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Final Report

AN EVALUATION OF THE AVERAGE IMPACT PRESSURE PRODUCED BY A SUPERSONIC NOZZLE OPERATING AT CONDITIONS SPECIFIED FOR THE OXYGEN CONVERSION PROCESS

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ABSTRACT

A series of tests was conducted with a supersonic nozzle ($M = 1.5$; throat diameter = 0.377 in.) operating at 50-psi gauge. The jet diameter was determined at positions 20 in. and 13 in. downstream of the nozzle and can be estimated by the equation:

$$D_j = (.32\,x + .41) \text{ in.}$$

where x is the distance from the nozzle and equal to or less than 20 in.

The average impact pressure (P_i) within the air jet was found to be approximately determinable by the equation:

$$P_i = 9.68/(.32\,x + .41)^2 \text{ psi.}$$

This equation applies with reasonable accuracy to both air and oxygen.

The results of this study are based on a particular nozzle and cannot be extended to nozzles of other sizes and shapes without modification.

OBJECTIVE

To evaluate, by experiment and analysis, the average impact pressure at several positions in a jet stream issuing from a supersonic nozzle under fixed operating conditions.

INTRODUCTION

The Oxygen Conversion Process, of interest in this study, requires the use of a high-velocity gaseous jet which is directed toward the surface of a molten metal. The necessary or recommended characteristics of this jet may be defined in different ways. In particular, the range of nozzle operating conditions and positions relative to the metal surface may be defined as in British Patent 642,084; or the range of average impact pressures at the metal surface may be defined as in United States Patent 2,741,555.

This report presents the results of tests and analysis of one nozzle operated in accordance with British Patent 642,084 expressed in terms of characteristics defined in the United States Patent 2,741,555.

DISCUSSION OF GENERAL NOZZLE CHARACTERISTICS

A. NOZZLE FLOW RATE

The flow rate of any given compressible fluid through a supersonic nozzle is primarily a function of: (1) the throat area (i.e., minimum cross-sectional area) of the nozzle; (2) the driving pressure (i.e., pressure at the nozzle inlet); and (3) the reservoir temperature of the compressible fluid. The above presupposes that the nozzle is exhausting into ambient pressure less than one-half the magnitude of the driving pressure.

$$\text{Thus, air flow rate } (w_a) \cong .53\, P_d A^* T_d^{-1/2} \text{ lb/sec}$$

$$O_2 \text{ flow rate } (w_o) \cong .56\, P_d A^* T_d^{-1/2} \text{ lb/sec,}$$

where

P_d is the driving pressure, psia,
A^* is the throat area, sq. in., and
T_d is the reservoir temperature, °Rankine.

Reference 1 includes a derivation of these equations as well as further discussion on flow through nozzles. References 2 and 3 and similar books deal extensively with this subject. References 4 and 5 include tables useful in computing flow through nozzles. In these tables γ (the ratio of specific heats is $\gamma = C_p/Cv$) is 1.4, which is approximately the case for both oxygen and air throughout the range of conditions considered in this report. Values of specific heat for air and oxygen are listed in reference 6.

Throughout these studies it has been assumed that the oxygen would be stored and delivered to the nozzle inlet at approximately room temperature, i.e., 60°F (520°R). Since flow rate is inversely proportional to the absolute temperature to the one-half power, a 50° variation in delivery temperature will produce less than a 5% variation in the fluid flow rate. Thus, for our purposes any given supersonic nozzle having a particular throat area will have an oxygen flow rate that is essentially a direct function of driving pressure only.

B. CORRECT NOZZLE DRIVING PRESSURE

The purpose of a nozzle is to accelerate the compressible fluid from an initial condition of high pressure and low velocity to one of low pressure and high velocity. The pressure at the exit of a supersonic nozzle is, within limits, a function of: (1) the nozzle driving pressure and (2) the ratio of nozzle exit area to nozzle throat area, for any particular compressible fluid. By definition, a given supersonic nozzle is operating at the design point when the driving pressure is such that the pressure at the nozzle exit is exactly equal to the static pressure surrounding the nozzle. Thus, the exit static pressure of a nozzle, operating at the design point and exhausting into the standard atmosphere, is essentially 14.7 psia. Conversely, any given driving pressure determines the ratio of exit area to throat area of a nozzle which is to be designed for that particular driving pressure.

