UMM-13

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Project "Wizard"

"A Simplified Method of Calculating Ram-Jet Performance Applicable To Low Mach Numbers"

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LIST OF FIGURES

This memorandum is published for use in conjunction with and as an extension of University of Michigan External Memorandum No. 7, "A Simplified Method of Calculating Ram-Jet Performance Applicable to High Mach Numbers", (UMM-7). The present memorandum may be considered as Appendix III of UMM-7, in which the theory is derived and the method developed.

I. INTRODUCTION AND DISCUSSION

The report for which this appendix was written, "Simplified Method of Calculating Ram-jet Performance Applicable to High Mach Numbers", (Reference 1) shows that the combustion chamber parameter, S_a, is not affected appreciably by variations in pressure in the combustion chamber. Consequently, a solution for conditions at the combustion chamber exit can be obtained independently of the pressure, at Mach numbers 2.0 to 6.0, mentioned in the report.

However, for a particularly low range of Mach numbers, a solution for conditions at the combustion chamber exit (station 3, Figure A of Reference 1) is necessarily obtained in a less direct manner. In this low range of Mach numbers (1.0 - 1.25) the recovered pressure at the combustion chamber inlet, P_2, is quite low, so that its ratio to the atmospheric pressure, P_2/P_o, is no longer sufficient to allow the particular nozzle design to determine M_3 independently. That is, P_2 has decreased to a point where its ratio to atmospheric pressure limits the available pressure drop in the combustion chamber and thus limits M_3 (for a given M_2) to some value less than one. It is assumed that P_3 would be atmospheric if sonic velocity were not present at the combustion chamber "open tail" exit.

As shown in Reference 1, for given conditions at station 2; namely, T_2, V_2, and the fuel-air ratio, M_3 can be determined independently of the pressure, P_2, when working with the higher Mach numbers. The pressure drop through the combustion chamber is given by

$$\frac{P_2}{P_3} = \frac{1 + \gamma_3 M_3^2}{1 + \gamma_2 M_2^2} \qquad (1)$$

P_3 can be any value above atmospheric when the exit velocity of the exhaust gases is sonic but, when P_2/P_0 is insufficient to accelerate the flow to $M_3 = 1$, P_3 will necessarily be atmospheric.

Figure 1 shows the ratio P_2/P_0 plotted versus Mach number of flight for two values of V_2. This curve obviously includes an assumption as to diffuser efficiency, η_D, the values of which were taken from Figure 2B, Reference 1. The fact that V_2 may be related closely to the diffuser efficiency has been sidestepped until more definite information is made available, so that the diffuser efficiency is considered only a function of flight Mach number. Assuming a specific heat ratio (γ) of 1.3, the maximum P_2/P_3 occurs for choking at station 3 and can be calculated from Equation 1, as shown on Figure 1. If P_2/P_0 falls below this approximate "critical" value, sonic velocity cannot be maintained at the combustion chamber exit. A line is drawn on Figure 1 so that its intersection with the curves represents the approximate critical pressure ratio, P_2/P_0, and flight Mach number, below which P_2/P_0 must be considered in calculating Sa and $\phi(M_3)$. The method of solution is one of trial and error for these low Mach numbers (1.0 to approximately 1.25) and is outlined below with references to Figures 2, 3, and 4.

II. METHOD OF SOLUTION

Assume a V_2 and calculate h_2 from

$$h_2 = \frac{\eta_D}{2gJ}\left(V_1{}^2 - V_2{}^2\right) + h_0 \qquad (2)$$

Find P_{r_2} corresponding to h_2 from the Air Tables (Reference 2). P_2/P_3 is equal to P_{r_2}/P_{r_0}, since P_3 would be atmospheric.

Calculate M_2 from V_2 and T_2.

Knowing P_2/P_3 and M_2, read M_3 from Figure 2.

For T_2 and V_2, read F_2/w_a from Figure 3.

Knowing M_3 and the fact that

$$S_a \phi (M_3) = F_2/w_a \qquad \text{(for zero velocity heads loss at the flame holders)} \qquad (3)$$

read S_a from Figure 4. Probably S_a will be out of the possible range and the above steps will have to be repeated until the desired S_a is obtained. To account for flame-holder losses, subtract $\frac{nV_2}{2g}$ from $\frac{F_2}{w_a}$, where n is the number of velocity heads loss. $\phi(M_3)$ can be calculated and is equal to $\phi(M_5)$, so that C_T can now be evaluated as outlined in Reference 1.

