of the moving part. In this way it always causes some mechanical loading.

2.3 DIFFERENTIAL TRANSFORMER

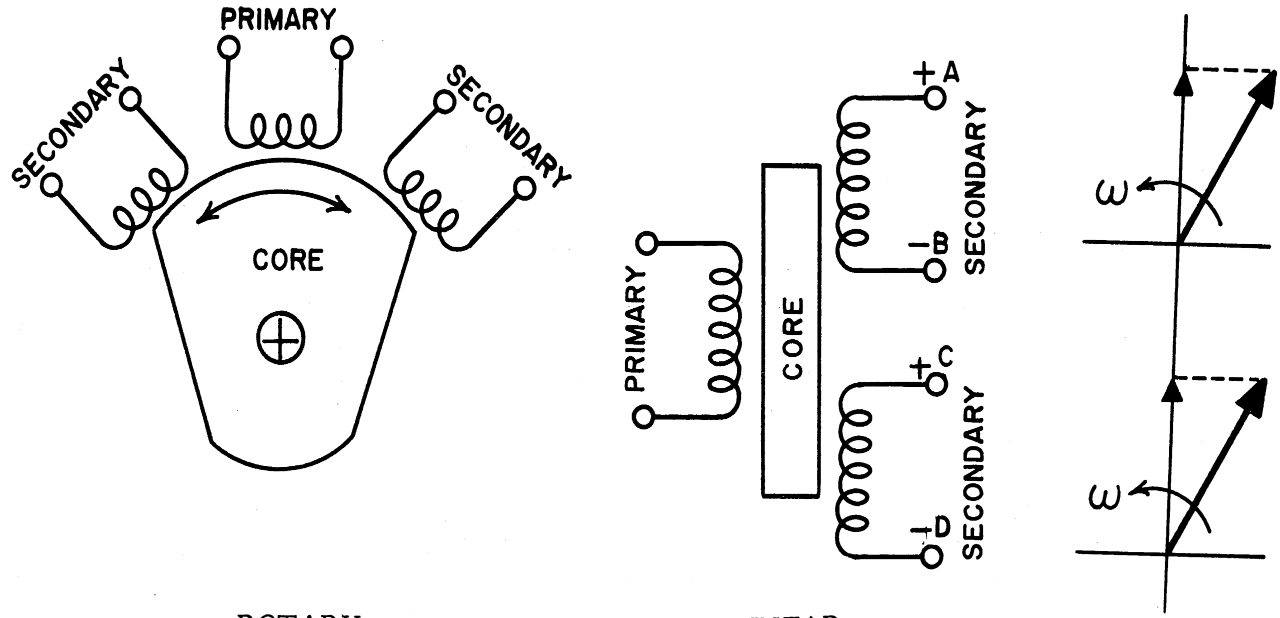
2.3.1 PRINCIPLE AND CIRCUITS OF THE DIFFERENTIAL TRANSFORMER. The differential transformer is a displacement transducer, and depending upon its construction can be used to measure either linear or angular displacement. Figure 2.3.1.1 shows both types of differential transformers.

The schematic arrangements of these two types of differential transformers are shown in figure 2.3.1.2. In either construction, a soft iron core provides a magnetic coupling, and when the core is centrally positioned, essentially identical alternating voltages are induced in the two secondary windings when an alternating voltage is applied to the primary.

Since these induced alternating voltages are sinusoidal, they can each be represented by the projection of a rotating vector on a vertical axis as shown in part (c) of figure 2.3.1.2.



DIFFERENTIAL TRANSFORMERS
FIGURE 2.3.1.1



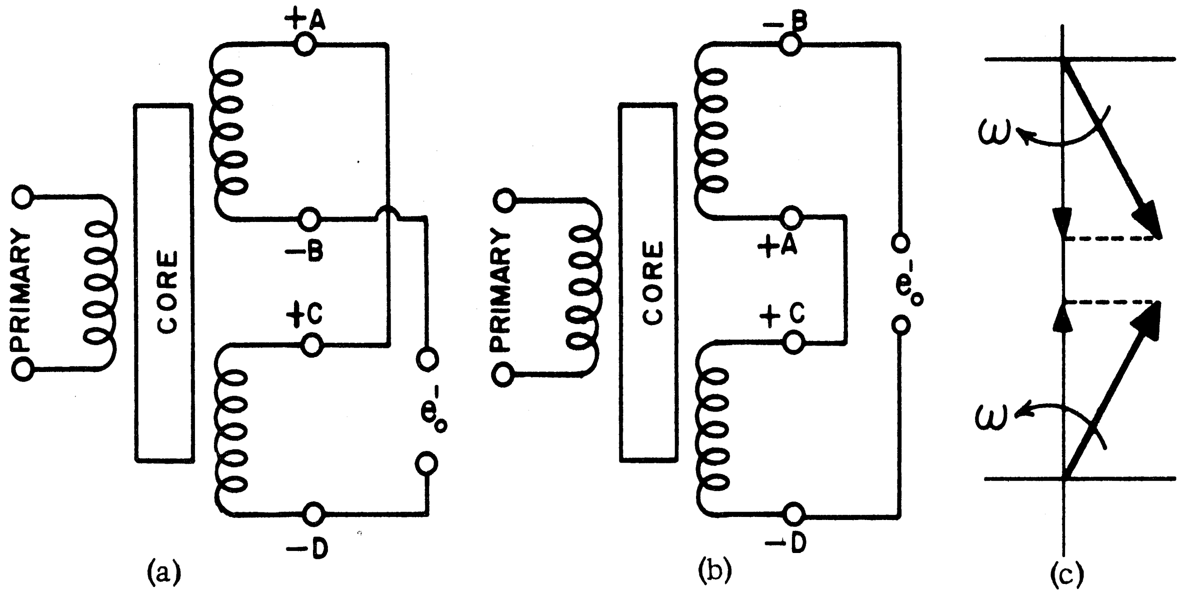
ROTARY
(a)

LINEAR
(b)

(c)

DIFFERENTIAL TRANSFORMER SCHEMATICS
FIGURE 2.3.1.2

Suppose now that the ends of the secondary windings are connected as shown in figure 2.3.1.3 a, which is schematically the same as in figure 2.3.1.3 b. The projections of the rotating voltage vectors now oppose each other, and the no-load output voltage e_o' will be the difference of these two projections. It is from this feature that the device gets the name differential transformer.

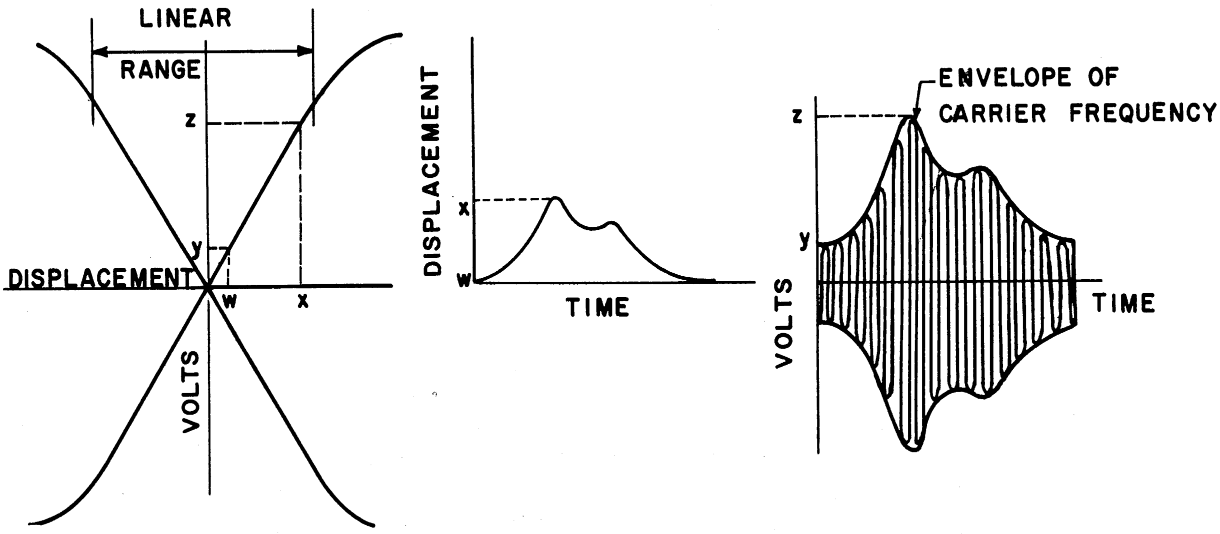


DIFFERENTIAL TRANSFORMER CIRCUIT
FIGURE 2.3.1.3

When the voltages in the two secondary windings are equal, the no-load output voltages will be zero. This condition exists when the core occupies a central, or null, position and so equally favors each secondary winding. However, if the core is displaced from the null position, it will weaken one primary to secondary coupling and strengthen the other, thus decreasing the induced voltage in one of the secondary windings while increasing it in the other secondary winding. This has the effect of shortening one of the rotating vectors and lengthening the other, and the no-load output voltage will have an amplitude proportional to the displacement of the core from the null position.

It follows that to use the differential transformer as a displacement transducer, the core must be attached to the moving part on which the displacement measurements are to be made, and the windings connected as shown in figure 2.3.1.3 a. An audio-frequency oscillator can be used as an exciter for the primary winding.

The differential output voltage from the secondary windings will have the same frequency as that supplied to the primary and its amplitude will be proportional to the displacement of the core from the null position and to the amplitude of the primary excitation. When the differential transformer is used to measure displacement, the output voltage will appear as an envelope of the exciter, or carrier, frequency. Figure 2.3.1.4 a shows the variation of the no-load output voltage envelope with displacement of the core from the null position. It will be noticed that this voltage-displacement relation is linear for only a short distance on either side of the null position. Most rotary differential transformers are linear for ranges from 10 to 40 degrees on either side of the null.

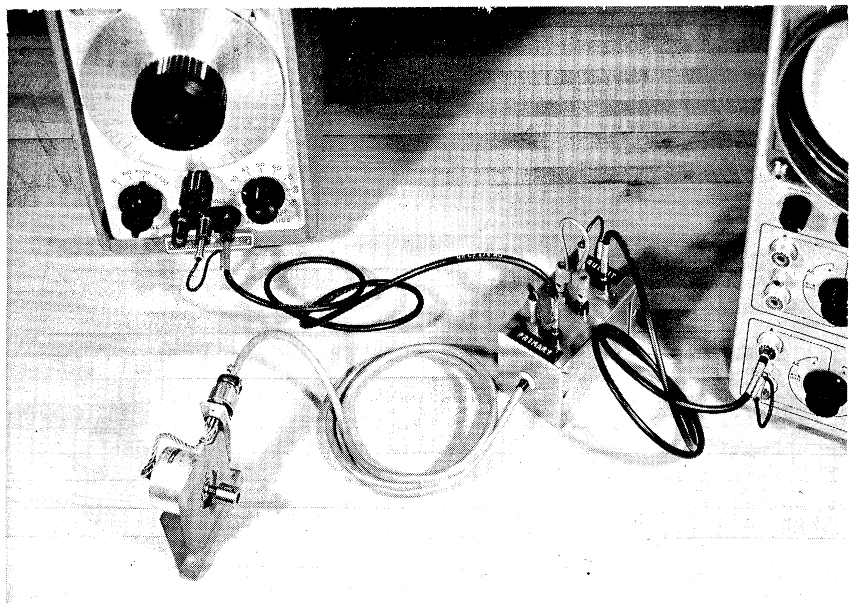


VOLTS VS. DISPLACEMENT (a) | DISPLACEMENT VS. TIME (b) | VOLTS VS. TIME (c)

DIFFERENTIAL TRANSFORMER OUTPUT
FIGURE 2.3.1.4

In actual operation the two secondary windings will not be exactly identical, hence the null point will seldom be as sharply defined as is indicated in figure 2.3.1.4 a. For this reason it is always advisable to use the differential transformer in such a way that the core does not too closely approach the null position.

The appearance of the transformer output is also shown in figure 2.3.1.4, where (b) represents a possible motion of the core between points w and x on (a). The output voltage varies between y and z in proportion to the core displacement, as shown in (c), and the top edge of the carrier frequency envelope represents the displacement of the core.



DIFFERENTIAL TRANSFORMER TRANSDUCER
FIGURE 2.3.1.5

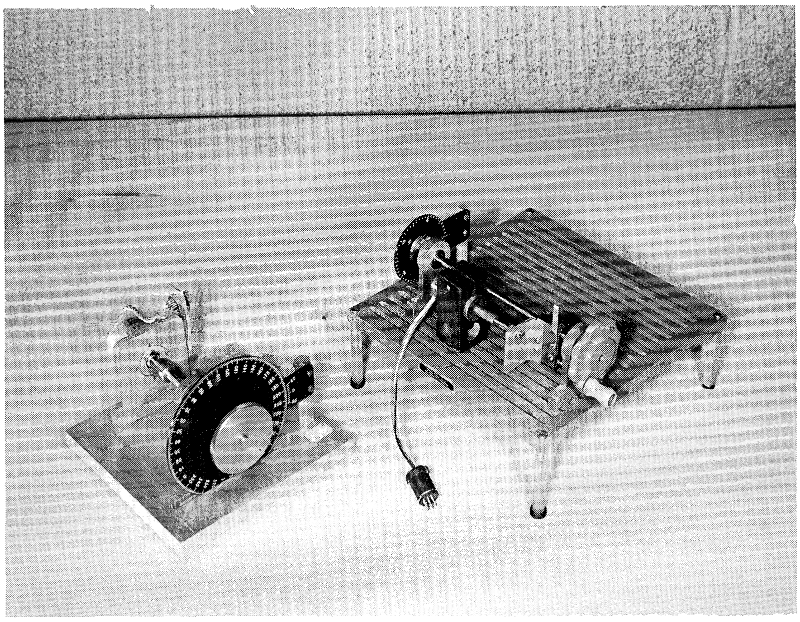
A differential transformer with an audio-frequency oscillator to provide the primary excitation, and with a laboratory build connection box is shown in figure 2.3.1.5. The transformer is mounted onto a hanger bracket which supports a six-pin socket connected to the ends of the three windings. A 6-wire cable with plug connects the socket, and hence the transformer windings to the 6 banana jacks on the box. Connections to the oscillator, between windings, and to the display instrument are made at the connection box with patch cords.

2.3.2 IMPEDANCE OF THE DIFFERENTIAL TRANSFORMER. The source impedance analysis of the differential transformer circuit is rather complicated and is not presented here. However, the source impedance is essentially pure resistance when the recommended exciting frequency is used, and it is approximately equal to the resistance of the secondary coils. The value of this resistance is generally made known by the manufacturer of the transformer, and is often in the neighborhood of 1000 ohms.

2.3.3 CALIBRATION OF THE DIFFERENTIAL TRANSFORMER. Both the linear and rotary differential transformers can be calibrated before mounting in place and attaching to the moving part on which measurements are to be made. However, due to the difficulty of locating the core relative to the windings, the linear type must often be calibrated in place, using a dial indicator or similar device.

Calibration of either type can be accomplished by mounting it on a suitable stand which has a means of providing andindicating displacement, as shown in figure 2.3.3.1. While the transformer core is displaced known amounts on the calibration stand, the output voltage is read on the display or recording instrument.

The proportionality constant between core displacement and amplitude of output voltage is adjusted by varying the excitation amplitude at the oscillator. However, the primary excitation should not exceed the maximum recommended by the manufacturer.



CALIBRATION STANDS
FIGURE 2.3.3.1

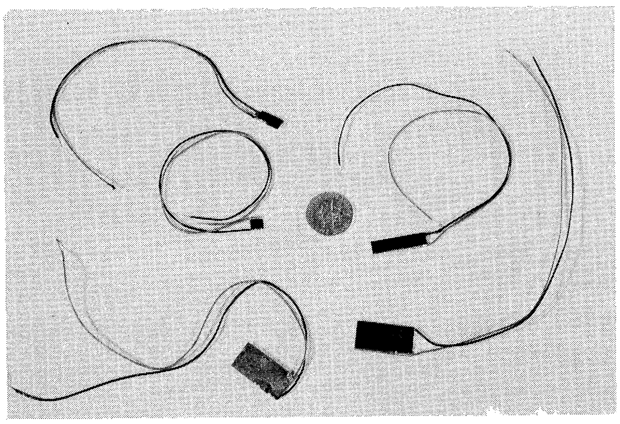
The frequency of the primary excitation, or carrier frequency, can also be adjusted at the oscillator. The exact value of the carrier frequency is seldom of any importance, so long as it is high enough to establish a reasonably smooth envelope without exceeding the operating capabilities of the transformer. However, the amplitude of the transformer output will generally vary with the carrier frequency, hence it is important that the carrier frequency be set before calibration, and not changed after calibration has been completed. Frequency recommendations are also supplied by the transformer manufacturer.

2.3.4 ADVANTAGES AND DISADVANTAGES OF THE DIFFERENTIAL TRANSFORMER TRANSDUCER. The differential transformer is best suited for measuring small displacements because of its limited range of linearity. It can readily measure linear displacements of a few ten-thousandths of an inch. It has an alternating output voltage which must be de-modulated if a direct current signal is needed. Its output voltage is generally high enough to be well above common noise and its output is infinitely variable within the limits imposed by the carrier frequency. Since there is no contact between the moving parts its life is not limited by mechanical wear.

The frequency response is limited by the carrier frequency, and by the ability to move the iron core at high frequency. The core must be attached to the moving part on which the measurements are to be made, adding the inertia of the core to the moving part. Providing a proper physical mounting for the linear displacement transformer on the mechanical device is often troublesome.

2.4 SOLAR CELLS

2.4.1 PRINCIPLE AND CIRCUITS OF THE SOLAR CELL. Solar cells are silicon diodes which can utilize a beam of light to measure either displacement or velocity, depending upon the circuitry and physical set-up with which they are used. Solar cells are formed by fusing layers of P and N type silicon to form a P-N junction semi-conductor. In physical appearance the solar cell resembles a rectangular piece of stiff paper about 0.025" thick, black on the positive side and silver on the negative side, with a red and black lead wire. A collection of solar cells is shown in figure 2.4.1.1



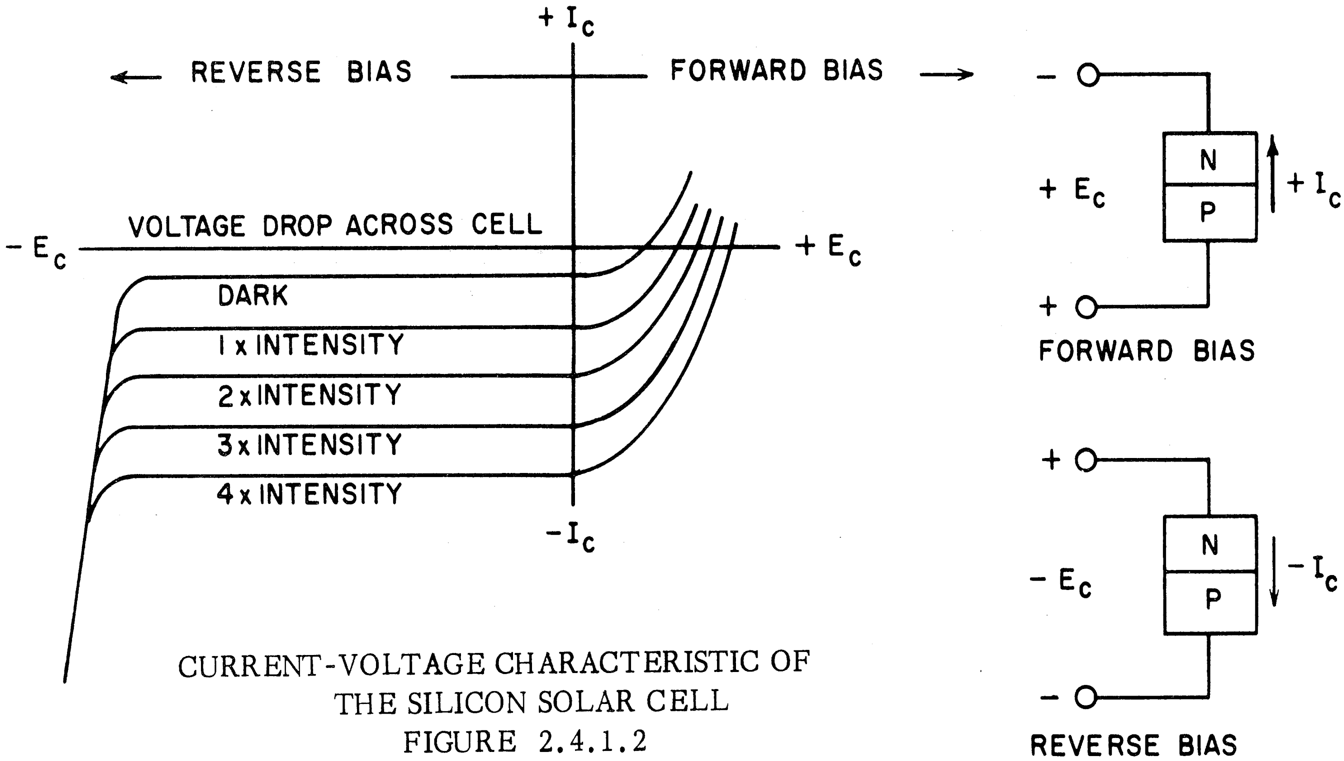
SILICON SOLAR CELLS
FIGURE 2.4.1.1

The silicon solar cell exhibits the non-linear current- voltage characteristics typical of diodes. If energy is supplied to the P-N junction the current-voltage characteristic will be altered. In the case of the solar cell, energy will be supplied in the form of light, and since the junction area is as large as the whole cell itself, a substantial change in the current—voltage characteristics can be caused by the light. This feature of the solar cell is utilized to make it into a displacement or velocity transducer.

When illuminated the cell also acts as a solar battery having a high internal resistance, and can thus be thought of as a current generator which gets its energy from the light. This feature will help to explain some of the characteristics of the solar cell, although it is not the characteristic to be exploited in the transducer circuit to follow.

When used as a displacement transducer the solar cell is mounted so that the mechanical part on which the displacement is to be measured passes between the cell and its light source. The mechanical part will thus shadow a portion of the cell, and if the motion of the shadow is the same as the motion of the part, the area of the cell which is illuminated will vary directly with the displacement of the mechanical part. It is necessary to establish a circuit which will develop a voltage proportional to the illuminated area of the cell.

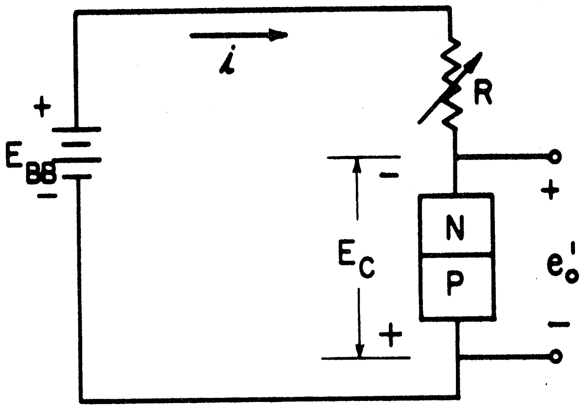
Figure 2.4.1.2 shows the current-voltage characteristics of the solar cell when connected in a forward-bias and in a reverse-bias circuit, and illuminated with various intensities. Notice that in figure 2.4.1.2 voltage and current are considered to be positive when current flows from P to N in the cell, and negative when current flows from N to P.



CURRENT-VOLTAGE CHARACTERISTIC OF THE SILICON SOLAR CELL
FIGURE 2.4.1.2

The lines of figure 2.4.1.2 are parallel and have essentially zero slope for that part of the plot where the diode has a reverse bias, and in this region the current flowing through the cell at any given voltage is proportional to the intensity of the light.

The current flowing through the cell at any given combination of voltage and light intensity is also proportional to the area of the cell which is illuminated, and while this is not shown by figure 2.4.1.2 it is characteristic which shall later be utilized.



SOLAR CELL CIRCUIT

FIGURE 2.4.1.3

Figure 2.4.1.3 shows a battery and a reverse biased solar cell connected in series with a resistance load. It follows that the current-voltage relation at the solar cell must not only satisfy the curves of figure 2.4.1.2, but must also satisfy the requirements of this circuit. Applying Kirchoff's law of voltages:

$$E_{BB} - iR - e_o' = 0 \qquad (2.4.1.1)$$

and therefore

$$e_o' = E_{BB} - iR \qquad (2.4.1.2)$$

where

e_o' = no-load output voltage across the solar cell.

In the circuit diagram of figure 2.4.1.3, the current and voltage are considered to be positive or negative in terms of the battery polarity, rather than in terms of solar cell polarity as was done in figure 2.4.1.2. By comparing the two notations:

$$e_o' = -E_c \qquad (2.4.1.3)$$

$$i = -I_c \qquad (2.4.1.4)$$

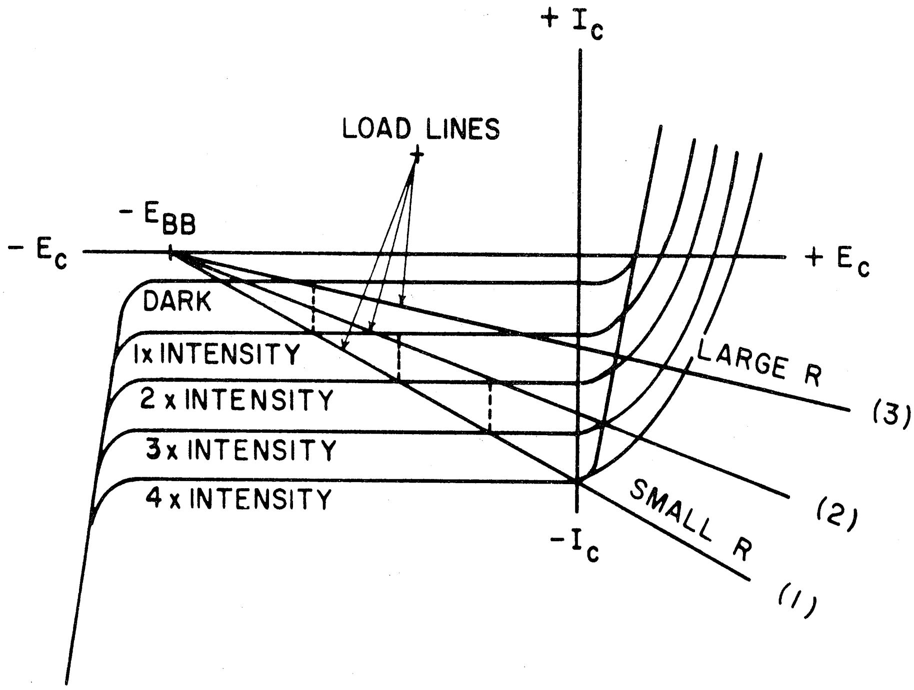
where e_o' = no load output volts across solar cell in circuit, figure 2.4.1.3

$-E_c$ = reverse bias voltage drop across solar cell, figure 2.4.1.2

i = circuit current, figure 2.4.1.3

$-I_c$ = reverse bias current, figure 2.4.1.2

For any given value of E_{BB} and R in equation 2.4.1.2, a straight line results when current i is plotted against the no-load output voltage e_o'. Making the substitutions indicated by equations 2.4.1.3 and 2.4.1.4, three such lines are shown in figure 2.4.1.4, super-imposed upon the characteristic curves of figure 2.4.1.2. The three straight lines are called load lines, and would be obtained by using three different values of R in the circuit.



SOLAR CELL CHARACTERISTICS AND LOAD LINES
FIGURE 2.4.1.4

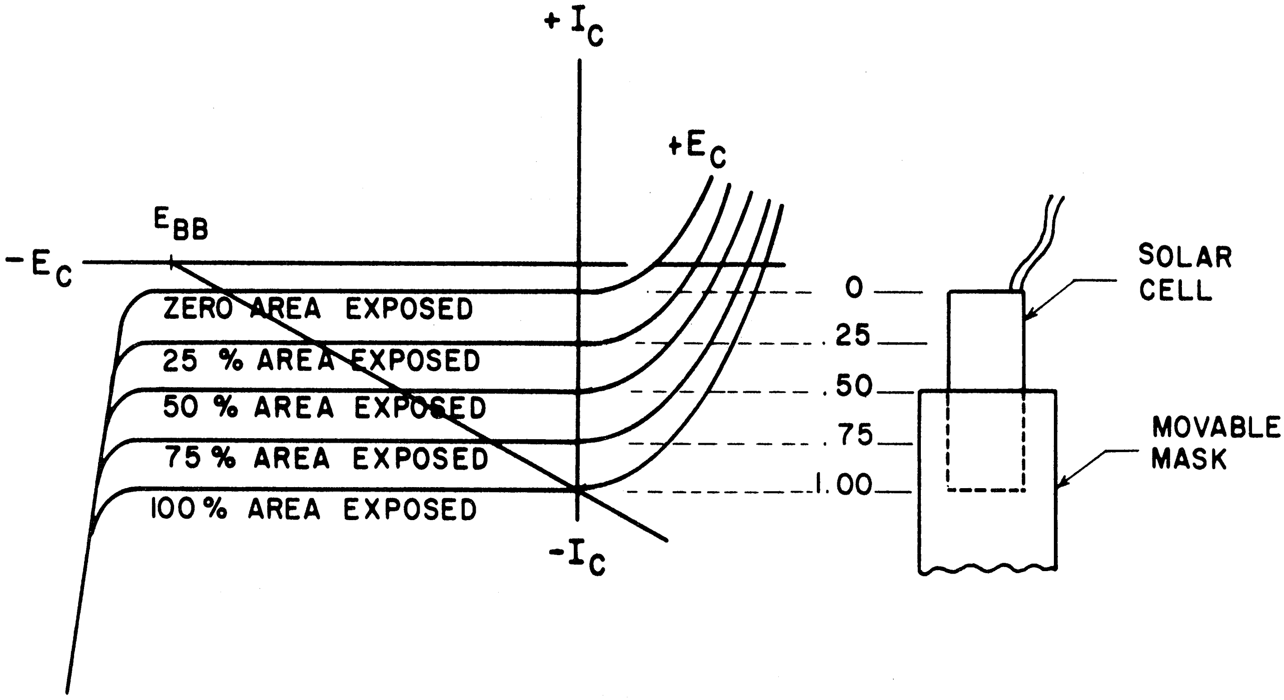
A load line shows the current-voltage relationship for the circuit, while the characteristic curves show the current-voltage relationship for the solar cell at various illumination levels. Since the solar cell is in the circuit, it must simultaneously satisfy the load line characteristic and its own characteristic, hence the intersections of the load line and the characteristic curves establish the operating points for the cell in terms of E_c and I_c for any given combination of circuit resistance R and light intensity on the cell.

If the light intensity is caused to vary, then the solar cell characteristic will correspondly be changed, and the intersection point will move. The operating point will shift up and down the fixed load line as the illumination varies.

If the region in which the operating point shifts up and down is located in the lower left quadrant where the characteristic curves are essentially parallel lines, then the solar cell voltage variations, E_C, will be linearly related to the light intensity. If, however, the region is in the lower right quadrant where the characteristic lines are no longer parallel, then the voltage variations of the solar cell will not be proportional to changes in light intensity.

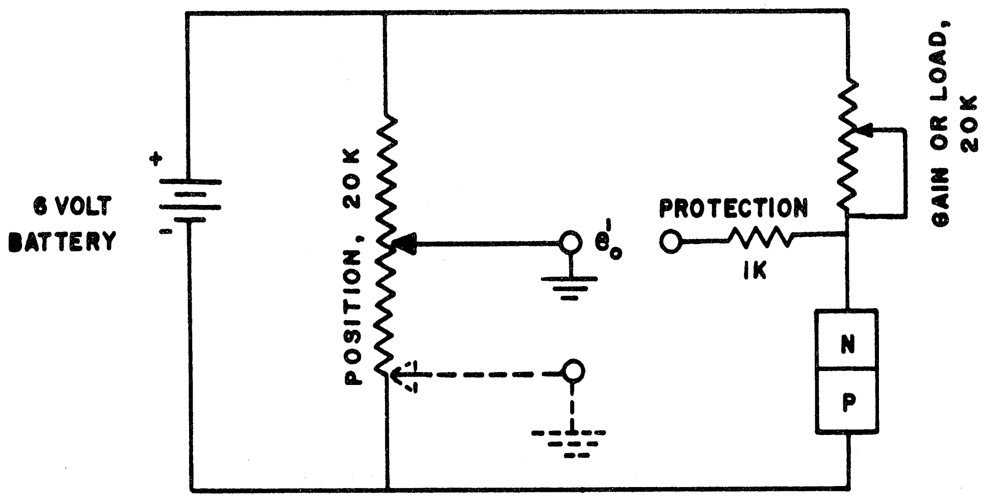
Consider a circuit having the steepest load line of figure 2.4.1.4. If the intensity of illumination on the cell is increased by the increments shown, it follows that the current will increase in direct proportion to the intensity as the current-voltage relation of the circuit moves along the load line to the corresponding intersections. However, due to the constant slope of the load line, the voltage drop across the cell decreases in direct proportion as the intensity increases.

It was previously stated that at any given voltage the current shown in the characteristic curves of figure 2.4.1.2 was proportional to the cell area exposed to light at a uniform intensity. Thus it follows that keeping the illuminated area constant and increasing the intensity provides the same results as keeping the intensity constant and increasing the area. Figure 2.4.1.4 is re-drawn as shown in figure 2.4.1.5, from which it can be seen that the voltage drop across the cell, for any given illumination, will be proportional to the illuminated area, as long as the load line of the circuit intersects the intensity or area lines on the left side of the vertical ordinate of figure 2.4.1.5. Thus we see that the circuit of figure 2.4.1.3 can be used to make the solar cell into a displacement transducer, since the illuminated area is directly proportional to the displacement of the mask in figure 2.4.1.5.



SOLAR CELL CHARACTERISTICS WITH VARIABLE AREA
FIGURE 2.4.1.5

To be used as a transducer, the simple circuit of figure 2.4.1.3 is expanded to the bridge circuit shown in figure 2.4.1.6 with typical resistance values. A variable resistor has been used for the load or gain, and a variable position resistor and a fixed protection resistor



SOLAR CELL DISPLACEMENT TRANSDUCER CIRCUIT
FIGURE 2.4.1.6

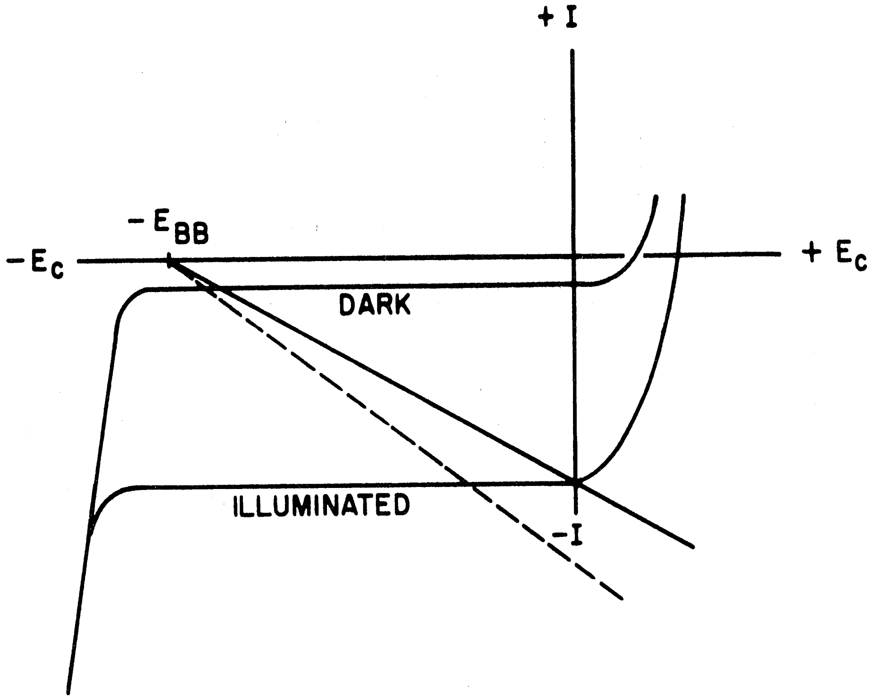
have been added. The functions of the position and protection resistors are the same as in the potentiometer transducer circuit which is described in detail in section 2.2.1 of this chapter.

To obtain a linear relation between displacement, or illuminated area, and no-load output voltage, it is essential that the gain or load resistance be such that the load line will intersect the characteristic lines in the lower left hand quadrant of figure 2.4.1.5. To establish this linear relation without the necessity of actually knowing the light intensity, or the value of the load resistance, the following procedure is followed:

1. Set the load or gain resistor so that the maximum resistance is in the circuit.

2. Set the position resistor as shown by the dashed lines of figure 2.4.1.6.

3. Keep the entire area of the cell uniformly illuminated with any arbitrary intensity and assume this intensity is represented by the illuminated current-voltage curve of figure 2.4.1.7.



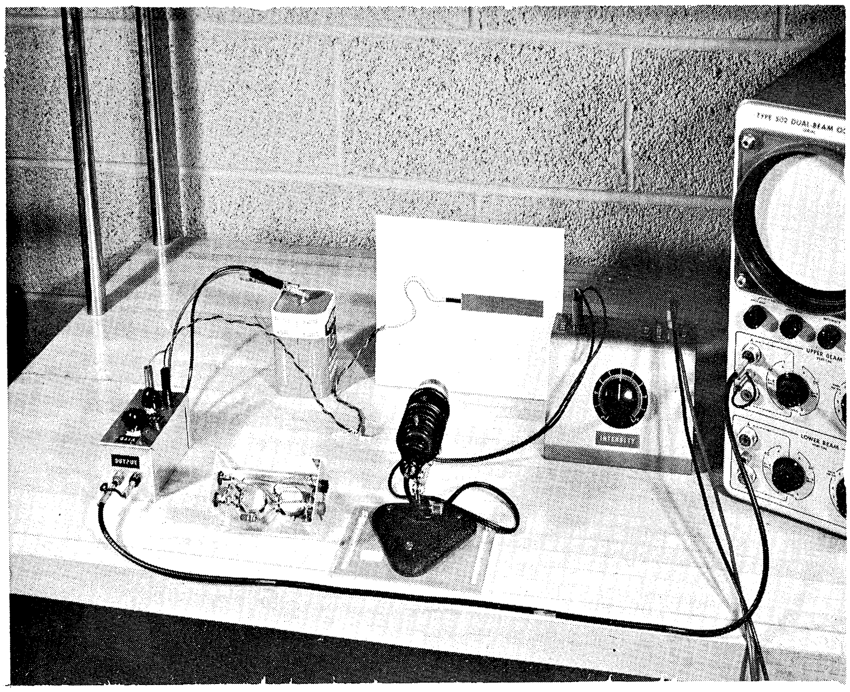
LINEAR RANGE OF SOLAR CELL DISPLACEMENT TRANSDUCER
FIGURE 2.4.1.7

4. Reduce the load resistance until the output voltage is zero. The circuit is now operating at the condition indicated by the intersection of the load line and the illuminated current-voltage curve of figure 2.4.1.7. Since this has been obtained with the whole area of the cell illuminated, all possible current-voltage lines corresponding to smaller illuminated areas will intersect the load line to the left of the vertical ordinate, and should provide a linear relation between output volts and illuminated area.

5. It is often wise to reduce the load resistance slightly further to better insure linearity, as indicated by the dashed line of figure 2.4.1.7. This tends to improve linearity, but reduces the output voltage, so should not be overdone.

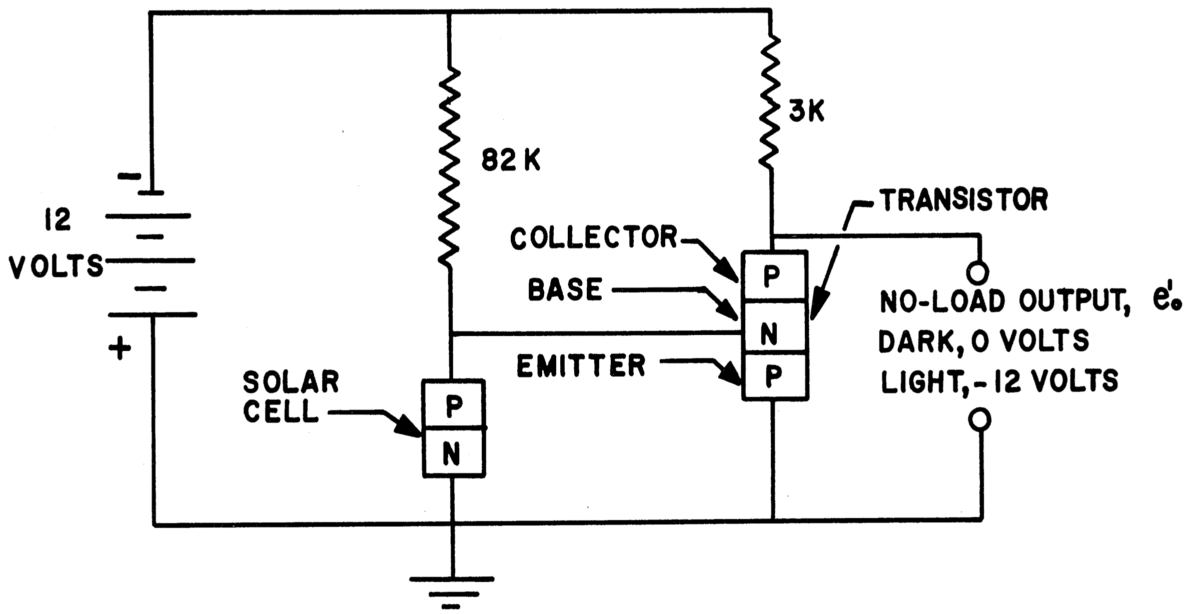
After completing the above procedure the position resistor can be adjusted as necessary to allow the output voltage to be displayed or recorded. The final check on the linearity is obtained by calibration as explained in section 2.4.3.

Figure 2.4.1.8 shows a solar cell displacement transducer with a laboratory constructed circuit box. The interior of a circuit box is also shown. The variable position and gain or load resistances are one-half watt ratio volume control resistors. The aluminum chassis box holds black and red banana jacks for the solar cell leads, battery connections, and output to the display instrument. Because of the importance of polarity in this circuit, the red jacks indicate



SOLAR CELL DISPLACEMENT TRANSDUCER
FIGURE 2.4.1.8

positive potential, while the black jacks indicate negative potential and ground. A microscope light powered by a 6-volt automobile battery through a 2 ohm, 25 watt variable resistor as an intensity control is used as a light source. To obtain a reasonably uniform intensity on the surface of the cell, the light must be placed quite far back from the cell, say 10" or so, and the focus blurred. A rather low level of intensity generally gives best results.



