# Bilateral Negotiations with Fees

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**Abstract**

Bilateral negotiation over a single good or service is a fundamental problem for automated systems, and is surprisingly resistant to general solutions. In this paper we offer advice and new results for the design of electronic negotiation and market systems. We review the theoretical and experimental literature as a guide to pragmatic design. We then investigate how some well-studied simple mechanisms could be extended with transaction and entry fees to improve their efficiency or their budget balance. The goal is to support pragmatic design for online automated transactions. We find that an iterated Generalized Vickrey Auction with fees can maintain budget balance and improve trading efficiency over a single-shot GVA. For k-double auctions we find that when processing costs are a function of the number of bids then efficiency favors entry fees, while transactions fees are favored if processing costs are a function of the number of transactions. We present simulations to support our theoretical conclusions.

## 1 Introduction

When we design electronic commerce infrastructure, we are typically pursuing a constrained optimization problem (though often implicitly): optimize some objective function — the design goal — subject to a constraint set. Over the past twenty years, one particular formulation of this design problem has emerged as a canonical benchmark. To the frustration of many, there are strong negative results on the possibility of satisfying the canonical constraint set in a general class of fundamental settings. One such setting is the *bilateral bargain* over a single good or service: under modest conditions, *there exists no mechanism* that satisfies basic desiderata and simultaneously ensures that the party who more greatly values the good or service always obtains it.

In this paper we present analytic and simulation results on the performance of mechanisms that naturally arise when we relax one of the standard design constraints. In one formulation of the bilateral bargaining problem it is impossible to ensure that all efficient trades take place unless the mechanism designer injects a subsidy into the negotiation from the outside. Therefore, we examine bargaining mechanisms in which participants pay fees in order to balance the budget. We characterize how well these mechanisms perform given the relaxed constraint set. Then we compare their performance when the fees may be charged to cover real costs of operating a bargaining institution.

Our results provide guidance to the developers of electronic commerce (and other bargaining) systems. We characterize design tradeoffs to consider, and organize prior literature according to such tradeoffs. We then present two particular mechanisms that are simple to understand and implement, and characterize some circumstances under which they perform well.

## 2 Mechanism Design

A *mechanism* comprises the rules of a resource allocation process, specifying (1) permissible messages and (2) resource allocations, as a function of messages (or entire message patterns) from mechanism participants (agents). It is often difficult in complex environments to find a mechanism that implements a desired system goal while satisfying even a few, seemingly reasonable goal constraints. Indeed, in very general classes of problems, no such mechanisms of any sort exist.

Before discussing results from the literature we define some terms. A mechanism is *Pareto efficient* if it has an equilibrium allocation such that there is no other allocation that makes some agent better off without making at least one agent worse off. There are three possible time periods at which efficiency (or other variables of interest) can be assessed: *ex ante*, *interim*, and *ex post*. These respectively refer to: before any participant learns its valuation, after all learn their valuations but before they take part in the mechanism (before negotiation), and after the results of the mechanism have been determined.[1] In addition we could define efficiency in a restricted environment (constrained efficiency) so that we restrict ourselves to decision rules that have certain characteristics, i.e. that result from individually rational mechanisms. Finally, if a mechanism is ex post efficient then when a numeraire good (such as money) can be freely exchanged among participants the mechanism also maximizes the gains from trade.

We now turn to strong impossibility results from the literature that motivate our search for feasible mechanisms. Gibbard and Satterthwaite [6],[20], make weak assumptions about agent rationality and preferences: (1) agents pursue

[1] An inclusion relation holds between the three notions of efficiency: any decision rule that is ex ante efficient is necessarily interim efficient, and any decision rule that is interim efficient is necessarily ex post efficient. This relation also applies to constrained efficiency, defined below. See Holmström and Myerson [12]

dominant strategies when they have them[2]; and (2) preferences are rational in the sense of ordinal, transitive, and complete. Then in general the *only* decision functions that can be implemented are those that are dictatorial, which is to say, that give one agent its most preferred settlement under all circumstances. Rarely will the desired system goal coincide with a choice function that puts all of the weight on the preferences of a single agent, and thus it will be strictly impossible to design mechanisms that fully implement desired system goals in general, relatively unrestricted problems.

In the face of such a negative result, mechanism design theory has proceeded by looking for good solutions for more narrowly restricted problem domains. One view of the method is to look for a mechanism that satisfies a less demanding set of constraints. A related approach is to study a particular mechanism and determine which properties it does fulfill.

The fundamental problem in the Gibbard-Satterthwaite setting is that little is assumed about the way in which autonomous agents formulate strategies: only that if agents have a dominant strategy, they pursue it. Most work since has made stronger assumptions about how agents formulate their strategies. The most common assumption is that agents play Bayesian-Nash rational strategies. Roughly, this means that agents calculate their expected payoffs from pursuing all possible strategies, and play the strategy that gives the best expected payoff assuming that all other agents are also Bayesian-Nash rational (see [15] for a more formal definition). In addition to this more restrictive assumption about how agents strategize, the canonical design problem imposes three other constraints: *incentive compatibility* or IC (participants are assumed to act in their own self interest, e.g., they will lie if it is in their interest to do so); *individual rationality* or IR (that participants voluntarily abide by the announced allocation and payments); and *budget balance* or BB (that no external subsidy is required). These requirements are widely accepted as modest desiderata for a good mechanism, although there are of course circumstances in which they are relaxed.

Unfortunately, for our very simple problem of bilateral negotiation over a single good or service, Myerson and Satterthwaite [17] have shown that no mechanism exists that satisfies these constraints on strategies and outcomes that also ensures trade efficiency (that all mutually beneficial exchanges take place). We explain this result in some detail because it is fundamental to negotiation theory.

Assume the mechanism designer believes that the two parties draw their value for the object from a distribution $v^i$, $i \in \{b, s\}$, where $v^i$ has support $[v^i_{\min}, v^i_{\max}]$. If the supports of the two value distributions are disjoint, then we can design efficient mechanisms that are IR, IC and BB. Suppose, without loss of generality, that $v^s_{\min} < v^s_{\max} < v^b_{\min} < v^b_{\max}$. As shown in Figure 1, the mediator can announce any trading price $p^*$ in the interval $[v^s_{\max}, v^b_{\min}]$. If $s$ holds the good initially, trade takes place with the buyer paying $p^* < v^b_{\min}$ and

[2] A weakly dominant strategy guarantees an agent at least as high a utility as any other strategy for all realizations of the random variables of interest, regardless of the strategies played by other agents.

