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HUMID AIR AND SUPERHEATED STEAM AS DRYING MEDIA FOR RIGID POROUS SOLIDS

PROGRESS REPORT NO. 2
ON
SUPERHEATED STEAM IN DRYING

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REVIEWED BY
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ABSTRACT

The relative importance of the various factors involved in the commercial drying of a porous permeable solid are discussed and related to the cost of the operation and the quality of the product.

A basic mechanism and quantitative theory of air or superheated steam drying by direct surface contact is presented, the essential similarity between the two fluids being stressed. Four "periods" of drying are discussed, and fundamental equations are developed to describe the flow of heat, the flow of liquid and vapor phase water, and the equilibrium involved in each period. The present utility of these equations is limited because of the lack of experimental data needed both for the evaluation of coefficients and for the determination of the nature of certain relationships now known only in functional form.

Experimental data made available in the Shuman and Fraser reports are analyzed in the light of the theory developed. These data are also used to develop simplified relations, which, although not accurately descriptive of the drying operation, are believed sufficiently sound for a preliminary study. The simplified equations assume three periods of drying: a constant rate period, followed by two periods in which the rate decreases linearly with the free moisture content of the stock.

A quantitative comparison of the relative merits of humid air and steam as drying media is made. The comparison covers air at various humidities and steam at various pressures, both over a temperature range of 300 to 500°F, the basis chosen being constant power expenditure across the stock and stock (of 1-3/4 inch thickness, 20 pcf density) being dried from both sides.

The analysis shows the effect of the partial pressure of water vapor (related to the humidity in the case of humid air and equal to the pressure in the case of superheated steam) and drying temperature upon the following cost-determining factors:

(a) Drying rate and drying time in each of the drying periods
(b) Total drying time
(c) Heat consumption
(d) Electrical energy consumption

The restrictions of constant power expenditure and stock thickness are then partially removed by a further analysis showing the effect of these variables upon the drying rates. The more complex relationships between power and thickness and the drying times are mentioned but not elaborated.

The influence of partial pressure of water vapor and drying temperature upon the following quality-determining factors is discussed:

(a) Equilibrium moisture content
(b) Drying rate
(c) Temperature and moisture gradients in the stock
(d) Temperature gradients in the drying gas

A final analysis of the composite influence of pressure and temperature upon the cost and quality factors indicates superheated steam to be preferable to air as a drying medium, provided good quality is obtainable at temperatures above 400°F and in drying times of eight hours or less. Experimental data is desirable to substantiate the theory upon which this conclusion is based, and to determine the effect of operating conditions on the quality of the product.

HUMID AIR AND SUPERHEATED STEAM
AS DRYING MEDIA FOR RIGID POROUS SOLIDS

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I. INTRODUCTION

The study reported here has been made for the purposes of developing basic theory and preparing a preliminary evaluation of the merits of superheated steam as an agent for direct-contact drying of a rigid porous insoluble solid. The agents normally considered for this type of drying are moist air or moist flue gases (whose properties approximate those of air). Such agents are employed at or near atmospheric pressure. In the case of superheated steam, however, it is reasonable to consider also operation at pressure above atmospheric. The analysis covers, therefore, a detailed discussion of the basic theory of drying and the relative merits of air at various humidities and steam at various pressures, both over the temperature range of commercial interest.

An evaluation of the "merits" of a drying agent must be preceded by a decision as to what constitutes merit in such a case. In drying, as in most industrial operations, merit is obviously associated with low cost of operation and high quality of product. The cost referred to is the total drying cost per unit of product, which in general may be considered to consist of:

c_c - Interest on capital investment, per unit product

c_d - Insurance, depreciation, obsolescence, taxes, etc., all assumed proportional to the capital investment, per unit product

c_o - Fixed overhead, as management, etc., assumed independent of the capital investment, per unit product

c_l - Direct labor cost, per unit product

c_h - Cost of heat, per unit product

c_p - Cost of electrical energy, per unit product

Costs c_c and c_d, the interest and the insurance, depreciation, etc., are proportional to the cost of installation and inversely proportional to the production capacity. Now, the production capacity of a given dryer unit is inversely proportional to the time required to dry the stock; thus