SOLAR CELL TRIGGER CIRCUIT
FIGURE 2.4.1.9

Solar cells can also be made into a velocity transducer by the use of two or more of the trigger type of circuits shown in figure 2.4.1.9, in which the solar cell acts as a trigger to switch the PNP transistor. The action of the circuit will be explained first, and it shall then be shown how it is used as a velocity transducer.

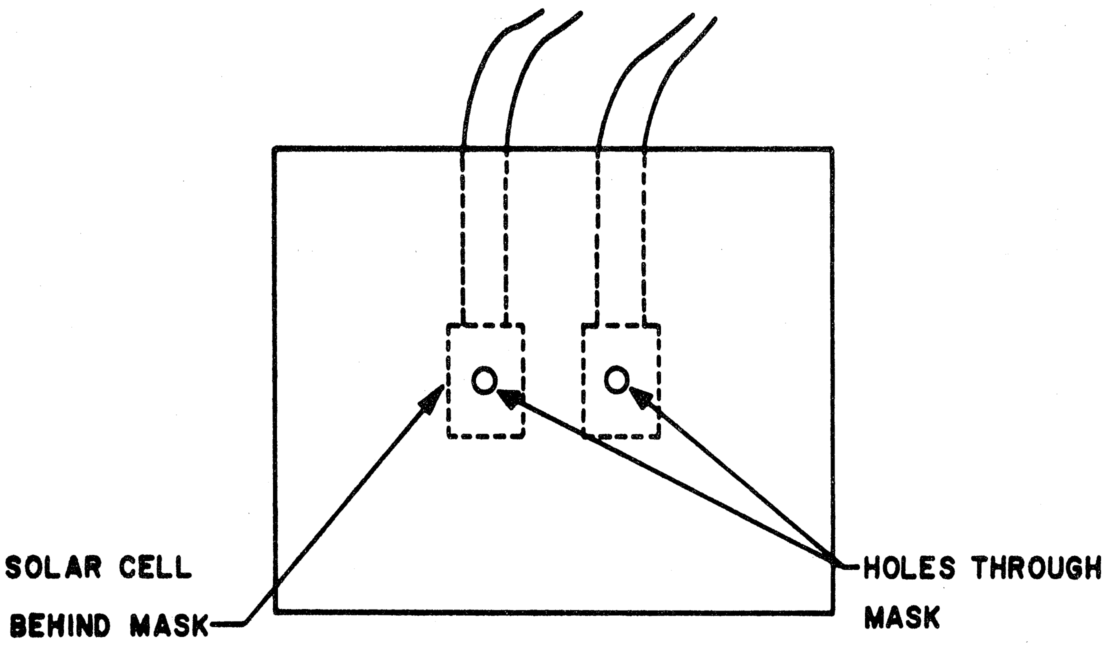
In that region of figure 2.4.1.2 where the lines are parallel and have essentially zero slope it might be considered that illuminating the solar cell has somewhat the same effect as reducing the resistance of the cell, since the greater the intensity of illumination the greater the current flow at any given voltage. This is not a correct interpretation of figure 2.4.1.2, for as previously mentioned the cell really acts as a high impedance battery, or current generator, when illuminated. However, thinking of the illumination as having essentially the same effect as reducing the resistance aids in the understanding of the trigger circuit.

In the trigger circuit of figure 2.4.1.9 the solar cell is connected in a reverse bias circuit in series with an 82 K ohm resistor. The N side of the solar cell is at ground, or zero potential, hence when the cell is not illuminated there is a voltage drop across the cell in a reverse bias direction. This same voltage drop occurs as a forward bias across the base-emitter junction of the transistor. This forward bias across the base-emitter junction drives the transistor into conduction, in which case the resistance of the transistor is small. Thus the no-load output voltage measured across the transistor is essentially zero when the cell is not illuminated.

As previously mentioned, illuminating the cell can be considered to reduce the resistance of the cell, with a corresponding reduction in the reverse bias voltage drop across the cell and in the forward bias voltage drop across the base-emitter junction of the transistor. If the illumination is sufficiently intense the forward bias across the base-emitter junction will be decreased enough to drop the transistor out of conduction, resulting in a very high resistance across the transistor. When this happens the no-load output voltage measured across the transistor will be approximately equal to the battery voltage.

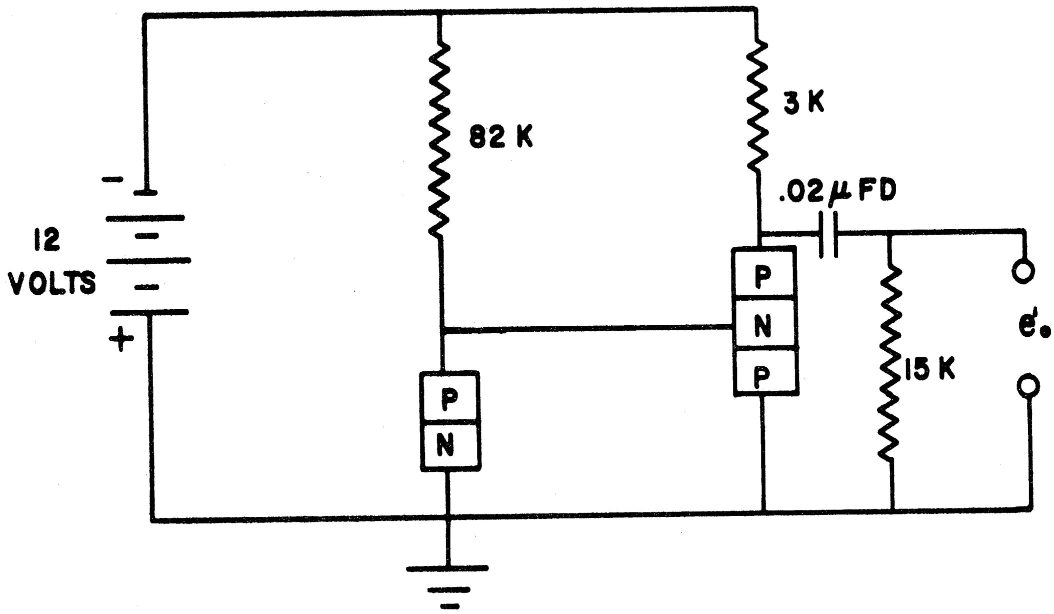
It might be said that in this circuit the solar cell acts as a switch or trigger, turning the no-load output voltage on or off with the solar cell illumination.

A velocity transducer is formed by having two solar cells mounted side by side, and covered with an opaque mask having two small holes a measurable distance apart and so that each hole coincides with one cell. Such a set-up is shown in figure 2.4.1.10. Each cell is connected into a separate trigger circuit, and the mask is then illuminated so that the light will pass through the holes onto the cells. When the shadow of the moving part moves across the holes, each circuit will switch, or trigger, as the edge of the shadow passes. If the



MASKED SOLAR CELLS
FIGURE 2.4.1.10

output voltages from the two circuits are fed into an oscilloscope with a calibrated time sweep, the time for the edge of the shadow to move from one hole to the other can be displayed, from which the average velocity between the holes can be calculated.

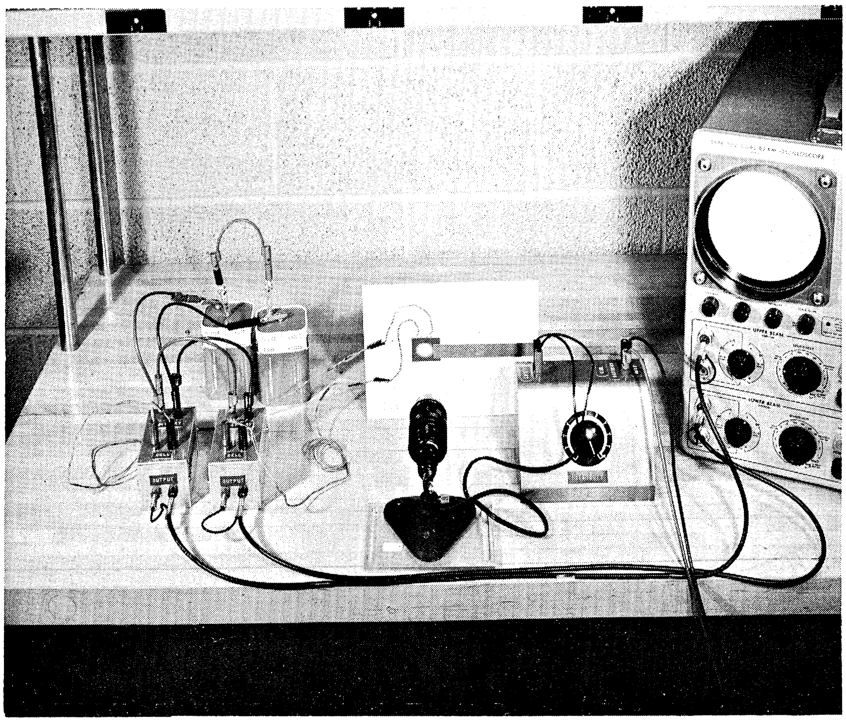


SOLAR CELL VELOCITY TRANSDUCER CIRCUIT
FIGURE 2.4.1.11

The trigger circuit used for the velocity transducer is shown in figure 2.4.1.11. To improve the output signal a differentiating RC circuit has been added. As explained in section 1.2.4, this can be called a peaking circuit, for it will result in a sharply peaked output on the oscilloscope when there is a step type change in the voltage drop across the transistor. The output peaks corresponding to the shadow passing the holes will permit a more accurate determination of the average velocity.

It is of interest to note that this transducer does not follow the general principle of generating a voltage proportional to the physical variable to be measured. Because of this no calibration is necessary.

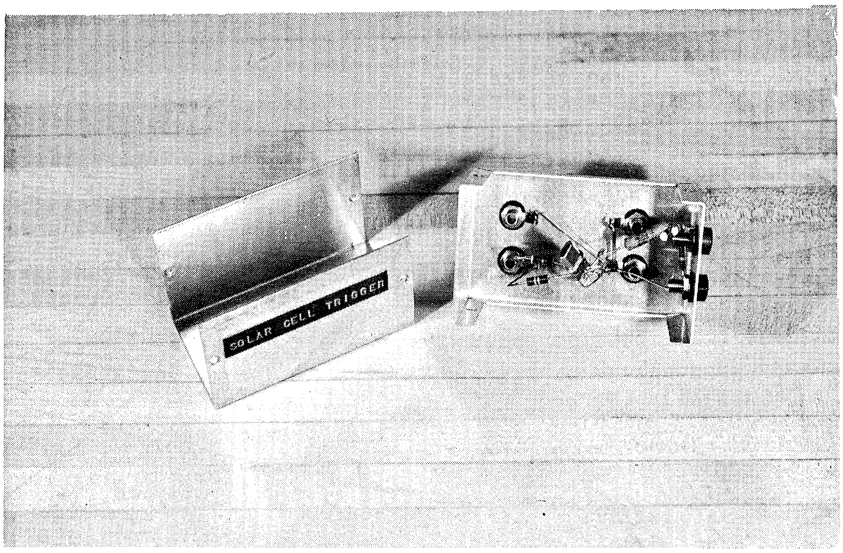
A velocity transducer with two laboratory constructed trigger circuit boxes are shown in figure 2.4.1.12, while the interior of the circuit box is shown in figure 2.4.1.13.



SOLAR CELL VELOCITY TRANSDUCER
FIGURE 2.4.1.12

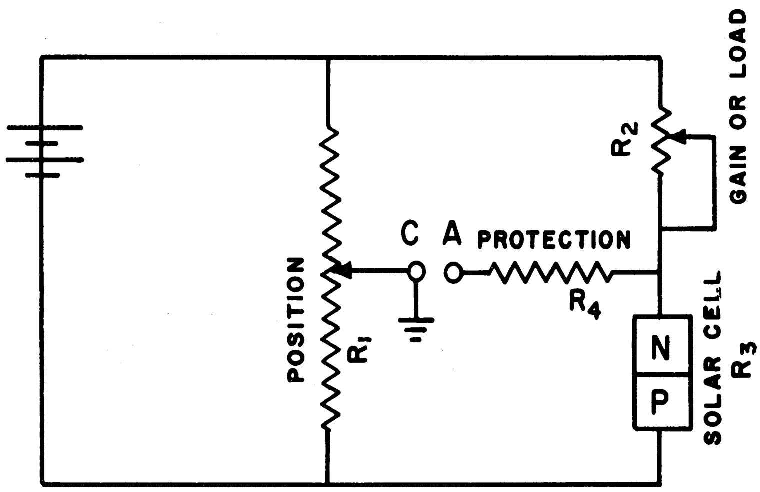
Red and black banana jacks on the aluminum chassis box serve to connect batteries, solar cell, and the output display instrument into the circuit. As with the displacement transducer, red jacks indicate positive potential, while black jacks indicate negative potential or ground.

All resistors are one-half watt, and the PNP transistor is an RCA 2N247.



SOLAR CELL VELOCITY TRANSDUCER CIRCUIT BOX
FIGURE 2.4.1.13

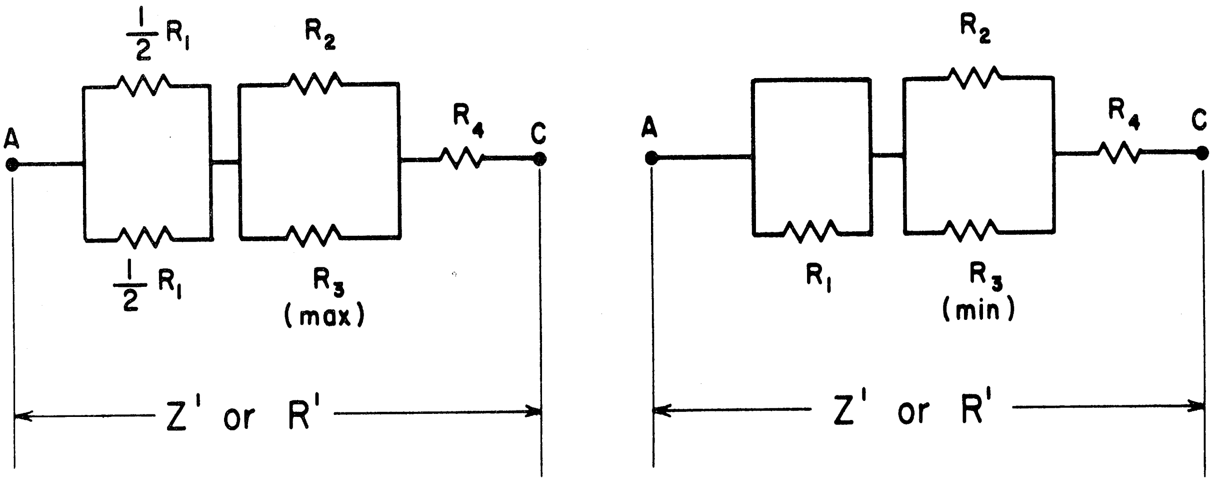
2.4.2 IMPEDANCE OF THE SOLAR CELL TRANSDUCERS. The source impedance of the solar cell displacement transducer circuit is the impedance from A to C with the voltage source short circuited as shown in figure 2.4.2.1.



CIRCUIT TO DETERMINE THEVENIN SOURCE RESISTANCE OF THE SOLAR CELL DISPLACEMENT TRANSDUCER
FIGURE 2.4.2.1

The impedance is essentially pure resistance, and will vary with the adjustment of the position resistance, the gain or load resistance, and to some extent with the variation of the solar cell equivalent resistance during operation. However, the effect of this variation will be reduced because of the parallel load resistance. For a typical circuit the load resistance, R_2, might be 1000 ohms and light variations cause the solar cell to "generate" currents from 2 to 20 milliamps resulting in a corresponding equivalent resistance variation from 11,000 ohms to zero ohms. The parallel resistance variation will be from approximately 1000 ohms to zero ohms respectively as the light energy falling on the cell goes from low to high extremes.

The equivalent circuits to determine the maximum and minimum values of the source impedance are shown in figure 2.4.2.2 a and b.



a

MAXIMUM IMPEDANCE

b

MINIMUM IMPEDANCE

EQUIVALENT CIRCUITS FOR SOURCE RESISTANCE
OF THE SOLAR CELL DISPLACEMENT TRANSDUCER
FIGURE 2.4.2.2

Maximum impedance occurs when the position resistor is at midpoint and the solar cell equivalent resistor has its full value in the circuit. Then from figure 2.4.2.2 a:

$$Z'_{max} = R'_{max} = \frac{1}{4} R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} + R_4 \qquad (2.4.2.1)$$

where: R_1 = position resistance, ohms

R_2 = gain resistance, ohms

R_3 = maximum solar cell resistance, ohms

R_4 = protection resistance, ohms

Minimum impedance occurs when both the position resistor and the solar cell equivalent resistance are at their extremes, as shown in figure 2.4.2.2 b. Then:

$$Z'_{min} = R'_{min} = \frac{R_2 \cdot R_3}{R_2 + R_3} + R_4 \qquad (2.4.2.2)$$

For a typical solar cell circuit as previously mentioned with a position resistor R_1 equal to 10 K ohms and a protecting resistor R_4 equal to 1 K ohms.

$$R'_{max} = 4.5 \text{ K ohms},$$

and

$$R'_{min} = 1 \text{ K ohm}$$

Due to the change in the equivalent resistance of the solar cell during operation, there can be distortion of the no-load output voltage. In addition, the attenuation of the output voltage could be changed if the position resistor is adjusted after calibration, thus making the calibration incorrect. Since even the maximum impedance of the circuit is quite small, none of these considerations would be of significance if a reasonably high impedance display instrument is used, as discussed in section 1.5.

The impedance of the solar cell trigger circuit is of no concern, since attenuation or phase lag of the output is of no consequence so long as the output is of a readable magnitude, and the phase lag is the same for the two circuits used for the velocity transducer.

2.4.3 CALIBRATION OF THE SOLAR CELL TRANSDUCERS. In most cases the displacement transducer must be calibrated in place, using a dial indicator or similar device to indicate displacement of the mechanical part as it is moved in small increments across the illuminated area of the cell. The corresponding output voltage is read on the display or recording instrument. The proportionality constant of displacement to output volts can be adjusted by varying the intensity of the light or by changing the setting of the gain or load resistor. The gain resistance must only be decreased in this operation, however, since increasing this resistance will destroy the linear relation between displacement and output voltage.

Care must be exercised to see that the physical set-up is such that the displacement of the shadow on the cell is the same as the displacement of the mechanical part itself, and that supplementary shadows or reflections do not vary the intensity of illumination on the cell as the mechanical part moves. It is often necessary to cement a small appendage to the mechanical part to obtain a clear shadow on the cell.

Remembering from section 2.4.1 that the relation between displacement and output volts will be linear only if the circuit is properly adjusted and if the cell is illuminated with a uniform intensity, it is almost always necessary to plot a calibration curve with a reasonable number of intermediate points so as to insure linearity.

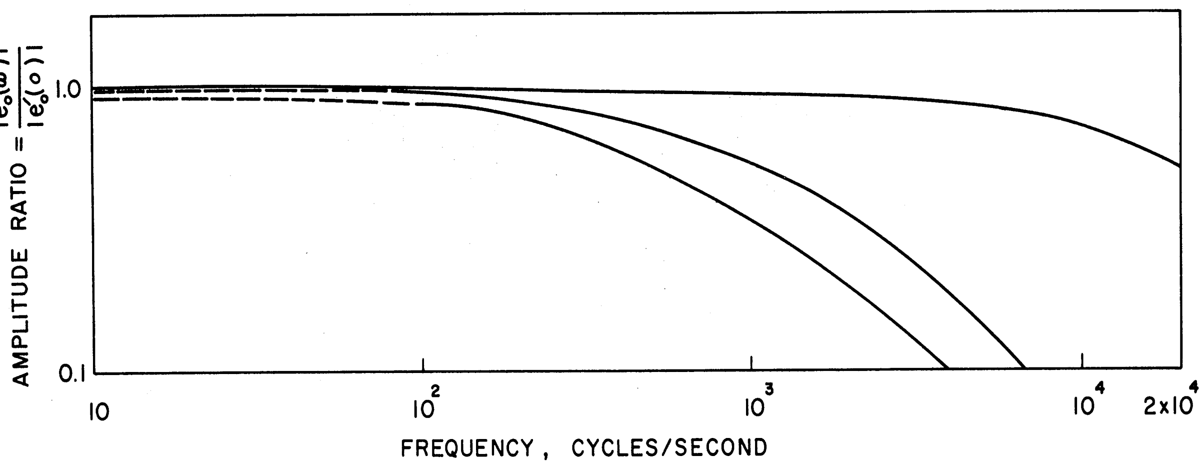
As previously mentioned, the solar cell velocity transducer requires no calibration.

2.4.4 ADVANTAGES AND DIS-ADVANTAGES OF THE SOLAR CELL TRANSDUCERS. Probably the main advantage of the solar cell transducers lies in the fact that at most they require no physical attachment other than a mask to the mechanical part which is moving. Both the displacement and velocity transducers are simple, rugged, and easily used, although the displacement transducer may take some manipulation of the light source to obtain a uniform intensity on the cell surface and hence a linear relation between displacement and output volts.

Rather large displacements can be measured. The largest cells are about 7/8" long, and can measure displacements up to about 3/4". When measuring large displacements the output voltage is sufficiently high so that noise presents no real problem.

If the intensity of the electric lighting in the room is too high as compared to the intensity of the direct current illumination of the cell from its light source, the room light may have to be reduced or it will appear as 60 cycle voltage variation.

The solar cells are remarkably sensitive, and so can also be used to measure small displacements. At the sensitivity needed to read small displacements noise presents more of a problem. However, since small displacements commonly occur at high frequency, the measurement of small displacement is often limited by the frequency response of the solar cell. Typical frequency response curves for three different silicon solar cells are shown in figure 2.4.4.1.



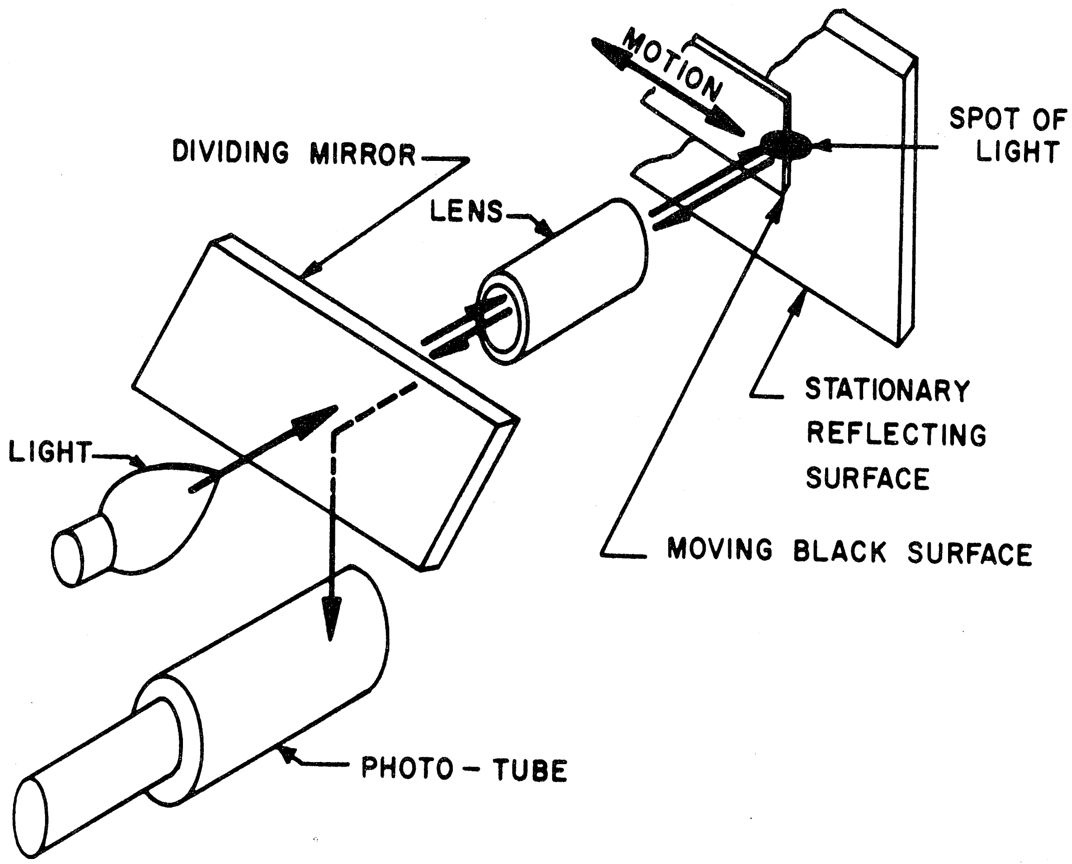
FREQUENCY RESPONSE OF SILICON SOLAR CELLS
FIGURE 2.4.4.1

2.5 REFLECTING TRANSDUCER

2.5.1 PRINCIPLE AND CIRCUITS. This is a linear displacement transducer, primarily intended for measuring small displacements.

The schematic physical set-up of the reflecting transducer is shown in figure 2.5.1.1. It consists of a light source, a lens system, a dividing mirror, a photo-tube, and a vacuum tube amplifier, all contained in a single housing. The light passes through the mirror and lens, and is focused in an elongated spot located on either the mechanical part on which the displacement measurements are to be made or on a stationary piece immediately behind the mechanical part. In either case a small reflecting surface is provided in such a way that the focused spot is about equally divided between the reflecting surface and a non-reflecting surface. In figure 2.5.1.1 this is done by focusing the spot on the stationary reflecting surface, and shadowing about one-half of the spot with the moving part which has a non-reflecting surface. This could also have been accomplished by cementing a small piece of reflecting tape on the moving part itself, and focusing the spot so that the edge of the reflecting tape approximately bisected the spot.

In either case the light striking the reflecting surface is reflected back through the lens and the dividing mirror deflects it onto the photo-tube. Thus the amount of light reaching the photo-tube is proportional to that area of the spot which falls on the reflective surface, and it follows that as the moving part is displaced the amount of light

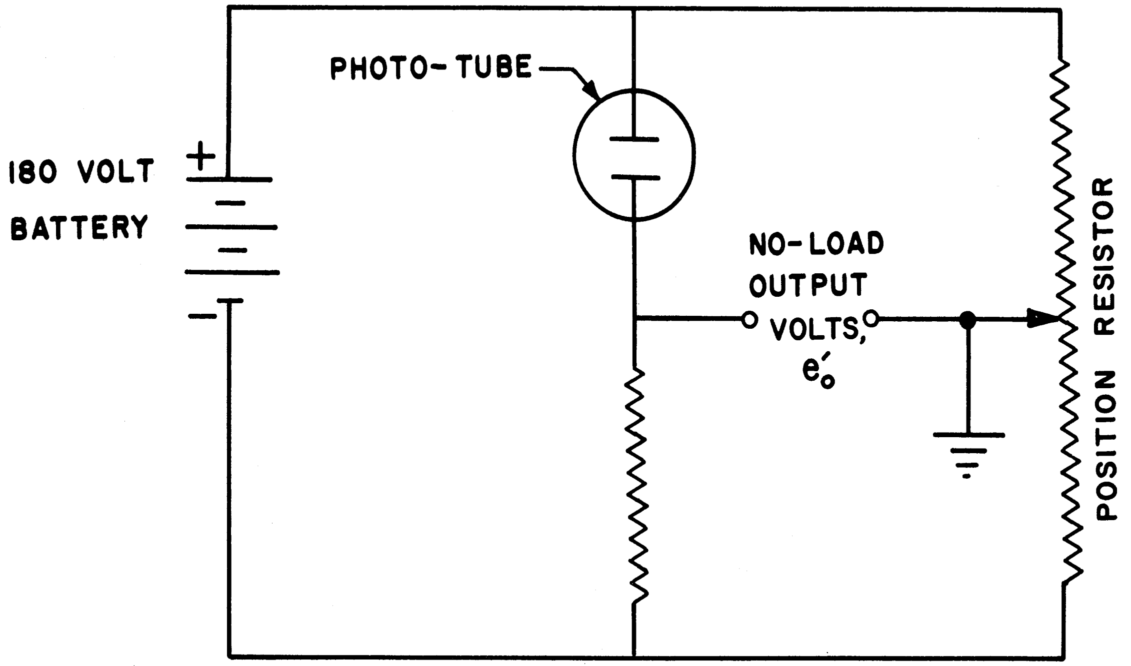


PHYSICAL SCHEMATIC OF THE REFLECTING TRANSDUCER
FIGURE 2.5.1.1

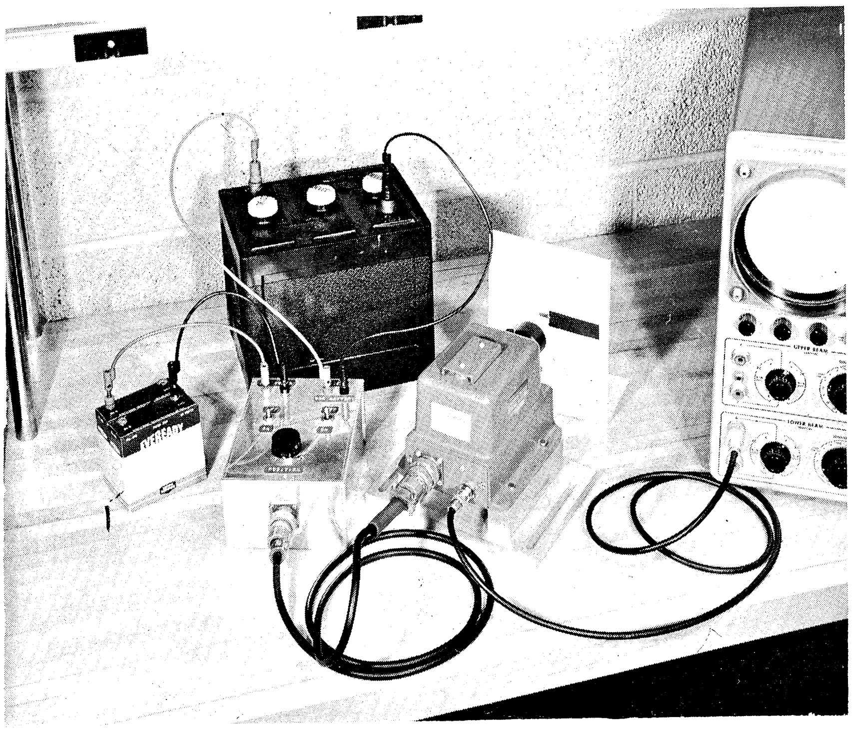
reaching the photo-tube will be proportional to this displacement, at least through the center portion of the elongated spot of light.

Photo-tubes, or photo-cells as they are sometimes called, have high electrical resistance which decreases when the tube is illuminated from an external source. In the reflecting transducer the resistance of the photo-tube will therefore vary with the displacement of the moving part. As with other transducers which utilize a changing resistance to develop a voltage, the photo-tube is used in a bridge circuit as shown in figure 2.5.1.2. The function of the position resistor is the same as in the potentiometer transducer circuit, which is explained in detail in section 2.2.1 of this chapter.

A reflecting transducer with a laboratory constructed circuit box is shown in figure 2.5.1.3. A 6-volt automobile battery powers the light and the vacuum tube filament, while two 90-volt dry batteries are used as the 180-volt source. Toggle switches are provided so that the 6-volt filament power can be turned on ahead of the 180 volt source to provide a warm up period. As in the circuit boxes of other transducers, the position resistor is a one-half watt radio volume control resistor.

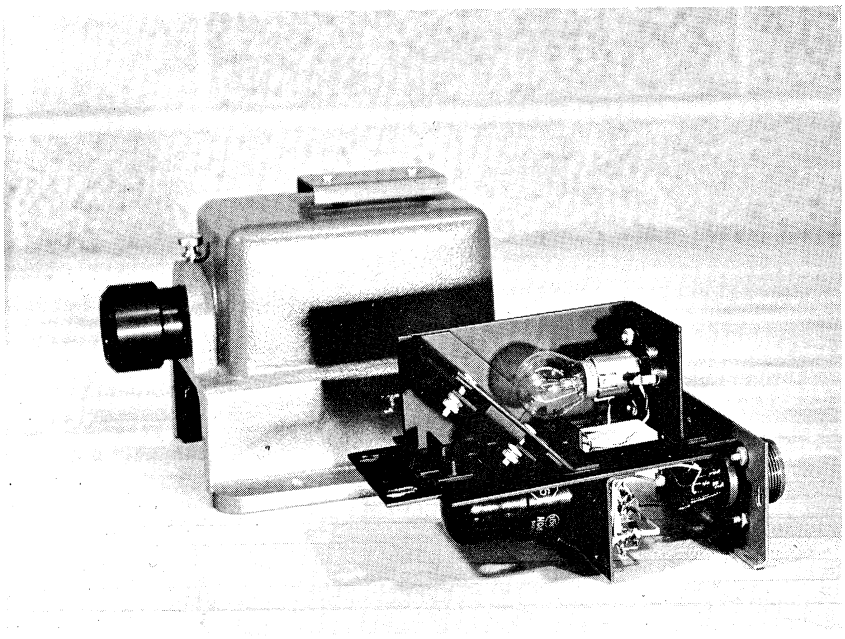


REFLECTING PICK-UP SCHEMATIC CIRCUIT
FIGURE 2.5.1.2



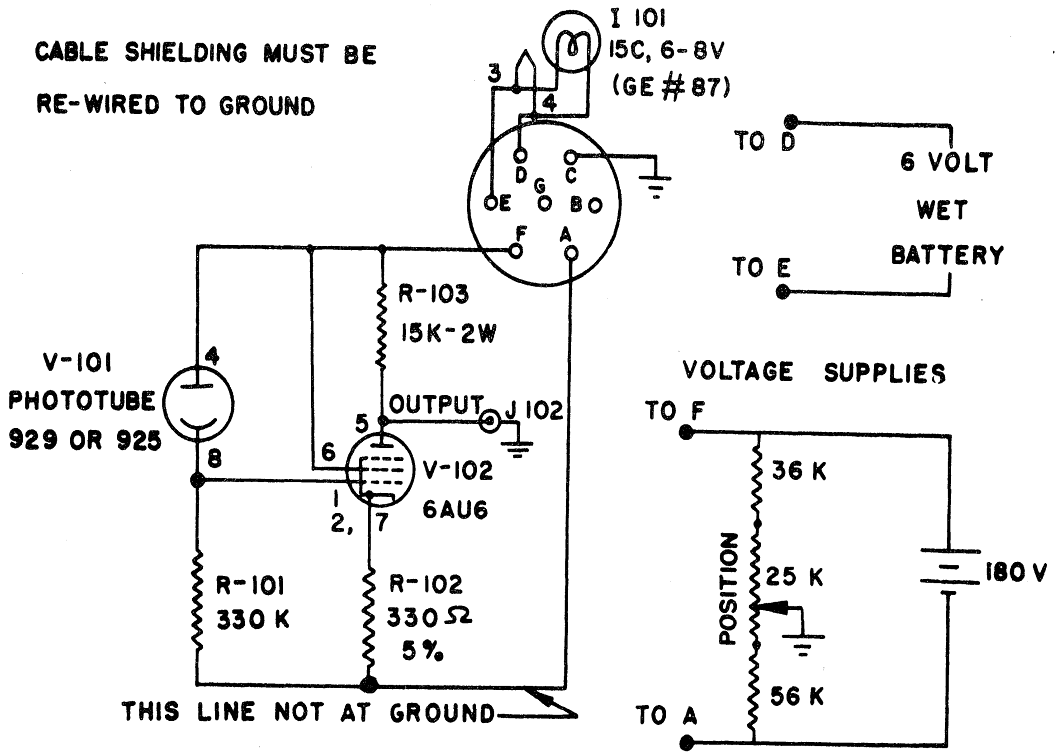
REFLECTIVE TRANSDUCER
FIGURE 2.5.1.3

Figure 2.5.1.4 shows the reflecting transducer with the housing removed so that the light source, the dividing mirror, and the photo-tube are visible.



INTERIOR OF REFLECTING TRANSDUCER
FIGURE 2.5.1.4

The reflecting transducer shown in the preceding figures has a linear range of about 0.060", hence is intended to measure small displacements only. This transducer is made by re-wiring a commercial pick-up used with a counter. The rewired circuit is shown in figure 2.5.1.5. The output from the amplifier is fed into the display or recording instrument.



RE-WIRING DIAGRAM FOR REFLECTION TRANSDUCER
FIGURE 2.5.1.5

2.5.2 <u>IMPEDANCE OF THE REFLECTING TRANSDUCER</u>. As mentioned in the previous paragraph, there is an amplifier built into the pick-up. This eliminates any impedance loading problem since the amplifier has been matched to the photo-tube circuit.

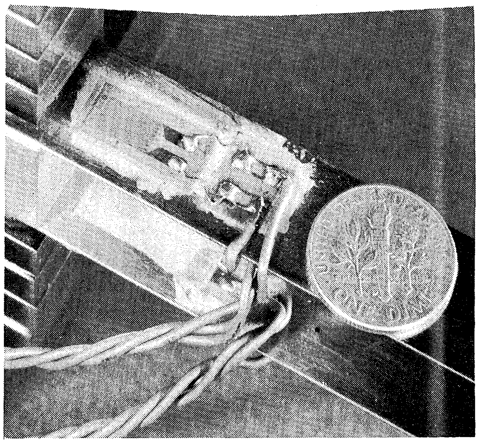
2.5.3 <u>CALIBRATION OF THE REFLECTING TRANSDUCER</u>. The reflecting pick-up must be calibrated in place, using a dial indicator or similar device to indicate displacement as the mechanical part is moved in small increments across the focused spot of light. The corresponding output voltage is read on the display or recording instrument. Since only small displacements can be measured with the reflecting pick-up, the calibration must be very carefully carried out and a calibration curve with a reasonable number of points must be plotted to insure linearity.

2.5.4 <u>ADVANTAGES AND DIS-ADVANTAGES OF THE REFLECTING TRANSDUCER</u>. An obious advantage of the reflecting pick-up is the fact that no physical connection to the moving part is required, hence the reflecting pick-up can measure displacements of very small parts without causing any mechanical loading. It is limited to small displacements, up to about 0.060" maximum. Small displacements commonly occur at high frequencies, and the reflecting pick-up has a sufficiently high frequency response to measure most vibrating types of motions in mechanical devices.

2.6 RESISTANCE STRAIN GAGES

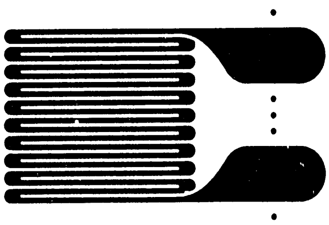
2.6.1 PRINCIPLE OF THE RESISTANCE STRAIN GAGE. As its name implies, the resistance strain gage is a strain transducer, but since for most materials, force, stress, and deflection are all proportional to strain within the elastic limit, the strain gage can also be used as a force, stress, or displacement transducer. In the following discussion the strain gages are considered as force transducers.

The most commonly used strain gage is the foil type shown in figure 2.6.1.1, mounted on a mechanical part. The gage consists of a fine metallic foil grid on a thin epoxy film backing which is cemented to the part. The grid is formed so that two wires can be soldered to the ends of the grid. The thickness of the gage is approximately 0.001".



MOUNTED FOIL GAGE
FIGURE 2.6.1.1

When a resistance element such as a wire is strained by an external force, the total resistance of the element changes in direct proportion to the total strain of the element. A tensile or positive strain increases the resistance, while compressive or negative strain decreases resistance. This is the basic principle upon which the resistance strain gage operates.



FOIL GAGE
FIGURE 2.6.1.2

An enlarged drawing of a foil type gage is shown in figure 2.6.1.2. It is essentially a small resistance element which can be cemented onto the mechanical part on which measurements are to be made. When the part is strained under load, the gage is equally strained, and its resistance will accordingly change. A bridge circuit will develop a voltage proportional to this change in resistance, in keeping with the principle of the basic measuring system described in section 1.1.

There is a wealth of information in the literature concerning strain gages and their uses, and no attempt is made to present a comprehensive coverage here. In the discussions which follow we shall be mostly concerned with the foil gages, with some comments on the newer semi-conductor gages. The art of attaching strain gages to mechanical parts is not covered here.*

The relation between strain and the change in resistance of a strain gage is given by the gage factor which is defined as follows:

$$f = \frac{\Delta R/R}{\epsilon} \qquad (2.6.1.1)$$

where

f = gage factor

ΔR = change in resistance, ohms

R = original gage resistance, ohms

ϵ = unit strain, inches/inch

It is apparent from equation 2.6.1.1 that for any given value of original gage resistance R and unit strain ϵ, the change in resistance ΔR will vary directly with the gage factor f. Hence the magnitude of the gage factor indicates the sensitivity of the gage.

Typical characteristics of foil gages and semi-conductor gages are shown in figure 2.6.1.3

Type of Gage	Gage Resistance, R, ohms	Gage Factor, f	Max. Current Cap., amps
Etched Foil	$120 \pm .2\%$	$2.1 \pm .5\%$	0.050
Silicon Semi-conductor	$120 \pm 2\%$	$110 \pm 5\%$	0.035

TYPICAL STRAIN GAGE CHARACTERISTICS
FIGURE 2.6.1.3.

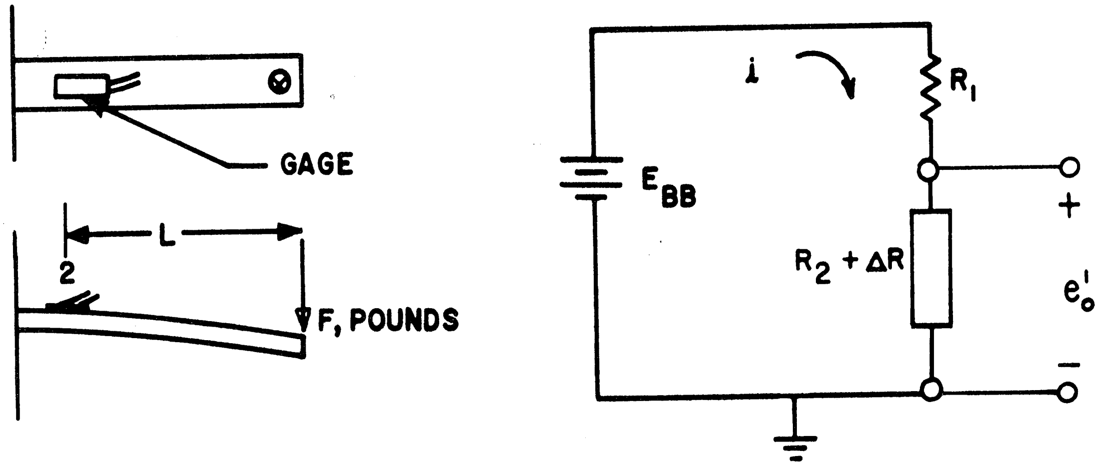
It is interesting to note that the semi-conductor gages have over 50 times the sensitivity of the foil gages. The semi-conductor gages are also more expensive, costing about ten times as much as the foil gages.