CONCLUSIONS

At a given flight velocity within this range of low Mach numbers, where the choking condition at the exit of the combustion chamber can not be obtained, V_2 is determined by S_a and the diffuser efficiency. Figure 5 shows a curve drawn through several points obtained by trial and error solutions to obtain a maximum S_a. From this curve it can be seen that as S_a increases, V_2 decreases; this would necessarily be accompanied by a change in diffuser efficiency or mass flow, probably both. There is one V_2 which will occur with the maximum possible S_a, as shown on the figure. Therefore, to obtain maximum performance, the inlet area ratio, A_1/A_2, will be determined by the V_2 which corresponds to the maximum S_a and diffuser efficiency, from the relation

$$\frac{A_1}{A_2} = \frac{P_{r_2} V_2 T_o}{P_{r_o} V_o T_2} \qquad (4)$$

where P_{r_2} corresponds to h_2 of Equation 2. If η_D, diffuser efficiency, is considered a function of V_2, a curve similar to the one on Figure 5 would probably be plotted through points obtained by several trial and error solutions, each point calculated using a different η_D for each assumed V_2. Then the V_2 (corresponding to the maximum S_a available for the particular fuel-air ratio) and T_2 would be the design point, thus determining A_1/A_2 and requiring a diffuser to obtain the assumed or predicted efficiency used in the calculation. An increase in A_1/A_2 will increase V_2 but will penalize the S_a term in the equation for C_T

$$C_T = 2\frac{A_1}{A_2}\left(\frac{gS_a\phi(M_5)}{V_1} - 1\right) - \frac{2A_5}{\gamma A_2 M_1{}^2}$$

The loss from the reduced S_a will be greater than the benefit from the A_1/A_2 term, resulting in a lower C_T. Thus the V_2 that gives the maximum S_a will also give the maximum C_T.

