

A new fractional D2-brane, G_2 holonomy and T-duality

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Abstract

Recently, a new example of a complete non-compact Ricci-flat metric of G_2 holonomy was constructed, which has an asymptotically locally conical structure at infinity with a circular direction whose radius stabilizes. In this paper, we find a regular harmonic 3-form in this metric, which we then use in order to obtain an explicit solution for a fractional D2-brane configuration. By performing a T-duality transformation on the stabilized circle, we obtain the type IIB description of the fractional brane, which now corresponds to D3-brane with one of its worldvolume directions wrapped around the circle.

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1. Introduction

The T-duality that relates the type IIA and type IIB strings allows one to establish a mapping between a Dp -brane in one theory and a $D(p+1)$ -brane in the other. Recently, more general classes of Dp -brane solution have been constructed, in which further fluxes for form fields are present, in addition to the one that carries the standard Dp -brane charge. An example of this type is the fractional D3-brane of Klebanov and Strassler [1]. Subsequently, a number of other fractional or resolved brane solutions have been constructed, including fractional D2-branes in which the usual flat transverse 7-metric is replaced by a smooth Ricci-flat metric of G_2 holonomy [2, 3]. It is therefore of interest to study the possibility of relating fractional branes, such as these D2-branes of type IIA, to branes in the T-dual type IIB description.

New families of complete eight-dimensional manifolds of Spin(7) holonomy have recently been constructed, which are asymptotically locally conical (ALC) [4]. In these, the geometry at large distance locally approaches the product of a circle of fixed radius, and an asymptotically conical (AC) 7-manifold. Deformed M2-branes using these Spin(7) manifolds were constructed in [4], and it was shown that dimensional reduction on the circle gives D2-brane solutions in $D = 10$ that are similar to those that can be built using G_2 manifolds.

This Spin(7) construction motivated an analogous construction of asymptotically locally conical seven-dimensional manifolds of G_2 holonomy, and an explicit isolated example has been obtained [5]. It has the same property as the Spin(7) example, of locally approaching the product of a circle of fixed radius, and an asymptotically conical manifold (of dimension 6 in this case), at large distance.

In this paper, we shall construct a new fractional D2-brane solution, using the new ALC metric of G_2 holonomy obtained in [5]. In order to obtain the fractional D2-brane using this metric, we first need to construct a suitable well-behaved harmonic 3-form $G_{(3)}$, which is regular at short distance and which falls off at infinity. The integral of $|G_{(3)}|^2$ diverges logarithmically with proper distance. (This is reminiscent of the behaviour of the harmonic 3-form used in the construction of the fractional D3-brane [1].)

Having obtained the harmonic 3-form, we then use it in order to construct the associated fractional D2-brane solution, which is supersymmetric. Remarkably, it turns out that the equations can be exactly integrated, and so we are able to give the result explicitly in closed form.

The fact that the new G_2 metric of [5] is asymptotically locally conical means that the circle whose radius tends to a constant at infinity is a natural candidate on which to perform a T-duality transformation. We carry out this transformation, and show how the fractional D2-brane is mapped into a supersymmetric wrapped D3-brane in the type IIB theory. In order to set the scene for this duality mapping, we first carry it out for the simpler situation of a ‘vacuum’ (Minkowski) $_3 \times \mathcal{M}_7$, where \mathcal{M}_7 denotes the Ricci-flat G_2 manifold.

The paper is organized as follows. In section 2, we review the non-compact cohomogeneity 1 G_2 manifolds and previously-known fractional D2-branes. In section 3, we review the recently-constructed new G_2 manifolds. In section 4, we obtain the regular harmonic 3-form for the new G_2 manifold and construct new fractional D2-branes. We perform T-duality and obtain the S^1 wrapped fractional D3-brane in section 5. Finally, we conclude the paper in section 6.