*A good source of information for this purpose is "Strain Gage Accessories and Techniques" by Wm. Bean, 18915 Grand River Ave., Detroit 23, Michigan, $2.00.

In addition to changing with strain, the resistance of the strain gage will also change with temperature, an obviously undesirable characteristic. Most gages exhibit a reduction in resistance with an increase in temperature, but since most materials produce a positive strain with an increase in temperature, it is theoretically possible for the combined effects to cancel.

This has been done to some extent, and temperature compensated gages are available for a number of materials. Temperature effects can also be reduced by circuit configurations, as shown in the next section.

2.6.2 CIRCUITS FOR RESISTANCE STRAIN GAGES. As with the other transducers which utilize a change in resistance to develop a proportional voltage, the strain gages are generally used in a bridge circuit as previously described in section 1.2.3, and shown in figure 1.2.3.1. However, an analysis will first be made of the circuit of figure 2.6.2.1, utilizing a single gage placed on a cantilever beam. Applying the rule from the top of page 4 for the voltage drop across a



SINGLE STRAIN GAGE CIRCUIT
FIGURE 2.6.2.1

resistor in series, the no-load output voltage will be:

$$e_o' = E_{BB} \frac{R_2 + \Delta R}{R_1 + R_2 + \Delta R} \qquad (2.6.2.1)$$

Dividing by R_2, and assuming that $R_1 = R_2 = R$:

$$e_o' = E_{BB} \frac{1 + \frac{\Delta R}{R}}{2 + \frac{\Delta R}{R}} = E_{BB} \frac{1 + \frac{\Delta R}{R}}{2(1 + \frac{\Delta R}{2R})} \qquad (2.6.2.2)$$

or,

$$e_o' = \frac{1}{2} E_{BB} \left(1 + \frac{\Delta R}{R}\right)\left(\frac{1}{1+\frac{\Delta R}{2R}}\right) \qquad (2.6.2.3)$$

The last term of equation 2.6.2.3 can be expressed as the sum of a geometric series such that:

$$e_o' = \frac{1}{2} E_{BB}\left(1 + \frac{\Delta R}{R}\right) \left[1 - \frac{\Delta R}{2 R} + \left(\frac{\Delta R}{2 R}\right)^2 \text{------}\right] \qquad (2.6.2.4)$$

When $\Delta R/R$ is small the higher terms of the series can be neglected, and the equation becomes:

$$e_o' = \frac{1}{2} E_{BB} \left(1 + \frac{\Delta R}{R}\right)\left(1 - \frac{\Delta R}{2 R}\right) \qquad (2.6.2.5)$$

By expanding:

$$e_o' = \frac{1}{2} E_{BB} \left[1 + \frac{\Delta R}{R} - \frac{\Delta R}{2 R} - \frac{1}{2} \left(\frac{\Delta R}{R_1}\right)^2 \right] \qquad (2.6.2.6)$$

Re-arranging, and dropping the squared term as insignificant:

$$e_o' = E_{BB} \left(\frac{1}{2} + \frac{1}{4} \frac{\Delta R}{R}\right) \qquad (2.6.2.7)$$

This equation establishes the no-load output voltage for the single gage circuit of figure 2.6.2.1 when $R_1 = R_2 = R$. However, the single gage circuit of figure 2.6.2.1 is seldom used, for reasons which are best illustrated by the following example:

Example 2-1: A single strain gage having a resistance of 120 ohms and a gage factor of 2.1 is mounted on a steel cantilever beam, and connected in series with a 120 ohm resistor and a 12 volt battery as shown in figure 2.6.2.1. The bending stress at the gage fluctuates from zero to 20,000 psi as the force F fluctuates. What is the corresponding variation in the no-load output voltage, and can this variation be magnified to full scale on an oscilloscope?

Solution: Solving equation 2.6.1.1 for $\Delta R/R$ when the stress is 20,000 psi:

$$\frac{\Delta R}{R} = f \cdot \epsilon$$

where

$$\epsilon = \frac{\sigma}{E}$$

σ = stress, 20,000 psi

E = modulus of elasticity, 30×10^6 psi

Then by substitution:

$$\frac{\Delta R}{R} = 2.1 \times \frac{20,000}{30 \times 10^6} = 0.0014$$

By equation 2.6.2.7:

$$e_o' = E_{BB} \left(\frac{1}{2} + \frac{1}{4} \frac{\Delta R}{R}\right)$$

$$= 12 \left(\frac{1}{2} + \frac{1}{4} \cdot 0.0014\right)$$

$$= 6.0042 \text{ volts}$$

It is apparent from the preceding equation that the no-load output voltage for zero stress will be

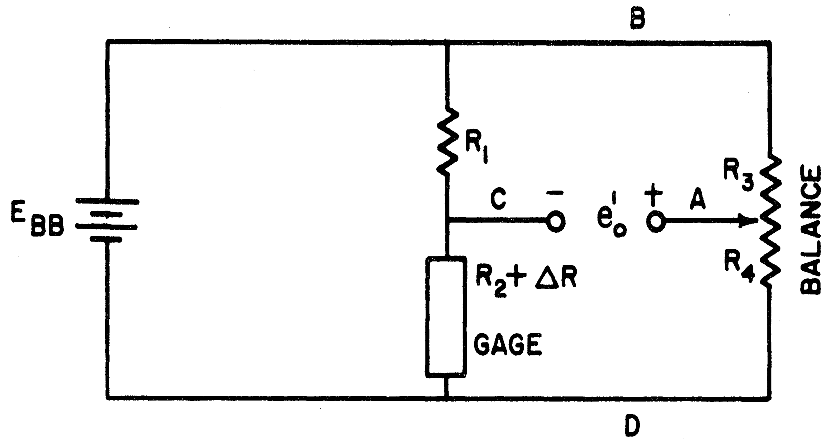
$$e_o' = 6.0000 \text{ volts}$$

The no-load output voltage varies from 6.0000 to 6.0042 volts as the stress in the beam varies from zero to 20,000 psi, a change in voltage of 4.2 millivolts. If the oscilloscope had a sensitivity of 0.5 millivolts per cm of trace deflection, the 4.2 millivolts could cover 8.4 of the 10 cm on the oscilloscope screen. However, the maximum output voltage is 6.0042 volts, and at 0.5 millivolts/cm, this would deflect the oscilloscope trace 12,008.5 cm, an obvious impossibility with a 10 cm screen. The trace would be driven off the screen, and could not be displayed on the oscilloscope screen unless the sensitivity was so reduced that the voltage variation would no longer be visible.

From the example it might be concluded that the voltage variation of 4.2 millivolts could be displayed on the oscilloscope screen if this voltage variation was close to the zero voltage level, say from plus 2.1 to minus 2.1 millivolts. This is the same problem encountered with the potentiometer transducer described in section 2.2.1, and it can be similarly solved by using the strain gage in a bridge circuit. The simplest way to form the bridge circuit is to include a variable resistor as shown in figure 2.6.2.2. This variable resistor is commonly called a balance resistor, as indicated on figure 2.6.2.2, since it can be adjusted to obtain the balanced bridge condition described in section 1.2.3. Notice also that the balance resistor serves the same purpose as the position resistor of the potentiometer transducer circuit of figure 2.2.1.4.

The effect of this circuit is to subtract the $\frac{1}{2} E_{BB}$ voltage from the output, giving:

$$e_o' = \frac{1}{4} E_{BB} \frac{\Delta R}{R_2} \qquad (2.6.2.8)$$



ADDITION OF A BALANCE POTENTIOMETER
TO FORM A BRIDGE CIRCUIT
FIGURE 2.6.2.2

The zero voltage reference level, or ground, for the strain gage bridge circuit can be located where most convenient. When there is no grounding, as in figure 2.6.2.2, the circuit is said to be floating, and it is necessary to have an instrument which can measure the output difference from A to C.

In practice it is usually best to have the strain gage circuit floating, and apply a ground later if a single-ended or grounded input to the display instrument must be used. The use of a floating circuit with a differential input generally results in minimum noise.

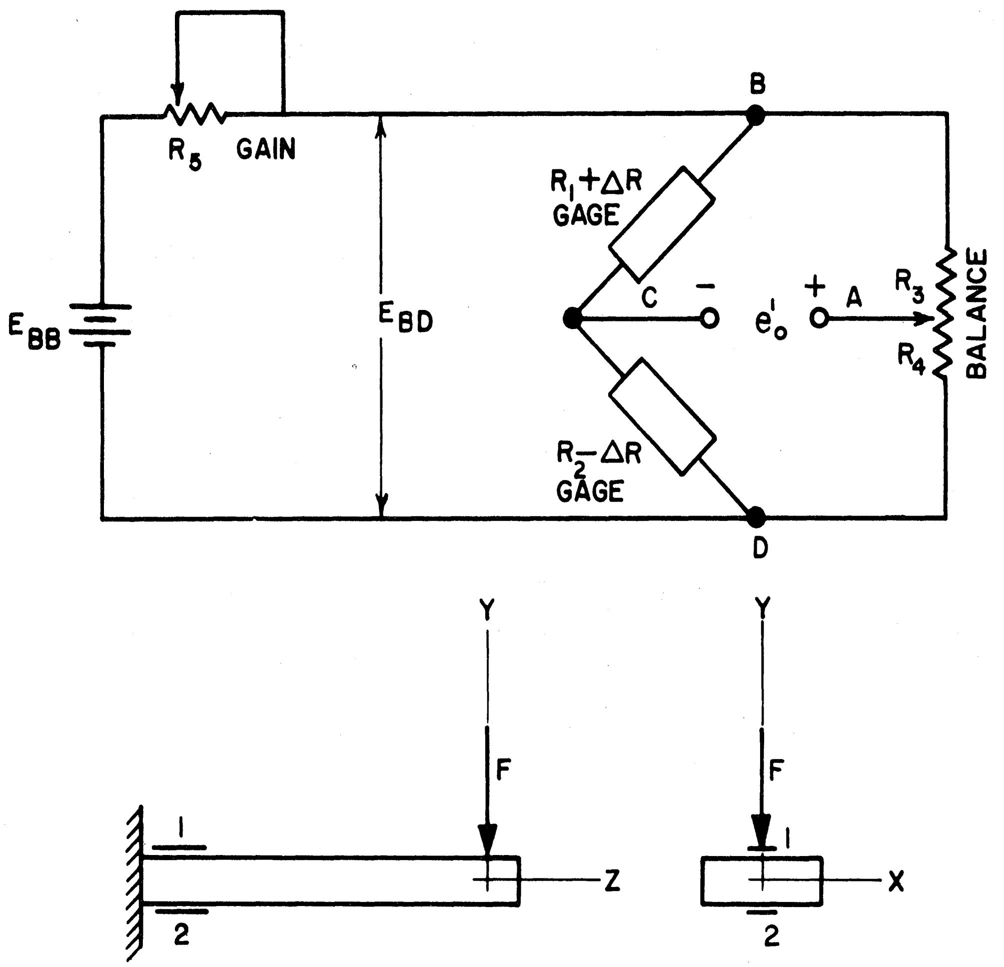
The balance resistor of figure 2.6.2.2 can be adjusted to give a zero output voltage when the force, and hence the strain at the gage, are both zero. Any subsequent voltage output will then directly represent the force. If the force is very small, the signal will be small and the sensitivity of the oscilloscope can be increased to magnify the signal without driving the signal off the screen.

It might be noticed that the function of the variable balance resistor of figure 2.6.2.2 could also be thought of as providing 6 volts at C to subtract from or to null out the non-signal part of the 6.0042 volt output which resulted when the single gage circuit of figure 2.6.2.1 was used in the example.

In practice a single strain gage bridge circuit is seldom used in a measuring circuit. There are several reasons for this. Additional gages will increase the output voltage, and judicious placement of the gages on the mechanical part and in the circuit will often reduce or eliminate unwanted outputs resulting from temperature effects or from unwanted load effects. This will be shown in some of the circuits which follow.

Another reason for not using the single gage circuit is the possibility of a non-linear relation between strain and output volts, especially with relatively large strains. An analysis is not presented here, but it can be shown that under these same conditions the source impedance would not remain constant, and this could also effect the linearity.

A commonly used two-gage bridge is shown in figure 2.6.2.3, using a cantilever beam for illustration. Gage R_1 is strained in tension which will increase its resistance, while gage R_2 is strained in compression which causes its resistance to decrease.



TWO GAGE BRIDGE CIRCUIT
FIGURE 2.6.2.3

The two-gage bridge circuit of figure 2.6.2.3 is essentially the same as that shown in figure 1.2.3.1 of chapter 1, and the bridge circuit analysis presented in section 1.2.3 can be utilized here.

To avoid confusion it must be remembered that the term "no-load output voltage" refers to a condition of no electrical load on the transducer output, and does not refer to a condition of no force applied to the member on which the gages are mounted. A force, rather than a load, will be described as being applied to the beam or other physical member to strain the gages.

Before applying the force F to strain the gages the bridge must be balanced by adjusting the balance resistor of figure 2.6.2.3 until the no-load output voltage e'_o equals zero. When the bridge is balanced, the relationship of the bridge resistances is given by equation 1.2.3.7 as:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (1.2.3.7)$$

If the two gages are identical, then $R_1 = R_2$, and $R_3 = R_4$.

When the gages are now strained by the application of force F, the no-load output voltage can be found by applying equation 1.2.3.6 to the strained bridge to obtain the following:

$$e'_o = E_{BD}\left(\frac{R_4}{R_3 + R_4}\right) - E_{BD}\left(\frac{R_2 - \Delta R}{R_1 + \Delta R + R_2 - \Delta R}\right) \qquad (2.6.2.9)$$

$$e'_o = E_{BD} \cdot \frac{1}{2} - E_{BD}\left(\frac{R_2 - \Delta R}{R_1 + R_2}\right) \qquad (2.6.2.10)$$

Letting R = original gage resistance = $R_1 = R_2$, then the no-load output of the two gage bridge will be:

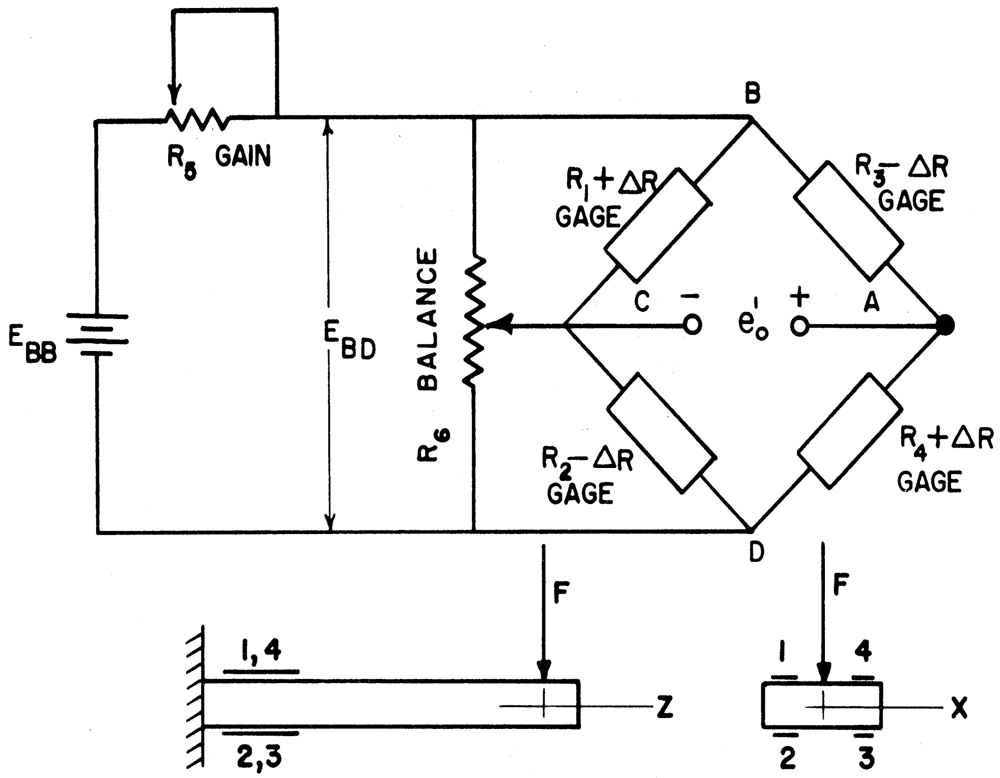
$E_{BD} \times \frac{1}{2} - E_{BD}\left(\frac{1 - \Delta R/R}{2}\right)$

$$e'_o = \frac{1}{2} E_{BD} \frac{\Delta R}{R} \qquad (2.6.2.11)$$

$E_{BD} \times \frac{1}{2} - E_{BD} \frac{1}{2} + E_{BD} \times \Delta R/R$

The maximum allowable battery voltage is found from the gage resistance and the maximum allowable current through the gages, as stated in figure 2.6.1.3.

Two more gages have been added to the circuit to form the four gage bridge of figure 2.6.2.4. A balancing resistor has been included to compensate for slight variations in the resistance values of the four gages. Following the method used for analysis of the two gage bridge,



FOUR GAGE BRIDGE CIRCUIT
FIGURE 2.6.2.4

the no-load output of the strained bridge can be expressed as:

$$e_o' = E_{BD}\left(\frac{R_4 + \Delta R}{R_3 - \Delta R + R_4 + \Delta R}\right) - E_{BD}\left(\frac{R_2 - \Delta R}{R_1 + \Delta R + R_2 - \Delta R}\right)$$

(2.6.2.12)

Again letting R = original gage resistance, and considering all gages to be the same, the no-load output of the four gage bridge will be:

$$e_o' = E_{BD} \frac{\Delta R}{R} \qquad (2.6.2.13)$$

Comparing equations 2.6.2.11 and 2.6.2.13 shows that for the same strain the four-gage bridge will develop twice the output of the two-gage bridge, and that the linear relation between ΔR and e_o' is not affected by the magnitude of ΔR. A general equation for the no-load output voltage of any strain gage bridge circuit can be stated as:

$$e_o' = E_{BD} \cdot \frac{n}{4} \cdot \frac{\Delta R}{R} \qquad (2.6.2.14)$$

where

E_{BD} = voltage applied to the bridge

n = number of gages subjected to strain

ΔR = change in resistance of each gage, ohms

R = resistance of each gage, ohms

It is important to notice that in figure 2.6.2.3 and 2.6.2.4, the circuits were so arranged that gages being strained in opposite directions were placed in adjacent arms of the bridge. A review of the analysis shows that the output voltage would have been zero had the gages being strained in opposite directions been placed in opposite arms of the bridge. From this a simple but very useful rule can be formulated:

> So far as the output voltage is concerned, the effect of any changes in resistance in opposite arms of the bridge are additive, while the effects of resistance changes of adjacent arms subtract from each other.

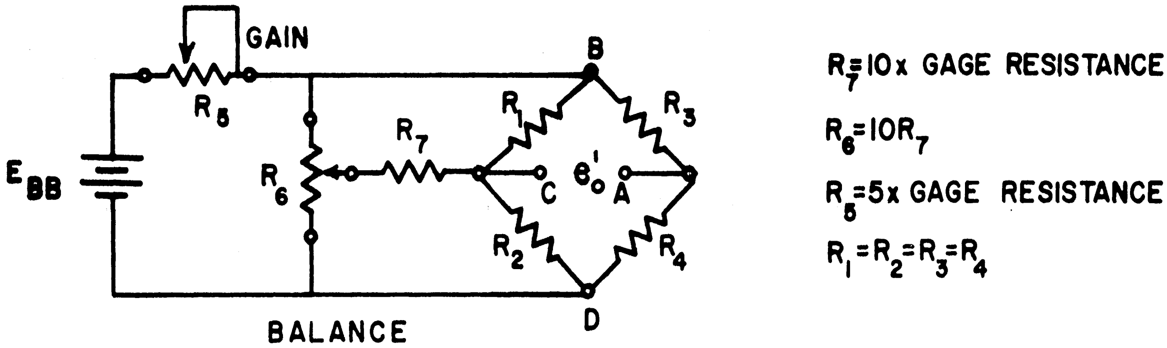
This rule can be used in the place of a detailed analysis in some of the explanations which follow.

It was mentioned in section 2.6.1 that in addition to changing with strain, the resistance of the strain gages will also change with temperature. Both the two-gage bridge and the four-gage bridge are inherently temperature compensated, providing the temperature of all the gages fluctuate together, since the resistance change due to a change in temperature will be the same for all gages. Applying the rule previously stated, the change of resistance in any gage is cancelled by the same change in the gage located in the adjacent arm of the bridge.

For this same reason the bridges of figures 2.6.2.3 and 2.6.2.4 will respond only to forces which bend the beam in the vertical direction shown. For example, if a tensile force was applied, it would increase the resistance of all gages the same amount, and according to the rule stated the effects will cancel.

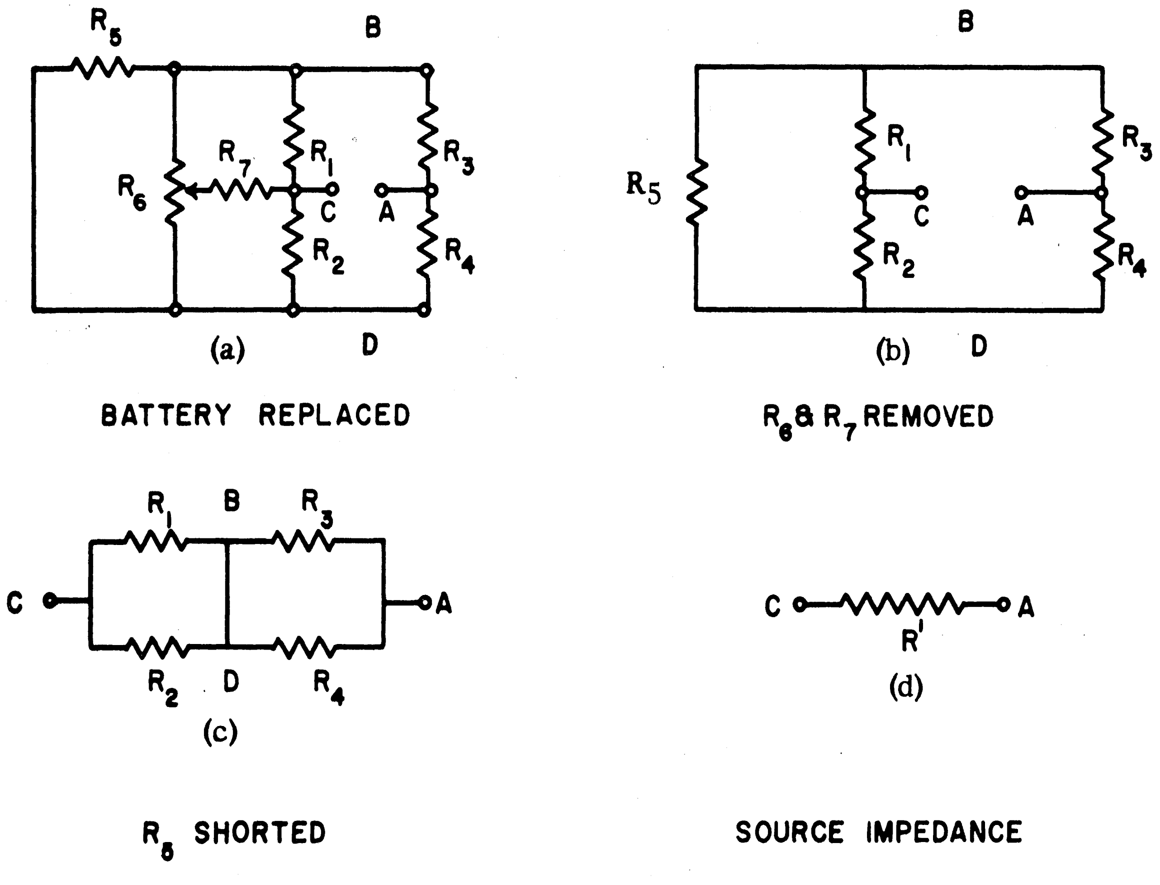
Care must also be taken to follow the rule when correlating the physical location of the gages with their positions in the circuit.

2.6.3 <u>IMPEDANCE OF RESISTANCE STRAIN GAGE CIRCUITS</u>. Figure 2.6.3.1 represents almost any strain gage bridge circuit, with R_1, R_2, R_3, and R_4 being gages or fixed resistors. A comparative order of magnitude of the various resistors is also shown in the figure.



TYPICAL CIRCUIT FOR STRAIN GAGES
FIGURE 2.6.3.1

The source impedance will be the resistance from A to C with the battery shorted, but to simplify the impedance analysis the circuit itself can be simplified as shown in figure 2.6.3.2.



SOURCE IMPEDANCE OF A STRAIN GAGE BRIDGE
FIGURE 2.6.3.2

The balancing resistors R_6 and R_7 are large compared to the gage resistance, and they may be removed as shown in b. The justification for removing R_5 is more subtle, and time will not be taken to justify it here other than to say that almost no difference would result in the calculation of the source impedance R' if R_5 was first considered to be zero, and then was considered to be infinite. The actual value of R_5 will be between these extremes, but here it is considered a short circuit as shown in C. From figure 2.6.3.2 c the source impedance is calculated as:

$$Z' = R' = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \qquad (2.6.3.1)$$

In a 4-gage bridge,

$$R_1 = R_4 = R + \Delta R \qquad (2.6.3.2)$$

and

$$R_2 = R_3 = R - \Delta R \qquad (2.6.3.3)$$

The impedance of the 4-gage bridge is found by substitution into equation 2.6.3.1:

$$Z' = R' = \frac{(R + \Delta R)(R - \Delta R)}{(R + \Delta R)+(R - \Delta R)} + \frac{(R + \Delta R)(R - \Delta R)}{(R + \Delta R)+(R - \Delta R)} \qquad (2.6.3.4)$$

$$Z' = R' = R \left[1 - \left(\frac{\Delta R}{R}\right)^2\right] \qquad (2.6.3.5)$$

The impedance varies with the term $\Delta R/R$, and will be maximum when ΔR is zero, or when the gages are not strained.

By a similar method for a 2-gage bridge, where:

$$R_1 = R + \Delta R \qquad (2.6.3.6)$$

$$R_2 = R - \Delta R \qquad (2.6.3.7)$$

$$R_3 = R_4 = R \qquad (2.6.3.8)$$

The impedance for the two-gage bridge is found to be:

$$Z' = R' = R \left[1 - \frac{1}{2}\left(\frac{\Delta R}{R}\right)^2\right] \qquad (2.6.3.9)$$

Repeating for a one-gage bridge:

$$Z' = R' = R\left(1 - \frac{1}{2}\frac{\Delta R}{R}\right) \qquad (2.6.3.10)$$

To provide an undistorted output voltage, the impedance of any transducer should remain constant, which is not the case for any of the strain gage bridges. However, the change in impedance will be small and hence can be neglected, as shown by the following example:

Example 2-2: Foil gages listed in figure 2.6.1.3 are used in the four gage bridge of figure 2.6.2.4. A variable force applied to the steel cantilever beam causes the stress at the gages to vary from zero to 30,000 psi. Is there a significant change in bridge impedance as the strain varies?

Solution: From equation 2.6.1.1:

$$\frac{\Delta R}{R} = f \cdot \epsilon = 2.1 \times \frac{30{,}000}{30 \times 10^6} = 0.0021$$

Applying equation 2.6.3.5 for a four-gage bridge:

$$Z' = R' = R\left[1 - \left(\frac{\Delta R}{R}\right)^2\right]$$

$$= 120\left[1 - 0.00000442\right]$$

It would seem apparent that the change in impedance due to $\Delta R/R$ is insignificant.

Example 2-3: A single semi-conductor gage as listed in figure 2.6.1.3 is used in a bridge with the 3 remaining arms being 120 ohm fixed resistors. The single gage is strained from zero to 0.001 inches/inch, and this strain is to be read on a display instrument having a load impedance of 30 ohms. Will distortion be serious?

Solution: According to equation 2.6.3.10 for a single gage bridge, the minimum source impedance will be:

$$Z'_{min} = R'_{min} = R(1 - \frac{1}{2}\frac{\Delta R}{R})$$

where

$$\frac{\Delta R}{R} = f \cdot \epsilon = 110 \times 0.001 = 0.110$$

$$Z'_{min} = 120\,(1 - \frac{1}{2} \times 0.110) = 120(0.945)$$

$$= 113.4 \text{ ohms}$$

The source impedance will be maximum when ΔR is zero, then

$$Z'_{max} = R'_{max} = R = 120 \text{ ohms}$$

Using equation 1.5.8 for maximum and minimum values of Z':

$$\frac{e_o}{e'_o} = \frac{1}{1 + \frac{Z'}{Z_L}} = \frac{1}{1 + \frac{120}{30}} = 0.200 \text{ (min)}$$

Similarly:

$$\frac{e_o}{e'_o} = \frac{1}{1 + \frac{113.4}{30}} = 0.209 \text{ (max)}$$

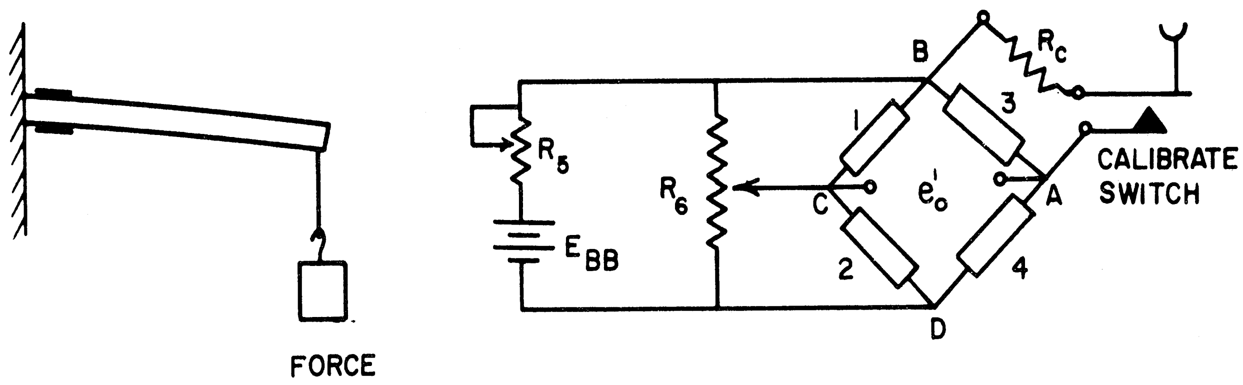
The output voltage is substantially attenuated, but distortion is not great, the ratio of the output voltages varying only 4.5% which is sufficiently accurate for most work. As mentioned in section 2.6.2, a single strain gage bridge is seldom used, and furthermore this example is about the worst extreme possible since the most sensitive single gage is used in conjunction with a display of exceptionally small load impedance.

It can be concluded that the source impedance of strain gage bridges is low and essentially constant, hence doesn't present serious impedance loading problems.

An exception might occur with very long lead lengths, say over 200 ft., which can cause high frequency deterioration and some amplitude attentuation. Knowing the lead line impedance, these effects can be calculated by the method of section 1.6 on frequency response. For lead lengths up to 50 ft. there are no loading problems with strain gage circuits.

2.6.4 CALIBRATION When the gages are used as force transducers, it is generally necessary to calibrate the gages by applying known forces to the part on which the gages are cemented, and record the resulting output voltage or display instrument deflection. For example, the bridge of figure 2.6.4.1 could be calibrated by applying known forces to the bar, and plotting this force against the bridge output voltage. This could be done statically, even though the force to be measured may be dynamic, or rapidly changing.

This is commonly called a mechanical calibration, and is often a tedious job, especially if the part onto which the gages are cemented has a complex shape, and is difficult to load and support. Furthermore, the setting of the gain might accidentally be changed between periods of use, thus requiring a frequent check of the calibration.



BRIDGE CIRCUIT WITH CALIBRATING RESISTOR
FIGURE 2.6.4.1

To eliminate the necessity of performing the mechanical calibration more than once, or in some cases of performing it at all, calibrating resistors can be used.

If a resistor is connected in parallel with one gage of a balanced bridge, as shown in figure 2.6.4.1, it will decrease the resistance of that arm of the bridge, and cause an output voltage to be developed. This has the same effect as straining that gage, hence connecting any given value of calibrating resistance into the bridge is equivalent to applying some force to the piece onto which the gages are cemented. The relation between the value of the calibrating resistor and the bridge output voltage can be found as follows:

Let $(e'_o)_c$ = bridge output from calibration

e'_o = bridge output from a known force

n = number of active gages

R = gage resistance, ohms

ΔR = change in gage resistance from force, ohms

R_c = calibrating resistance, ohms

ΔR_c = change in resistance of bridge arm when shunted by R_c, ohms

The bridge output, as stated in equation 2.6.2.14 and repeated here, will be:

$$e_o' = E_{BD} \frac{n}{4} \frac{\Delta R}{R} \tag{2.6.2.14}$$

Since the calibration resistance is switched across one gage only the output is,

$$(e_o')_c = E_{BD} \frac{1}{4} \frac{\Delta R_c}{R} \tag{2.6.4.1}$$

The change in resistance of the shunted bridge arm is the original resistance minus the parallel resistance, and

$$\Delta R_c = R - \frac{R\,R_c}{R + R_c} = \frac{R\,(R + R_c) - R\,R_c}{R + R_c}$$

$$\Delta R_c = \frac{R^2}{R + R_c} \tag{2.6.4.2}$$

and the relative change becomes

$$\frac{\Delta R_c}{R} = \frac{1}{1 + R_c/R} \tag{2.6.4.3}$$

It turns out that R_c/R will be very much larger than unity, and the unity term in the denominator can be neglected. Then with negligible error,

$$\frac{\Delta R_c}{R} = \frac{R}{R_c} \tag{2.6.4.4}$$

where the relative change produced by a calibration resistance is equal to the ratio of the gage resistance to the calibration resistance, a relationship fairly easy to remember.

Since the idea behind electrical calibration is to find an R_c that will produce an output equivalent to the output of a known force,

$$(e_o')_c = e_o' \tag{2.6.4.5}$$

Substituting from equation 2.6.2.14 and 2.6.4.1 gives

$$E_{BD} \frac{1}{4} \frac{\Delta R_c}{R} = E_{BD} \frac{n}{4} \frac{\Delta R}{R} \tag{2.6.4.6}$$

and, upon cancelling like terms,

$$\Delta R_c/R = n \cdot \Delta R/R \tag{2.4.2.7}$$

Since $\Delta R_c/R = R/R_c$, we can substitute, getting

$$R/R_c = n \cdot \Delta R/R \qquad (2.6.4.8)$$

from which we get the calibration resistance,

$$R_c = \frac{1}{n} \frac{R}{\Delta R/R} \qquad (2.6.4.9)$$

The calibration resistance is related to strain by the gage factor equation 2.6.1.1, repeated here:

$$\Delta R/R = f \epsilon \qquad (2.6.1.1)$$

from which the relationship of the calibration resistance and force or stress can be established.

<u>Example 2-4</u>: Foil gages as listed in figure 2.6.1.3 are used on the steel cantilever beam of figure 2.6.4.1. Determine the required magnitude of the calibrating resistor to be equivalent to a force which causes stress in the beam of 30,000 psi at the gages.

<u>Solution</u>: By equation 2.6.1.1:

$$\frac{\Delta R}{R} = f \cdot \epsilon = f \cdot \frac{\sigma}{E}$$

where f = gage factor = 2.1

σ = stress, = 30,000 psi

E = modulus of elasticity = 30×10^6

$$\frac{\Delta R}{R} = 0.0021$$

Then using equation 2.6.4.9:

$$R_c = \frac{1}{n} \frac{R}{\Delta R/R} = \frac{1}{4} \cdot \frac{120}{0.0021}$$

$$R_c = 14.3 \text{ K ohms}$$

The proceeding method shows how a calibration resistance can be calculated to correspond to a chosen force. In practice this is seldom done, since it is usually easier to experimentally determine the required value of the calibration resistance.

This is done by first completing the mechanical calibration, and then removing the weights or forces used for the calibration. Shunting an arbitrarily selected resistor across one of the gages will now cause a change in output voltage, and the resistor used can be said to be equivalent to the calibrating weight or force which caused that same change in output voltage during calibration.

Example 2-5: Experimentally find a calibration resistance that will correspond to a force in the calibration range of zero to 20 lbs. for the cantilever beam of figure 2.6.4.1.

Solution: The gain of the bridge circuit and oscilloscope are adjusted so that a 20 lb. force at the end of the beam causes the trace to deflect exactly 10 cm. After removing the 20 lb. force an arbitrarily selected resistor is shunted across one of the gages and the resulting trace deflection is observed. For this example, suppose the resistor was 50 K ohms, and the trace deflection 6.4 cm. Then from the original calibration:

$$F_c = 2 \text{ lbs/cm} \times 6.4 \text{ cm}$$

$$F_c = 12.8 \text{ lbs.}$$

In this case a 50 K ohm resistor corresponds to 12.8 lbs.

For future reference it would be wise to mark the beam of problem 2-5:

$$R_c = 50 \text{ K}; \quad F_c = 12.8 \text{ lbs.}$$

If the strain gages are precisely matched it will not matter which gage had been shunted with the calibrating resistance. However, since most gages are manufactured with an allowable variation in resistance, it would be wise to also indicate which gage had been shunted with the calibrating resistor.

Noting the linear relationship between the value of the calibrating resistor and the inverse of $\Delta R/R$ in equation 2.6.4.9, the following relation can be obtained for any given circuit:

$$F_{c_1} \cdot R_{c_1} = F_{c_2} \cdot R_{c_2} = \text{constant} \qquad (2.6.4.10)$$

where

F_{c_1} = known calibrating force corresponding to a known calibration resistor R_{c_1}

F_{c_2} = any other calibrating force corresponding to another calibrating resistor R_{c_2}

The use of this equation is illustrated in the following example.

Example 2-6: What calibrating resistors correspond to forces of 10 lbs. and 20 lbs. respectively for the strain gage bridge of example 2-5?

Solution: Applying the data from example 2-5 to equation 2.6.4.10:

$$F_{c_1} \cdot R_{c_1} = \text{constant}$$

$$12.8 \times 50\ K = 640 \times 10^3 = \text{constant, lb-ohms}$$

Then the resistor equivalent to 10 lbs. will be:

$$R_{c_{10}} = \frac{640 \times 10^3}{10} = 64\ K\ \text{ohms}$$

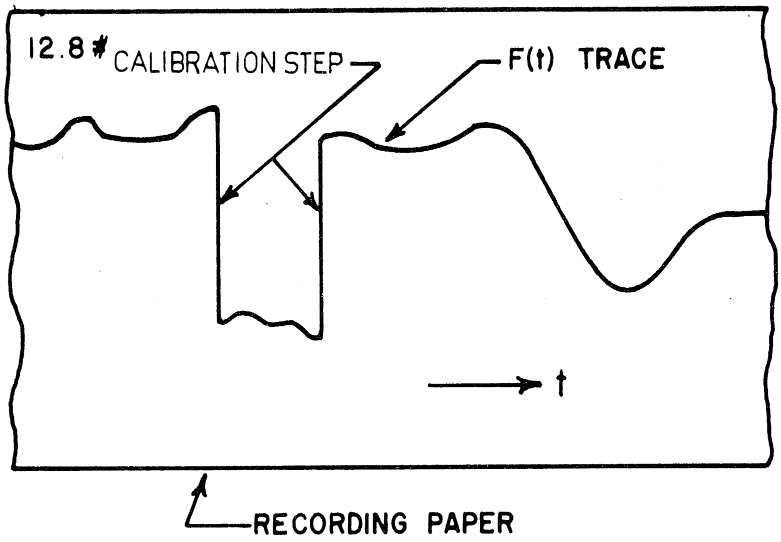
and similarly:

$$R_{c_{20}} = 32\ K\ \text{ohms}$$

The 50 K ohm resistor used in example 2-5 might seem like a rather good choice for an arbitrary selction, and one that probably would not occur on the first trial. It follows that a few trials might be necessary to get a resistor which causes a readable deflection of the display instrument without exceeding the calibration range.

One of the advantages of this calibration technique is that as long as the load beam is not changed the calibration holds for any display system regardless of gain setting or even battery power. For example, if at a future date a pen recorder, was used as the recording instrument, calibration would just require shunting the proper gage with R_c = 50 K. A step response corresponding to 12.8 pounds would result, and all signals could be linearly compared with the step amplitude without further adjustment. It might, for analysis purposes, be more convenient to take the time to experimentally adjust the circuit gain or pen recorder gain until a calibration deflection corresponded to a convenient pen response such as 6.4 cm., but this is not necessary.

Figure 2.6.4.2 illustrates an example of a continuously varying dynamic signal which has had a calibration step applied and released during the interval of recording. The speed and convenience of electrical calibration becomes obvious.



FORCE CALIBRATION
FIGURE 2.6.4.2

2.6.5 ADVANTAGES AND DISADVANTAGES OF STRAIN GAGES AS A FORCE TRANSDUCER. If it is at all possible to measure a force, it can usually be done with strain gages. Foil gages in a four-gage bridge can quite easily indicate forces which cause unit strains as small as 2×10^{-6} inches/inch, while semi-conductor gages can do the same for unit strain in the neighborhood of 1.5×10^{-7} inches/inch. The gages are small and are light in weight, hence rarely cause significant mechanical loading when attached to a mechanical part. However, care must be exercised to see that the introduction of special mechanical parts on which the gages are mounted does not appreciably reduce the spring rate of the mechanical device, as mentioned in section 1.4.

With an appropriate beam strain gages can also be used as a low inertia and frictionless displacement transducer, and are frequently used in accelerometers.

Impedance loading is virtually never a problem with strain gages, even when a very low impedance recorder is used. They are one of the few transducers which can be used directly with low frequency high sensitivity mirror galvonometers in an optical oscillograph without any intervening impedance adjusting amplifiers.