$$c_c + c_d = k_c C_c \theta \qquad (1)$$

where

k_c = proportionality factor

C_c = total capital investment in dryer

θ = drying time, hours

The variation of the total capital investment (C_c) with drying agent and conditions is hardly predictable at this time. As a first approximation, however, it may be considered that, for installations of the same production capacity and the same degree of refinement, this cost will be related directly to the operating temperature and approximately to the partial pressure of the water vapor* in the drying agent. Thus,

$$C_c \simeq k_c' (a + bT_g)\ (d + eP_g) \qquad (2)$$

where

a, b, d and e are unknown constants

T_g = temperature of the drying gases

P_g = partial pressure of water vapor in the drying gases

Labor cost (c_l) may be taken, for installations of the same production capacity and equivalent refinement, as approximately independent of the drying time; i.e.,

$$c_l \simeq \text{constant}$$

Thus the total unit cost may be represented by

$$C = k_c C_c \theta + c_l + c_o + c_h + c_p \qquad (3)$$

$$C = k_c''(a + bT_g)(d + eP_g)\ \theta + c_l + c_o + c_h + c_p \qquad (4)$$

and the cost determining elements which are influenced by the dry-agent and its conditions are T_g, P_g, θ, c_h and c_p. Low unit cost is therefore favored by:

1. Low operating temperature
2. Low partial pressure of water vapor
3. Short drying time
4. Low heat consumption

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*The partial pressure of water vapor, as will be shown later, determines the humidity of the air in the case of air drying at atmospheric pressure, and the total pressure in the case of superheated steam drying. Thus it serves as a measure of composition in the first instance and as a measure of operating pressure in the second.

5. Low electrical energy consumption

The influence of the drying agent and its partial pressure, temperature conditions upon product quality is not so readily visualized. It may be expected that the important factors influencing quality will be moisture content of the stock*, drying rate, temperature and moisture gradients through the stock, and uniformity of temperature of drying gas. The real influence and importance of such factors must be determined experimentally. For the time being, however, it may be assumed that good quality is favored by:

1. High moisture content of stock
2. Low drying rate
3. Low stock temperature gradients
4. Low stock moisture gradients
5. Low temperature gradient of drying agent parallel to its direction of flow

These factors favoring low cost of operation and high quality of product are therefore of importance in evaluating the relative merits of drying agents and drying conditions. An understanding of the influence of the properties and the partial pressure, temperature conditions of the drying agent upon these "factors of merit" is the next essential step. This may best be done through a study of the relationship between humid air and superheated steam and the mechanism and basic theory of the drying operation.

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*It is believed that the stock is much more liable to shrinkage cracking when below a certain critical range of moisture contents.

II. THE RELATIONSHIP BETWEEN HUMID AIR AND SUPERHEATED STEAM

It is desirable at the outset to clarify the relationship between humid air at atmospheric pressure and superheated steam.

Humid air consists, in "Phase Rule" language, of a two-component system: air and water. The variance of such a system present at constant atmospheric pressure in a single phase is $V_p = C + 1 - P = 2$, where V_p = variance of system at constant pressure, C = number of components, and P = number of phases. Or, in the presence of liquid water, where P = 2; $V_p = 1$. This means that fixing two variables, e.g., temperature and phase composition, completely defines the single-phase system. Similarly, fixing one variable, e.g., temperature, determines the two-phase system. Of the various ways in which phase composition may be represented (i.e., mol fraction, absolute humidity, etc.) it is expedient here to represent it by the partial pressure of water vapor. That this is a true measure of phase composition is evident from the relationships (22):

$$\text{mol fraction} = \frac{P_g}{14.7} \qquad (5)$$

and

$$\text{absolute humidity} = \frac{P_g}{14.7 - P_g}\left(\frac{18.06}{28.97}\right) \text{ lbs/lb} \qquad (6)$$

where P_g is the partial pressure of water vapor, psi.abs. Thus, in accord with the phase rule, the two-phase water-air system at atmospheric pressure may be represented by a single line on a plot of partial pressure of water vapor (P_g) vs. temperature (T_g). This line is the vapor pressure curve for water, designated later as $P_g = f_E (T_g)$. Similarly, the single-phase system may be represented by a family of lines on this same plot, one line for each constant value of $\frac{P_g}{(P_g)_{sat.}} \times 100$; i.e., one line for each value of per-cent relative humidity.