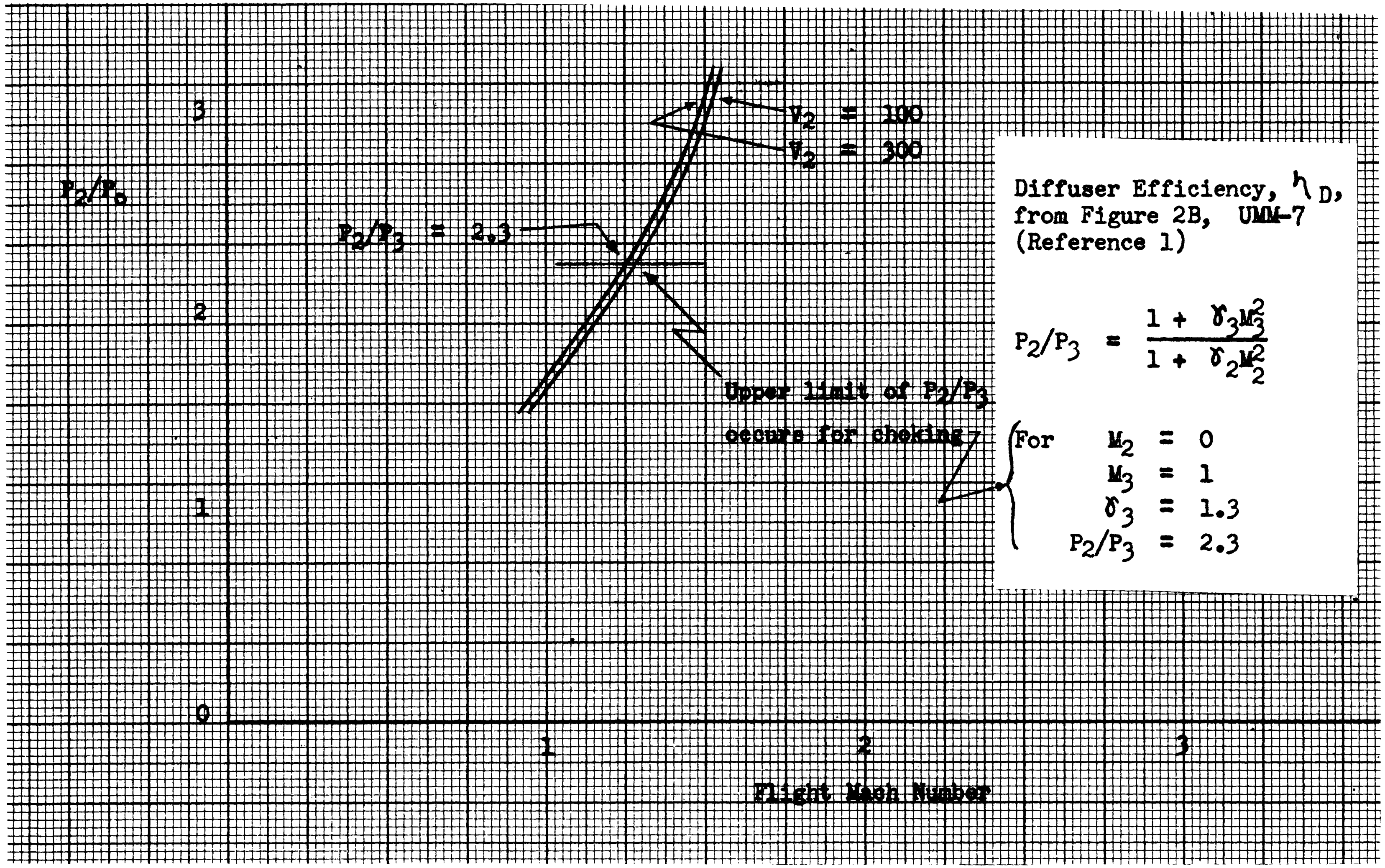

Figure 1
P_2/P_o vs FLIGHT MACH NUMBER
For any altitude

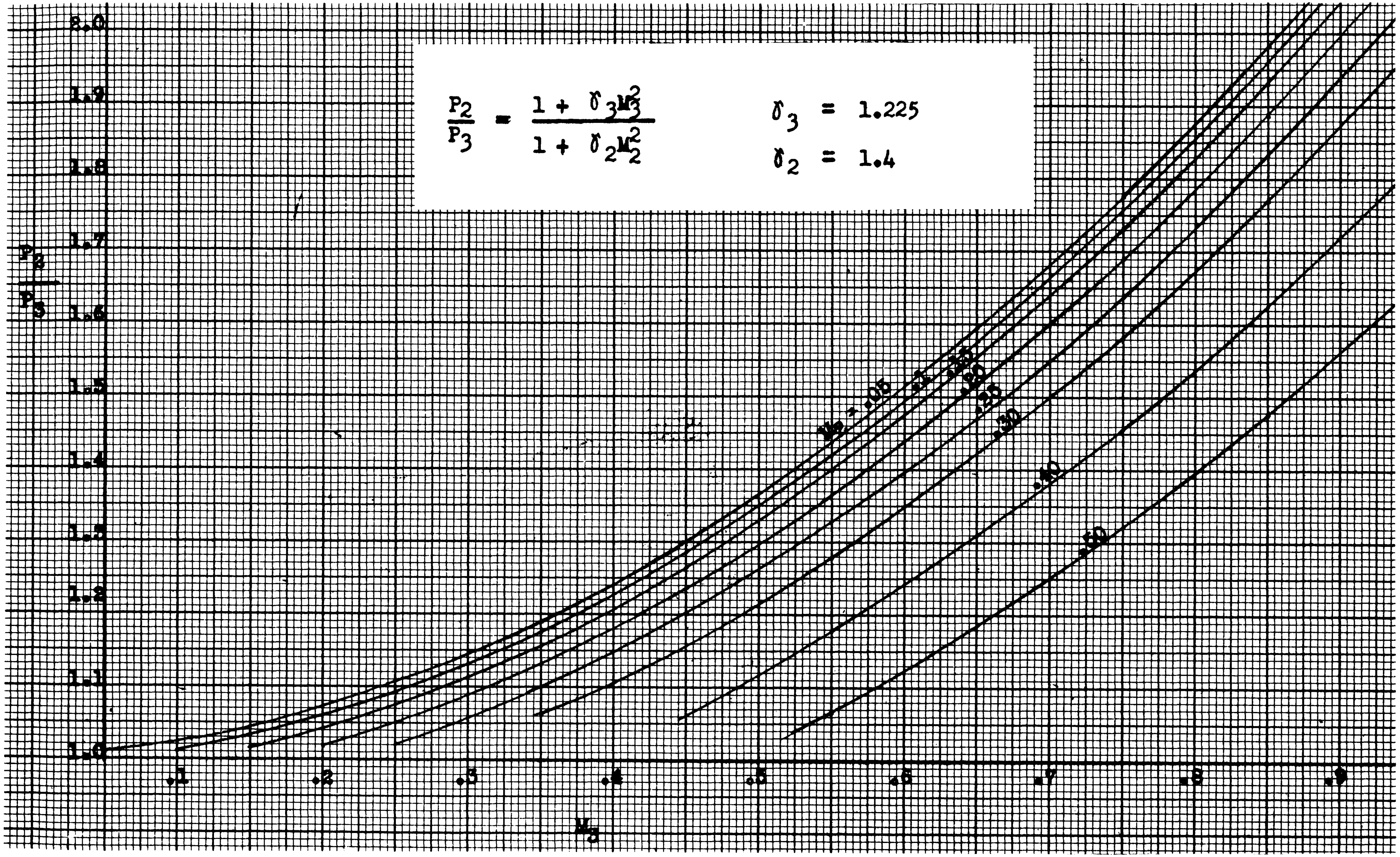

Figure 2

P_2/P_3 vs M_3 FOR VARIOUS M_2