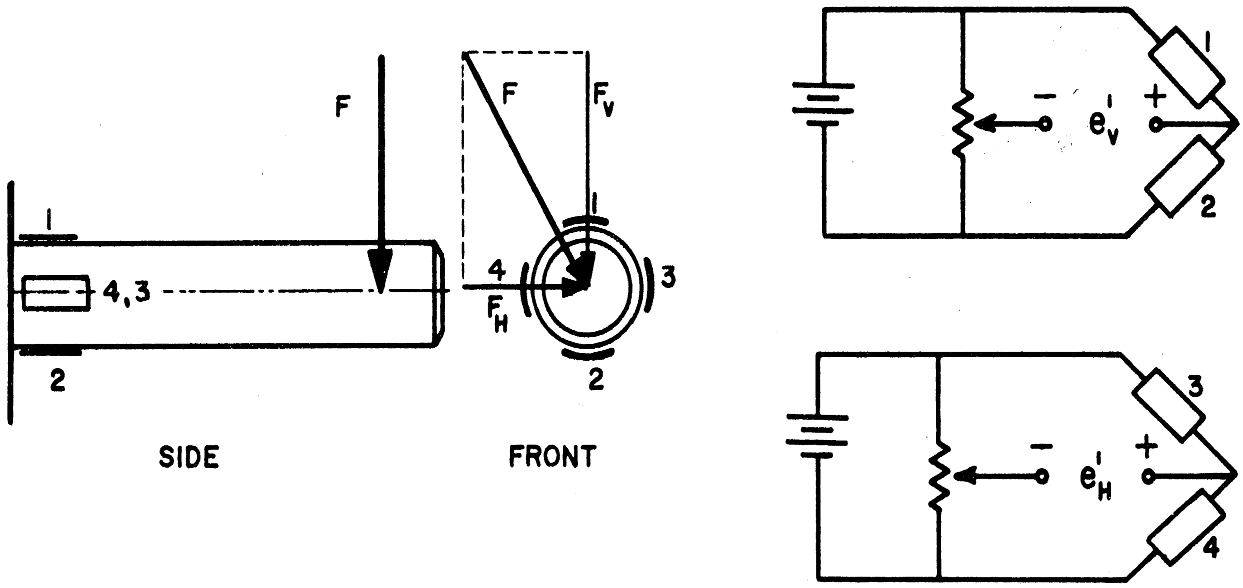
The output voltages are often very low, and distortion due to noise is often a problem.

The cost of semi-conductor gages is about ten times that of foil gages but since the semi-conductor gages produce about forty times the output signal, they often reduce the total system cost by eliminating an amplifier or some other electrical instrument.

Care is required in cementing gages to mechanical parts. Skill helps but is not required --- fortunately!

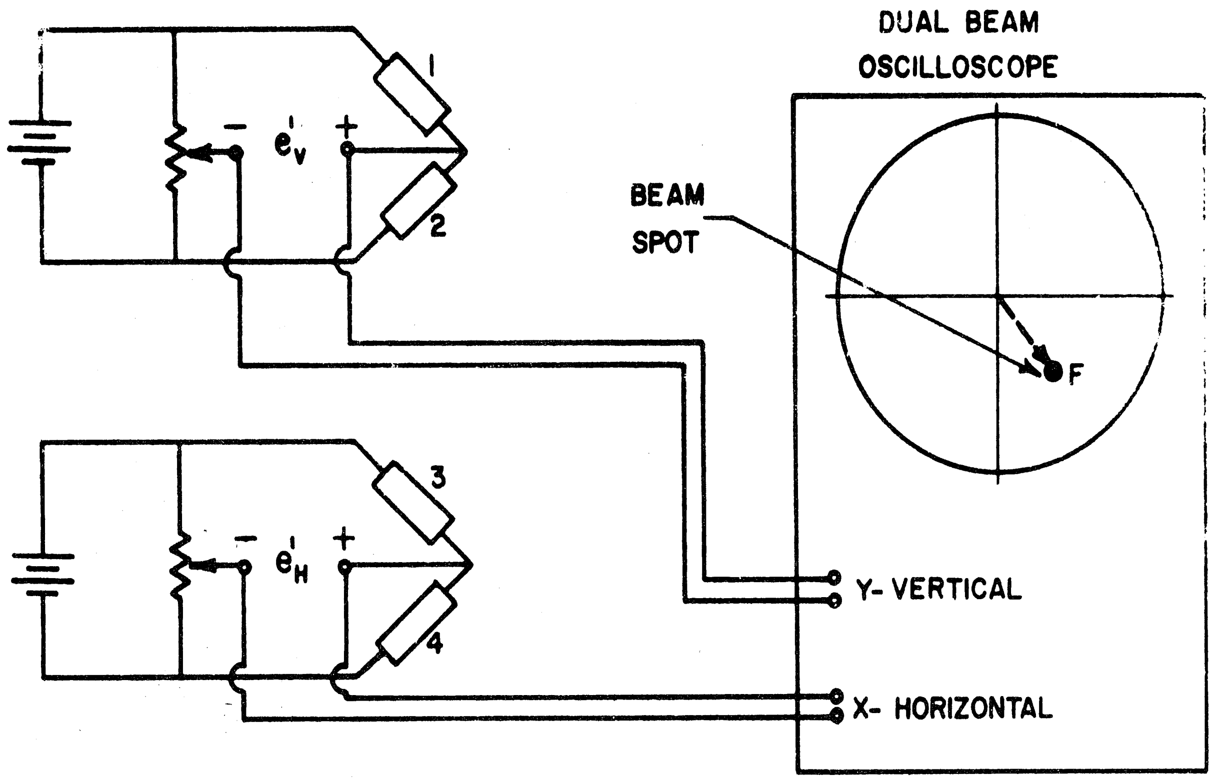
2.6.6 APPLICATIONS. The following paragraphs consider the use of the strain gages as force transducers. Since a force acting on a lever arm produces a twisting moment or bending moment, the measurement of such moments is included in the discussion of force measurements.

In a general case, one might wish to measure bending moments, twisting moments, or axial loads, and be able to tell one from the other although all might be present at the same time. A circuit for each will be considered, with a cylindrical bar used as an illustration.



BENDING MOMENT FORCE TRANSDUCER
FIGURE 2.6.6.1

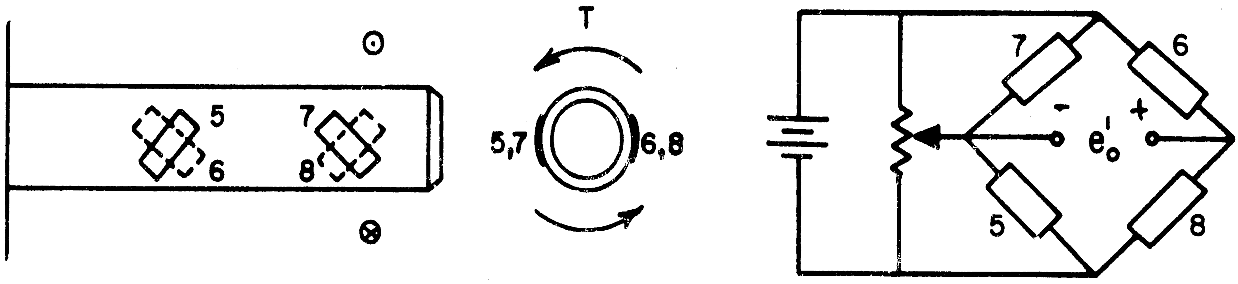
Figure 2.6.6.1 shows a cantilever beam with strain gages to measure the magnitude and direction of a bending moment, or of any force applied a known distance from the gages. Notice that two 2-gage bridge circuits are used, with a variable resistor to complete each bridge. Gages 1 and 2 measure the vertical component of the bending moment or of force F, while gages 3 and 4 measure the horizontal components. If the output of the two bridges are connected to the respective horizontal and vertical inputs of an oscilloscope, as shown schematically in figure 2.6.6.2 the magnitude and direction of the applied force F, or of the resulting bending moment, will be plotted by the displacement of the illuminated spot on the oscilloscope screen.



X-Y DISPLAY OF FORCE VECTOR
FIGURE 2.6.6.2

Observing the rule stated in section 2.6.2, the gages are used in adjacent arms which subtract, but since one gage is strained in tension and the other in compression, the results are additive. The bridges are temperature compensated, since a change in temperature will cause the same resistance change in both gages, and the effects will cancel. For this same reason the bridges will not respond to either torsion or axial loading, but only to bending.

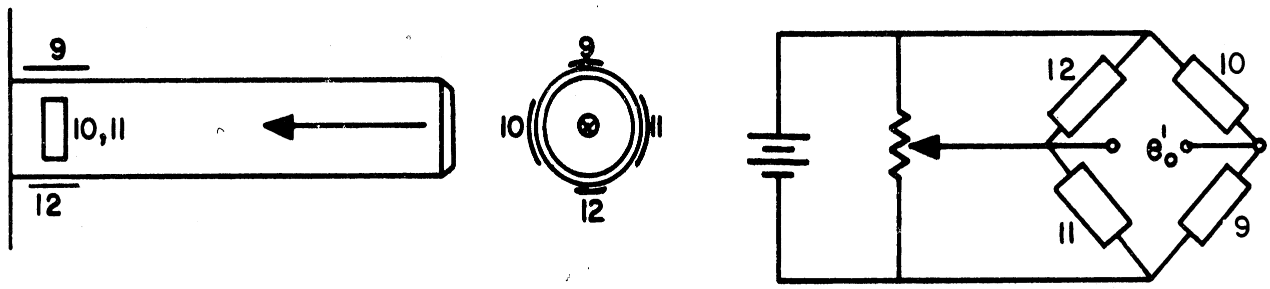
Figure 2.6.6.3 shows a 4-gage torsion bridge used to measure twisting moments only. Since the gages respond to tensile or compressive strains only, they are placed on the 45^{o} planes as shown. Gages 5 and 6 are 180^{o} apart on a left-hand helix, while gages 7 and 8 are 180^{o} apart on a right-hand helix. The bridge is connected as shown, with an external balance resistor. As previously explained, the 4-gage bridge is temperature compensated. This bridge reads torsion, but will not respond to bending, since whatever effect the bending might have on gage 5 it will have the opposite effect on gage 6, and since these effects are additive they will cancel. For this same reason gages 7 and 8 will not respond to bending. Axial forces, tension or compression will affect all gages the same, hence will cause no output. Thus it is seen that this bridge will not respond to either bending or axial forces, but only to torsion.



FOUR GAGE TORSION TRANSDUCER

FIGURE 2.6.6.3

Figure 2.6.6.4 shows a four-gage bridge used to measure axial forces only. Two gages, number 9 and 12, are placed parallel to the direction of the force, and the other two, number 10 and 11, are placed at right angles to the direction of the force.



FOUR GAGE AXIAL FORCE TRANSDUCER

FIGURE 2.6.6.4

Gages 10 and 11 are necessary for temperature compensation. If all four gages were parallel to the direction the force, the output would be zero, so gages 10 and 11 are rotated 90° so they will be strained in a direction opposite to that of gages 9 and 12. Gages 9 and 12 are fully active gages, and since both would be strained in the same direction when measuring axial forces, they are in opposite arms of the bridge. This circuit will not respond to either torsion or bending, but only to axial forces. Notice that the sidewise placed gages, which give temperature compensation, sense a lateral strain related to the longitudinal strain by Poisson's ratio but opposite in sign. They not only give temperature compensation but an increased output, hence they are an active gage but not fully active. Because of this they must not be counted as a full gage in an n gage bridge, but as a portion of a full gage according to Poisson's ratio. This example gives n = 2.6 for use in the output equation 2.6.2.14. For a Poisson's ratio of 0.3, n accounts for two fully active gages (9 and 12) and two partially active gages (10 and 11) giving respectively n = 2 + 2 (.3) = 2.6

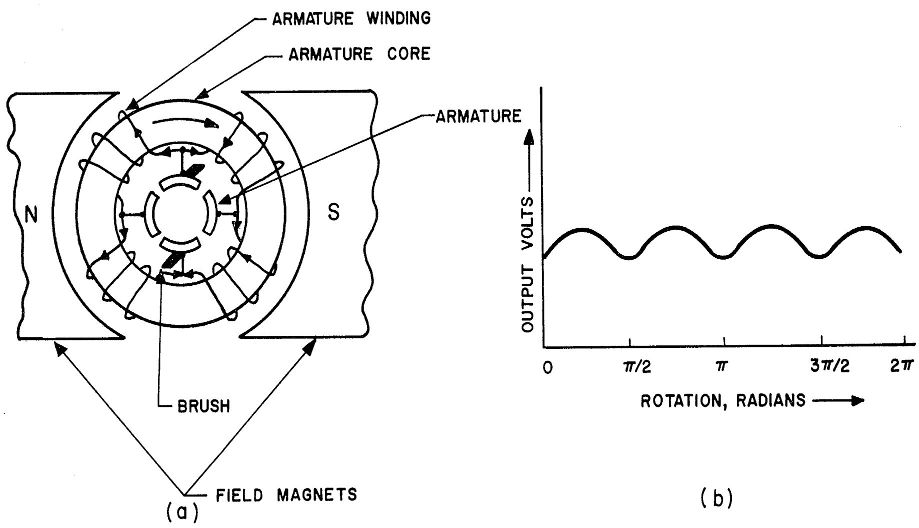
If all 12 of the gages used in the circuits just discussed were placed on the same bar, it would be possible to independently measure forces, bending moments, and twisting moments at any instant, although all were present. This same thing could be done for any part in an actual device, and the forces could be measured as the device operated.

As with the other transducers, it is desirable to put a variable gain resistor in series with the battery when the gages are used as force transducers, so the relationship between force and output voltage can be adjusted as desired. If the piece onto which the gages are cemented is only slightly strained, the output voltage will be very small, and must be put through an amplifier before being displayed. When this is so, a bridge-amplifier made specifically for strain gage bridges, such as the one shown in figure 3.3.1 can be used. A single instrument includes fixed resistors, a variable gain resistor, balancing resistors, calibration resistors, an amplifier, output meter and a convenient calibration switch.

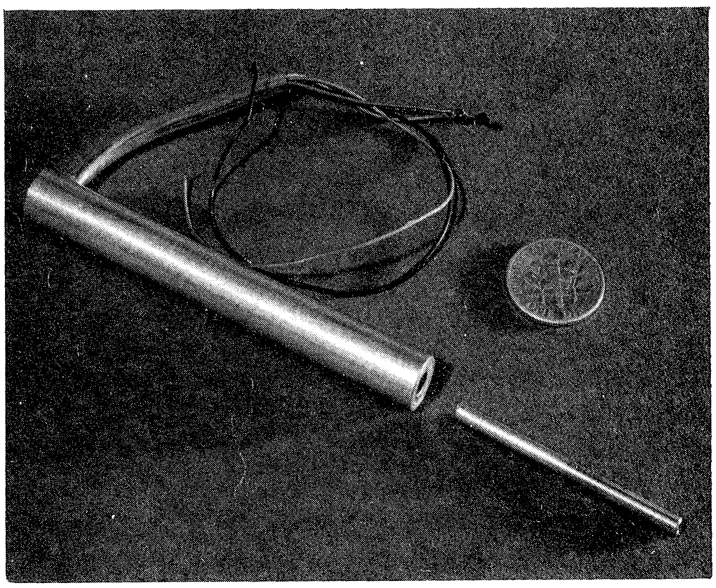
2.7 DIRECT CURRENT GENERATOR

2.7.1 PRINCIPLE AND CIRCUITS OF THE DC GENERATOR. A DC generator operates on the well known principle that if a conductor cuts magnetic flux lines, a voltage is induced in the conductor proportional to the rate at which the magnetic flux lines are cut. It can therefore be used as a velocity transducer, and depending upon its construction can measure either straight line velocity or rotary velocity, commonly expressed as rotary speed in revolutions/minute.

The rotary speed transducer is called a DC tachometer generator, and in physical appearance is similar to the AC tachometer generator shown in figure 2.8.1.1. The schematic arrangement of a construction using a permanent magnet and distributed windings is shown in figure 2.7.1.1 (a) while (b) shows the pulsations of the no-load output voltage taken from the brushes.



SCHEMATIC OF DC TACHOMETER GENERATOR
FIGURE 2.7.1.1

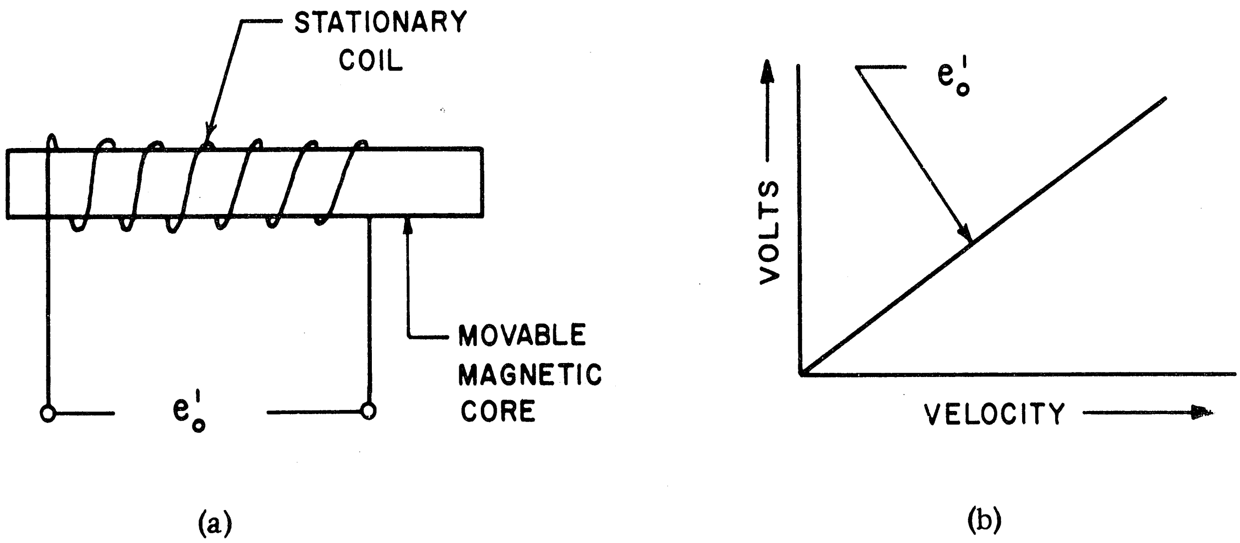


STRAIGHT LINE VELOCITY TRANSDUCER
FIGURE 2.7.1.2

The inherent ripple in the no-load output is minimized by having a large number of properly distributed coils.

Although any DC generator could be used as a speed transducer, the DC tachometer generators are designed and built with special features and extra precision to obtain the desired accuracy and linearity. The shaft of the tachometer generator must be coupled to the rotating shaft of which the speed is to be measured.

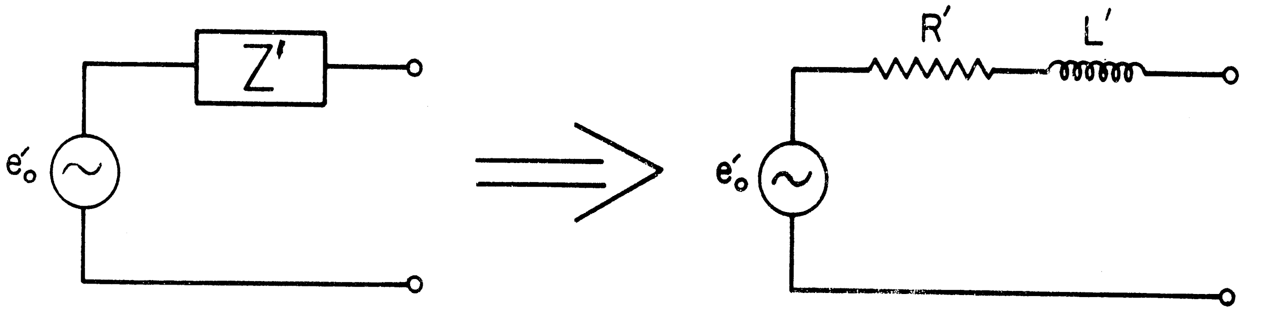
A DC generator used as a straight line velocity transducer is shown in figure 2.7.1.2. The transducer consists of a movable permanent magnet and a stationary coil. The schematic diagram and output voltage characteristics are shown in figure 2.7.1.3. The magnet must be physically attached to the moving part on which the velocity measurements are to be made.



SCHEMATIC OF VELOCITY TRANSDUCER
FIGURE 2.7.1.3

Since the DC tachometer and the velocity transducer are electric generators, no battery or other excitation is needed.

2.7.2 IMPEDANCE OF THE DC VELOCITY TRANSDUCER. The equivalent circuit of the dc linear velocity transducer is shown in figure 2.7.2.1.



CIRCUIT TO DETERMINE THEVENIN SOURCE RESISTANCE
OF THE VELOCITY TRANSDUCER
FIGURE 2.7.2.1

The source impedance is inductance and resistance in series. Referring to figure 1.2.1.1, the impedance expressed in operator form will be:

$$Z' = R' + p \cdot L' \qquad (2.7.2.1)$$

where
Z' = source impedance, ohms
R' = series resistance of the coil, ohms
L' = coil inductance, henries

The ratio of the output voltage under load to the no-load output voltage was developed in equation 1.5.8 repeated here for reference:

$$\frac{e_o}{e'_o} = \frac{1}{1 + \frac{Z'}{Z_L}} \qquad (1.5.8)$$

Then by substitution:

$$\frac{e_o}{e'_o} = \frac{1}{1 + \frac{R'}{L'} + p\,\frac{L'}{R_L}} \qquad (2.7.2.2)$$

This can be re-arranged to:

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{R'}{R_L}} \cdot \frac{1}{\frac{1}{1 + \frac{R'}{R_L}} \cdot \frac{L'}{R_L} p + 1} \qquad (2.7.2.3)$$

This equation is directly analogous to equation 1.6.2, which was developed in section 1.6 describing frequency response, and can be further reduced to a more general form:

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{R'}{R_L}} \cdot \frac{1}{\tau p + 1} \qquad (2.7.2.4)$$

where

$$\tau = \frac{1}{1 + \frac{R'}{R_L}} \cdot \frac{L'}{R_L},$$ the time constant of the circuit, seconds.

In those cases where the ratio R'/R_L is small and can be neglected, the above equations reduce to:

$$\frac{e_o}{e_o'} = \frac{1}{\tau p + 1} \qquad (2.7.2.5)$$

where

$$\tau = \frac{L'}{R_L},$$ the time constant of the circuit, seconds

It will be noted that equation 2.7.2.5 is the same as equation 1.6.6 in section 1.6. The frequency response for a sinusoidal input voltage is calculated in section 1.6, and is not repeated here.

Example 2-7: A linear velocity transducer having the following characteristics is used with an oscilloscope as a display instrument:

sensitivity: $e_o' = 500$ millivolts/inch/second

coil resistance: $R' = 11$ K ohms

coil inductance: $L' = 1.4$ henries

nominal working range: 0.8 inches

Will impedance loading cause significant distortion at low frequencies? At what frequency of sinusoidal motion of the magnet will the amplitude of the output voltage be attenuated to about 90% of the amplitude of the no-load output voltage?

Solution: Most oscilloscopes have an impedance of about 1 megohm in parallel with a very small capacitance which can be neglected. At low frequencies the impedance of the transducer coil can be considered to be essentially resistance. Applying equation 1.5.8 for low frequencies only:

$$\frac{e_o}{e_o'} = \frac{1}{1 + \frac{Z'}{Z_L}} = \frac{1}{1 + \frac{R'}{R_L}} = \frac{1}{1 + \frac{11 \times 10^3}{10^6}}$$

$$\frac{e_o}{e_o'} = 1.011$$

The attenuation of the output voltage at low frequencies is negligible.

Applying the analysis of section 1.6 to equation 2.7.2.5:

$$\omega_o = \frac{1}{\tau} = \frac{R_L}{L'} = \frac{10^6}{1.6} \text{ radians/sec.}$$

where ω_o = "break point" frequency

The corresponding frequency in cycles/sec. will be

$$f_o = \frac{\omega_o}{2\pi} = \frac{10^6}{2\pi\, 1.6} = 100{,}000 \text{ cycles/sec.}$$

Figure 1.6.4 shows that the amplitude ratio is attenuated to 89.2% at one-half of the break point frequency, so that:

$$f_{max} = \frac{1}{2} f_o = 50{,}000 \text{ cycles/sec.}$$

If the velocity variation of the transducer magnet was of some form other than sinusoidal, the output voltage under load could still be determined by solving the differential equation which is represented by equation 2.7.2.4 in operator form.

A similar method could be used for the DC tachometer generator, but it is not presented here.

2.7.3 CALIBRATION OF THE DC TACHOMETER GENERATOR AND THE VELOCITY TRANSDUCER. The tachometer can be calibrated by driving it from a variable speed output and using some other device such as a precision mechanical tachometer to measure speed. The reflecting pick-up described in section 2.5 might also be used to measure the speed during calibration by focusing its beam onto the rotating shaft or coupling of the tachometer and having a small reflecting surface cemented to the shaft or coupling. The output from the reflecting pick-up will appear as a series of pulses, and when fed into the oscilloscope the calibrated time sweep will allow the rotational speed to be calculated. In any system used for calibration care must be taken that the DC tachometer generator does not cause excessive mechanical loading, and so reduce the accuracy of the calibration.

Calibration of the straight line velocity transducer is commonly carried out by moving the magnet with a shake table having sinusoidal motion of known amplitude and frequency. The velocity of the shake table can then be calculated and compared with the voltage output of the transducer. Fortunately, the manufacturer of the velocity transducer supplies calibration data for each individual transducer.

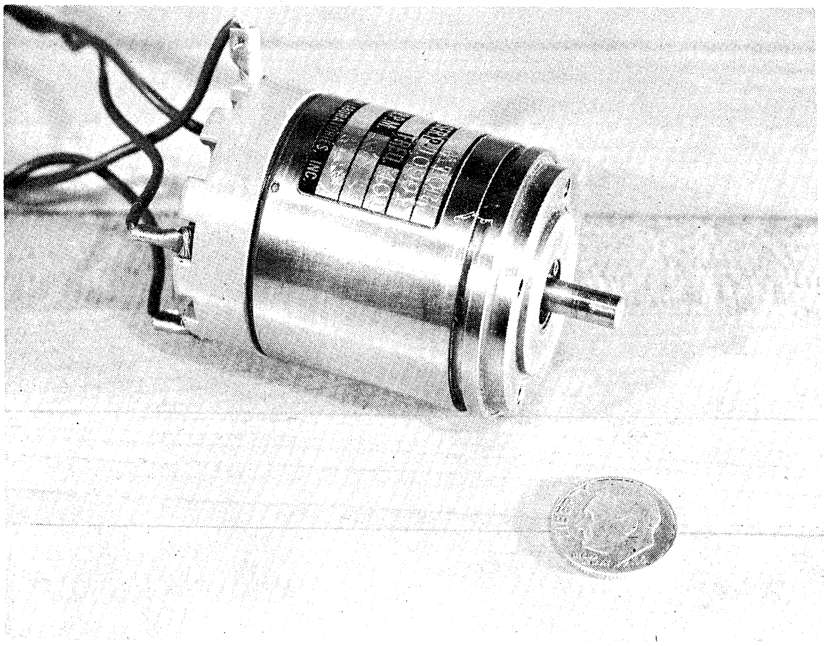
2.7.4 ADVANTAGES AND DISADVANTAGES OF THE DC TACHOMETER GENERATOR AND THE VELOCITY TRANSDUCER. Since both of these transducers are generators, the energy or power required to develop the voltage output is taken from the mechanical device to which the transducer is attached. Thus the development of the output voltage causes mechanical loading in addition to the mechanical loading caused by the addition of the inertia and friction of the transducers, which are apt to be significant in the DC tachometer generator.

The straight line velocity transducer can be used to measure vibratory type of motion. If the output voltage which represents velocity is fed into an integrating circuit as described in section 1.2.4, the output from the circuit will represent displacement. The velocity transducer could supposedly be used with a differentiating circuit to indicate acceleration, but as pointed out in section 1.2.4 this is seldom feasible because of the problem of distortion due to high frequency electrical noise.

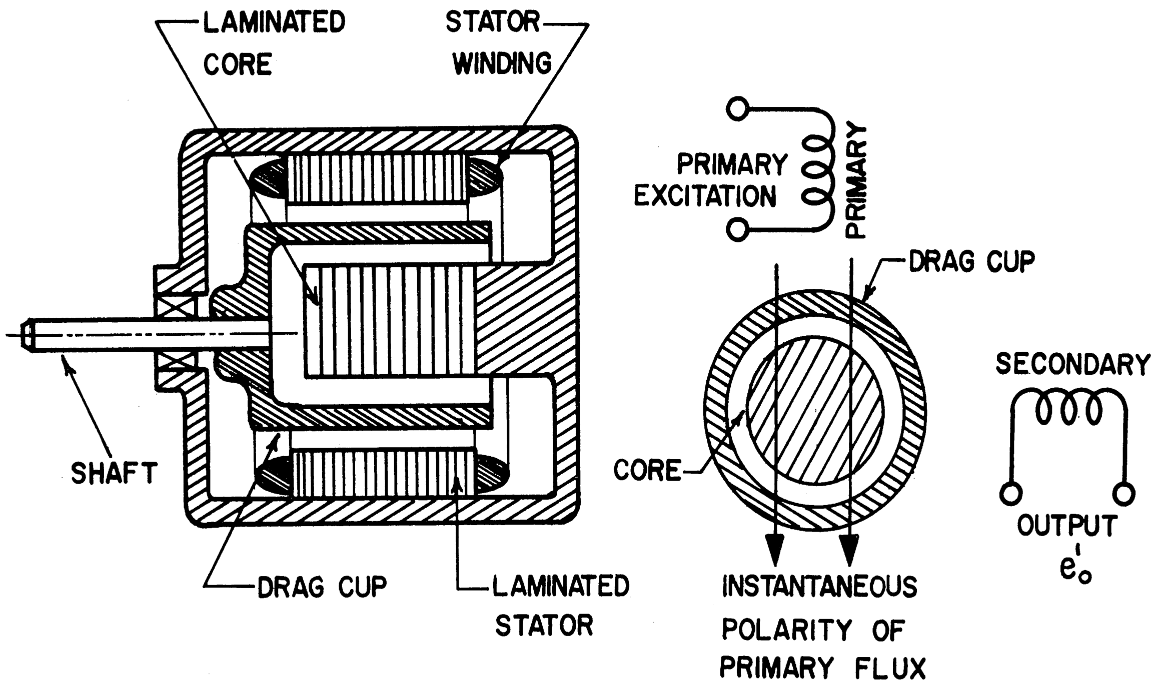
2.8 AC TACHOMETER GENERATOR

2.8.1 PRINCIPLE AND CIRCUITS OF THE AC TACHOMETER GENERATOR. The AC tachometer generator is a speed transducer, with a physical appearance as shown in figure 2.8.1.1. It is essentially a precision made two-phase drag cup induction motor, one winding of which is excited with a carrier frequency. A voltage of this same frequency but with an amplitude proportional to the speed of rotation of the drag cup is generated in the other winding, and this is the tachometer output.

Figure 2.8.1.2 shows the schematic arrangement of the AC tachometer generator. An audio-frequency oscillator can be used for excitation, and the shaft of the tachometer must be coupled to the rotating part on which the speed measurements are to be made.



AC TACHOMETER GENERATOR
FIGURE 2.8.1.1



PHYSICAL
(a)

CIRCUIT
(b)

SCHEMATIC OF AC TACHOMETER GENERATOR
FIGURE 2.8.1.2

2.8.2 IMPEDANCE OF THE AC TACHOMETER GENERATOR. The source impedance analysis of the AC tachometer generator is rather involved and is not presented here. However, there is no problem of impedance loading when used with a high impedance display instrument such as an oscilloscope.

2.8.3 CALIBRATION OF THE AC TACHOMETER GENERATOR. This transducer can be calibrated in the same way as described for the DC tachometer generator in section 2.7.3.

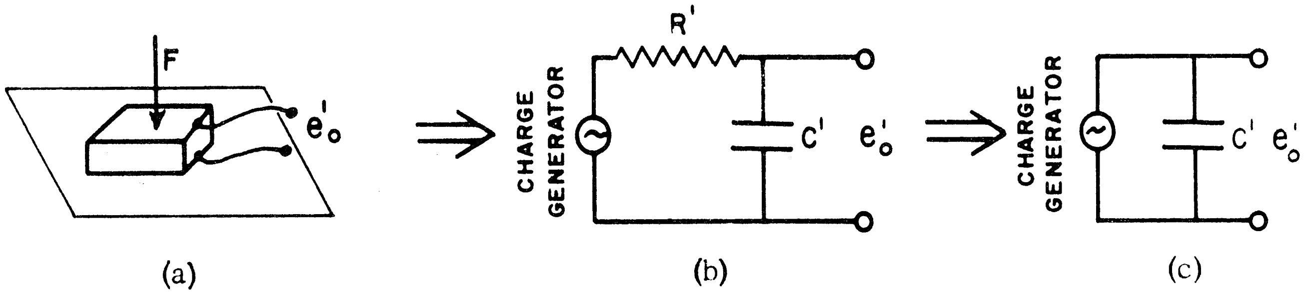
2.8.4 ADVANTAGES AND DISADVANTAGES OF THE AC TACHOMETER. The drag cup construction has very little inertia or friction, hence mechanical loading is generally less than when the DC generator is used. A well made tachometer gives a clean output, generally quite free of noise.

2.9 PIEZOELECTRIC CRYSTALS

2.9.1 PRINCIPLES AND CIRCUITS OF PIEZOELECTRIC CRYSTALS. There are a number of crystalline materials which produce an electrical charge when physically distorted. This characteristic, the piezoelectric effect, is often used for accelerometers and load cells. The equivalent circuit for a piezoelectric crystal is shown in figure 2.9.1.1. Typical values for the source capacitance C' and the source resistance R' are 5×10^{-14} farads and 2×10^{10} ohms respectively.

The crystal behaves as though it were a capacitor carrying a charge proportional to the applied force. Even though the resistance R' is large, it provides a short circuit path that will drain the capacitor charge. This means the crystal alone cannot be used to measure static or unchanging forces, but can only be used to measure dynamic forces. However, there is at least one exception, where a manufacturer has built a transducer-cable-amplifier package which prevents the charge from leaking off. This allows the crystal to be used for static measurements while retaining its other characteristics.

If a measurement circuit is connected to the transducer this will also drain the charge and cause impedance loading. For this reason special amplifiers with high input impedance are designed to be used with piezoelectric transducers. Cathode



EQUIVALENT CIRCUIT FOR A PIEZOELECTRIC TRANSDUCER
FIGURE 2.9.1.1

follower amplifiers with an input impedance of 10^{10} ohms and higher are often used. These are also called charge amplifiers. Unless a person is fairly knowledgeable with electronic circuitry it is usually best, when using these transducers, to buy a complete package including the transducer, connecting cable, and amplifier from one manufacturer. The output of the amplifier can then be used to feed the transducer signal to an oscilloscope or other display instrument.

2.9.2 ADVANTAGES AND DISADVANTAGES OF PIEZOELECTRIC TRANSDUCERS. In spite of some of the above drawbacks, piezoelectric transducers are very useful.

They are very sensitive, giving large signals for small forces. For example, a single piezocrystal load cell can produce signals large enough to measure forces from .01 to 5000 pounds.

The transducer is small which is especially useful for accelerometers where added mass will mechanically load a mechanism.

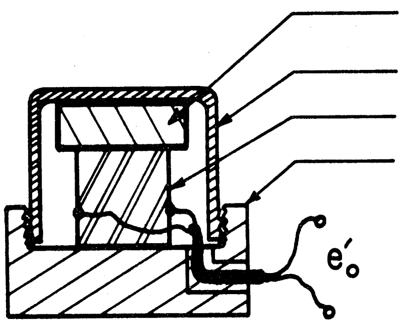
Because piezocrystals are very stiff they allow transducers to be built which have a high mechanical frequency response. For example, it is common for piezocrystal accelerometers to have a mechanical frequency response up to 35,000 cps.

Piezoelectric transducers are most often used for accelerometers, pressure cells and force cells in that order.*

2.10 ACCELEROMETERS

2.10.1 PRINCIPLES OF ACCELEROMETERS. All accelerometers produce voltages by directly or indirectly sensing the inertial reaction force acting on a mass subjected to an unknown acceleration. For this reason accelerometers utilize one of the transducers previously described in this chapter. Their circuits, impedances, calibration, advantages and disadvantages depend on the particular transducer used and have already been discussed in previous sections of this chapter.

There are two basic types of accelerometers which will be discussed in the following sections.

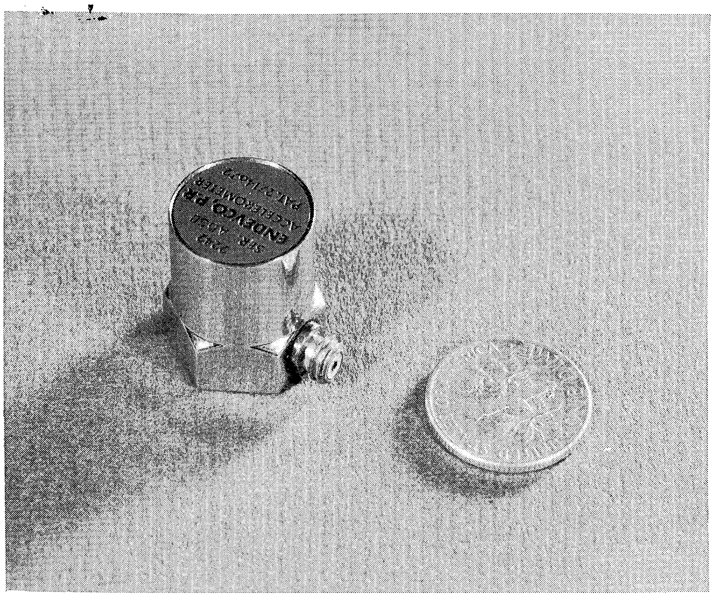


SCHEMATIC OF A CRYSTAL ACCELEROMETER
FIGURE 2.10.2.1

*The Endevco Corporation, 801 S. Arroyo Parkway, Pasadena, California, puts out an informative booklet on piezoelectric accelerometers which is helpful in describing their use and applications.

2.10.2 THE CRYSTAL ACCELEROMETER. Probably the most common is the crystal accelerometer employing a piezoelectric material as a voltage transducer which directly senses inertial reaction forces as illustrated in figure 2.10.2.1. The can, threaded to the base acts as a spring, which squeezes the mass against a properly oriented piezoelectric material. The stress on the piezoelectric material is approximately half the maximum allowable stress. If the base were to accelerate downward the inertial reaction force on the mass would act upward against the top of the can relieving the stress on the piezocrystal producing an output voltage, e_o' proportional to the acceleration. An acceleration in the upward direction would increase the stress on the crystal giving an output in the opposite direction.

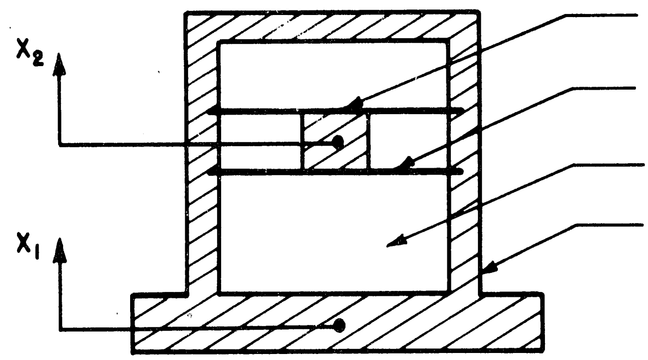
The main advantage of a crystal accelerometer is its small size, high output and very high frequency response. For example, the accelerometer shown in figure 2.10.2.2 weights one ounce, giving a seven millivolt no-load signal for one g of acceleration, measures up to $\pm$ 10,000 g's, has a low frequency response of 2 cps for a load impedance of 10^8 ohms, and has a high frequency response of 6000 cps. Accelerometers with a high frequency response of 35,000 cps are common. Most all are limited to a low frequency response of 1 cps or higher depending on the load.



CRYSTAL ACCELEROMETER
FIGURE 2.10.2.2

The disadvantages are the requirement of an amplifier with an unusually high load impedance and the inability to measure low frequency accelerations which are commonly encountered.

2.10.3 DISPLACEMENT SENSING ACCELEROMETERS. The second type of accelerometer measures the inertial force indirectly by sensing the deflection of a spring. Figure 2.10.3.1 is a schematic of a common spring-mass-damper system which accomplishes this.



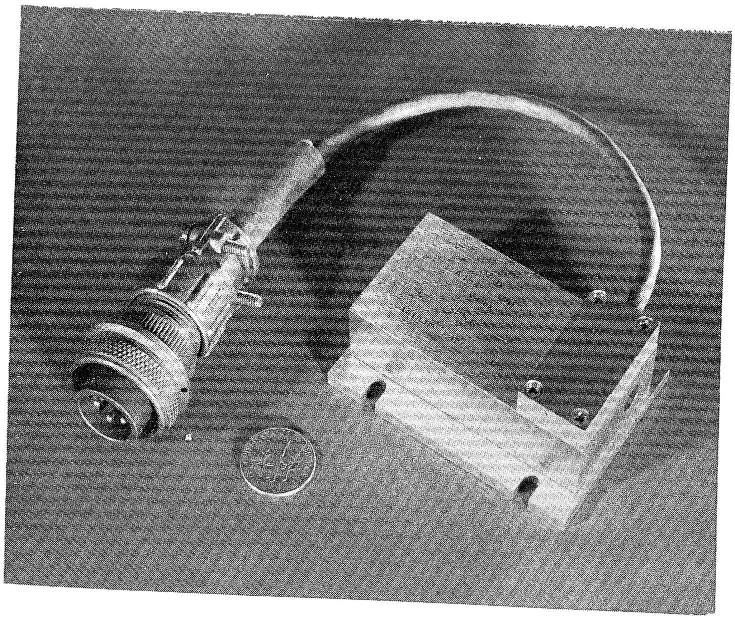
DISPLACEMENT SENSING ACCELEROMETER
FIGURE 2.10.3.1

An acceleration of the container will accelerate the inertial mass through the leaf springs. The springs will deflect due to the dead weight of the mass plus any inertial reaction force. If the accelerometer remains in the same orientation, the dead weight deflection, termed the static deflection, will be constant and any changes in deflection will be due to the inertial force alone which is proportional to acceleration. Any sensitive displacement transducer can be used to detect this deflection. Precision potentiometers and differential transformers are commonly used. Strain gages attached to the beam accomplish the same result. In fact, one of the most popular accelerometers of this type uses unbonded strain gage wire to function both as a spring and displacement sensor. This gives a compact design with higher natural frequencies. However, the frequency response of this type of accelerometer is much lower than the crystal type and the user must be cautious in their application.

The frequency content of the acceleration being measured must not exceed approximately one-half of the natural frequency of the sensing mass-spring-damper system. Accelerometers of this type generally have natural frequencies of 700 cps or below.

If there were no means for damping, such a device would tend to "ring", or oscillate for long periods when subjected to a sudden change in acceleration. Damping is accomplished by filling the container with a silicon fluid of the proper viscosity that is relatively insensitive to temperature.

The displacement sensing accelerometer has two important advantages. The first is its ability to measure low frequency accelerations down to zero or constant accelerations. This is especially helpful when studying zero gravity environments or low frequency vibrations such as occur in vehicle suspensions systems or building structures.



STRAIN GAGE ACCELEROMETER
FIGURE 2.10.3.2

The second important advantage is the simplicity which results from a low source impedance. They do not require special amplifiers to prevent distortion. This is especially true of the strain gage type.