The fundamental problem arises when the value distribution supports overlap, and the realized values drawn from these distributions are private information; see Figure 2. There is no ex ante price $p^*$ that simultaneously guarantees all efficient trades take place while satisfying IR, IC and BB. For example, for any $v^b_{\min} < p^* < v^s_{\max}$ there will be situations in which $p^* < v^s < v^b$: the seller will not agree to trade ($p^* < v^s$) yet trade would be efficient ($v^b > v^s$). More generally, Myerson and Satterthwaite [17] have shown that *no* mechanism (price-based or otherwise) exists that satisfies the conditions above while respecting the agents' private information.[3]

In the face of these impossibility theorems, several authors have tried to determine which mechanisms satisfy a more relaxed set of constraints. We review some of this prior work on pragmatic mechanism design for bilateral negotiations in the next subsection, largely to provide a guide to electronic commerce infrastructure designers.

## 2.1 Bilateral Trade Theory

The authors of papers reviewed in this section assume that each party to the negotiation has private information concerning the value of the good or service to them. They further assume that participants learn of their valuations for the object before they take part in the mechanism. We maintain these reasonable assumptions throughout this paper.

[3]The result requires that the supports of the two value distributions overlap, and that there be positive density throughout the region of overlap.

Myerson and Satterthwaite [17], who proved the impossibility of finding a Bayesian-Nash mechanism that is efficient while satisfying IC, IR and BB, also provide some constructive results. For a wide class of problems, they are able to derive a condition on when trade should take place if a mechanism is optimal (as efficient as possible subject to the IC, IR, BB constraints).

d'Aspremont and Gérard-Varet suggest a mechanism that is *ex post* efficient, incentive compatible, and balanced budget, but not individually rational. Varian and MacKie-Mason [22] present a Generalized Vickrey Auction mechanism that is *ex post* efficient, incentive compatible, and individually rational, but in some situations requires a subsidy.[4]

Satterthwaite and Williams (SW) [21] focus on a simple and intuitive mechanism, the $k$-double auction. In a $k$-double auction the buyer submits one bid, $b$, and the seller submits one ask, $s$. If the buyer's bid is greater than the seller's ask then the buyer gets the object and pays the seller a price equal to $kb+(1-k)s$. In the $k$-double auction buyers have an incentive to lower their bid below their true value, while sellers have an incentive to raise their asks above their true value in order to have a favorable effect on price. As a consequence, not all efficient trades will take place (since there will be cases in which the buyer's value is greater than the sellers, but $b < s$ so no trade takes place). SW characterize the set of Bayesian Nash equilibria of this game, showing that there is a continuum. SW also show that with $k = 0$ or 1 the k-double auction is *ex ante* constrained efficient (among those mechanisms that satisfy IC, IR, and BB).[5]

The k-double auction, as well as other simple mechanisms discussed later, is *ex post* IR. Gresik [7] shows that imposing *ex ante* rather than *ex post* IR does not increase the efficiency of an optimal mechanism. There are of course other ways in which the constraints could be relaxed in the timing dimension, but these have not proved to be particularly fruitful. For example, although Myerson and Satterthwaite [17] state their impossibility theorem in terms of *ex post* budget balance, in fact their proof also suffices to establish impossibility under *ex ante* budget balance (which means that a particular negotiation could run a deficit, but not on average).

For a negotiation in which there are multiple buyers and sellers, McAfee [16] suggested a simple mechanism (hereafter the dual price mechanism) in which it is a dominant strategy for each agent to reveal his/her true valuations. Buyers are ordered by their bids and sellers by their asks from low to high on a graph. The crossing point on this graph determines a range of potential market clearing prices, from the ask of the next-to-last seller below the crossing to the bid of the next to last buyer above the crossing. The mechanism chooses any price in

[4]There are entire classes of problems in which the GVA satisfies all the constraints, including budget balance; see, e.g., Walsh et al. [24] for an example. The GVA has also been shown to satisfy the same criteria for more complex problems including those with multiple goods, multiple units, and externalities across participants.

[5]Myerson and Satterthwaite [17] show that if buyer and seller valuations are uniformly distributed over the unit square, then the k-double auction with k=0.5 has an equilibrium in which all traders' bids and asks are linear functions of their valuations, and which maximizes the gains to trade among all mechanisms that are IC, IR and BB.

this range and all buyers who bid above this price, save the one with the lowest bid in this set, trade with the sellers whose asks lie below this price, save the seller with the highest ask in this set. (This is a simplified characterization to illustrate the main point.) Note that no participant who actually trades can affect the price at which they trade since the buyer and seller whose bid and ask determined the range of allowable prices were not allowed to trade. This mechanism has several advantages. It satisifies IR, IC and BB. Strategies are extremely simple for participants to calculate. There is a unique equilibrium. Only the least profitable trade is eliminated, and thus the relative efficiency loss is not large when the number of traders is large.[6]

## 2.2 Comparing Mechanisms

In this section we compare four mechanisms: fixed price, dual price, the k-double auction, and the Myerson-Satterthwaite constrained optimal mechanism. As seen above much of the literature focuses on efficiency concerns. We also discuss the complexity of a mechanism for both participants and automated system designers. We distinguish between complexity arising from knowledge requirements and from computational burden.

The dual price mechanism seems to offer less in the way of efficiency than the k-double auction. For example, Rustichini et al. (RSW) [19] show that even the least efficient equilibria of the k-double auction converge (as the number of participants increases) to maximal gains from trade at least as fast as dual price equilibria, and as fast as the constrained optimal mechanism for the case of uniform buyer/seller value distributions. They support this with simulation evidence (based on uniform distributions). Indeed, with six buyers and six sellers, even the most inefficient equilibria in the k-double auction capture more than 99% of the gains to trade in the RSW simulations. However, these results should be greeted with some skepticism, since they only report results for the uniform distribution with k=0.5. This may be an especially favorable case for the k-double auction, because it is known that at least one of the equilibria in this k-double auction maximizes the gains to trade among all IR, IC and BB mechanisms. For other studies of convergence to efficiency see Wilson [27] and Williams [25] [26].