2. Review of G_2 manifolds and fractional D2-branes

Three explicit metrics of cohomogeneity 1 for seven-dimensional manifolds with G_2 holonomy have been known for some time [6, 7]. The first two have principal orbits that are $\mathbb{C}\mathbb{P}^3$ or $SU(3)/(U(1) \times U(1))$, viewed as an S^2 bundle over S^4 or $\mathbb{C}\mathbb{P}^2$, respectively. The associated 7-manifolds have the topology of an $R^{(3)}$ bundle over S^4 or $\mathbb{C}\mathbb{P}^2$. The third manifold has principal orbits that are topologically $S^3 \times S^3$, viewed as an S^3 bundle over S^3 , and the 7-manifold is topologically $\mathbb{R}^4 \times S^3$. Recently, there has been considerable interest in studying $D = 4, \mathcal{N} = 1$ theory from the compactification of M-theory on G_2 holonomy spaces [5, 8–19]. This provides a geometric description of M-theory on G_2 manifolds for the strongly coupled $D = 4, \mathcal{N} = 1$ dual field theory arising from the wrapped D6-brane on conifolds.

Supersymmetric M3-branes, arising as deformations of (Minkowski) $_4$ times (G_2 holonomy) backgrounds, with non-vanishing 4-form field in eleven-dimensional supergravity, were constructed in [20, 21].

Another natural application for the spaces of G_2 holonomy is in the construction of fractional D2-branes, since they have non-vanishing Betti numbers b_3 and b_4 . The fractional D2-brane solutions of type IIA supergravity using the three above-mentioned spaces of G_2 holonomy are given by [2, 3]

$$\begin{aligned} ds_{10}^2 &= H^{-5/8} dx^\mu dx^\nu \eta_{\mu\nu} + H^{3/8} ds_7^2, \\ F_{(4)} &= d^3x \wedge dH^{-1} + mG_{(4)}, \quad F_{(3)} = mG_{(3)}, \quad \phi = \frac{1}{4} \log H, \end{aligned} \quad (1)$$

where $G_{(3)}$ is an harmonic 3-form in the Ricci-flat 7-metric ds_7^2 , and $G_{(4)} = *G_{(3)}$, with $*$ being the Hodge dual with respect to ds_7^2 . The function H satisfies

$$\square H = -\frac{1}{6}m^2 G_{(3)}^2, \tag{2}$$

where \square denotes the scalar Laplacian in the transverse 7-metric ds_7^2 .

The fractional D2-brane for the \mathbb{R}^4 bundle over S^3 was constructed in [2], and its supersymmetry was demonstrated in [3]. The harmonic 3-form is square integrable at small distance, but linearly non-normalizable at large distance. Correspondingly, the solution is regular everywhere, with the small distance structure $(\text{Minkowski})_3 \times \mathbb{R}^4 \times S^3$, whilst at large distance, the function H behaves like [2]

$$H \sim 1 + \frac{m^2}{4r^4} - \frac{4m^2}{15r^5}. \tag{3}$$

Note that this leading-order $1/r^4$ behaviour is more like that for a standard D3-brane, which has a six-dimensional transverse space, rather than usual $1/r^5$ behaviour for the seven-dimensional transverse space that we have here.

The fractional D2-branes in which the transverse space is the R^3 bundle over S^4 or \mathbb{CP}^2 were constructed in [3]. In these cases, the harmonic 4-form is L^2 -normalizable, and consequently, the solution is regular everywhere. Its large-distance asymptotic behaviour is

$$H \sim 1 + \frac{cm^2}{r^5} - \frac{m^2}{4r^6}, \tag{4}$$

where c is a certain numerical constant.

3. Review of the new G_2 manifolds

Recently, more general metrics with the structure of an R^4 bundle over S^3 have been considered [5, 21], with a view to obtaining further examples of metrics of G_2 holonomy. This would be analogous to the recent construction of more general eight-dimensional metrics of Spin(7) holonomy in [4]. One can consider the following ansatz for seven-dimensional metrics:

$$ds_7^2 = h^2 dr^2 + a_i^2 \tilde{h}_i^2 + b_i^2 h_i^2, \tag{5}$$

where

$$h_i \equiv \sigma_i + \Sigma_i, \quad \tilde{h}_i = \sigma_i - \Sigma_i. \tag{6}$$

Here σ_i and Σ_i are the left-invariant 1-forms on two $SU(2)$ group manifolds, S_σ^3 and S_Σ^3 , and a_i, b_i and h are functions of the radial coordinate r . The principal orbits are therefore S^3 bundles over S^3 , and since the bundle is topologically trivial, they have the topology $S^3 \times S^3$. The ansatz is a generalization of the one in [6, 7] that was used for obtaining the original complete metric of G_2 holonomy on the \mathbb{R}^4 bundle over S^3 .