On the other hand, superheated steam is a single-component system, and without restriction of constant pressure, exhibits a variance of $V = C + 2 - P = 2$, where V = variance of system. In the presence of liquid water, or when saturated, $P = 2$ and the variance becomes $V = 1$. Thus, the phase rule predicts similar descriptions of superheated steam and atmospheric pressure humid air. The saturated steam system may be represented by a single line on a plot of P_g vs. T_g, and the superheated system by a family of lines, one for each value of $\frac{P_g}{(P_g)_{sat.}} \times 100$, on the same figure.

Figure 1 is therefore a two-dimensional representation of humid air at atmospheric pressure ($P_g < 14.7$ psi.abs.) and superheated steam ($P_g > 14.7$ psi.abs.). The essential continuity between humid air and superheated steam is clearly brought out. It is to be noted that for $P_g < 14.7$ psi.abs., P_g is the partial pressure of water vapor in the air-water system under a total pressure of 14.7 psi.abs., and is therefore a measure of composition (mol fraction, humidity, etc.). For $P_g > 14.7$ psi.abs., P_g, although still the partial pressure of water vapor, is also the total pressure. The significance of the adiabatic saturation lines shown on Figure 1 will be discussed later.

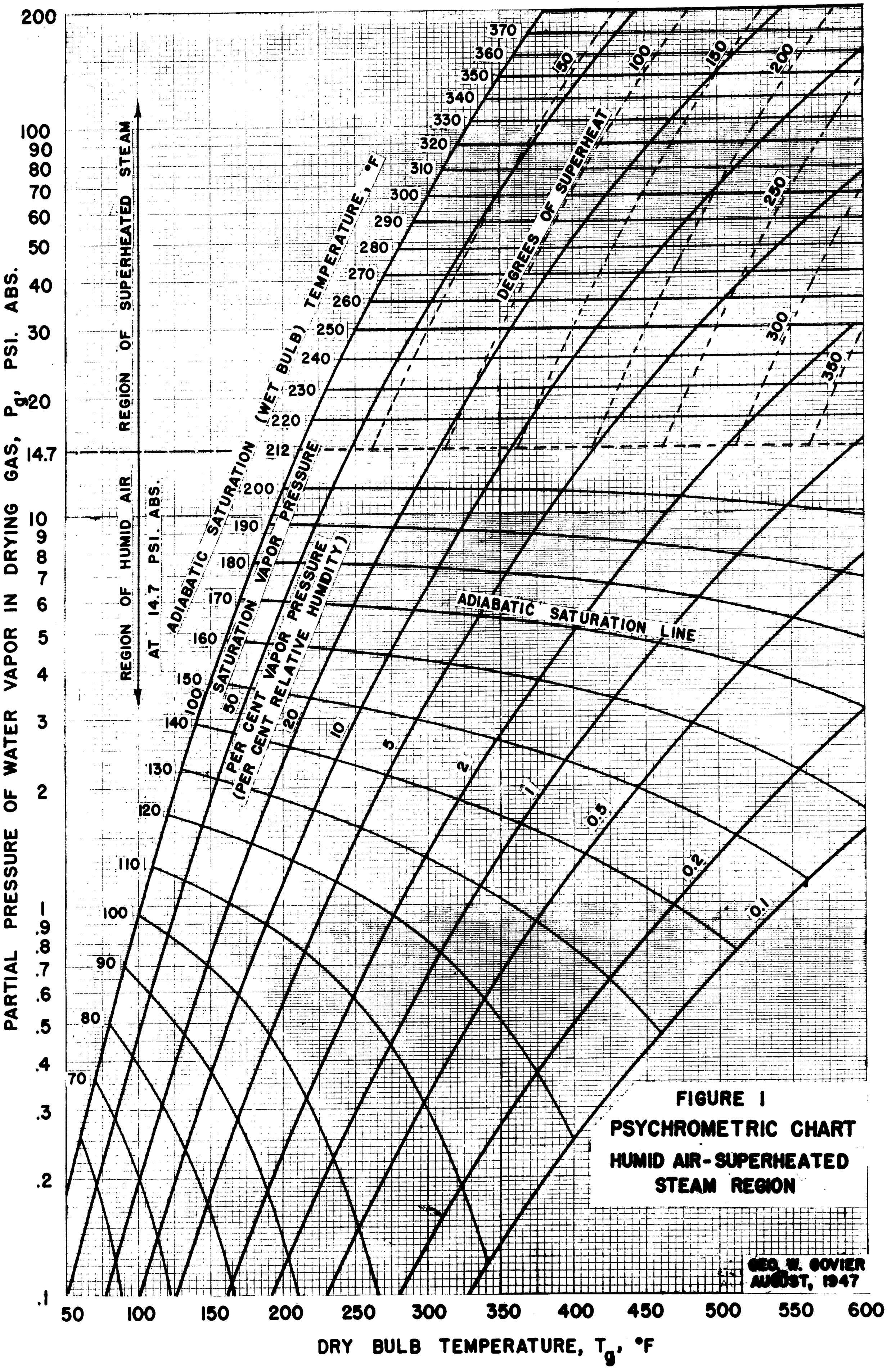

FIGURE I
PSYCHROMETRIC CHART
HUMID AIR-SUPERHEATED
STEAM REGION

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III. BASIC MECHANISM AND THEORY

The type of drying under consideration is that of a rigid, porous, insoluble solid by direct exposure to a gaseous medium which is unsaturated with respect to water vapor. All heat for vaporization, heating of stock, etc. is supplied directly from the gas which is assumed to flow parallel to the surface of the stock in turbulent motion and at constant velocity, temperature, and partial pressure of water vapor.

The literature related to this general problem is fairly extensive, although none too revealing. Most of the published work falls into the following categories:

1. Experimental drying rate data for specific materials; mostly in the form of instantaneous rate of drying per unit of exposed surface as a graphical function of the instantaneous moisture content and the conditions of drying (3)(9)(14).