We have summarized the theoretical constrained efficiency results for all mechanisms save the optimal mechanism in Table 1. The class of mechanisms under consideration are those that are IR, IC and do not run a deficit on average. Note that the k-double auction is always constrained efficient for $k \in \{0, 1\}$. For any other k, the k-double auction achieves interim constrained efficiency (and thus ex post constrained efficiency), for distributions of buyer/seller valuations "close enough" to the uniform.[7]

[6] Of course, in the bilateral case on which we focus, the efficiency loss is complete, since no trade would take place under these rules.

[7] For the precise sense in which a distribution must be "close enough" see Satterthwaite and Williams [21].

**Table 1: Constrained Efficiency**

| Mechanism | Ex Ante | Interim | Ex Post |
|---|---|---|---|
| Fixed Price | No | No | No |
| Dual Price | No | No | No |
| k-Double | Yes if k=0,1 | Yes if k=0,1 | Yes if k=0,1 |
| | | Yes $\forall$ k if $v^i \approx$ uniform | Yes $\forall$ k if $v^i \approx$ uniform |

Although the fixed and dual price mechanisms do not perform as well in terms of efficiency as the k-double auction (or the optimal mechanism), the situation is reversed when one considers the level of sophistication required of participants. The results above on the k-double auction are based on the assumptions that participants know the distribution over valuations for both buyers and sellers and which of many possible equilibria is being played, in addition to being able to perform extremely complicated calculations.[8] In the fixed price and dual price mechanisms it is a dominant strategy for participants to bid (or ask) their values. The mechanisms are extremely simple to understand. One should be confident that the participants would follow these dominant strategies.

Next we consider the level of complexity for those running the mechanism, the mediator (MD). The dual price mechanism does not require the MD to know anything about the distribution of valuations ex ante, nor does it require the mediator to perform any complicated calculations. The fixed price mechanism requires that the MD know the distribution of valuations and the results of the mechanism are extremely sensitive to this assumption. Likewise, designing an optimal mechanism requires that the MD know the distribution of buyer and seller valuations: designing the mechanism is a nontrivial computational task which would be difficult to automate, and the results of the mechanism are very sensitive to the distributional assumptions used. To run a k-double auction the MD need not know the distribution of buyer and seller valuations, although the value of k that maximizes the gains from trade depend on these distributional assumptions.[9] Table 2 summarizes the knowledge requirements of each mechanism.

[8] This is not to say that we should not expect equilibria to obtain, since equilibria can (and do) arise from a large spectrum of simple learning behaviors with repeated interactions. However, the process by which one equilibrium is selected over another as real participants interact in a double auction setting is not yet well understood. Given that there are multiple equilibria in the k-double auction, one should take care in evaluating the above claims of efficiency, given that they are merely statements that an equilibrium exists.

[9] Theoretically all that is known regarding the choice of k is that higher values of k are better for the buyer and that in the uniform case k=0.5 maximizes the gains from trade when the buyer and seller draw their valuations from identical supports. Theoretical analysis of the optimal k is complicated by the fact that closed form equilibrium strategies have not been found except in particularly simple cases (such as the uniform distribution) and that multiple equilibria make outcome ranking difficult. For example, in the bilateral case, regardless of the distributional assumptions and the choice of k there will always be equilibria in which no one ever trades.

**Table 2: Knowledge Requirements**

| Mechanism | Mediator | Agents |
|---|---|---|
| Fixed Price | Extreme | None |
| Dual Price | None | None |
| k-Double | Moderate | Extreme |
| Optimal | Extreme | Varies |

# 3 Negotiation Mechanisms with Transaction or Entry Fees

Our review of the literature provides us with two reasons to consider mechanisms with transaction fees. First, as Varian and MacKie-Mason [22] have shown, the Generalized Vickrey Auction implements all efficient trades while maintaining IR and IC, at the cost of a budget deficit for the mediator. In many practical situations, a budget deficit is unacceptable. This suggests introducing some fees to raise revenue.

Participation fees do not enable us to undo the Myerson and Satterthwaite impossibility theorem. Rather, they convert the budget deficit into an efficiency loss: when faced with a participation fee, some agents for whom trade would otherwise be efficient will not participate. For example, suppose there is a fee of $t$ on each participant in a trade. Then if $v^b - 2t < v^s < v^b$, there is no price at which trade will take place (since $v^s < p^* - t$, $v^b > p^* - t$) yet trade would be efficient ($v^s < v^b$).

Merely converting the mediator's budget deficit into an efficiency loss is not a compelling reason to explore transaction fees. However, transaction fees in a GVA allow us to reduce cases with overlapping supports (Figure 2) into a sequence of problems with disjoint intervals. Therefore, we study an iterative GVA with transaction fees in order to determine the gains from trade that are possible.

Our second reason for considering transaction fees is that running an auction can be costly. We will refer to any costs associated with running the auction as processing costs. An efficient trading mechanism would recognize and account for these costs. Two simple ways to extend an auction to account for costs would be: 1) charging an entry fee; or 2) levying a transactions fee for each completed exchange.[10]

For simplicity we shall focus on the bilateral case, although we expect that the basic intuition extends to the case with many buyers and sellers. Throughout we shall assume that bidder and seller valuations, $v^b$ and $v^s$ respectively, are drawn independently from a commonly known uniform distribution. We set $k$ at 0.5. The timing of either game will be as follows:

[10] So far we have been considering only single-round sealed bid mechanisms. In this case, an entry fee is equivalent to a fee for each bid submitted. If a multiple-round mechanism is considered (as we do below) then a per-bid fee could be distinguished from an entry fee, but we have not yet explored the per-bid fee in this situation.