It was shown in [21, 5] that the conditions for Ricci flatness for the ansatz (5) admit as first integrals a system of first-order equations that can be derived from a superpotential. In [21], it was shown that these first-order equations are the integrability conditions for the existence of a covariantly-constant spinor, and hence for G_2 holonomy. An equivalent demonstration of G_2 holonomy was given in [5], by showing that the first-order equations follow by requiring the covariant constancy of a 3-form. The general explicit solution is not known. If one specializes to $a_i = a$ and $b_i = b$, then the general solution becomes the previously-known G_2 manifold.

In [5], a less restrictive specialization was made, involving setting $a_1 = a_2$ and $b_1 = b_2$. Writing the ansatz now as

$$ds_7^2 = h^2 dr^2 + a^2(\tilde{h}_1^2 + \tilde{h}_2^2) + c^2 \tilde{h}_3^2 + b^2(h_1^2 + h_2^2) + f^2 h_3^2, \tag{7}$$

one again obtains first-order equations, which have not been solved in general. However, a special solution was found in [5], which may be written as

$$\begin{aligned} a &= \frac{1}{\sqrt{8\ell}} \sqrt{(r-\ell)(r+3\ell)}, & b &= \frac{1}{\sqrt{8\ell}} \sqrt{(r+\ell)(r-3\ell)} \\ c &= -\frac{r}{\sqrt{6\ell}}, & f &= \sqrt{\frac{2\ell}{3}} \frac{\sqrt{r^2-9\ell^2}}{\sqrt{r^2-\ell^2}}, \end{aligned} \quad (8)$$

with $h = 1/f$. (Note that the minus sign in the expression for c is purely conventional.) The radial coordinate runs from $r = 3\ell$ to infinity. The metric (7) is then complete on a manifold with the same $\mathbb{R}^4 \times S^3$ topology as the original metric in [6, 7]. However, an important difference is that in the new metric the radius in the S^1 direction associated with h_3 becomes a constant asymptotically at large r . Thus, the metric is no longer asymptotically conical, but instead it locally approaches the product of a circle and an asymptotically-conical 6-metric at infinity. This is analogous to the new Spin(7) manifolds obtained in [4]. Arguments were also given in [5] for the existence of more general solutions of the first-order equations for the G_2 metrics (7), which would be analogous to the more general Spin(7) solutions in [4].

4. New fractional D2-brane

4.1. Harmonic 3-form

As was discussed in section 2, the construction of a fractional D2-brane requires a well-behaved harmonic 3-form in the G_2 holonomy space. In this subsection, we construct a harmonic 3-form $G_{(3)}$ in the new G_2 metric. We begin by choosing the natural vielbein

$$\begin{aligned} e^0 &= hdr, & e^1 &= a\tilde{h}_1, & e^2 &= a\tilde{h}_2, & e^3 &= c\tilde{h}_3, \\ e^4 &= bh_1, & e^5 &= bh_2, & e^6 &= fh_3 \end{aligned} \quad (9)$$

for the metric (7). Motivated by the ansatz for the harmonic form in the original G_2 metric on $\mathbb{R}^4 \times S^3$ [2, 3, 21], we make the following ansatz for the 4-form $G_{(4)} = *G_{(3)}$ dual to $G_{(3)}$:

$$\begin{aligned} G_{(4)} &= u_1 e^1 \wedge e^2 \wedge e^4 \wedge e^5 + u_2 e^2 \wedge e^3 \wedge e^5 \wedge e^6 + u_2 e^3 \wedge e^1 \wedge e^6 \wedge e^4 \\ &\quad + u_3 e^0 \wedge e^4 \wedge e^5 \wedge e^6 + u_4 e^0 \wedge e^1 \wedge e^2 \wedge e^6 \\ &\quad + u_5 e^0 \wedge e^2 \wedge e^3 \wedge e^4 + u_5 e^0 \wedge e^3 \wedge e^1 \wedge e^5. \end{aligned} \quad (10)$$