Figure 2.10.3.2 shows a typical unbonded strain gage type of accelerometer.

The range of this accelerometer is ± 5 g's. It has a frequency response from 0 to approximately 40 cps, weighs 5 ounces, gives a 25 millivolt signal at full scale, and has a source impedance of 120 ohms. When connected into the simple circuit given for a four-armed strain gage bridge of section 2.6.2, it can be used directly with an oscilloscope, x-y plotter, or an optical oscillograph.

CHAPTER 3

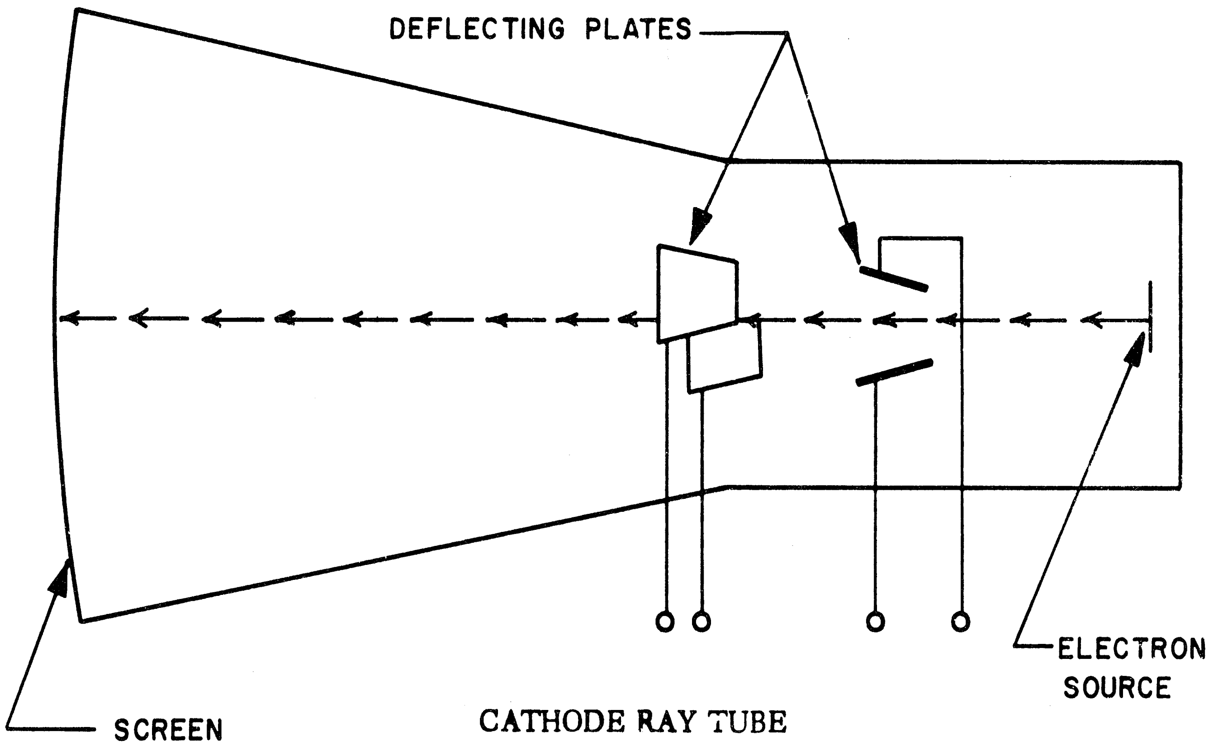
ASSOCIATED INSTRUMENTATION

3.1 INTRODUCTION

A certain amount of associated instrumentation and other equipment must be used with the transducers to complete the measuring system. Such instrumentation is often rather complex to understand in detail, but is fairly simple to use. Some of this instrumentation is briefly described in the following paragraphs, and operating instructions are given. It is advisable to study these instructions before using the instrumentation, and to carefully follow the instructions for any given piece of equipment.

3.2 CATHODE RAY OSCILLOSCOPE

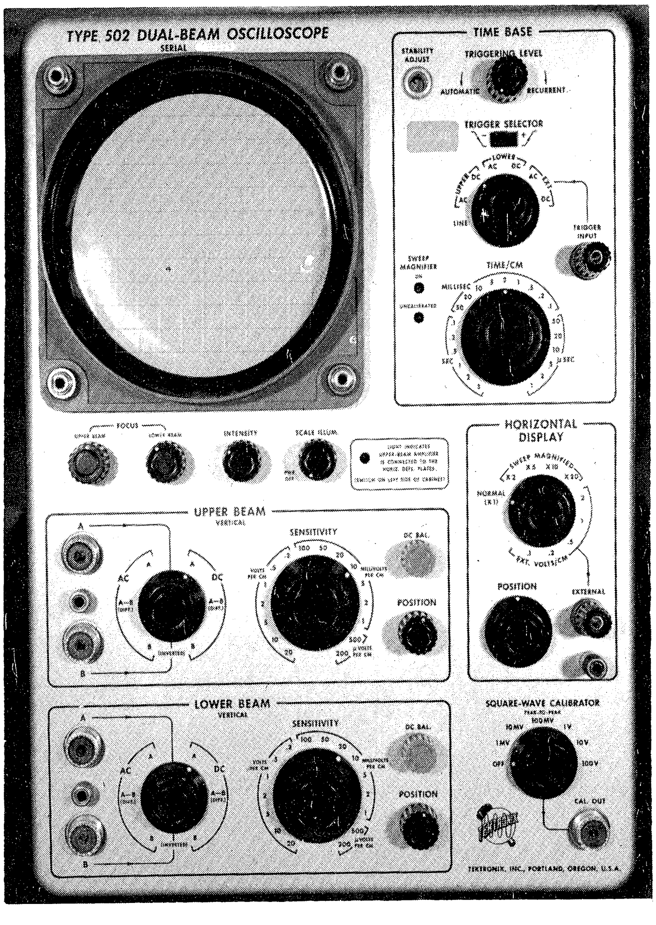
The basic parts of the oscilloscope consist of a glass tube with an electron gun at one end, a luminescent screen at the other, and pairs of deflecting plates in between. The electron gun shoots a stream of electrons at the screen, and as they impinge on the screen it glows brightly. The electron stream passes between two pairs of deflecting plates, one pair vertical and the other horizontal. Voltages applied to the pairs of plates will deflect the beam in proportion to the magnitude of the voltage. Figure 3.2.1 shows the schematic arrangement of the tube. With the two sets of plates it is possible to deflect the stream in any direction.



CATHODE RAY TUBE
FIGURE 3.2.1

A voltage which varies with time is applied to the plates which deflect the stream horizontally, thus causing the illuminated spot where the electrons strike the screen to move across the screen from left to right at a uniform rate which can be selected by a control on the panel of the oscilloscope. At high sweep rates the visible signal on the screen appears to be a continuous beam of light, and for this reason we shall refer to it as a beam.

The oscilliscope used in the laboratory, which is illustrated in figure 3.2.2, has two electron guns in the tube, thus providing two beams with a common time sweep but independent vertical deflections. The screen of the oscilloscope is graduated in centimeters.



DUAL BEAM OSCILLOSCOPE
FIGURE 3.2.2

One best learns to use the oscilloscope by using it, but a few instructions are helpful to get started.

Figure 3.2.2 shows the front panel of the oscilloscope. On the actual oscilloscope each knob is labelled, and this is all the explanation need for most of them. The others are explained as follows:

1. Directly underneath the screen are four small knobs in a horizontal row. The right hand one, labelled "scale illumination" is also the knob which turns the oscilloscope on and off. All work with the oscilloscope should begin and end with this knob!

2. There are two identical input panels, one for each beam, at the lower and middle left. These are plainly outlined by a solid black line, the top one labelled "UPPER BEAM", and the bottom one labelled "LOWER BEAM". At the left of each input panel are 3 terminals in a vertical row, the top one labelled A, the bottom one B, with the center one not labelled. This center terminal is the ground. The A and B terminals are connected in opposite directions to the vertical deflection plates. Thus a two wire input plugged into A and ground may show voltage above ground on the screen. This is commonly called a single ended input. The same input plugged into B and ground will show this same voltage, but in the opposite direction, hence the input is inverted. Using A and B together provides a differential input, and the screen will display the difference between the voltages supplied to A and to B.

3. Immediately to the right of the input plugs is an input selector knob which must correspond to the input being used. If placed on AC, only changes in voltage will appear on the screen. In all of your experiments in this course this knob should be reading DC.

4. Next right is the sensitivity control, which establishes the number of volts per cm deflection of the beam.

5. At the extreme right of the input panel are two small knobs, one above the other, labelled "DC Balance" and "Position". The position knob will change the position of the zero voltage level, hence will move the beam up or down on the screen. The DC balance knob is used to balance the vertical amplifier.

   This amplifier should be balanced each time before using the oscilloscope, as follows:

   > With no input into the oscilloscope, set the sensitivity knob at some low sensitivity, say 5 volts per cm. and center the beam on the screen by adjusting the position knob. Without further touching the position knob, slowly turn the sensitivity knob clockwise a step at a time, watching the beam on the

screen. As the beam moves from its original center position bring it back with the DC balance knob. Continue this until the maximum sensitivity is reached and the beam is still centered. The amplifier should now be balanced, and it should be possible to turn the sensitivity knob from one end to the other without any substantial change in the position of the beam on the screen.

After balancing the amplifier, the DC balance knob should not be further adjusted.

6. To the right of the screen is the time base panel, outlined by a solid black line and plainly labelled. The bottom knob in this panel controls the sweep rate, and is labelled time/cm.

7. The upper knobs in this time base panel control the triggering operation, and are very important in using the oscilloscope. When the triggering level knob is fully clock-wise at the re-current position the beam sweeps across the screen from left to right, quickly returns to the left and repeats. At any other position of the triggering level knob, the beam will not start its sweep until the voltage has reached some specific level and this level can be varied by adjusting the triggering level knob. The further this knob is turned counter-clockwise, the lower the voltage level at which it will trigger, or start its sweep.

   If the voltage is to start low and increase to the triggering level, the trigger selector switch must be on plus. If it is to start high and decrease to the triggering level this switch must be on minus. Thus it triggers on a plus or minus slope of the trace. Just below this switch is the trigger selector knob which selects either the upper or lower beam for triggering, or an external source. An external voltage, connected at the proper time by a switch, can also be used to trigger the oscilloscope by plugging it into the external trigger input jack.

8. Underneath the time-base panel is the horizontal display panel. This contains the sweep magnifier, which we shall probably have no occasion to use. Be sure it is at its normal position when using the horizontal scale for time measurements.

   An external jack is provided to allow the horizontal sweep to be driven by an external source such as a transducer. When this is used the sweep magnifier knob must be set in the external volts/cm range.

   The position knob in this panel adjusts both beams horizontally.

9. In the lower right corner is the square wave calibrator. If the calibrator output jack is connected to the A or B inputs of either beam a square wave will appear on the screen having the amplitude indicated by the square wave calibrator knob setting. This can be used to check the accuracy of the sensitivity settings. The square wave calibrator should always be turned off when not in use.

As a means of getting acquainted with the use of the oscilloscope it is a good idea to first balance both vertical amplifiers after they have been deliberately unbalanced by turning the DC balance knobs, then calibrating with the square wave calibrator. The square wave should be triggered, and the effect on the starting point of the triggered wave should be observed as the triggering level and triggering slope are changed.

On the left hand side of the oscilloscope case toward the rear of the instrument there is a switch which disconnects the time sweep and at the same time connects the upper beam input to the horizontal deflection plates. This converts it to a single beam oscilloscope in which the input to the upper beam panel drives the beam horizontally, while the input to the lower beam panel drives the beam vertically. Thus the oscilloscope becomes a high speed x-y plotter, plotting the two inputs against one another. When there is no input the beam becomes a stationary dot. If allowed to stand in one placewith high intensity the dot will burn a permanent hole in the screen coating. Care must always be exercised to prevent this by either not allowing the beam to remain a dot when not in use, or by turning down the intensity, or by dulling the focus.

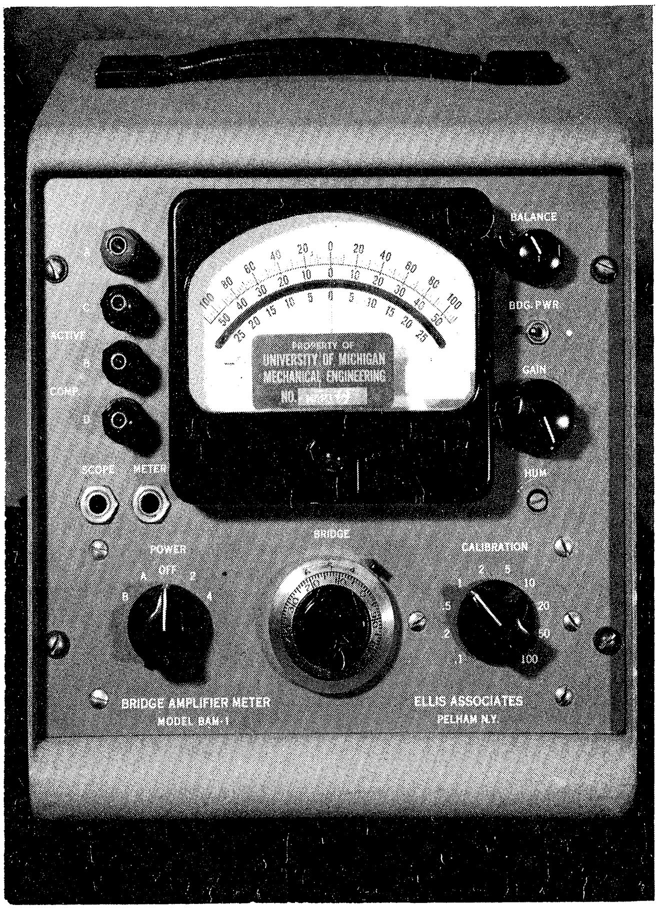
3.3 BRIDGE AMPLIFIER

When discussing resistance strain gages it was mentioned that an amplifier was often needed, and that this amplifier was contained with variable resistors to complete and balance the bridges, batteries, and calibrating resistors in a common housing. The front panel of such an amplifier unit is shown in the photograph of figure 3.3.1. In figure 3.3.2 a sketch of the front panel is repeated along with simplified schematic circuits containing a two gage bridge and a four gage bridge.

Instructions for operating are as follows:

1. In the lower left hand corner is a knob labelled POWER. This is used to turn the amplifier on and off, and to check the batteries which are inside.

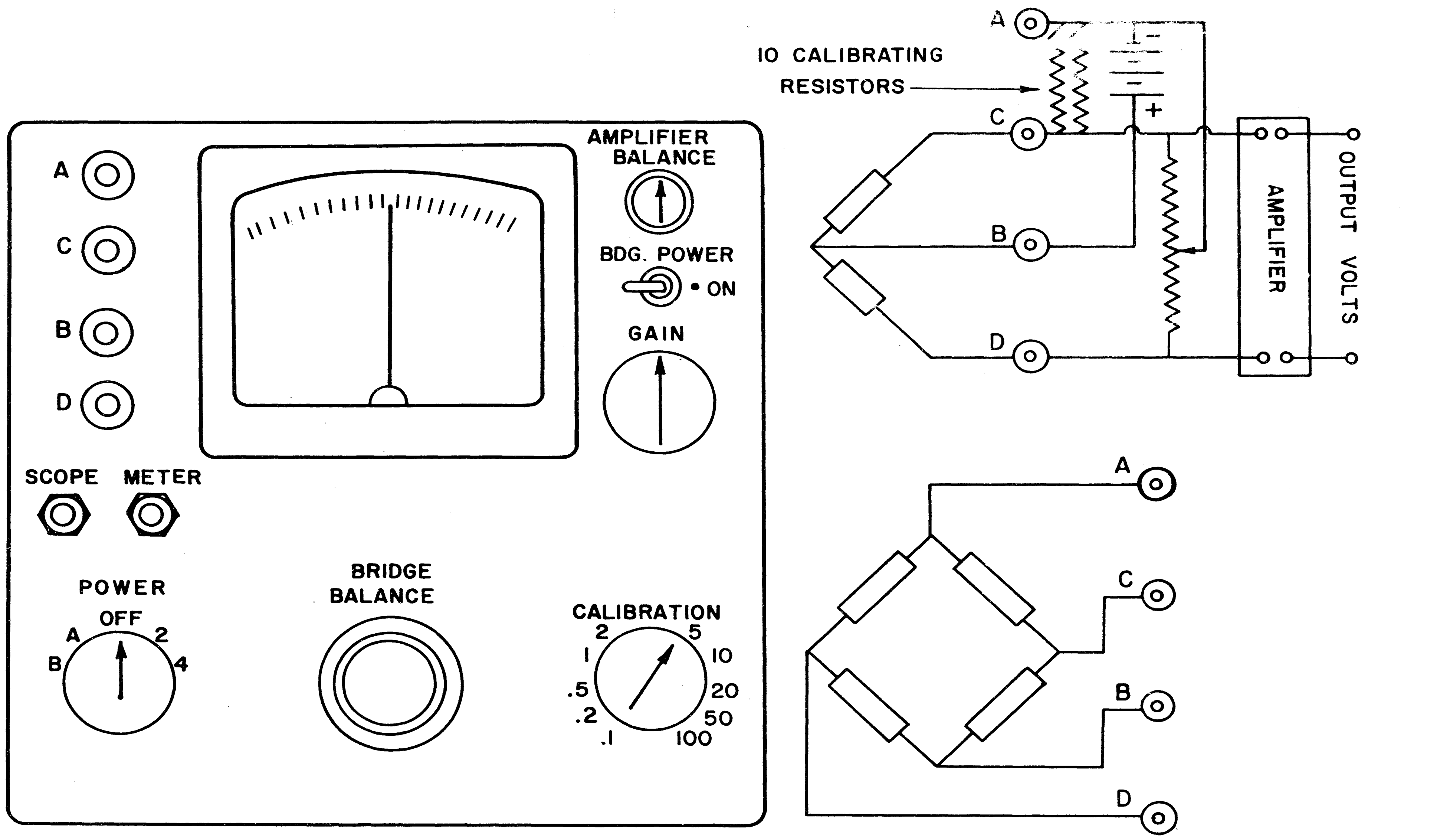
   The batteries should always be checked before using by turning the power knob first to A, then to B. At each position the meter should read at least 75 on the 0-100 scale. If either reading is below this, the corresponding battery should be replaced.



BRIDGE AMPLIFIER
FIGURE 3.3.1

The amplifier should be turned on to either the 2 or 4 position, depending upon whether it is to be used with a 2 or 4 gage bridge. It should warm up for at least 15 minutes before being used.

2. The strain gage bridge is connected to the terminals labelled A, B, C, D in the upper left corner, as



FRONT PANEL OF BRIDGE AMPLIFIER WITH SIMPLIFIED SCHEMATIC
TWO AND FOUR GAGE BRIDGE CIRCUITS
FIGURE 3.3.2

shown in the circuits of figure 3.3.2. The bridge is connected to the battery by moving the BRIDGE POWER toggle switch to the right. This switch is located near the upper right corner. The bridge power switch must be turned off while the bridge is being connected to the terminals of the amplifier.

3. After suitable warm-up, the amplifier must be balanced with the amplifier balance knob in the upper right corner. This is done by turning off the bridge power switch, then turning the amplifier balance knob until the meter reads zero. The bridge power switch can then be turned on again.

4. After balancing the amplifier, the bridge must be balanced. To do this all load must be removed from the piece onto which the gages are cemented so that the bridge is not strained. The lower center knob labelled BRIDGE is then turned until the meter reads zero. This bridge knob adjusts the balance resistors of figure 3.3.2.

5. The meter has 3 arbitrary scales which can be used to read pounds, torque, etc., depending upon the calibration. When calibrating the gages and reading on the meter, the GAIN knob at the right can be adjusted to obtain the desired relation between scale reading and load.

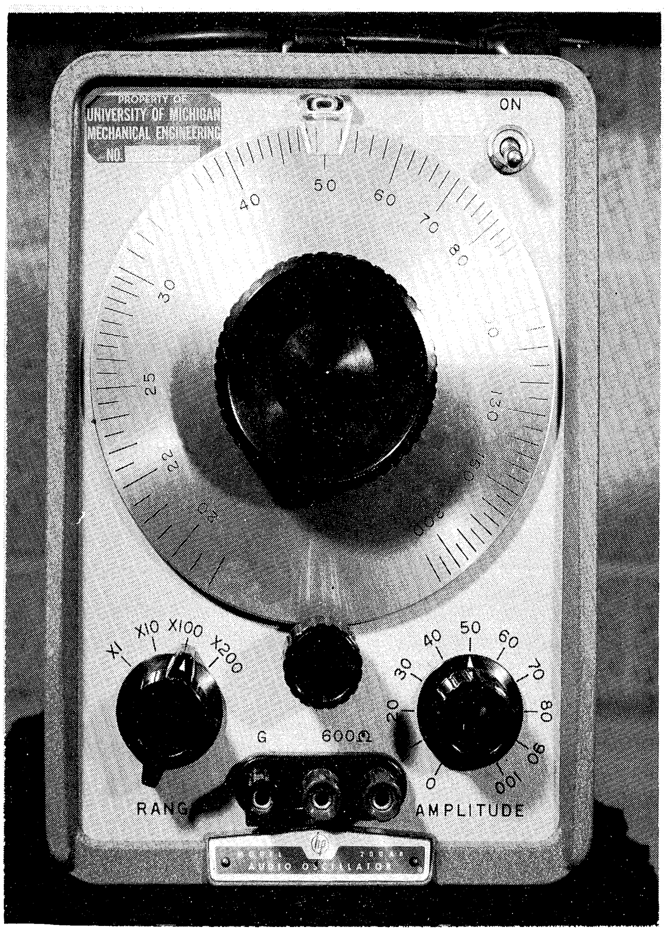
6. In the lower right corner is the calibration switch which selects and connects the calibration resistors into the circuit. There are 10 such resistors, designated by number from 0.1 to 100. So far as our use is concerned we can consider these as simply identification numbers. The desired resistor is selected by turning the calibration knob. The selected resistor is connected into the circuit by holding the knob in, and is disconnected by simply releasing the knob.

7. If it is desirable to use the oscilloscope as the display device rather than the meter on the amplifier, a cable to the oscilloscope input is plugged into the jack labelled SCOPE. This disconnects the meter from the circuit. All previous instructions are still valid, except that the zero voltage for the balanced bridge is read on the oscilloscope, etc.

8. When finished with the amplifier, both the bridge power switch and the power switch are to be turned off.

3.4 AUDIO OSCILLATOR

An audio oscillator develops an AC voltage, the frequency and amplitude of which can be varied. The front panel is shown in figure 3.4.1. The frequency is adjusted by the large dial knob in the center, and the RANGE knob at the lower left. The actual frequency in cycles per second is the product of the range setting and the number on the large dial, and can be varied from 20 cycles per second to 40,000 cycles per second. The amplitude is adjusted by the AMPLITUDE knob in the lower right corner.

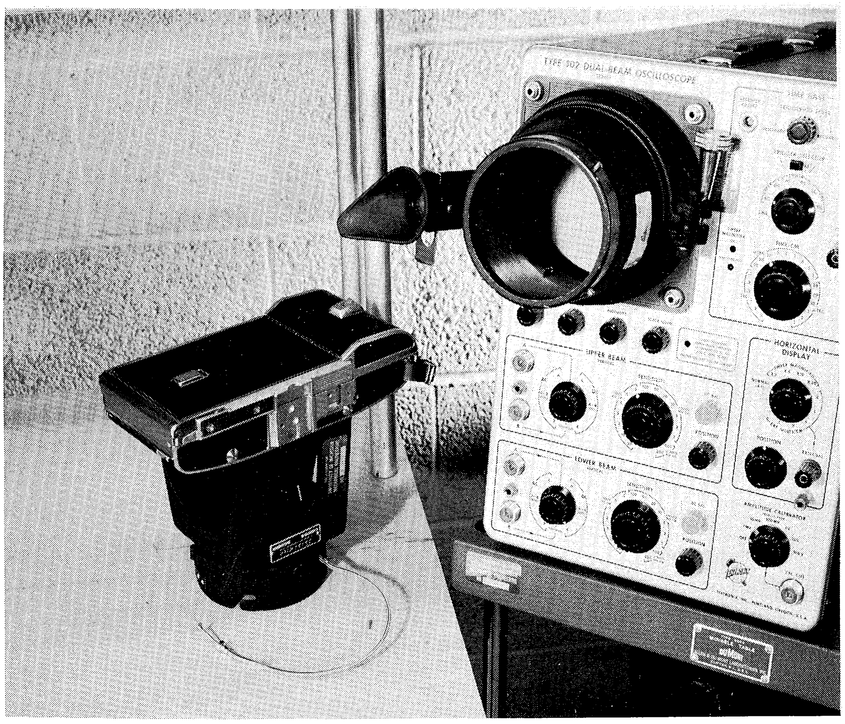


AUDIO OSCILLATOR
FIGURE 3.4.1

The oscillator output voltage is available at the 3 binding posts at the center near the bottom. The ground terminal on the left and the middle terminal must be connected together, and the output is taken between the right hand terminal and ground. In this course we shall use the oscillator as an exciter for the differential transformer transducers.

3.5 OSCILLOSCOPE RECORDING CAMERA

A Polaroid camera is used to record what is displayed on the oscilloscope. This is a fixed-focus camera with an adapter which clamps onto the oscilloscope as shown in figure 3.5.1. There is a small window in the adapter through which you can view the display to be photographed.



POLAROID CAMERA
FIGURE 3.5.1

The camera must be handled very carefully at all times, whether or not it is being used to take pictures. The following rules should be followed:

1. Carefully mount the adapter only on the oscilloscope so that the knurled head clamping screw is vertical and on the right. Push the adapter all the way onto the mounting sleeve of the oscilloscope but do not over do it, then tighten clamping screw.

   Notice that there are two clamping screws on the adapter, one for holding the two parts of the adapter together, and the other for clamping the adapter to the oscilloscope. Do not loosen the screw which holds the two parts of the adapter together, since this will allow the camera to get out of focus.

2. Mount the camera on the adapter and twist into place. Adjust adapter and camera positions until the vertical edges of the camera are parallel to the edges of the oscilloscope case. Always hold onto the camera if the adapter clamping screw is loosened for adjustment while the camera is mounted on the adapter.

3. If a single shot triggered trace is to be phtographed, set the lens between f4 and f5.6 and the time at B. If instead the trace is repetitive or constant, the shutter shutter should not be set at B, but should be timed so as to get one full sweep of the trace. Do not touch the glass lens.

4. To photograph a single shot triggered trace, hold the camera shutter open with the cable, trigger the trace, then close the shutter. For the repetitive or constant trace just snap the shutter. In either case a little practice may be necessary to get the beam intensity and the scale illumination intensity such that you will get clear, fine lines on the picture.

5. After taking the picture, flip the red button to its opposite position to release the film, pull back the cutter bar, support the camera by holding up on the strap with your left hand, pull down on the film with your right hand until it stops, close the cutter bar and tear off. Wait ten seconds, then open the back of the camera and peel off the picture. Close the back before taking the next picture.

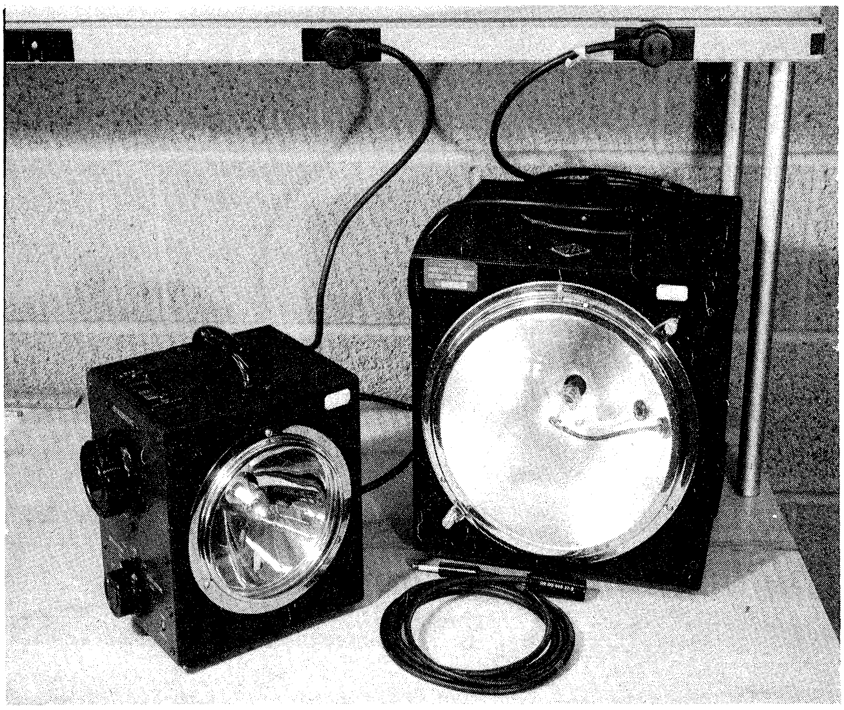
6. To remove the camera from the adapter, press the release clip, rotate, and remove. Do not set the camera down on the red button nor on the cable, but instead stand it on its bracket end whenever it is not being used, as shown in figure 3.5.1.

7. When new film is needed, stand the camera on its bracket, open the back, and follow the instructions on the new film.

3.6 STROBOTAC AND STROBOLUX

The strobotac is a stroboscopic tachometer consisting of a flashing light and a means of varying the rate of flashing. When the rate of flashing coincides with the speed of any rotating or oscillating mechanical device, the device appears to be stationary when the light is played on the device. The rate of flashing is then read directly from the strobotac, thus establishing the speed of the mechanical device.

The strobolux is a separate unit which can be connected to the strobotac to provide a high intensity light source which will flash with the strobotac. The strobolux is used where the surroundings are very bright, or for measuring low speeds. The strobolux and strobotac are shown in figure 3.6.1.



STROBOTAC AND STROBOLUX
FIGURE 3.6.1

Calibrating instructions and operating instructions are printed on the back of the strobotac, and so are not repeated here.

When using the strobotac it is best to start at a flashing speed greater than the speed to be measured, and work down until the part on which it is focused appears to stand still. When the flashing rate is a whole multiple of the speed of the mechanical device, multiple images will appear, all seeming to be stationary, but out of phase with one another. For example, when the flashing rate is twice the speed of the device two images will appear 180 degrees apart. As the flashing rate is reduced from a rate greater than the speed of the device, the flashing rate coincides with the speed of the device the first time a single image occurs.

If the light is flashed at a rate which is a sub-multiple of the speed of the mechanical device, such as 1/2, 1/3, 1/4. etc., a single image will appear just as though the flashing rate was the same as the speed of the device. Thus it follows that the highest flashing rate at which a single image occurs is the correct one, and coincides with the actual speed of the device.

When measuring the speed of gears, ball bearings, wheels with spokes, etc., a chalk mark or some other similar reference mark must be put on one tooth, ball, or spoke, and this mark used as an image. Otherwise every multiple or sub-multiple of the actual speed will provide what appears to be a single image.

CHAPTER 4

LABORATORY TECHNIQUES

4.1 INTRODUCTION

The transducers and associated instrumentation described in the previous chapters are useful only if they can be put to work in the laboratory to obtain valid measurements. The usefulness of the measuring system and the validity of the results obtained will largely depend upon the laboratory techniques which are used. It is generally necessary towork with rather exacting care, for the physical variables to be measured are apt to be very small, the rate of change very large, the instrumentation very sensitive, and the mechanical device very cantankerous. If on top of this the worker is all thumbs, acts like the proverbial bull in the china shop, or is just plain careless, the results will be mediocre at best.

Proper laboratory techniques are largely predicated on the simple rules of good housekeeping and common sense, and so should not be difficult to understand or to follow. It is just as easy to learn and use correct techniques as it is to learn and use the wrong ones, but the results are apt to differ considerably.

4.2 CARE OF EQUIPMENT AND LABORATORY

Proper laboratory technique starts with proper care of the laboratory and all of its equipment. The yellow numbers found on much of the equipment is the price in dollars---a gentle reminder to treat it carefully and well!

The following are some specific requests concerning the care of equipment and of the laboratory:

1. Handle all equipment carefully at all times. The equipment is not easily damaged electrically, but it can be very easily damaged mechanically by handling it roughly, dropping it, turning knobs beyond their limit of travel, improper clamping, over tightening, running at excessive speeds, etc.

2. Return all instruments, tools, patch cords, etc., to their proper place as soon as you have finished with them.

3. Keep instruments in cases or holders when these are provided.

4. Report any breakage or mal-functioning to your instructor at once.

5. Abide by the common rules of good housekeeping by keeping your work bench clean and neat at all times, by neatly putting everything away at the end of each period, and by disposing of all waste-paper, trash, etc., before you leave.

4.3 EXPERIMENTAL PROCEDURE

4.3.1 INSTRUCTIONS. The write-up for each experiment covers the equipment to be used and the measurements to be taken. Proper experimental procedure begins by carefully studying and following all of the instructions for any assigned experiment. It is also often necessary to study or review the information and instructions given in the preceeding chapters when trying to use any given transducer or associated instrument.

After the transducers have been connected into their circuits and are properly mounted, it is generally a good idea to first try to display on the oscilloscope the variables to be measured, in the manner called for by the instructions. This will show if your measuring system is working properly, and will provide some familiarity with what the output will look like. The transducers can then be calibrated, removing them from the model for calibration if this is necessary. Following this the desired data can be taken.

4.3.2 LABORATORY MODELS. The experimental models which you shall use have been constructed specifically for this laboratory, and perform quite well if handled with reasonable care. It is requested that the motor driven models be run only when readings are being taken, since the life of some of the transducers on these models is limited by wear of their sliding contacts.

4.3.3 USE OF PATCH CORDS. Almost all of the electrical connecting and wiring called for in the experiments will be done with patch cords which plug into suitable jacks, or are clipped onto terminals. Unless special colors are called for by the instructions, the black and the red patch cords should be used. Detachable black and red alligator clips can be slipped onto the ends of the cords when needed.

Where polarity and grounding are important, the red cords and jacks are used for positive potential, while the black cords and jacks are used for negative potential. The black cords and jacks also indicate ground potential. Good laboratory technique calls for the color of the patch cords and alligator clips to match the color of the jacks into which they are plugged. This is often very helpful when trouble shooting the circuits, especially if the circuits are numerous and complicated. The circuits used for the experiments in this course are relatively simple, but this technique should still be learned and followed.

The wiring should be done in a neat and orderly fashion, using the long and short cords as needed. A long cord should not be formed by patching a number of short cords together, nor should a long cord be used where a short cord will suffice.

An important aid to trouble free instrumentation is a single sketch of the complete wiring diagram placed in a convenient location for reference during wiring, trouble shooting or interpreting results. Sketching the diagram before the experiment is also a good form of preparation.

Static electricity, or electrical noise, tends to be gathered through the cords, another reason for keeping the cords short and orderly. As much as possible the cords should be kept away from such noise producers as AC power cords, the brushes of DC motors, etc. This is especially important when the output voltage is of a very small magnitude, as from strain gages, and hence easily effected.

4.3.4 OUTPUT CORDS. Shielded cables are provided for use as output cords, to connect the transducer circuit to the oscilloscope. These appear as a single cable with 2 wires at each end. The large wire at each end is the main or central wire, while the small wires, or pigtails, are connected to the metal shield of the cable. In these cables the metal shield surrounds the central wire, thus protecting it from electrical noise or static. The shield is also utilized as a conductor, so the shielded cable can be used as any 2 wire connector might be used. However, the pigtails must be used as the ground side of the circuit to utilize the shield as a means of static prevention.

4.3.5 GROUNDING. Static electricity, or electrical noise, can generally be reduced or eliminated by proper grounding of the equipment being used. Most of the oscilloscopes in the laboratory are grounded through a third wire in the power cord. Before using the oscilloscope the power cord should be checked, and if there is no ground plug a separate ground wire must be provided by connecting a long patch cord from one of the external ground plugs on the front panel of the oscilloscope to the round or bottom hole of the electrical outlet. It is best to use an alligator clip to fasten the patch cord to the electrical outlet.

In addition to grounding the oscilloscope, the mechanical devices, and in some cases the circuit boxes, must also be grounded. This is accomplished by using patch cords to connect these parts to the ground terminal of the oscilloscope.

There are holes in the base of each mechanical model used in the laboratory into which grounding cords can be plugged. Clips can be used to attach cords to other devices as needed.

It must be remembered that all points in the ground circuit will be at zero potential only if no current flows in the ground circuit. To prevent any possibility of such current flow, the ground wiring should be arranged so as not to have parallel paths to ground from any point in the circuit, since this will form a loop in which current could possibly flow.

The prevention of ground loops becomes especially important when long leads are used with transducers which develop very small output voltages and are especially sensitive to distortion by static. It sometimes helps to think of the ground circuit as a tree, with its trunk at the ground connection. The grounding wires form the branches of the tree, leading out from the trunk, but with none of the tips touching one another to form parallel paths to the ground.

4.3.6 USE OF BATTERIES. Everyone is familiar with the limited life of batteries, and it is common sense to disconnect the batteries whenever they are not in use. Disconnecting should always be done at the battery, and not at the other end of a cord which may inadvertantly touch a point of opposite polarity and drain the battery. This is especially important with the automotive type batteries, since an accidental short circuit will seriously burn the equipment.

When connecting patch cords to dry batteries by means of alligator clips care must be exercised to see that the clips of opposite polarity are not touching one another, and are not touching the metal case of the battery.

The automotive type batteries must always be used to power the lamps used with the light sensitive transducers, since the dry batteries do not have sufficient capacity and will be quickly depleted.

4.3.7 CALIBRATION. Methods of calibration are covered with the discussion of each transducer in chapter 2, and will not be repeated here. To insure maximum accuracy, the calibration should always be carried out over as large a range as reasonably possible. For example, if a rotary displacement transducer is being calibrated to provide one centimeter deflection of the oscilloscope beam per 10 degrees of rotation, the calibration should be carried through 80 to 100 degrees, deflecting the oscilloscope beam 8 to 10 centimeters or approximately full scale, rather than through just 10 degrees and the one centimeter deflection. This reduces the errors inherent in reading degrees on the calibrating stand and centimeters on the oscilloscope screen, with a resulting increase in the overall accuracy.

When the quantity to be measured is small and the instrumentation sensitivity is quite high, or when the linearity is uncertain, it is often necessary to plot a curve of the calibration data to reduce or eliminate the effects of small variations in reading the instruments and in reading dial indicators, etc., which may be used for calibration. For example, if a solar cell displacement transducer is being calibrated with a dial indicator, and a total travel of 0.08" is to be equivalent to 8 cm deflection of the oscilloscope, small variations are very apt to creep in as we read 0.01 inch increments of travel on the indicator and the corresponding deflection on the oscilloscope. Plotting the readings will help determine the relation between displacement and oscilloscope beam deflection, and will also show if the relationship is linear.

The proportionality between the physical variable and the oscilloscope deflection should be such that the trace will reasonably well fill the oscilloscope screen when the variable is displayed or recorded. The larger the trace the more accurately it can be scaled for numerical values.

Calibration is very important, for none of the results can be very good if the calibration is poorly done.

4.3.8 RECORDING. The final result of all the laboratory work shows up in a recording of the measured variable in some form. In our laboratory most of the recording will be done by photographing the oscilloscope trace with the Polaroid camera described in section 3.5.

When recording, both the trace and the grid lines should be thin and distinct, the trace should be so located on the oscilloscope screen that it can be easily referenced to the graduated co-ordinates on the screen, and the grid lines should be parallel to the edges of the photograph. The trace should be as large as reasonably possible. However, the camera does not photograph the entire screen, hence the trace should not extend to the edges of the screen.

If high frequency static or noise prevents the obtaining of a clear trace, a small capacitor can be placed across the oscilloscope input terminals to act as a filter. The distortion caused by capacitance is covered in section 1.7, and it is important that the capacitance does not cause undesirable distortion of the main signal.

All photographs should be immediately identified by writing a title, the amplitude and time scales, the date, and any other pertinent information on the back of the photograph. The amplitude must be expressed in terms of the physical variable being measured, not just in terms of volts, for the purpose of the whole procedure is to measure the physical variable.

CHAPTER 5

EXPERIMENTAL DETERMINATION OF INERTIA, SPRING RATES, AND DAMPING

5.1 INTRODUCTION

In addition to measuring such physical variables as displacement, force, etc. with the instrumentation systems described in the previous chapters, it is often necessary to determine inertia, spring rates, and damping of the mechanical device. Values for these parameters are essential for any analytical work which must often be utilized to fully understand what the instrumentation has revealed, to improve a design which is not completely satisfactory, or to predicate new designs of similar devices.

For parts having simple shapes such as disks, bars, etc., it is quite possible and often convenient to calculate moments of inertia and spring rates by using well known equations of solid mechanics. For certain devices it is also possible to calculate the damping with reasonable accuracy. However, the simple configurations which allow easy calculation of these parameters are not often encountered in practice, and in such cases experimental methods are very useful.

There are many ways of experimentally determining inertia, spring rates, and damping, and it is not intended that this is a complete coverage of the subject. However, the methods and techniques which follow should probably suffice for the majority of cases commonly encountered, and should provide sufficient background so that other methods can be devised if the need arises. Careful attention should be paid to the assumptions and limiting conditions of each case described so that the actual physical set-up corresponds reasonably well to the theoretical model.

5.2 EXPERIMENTAL DETERMINATION OF MOMENT OF INERTIA

5.2.1 DEFINITIONS. A rotating body tends to resist any change in motion, and any change in motion which is brought about by the action of external forces or torques is inversely proportional to a property of the rotating body known as its moment of inertia. The moment of inertia is proportional to the mass of the body and to the square of the distance of the mass from the center of rotation.

For a particle of mass d(m) located a distance r from the center of rotation, the moment of inertia will be:

$$I = r^2 \, d(m) \qquad (5.2.1.1)$$

The moment of inertia of a body is the summation of such expressions for all elements in the body:

$$I = \int r^2 \, d(m) \qquad (5.2.1.2)$$

The radius of gyration of a mass may be defined as the distance from the axis at which the entire mass may be concentrated and still have the same moment of inertia that it has in its distributed form. This can be expressed as:

$$I = m K^2 \qquad (5.2.1.3)$$

where

m = total mass of the body

K = radius of gyration

Expressing this in common engineering units:

$$I = \frac{W_t}{g} \cdot K^2 \qquad (5.2.1.4)$$

where W_t = weight of body, lbs., ounces, grams

g = acceleration of gravity

= 32.2 ft/sec.^{2}

= 386 in/sec.^{2}

K = radius of gyration, feet, inches, cm.