**Stage 0** The buyers and the sellers discover their own (private) valuations. The mediator announces the fee schedule.

**Stage 1** The buyer and the seller simultaneously submit their sealed bid or ask if they choose to participate.

**Stage 2** The results of the mechanism are determined.

### 3.1 A Generalized Vickrey Auction with Transaction Fees

The Generalized Vickrey Auction [22] mechanism requires each agent to report to an auctioneer a utility function in the form of the maximum price it is willing to pay for each available good. The auctioneer solves a maximization problem based on the revealed valuations, and assigns the goods to the agents whose bids maximize the total surplus. Agents are then required to pay a sum that is independent of their own bids. Reporting the true utility function is a dominant strategy. In addition to this incentive-compatibility property, we focus our attention on this type of auction because it can be used to implement optimal allocations in a broad class of problems. The GVA can be applied to problems with multiple goods, multiple units, contingencies, externalities, etc. In addition, it solves for the optimal allocation when a competitive price equilibrium does not exist.

When the value distributions overlap, the GVA for a bilateral bargain will not satisfy budget balance (BB). Indeed, the buyer pays the seller's ask, $p_b = s$, and the seller receives the buyer's bid, $p_s = b$ (and trade only occurs if $b > s$). Therefore, we now modify the GVA to incorporate a transaction fee charged to buyers and sellers, $t_b$ and $t_s$ respectively.

If an agent declines to participate in a GVA because the transaction fee is higher than her expected gross gains from participation, the information revealed by non-participation could be used to update the common knowledge about the probability density of the agent's valuation. Additional rounds of the auction, based on the updated knowledge, will then help regain some of the efficiency losses that were incurred when we induced the participation pattern required to have disjoint support intervals for the probability density of the valuations.[11]

The outcome of a round of the modified GVA is: trade if and only if $b > s$, with the buyer paying $s + t_b$ and the seller receivinv $b - t_s$. The participation decisions are (see Figure 3):

- The buyer participates if its valuation $v^b$ of the good satisfies: $v^b \geq v^{b*}$, where $v^{b*} = v^{b*}(t_s, t_b)$, i.e. participation will depend on the amount of the fees $t_s$ and $t_b$.
- The seller participates if its valuation $v^s$ of the good satisfies: $v^s \leq v^{s*}$, where $v^{s*} = v^{s*}(t_s, t_b)$.

[11] Our mechanism is different than the iterative Vickrey-Groves process suggested by Banks, Ledyard, and Porter (1989), which is designed to reduce the information complexity, and is based on the communication of a single demand point at each step of the iteration.

We want the mediator to have nonnegative expected gains from participation. The expected value of mediator's net revenue is:

$$E[R(b,s,t_s,t_b)] = E[s - b + t_s + t_b | s \leq v^{s*}, b \geq v^{b*}, s \leq b] = f(v^{s*}, v^{b*}, t_s, t_b),$$

which depends on the bounds that describe agents' participation and on the transaction fee. Since $v^{s*} = v^{s*}(t_s, t_b)$, and $v^{b*} = v^{b*}(t_s, t_b)$ we can rewrite the above expression as follows:

$$E[R(b,s,t_s,t_b)] = f(v^{s*}, v^{b*}, t_s, t_b) = f(v^{s*}(t_s,t_b), v^{b*}(t_s,t_b), t_s, t_b) \equiv \phi(t_s, t_b).$$

We can find $t_s^1$ and $t_b^1$ such that $\phi(t_s^1, t_b^1) = 0$, i.e. so that the mediator's budget is balanced (in expectation). Ideally, we will also have $v^{s*} \leq v^{b*}$, such that we know that the agents who participate in this auction have valuations distributed over disjoint intervals. Our mechanism is very flexible and allows the designer to choose the values of $t_s^1$ and $t_b^1$ to serve the ultimate goal. If one is interested only in budget balanced auctions, then the procedure presented above is sufficient to guarantee ex ante a zero net subsidy auction, at the expense of an efficiency loss.

Having solved implicitly for the fees that balance the mediator's budget in the GVA mechanism, we now turn to the task of making the mechanism more efficient by the choice of transaction fees from the set that balance the budget. If we choose the transaction fees such that $v^{s*}$ and $v^{b*}$ are equal, we have the following four cases:

**1.** $s < v^{s*} = v^{b*} < b$ Both agents participate, trade occurs.

**2.** $b < v^{s*} = v^{b*} < s$ No participation, no trade.

**3.** $b, s < v^{s*} = v^{b*}$ Seller participates, buyer does not participate, no trade.

**4.** $v^{s*} = v^{b*} < b, s$ Seller does not participate, buyer participates, no trade.

In case 1, agents' valuations are actually separated by the "demarcation" line $v^{s*} = v^{b*}$. Since this is the only case in which trade occurs, we guarantee that when both agents participate in the auction, their valuations will be distributed over two disjoint intervals. Then a direct (incentive compatible) mechanism can be implemented and the resulting trade, conditional on being in case 1, will be efficient. In case 2, trade is inefficient, no agent participates and the auction stops. No trade occurs in cases 3 and 4 either, but here we are losing potentially efficient trades (it could be that $b > s$).

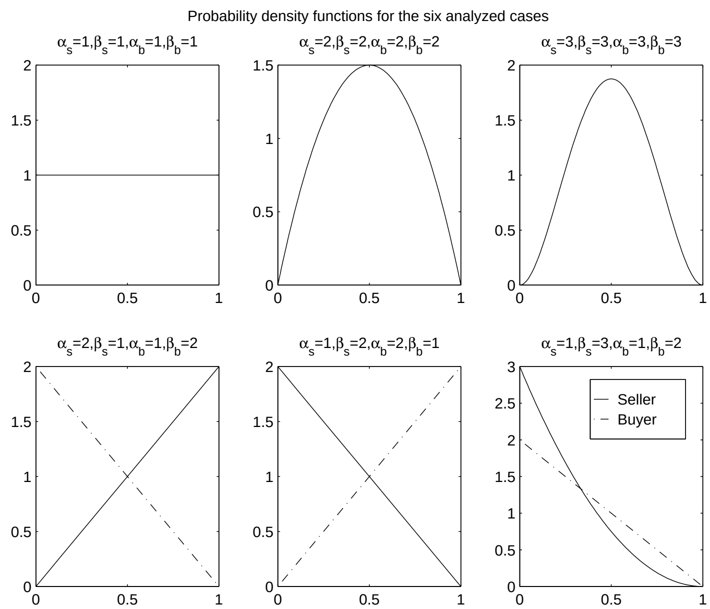
As can be seen, each combination of possible bidding participation is a distinguishable case, and thus the agents reveal information that can be used by the mediator in subsequent rounds of bargaining. Therefore, efficiency may be improved by organizing new rounds of the auction, with updated knowledge about the distribution of the valuations. A second round will be held only in the cases 3 and 4, when only one of the agents has submitted a bid, and there are potentially efficient trades. For example, in case 3 the second round will have the seller's valuation included in the interval $[s_{\min}, v^{s*}]$ and the buyer's valuation in the interval $[b_{\min}, v^{b*}]$. Using the same approach, second round transaction fees $t_s^2$ and $t_b^2$ will be determined such that the auction has balanced payments in expectation, and such that participation is restricted to disjoint intervals separated by $v^{s**} = v^{b**}$. The outcome of this second round is described by the four cases mentioned above, with the difference that now the separation line will be at $v^{s**} = v^{b**}$. Subsequent rounds may be held as needed based on the same procedure.