Reference 1 includes the test results of several supersonic nozzles operated at driving pressures other than the design driving pressure. These tests have shown that a nozzle designed for any given driving pressure can actually be operated over an appreciable range of driving pressure with little deleterious effect regarding jet characteristics.

C. IMPACT FORCE OR TOTAL MOMENTUM AT THE NOZZLE EXIT

Momentum is defined as mass times velocity. The rate of change of momentum of a system is proportional to the unbalanced forces acting on the system. An extension of this basic law of motion is that commonly referred to as the "momentum theorem." This states that under steady flow conditions through a given closed region in space the integral of inwardly acting pressure over the surface surrounding the region is equal to the integral of the outward component of momentum over the surface. In the one-dimensional case (i.e., all components of velocity except the one parallel to the longitudinal axis are negligible as compared to the longitudinal velocity in calculating momentum, kinetic energy, etc.), and as applied here, this theory reduces to the statement that the momentum at the nozzle exit, plus the nozzle exit area times the difference between exit and static pressures is equal to the momentum downstream and is herein referred to as "total momentum." Page 178 of reference 2 contains the same expression for total momentum (in this reference the total momentum is, perhaps more correctly, referred to as impulse). The above is presented to establish the fact that the total momentum (or impulse) at any point downstream of a nozzle is essentially independent of the distance from the nozzle.

If a solid surface of infinite area is placed in the jet stream and normal to the jet-stream axis, all components of velocity parallel to the axis will be reduced essentially to zero at this surface. Then the net impact force on the surface will be equal to the total longitudinal momentum of the jet stream approaching the surface. Obviously, an infinite surface is not required in an actual case to reduce the longitudinal total momentum to approximately zero. A surface with an area appreciably larger than that of the jet stream can be expected to serve this purpose.

D. NOZZLE OPERATING CONDITIONS AS DEFINED BY BRITISH PATENT 642,084

Claim 3 of British Patent 642,084 covers the operation of an oxidizing nozzle with a driving pressure of <u>50 psi or greater</u>. Claim 4 of this same patent covers the condition wherein the oxidizing gas has a velocity of <u>1250 ft per sec or greater</u> at the nozzle exit. These two claims establish the <u>lower limit</u> of driving pressure and nozzle exit velocity, but there is no upper limit specified within the claims of this patent. However, within provisional specification No. 11236, in the year 1945, the suggestion is made that 7 or 8 atmospheres (107 or 118 psi) would be used. The use of these higher driving pressures could more than double the average impact pressure obtained at any given position downstream from the same nozzle over that obtained at 50 psi.

The position of the oxidizing nozzle relative to the metal is not specified in any of the claims of British Patent 642,084. In the complete specification of this patent, however, a range of nozzle positions is recommended; i.e., the nozzle should be between 1 to 20 in. from the metal, preferably between 4 and 12 in. Thus, the range of jet-flow characteristics prescribed in British Patent 642,084 and provisional specifications will be that which results from the use of the following ranges of nozzle design, operating condition, and locations.

Characteristic	Minimum	Maximum	Remarks
Nozzle driving pressure	50 psi	not specified	Up to 118 psi recommended
Nozzle delivery velocity	1250 ft/sec	not specified	Nozzle exit velocity is within limits, a function of design and driving pressure (at a given temperature)
Nozzle shape			Round or rectangular
Nozzle throat area	not specified	not specified	As required by application
Nozzle exit area	not specified	not specified	Presumably as required by driving pressure
Nozzle position relative to metal surface	1 in.	20 in.	4 in. to 12 in. recommended
Impact pressure at surface of metal	not specified	not specified	Determined by nozzle size, driving pressure, and position

E. NOZZLE OPERATING CONDITIONS AS DEFINED BY UNITED STATES PATENT NO. 2,741,555

This patent does not define specifically the nozzle size, design, or driving pressure. Rather, it recommends a nozzle position relative to the metal surface and the impact pressure (as defined in column 8 and 9 of this patent) at the metal surface.