I = moment of inertia

It will be noticed that the units of the moment of inertia will depend upon the units used for the terms on the right hand side of equation 5.2.1.4. In general the units will be weight - length-(time)^2, such as lb-ft-sec^2. It is essential that identical units of length be used for g and K.

It is often necessary to determine the moment of inertia of a body with respect to some other axis than that for which the moment of inertia is already known. If the two axes are parallel, and one passes through the center of gravity, it is possible to use the transfer theorem as follows:

$$I_o = I_G + \frac{W_t}{g} d^2 \qquad (5.2.1.5)$$

where I_G = moment of inertia about an axis passing through the center of gravity.

I_o = moment of inertia about any axis parallel to the axis passing through the center of gravity

W_t = weight of the body

g = acceleration of gravity

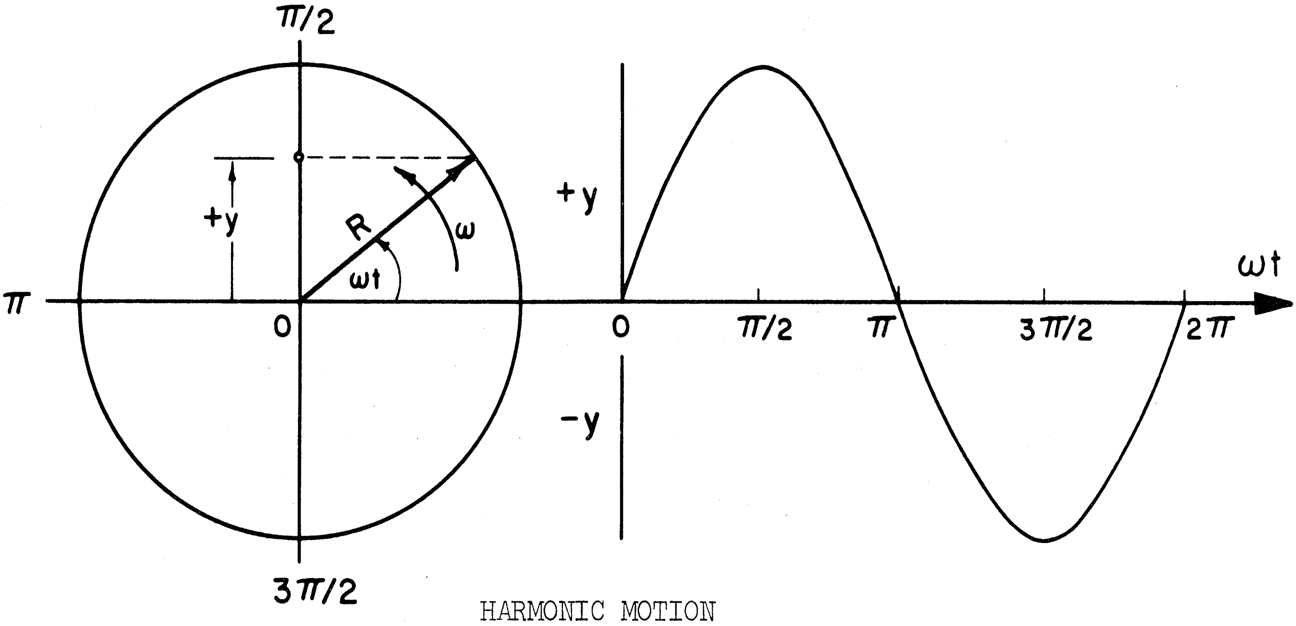
d = perpendicular distance between the two axes

In this equation I_G must pass through the center of gravity. If it is desired to transfer the moment of inertia of a body from one axis to another when neither axis passes through the center of gravity, it must be accomplished in two steps, the moment of inertia about the axis through the center of gravity being an intermediate result.

It can be concluded from equation 5.2.1.5 that the moment of inertia about the axis passing through the center of gravity is the smallest possible moment of inertia of the body.

5.2.2 HARMONIC MOTION. One common technique of experimentally determining the moment of inertia of individual mechanical parts uses a pendulum of one sort or another. Some specific pendulums are described in the sections which follow, but all of the pendulums utilize the principles of harmonic or sinusoidal motion to determine the moment of inertia of a given mechanical part.

Harmonic motion can be described as motion in which the acceleration is directly proportional to the displacement from a given origin, but opposite in direction to this displacement. A physical representation of harmonic motion is shown in figure 5.2.2.1, where a vector R



HARMONIC MOTION
FIGURE 5.2.2.1

rotates with a constant angular velocity ω, and the point which represents the end of the projection of R moves on either the vertical or horizontal axis with harmonic motion. Considering the vector R in the position shown, displaced an angle ωt from the horizontal axis, and the end of its projection displaced a linear distance y from the origin O:

$$y = R \cdot \sin \omega t \qquad (5.2.2.1)$$

Differentiating with respect to time, and designating the first derivative as $\dot{y}$:

$$\dot{y} = \omega R \cos \omega t \qquad (5.2.2.2)$$

Repeating the process and designating the second derivative as $\ddot{y}$:

$$\ddot{y} = -\omega^2 R \sin \omega t \qquad (5.2.2.3)$$

Then by combining the first and last equation:

$$\ddot{y} = -\omega^2 y \qquad (5.2.2.4)$$

The first derivative $\dot{y}$ is the velocity and the second derivative $\ddot{y}$ is the acceleration of the point which represents the end of the projection of the vector R on the vertical axis. Equation 5.2.2.4 shows that in keeping with the previous description of harmonic motion, the acceleration is proportional to the displacement, but is opposite in direction. The proportionality factor is ω^2, the square of the angular velocity of the rotating vector R expressed in radians/second.

The frequency of rotation of the vector R will be:

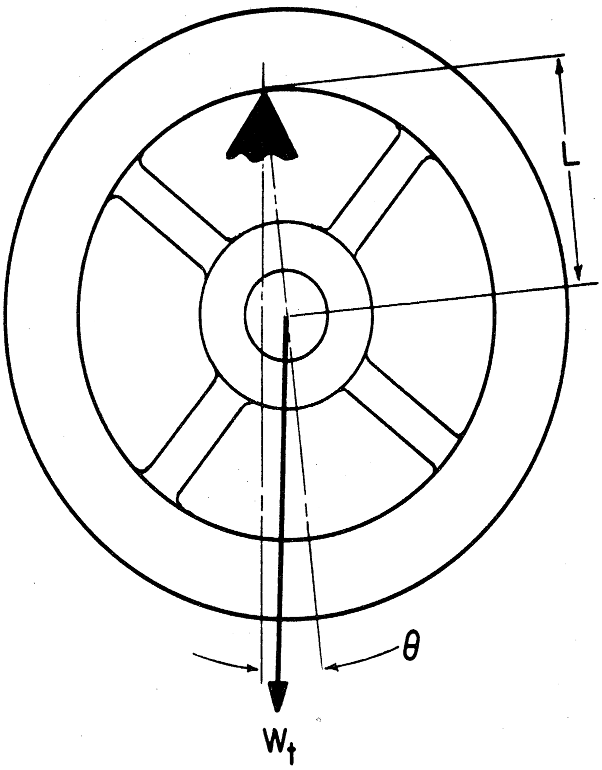
$$f = \frac{\omega}{2\pi} \qquad (5.2.2.5)$$

where f = frequency of rotation of vector R, revolutions/second, or cycles/second.

ω = angular velocity of vector R, radians/second

Equation 5.2.2.4 is the equation for harmonic motion. If the equation of motion for any device has the same form as equation 5.2.2.4, it can be concluded that the device moves with harmonic motion, that the coefficient of the displacement represents ω^2 in the harmonic equation, and that the frequency can therefore be found by equation 5.2.2.5. This principle will be utilized with the various pendulums described in the sections which follow.

5.2.3 COMPOUND PENDULUM. It is desired to find the moment of inertia of the small flywheel of figure 5.2.3.1 about its axis of rotation. The flywheel has been suspended on a knife edge or small pin as shown, and if it is displaced through a small angle and released it will oscillate as a pendulum. Since the flywheel is symmetrical, it is assumed that the geometric center of the flywheel is also the center of gravity. It is also assumed that the flywheel oscillates through small angles only, and that the friction at the support is negligible. Considering the flywheel to have just been released in the position of figure 5.2.3.1, and taking counterclockwise torque and displacement to be positive, the differential equation of motion can be developed from the basic relationship:



COMPOUND PENDULUM
FIGURE 5.2.3.1

$$-T = I_k \cdot \ddot{\theta} \qquad (5.2.3.1)$$

where T = clockwise restoring torque about the knife edge

I_k = moment of inertia about the knife edge

$\ddot{\theta}$ = angular acceleration about the knife edge, radians/sec.^{2}

From the figure the magnitude of the restoring torque will be:

$$T = W_t \cdot L. \sin \theta \qquad (5.2.3.2)$$

Taking $\theta = \sin \theta$ for small angles, and substituting into equation 5.2.3.1, the differential equation of motion will be:

$$-W_t \cdot L \cdot \theta = I_k \cdot \ddot{\theta} \qquad (5.2.3.3)$$

or

$$\ddot{\theta} = -\left(\frac{W_t \cdot L}{I_k}\right) \cdot \theta \qquad (5.2.3.4)$$

This is in the same form as the harmonic equation 5.2.2.4, expressed in terms of angular displacement θ instead of linear displacement y. It follows that the flywheel oscillates about the knife edge with harmonic motion, and that the term in the parenthesis of equation 5.2.3.4 corresponds to ω^2 of equation 5.2.2.4. Then from equation 5.2.2.5 the frequency of oscillation in cycles per second will be:

$$f = \frac{1}{2\pi}\sqrt{\frac{W_t \cdot L}{I_k}} \qquad (5.2.3.5)$$

or

$$I_k = \frac{W_t \cdot L}{4\pi^2 f^2} \qquad (5.2.3.6)$$

where I_k = moment of inertia about the knife edge, or the axis of oscillation as a pendulum

W_t = weight of the body

L = distance from the knife edge to the center of gravity

f = frequency of oscillation as a pendulum, cycles/second

The frequency could be found by actually supporting the flywheel on the knife edge or small pin and timing a reasonable number of oscillations with a stop watch. Length L can be measured, the body can be weighed, and the moment of inertia about the supporting knife edge can be calculated from equation 5.2.3.6.

In keeping with the discussion in section 5.2.1, no units are given for I_k, W_t, and L of equation 5.2.3.6. However, if the weight is expressed in lbs. and L in inches, the units for I_k will be lb-inch-sec^2.

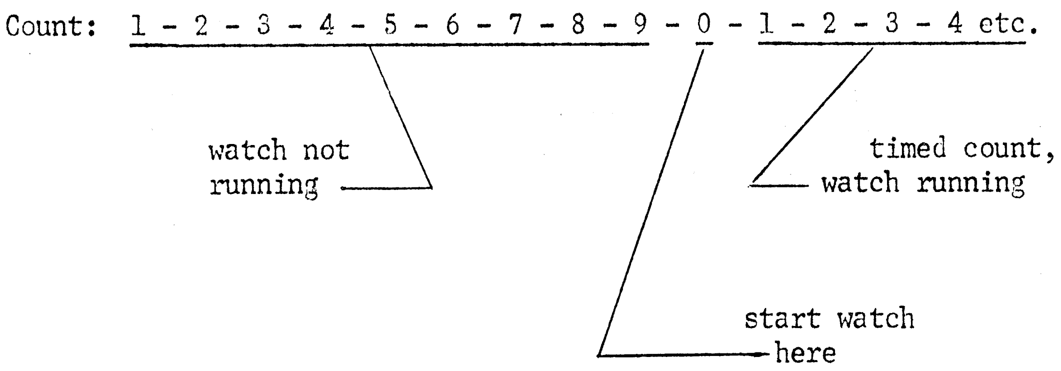
To determine the moment of inertia of the flywheel about its axis of rotation, which axis passes through its center of gravity, the transfer theorem stated in equation 5.2.1.5, can be used. Applied to our flywheel this becomes:

$$I_G = I_k - \frac{W_t}{g} \cdot L^2 \qquad (5.2.3.7)$$

In the case of the flywheel it could be logically assumed that the center of gravity was at the geometric center of the flywheel. For most mechanical parts the center of gravity must be located before equations 5.2.3.6 or 5.2.3.7 can be used.

The center of gravity can generally be found with sufficient accuracy by balancing the part on a pin point. The point at which it balances is the center of gravity. The part could also be balanced on a horizontal straight edge, the balance line marked, and the part rotated about 90 degrees and balanced again. The intersection of the two lines on which it balanced is the center of gravity. It hardly needs saying that this work must be carefully done.

Timing the oscillations to obtain the frequency of the pendulum can also present some problems. Small oscillations are not easy to count, and considerable error can be introducted if this is not carefully done. Starting the watch and the count at the proper time is often difficult, and it is suggested that you start the counting before starting the watch so as to get the proper rhythm as shown in figure 5.2.3.2.



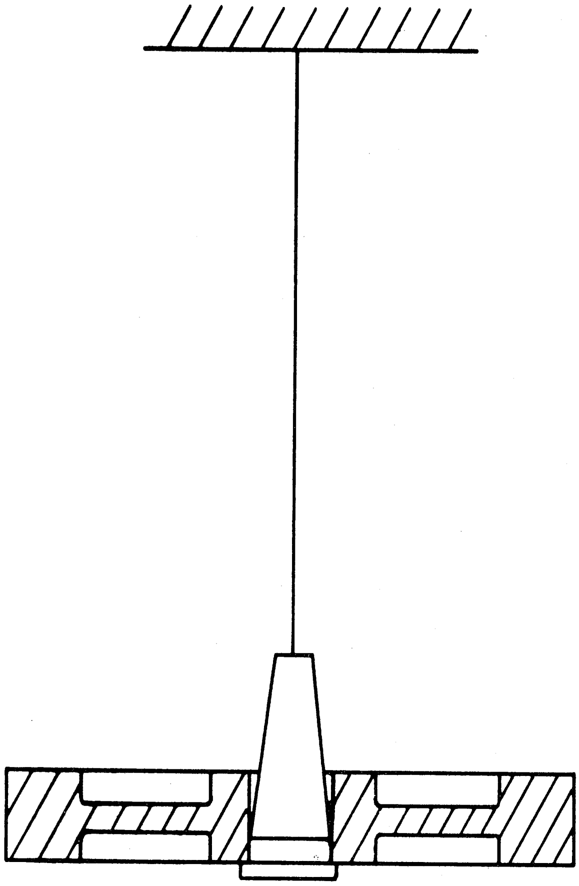
METHOD FOR TIMING OSCILLATIONS

FIGURE 5.2.3.2

Timing the oscillations for any one part should be repeated until at least three readings with reasonable agreement are obtained. Fifty cycles is often a reasonable number of cycles to time.

5.2.4 <u>SINGLE WIRE TORSIONAL PENDULUM</u>. The moment of inertia of a symetrical part such as the flywheel could also be found by suspending it on a single wire through its center as shown in figure 5.2.4.1. A small torsional displacement will cause the flywheel to oscillate in the horizontal plane about its own axis of rotation. Assuming there is no damping in the system, and limiting the oscillation to small angles, the differential equation of motion can again be developed from the basic relationship:

$$-T = I_G \ddot{\theta} \qquad (5.2.4.1)$$



SINGLE WIRE TORSIONAL PENDULUM
FIGURE 5.2.4.1

where T = restoring torque

I_G = moment of inertia about the center of oscillation, or center of the flywheel

$\ddot{\theta}$ = angular acceleration, radians/sec.^{2}

But the magnitude of the restoring torque will be:

$$T = K\,\theta \qquad (5.2.4.2)$$

where K = torsional spring rate of the wire

θ = angle of displacement from central position, radians

The value of K for a uniform wire with a circular cross section can be established from equations of mechanics to be:

$$K = \frac{\pi d^4 G}{32 L} \qquad (5.2.4.3)$$

where
K = torsional spring rate, in-lb/radian

d = wire diameter, inches

G = shear modulus of wire material, psi

L = length of wire, inches

Combining the preceding equations, the differential equation of motion will be:

$$-\frac{\pi d^4 \cdot G}{32\,L} \cdot \theta = I_o \cdot \ddot{\theta} \qquad (5.2.4.4)$$

or

$$\ddot{\theta} = -\left(\frac{\pi d^4 \cdot G}{32.L.I_G}\right) \cdot \theta \qquad (5.2.4.5)$$

This is in the same form as the harmonic equation 5.2.2.4, and the term in the parenthesis of equation 5.2.4.5 must represent ω^2. Then from equation 5.2.2.5 the frequency of oscillation will be:

$$f = \frac{1}{2\pi}\sqrt{\frac{\pi\ d^4.G}{32.L.I_G}} \qquad (5.2.4.6)$$

or

$$I_G = \frac{d^4.G}{128\pi f^2 L} \qquad (5.2.4.7)$$

where
I_G = moment of inertia of flywheel, lb-in-sec.^{2}
d = wire diameter, inches
G = shear modulus of wire material, psi.
f = frequency of oscillation of the pendulum, cycles/second
L = length of wire, inches

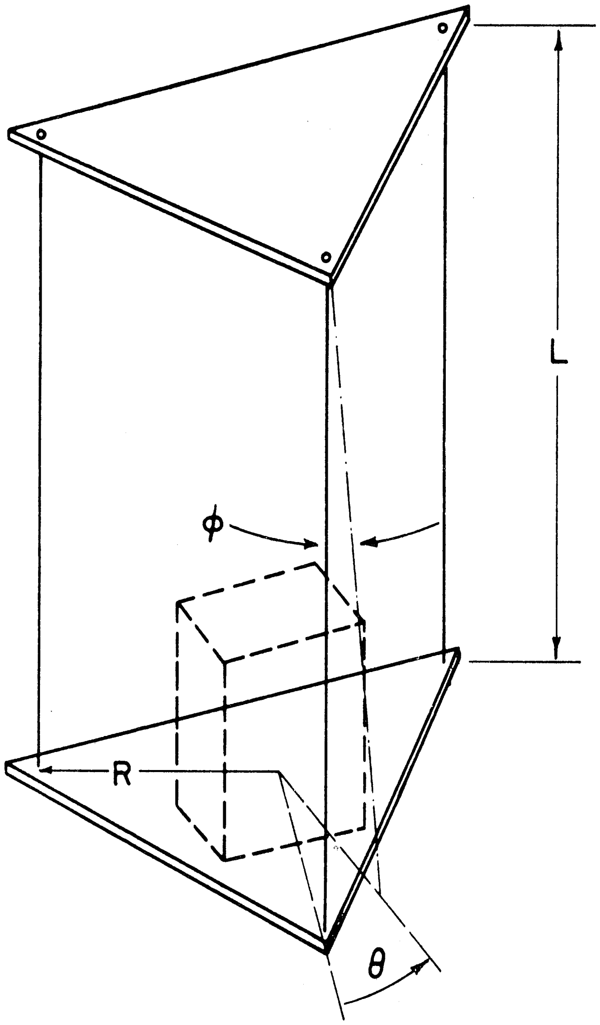
Units have been given above, but in keeping with the discussion of section 5.2.1 other units than those given also can be used.

The value of G can be taken as approximately 11.6×10^6 psi. for steel wire. As with the compound pendulum described in section 5.2.3, the frequency is found by timing a representative number of oscillations with a stop watch. The technique for doing this is described in section 5.2.3.

The single wire pendulum is often difficult to use, since the center of gravity of the part must coincide with the center of oscillation. The pendulum must be constructed so that the wire is attached to the pendulum above the mechanical part as shown in figure 5.2.4.1. If the wire passed through the flywheel and was attached to a plate beneath the flywheel the assembly would be unstable, and would tend to tip away from the horizontal plane.

5.2.5 THREE-STRING TORSIONAL PENDULUM. This pendulum consists of a platform supported by three strings equally spaced about its center, as shown in figure 5.2.5.1. The mechanical part is placed on the platform so that the center of gravity of the part coincides with the center of the platform, and the platform and part are oscillated through a small angle in the horizontal plane.

If the oscillations are limited to small angles and the strings are of reasonable length, the vertical displacement of the platform can be neglected. Assuming there is no damping in the system, and proceeding in a manner similar to that used for the other two



THREE STRING TORSIONAL PENDULUM
FIGURE 5.2.5.1

two pendulums discussed, the equation of motion can be derived from figure 5.2.5.1 as follows:

$$- R \cdot W_t \cdot \sin \phi = I_G \ddot{\theta} \qquad (5.2.5.1)$$

Or for small angles of oscillation:

$$- R \cdot W_t \cdot \phi = I_G \ddot{\theta} \qquad (5.2.5.2)$$

where

R = distance from center of platform to any string, inches.

W_t = combined weight of part and platform, lbs.

ϕ = angular displacement of string in vertical plane, radians

I_G = combined moment of inertia of part and platform about center of gravity, lb-in-sec.^{2}

$\ddot{\theta}$ = angular acceleration in horizontal plane, radians/sec.^{2}

From the geometry of the pendulum:

$$\phi = \frac{R}{L} \theta \qquad (5.2.5.3)$$

where L = length of string, inches

So by substitution:

$$- \frac{R^2}{L} W_t \cdot \theta = I_G \ddot{\theta} \qquad (5.2.5.4)$$

or

$$\ddot{\theta} = - \frac{R^2 \cdot W_t}{I_G L} \theta \qquad (5.2.5.5)$$

As with the other pendulums, this is the harmonic equation, for which the frequency will be:

$$f = \frac{1}{2\pi} \sqrt{\frac{R^2 W_t}{I_G \cdot L}} \qquad (5.2.5.6)$$

or

$$I_G = \frac{R^2 \cdot W_t}{4\pi^2 f^2 L} \qquad (5.2.5.7)$$

where

I_G = combined moment of inertia of mechanical part and platform about the center of gravity, lb-in-sec.^{2}

R = distance from center of platform to any string, inches.

W_t = combined weight of part and platform, lbs.

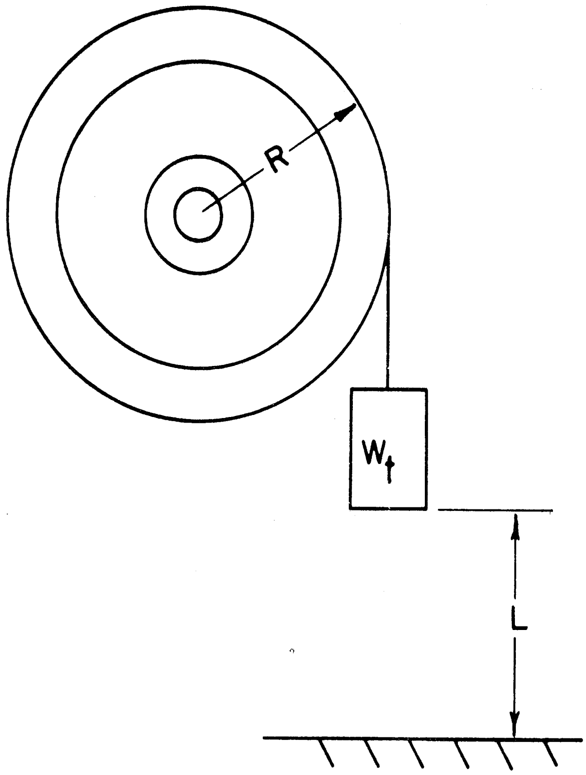
f = frequency of oscillation of pendulum, cycles/sec.

L = length of string, inches.

Techniques for locating the center of gravity and for accurately timing the oscillations to obtain the frequency are discussed near the end of section 5.2.3.

To properly use the three-string pendulum the weight and moment of inertia of the empty platform should first be determined. The process can then be repeated using the combined weight of the platform and mechanical part, and obtaining the combined moment of inertia. Subtracting the moment of inertia of the empty platform from the combined moment of inertia gives the moment of inertia for the part only. It is best if the moment of inertia of the platform is small compared to that of the mechanical part, so that a sizable difference exists between the combined moment of inertia and that of the empty platform. Otherwise small errors in these two moments of inertia will result in a large error in their difference.

5.2.6 **STRING AND WEIGHT**. Another method of determining the moment of inertia for rotating parts is shown in figure 5.2.6.1.



STRING AND WEIGHT
FIGURE 5.2.6.1

A light string is wrapped around the part which is supported in bearings, and a known weight is attached to the string. By accurately measuring the time required for the weight to fall a known distance from its point of release, the moment of inertia of the rotating part can be established. Starting with the basic relationship:

$$T = -I\ \ddot{\theta} \qquad (5.2.6.1)$$

where T = torque applied to rotating part by the falling weight, lb-in.

I = moment of inertia of the rotating part, lb-in-sec.^{2}

$\ddot{\theta}$ = angular acceleration of the rotating part, radians/sec.^{2}

But

$$T = (W_t - \frac{W_t}{g}\ddot{y})\ R \qquad (5.2.6.2)$$

where

W_t = falling weight, lbs.

g = acceleration of gravity, 386 in/sec.^{2}

y = linear acceleration of the falling weight, in/sec.^{2}

R = radius of surface on which string is wrapped, inches

The relationship between the angular acceleration $\ddot{\theta}$ and the linear acceleration $\ddot{y}$ will be:

$$\ddot{y} = \ddot{\theta} \cdot R \tag{5.2.6.3}$$

By suitable substitution

$$\left(W_t - \frac{W_t}{g} \cdot \ddot{y}\right) R = I \cdot \frac{\ddot{y}}{R} \tag{5.2.6.4}$$

and

$$I = \left(1 - \frac{\ddot{y}}{g}\right) \frac{W_t R^2}{\ddot{y}} \tag{5.2.6.5}$$

When the falling weight starts from rest, the distance L travelled in any time t will be:

$$L = \frac{1}{2} \ddot{y} \; t^2 \tag{5.2.6.6}$$

From which

$$I = \left(\frac{t^2}{L} - \frac{2}{g}\right) \frac{W_t \cdot R^2}{2} \tag{5.2.6.7}$$

where

I = moment of inertia of rotating part, lb-in-sec^2.

t = time for weight to fall distance L from time of release, seconds

L = distance weight falls, inches

g = acceleration of gravity, 386 in/sec.^{2}

W_t = falling weight, lbs.

R = radius of surface on which string is wrapped, inches.

The preceding analysis assumes that there is no stiffness to the string, and neglects friction. If there is so much friction that it cannot be neglected it can be measured by experimentally determining what magnitude of weight will fall at a constant velocity, thus just overcoming friction. This amount of weight can then be subtracted from the weight W_t in equation 5.2.6.7.

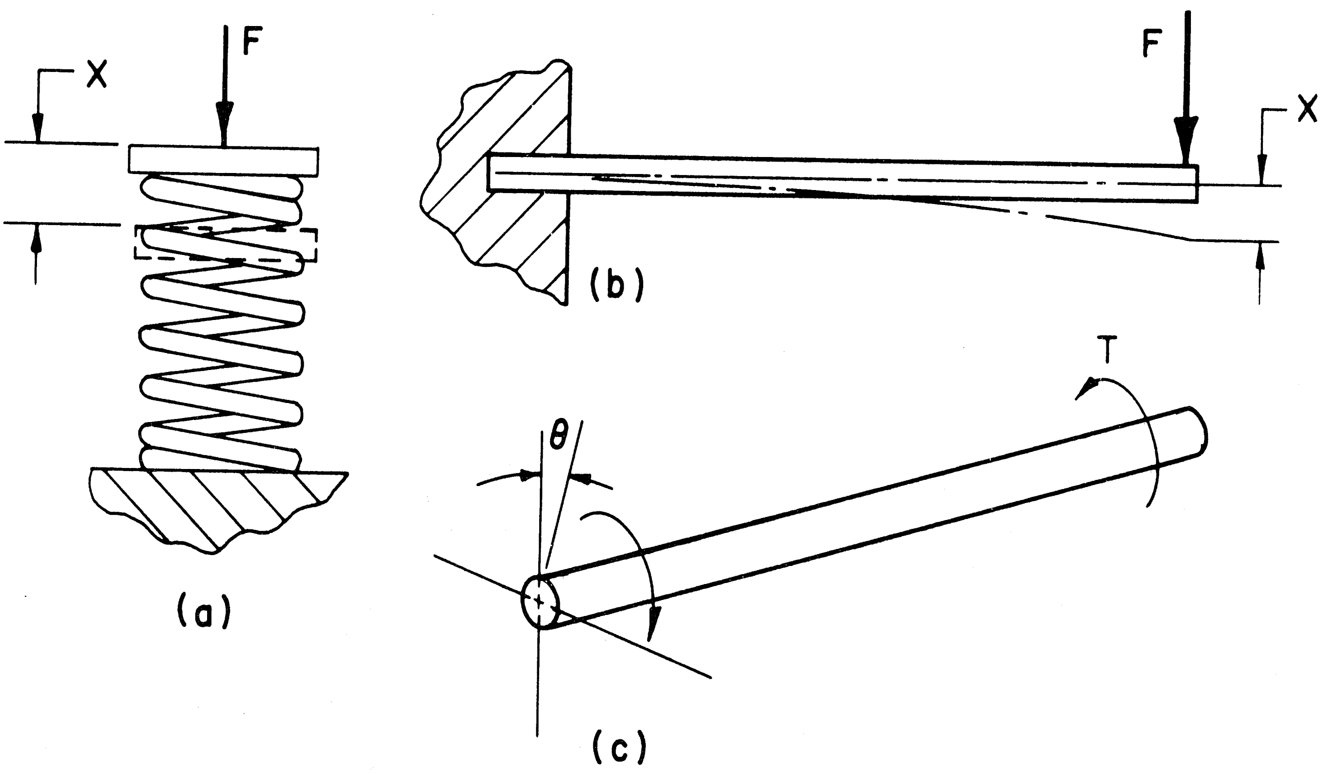
This method of determining the moment of inertia is best suited to large parts which cannot be easily removed from their installed position, or would be difficult to handle on a pendulum. This method can also determine the effective moment of inertia of a complete rotating assembly as referred to the rotating part onto which the string is wrapped.

A more accurate method of timing than an ordinary stop watch is generally required, since the time involved is apt to be quite small. Since the time t is squared in equation 5.2.6.7, any error is compounded. The problem of friction, which was mentioned earlier, is also apt to be troublesome.

5.2.7 TRANSIENT RESPONSE METHOD. Another method of determining the moment of inertia of individual mechanical parts or of an assembly is described in chapter 6. This method consists of holding or anchoring one end of the part or of the assembly, measuring the spring rate of the part or assembly, then striking or otherwise exciting the free end and observing the transient vibration, or response, displayed from a transducer attached to the free end. The inertia of the part or assembly can then be calculated from the transient response by using the basic principles of vibration. Since this is covered in chapter 6 it is not repeated here.

5.3 DETERMINATION OF SPRING RATES

5.3.1 DEFINITIONS. The spring rate of any mechanical part or assembly is the ratio of the external force or torque to the elastic deflection caused by that force or torque.



SPRINGS
FIGURE 5.3.1.1

Then for a linear displacement spring such as shown in figure 5.3.1.1 a and b:

$$K = \frac{F}{X} \tag{5.3.1}$$

where

K = spring rate, lbs/inch

F = external force, lbs.

X = linear deflection of the spring, inches

For a torsional spring as shown in figure 5.3.1.1 c

$$K = \frac{T}{\theta} \tag{5.3.2}$$

where

K = spring rate, lb-in/radian

T = external torque, lb-in.

θ = torsional deflection of the spring, radians

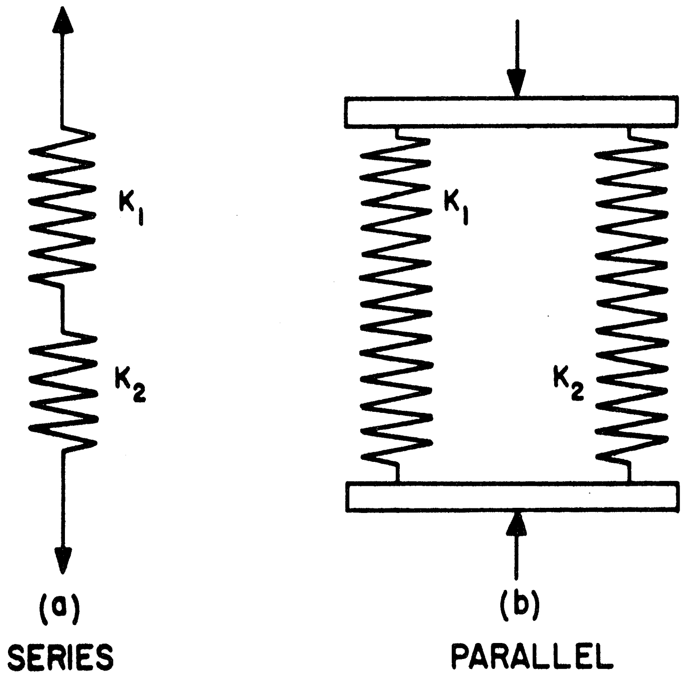
Springs can be used in series or in parallel as shown in figure 5.3.1.2 a and b. The combined spring rates can be derived from the equations of mechanics. For springs in series:

$$K = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2}} \tag{5.3.3}$$

For springs in parallel:

$$K = K_1 + K_2 \tag{5.3.4}$$

If there is a linear relation between the force applied to the spring and the resulting deflection, the spring rate will be constant, and the spring is said to be linear. In similar manner a non-linear relation between force and deflection results in a variable spring rate, and the spring is said to be non-linear. Most metallic springs are linear, although it is possible to have metal springs be non-linear by virtue of their shape.



COMBINED SPRINGS
FIGURE 5.3.1.2

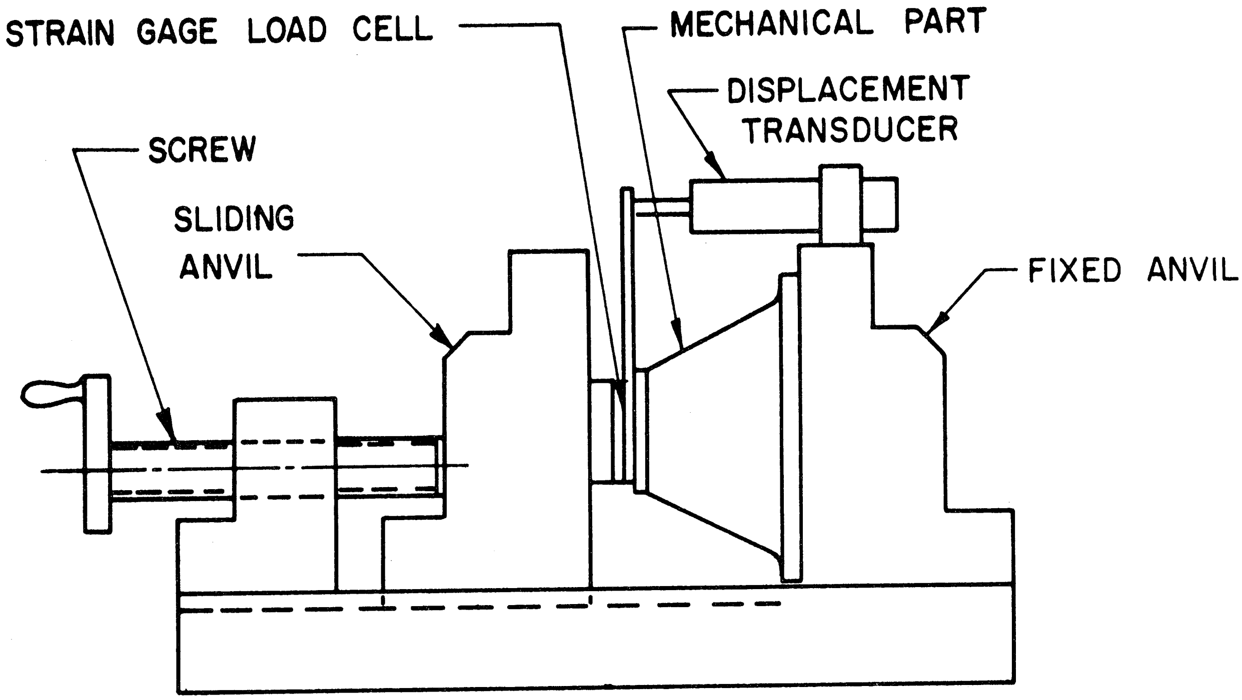
Springs made from rubber and other non-metallic materials are often non-linear.

5.3.2 EXPERIMENTAL DETERMINATION OF SPRING RATES. The illustrations of figures 5.3.1.1 and 5.3.1.2 show springs in the form of a coil, a cantilever beam, and a bar. These and other simple configurations are often used for springs, and in such cases the spring rate can generally be calculated from the physical dimensions and the modulus of elasticity. If these simple shapes are used in series or parallel as shown in figure 5.3.1.2, or in a combination of these two arrangements, it is still possible to calculate the spring rate if it is possible to establish the way in which the load is distributed among the various springs.

However, all mechanical parts act as springs when a load is applied, and most mechanical parts are not made in shapes that lend themselves to easy calculation of their spring rate. When a mechanical device is made up of a number of interconnected parts, it becomes even more difficult to establish an overall spring rate by analytical means only. The assembled parts could represent combinations of series and parallel springs, each with a complicated configuration, some linear and some non-linear. In such cases as these it is often expedient to experimentally determine the spring rates for the individual parts or for the whole assembly.

Since the spring rate is the ratio of force or torque to deflection, it follows that experimentally determining the spring rate consists of simply applying known forces or torques and measuring the resulting deflections. For many parts or assemblies the force can be applied with dead weights, or a torque applied with the weights on a lever arm. The resulting deflection can be measured with dial indicators, or with transducers which may be all ready mounted on the assembled device. By applying a series of loads or torques and recording the corresponding deflections a force-deflection curve can be plotted to establish the spring rate, and to see whether or not the spring is linear.

For mechanical parts which have unusual shapes, or have a very high rate, or both, dead weight loading is often unsatisfactory. The large forces needed to obtain readable deflections are difficult to obtain with easily handled weights, and it is often difficult to load and support the part in the desired position. In such cases a mechanical device which can apply large loads of known magnitudes, and which can measure deflections can be used. One such possible device is shown schematically in figure 5.3.2.1, where the load is applied by a screw, and is measured by a strain gage load cell. If a graduated screw is used the deflections could be read from the screw itself. It must be noticed, however, that if this is done the load cell acts as a spring in series with the mechanical part, and the screw indicates the combined deflection of the load cell and the mechanical part. To utilize this method of measuring the deflections, it would first be necessary to measure the spring rate of the load cell alone, and use this to correct the results obtained with the load cell and mechanical part in series.



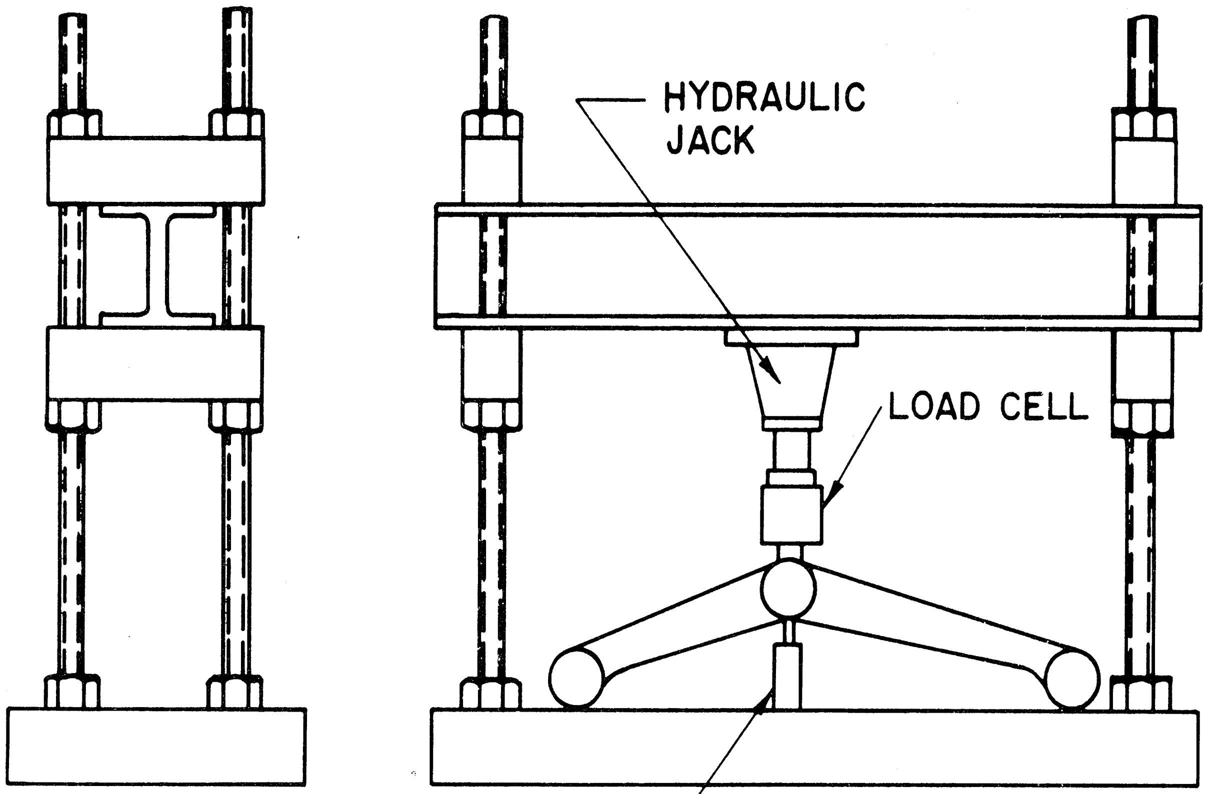
LOAD PLOTTER
FIGURE 5.3.2.1

In figure 5.3.2.1 a displacement transducer such as a linear potentiometer or a linear differential transformer is used to measure the deflection. If the outputs of the load cell and the displacement transducer are fed into an x-y plotter or similar recording device, the force deflection curve will be drawn out as the screw is turned to load the mechanical part. Notice that the displacement transducer in the figure measures the deflection of the mechanical part only, and does not include the deflection of the load cell.

A device such as shown in figure 5.3.2.1 is known as a spring rate tester, or as a load plotter. For larger parts or assemblies which might not fit into a screw type load plotter, one similar to figure 5.3.2.2 might be used.

This consists of an adjustable frame, a hydraulic jack applying a load through the strain gage load cell, and the displacement transducer measuring deflection.

A tensile-compressive testing machine also makes a good load plotter for large parts or assemblies.



LOADING FRAME
FIGURE 5.3.2.2

When determining the spring rates of mechanical parts it is essential that they be supported and loaded in the same manner as when they are in use in the mechanical device. This often requires some extra fixtures as well as considerable ingenuity.

The effects of friction can sometimes influence the measurement of spring rates. For example, in figure 5.3.2.2 the ends of the mechanical part which rest on the base of the loading frame must slide horizontally as the part is deflected, and friction at the sliding surfaces will influence the force required to provide any given deflection. The effect of friction can be eliminated by plotting the force-deflection characteristics as the load is applied, and again as it is released, and taking the average of the two curves.