We now wish to characterize the trade-off between mediator deficit and efficiency for the iterated GVA with transaction fees. The benchmark is known: a single-round GVA applied to a bilateral negotiation with no transaction fees, and with both agents drawing their valuation from a uniform distribution on [0,1], the expected gains from trade are 0.17 and the expected budget deficit is also 0.17. We have analytic results for a single round GVA with transaction fees, because truth-telling is the incentive-compatible (indeed, dominant) strategy. We do not have analytic results when there is more than one round in an iterated GVA with transaction fees. With two or more rounds it is no longer incentive compatible for agents to tell the truth, and we have not yet been able to derive a general expression for equilibrium Bayesian-Nash incentive compatible strategies. Therefore, we resort to evaluating the upper bound of mechanism performance, obtained by assuming that agents truthfully bid their values. Since we know that rational agents would shave their bids, the actual efficiency of the mechanism will be somewhat less. However, the trend of increasing efficiency with increasing rounds should continue to hold.

For our numerical evaluations, we assume that agents draw their value for the object from a beta distribution, with probability density function

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1,$$

where $\Gamma$ denotes the "gamma" function. The beta distribution describes a wide variety of value distributions for different choices of the parameters; see Figure 4. For example, when all four parameters are equal to unity, the beta collapses to a uniform distribution on [0,1] for both agents. When all four parameters are equal to three, the distribution has a bell-shape, but with a finite support. Other variations are shown in the figure.




We evaluate the relative efficiency of the mechanism when both agents truthfully bid their valuations, and with both agents paying the same transaction fee. The fee is chosen in each period before bidding. In Table 3 we report the results when the fees are chosen to set expected mediator revenue to zero (budget balance); the results in Table 4 are for auctions in which the fees are chosen to raise revenue equal to 1% of the gains from trade, to show performance if it is necessary to cover costs from running the mechanism. The columns in each table represent the relative efficiency of a mechanism that is permitted to iterate for the indicated number of rounds.[12]

The first row in each table is the familiar case of values drawn from a uniform distribution on [0,1] for both agents. The relative efficiency of this mechanism is 50% in a single-round auction (Table 3). We can compare this to the benchmark of a GVA without transaction fees, which obtained expected gains of 0.17. With a transaction fee, we obtain expected gains of 0.09, but with expected budget balance rather than an expected deficit of 0.17. Thus, the cost of balancing the budget is half the gains.

[12]That is, the first column is for a single-round auction; the second is for a two-round auction; and so forth.

From Table 3 we can see that for a wide range of value distributions, a four-round auction with an expected budget balance can typically obtain about 94% of the total trading gains. Thus, if there is no cost to running the mechanism, and if agents report their valuation truthfully, we can balance the budget by giving up only about 6% of the trading gains. In Table 4 we see that even when we raise surplus revenue of 1% of expected trading gains, a four-round auction can typically obtain over 90% of the total trading gains. Therefore, it appears that solving the GVA budget deficit problem in bilateral negotiations with transaction fees is rather effective, with only four rounds required to capture 90% or more of the trading gains even when there are modest costs of running the mechanism.

### 3.2 A k-Double Auction with Transaction Fees

We now turn to the analysis of a k-double auction with transaction fees. As described in section 2 above, when the buyer's bid is higher than the seller's ask, the buyer gets the object and pays the seller $kb + (1-k)s$. Thus, the simple k-double auction is budget balanced but does not achieve unconstrained full efficiency, in contrast to the simple GVA which is fully efficient but incurs a budget deficit. The simple k-double auction has been thoroughly explored. We study the case with transaction fees to compare to the GVA with fees, and because real implementations of mechanisms will often have costs to recover.

There are multiple incentive-compatible bidding equilibria in a typical k-double auction, but we restrict ourselves to strategies that are piecewise linear in the agent's true valuation because the resulting equilibria have several nice properties.[13] For example, with k=0.5 the linear equilibrium is the most (constrained) efficient among all equilibria of all possible mechanisms satisfying IC, IR and BB (Myerson and Satterthwaite [17]), as long as the supports of the value distributions are identical. There is also experimental evidence suggesting that agents tend to select linear strategies when available in the set of equilibrium strategies [5], [18]. The linear equilibrium is also relatively easy to calculate.

To introduce transction fees to the k-double auction, we assume that the mediator keeps a percentage, $f$, of the reported surplus for each trade. That is, if a buyer bids $b$ and a seller asks $s$ then a transaction is executed as long as $b \geq s$ and the mediator receives $(b-s)f$. Given any arbitrary fee we can solve for the linear equilibrium. We set up the problem for any $k$, although we only anaylze the case of $k = 0.5$.

In a simple k-double auction the buyer receives its true valuation minus the payment to the mediator:

$$v^b - kb - (1-k)s.$$

[13] We will show below that in the case of entry fees there does not exist an equilibrium in which strategies are linear over the whole support, but the equilibrium on which we focus is piecewise linear with only one kink as long as the fee is not set so high that there is never trade.