2. Empirical equations based directly upon experimental data expressing, for specific materials, drying rates in terms of the more important variables (3)(14)(18)(19).
3. Studies of the mechanism of drying (3)(6)(5)(10)(19).
4. General articles dealing with methods of operation, practical considerations, etc. (2)(21)(5).

Information from the first and third categories has proved most helpful in developing the following pictures of the mechanism of drying for the particular problem at hand.

It is now well established (3)(6)(22) that under constant drying conditions (i.e., constant temperature, water vapor pressure, and flow velocity) several distinct periods of drying are observed as the moisture content of a solid is reduced from a high initial to a low final value. These periods are best distinguished through a study of a typical drying curve. Figure 2a is such a curve, representing the instantaneous rate of drying per unit of exposed surface as a function of the instantaneous moisture content. In this figure:

W_0 = initial water content, pounds of water per pound of dry stock

W_1 = water content at beginning of Period I, lb/lb

W_2 = water content at beginning of Period II, lb/lb

W_3 = water content at beginning of Period III, lb/lb

W_4 = final water content, lb/lb

W_5 = water content corresponding to zero drying rate

R_0, etc. = instantaneous drying rates, lbs/ft^{2}hr at corresponding water contents

It is to be noted that the drying operation divides itself naturally into four periods as follows:

Period I^0: An initial stage wherein the drying rate may increase (or decrease) rapidly from R_0 (or $R_0{}'$) to $R_1 = R_2$. This period is usually short and may be unobserved.

Period I: An early stage during which the drying rate remains substantially constant.

Period II: A stage immediately following Period I during which the drying rate decreases, more or less uniformly, with continued decrease in water content.

Period III: A stage immediately following Period II, but not always clearly distinguished from it, during which the drying rate decreases, also approximately linearly with the moisture content but in general at a different rate than in Period II.