Thus, the range of jet-flow and nozzle operating characteristics prescribed in the United States Patent Number 2,741,555 is as follows:

Characteristics	Minimum	Maximum	Remarks
Nozzle driving pressure	not specified	not specified	Presumably as required
Nozzle delivery velocity	not specified	not specified	Within limits determined by driving pressure, design, and temperature
Nozzle shape	not specified	not specified	Presumably circular
Nozzle throat area	not specified	not specified	Presumably as required
Nozzle exit area	not specified	not specified	Presumably as required by throat area and driving pressure
Nozzle position relative to metal surface	150 mm (5.9 in.)	2,000 mm (78.7 in.)	The position is to be varied as required to obtain prescribed impact pressure
Impact pressure at surface of metal	0.02 kg/sq cm (.284 psi)	0.5 to 0.75 kg/sq cm (7.11 to 10.67 psi)	

TEST PROCEDURES AND EQUIPMENT

A series of tests was desired which would determine, to some extent, the jet characteristics produced by a nozzle operating according to the prescribed conditions in British Patent 642,084. The characteristics of particular interest, defined as the average impact pressure at recommended locations, were determined according to U. S. Patent 2,741,555.

This U. S. patent makes a particular point of keeping average impact pressure below certain values. Therefore, it was logical to test a nozzle in accordance with the minimum driving pressure set forth in the British patent in order to determine the minimum impact pressure indirectly prescribed by that patent. The nozzle driving pressure used in the tests described below was approximately 50 psi (held to within ± 2 psi; the test gauge was calibrated by a dead-weight tester).

The nozzle design regarding divergent angle, throat area, or exit area was not specified in either of the two patents under consideration. It is certainly reasonable to assume that a nozzle used at any given driving pressure should be designed at least approximately for that pressure. Thus, a nozzle exhausting into a standard atmosphere with a driving pressure of 13 psi or less would ideally be simply a convergent nozzle, i.e., the minimum nozzle cross-sectional area would be reached at the nozzle exit. The nozzle exit velocity would then be equal to the speed of sound or less. Any nozzle driving pressure above 13 psi would normally call for a Convergent-Divergent nozzle where the ratio of exit area to minimum area is approximately determined by the ratio of driving pressure to the ambient pressure. The exit velocity will then be supersonic.

A driving pressure of 50 psi (i.e., approximately 64.3 psia) for a nozzle exhausting into the local atmosphere at the time of these tests indicates that the exit area should be approximately 1.28 times the throat area (as per reference 5). A nozzle whose exit area was 1.18 times the throat area was already available at this laboratory.

Since the precise nozzle design is not specified by British Patent 647,084, it was considered that the available nozzle would certainly be suitable for the required tests.

The throat diameter of this nozzle was 0.377 in., which is by coincidence very near the nozzle size mentioned in the British Patent, i.e., 0.32 in.

A high-pressure air system including compressors, air storage tanks, air lines, and shut-off and control valves was used in conducting these tests. The validity of using air instead of oxygen is discussed below.

Instead of using a single traversing pitot or total head tube to determine the jet boundary, two stationary total head rakes were used. The position of the rakes was adjustable between runs. Thus, a series of runs was made in order to determine the correct rake position. The individual tubes on each rake were about 3/16 in. apart. The location of each of the jet boundaries on horizontally opposed sides was therefore accomplished within an accuracy of 3/16 in., when one of two adjacent probes indicated a pressure above 1 mm of water while the other probe indicated a pressure below 1 mm of water. This is the definition of the jet boundary given in U. S. Patent 2,741,555. Inclined water manometers, one in. scale, were used to indicate total head pressure.

The total head tubes used to indicate the jet boundary were all located along a straight horizontal path (Fig. 1A). The vertical position of the rake was first established by measuring from the nozzle exit face. Two additional probes equidistant above and below the horizontal plane of the rake were connected to the legs of a "U" tube manometer. When the "U" tube manometer was in balance the rakes were located essentially along the diameter of the jet. The vertical position of the rake was certainly determined to be within less than 3/8 in. of the correct position by this technique. On a four-inch jet an error of 3/8 in. in the vertical position of the rake would introduce an error of 1/16 in. in rake determination of jet diameter. The nozzle and press rake are shown in Figs. 1A and 1B. (sketched)

The total impact force of the jet was measured by mounting a cylinder-piston assembly in the jet. The centerline of the cylinder was essentially colinear with the jet centerline. A flat disc 14 in. in diameter was fastened to the piston shaft at 90° to the jet. The inside diameter of the cylinder was 0.865 in. (area = .588/sq in.). A pressure gauge was directly connected to the closed end of the cylinder. The oil pressure in this closed end of the cylinder times the area of the piston was the only force opposing the impact force. Thus impact force = .588 times gauge reading. The friction of the cylinder piston, evaluated at zero jet-velocity, appeared negligible. Under test conditions, however, some indication of an increase in force was obtained when the disc was manually rotated. This variation of the impact force data is taken account of later. A sketch of this assembly for measuring the total impact force is shown in Fig. 2.