Table 3. Efficiency as a function of the number of auction rounds when both the seller and the buyer truthfully report their valuations

| Distribution parameters | | | | Number of rounds | | | |
|---|---|---|---|---|---|---|---|
| Seller | | Buyer | | | | | |
| $\alpha_s$ | $\beta_s$ | $\alpha_b$ | $\beta_b$ | 1 | 2 | 3 | 4 |
| 1 | 1 | 1 | 1 | 50% | 75% | 87.5% | 93.75% |
| 2 | 2 | 2 | 2 | 50% | 74.4% | 87.02% | 93.47% |
| 3 | 3 | 3 | 3 | 50% | 73.85% | 86.62% | 93.25% |
| 2 | 1 | 1 | 2 | 38.82% | 65.46% | 82.45% | 92.34% |
| 1 | 2 | 2 | 1 | 67.62% | 87.42% | 94.57% | 97.55% |
| 1 | 3 | 1 | 2 | 50% | 75% | 87.5% | 94% |
| 1 | 1 | 2 | 2 | 51.94% | 76.22% | 88.07% | 94% |
| 1 | 1 | 3 | 3 | 54.60% | 77.66% | 87.70% | 94.26% |

Table 4. Efficiency as a function of the number of auction rounds when both the seller and the buyer truthfully report their valuations, and the auctioneer has a revenue target of 1 percent of the expected surplus

| Distribution parameters | | | | Number of rounds | | | |
|---|---|---|---|---|---|---|---|
| Seller | | Buyer | | | | | |
| $\alpha_s$ | $\beta_s$ | $\alpha_b$ | $\beta_b$ | 1 | 2 | 3 | 4 |
| 1 | 1 | 1 | 1 | 49.80% | 74.86% | 87.48% | 93.74% |
| 2 | 2 | 2 | 2 | 49.38% | 73.66% | 86.30% | 92.81% |
| 3 | 3 | 3 | 3 | 49.14% | 72.74% | 85.47% | 92.13% |
| 2 | 1 | 1 | 2 | 35.69% | 60.31% | 76.10% | 85.35% |
| 1 | 2 | 2 | 1 | 67.15% | 87.03% | 94.26% | 97.30% |
| 1 | 3 | 1 | 2 | 49.16% | 74.58% | 87.09% | 93.25% |
| 1 | 1 | 2 | 2 | 51.23% | 75.36% | 87.22% | 93.19% |
| 1 | 1 | 3 | 3 | 54.05% | 77.07% | 87.17% | 93.79% |

- %

$f$ is set so that the buyer receives in net

$$v^b - fb - k(1-f)b - (1-k)(1-f)s.$$

By setting this expression equal to $v^b - P_B$ we can implicitly define the price the buyer pays:

$$P_B = [1-(1-k)(1-f)]b + (1-k)(1-f)s.$$

To see why we define the transaction fee in this way, consider the net value received by a buyer who truthfully bids her value ($b = v^b$). With no fee, a truthful bidder receives $1-k$ of the total surplus, $(1-k)(b-s)$. With the transaction fee, a truthful bidder receives the analogous amount, $(1-k)(1-f)(b-s)$. That is, we have defined the transaction fee to be a percentage $f$ of the *surplus* the buyer would receive if she bid truthfully. This formulation has the convenient feature that the price faced by the buyer is the same as in a $k_B$-double auction with no transaction fee, where $k_B = 1-(1-k)(1-f)$.

Following the same approach, we define the transaction fee paid by the seller so that the after-fee price received is

$$P_S = k(1-f)b + [1-k(1-f)]s.$$

Thus, the seller perceives the price that he receives to be the same price that he would receive in a $k_S$-double auction with no transaction fees, where $k_S = k(1-f)$.

Assume that the seller and buyer use linear (strictly speaking, affine) strategies: $S(v^s) = \sigma_1 + \sigma_2 v^s$ and $B(v^b) = \beta_1 + \beta_2 v^b$. A buyer with a valuation of $v^b$ contemplating an arbitrary bid, $b$, expects a payoff of:

$$(v^b - E[P \mid b \geq S(v^s)])Pr[b \geq S(v^s)]. \tag{1}$$

Substituting for the buyer's probability of trade and the expected price conditional on trade, and optimizing with respect to $b$ yields the following necessary condition:

$$b = \sigma_1 \frac{k_B}{1+k_B} + \frac{1}{1+k_B} v^b. \tag{2}$$

A seller with a valuation of $v^s$ contemplating an arbitrary bid, $s$, expects a payoff of:

$$(E[P \mid s \leq B(v^b)] - v^s)Pr[s \leq S(v^b)]. \tag{3}$$

Substituting for the seller's probability of trade and the expected price conditional on trade, and optimizing with respect to $s$ yields the following necessary condition:

$$s = (\beta_1 + \beta_2)\frac{1-k_S}{2-k_S} + \frac{1}{2-k_S} v^s \tag{4}$$

Using the necessary conditions for the buyer and the seller we can easily solve for the coefficients of the buyer's and seller's strategies and verify that the equilibrium determined is in fact linear. In this equilibrium:

$$\beta_1 = \frac{(1-k_S)k_B}{(2-k_S+k_B)(1+k_B)} \quad \beta_2 = \tfrac{1}{1+k_B}$$
$$\sigma_1 = \frac{1-k_S}{2-k_S+k_B} \quad \sigma_2 = \tfrac{1}{2-k_S}. \tag{5}$$

The trading boundary defined as $\{(v^b, v^s) \mid B(v^b) = S(v^s)\}$ will be used extensively in comparing the two payment mechanisms. For the transactions fee the trading boundary is:

$$v^b = \frac{\sigma_1 - \beta_1}{\beta_2} + \frac{\sigma_2}{\beta_2} v^s \tag{6}$$

which for $k = 0.5$ yields $v^b = \frac{1+f}{2(2+f)} + v^s$.[14]

### 3.3 A k-Double Auction with Entry Fees

In a k-double4 auction with entry fees, the mediator charges some entry fee $F$ to each participant who submits a bid or an ask. We set $k = 0.5$.

The timing of the mechanism will be identical to that for the transactions fee mechanism. Here we cannot focus on a strictly linear equilibrium. Given that there is an entry fee there will be a minimum buyer valuation $\underline{v}^b$ and a maximum seller valuation $\bar{v}^s$ such that no buyer with a valuation below this threshold will enter and no seller above this threshold will enter.[15] Let $B(v^b)$ be the equilibrium strategy for the buyer; it must be nondecreasing [3]. Consider the seller's strategy. Clearly the seller would never ask for an amount less than $B(\underline{v}^b)$. In order to accommodate this we consider strategies that are constant for sellers from 0 to the seller's value at which its linear bid strategy would yield a bid equal to the lowest possible buyer's bid (a bid that the seller would never want to ask below), and then linear increasing above this point (to $\bar{v}^s$). Likewise for the buyer we consider strategies that are linear from $\underline{v}^b$ to $S(\bar{v}^s)$ and constant above this point.