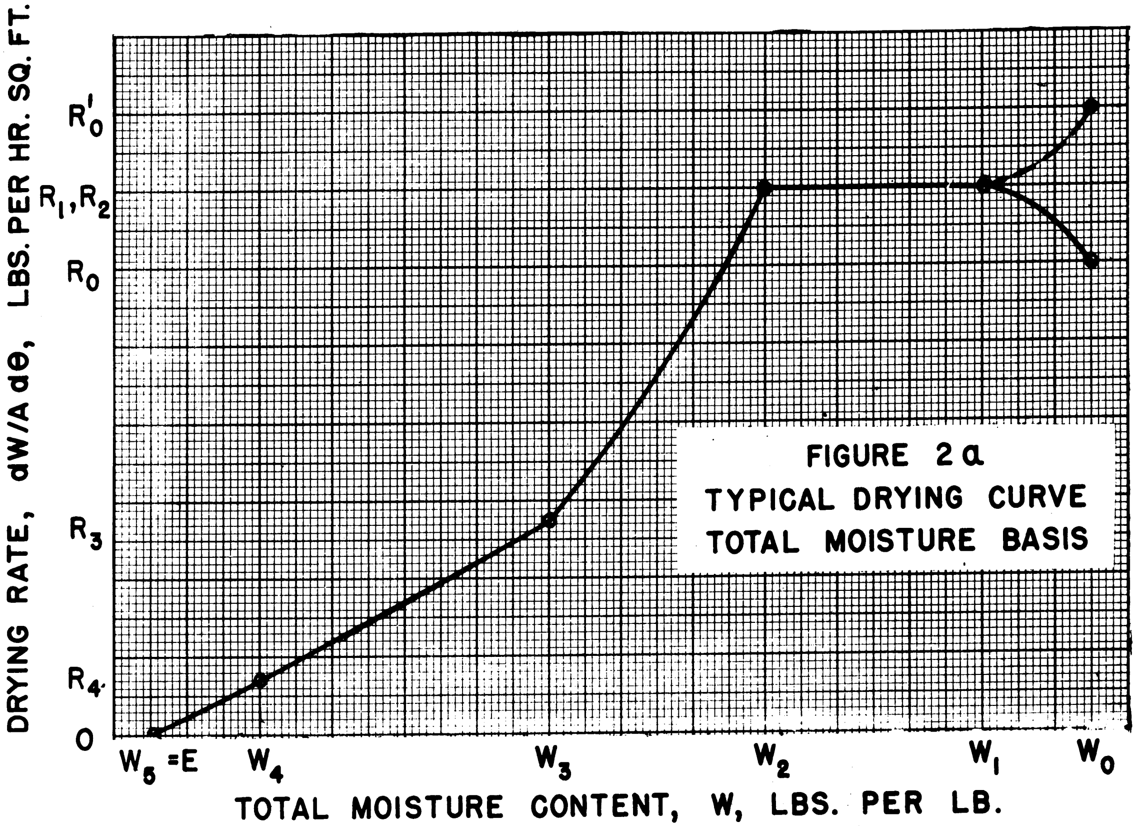

FIGURE 2 a
TYPICAL DRYING CURVE
TOTAL MOISTURE BASIS

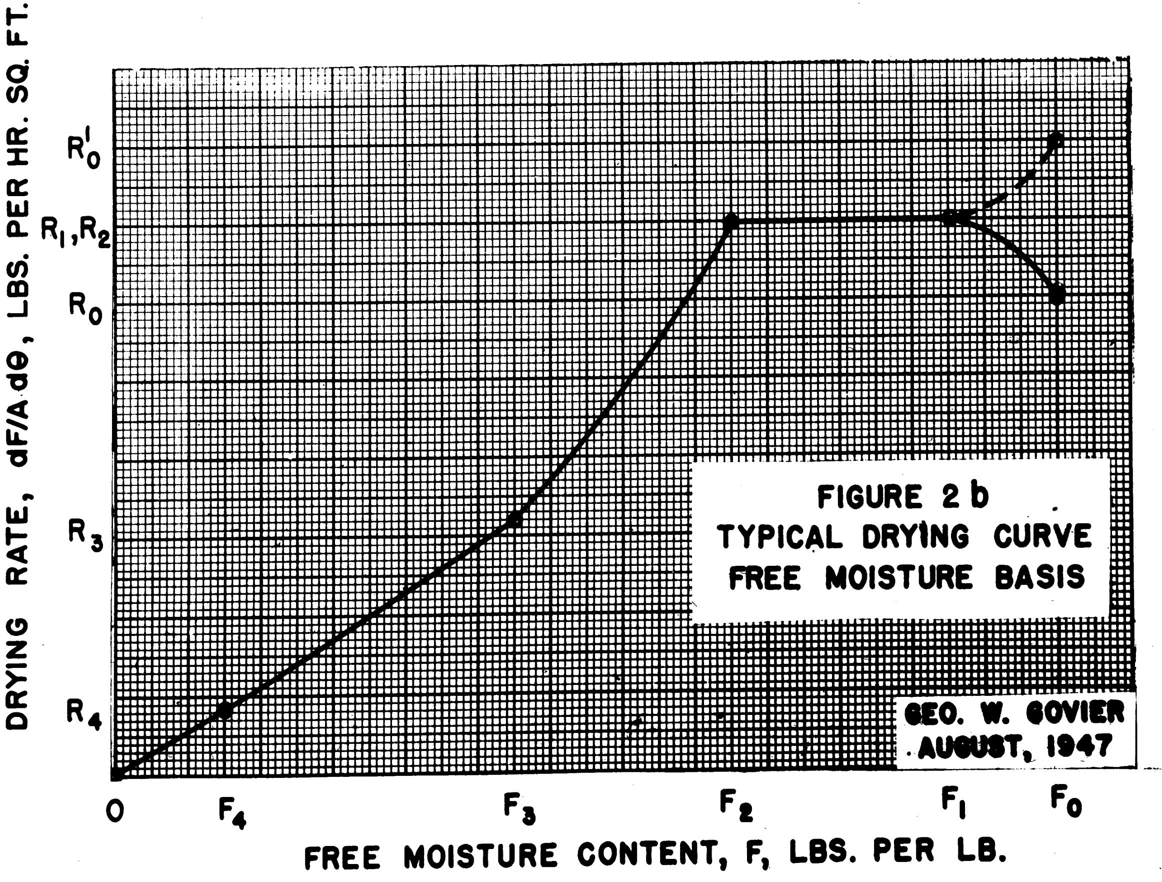

FIGURE 2 b
TYPICAL DRYING CURVE
FREE MOISTURE BASIS

On prolonged drying, Period III is terminated at W_5, where the drying rate becomes equal to zero. W_5 is known as the equilibrium moisture content of the material (22), designated as E, and represents the minimum moisture content attainable under the conditions of drying. This equilibrium moisture content is determined by the nature of the stock being dried, the temperature of the drying gases, and the partial pressure of water vapor in the drying gases; i.e.,

$$E = f\ (\text{stock}, T_g, P_g) \qquad (7)$$

The nature of this relationship, even for a given stock, is complex and must be determined experimentally, but it is known generally that E decreases rapidly with increasing T_g and increases with increasing P_g (22). The difficulties associated with the evaluation of E may be circumvented <u>to some extent</u> through the definition of $F = W - E$ where F is the so-called free water content of the stock in lbs/lb. This means, of course, that for a given total moisture content W, the free moisture content F will depend upon the factors controlling E. Usually, however, E is small compared with W, so that the variation of free moisture content with P_g and T_g at constant total moisture content is reasonably small. It should be noted that at given conditions of pressure (P_g) and temperature (T_g) and with a given stock,

$$E = \text{constant}$$

and $$\frac{dW}{Ad\theta} = \frac{dF}{Ad\theta} = \text{drying rate, lbs/ft}^2\text{hr}$$

Figure 2b shows the drying rate curve of Figure 2a reported in terms of F, the free water content. Further discussion of drying rates will be based upon free water contents.

The mechanism of drying and the descriptive mathematical relations are different in each of the different periods. In the following discussion of these factors, it is convenient, therefore, to consider each period separately.

Period I^o - Initial Period:

This is essentially a period of unsteady state operation during which conditions are adjusting themselves to the steady state represented by Period I.

Assuming that the stock is initially cold, i.e., below the adiabatic saturation temperature of the gas,* the mechanism is believed to be as follows (22). Heat is transferred to the cool wet surface of the stock from the hot gases. This heat supplies preheat and latent heat of evaporation to vaporizing water and also serves to increase the surface (and the average) temperature of the stock. Simultaneously evaporation is taking place at a rate controlled by the instantaneous difference in vapor pressure of water at the surface and in the drying gases. As this situation continues the surface temperature increase causes a corresponding increase in the rate of evaporation and a decrease in the rate of heat supply. After a short time the decreased heat supply just balances the increased heat requirement for evaporation and conditions become steady. This is the end of the initial period.