(The ansatz for the original G_2 metric is like this, with $u_2 = u_1$, and $u_5 = u_4$.) Its Hodge dual is given by

$$\begin{aligned} G_{(3)} &= -u_1 e^0 \wedge e^3 \wedge e^6 - u_2 e^0 \wedge e^1 \wedge e^4 - u_2 e^0 \wedge e^2 \wedge e^5 + u_3 e^1 \wedge e^2 \wedge e^3 \\ &\quad + u_4 e^3 \wedge e^4 \wedge e^5 + u_5 e^1 \wedge e^5 \wedge e^6 - u_5 e^2 \wedge e^4 \wedge e^6. \end{aligned} \quad (11)$$

After straightforward manipulations, we can obtain the first-order equations that follow from imposing $dG_{(4)} = 0$ and $d*G_{(4)} = 0$. In order to simplify the task of solving these, it is useful to note that in previous examples, the harmonic forms that could give rise to supersymmetric fractional branes all had the feature that some constant linear combination of the functions u_i in the ansatz for the harmonic form vanished [1–3, 22, 23]. We find that in this case too, the system of first-order equations following from $dG_{(4)} = 0$ and $d*G_{(4)} = 0$ imply that a certain linear combination of the u_i functions in (10) is a constant, which we can choose to be zero. Thus, we are led to impose

$$u_1 - 2u_2 - u_3 + u_4 - 2u_5 = 0. \quad (12)$$

This linear relation ensures that the fractional D2-brane is supersymmetric. We are left with four first-order equations for the remaining undetermined functions, and so the general

solution has four constants of integration. Requiring that the 4-form is well-behaved at the origin $r = 3\ell$, and that it falls off at large r , fixes these integration constants completely, up to an overall scale. We then find that the functions u_i are given by

$$\begin{aligned}
 u_1 &= \frac{4[1053 - 441r^2 + 27r^4 + r^6 - 36(-9 - r^2 + 2r^4) \log(\frac{r}{3})]}{(r^2 - 9)^3(r^2 - 1)^2}, \\
 u_2 &= \frac{3(r^2 + 3)[r^4 - 81 - 36r^2 \log(\frac{r}{3})]}{r^2(r^2 - 9)^3(r^2 - 1)}, \\
 u_3 &= \frac{-(r^2 - 9)(-9 + 15r + 17r^2 + 15r^3 + 2r^4) + 36r^2(3r + 1) \log(\frac{r}{3})}{(r + 3)^3(r + 1)r^2(r - 1)^2(r - 3)}, \\
 u_4 &= \frac{(r^2 - 9)(9 + 15r - 17r^2 + 15r^3 - 2r^4) - 36r^2(3r - 1) \log(\frac{r}{3})}{(r - 3)^3(r - 1)r^2(r + 1)^2(r + 3)}, \\
 u_5 &= \frac{4[r^4 - 81 - 36r^2 \log(\frac{r}{3})]}{(r^2 - 9)^3(r^2 - 1)}.
 \end{aligned} \tag{13}$$

Note that without losing generality, we have set the scale parameter ℓ in the metric functions in (8) to $\ell = 1$ here, to simplify the writing somewhat. It should be remarked that despite appearances, the functions u_i in (13) are actually non-singular at the minimum radius $r = 3$, and they have regular Taylor expansions there. Note that the harmonic 3-form we have constructed here, which is localized near the origin, is quite distinct from the covariantly-constant calibrating 3-form discussed in [5].