Deriving the equilibrium strategies in this case is slightly (but not much) more difficult than for transactions fees. For now let $\underline{v}^b$ and $\bar{v}^s$ be arbitrary. Since the model is completely symmetric it will be the case that $\underline{v}^b = 1 - \bar{v}^s$. Additionally define seller and buyer strategies as follows:

[14] It may appear strange that even with $f = 1$ there is still trading. This is because the fee has been constructed as a fee on reported surplus. Since buyers underbid and sellers overask, reported surplus is strictly less than actual surplus.

[15] To see that that buyer and seller entry strategies must involve a cutoff consider two potential buyer valuations $v_1^b < v_2^b$. If it is profitable for $v_1^b$ to enter then it must be profitable for $v_2^b$ to enter since $v_2^b$ could always submit the same bid as $v_1^b$. Therefore, if it is not profitable for $v_1^b$ to enter, it must not be profitable for any $v^b < v_1^b$ to enter. Similar reasoning holds for any seller valuations.

$$
\begin{aligned}
S(v^s) &= B(\underline{v}^b) \; \forall \; v^s < \frac{B(\underline{v}^b)-\sigma_1}{\sigma_2} \\
& \quad \sigma_1 + \sigma_2 v^s \; \forall \; v^s \in [\frac{B(\underline{v}^b)-\sigma_1}{\sigma_2}, \bar{v}^s] \qquad (7) \\
B(v^b) &= \beta_1 + \beta_2 v^b \; \forall \; v^b \in [\underline{v}^b, \frac{S(\bar{v}^s)-\beta_1}{\beta_2}] \\
& \quad S(\bar{v}^s) \; \forall \; v^b > \frac{S(\bar{v}^s)-\beta_1}{\beta_2} \qquad (8)
\end{aligned}
$$

The inequalities define kinks in the piecewise linear strategies, and are determined by the bounds on strategies implied by the buyer and seller threshold bidding strategies. For example, the kink in the seller strategy occurs where $v^s = (B(\underline{v}^b) - \sigma_1)/\sigma_2$.

In order to determine the coefficients of these strategies, note that the probability of trade is identical to the transactions fee case above for the same coefficients. Consider a buyer contemplating an arbitrary bid, $b$. The expected price conditional on trade is:

$$
E[P \mid b \geq S(v^s)] = \frac{1}{2}b + \frac{1}{2}E[S(v^s)|b \geq S(v^s)] \qquad (9)
$$

where

$$
E[S(v^s)|b \geq S(v^s)] = \frac{\underline{v}^b - \sigma_1}{b - \sigma_1}\underline{v}^b + (\frac{1}{2}(\frac{b-\sigma_1}{\sigma_2} + \frac{\underline{v}^b - \sigma_1}{\sigma_2})\sigma_2 + \sigma_1)\frac{b - \underline{v}^b}{b - \sigma_1} \qquad (10)
$$

or

$$
E[S(v^s) \mid b \geq S(v^s)] = \frac{b^2 + (\underline{v}^b)^2 - 2\sigma_1 \underline{v}^b}{2(b-\sigma_1)}. \qquad (11)
$$

Substituting this into the expected profits of the buyer, taking derivatives, setting them equal to zero and rearranging yields: $b = \frac{1}{3}\sigma_1 + \frac{2}{3}v^b$. Performing the same calculations for the seller yields: $s = \frac{1}{3}(\beta_1 + \beta_2) + \frac{2}{3}v^s$. Thus, the linear portion of the equilibrium strategies for the buyer and sellers are:

$$
b = \frac{1}{12} + \frac{2}{3}v^b \qquad (12)
$$

$$
s = \frac{1}{4} + \frac{2}{3}v^s. \qquad (13)
$$

These are precisely the same as the well-known linear strategies with no entry fees [3], i.e. the entry fees distort the entry decision and the bid of the highest buyers and lowest sellers, but all other buyers and sellers bid exactly as they would in the absence of the entry fees. The trading boundary is horizonal from $(v^s = 0, v^b = \underline{v}^b)$ until intersecting with $v^b = v^s + \frac{1}{4}$, which the boundary follows until it becomes vertical at $\bar{v}^s$.

In order to complete the description of the equilibrium we need to determine $\underline{v}^b$ and $\bar{v}^s$. As before we will do the derivation for the buyer. The derivation for the seller is symmetric. Given that no seller will ever ask less than $B(\underline{v}^b)$ the expected profits conditional on trade for a buyer with valuation $\underline{v}^b$ is $\underline{v}^b - B(\underline{v}^b) = \frac{1}{3}\underline{v}^b - \frac{1}{12}$. The probability of trade is: $\underline{v}^b - \frac{1}{4}$ for all $\underline{v}^b \geq \frac{1}{4}$ because the highest seller willing to trade with $\underline{v}^b$ is on the intersection of the linear and horizontal portion of the trading boundary. Thus, the expected profits of buyer $\underline{v}^b$ given an entry fee, $F$ are:

$$\frac{1}{3}(\underline{v}^b)^2 - \frac{1}{6}\underline{v}^b + \frac{1}{48} - F. \tag{14}$$

Since $\underline{v}^b$ is the marginal buyer it must be the case that this buyer earns zero expected profits and thus setting (14) equal to zero yields[16]:

$$\underline{v}^b = \frac{1}{4} + \sqrt{3F}. \tag{15}$$

# 4 Comparison of the Two k-Class Mechanisms

## 4.1 Bid Processing Costs

We assume that bid processing costs are some increasing function of the expected number of *serious* bidders, i.e. bidders who have a positive probability of trading.[17]

Define the *net benefit* in a particular equilibrium of a mechanism with bid processing costs to be the expected gains from trade in that equilibrium minus the expected bid processing costs. Our main result in this section is the following:

**Theorem 1** *Choose any arbitrary $f$, determining a transactions fee mechanism. If costs are an increasing function of the expected number of bidders, for whom the probability of trade is positive, then there exists an $F$ such that the equilibrium described above for the entry fee mechanism has at least as high a net benefit as the linear equilibrium in the transactions fee mechanism.*

Although we could offer a more formal proof of this theorem, the following graphical argument is clear and makes a more formal proof transparent.