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*See Period I.

With the stock initially hot, i.e., above the adiabatic saturation temperature of the gas, the mechanism is similar but the latent heat requirement exceeds the heat supply from the gas, causing a decrease in the surface (and the average stock) temperatures, ultimately to the same steady state.

Under special conditions where the initial temperature of the stock is at the adiabatic saturation point, no initial period is observed.

The important physical phenomena of this period are:

1. Heat transfer from the drying gas to the stock surface.
2. Heat transfer from the stock surface to its interior.
3. Transfer of water vapor from the stock surface into the drying gas.
4. Thermal and pressure equilibrium established at the drying interface.

The mathematical relationships describing these phenomena and the balance of heat energy are presented and discussed in the Appendix.

Period I - Constant Rate Period:

This period, which starts at the free moisture content F_1 and ends at F_2, is characterized by a uniform rate of drying and constancy of surface and interior temperatures. It is the steady state reached at the end of Period I^o.

The period continues so long as water is supplied to the surface as rapidly as evaporation can take place (22). Up until this point, no reference has been made to the mechanism of water transfer from the stock interior. During Period I^o this mechanism is of little importance, since it has no significant effect upon the drying rate or other factors. In Period I, however, the mechanism of water transfer from the stock interior is important, since it determines the duration of the period (3)(6).

It is now believed established that in the drying of porous, insoluble solids, the mechanism of liquid water transfer from the interior to the surface is by capillary action (3)(6)(22). The origin of the capillary forces is discussed by Comings and Sherwood (6) and again by Ceaglske and Hougen (3). It is assumed that surface menisci of small radii exert sufficient capillary pull to draw water through intricate interior passages ending in gas-water interfaces of larger radii.

To quote directly from Comings and Sherwood (6):

> "The water drawn to the surface is necessarily replaced by air which enters the solid through the larger passages connected with the larger openings at the surface. Because of the complicated interconnecting passages beneath the surface, it is possible for the necessary air to enter through a relatively few surface openings and thus for the moisture concentration near the surface to remain relatively high... The water will continue to rise to the surface through any system of interconnecting passages until all of the various menisci at the lower ends of the water column have the same radius of curvature as the small menisci at the surface from which evaporation is taking place. When this stage is reached, a small amount of evaporation from the surface menisci may result in a retreat of these surface menisci into passages of smaller cross section, and the increased capillary tension is sufficient to draw additional water to the surface. It is possible, in fact, for the increased tension caused by this retreat of the surface menisci to draw some of the menisci at the lower ends of the water column through the narrow constrictions into larger cavities and thus reduce the tension necessary to cause movement toward the surface. The menisci in the passages at the surface can then rise to the former position and the process can continue.
>
> "As the drying process proceeds, a time will be reached when the menisci at the lower ends of the water column in any system of interconnecting passages, are, in general, about the same size as the smallest cross section of

the surface openings, and water will not longer be drawn to the surface through these passages."

Continued evaporation then results in the depletion of the surface moisture, a decrease in drying rate, and the end of Period I.

The important physical phenomena of Period I are:

1. Heat transfer from the drying gas to the stock surface.
2. Transfer of liquid water from the stock interior to its surface.
3. Transfer of water vapor from the stock surface into the drying gas.
4. Thermal and pressure equilibrium established at the drying interface.

The mathematical equations describing these processes, and the balances of heat energy and water, are presented and discussed in the Appendix.

It is also pointed out in the Appendix that F_2, the first critical moisture content, depends upon the nature of the stock, the drying rate in Period I, and the stock thickness. This may be expressed functionally by the relationship:

$$F_2 = f_2\left(\text{stock}, \frac{dF}{Ad\theta}, X_o\right) \tag{22}$$

It is important to note that this critical moisture content is not a "property" of the stock being dried, but that it is dependent also upon the drying rate and the stock thickness.