5.4 DETERMINATION OF DAMPING

5.4.1 DEFINITION. In any mechanical part or assembly there will be forces which tend to resist motion, and such forces are known as damping forces. Friction is a common cause of damping, but shearing of oil films in bearings, motion of parts through air or fluids, and energy absorption by the deflecting materials also cause damping forces to exist.

Since the phenomenon is often expressed in terms other than force, it is commonly referred to simply as damping, rather than as a damping force.

Some of the characteristics of common types of damping are covered in the section on vibrations in chapter 6, and so are not repeated here.

5.4.2 EXPERIMENTAL DETERMINATION OF DAMPING. Damping due to static friction, or Coulomb damping as it is commonly called, can be measured in an assembly by using the same type of load plotters that are shown in figures 5.3.2.1 and 5.3.2.2. If a force vs. travel or deflection curve is plotted first with the force being applied and then with the force being released, the area between the two curves will represent the work of friction.

However, this will not measure damping which is proportional to velocity, and for this the transient response method is recommended. This was described in section 5.2.7 as a means of determining the inertia of the system. As explained in chapter 6, the same transient response can be used to calculate damping.

CHAPTER 6

EQUATIONS FOR VIBRATIONS

SYSTEM FOR THE GENERAL CASE

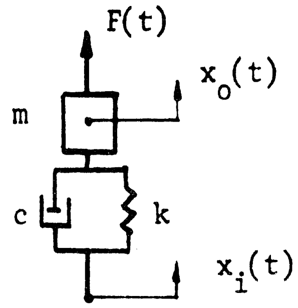


FIGURE 6.1

SYSTEM CONSTANTS:

m = mass, # sec^2/in.

c = damping coefficient, # sec/in.

k = spring constant, #/in.

VARIABLES WITH TIME:

x_o = output displacement of the system, in.

x_i = input displacement excitation, in.

F = force excitation at the mass, #

SYSTEM EQUATIONS

From dynamic equilibrium,

$$m\,\ddot{x}_o + c\,\dot{x}_o + k\,x_o = c\,\dot{x}_i + k\,x_i + F \qquad (6.1)$$

When elasticity dominates, $(c/2m)^2 < k/m$, and the system vibrates. A good form to describe the system is:

$$\ddot{x}_o + 2\,\zeta\,\omega_n\,\dot{x}_o + \omega_n^2\,x_o = 2\,\zeta\,\omega_n\,\dot{x}_i + \omega_n^2\,x_i + (1/m)F \qquad (6.2)$$

where the system constants are:

ζ = damping ratio always less than unity for vibrating systems

ω_n = undamped natural frequency, rad/sec.

EQUATIONS RELATING THE TWO FORMS OF SYSTEM EQUATIONS

C_c = the critical damping coeficient, # sec/in.

$$C_c = 2\,m\,\omega_n \tag{6.3}$$

$$C_c = 2\sqrt{k\,m} \tag{6.4}$$

$$C_c = 2\,k/\omega_n \tag{6.5}$$

$$\zeta = C/C_c \tag{6.6}$$

$$\omega_n = \sqrt{k/m} \tag{6.7}$$

SOLUTION FOR THE SYSTEM EQUATIONS

$$x_o(t) = x_{ts}(t) + x_{ss}(t) \tag{6.8}$$

where:

x_{ts} = transient solution which depends on initial conditions, $x_o(0)$, and $\dot{x}_o(0)$.

x_{ss} = steady state solution which depends on the exciting functions F and x_i, and equals $x_o(t)$ after a sufficiently long time for the transient to die out.

Notes:

(1) In differential equation theory, x_{ts} corresponds to the homogeneous solution or complimentary solution, whereas, x_{ss} corresponds to the particular solution.

(2) When the excitation is applied unchanged for a long period of time, it is common to be concerned only with the steady state solution since it represents the output after decay of the transient.

THE TRANSIENT SOLUTION FOR ALL CASES

$$x_{ts} = e^{-\zeta\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \tag{6.9}$$

where: A and B are arbitrary constants which must be determined to satisfy initial conditions $x_o(0)$ and $\dot{x}_o(0)$.

ω_d = the damped natural frequency of the system, rad/sec.

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \tag{6.10}$$

<u>THE STEADY STATE SOLUTION</u> will depend on the exciting functions $x_i(t)$ and $F(t)$, and must be determined for each case.

SOLUTIONS FOR SOME COMMON CASES

<u>FREE TRANSIENT VIBRATION</u> (see Figure 6.6, page 148)

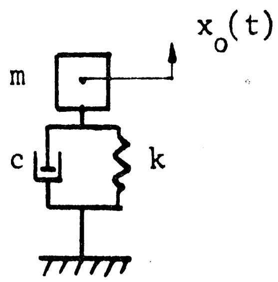


FIGURE 6.2

Differential equation

$$m\ddot{x}_o + c\dot{x}_o + k\,x_o = 0 \tag{6.11}$$

or

$$\ddot{x}_o + 2\zeta\omega_n \dot{x}_o + \omega_n^2 x_o = 0 \tag{6.12}$$

No excitation:

$$F = x_i = 0 = \text{constant}$$

Initial conditions:

$$x_o(0),\ \dot{x}_o(0)$$

Consider the special case where $\dot{x}_o(0) = 0$, $x_o(0) = X_o$

Solution:

$$x_o(t) = x_{ts} + \cancelto{0}{x_{ss}}$$

$$x_o(t) = \frac{X_o}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t} \cos\left(\sqrt{1-\zeta^2}\,\omega_n t - \phi\right) \tag{6.13}$$

$$\phi = \tan^{-1} \frac{\zeta}{\sqrt{1-\zeta^2}} = \sin^{-1}\zeta \tag{6.14}$$

When $\zeta < .2$, a common case, $\sqrt{1 - \zeta^2} \approx 1$ within a 2% error and the solution becomes,

$$x_o(t) = X_o e^{-\zeta \omega_n t} \cos(\omega_n t - \phi) \qquad (6.15)$$

$$\phi = \tan^{-1} \zeta \qquad (6.16)$$

SINUSOIDAL FORCE EXCITATION AT THE MASS

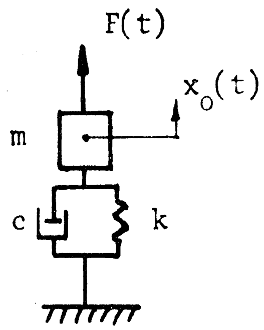


FIGURE 6.3

Differential equation

$$m \ddot{x}_o + c \dot{x}_o + k x_o = F \qquad (6.17)$$

or

$$\ddot{x}_o + 2 \zeta \omega_n \dot{x}_o + \omega_n^2 x_o = \frac{1}{m} F \qquad (6.18)$$

Excitation:

$$F = F_o \sin \omega t \qquad (6.19)$$

where,

ω = the excitation frequency, rad/sec.

F_o = 0-peak amplitude of the exciting force, #

Initial conditions:

$x_o(0)$, $\dot{x}_o(0)$ do not effect the steady state.

Solution:

IGNORE

$$x_o(t) = x_{ts}(t) + x_{ss}(t) \qquad (6.20)$$

$$x_o(t) = X_o \sin(\omega t - \phi) \qquad (6.21)$$

in terms of m, c, k,

$$X_o = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \qquad (6.22)$$

$$\phi = \tan^{-1} \frac{c\,\omega}{(k - m\,\omega^2)}, \quad 0 \leq \phi \leq 180^\circ \tag{6.23}$$

in terms of ζ, ω_n,

$$X_o = \frac{F_{o/k}}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}} \tag{6.24}$$

$$\phi = \tan^{-1} \frac{2\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}, \quad 0 \leq \phi \leq 180^\circ \tag{6.25}$$

In the dimensionless form,

$$\frac{X_o}{F_{o/k}} = \frac{\text{output amplitude}}{\text{"input" amplitude}} \tag{6.26}$$

$$\frac{X_o}{F_{o/k}} = \frac{1}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}} \tag{6.27}$$

The "input" is the deflection amplitude that would result if only the spring existed.

SINUSOIDAL DISPLACEMENT EXCITATION WITH THE DAMPER GROUNDED

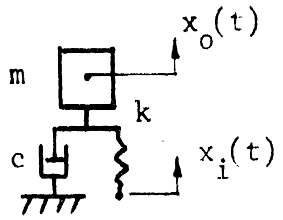


FIGURE 6.4

Differential equation

$$m \ddot{x}_o + c \dot{x}_o + k x_o = k x_i \qquad (6.28)$$

or

$$\ddot{x}_o + 2 \zeta \omega_n \dot{x}_o + \omega_n^2 x_o = k x_i \qquad (6.29)$$

Excitation:

$$x_i = X_i \sin \omega t \qquad (6.30)$$

where,

ω is the excitation frequency, rad/sec.

Initial conditions:

$x_o(0)$, $\dot{x}_o(0)$ do not effect the steady state.

Solution: IGNORE

$$x_o(t) = x_{ts}(t) + x_{ss}(t) \qquad (6.31)$$

$$x_o(t) = X_o \sin(\omega t - \phi) \qquad (6.32)$$

in terms of m, c, k,

$$X_o = \frac{k X_i}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \qquad (6.33)$$

$$\phi = \tan^{-1} \frac{c\omega}{(k - m\omega^2)}, \quad 0^o \leq \phi \leq 90^o \qquad (6.34)$$

in terms of ζ , ω_n

$$X_o = \frac{X_i}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}} \qquad (6.35)$$

$$\phi = \tan^{-1} \frac{2\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \qquad (6.36)$$

IN THE DIMENSIONLESS FORM OF THE AMPLITUDE RATIO

$$\frac{X_o}{X_i} = \frac{\text{output amplitude}}{\text{input amplitude}} \qquad (6.37)$$

$$\frac{X_o}{X_i} = \frac{1}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}} \qquad (6.38)$$

Note:

The results for this case and the previous one are similar. The previous case for the applied force can be compared by the relation:

$$X_i = F_o/k$$

where the force, if applied only to the spring, would give a deflection equivalent to X_i. Do not forget the damper input end is grounded, and this solution is only true after the transient has died out.

SINUSOIDAL DISPLACEMENT EXCITATION

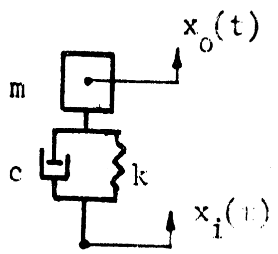


FIGURE 6.5

Differential equation

$$m\ddot{x}_o + c\,\dot{x}_o + k\,x_o = c\,\dot{x}_i + k\,x_i \tag{6.39}$$

or,

$$\ddot{x}_o + 2\zeta\omega_n\dot{x}_o + \omega_n^2 x_o = 2\zeta\omega_n\dot{x}_i + \omega_n^2 x_i \tag{6.40}$$

Excitation:

$$x_i = X_i \sin\omega t \tag{6.41}$$

$$x_i = X_i \cos\omega t \tag{6.42}$$

where,

ω is the excitation frequency, rad/sec.

Initial conditions:

$x_o(0)$, $\dot{x}_o(0)$ do not effect the steady state.

Solution: IGNORE

$$x_o(t) = x_{ts}(t) + x_{ss}(t) \tag{6.43}$$

$$x_o(t) = X_o \sin(\omega t - \phi + \theta) \tag{6.44}$$

For X_o in terms of m, c, k,

$$X_o = \frac{k X_i \sqrt{1 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{6.45}$$

$$\phi = \tan^{-1}\frac{c\omega}{k - m\omega^2}, \qquad 0^\circ \leq \phi \leq 180^\circ \tag{6.46}$$

$$\theta = \tan^{-1} c\,\omega, \qquad 0^\circ \le \theta \le 90^\circ \tag{6.47}$$

For X_o in terms of ζ, ω_n

$$X_o = X_i \frac{\sqrt{1 + (2\zeta(\omega/\omega_n))^2}}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}}$$

or in dimensionless form of the amplitude ratio,

$$\frac{X_o}{X_i} = \frac{\sqrt{1 + (2\zeta(\omega/\omega_n))^2}}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}} \tag{6.49}$$

THE DETERMINATION OF THE DAMPING RATIO FROM THE FREE TRANSIENT

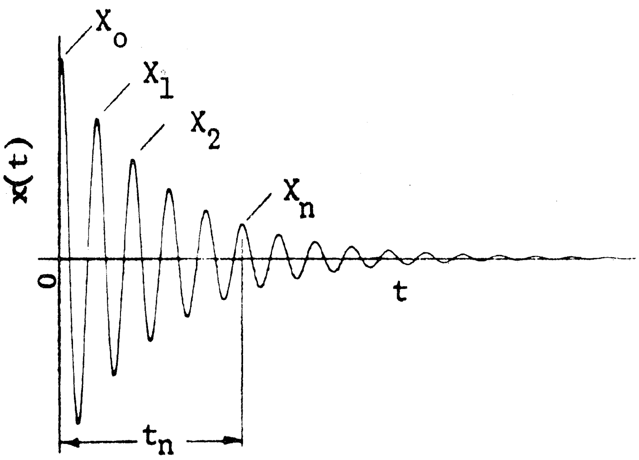


FIGURE 6.6

where:

X_o = initial 0-peak amplitude where t is set = 0, in.

X_1 = first 0-peak amplitude after one oscillation period, in.

X_n = n^{th} 0-peak amplitude after n oscillation periods, in.

T_d = damped natural period, sec.

n = number of oscillations

$$T_d = 2\pi/\omega_d = \frac{t_n}{n} \tag{6.50}$$

This is the free transient response when initial conditions are

$$x(0) = X_o$$

$$\dot{x}(0) = 0$$

and from before as stated in equation 6.13 repeated below,

$$x_o(t) = \frac{X_o}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n} \cos\left(\sqrt{1-\zeta^2}\,\omega_n t - \Phi\right) \tag{6.13}$$

Calculate the damping ratio knowing X_o, X_n, n:

$$\frac{X_o}{X_n} = \frac{x_o(0)}{x_o(t_n)} = \frac{x(0)}{x_o(n \times T_d)} \tag{6.51}$$

which becomes after substitution

$$\frac{X_o}{X_n} = e^{\frac{\zeta}{\sqrt{1-\zeta^2}} 2\pi \cdot n} \tag{6.52}$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{2\pi n} \log_e \frac{X_o}{X_n} \tag{6.53}$$

when $\zeta < .2$, $\sqrt{1-\zeta^2} = 1$ within 2% and

$$\frac{X_o}{X_n} = e^{\zeta 2\pi \cdot n} \tag{6.54}$$

$$\zeta = \frac{1}{2\pi n} \log_e \frac{X_o}{X_n} \tag{6.55}$$

Note:

The term $\log_e X_o/X_n$ is often called the log decrement.

For optimum measurement error chose n so

$$X_n/X_o \text{ is approximately } 1/4 \tag{6.56}$$

If the zero displacement reference is not known, draw an envelope and measure across it at X_o and X_n and use those values for the ratio.

CALCULATE THE UNDAMPED NATURAL FREQUENCY

From the measured t_n of the transient

$$t_d = \frac{t_n}{n} \tag{6.57}$$

$$\omega_d = \frac{2\pi}{T_d} \tag{6.58}$$

$$\omega_n = \frac{1}{\sqrt{1-\zeta^2}}\omega d \tag{6.59}$$

CALCULATE THE EQUIVALENT MASS FROM THE TRANSIENT KNOWING, k, THE SPRING CONSTANT

The equivalent mass can be calculated from equation 6.7 rearranged gives

$$m = k/\omega_n^2 \tag{6.60}$$

It is usually a simple experiment to determine the spring constant, k, of a system. For example, measurement of the deflection resulting from applied forces by means of dead weights is one method.

CALCULATE THE DAMPING COEFFICIENT, c, KNOWING ζ AND K

From equation 6.6,

$$c = \zeta C_c \tag{6.61}$$

Substituting for C_c using equation 6.5,

$$c = (2 k \zeta) / (\omega_n) \tag{6.62}$$

Example Problem: Calculate ζ, ω_n, c, and m given the spring constant, k and the transient response of figure 6.6.

Solution: From the transient response choose n cycles for calculation of the damping ratio such that the n^{th} amplitude is approximately 1/4th the first peak amplitude and use equation 6.55. For,

$$n = 4 \text{ cycles}$$

$$X_n = 1.25$$

$$X_o = 5.00$$

the damping ratio,

$$\zeta = \frac{1}{2\pi n} \log_e \frac{X_o}{X_n}$$

$$= \frac{1}{2\pi 4} \log_e \frac{5.00}{1.25}$$

$$= 0.055$$

The natural undamped frequency will be approximately the same as the damped value since

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

where $\sqrt{1 - \zeta^2}$ is approximately equal 1

hence

$$\omega_n = \omega_d$$

and from Equation 6.50 it can be seen

$$\omega_n = \frac{2\pi}{T_n}$$

$$\omega_n = \frac{2\pi n}{t_n}$$

where for this example it will be assumed that the time for four oscillations is 0.008 seconds.

$$t_n = 0.008 \text{ seconds}$$

which gives,

$$\omega_n = \frac{2\pi \times 4}{0.008}$$

$$\omega_n = 3140 \text{ rad/sec} .$$

for the undamped natural frequency.

The equivalent mass of the system can now be calculated knowing the natural frequency and the spring constant from equation 6.60,

$$m = k/\omega_n^2$$

where:

$$k = 1976 \text{ \#/in. (given)}$$

$$m = 1976/(3140)^2$$

$$m = 2.0 \times 10^{-4} \text{ \# sec.}^2\text{/in.}$$

The coefficient of viscous friction is calculated from equation 6.62

$$c = (2 k \zeta) / (\omega_n)$$

$$= 2 \times 1976 \times 0.055/3140$$

$$c = 6.95 \times 10^{3} \text{ \# sec./in.}$$

The differential equation which describes the system of the example problem is

$$2.0 \times 10^{-4} \ddot{x} + 6.95 \times 10^{3} \dot{x} + 1976 x = 0$$

where the coefficients are m, c, k or,

$$\ddot{x} + (2)(0.055)(3140) \dot{x} + (3140)^2 x = 0$$

$$\ddot{x} + 345 \dot{x} + 9.86 \cdot 10^{6} x = 0$$

where the coefficients are 1, $2 \zeta \omega/\omega_n$, and ω_n^2.

ELECTRONIC DIFFERENTIAL ANALYZER

L. L. Evans

ELECTRONIC DIFFERENTIAL ANALYZER

INTRODUCTION

The use of computing devices in design and analysis is becoming widespread, and with their continued use, they are becoming less instruments to be operated by a specialist and more common tools to be used by the engineer. It is thus advantageous for the engineering student to familiarize himself with the principles of their operation, and also to become aware of the sort of problems amenable to solution by computer methods.

One of these computing devices, known as the analog computer, has proved useful in applied mathematics and engineering where only slide-rule accuracy is required. It often happens that the mathematical laws which govern the variables in different systems are similar in form. These variable physical quantities may be voltages, currents, shaft rotations, temperatures, or any number of phenomena which are subject to measurement. The computer is thus substituted for an analogous, often unwieldy physical system, and one must only relate the variables of the computer system to the analogous variables in the system which is being simulated.

There are several types of analog computers. A special purpose type may be used in fire control systems in anti-aircraft installations; and there are a number of general purpose types which aid in the solution of a variety of problems. Our attention shall be fixed upon the latter type, specifically with that sub-type known as the electronic differential analyzer (EDA).

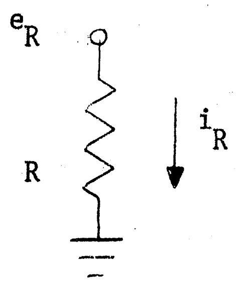
The EDA uses electronic elements to solve linear ordinary differential equations or to solve systems of equations in which the independent variable is time. With the use of auxiliary equipment, the EDA can be used to solve non-linear ordinary differential equations; and its application can be further increased to include partial differential equations, when it is permissible to approximate a partial differential equation by a system of ordinary differential equations using some mathematical subterfuge such as finite difference techniques.

To follow this discussion, the student must first grasp certain fundamental concepts of circuit analysis. It is hoped that a brief review of the concepts here employed will facilitate such an understanding.

ELECTRIC CIRCUITS

Ohm's Law

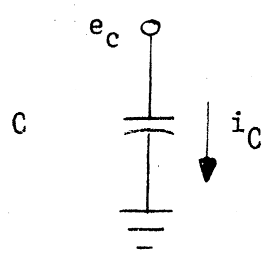
Ohm's law states that the voltage across a resistor is equal to the product of the current and the resistance.



$$e_R = i_R \cdot R \tag{1}$$

Figure 1

For a capacitor, the voltage is given by the expression:



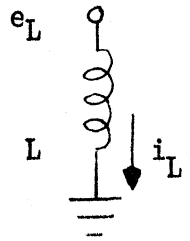
$$e_c = \frac{1}{C}\int i_c \, dt$$

and:

$$i_c = C \frac{de_c}{dt} \tag{2}$$

Figure 2

Similarly, for an inductor:



$$e_L = L\frac{di}{dt}$$

and: (3)

$$i_L = \frac{1}{L}\int e_L\, dt$$

Figure 3

KIRCHOFF'S CURRENT EQUATION

Kirchoff's current equation states that the sum of the currents entering a node in a circuit is equal to the sum of the current leaving that node, or, the algebraic sum of all currents entering a node is zero.

In Figure 4, if the voltage at node A is equal to zero, then e_1 is the voltage across resistor R_1, and $i_1 = \frac{e_1}{R_1}$

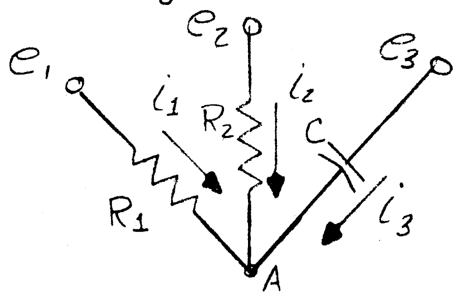


Figure 4

similarly, $i_2 = \frac{e_2}{R_2}$ and $i_3 = C\frac{de_3}{dt}$

According to Kirchoff's current law, the sum of the currents at node A is zero.

$$i_1 + i_2 + i_3 = 0$$

or

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + C\frac{de_3}{dt} = 0$$

THE HIGH-GAIN DIRECT CURRENT AMPLIFIER

The high-gain dc amplifier is a fundamental component of the EDA. Such an amplifier is represented schematically in Figure 5. The input voltage is designated as e_s, and the output voltage as e_o.

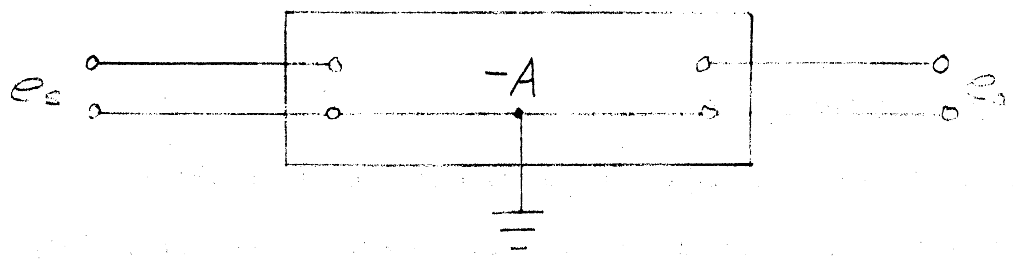


Figure 5

Note that the voltages e_s and e_o are measured with respect to ground potential, as are all voltages which we shall measure in the computer circuit. It is common practice to omit the lower leads and the schematic representation of the amplifier in computer circuits is represented thus:

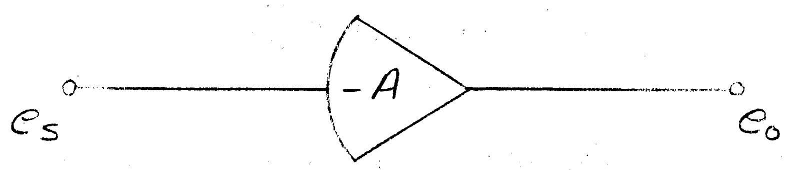


Figure 6

The gain (A) of the amplifier is defined as the ratio,

$$A = -\frac{e_o}{e_s}, \tag{4}$$

and this gain in the computer amplifier is negative. For modern computers, the value of /A/ is on the order of 10^6.

We can now consider a circuit such as that found in Fig. 7, where a feedback resistance (R_o) is connected across the amplifier, and a voltage (e_1) is applied to the input of the amplifier through a resistor (R_1).

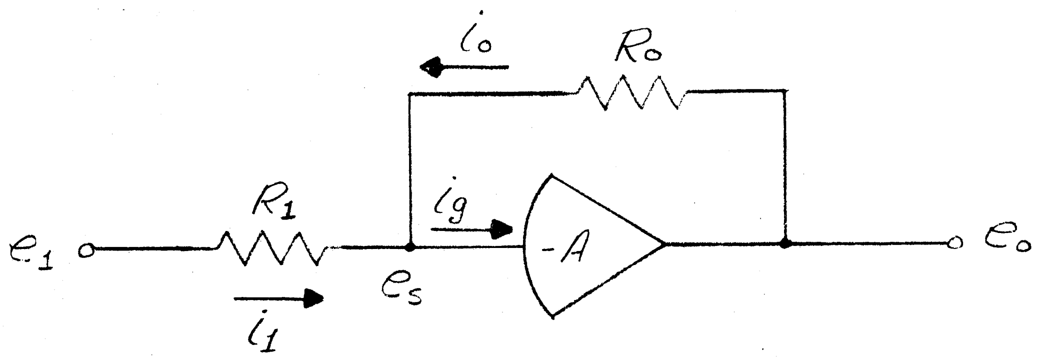


Figure 7

In the circuit of Fig. 7, certain assumptions will be made which for full justification would require a thorough treatment of the operational characteristics of the dc amplifier. For additional information in this regard the student should consult the references listed. Assumptions are as follows:

1. The gain (A) of the amplifier is assumed equal to infinity.
2. The voltage e_s is assumed equal to zero.
   This might logically follow under assumption 1 if one refers to the definition of the gain as given in Eq. (4),

$$A = -\frac{e_o}{e_s}$$

   For finite values of e_o, the value of e_s must be very small if A is to be very large.

3. The current i_g is assumed to be equal to zero.

   The current i_g would be a current flowing into the grid of the first stage of amplification of the amplifier. Under normal operating conditions the impedance at this point is extremely high and the current is negligibly small.

Using the notation indicated in Fig. 7, Kirchoff's current equation can be written at the node e_s, which will be referred to hereafter as the summing-junction.

$$i_1 + i_o = 0 \qquad i_g = 0 \tag{5}$$

Substituting these values in Eq. (5), the expression becomes

$$\frac{e_1}{R_1} + \frac{e_o}{R_o} = 0 \tag{6}$$

Solving for e_o,

$$e_o = -\frac{R_o}{R_1} e_1 \tag{7}$$

Thus Fig. 7 is a computer circuit which multiplies by an arbitrary negative constant, $\frac{R_o}{R_1}$, an arbitrary voltage (e_1) fed into the circuit.

SIGN-CHANGER AMPLIFIER

When the ratio $\frac{R_o}{R_1}$, in (7) above is equal to unity, the circuit of Fig. 7 serves as a sign changer or inverter. Equation (7) then becomes:

$$e_o = -e_1 \tag{8}$$

SUMMATION

For several input voltages as shown in Fig. 8 Kirchoff's current equation can be written at the summing junction and the output voltage e_o is proportional to the sum of the input voltages.

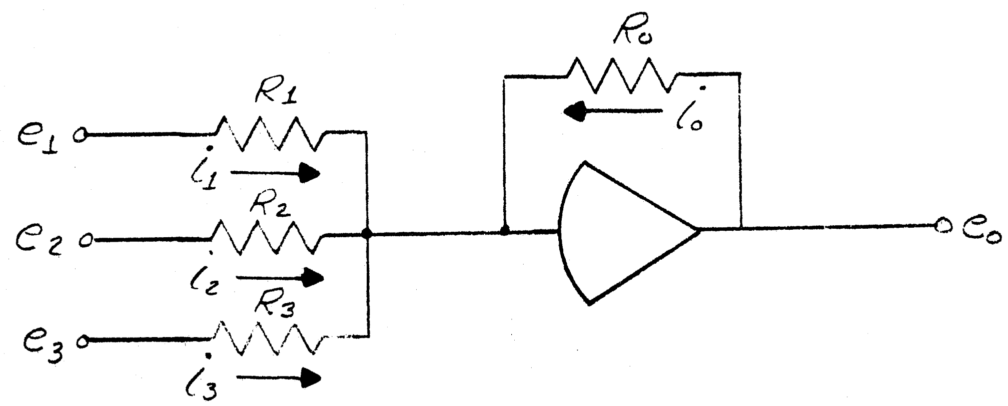


Figure 8

By adjusting the values of the R's, multiples or fractions of input voltages may be summed.

For the general case of 'n' input voltages,

$$e_o = - R_o \sum_{i=1}^{n} \frac{e_1}{R_i} \qquad (12)$$

INTEGRATION

When the feedback element is a capacitor, the Kirchoff equation shows that the output voltage is proportional to the time integral of the input voltage.

At the summing junction,

$$i_1 + i_o = 0$$

$$\frac{e_1}{R_1} + C \frac{de_o}{dt} = 0$$

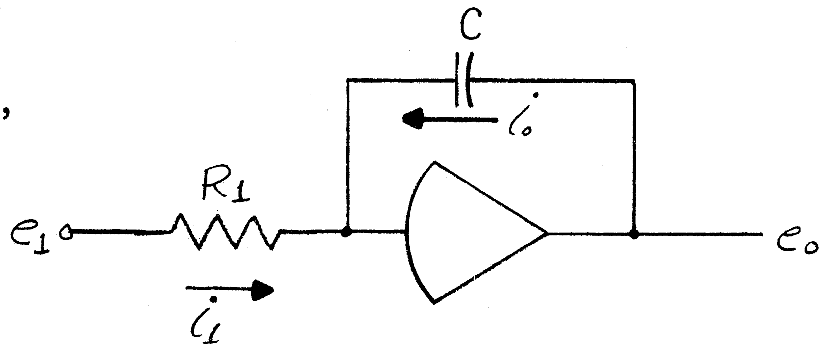


Figure 9

(13)

Rearranging Eq. (13) and separating the variables,

$$de_o = \frac{1}{R_1 C} e_1 \, dt \qquad (14)$$

Integrating (14)

$$e_o = - \frac{1}{R_1 C} \int e_1 \, dt \qquad (15)$$

It can be shown that for the general case of n input voltages, the output voltage is proportional to the time integral of the sum of the inputs.

$$e_o = - \frac{1}{C} \int \left[\sum_{i=1}^{n} \frac{e_i}{R_i} \right] dt \qquad (16)$$

UNITS

The units of resistance employed in the EDA are designated in terms of megohms (10^6 ohms) and the units of capacitance are designated as micro-farads (10^{-6} farad).

If, in Eq. (15), the value of R_1 is 1 megohm and the value of C is 1 micro-farad the constant, $\frac{i}{R_1 C}$ becomes $\frac{1}{10^6 \times 10^{-6}} = 1$.

In schematic representations of computer circuits, it is customary to omit the factors 10^6 and 10^{-6} and it is understood that the values of resistance and capacitance indicated are megohms and microfarads, respectively.

CIRCUIT LAYOUT PROCEDURE

A general step-by-step procedure for deriving a computer circuit corresponding to a differential equation is suggested below.

1. Solve the equation for the highest derivative.
2. Assume that the sum of the currents flowing into the summing junction of an amplifier through the input circuits is proportional to this highest derivative.

3. Generate voltages proportional to the lower derivatives by successive integrations, obtaining the proper signs as required by the equation.
4. Use the voltages from 3 to generate the input currents as required by 2.
5. Provide for specified initial conditions.

Example No. 1

This procedure will now be followed to derive a computer circuit which would solve a linear second-order differential equation.

Consider the equation,

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0 \tag{17}$$

with

$$\frac{dx}{dt} = 0 \text{ and } x = 1 \text{ for } t = 0$$

This might be the equation of motion for a simple vibrating system composed of a mass, a spring, and a damper where the mass has an initial displacement,

For convenience Eq. (17) will be written:

$$\ddot{x} + \dot{x} + x = 0 ,$$

$$x(0) = 1, \quad \dot{x}(0) = 0 \tag{18}$$

where the number of dots appearing above the dependent variable indicates the order of the derivative of the variable.

Following the step-by-step procedure listed above:

Step 1: Solve for the highest order derivative.

$$\ddot{x} = -\dot{x} - x$$

Step 2: Assume a current proportional to $\ddot{x}$ is flowing into the summing junction of an amplifier.

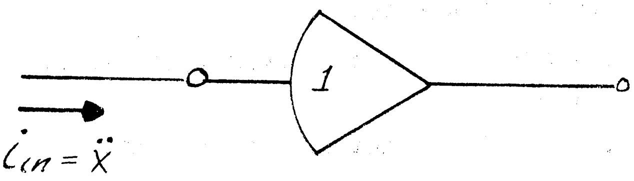


Figure 10

Step 3: If a feedback capacitor is placed across this amplifier and Kirchoff's current law is written at the summing junction the output of the amplifier is a voltage proportional to -x.

$$i_{in} + i_o = 0$$

$$\ddot{x} + C \frac{de_o}{dt} = 0, \quad C = 1$$

$$de_o = -\ddot{x}\,dt.$$

$$e_o = - \int \ddot{x}\,dt = -\dot{x} \text{ volts}$$

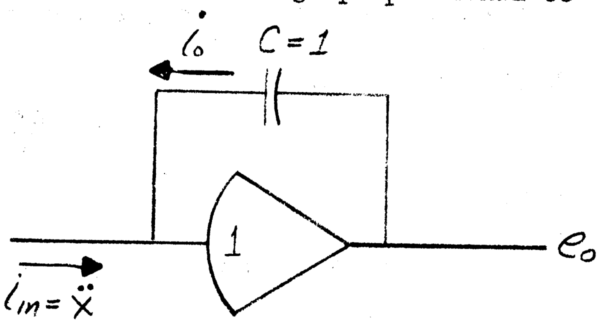


Figure 11

Hence if the current into amplifier No. 1 is proportional to $\ddot{x}$, the output voltage at amplifier No. 1 is proportional to $-\dot{x}$.

By repeating the integration process a voltage proportional to the dependent variable (x) appears as the output of amplifier No. 2. (Fig. 12).

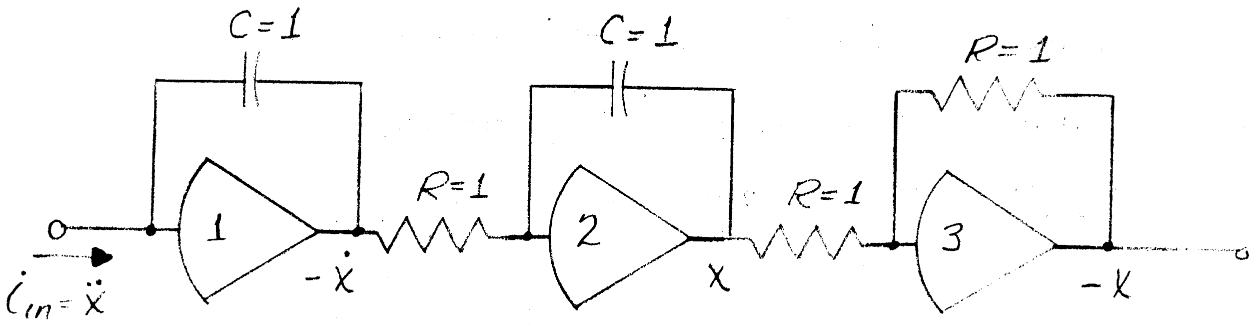


Figure 12

Unless the dependent variable is accompanied by an arrow, as at the imput to amplifier No. 1, the variable will represent a voltage at that point in the system rather than a current. It is important to note that voltages always appear at the output of an amplifier, since only voltages are available to be applied to recording equipment.

From the differential equation it is noted that currents equal to $-\dot{x}$ and $- x$ are required such that their sum will equal $\ddot{x}$. Hence the need for inverter amplifier No. 3.

Step 4: If the voltages $-\dot{x}$ and $-x$ at the outputs of amplifiers 1 3 respectively are now fed back through input resistors to amplifier No. 1, the currents i_1, and i_2 (Fig. 13) add to give the current $i_{in} = \ddot{x}$.

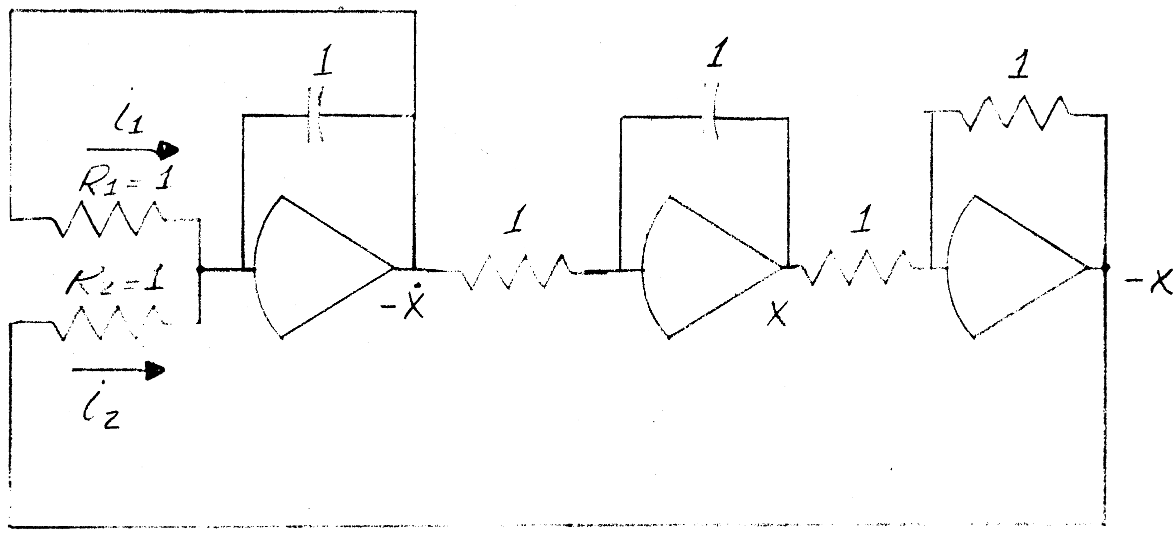


Figure 13

Figure 13 is the computer circuit for Eq. (18).

Step 5: Initial Conditions

The initial condition corresponding to $x(0) = 1$ remains to be incorporated in the circuit. This is accomplished by placing an initial charge on the capacitor of amplifier 2.

The initial voltage at the output of amplifier 2 is the voltage across the feedback capacitor and would correspond to an initial displacement of the mass for the mass, spring, and damper system. The act of releasing the mass from its initial displacement is accomplished on the computer by a switch which closes the feedback loop on amplifier No. 2 allowing the capacitor to discharge and place the system in motion.

Since we have specified a zero initial velocity, no initial charge is placed upon the feedback capacitor of amplifier No. 1.

UNIFORM GRAPHIC STRUCTURE

In the preceeding derivation of a computer circuit, the complete schematic for the operational amplifier and the associated circuit elements has been used. For example, a complete integrator is drawn as shown in Fig. 14.

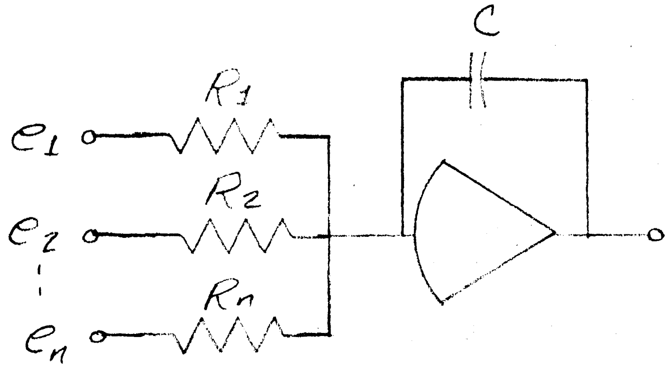


Figure 14

This schematic has the advantage of being very explicit. However, it is cumbersome and in fact unnecessary to use all of the elements on a computer diagram. A compact notation has been developed for all standard analog computing elements as shown in tabulated form in Fig. 15. The compact notation has several advantages which we shall exploit in the next section on scaling. In particular, note that in all cases the output voltage is related to the input voltage (s) by an operation such as summing or integration and a gain. (multiplication)

In the case of a summing amplifier each input is scaled by the gain R_f/R in. In the case of an integrating amplifier each input is scaled by the gain $\frac{1}{R \text{ in } C}$.[1] Note that for both the summer and integrator the gain of each input can be individually adjusted by selecting a proper R in.

The two remaining standard computing elements are the inverter and the potentiometer. These are shown in Fig. 15 and are self explanatory.

[1] An integrator can be viewed as a "unity integrator" and a gain. A "unity integrator" has a unity gain so that a constant one volt will integrate to a range of 1 volt/sec.

7

SCALING

In the preceding derivation of a computer circuit it was noted that voltages proportional to the velocity and position of the mass of a vibrating system were available as the output voltages of indicating or recording devices such as an oscilloscope, an x-y plotter, or a strip chart recorder. It remains to relate a given voltage to the physical variable which it represents.

This raises the problem of scaling. When making an engineering drawing, it is often necessary to incorporate a scale factor, since the subject of the drawing may be very large making a full sized drawing impossible or it may be so small that an actual size drawing would be impractical. This in principle holds true for the computer. Scaling on the computer involves two variables: the dependent variable, a voltage; and the independent variable, time. A change of the time base, or time scaling as it is commonly called, is a very simple operation that approaches triviality. For this reason we will break this section into two parts, the first part being the technique of amplitude or voltage scaling and the second being time scaling.