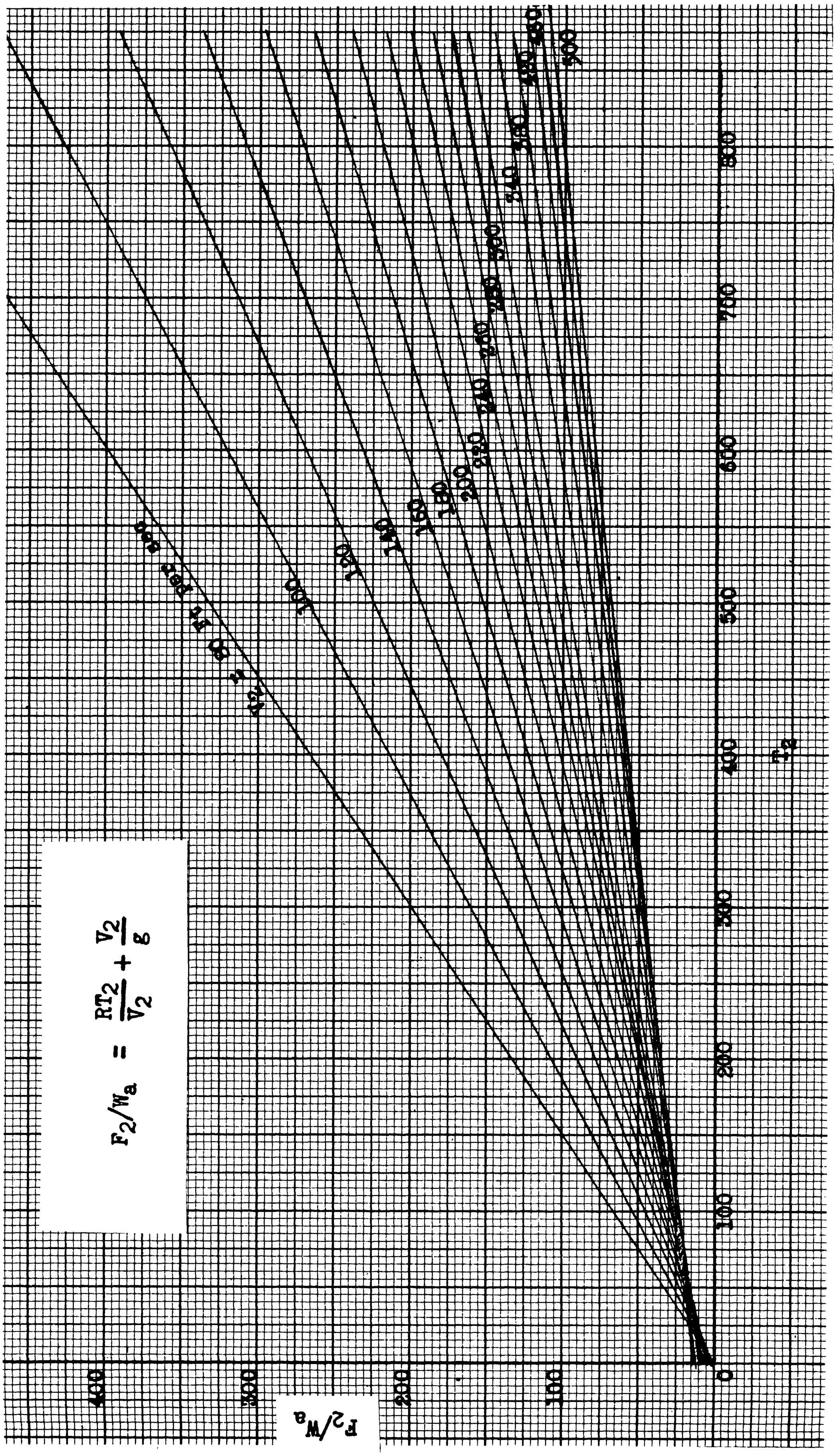

Figure 3
F_2/W_a vs T_2 FOR VARIOUS V_2

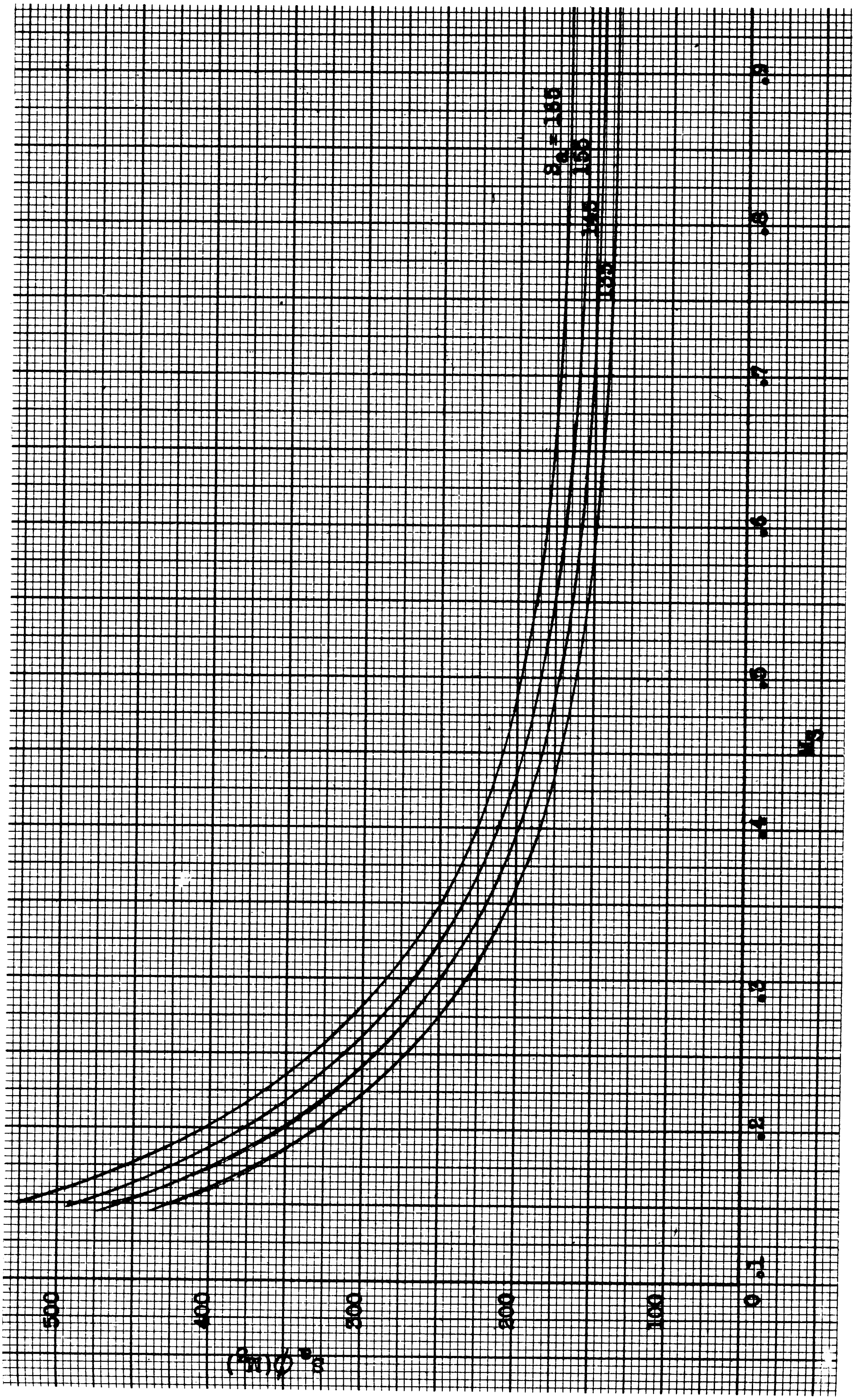

Figure 4

$S_a\ \phi\ (M_3)$ vs M_3 FOR VARIOUS S_a

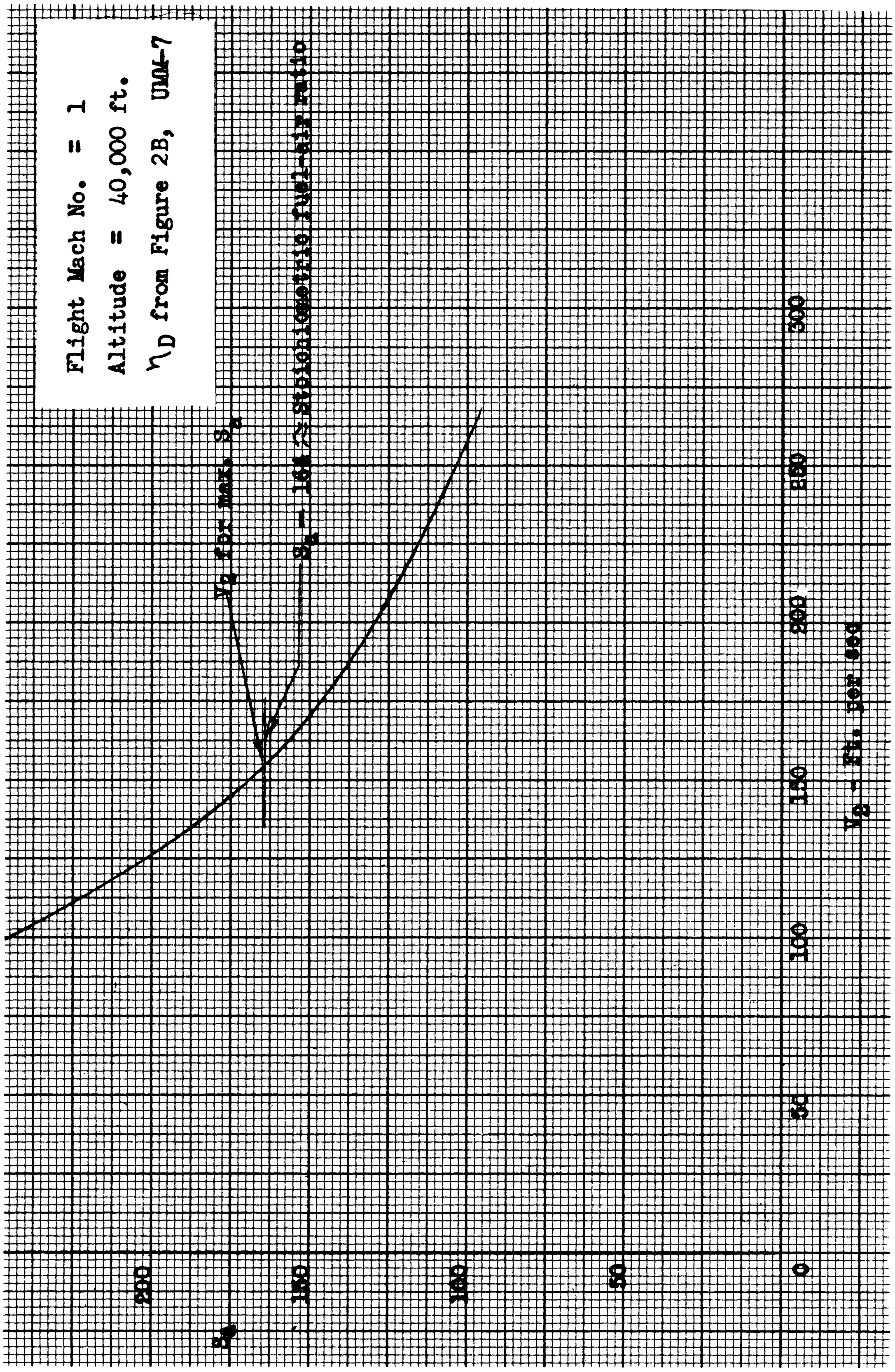

Figure 5
V_2 vs S_a

REFERENCES

1. James R. Garnett, "A Simplified Method of Calculating Ram-Jet Performance Applicable to High Mach Numbers", University of Michigan External Memorandum No. 7, (UMM-7), July 23, 1947.

2. J.H. Keenan and J. Kaye, "Thermodynamic Properties of Air", First Edition, John Wiley and Sons, Inc., New York, 1945.

DISTRIBUTION

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