The magnitude of $G_{(3)}$ is given by

$$|G_{(3)}|^2 = 6(u_1^2 + 2u_2^2 + u_3^2 + u_4^2 + 2u_5^2). \tag{14}$$

Substituting the above expressions for the u_i into this, we find that at a short distance, near to $r = 3$, we have

$$|G_{(3)}|^2 = \frac{7}{81} - \frac{49(r - 3)}{1243} + \frac{4973(r - 3)^2}{17496} + \dots \tag{15}$$

At a large distance, we have

$$|G_{(3)}|^2 = \frac{48}{r^6} + \frac{120}{r^8} + \frac{96[241 - 252 \log(\frac{r}{3})]}{r^{10}} + \dots \tag{16}$$

It follows from this, and the fact that $\sqrt{g} \sim r^5$ at large r , that $\int \sqrt{g}|G_{(4)}|^2$ diverges logarithmically at large r . The harmonic 4-form is therefore not L^2 -normalizable, and the rather slow logarithmic divergence in the integral of $|G_{(4)}|^2$ is very similar to that encountered in the fractional D3-brane construction of Klebanov and Strassler [1].

4.2. New fractional D2-brane

In this section, we show that the metric function H in the deformed D2-brane solution (1), (2) using this harmonic form can be obtained explicitly. First we note that

$$\begin{aligned}
 \int_3^r \sqrt{g}|G_{(3)}|^2 &= \frac{\sqrt{6}(2r^8 - 49r^6 + 729r^4 + 1161r^2 - 243)}{32r^2(r^2 - 3)(r^2 + 3)^2(r^2 - 1)^2} \\
 &\quad - \frac{\sqrt{6}}{8(r^2 - 9)^4(r^2 - 1)^2} \left(-324r^2(11r^4 + 2r^2 - 45) \left(\log\left(\frac{r}{3}\right) \right)^2 \right. \\
 &\quad \left. + (r^2 - 9)(r^{10} - 29r^8 + 424r^6 + 1944r^4 - 5913r^2 - 2187) \log\left(\frac{r}{3}\right) \right). \tag{17}
 \end{aligned}$$

From this, we find that H is given by

$$\begin{aligned}
 H = c_0 + & \frac{m^2(22r^8 - 589r^6 + 5247r^4 - 19035r^2 - 3645)}{30r^2(r^2 - 9)^3(r^2 - 1)} \\
 & + \frac{2m^2(11r^8 - 335r^6 + 3645r^4 - 14661r^2 + 43740)}{15(r^2 - 9)^4(r^2 - 1)} \log\left(\frac{r}{3}\right) \\
 & - \frac{1944m^2(11r^2 + 9)}{5(r^2 - 9)^5(r^2 - 1)} \left(\log\left(\frac{r}{3}\right)\right)^2 + \frac{22m^2}{135} \left[\psi\left(-\frac{r}{3}\right) - \psi\left(1 - \frac{r}{3}\right) \right. \\
 & \left. + \log\left(\frac{r}{3}\right) \log\left(1 + \frac{r}{3}\right) - \left(\log\left(\frac{r}{3}\right)\right)^2 \right], \tag{18}
 \end{aligned}$$

where c_0 is a constant and $\psi(x) \equiv -\int_0^x y^{-1} \log(1 - y) dy$ is the dilogarithm.

The function H becomes a constant at a small distance $r \rightarrow 3$, and at a large distance it has the asymptotic form

$$H = c_0 + \frac{3m^2(4 \log(\frac{r}{3}) - 1)}{4r^4} + \frac{3m^2(24 \log(\frac{r}{3}) - 3)}{2r^6} + \frac{m^2(8856 \log(\frac{r}{3}) - 2229)}{16r^8} + \dots \tag{19}$$

The solution is supersymmetric, and regular everywhere, with no horizon. It does not give a well-defined ADM mass per unit 3-volume; we have $M \sim r^5 H'$ in the limit $r \rightarrow \infty$, and this diverges logarithmically. The situation is directly analogous to that in the fractional D3-brane [1].

The NS-NS 3-form carries a non-vanishing magnetic charge. The integral of $F_{(3)}$ over the non-trivial 3-cycle can conveniently be evaluated by restricting it to the S^3 bolt at $r = 3\ell$ (which means $r = 3$ since we have set $\ell = 1$). From (11) we have

$$F_{(3)}|_{r=3} = mG_{(3)}|_{r=3} = \frac{m}{2\sqrt{6}} \tilde{h}_1 \wedge \tilde{h}_2 \wedge \tilde{h}_3, \tag{20}$$

and hence we find $\int_{S^3} F_{(3)} = 64m\pi^2/\sqrt{6}$.