TEST RESULTS

A. JET DIAMETER

After the rake spacing was arrived at, for any particular position relative to the nozzle, three separate runs were made wherein the pressure indicated by the appropriate total head rakes were observed. A typical set of data is shown below:

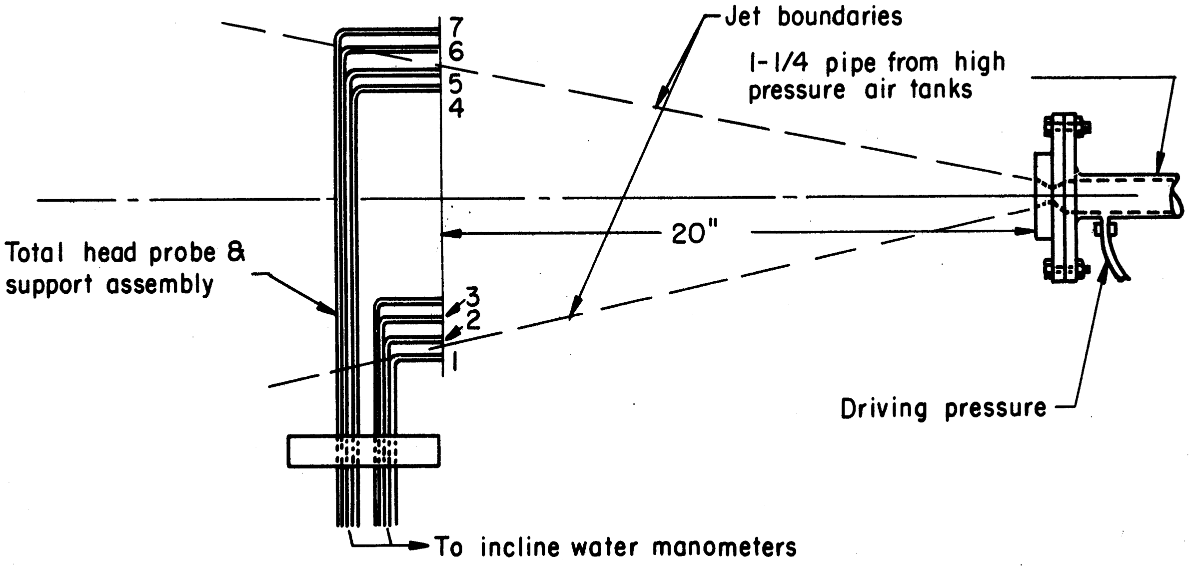

Fig. 1A. Top view of total head rake assembly and nozzle at 20-in. position.

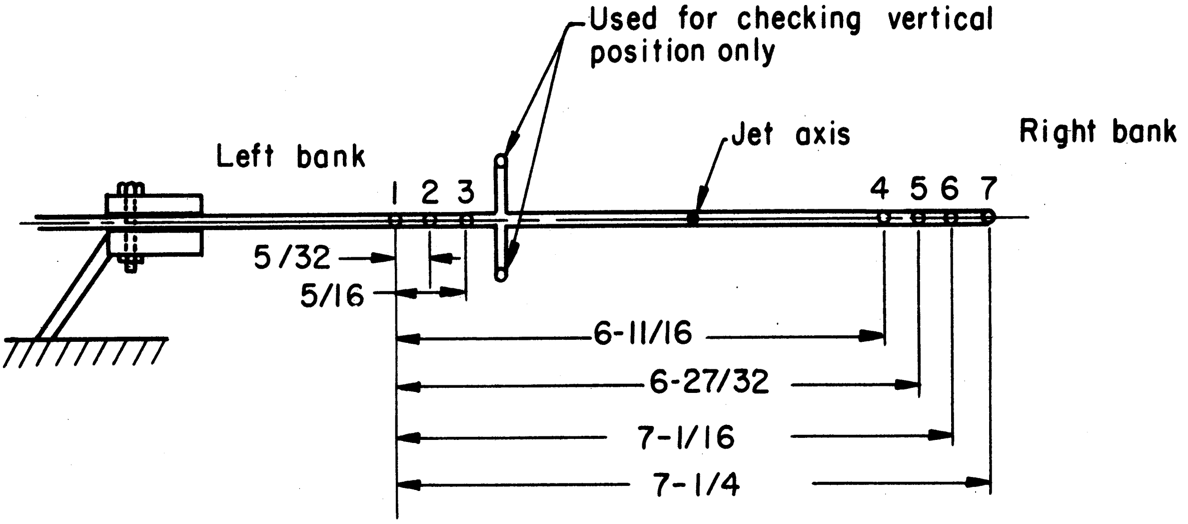

Fig. 1B. End view of total head rake assembly as viewed from the nozzle; the probe spacing is that arrived at for the 20-in. position.