*Proof:* There are two cases to consider. In the first case the net benefit from the transactions fee mechanism is zero. In that case the result is immediate because for a high enough $F$ there will be no bids, no trade, no costs and thus zero net benefit. To see that the result must also hold in the second case consider Figure 5. In this figure we have drawn the trading boundary for the described

[16]There are of course two roots to the quadratic equation in (14). However, the second root is not a profit-maximizing solution. This can be seen by showing that it appears to give better gains to trade than the linear equilibrium without fees, which is not possible because that equilibrium maximizes the gains to trade among all possible equilibria.

[17]Any nominal charge of $\epsilon > 0$ will remove the bidders from the market for whom the probability of trading is zero, without having a significant effect on the equilibrium.

[18] Since we have only considered a linear equilibrium for the mechanism with transaction fees, one might be concerned that a piecewise linear equilibrium exists that captures the same, or even a greater area of extra trades. It is simple to show that no piecewise linear equilibrium with flat segments exists for the transaction fee mechanism. We have not been able to prove that no transaction fee equilibrium exists with equal or greater trading.

Now suppose we define the optimal transactions fee mechanism as the mechanism in which $f$ is set to maximize the net social benefit in the equilibrium described from this mechanism.[19] Define the optimal entry fee mechanism similarly. The following corollary is then immediate from Theorem 1:

**Corollary 1** *The net benefit in the equilibrium described for the optimal entry fee mechanism is at least as high as the net benefit in the optimal linear transactions fee equilibrium.*[20]

These results are intuitive: if the source of the processing costs is the number of bidders then we should choose a mechanism that penalizes bidding or entry directly. In the next section we show, again intuitively, that if the source of processing costs is transactions rather than bidding, per se, we should charge for transactions directly.

## 4.2 Transaction Processing Costs

Suppose now that costs are an increasing function of the number of transactions. The following result establishes that when processing costs are a function of the number of transactions then the simple mechanism with the transaction fee should be preferred:

**Theorem 2** *Choose any arbitrary $F$, determining an entry fee mechanism. If costs are an increasing function of the expected number of transactions, then there exists an $f$ such that the linear equilibrium described above for the transactions fee mechanism has at least as high a net benefit as the equilibrium described above for the entry fee mechanism.*

*Proof:* There are two cases to consider. In the first case the net benefit from the entry fee mechanism is zero. In that case the result is immediate. To see that the result must also hold in the second case consider Figure 6. In this figure we have drawn the trading boundary for the described equilibrium for an entry fee mechanism for any arbitrary $F$ as line CDEF. Valuing (6) for $k = 0.5$ we see that the transactions fee $f$ can be set so that the total number of trades under a transactions fee mechanism are equal to the total number of trades in the specified entry fee equilibrium. Thus, the trading boundary AB in Figure 6 has been drawn so that Area ACG + Area HFB = GHED (given that $v^s$ and $v^b$ are distributed independently uniform on the same support the expected number of trades are equal if these areas are equal). Now recall that line segment AB is the graph of the equation $v^b = v^s + \delta$, where $\delta$ is some constant between 0.25 and 0.5. Thus, for all points above this line segment $v^b - v^s > \delta$ and vice versa below the line segment. Thus, the gains from trade for any point above this line segment are greater than for any point below this line segment. Thus, trades

[19] So that this mechanism is uniquely described, choose the lowest $f$ in the event of a tie.

[20] We cannot make the stronger claim that the most efficient equilibrium in the entry fee mechanism has a higher net benefit than the most efficient equilibrium in the transactions fee mechanism, since we have limited our analysis to linear equilibria.

**Corollary 2** *The net benefit in the equilibrium described for the optimal transactions fee mechanism is at least as high as the net benefit in the entry fee equilibrium, when costs are an increasing function of the number of transac-*

**Table 5: k-Double Auction with a Transaction Fee**

| Fee | Revenue as percent of expected gain | Total Gains | % Efficiency |
|---|---|---|---|
| 0.260831 | 0.01 | 0.125 | 81.0% |
| 0.683723 | 0.02 | 0.108 | 76.7% |
| 1.56613 | 0.03 | 0.087 | 70.5% |
| 6.9761 | 0.04 | 0.057 | 58.3% |
| infinity* | 0.0417 | 0.042 | 50.0% |

*Maximum revenue possible.

**Table 6: k-Double Auction with an Entry Fee**

| Fee | Revenue as percent of expected gain | Total Gains | % Efficiency |
|---|---|---|---|
| 0.008465 | 0.010 | 0.123 | 79.8% |
| 0.019737 | 0.020 | 0.101 | 72.6% |
| 0.035348 | 0.030 | 0.073 | 61.6% |
| 0.064474 | 0.040 | 0.026 | 39.8% |
| 0.08333* | 0.042 | 0.005 | 28.1% |

*Maximum revenue possible.

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## 5 Conclusion

The space of possible automated mechanisms for simple online negotiations is very large. At last count, the Michigan Internet AuctionBot (http://auction.ai.eecs.umich.edu) can be configured to run some 25 million different auction types. In this paper we have reviewed some of the analytic and experimental literature to provide general guidelines for electronic commerce infrastructure designers. We have also contributed results, both analytic and simulated, on a previously unstudied class of mechanisms: those with transaction and entry fees. At this writing we have characterized a budget-balanced and a revenue-raising iterated Generalized Vickrey Auction with transaction fees, and a revenue-raising k-double auctions with either transaction fees or entry fees.

Our results for the iterated GVA are quite promising: with four rounds of bidding, this mechanism can maintain budget balance (on average) but still capture over 90% of the maximal gains from trade. For the k-double auction, which has other desirable properties, we have shown that when costs of running the mechanism are associated with transactions, the transaction fee auction is preferred; when costs are associated with the number of bids, the entry fee auction is preferred.

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