The ‘standard’ 4-form electric flux is given by integrating the quantity $d(e^{\frac{1}{2}\phi} * F_{(4)})$ over a 7-volume bounded by a 6-surface \mathcal{M}_6 . Since $d(e^{\frac{1}{2}\phi} * F_{(4)}) = F_{(3)} \wedge F_{(4)}$, the electric flux can be written as $\int F_{(3)} \wedge F_{(4)}$. From (1), this gives $m^2 \int G_{(3)} \wedge G_{(4)}$, which is nothing but $12m^2 \int |G_{(3)}|^2$. As noted above, $G_{(3)}$ is not L^2 normalizable, and so if the 7-volume is taken to be the interior of the level set $r = \text{constant}$, the integral diverges logarithmically with proper distance. This is precisely what one expects, since the solution is supersymmetric and, as we saw above, the ADM mass per unit 3-volume diverges logarithmically too.

5. T-duality

The new G_2 metric (7) has a circle whose radius is stabilized at infinity. To see this explicitly, let us first write the $SU(2)$ left-invariant 1-forms σ_i and Σ_i in terms of Euler angles

$$\begin{aligned}
 \sigma_1 &= \cos \psi_1 d\theta_1 + \sin \psi_1 \sin \theta_1 d\phi_1, & \Sigma_1 &= \cos \psi_2 d\theta_2 + \sin \psi_2 \sin \theta_2 d\phi_2, \\
 \sigma_2 &= -\sin \psi_1 d\theta_1 + \cos \psi_1 \sin \theta_1 d\phi_1, & \Sigma_2 &= -\sin \psi_2 d\theta_2 + \cos \psi_2 \sin \theta_2 d\phi_2, \\
 \sigma_3 &= d\psi_1 + \cos \theta_1 d\phi_1, & \Sigma_3 &= d\psi_2 + \cos \theta_2 d\phi_2.
 \end{aligned} \tag{21}$$

Then the combination of $(\psi_1 + \psi_2)$ appears in the metric (7) only in h_3 , as $d(\psi_1 + \psi_2)$. Specifically, after making the following redefinition,

$$\psi = \psi_1 + \psi_2, \quad \tilde{\psi} = \psi_1 - \psi_2, \tag{22}$$

we have

$$\begin{aligned}
 ds_7^2 \equiv ds_6^2 + f^2 h_3^2 = h^2 dr^2 + (a^2 + b^2)(d\theta_1^2 + d\theta_2^2 + \sin^2 \theta_1 d\phi_1^2 + \sin^2 \theta_2 d\phi_2^2) \\
 + 2(a^2 - b^2)(\sin \tilde{\psi}(\sin \theta_1 d\theta_2 d\phi_1 - \sin \theta_2 d\theta_1 d\phi_2) + \cos \tilde{\psi}(d\theta_1 d\theta_2 \\
 + \sin \theta_1 \sin \theta_2 d\phi_1 d\phi_2)) + c^2(d\tilde{\psi} + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2)^2 \\
 + f^2(d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2.
 \end{aligned} \tag{23}$$

From this, it can be seen that there are three commuting $U(1)$ Killing vector fields of the metric, namely $\partial/\partial\phi_1$, $\partial/\partial\phi_2$ and $\partial/\partial\psi$. The Killing vectors $\partial/\partial\phi_1$ and $\partial/\partial\phi_2$ generate $U(1)$ subgroups $U(1)_L^\sigma$ and $U(1)_L^\Sigma$ of the left-acting $SU(2)_L^\sigma$ and $SU(2)_L^\Sigma$ groups in the two 3-spheres. The Killing vector $\partial/\partial\psi$ generates the subgroup $U(1)_D$ of the diagonal right-acting $SU(2)_R^D$.

If the G_2 manifold \mathcal{M}_7 is taken as a factor in an M-theory vacuum $(\text{Minkowski})_4 \times \mathcal{M}_7$, then we can reduce on any of the three circles $U(1)_L^\sigma$, $U(1)_L^\Sigma$ or $U(1)_D$ to give D6-branes in type IIA theory. In particular, the circle $U(1)_D$ with stabilized radius gives a distance-independent R-R vector potential

$$\mathcal{A}_{(1)} = \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2, \tag{24}$$

implying that

$$\mathcal{F}_{(2)} = d\mathcal{A}_1 = J_{(2)} \equiv \Omega_{(2)}^1 + \Omega_{(2)}^2. \tag{25}$$

(This reduction of the metric (7) with the solution (8) was discussed in [5].)