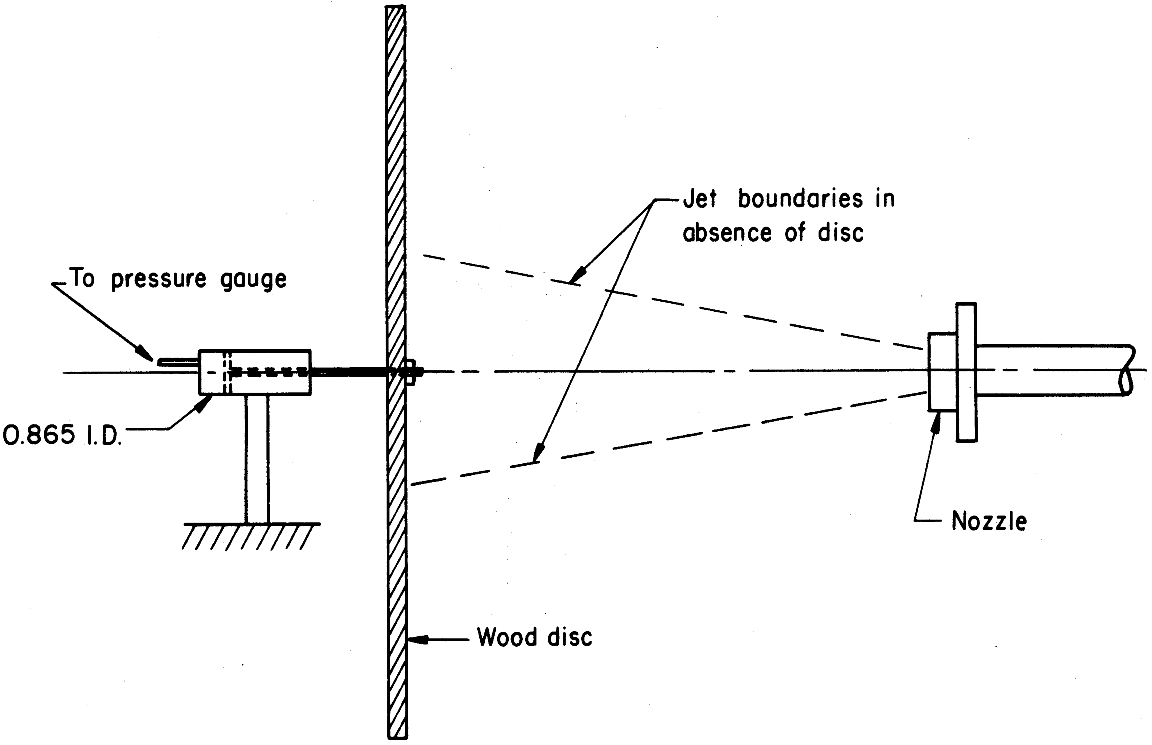

Fig. 2. Sketch of total impact measuring assembly.

Nozzle driving pressure reading—between 51 and 53 psi
Rake position—20 in. downstream of nozzle exit
Individual pitot tube position:

	Tube No.	Distance from No. 1 Tube		Pressure in H_2O (corrected for zero reading)
Left Bank	1	0		-.04
	2	5/32 in.	Minimum Jet Diameter	+.04
	3	5/16 in.		+.10
	4	6-11/16 in.		.11
Right Bank	5	6-27/32 in.		.06
	6	7-1/16 in.		.01
	7	7-1/4 in.		-.02

Thus, for this run the jet boundary lies between tubes 1 and 2 on the left side (facing downstream) and between tubes 5 and 6 on the right side. The jet diameter must be greater than 6-11/16 in. (6-27/32 in.- 5/32 in.) and less than 7-1/16 in. (7-1/16 in. - 0 in.). The minimum value indicated for the diameter was arbitrarily decided upon. Thus, at 26 inches from the nozzle the diameter (average of three tests) was determined to be 6.72 $^{+.375}_{-0}$ in. Similarly, at

13 in. the jet diameter was determined to be $4.48^{+.375}_{-0}$ in.