Amplitude Scaling: The technique of amplitude scaling the voltages of the analog computer so as to represent the variables of a physical system can be easily grasped if one appreciates the basic characteristics of the elements which comprise the analog computer. Up to this point in our discussion we have treated operational amplifiers as ideal devices with no limitations as to output voltage or current. In actual practice the amplifier will have a maximum amplitude of the output voltage which is 10 to 20 percent

SUMMARY OF BLOCK DIAGRAMS, SYMBOLS, AND COMPUTER CIRCUITS FOR ANALOG COMPUTER OPERATIONS

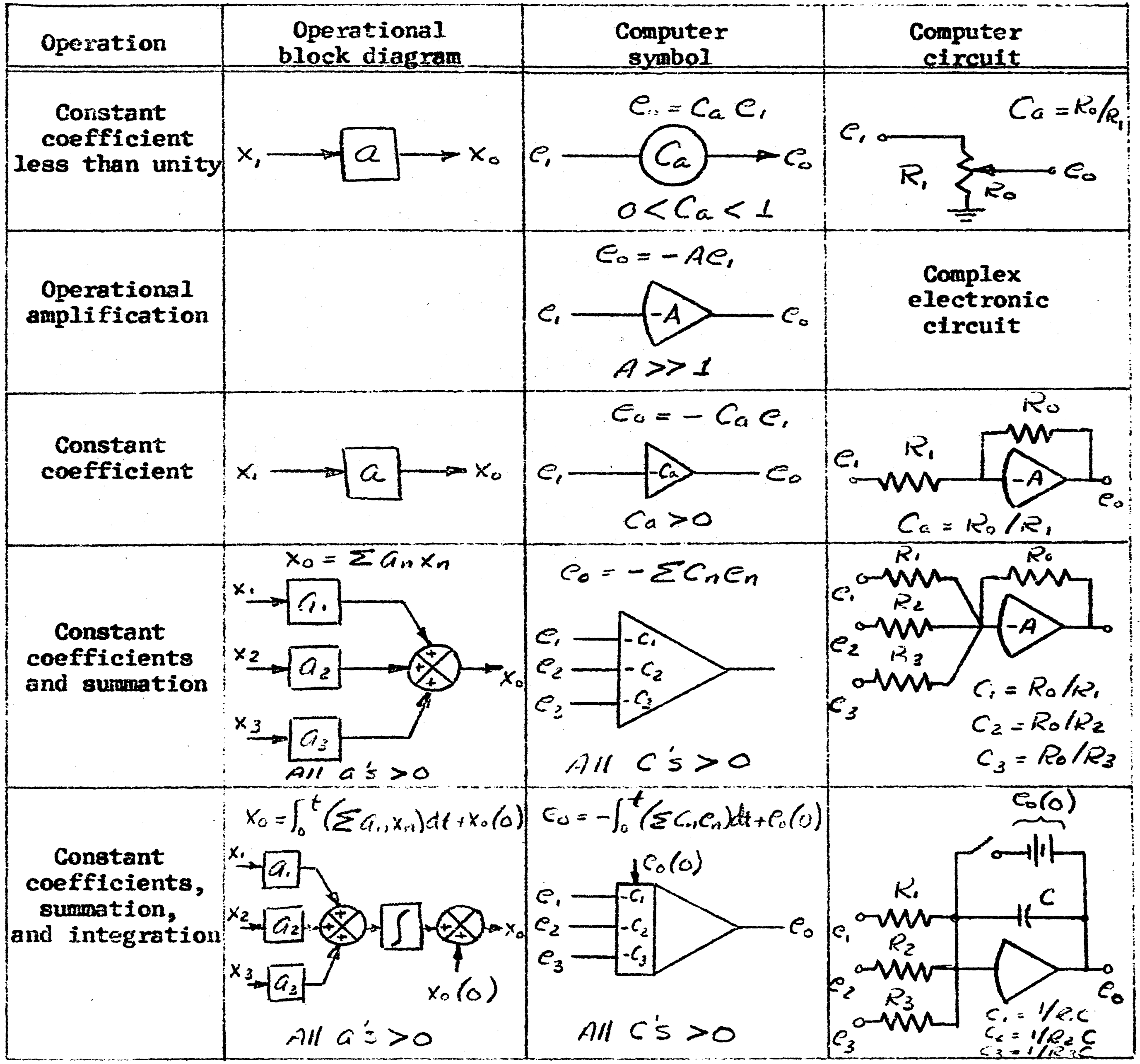


Figure 15

Reproduced from:
Shearer, Murphy, Richardson, "Introduction to Systems Dynamics," Addison-Wesley, Reading, Mass., 1967

greater than the rated machine voltage. Thus the amplifiers on a "100-volt machine" would have a maximum output voltage between 110 and 120 volts. Modern amplifiers have a current rating which is far in excess of any demands we shall make and we can ignore the current limitations.

The problems which arise when converting from physical units to voltages can be demonstated very easily with an example. Suppose we have a hydraulic piston and we wish to determine the force on the piston knowing the supply pressure and the piston area. A schematic of this system is shown in Fig. 16-a.

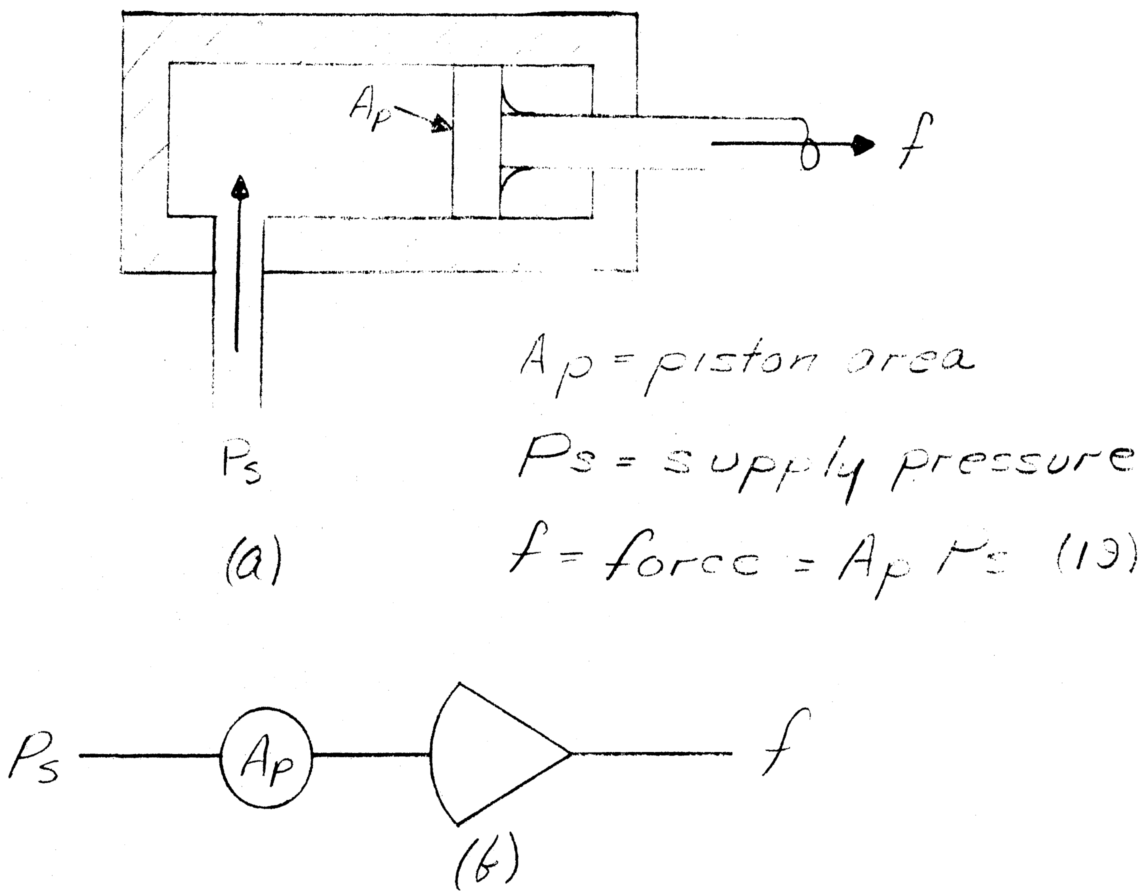


Figure 16

The analog structure for the system is shown in Fig. 16b. Note that 16b is the representation of Eq. (19). In order to get a numerical answer we need to provide a relationship between the physical variables represented and the voltages on the analog computer. To do this we will introduce the concept of a voltage or amplitude scale factor by the relationship:

$$\text{volts} = S_f \times \text{physical variable} \tag{20}$$

In simple terms the scale factor defines the relationship between a given voltage and its physical counterpart. For example a scale factor of 10 volts/unit indicates that 10 volts on the computer will represent one physical unit. A scale factor of 100 volts/unit indicates that 100 computer volts will represent one physical unit. A scale factor must be associated with every voltage on the computer. In general this amounts to the output of each amplifier and any input voltages.

With this definition in hand, lets look at the implementation of Fig. 16-b for the following typical case:

$$P_s = 100 \text{ lb/in.}^2$$
$$A_p = 89 \text{ in.}^2$$

Note that in Fig. 16-b the input and output voltages represent the pressure and force respectively. The scalar relationship between these two variables, A_p, is provided by the combination of the potentiometer setting and the voltage gain of the amplifier. A good choice for the scale factor to represent the pressure variable would be 1 V/u. Then the voltage proportional to the pressure is:

$$e_p = S_{f_p} \cdot \text{physical variable}$$
$$= 1 \text{ V}/u \cdot 100 \text{ units}$$
$$e_p = 100 \text{ volts}$$

This value effectively utilizes the range of a 100-volt computer.

Our next step is very important. If we set the physical value of A_p (in this case 89) by using a potentiometer set at .89 and an amplifier gain of 100, then the scale factor associated with the force voltage will also be 1 V/unit as shown in Fig. 17. However, there is a very important practical reason why we do not wish to leave the system as it stands. The required output voltage would be 100 volts x 89 or 8900 volts which is of course impossible. There are several techniques of scaling which can be used to rectify this dilemma as anyone who reads literature in this field will quickly discover. The following technique is preferred by the author.

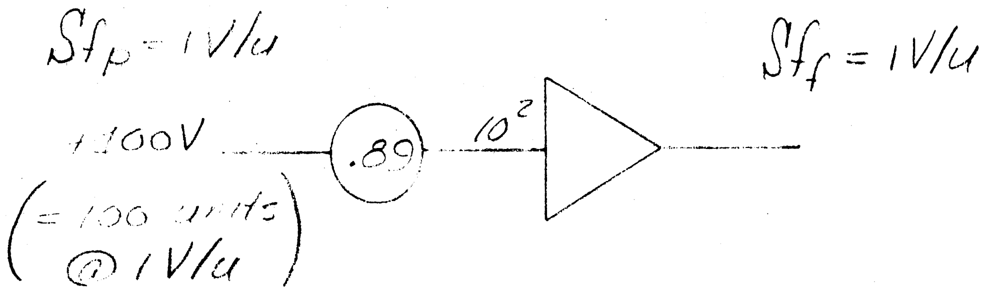


Figure 17

If we temporarily divorce ourselves from the physical problem and look only at the analog hardware a little thought will quickly point out that we would like the total gain of the pot-amplifier combination to be as close to unity as possible. If the set gain is close to unity then both the input and the output voltages can span the entire voltage range of the machine. Conversely, if the set gain is much different from unity then one or other of the voltages must be small in order for the other not to exceed the rated machine voltage. Fortunately it is always true that the computer gain can be different from the physical gain (within certain bounds to be discussed later) providing that a careful bookkeeping system is used. In Fig. 17 we can accomplish the desired unity computer gain by simply simultaneously changing the 10^2 gain to unity and also changing the way we read the output variable. The result is shown in Fig. 18. Note that in Fig. 18 the output now has a scale factor of 10^{-2} volts/unit. This scale factor was obtained from the following relationship between the input scale factor and the change of gain.

$$S_{f_{in}} \times \text{change in gain} = S_{f\ out} \qquad (21)$$

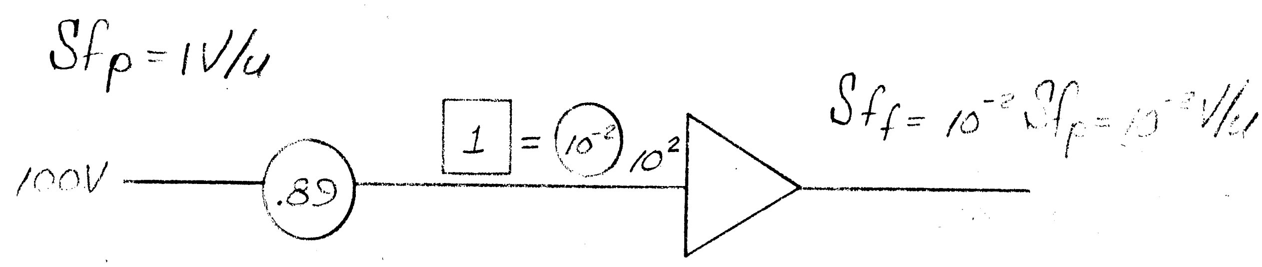


Figure 18

Equation (21), when properly applied, can reduce the scaling of an analog computer to a routine procedure. Note that the scale factors are related by the amount the gain was changed, not by the magnitude of the gain.

A quick check of the numerical result is in order. The output voltage from the amplifier in Fig. 18 will be 89 volts. The magnitude of the physical variable is:

$$f = \frac{\text{volts}}{S_f} = \frac{89 \text{ volts}}{10^{-2} \text{ volts/unit}}$$

$$f = 8900 \text{ units}$$
$$= 8900 \text{ lbs.}$$

The bookkeeping system which we will use is demonstrated in Fig. 18. There are in general three gain quantities associated with each input to an amplifier. The legend and explanation is as follows:

□ (Set gain) = ○ (Change in gain) x physical gain (22)

The physical gain is the gain which is derived from the problem statement.

The change in gain is the amount by which the gain is changed to satisfy certain requirements.

The set gain is the gain which will actually be programmed on the analog computer. If it is different from the physical gain there will naturally be a difference between the input and output scale factors and they should be related by the change in gain.

Also be sure to carefully record all three of the above quantities.

The technique described above can now be used along with the following guides to scale any problem for analog simulation. We will state first the complete procedures and then discuss the various steps in detail. Remember, we are discussing amplitude scaling only.

Scaling Procedure:

1. Check that all equations are complete and that consistent units have been used throughout.

2. Draw the preliminary computer diagram using summers, integrators, and inverters where necessary. All of the gains of the computing elements should be the physical gains associated with the system at this time.

3. The problem will in general have one to several closed paths or "loops" around which a signal can travel. Two rules which apply to the closed loops are:

   a) Each loop should have an odd number of computing amplifiers.

   b) The total gain in each loop must remain constant. This gain is determined by breaking the loop at any point and then determining the multiplication of the signal as it follows the path which returns it to the break.

4. Statement 3 requires only that the gain of any loop remain constant. It says nothing about the location of the gain within the loop. In general the total gain of a closed loop should be evenly distributed among the integrators within the loop and the gains on the summers and inverters should be approximately unity. This will be quite easy to accomplish using the bookkeeping technique previously described. These rules of thumb are listed as follows:

   For the scaled computer diagram:

   a) All summer and inverter inputs should have approximately unity gain.

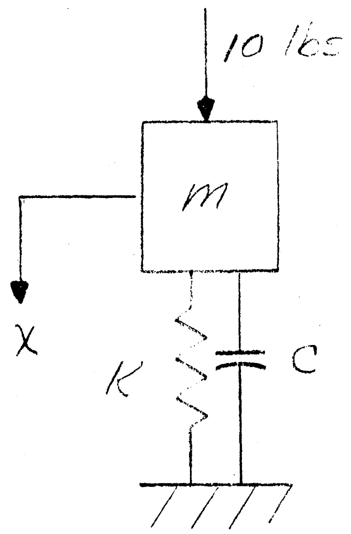
   b) All integrator inputs should have approximately the same gain - not necessarily unity.

   c) All of the inputs to a given integrator should have approximately the same gain.

One point that often causes confusion should be pointed out at this time. Equation (21) states the relationship between the input and output variables of a computing element for which we have changed the gain to satisfy requirement 4. It says nothing about the absolute magnitude of any of the scale factors, only their ratios. The inputs to the system (either inputs external to the closed loops or initial conditions) will determine the absolute magnitude of the various scale factors. If we have correctly proportioned the gains in the various loops according to step 4, then the specification of any one scale factor should be sufficient to determine the remaining scale factors of the problem. This is a very interesting property of a dynamic system. One further point, if the condition of step 4 cannot be met, then one should consider the possibility of some of the terms contributing an insignificant influence to the system.

Examples are:

Example #1 - A force of 10 lbs. is suddenly applied to the system as shown. The parameters are given below.



$$m = .1 \ \frac{lb\ sec^2}{ft}$$

$$K = 3.95 \ \frac{lb}{ft}$$

$$c = .251 \ \frac{lb\ sec}{ft}$$

$$x(0) = \dot{x}(0) = 0$$

Set up the analog computer circuit for solving this problem.

Solution:

a) Write the differential equation using consistent physical units.

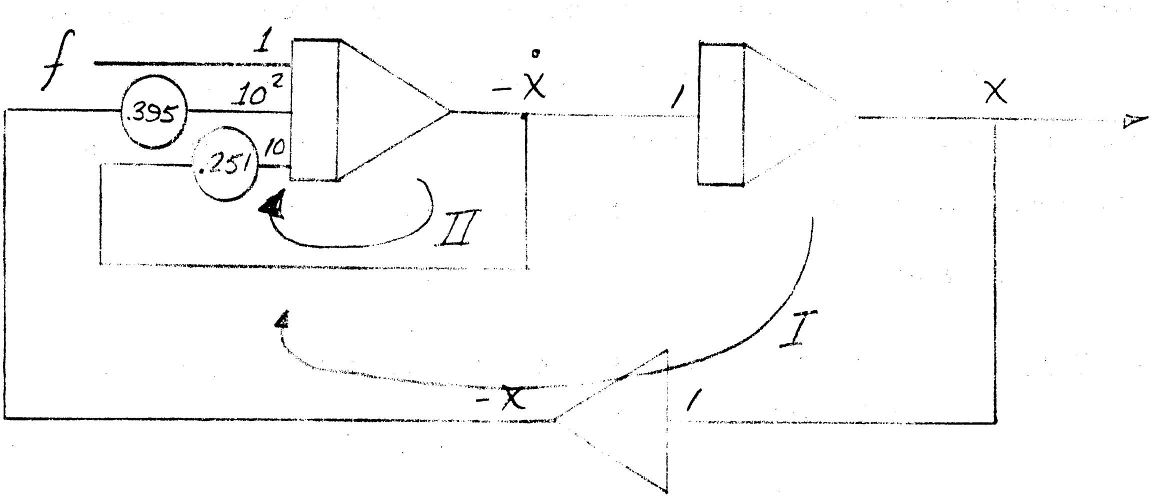
$$m\ddot{x} + c\dot{x} + kx = f$$

$$.1\ddot{x} + .251\dot{x} + 3.95x = f$$

b) Solve for the highest derivative:

$$\ddot{x} = -2.51\dot{x} - 39.5x + 10f$$

c) Set up the computer diagram in terms of physical gains.



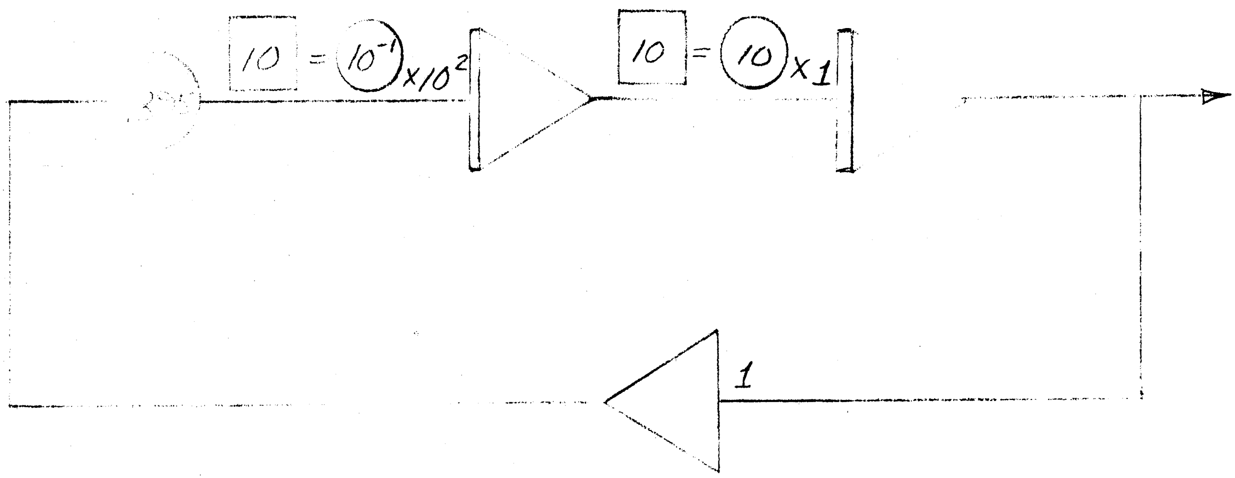
d) Redistribute the gains in the loops according to step 4. In this case there are two loops labeled I and II respectively.

Looking first at the outer loop (i.e. loop I) the total gain in this loop is 39.5. Since there is only one inverter in this loop and it has a gain of unity, we need distribute the integrator gains only. For the ideal case each integrator would have a gain of $\sqrt{39.5}$. However, this would require an additional potentiometer and would be more cumbersome to change any parameters. Our best bet is to work with powers of 10 only and leave the potentiometer at its present value. Thus we will change the loop I input on the first integrator to .395 x 10 and the input to the second integrator to 10. Each gain is specified by the legend:

Set gain = change in gain x original gain

□ = ○ x original gain

Remember that we need to keep track of the change in gain as it specifies the ratio between the scale factors whereas the set gain is the gain we will actually set on the computer. The correctly scaled outer loop is:



The total gain in this loop has not been altered.

The inner loop (loop II) has only one element. Therefore we cannot share this gain with other integrators and must use it as it stands.

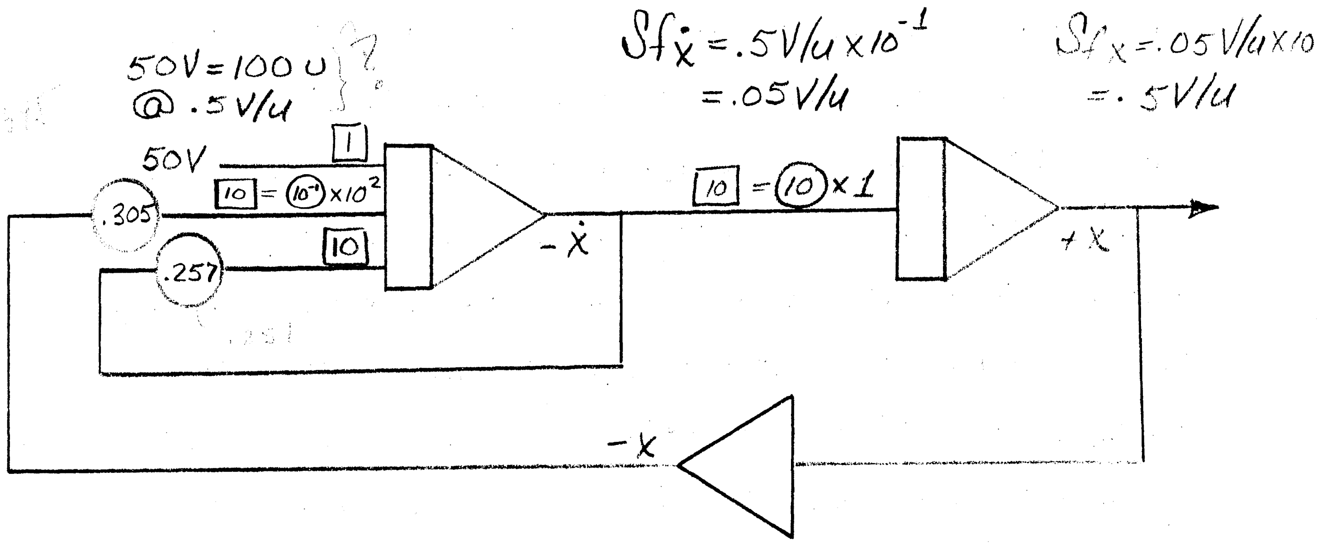
Both loops now have a satisfactory gain distribution as specified by step 4. All that remains is to specify the input which will in turn determine the scale factors of the system. A scale factor must be associated with the input quantity so that it can be set as a voltage on the computer. In this case the input is 10 f which has the magnitude 100 ft/sec.^{2} It is temping to choose 1 volt/unit as the scale factor as then the voltage representing this quantity would be the full 100 volts. However, it is usually true that the output will overshoot (exceed the final value during the transient) and if properly scaled will exceed 100 volts. Therefore we will choose .5 V/u as the scale factor realizing that it can be modified if need be.

The next step, which can easily cause confusion, is the selection of the gain to be used for the force input on the first integrator. A little thought will show that this gain can be changed from its present value of unity to any other value, the only effect being to change the magnitude of the scale factors of the problem. A good rule of thumb stated in step 4c was that all of the inputs to a given integrator should have approximately the same gain. Since factors of 10 can be easily set on the elements without the use of additional potentiometers we will require only that the gains of each input agree to the closest powers of 10. The outer loop input has a gain of 3.95 which is closer to unity than 10, therefore we shall leave the input gain at unity. Had the outer loop input gain been 8.31 for example, we would have multiplied the input gain by 10 by the same reasoning.

Once the scale factor and gain of the input have been selected the problem is complete. The remaining scale factors are determined by:

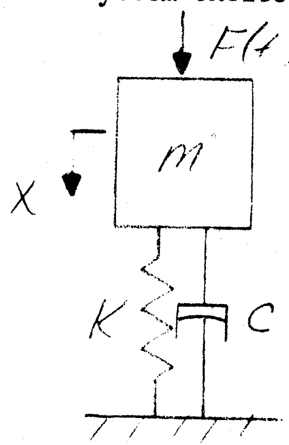
$$S_{fout} = \text{change in gain} \times S_{fin}$$

The final scaled diagram is shown below:



The quantities in square brackets are the values which would be set on the computer. Of course the ultimate test of the scaled solution is to actually set it upon the computer and look at the results. A clever person could quickly point out that the steady state value of x above will result in 16 volts for x. This value is low and can quickly be corrected by a simple combination of the external input magnitude and gain. (For example, what would be the effect of making the gain associated with the external input 2 insteady of unity?)

Example #2 - As a second example, look at a very high frequency mechanical system excited by a sinusoidal forcing function.

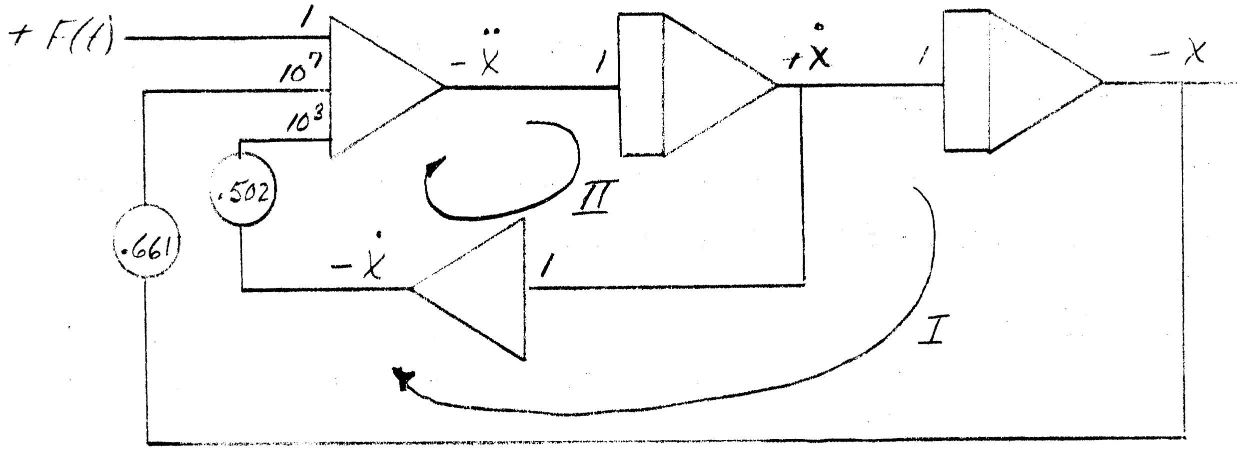


It has been determined that the governing differential equation is:

$$\ddot{x} + 5.02 \times 10^2 \dot{x} + 6.61 \times 10^6 x = 6.61 \times 10^4 \sin (1250t)$$

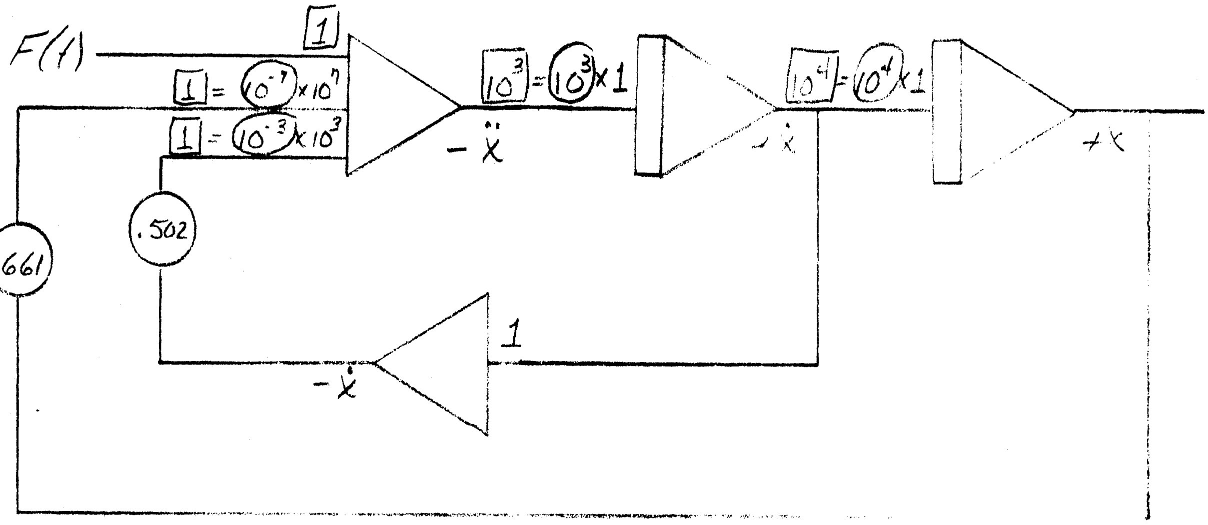
Set up and scale a computer diagram for this system which includes as output the acceleration, the velocity and the displacement.

Since we want to observe the acceleration it will be necessary to include a summer so as to allow looking at the acceleration before it is integrated. (In Example #1 a single amplifier provided both the summing and the integration, as such the acceleration was not directly observable). The computer diagram in terms of physical gains is:



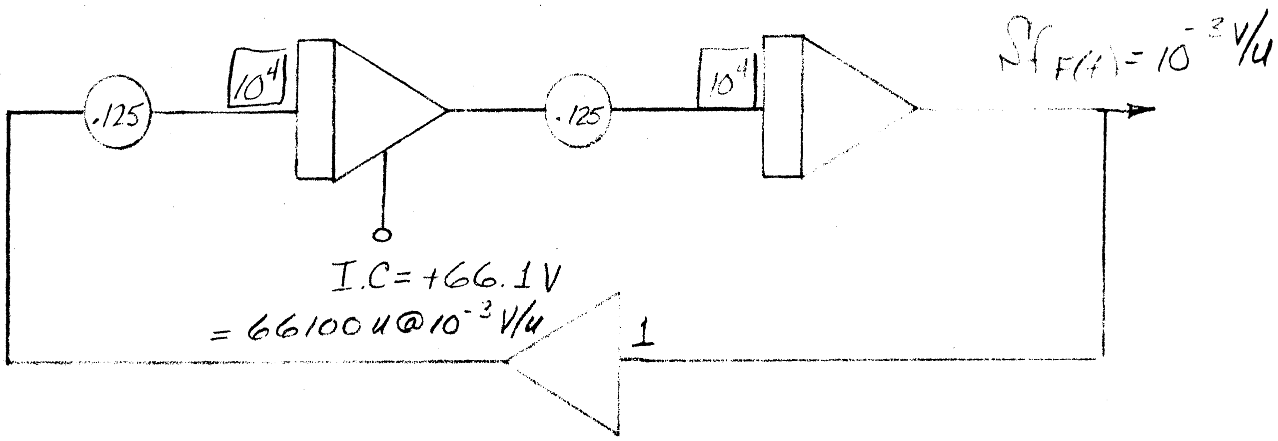
This problem can be approached in two parts. The first part consists of scaling the loops whereas the second part consists of scaling the forcing function.

In a similar manner to the previous example we will look first at the outer loop or loop I. This loop has a total gain of $.661 \times 10^7$ which must be maintained. However, the summer in this loop has an input gain of $.661 \times 10^7$ instead of the desired unity. The proper approach is to distribute the 10^7 part of the gain among the two integrators in the loop. The resulting diagram is shown below:



Note that once the outer loop gain was proportioned, it was necessary to change the gain of the inner loop so as to maintain the total gain of this loop constant.

The forcing function for this case is a sine wave whose maximum amplitude is 6.61 x 10^4 physical units. If we associate a scale factor of 10^{-3} V/unit with this amplitude, we will then need on the computer a sine wave whose maximum amplitude is 66.1 volts and whose frequency is 1250 rad/sec. The circuit required to generate this function is:



Since the scale factor on the forcing function is 10^{-3} V/u then by inspection of the previous diagram the remaining scale factors are:

$$S_{f_{\ddot{x}}} = 10^{-3} \text{ V/u}$$

$$S_{f_{\dot{x}}} = 1 \text{ V/u}$$

$$S_{f_x} = 10^4 \text{ V/u}$$

On a complete diagram these scale factors should be indicated at the output of their respective amplifiers.

Anyone who has had prior analog experience will recognize that in general gains of 10^4, such as utilized in Example 2, are not available on a standard analog computer. This is really not a problem as can be seen in the next section on time scaling.

Time Scaling: Time scaling on the analog computer is one of the hardest concepts to grasp yet one of the easiest to apply to a given situation. It was hinted in Example 2 that the allowable gain magnitudes on an analog integrator are quite small. One can quickly gain insight into a problem by looking at the integrator gains required for solving the problem (since in general all summer and inverter gains will be close to unity and the net gain across the integrators will dictate the dynamic behavior). A very slow system is indicated when the loop gains are very small such as 10^{-6}. An example would be a frost cycle which has a period of one year. At the other extreme, very high loop gains indicate a very fast system. This was the case in Example 2 which was an oscillatory system with a natural frequency of 2000 cps. A single computer which is capable of duplicating in real time such a range of systems would not only be prohibitively expensive but even more, extremely undesirable. Certainly nobody wants to wait one year for a slow solution. Also a very fast solution is well beyond the range of any recording instrument and thus equally undesirable.

Fortunately, virtually all dynamic systems can be compressed into a very narrow range of allowable computer gains by changing the independent variables of the problem, time. A standard computer will typically have integrators whose allowable gain settings cover the range 1 to 100.

Time scaling, or changing the time base, consists of programming the analog computer so that it performs a solution which has the exact amplitudes and wave shapes as the real time system but whose time base is some multiple of the real time base. In an analog simulation only the integrators have a time dependent behavior (summing can be viewed as an instantaneous operation). Thus only the integrators of a problem need be modified to cause a solution time which is different from the real time. It will turn out that time scaling can be accomplished merely by changing <u>all</u> integrator gains by the same amount. This can be easily derived from the equation for the integrator operation:

For an integrator

$$e_{out}(t) = -C_1 \int_0^t e_{in}(t)\,dt \tag{22}$$

and suppose we want a solution in terms of τ where $\tau = nt$. In other words we want the time base of the solution, τ, to be different from the time base of the real system t. For example if $n < 1$ then the solution time is slower than the real system while if $n > 1$ the solution time is faster than the real system. If we substitute nt for t in the above expression:

$$e_{out}(nt) = -C_1 \int_0^{nt} e_{in}(nt)\,d(nt)$$

$$e_{out}(nt) = nC_1 \int_0^{nt} e_{in}(nt)\,dt$$

$$e_{out}(\tau) = -nC_1 \int_0^{t} e_{in}(t)\,dt \tag{23}$$

This is a very subtle but interesting result. It states that by merely multiplying the non-time scaled integrator gain by n, the variables associated with the integrator will be in terms of τ instead of t. The gain which will actually be set for solving the problem is n C_1.

From this brief analysis we can make the following rule:

If <u>all</u> <u>integrator</u> <u>gains</u> <u>in</u> an analog solution are multiplied by the same arbitrary constant n, the solution which results has a time base τ where τ = nt and t is the real time base.

In general a standard computer will typically have allowable gain settings on the integrators which span the range 1 to 10^2. Referring back to example #2, we arrived at a final diagram which required gain settings of 10^3 and 10^4 in the integrators. Based on the above discussion we can change the problem to correspond to the standard computer by changing <u>all</u> integrator gains by some value n. <u>If only the integrator gains are changed, the original amplitude scale factors will still be valid.</u> In the case of example 2 we would choose $n = 10^{-3}$ which makes the solution time 1000 times slower than real time. (i.e. the solution proceeds at a rate 1000 times slower than real time).

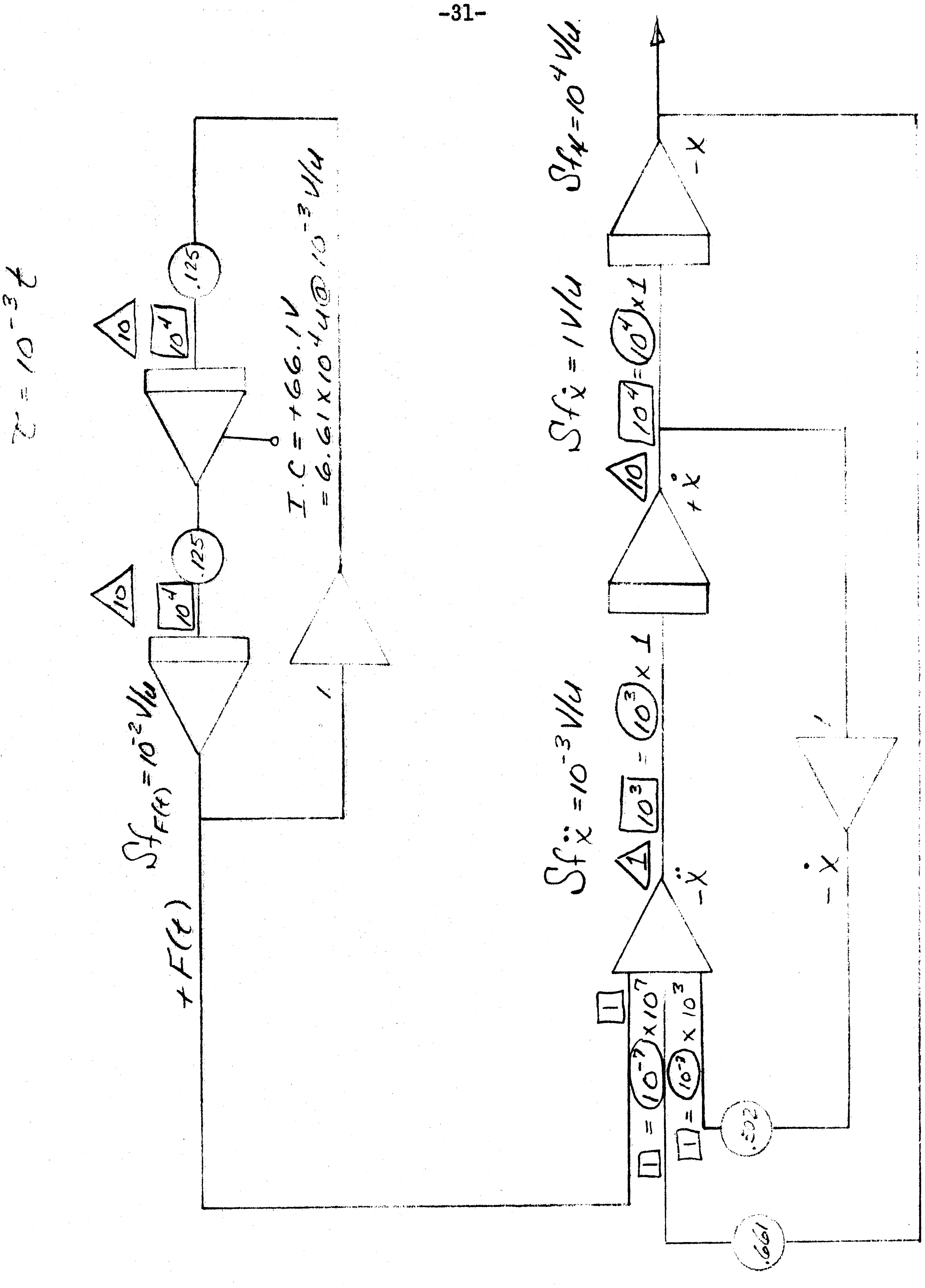
Since we are again using a change of gain to implement a desired condition, we need to extend our bookkeeping to keep pace. This will be done according to the following legend:

△ = time scaled integrator gain

△ = nx □ (24)

△ = n x set gain

Note that the triangular symbol will occur only at integrators. A complete diagram of Example #2, with time scaling included, is shown in Fig. 19.



τ = 10⁻³t
.125
10
10⁴
I.C = +66.1V
= 6.61 x 10⁴ u @ 10⁻³ V/u
+F(t)
Sf F(t) = 10⁻² V/u
Sf ẍ = 10⁻³ V/u
10³ = 10³ x 1
-ẍ
Sf ẋ = 1 V/u
10⁴ = 10⁴ x 1
+ẋ
Sf x = 10⁴ V/u
-x
-ẋ
1 = 10⁻⁷ x 10⁷
1 = 10⁻³ x 10³
.202
.661

One last word of explanation. The gain of an integrator is 1/RC and the gain can be changed for time scaling purposes by changing either the resistor or the capacitor (or both).

Summary: The previous scaling techniques can be summarized very compactly. Remember that the best technique is to first amplitude scale the computer diagram by forcing the system to have all non-unity gain settings on the integrators. For this part don't be concerned with the magnitude of the gains. With this diagram determine the correct scale factors for interpreting the variables. Finally time scale the system so as to make it fit the frequency requirements of a particular computer or possibly a recorder. The necessary steps are detailed as follows.

1. Derive equations using consistent physical units
2. Set up a computer diagram using physical gains
3. Amplitude scale the diagram so as to obtain the following:
   a) All summers and inverters should have approximately unity gain
   b) All integrators should have approximately the same gain
   c) All inputs to a given integrator should have roughly the same gain
   d) The total gain in any closed loop should not be changed
4. Assign a scale factor to the input or forcing function. Also make certain that the input gain associated with the forcing function is compatable with other input gains at the same amplifier.
5. Use the forcing function scale factor to determine the remaining scale factors.
6. Time scale the system if necessary. This is done by changing all integrator inputs by the same amount. (Note, this also includes the forcing functions).

Good Luck!

References:

Johnson, Clarence L., Analog Computer Techniques, New York, McGraw-Hill, 1956.

Korn, Granino A. and Korn, Theresa M., Electronic Analog Computers, New York, McGraw-Hill, 1956.

Smith, George W., and Wood, Roger C., Principles of Analog Computation, New York, McGraw-Hill, 1959.