If the G_2 manifold \mathcal{M}_7 is instead taken as a factor in a $D = 10$ vacuum $(\text{Minkowski})_3 \times \mathcal{M}_7$, we can then perform a T-duality on the ψ circle, and the corresponding dual solution becomes

$$\begin{aligned}
 ds_{10}^2 = e^{-\frac{1}{2}\phi} (dx^\mu dx_\mu + h^2 dr^2 + a^2(\tilde{h}_1^2 + \tilde{h}_2^2) + b^2(h_1^2 + h_2^2) + c^2 \tilde{h}_3^2 + f^{-2} d\psi^2), \\
 e^\phi = f^{-1}, \quad F_{(3)}^{\text{NS}} = J_2 \wedge d\psi.
 \end{aligned} \tag{26}$$

The 3-form $F_{(3)}^{\text{NS}}$ carries a magnetic 5-brane charge. As is typical with T-duals of regular configurations, this one is singular. Away from the singularity, the principal orbits have the geometry of those in the deformed conifold [24], and share with it the property that the S^2 fibres are shrinking relative to the S^3 base as the radius decreases. In other words the ratio b^2/a^2 decreases. Previously, a non-singular configuration using the 3-form field strength was considered in [25, 26], where the regular 6-metric was taken to be the *small resolution* of the cone over $T^{1,1}$. This has a smoothly embedded minimal S^2 at short distance (see [24, 28]). In this case it is the S^3 base, rather than the S^2 fibres, that shrinks as the radius decreases. Regularity requires turning on the $SU(2)$ Yang–Mills fields in the metric [27, 25].

To perform the analogous T-duality transformation on the fractional D2-brane, we first write the harmonic forms $G_{(4)}$ and $G_{(3)}$ as

$$G_{(4)} = L_{(4)} + \tilde{L}_{(3)} \wedge h_3, \quad G_{(3)} = L_{(3)} + L_{(2)} \wedge h_3, \tag{27}$$

where the forms $L_{(4)}$, $\tilde{L}_{(3)}$, $L_{(3)}$ and $L_{(2)}$ lying in the in the six-dimensional base manifold orthogonal to the h_3 fibres can be read off directly from expressions (10) and (11). They are therefore given by

$$\begin{aligned}
 L_{(4)} &= u_1 a^2 b^2 \tilde{h}_1 \wedge \tilde{h}_2 \wedge h_1 \wedge h_2 + 2u_5 h a b c dr \wedge \tilde{h}_3 \wedge (\sigma_1 \wedge \sigma_2 - \Sigma_1 \wedge \Sigma_2), \\
 \tilde{L}_{(3)} &= -u_2 f a b c \tilde{h}_3 \wedge (\tilde{h}_1 \wedge h_1 + \tilde{h}_2 \wedge h_2) + u_3 b^2 dr \wedge h_1 \wedge h_2 + u_4 a^2 dr \wedge \tilde{h}_1 \wedge \tilde{h}_2, \\
 L_{(3)} &= -u_2 h a b dr \wedge (\tilde{h}_1 \wedge h_1 + \tilde{h}_2 \wedge h_2) + u_3 a^2 c \tilde{h}_1 \wedge \tilde{h}_2 \wedge \tilde{h}_3 + u_4 b^2 c h_1 \wedge h_2 \wedge \tilde{h}_3, \\
 L_{(2)} &= -u_1 c dr \wedge (\sigma_3 - \Sigma_3) + 2u_5 f a b (\sigma_1 \wedge \sigma_2 - \Sigma_1 \wedge \Sigma_2).
 \end{aligned} \tag{28}$$