The velocity gradient near the jet boundary would be expected to increase as the nozzle is approached. Thus, any slight variation in flow conditions would make it difficult to read total heads to within .04 in. of water. This condition was beginning to cause trouble at the 13-in. position. At the 6-in. position the velocity fluctuation near the jet boundary and the relatively simple equipment used made any accurate pitot readings unfeasible. Hence, the jet diameter was measured only at the 13- and 20-in. positions. The nozzle-exit diameter provided a third point.

If we assume that the jet boundary forms a frustum of a right circular cone, at least for the first 20 in., then

D_j = $2x \tan \alpha + .409$;
D_j = local diameter of the jet (in.);
x = distance from the nozzle exit along jet axis (in.);
α = divergent angle of the jet (i.e., half of the total included case-angle)
$.409$ = diameter of nozzle exit (in.).

Operating on this premise and using the value of D_j indicated at x = 20 in., $\tan \alpha$ is found to be .1578 or α is approximately 9°. Now, using this value of $\tan \alpha$ at x = 13 in.,

$$D_j = (26)(.1578) + .409 = 4.51 \text{ in.}$$

as compared with 4.48-in. measured diameter. This would indicate that the jet diameter could be computed with reasonable accuracy for any particular point of interest by the equation

$$D_j = .32x + .41 \text{ in.}$$

B. TOTAL IMPACT FORCE

The total impact force was measured at positions 20 in., 13 in., and 6 in. downstream of the nozzle. Three readings were taken at each position. These data are presented below:

Distance from nozzle, in.	Gauge Reading, psi		
	Run No. 1	Run No. 2	Run No. 3
20	12.25	12.3	12.4
13	13.2	12.75	13.8*
6	12.2	13.9*	13.6*

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*Indicates disc rotated.

The average of all the above readings is 12.93 psi. The maximum variation from this reading is +.97 and -.73 or approximately +7.5% and -5.67%. The widest variation occurred during different runs at one position and is believed to be mainly due to friction effects. Note that whenever the disc was rotated, the pressure readings were consistently high.

Using the average pressure indicated above, the total impact force is 12.93 x .588 = 7.6 lb. Since the theoretical value calculated for total momentum of the jet was 7.602 lb (see discussion above), it is reasonable to conclude that a value of 7.6-lb impact force can be used to calculate impact force at any position of interest here. This close an agreement between test data and theoretical data must be attributed in part to coincidence since an accuracy of 10% would be very good for data of this kind.

The following equation can be used to indicate the average impact pressure:

$$P_i = (\text{impact force} / \text{jet area}) \text{ psi}$$

using the average measured value of impact force (here the same as the theoretical impact force) and the equation indicated previously which describes the jet diameter:

$$P_i = \frac{7.6 \text{ lb}}{(\pi/4)(.32x + .41)^2} = 9.68/(.32x + .41)^2 \text{ psi} \quad .$$

Average impact pressure (P_i) is computed for several positions downstream of the nozzle.

Distance from Nozzle x, in.	K/sq cm	P_i, psi
0	3.92	55.76
4	.24	3.38
12	.038	.54
13	.032	.46
20	.015	.21

The above values were determined for a nozzle driving pressure of 50 psi. Eight atmospheres (one of the driving pressures suggested in British Patent 642,084) driving pressure would produce impact pressures more than double those listed.

C. APPLICATION OF THESE TEST RESULTS TO OXYGEN JETS

All test data were obtained using compressed air which is available in large quantities at this laboratory. The theoretical total exit momentum of oxygen (delivered to the nozzle used for these tests at 50 psi and 60°F) is 7.622 lb as compared to 7.602 lb for air. This is much less than a 1% increase in impact force due to the use of oxygen over air. Thus, for practical purposes the data obtained above can be used interchangeably for air or oxygen.

It is estimated that the true average impact pressure might be appreciably less than the values listed above, but in no case will they exceed the values listed by more than 10%.

CONCLUSIONS

A nozzle can be designed and operated in accordance with conditions specified in British Patent 642,084 which will produce a range of impact pressures well within the range of impact pressures claimed for U. S. Patent 2,741,555.

Such a nozzle has been tested at 50 psi driving pressure. By the use of various driving pressures and nozzle positions relative to the plane of measurement this nozzle could produce a range of impact pressures which would cover most or all of the range of impact pressures claimed in U. S. Patent 2,741,555, all while operating within conditions defined in British Patent 642,084.

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