Then we can apply the T-duality map between type IIA and type IIB as given in [29, 30], thereby obtaining the following type IIB solution:

$$\begin{aligned} ds^2 &= H^{-1/2} (dx^\mu dx_\mu + f^{-2} (d\psi + m\mathcal{B}_{(1)})^2) + H^{1/2} ds_6^2, \\ e^\phi &= f^{-1}, \quad F_{(3)}^{\text{NS}} = m(L_{(3)} + L_{(2)} \wedge \mathcal{A}_{(1)}) + J_{(2)} \wedge d\psi, \\ F_{(3)}^{\text{RR}} &= m\tilde{L}_{(3)}, \quad F_{(5)} = (d^3x \wedge dH^{-1} + mL_{(4)}) \wedge (d\psi + m\mathcal{B}_{(1)}) + \text{Hodge dual}, \end{aligned} \quad (29)$$

where ds_6^2 can be read off from (23). Note that we have written the metric in the string frame, and that $\mathcal{B}_{(1)}$ is such that $d\mathcal{B}_1 = L_{(2)}$. It is straightforward to see from (28) that $\mathcal{B}_{(1)}$ can be taken to be

$$\mathcal{B}_{(1)} = -2u_5 \text{fab}(\sigma_3 - \Sigma_3). \quad (30)$$

Note that $\mathcal{B}_{(1)}$ falls off as $1/r^2$ at large distance. As well as the usual electric and magnetic D3-brane charge carried by $F_{(5)}$, there is also magnetic 5-brane charge, since both $J_{(2)} \wedge d\psi$ and $L_{(3)}$ give non-vanishing flux integrals. Like the fractional D2-brane itself, the T-dual solution is supersymmetric, since the Killing spinor is independent of the coordinate ψ on the circle.

It is worth mentioning that the fractional D3-brane solution (29) can be applied more generally than just for the metric (7) with the explicitly-constructed solution (8). In particular, another solution of the first-order equations obtained in [5] for the metric (7) is in a limit where the vector potential $\mathcal{A}_{(1)}$ is effectively set to zero. In this limit the 1-form h_3 becomes simply $d\psi$, its coefficient becomes constant, and the metric ansatz (7) reduces to an ansatz on the product of S^1 and six-dimensional metrics of the Stenzel type, as discussed in [21, 22]. The first-order equations for (7) then reduce to those for the Stenzel metrics, as obtained in [22]. In this case, which will in addition have $\mathcal{B}_{(1)} = 0$, the solution becomes precisely the previously-known fractional D3-brane on the deformed conifold [1]. In our new solution, by contrast, one direction in the worldvolume space is twisted, and $F_{(3)}^{\text{NS}}$ has an additional charge. However, the introduction of these charges does not affect the qualitative asymptotic behaviour of H at large distance.

6. Conclusions

In this paper, we have obtained the explicit solution for the fractional D2-brane whose transverse seven-dimensional space is the new Ricci-flat metric of G_2 holonomy that was constructed in [5]. (This metric is asymptotically locally conical, rather than asymptotically conical, and is analogous to one of the new ALC metrics of $\text{Spin}(7)$ holonomy constructed [4].) In order to obtain the fractional D2-brane, we first constructed the appropriate harmonic 3-form $G_{(3)}$ in the Ricci-flat metric. It is regular at short distance, and the integral of $|G_{(3)}|^2$ diverges logarithmically in proper distance at infinity. The equations determining the fractional D2-brane turn out to be exactly integrable, allowing us to obtain a fully explicit result for this solution. It describes D2-branes together with 5-branes wrapped over 3-cycles.

Since the Ricci-flat G_2 metric obtained in [5] is locally asymptotic to the product of a circle of stabilized radius and an asymptotically conical 6-metric, one can perform a dimensional reduction on the circle and thereby obtain a D2-brane solution in nine dimensions with an asymptotically conical six-dimensional transverse space. Having first studied this reduction for the simpler examples of vacuum solutions $(\text{Minkowski})_4 \times \mathcal{M}_7$ or $(\text{Minkowski})_3 \times \mathcal{M}_7$, we then repeated the dimensional reduction for the new fractional D2-brane solution. After doing so, by mapping into type IIB variables and then lifting back to $D = 10$, we obtained the T-dual description of the fractional D2-brane. This has an interpretation as a D3-brane in which one of the worldvolume directions is wrapped around the circle.

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