The constant surface temperature obtained during this period is shown (Appendix) to be equivalent to the wet bulb temperature in the case of humid air and to the saturation temperature in the case of superheated steam. Since in the case of humid air the wet bulb temperature is approximately equal to the adiabatic saturation temperature (22), it is convenient to refer to the constant surface temperature as the adiabatic saturation temperature without distinction between humid air and superheated steam.

Period II - The First Falling Rate Period:

This period starts at the free moisture content F_2 and ends at F_3. It is characterized by a decreasing rate of evaporation which results from the spot-wise recession of the evaporation surface into the first layers of the stock, with the consequent exposure of small radii of curvature (6). The momentarily undiminished heat supply causes an increase in the temperature at the receded zone of evaporation. This temperature approaches a changing equilibrium value determined primarily by the radii of curvature of the menisci slightly below the surface. As drying proceeds, the fractional area accounted for by the receded water menisci increases to unity and the fraction of "wetted surface" decreases to zero. At this time all evaporation becomes subsurface and Period II ends.

The important physical phenomena of this period are:

1. Heat transfer from the drying gases to the stock surface.
2. Heat transfer from the stock surface to its interior.
3. Transfer of liquid water from the stock interior to its surface.
4. Transfer of water vapor from the stock surface into the drying gas.
5. Thermal and pressure equilibrium established at the drying interface at the stock surface.

6. Thermal and pressure equilibrium established at the drying interface at the receded menisci.

The mathematical expressions describing these phenomena and the heat and water balances are given and discussed in the Appendix.

It is of interest to note here that the second critical moisture content, like the first, is not a "property" of the stock but is dependent also upon drying rate and stock thickness; i.e.,

$$F_3 = f_3 \left(\text{stock}, \frac{dF}{Ad\theta}, X_0\right) \tag{35}$$

Period III - The Second Falling Rate Period:

The second falling rate period starts at F_3 [when capillary flow to the surface has ceased (3)(6)] and continues, under prolonged time, to $F = 0$ when the stock is at its equilibrium moisture content E. This period is characterized throughout by subsurface evaporation from a continuously receding plane and by the necessity for the heat for evaporation to penetrate increasing thickness of partially-dried stock. The surface of the stock approaches, but does not reach, the temperature of the drying gases, T_g. The temperature at the receding plane of evaporation approaches a changing equilibrium value determined primarily by the radii of the exposed menisci.

The important physical phenomena of this period may be summarized as follows:

1. Heat transfer from the drying gas through the surface "film" and the partially-dried stock to the receded zone of evaporation.

2. Transfer of water vapor from the receded plane of evaporation through the partially-dried stock and the surface "film" into the drying gas.

3. Thermal and pressure equilibrium at the receded drying interface.

The mathematical relations describing these phenomena and the heat balance are presented and discussed in the Appendix.

Over-all Process:

The over-all drying process may include all periods or only parts of one or more, depending upon the initial and final moisture contents. A complete understanding of the process is best obtained in terms of the equations in the Appendix (or their counterparts) for the periods involved. These equations are useful in clarifying the mechanism of the operation and in bringing to the fore those factors of importance in any experimental study which is undertaken. Moreover, they suggest the proper way in which the experimental variables should be correlated.

As an immediate practical matter, however, the direct use of these particular equations is difficult. This is because many of them are at present known only in functional form and because of the limited knowledge of important properties and coefficients.

IV. APPROXIMATE PRACTICAL RELATIONS

The basic theory previously discussed is mathematically complex and impossible of direct quantitative application in the light of the present limited knowledge of the nature of certain relations and the lack of experimental data. Data made available in the Shuman (20) and Fraser (7) reports have been studied carefully with a view toward the simplification and modification of the basic theory into a tool of immediate practical utility.

It has been possible to develop a set of "practical" relations which, although preliminary and approximate at best, should serve to permit an intelligent comparison of air and superheated steam as drying media. The equations for the various periods of drying follow.

General:

$E = f(\text{stock}, P_g, T_g)$: insufficient data are available to permit any reasonable estimate of the effect of P_g and T_g upon E, although for the tests reported, E is close to and may be assumed equal to zero.

$F = W - E \simeq W$ for the tests reported.

where

E = equilibrium moisture content, lb/lb

F = free moisture content, lb/lb

P_g = partial pressure of water vapor in the drying gases, psi.abs.

T_g = temperature of the drying gases, °F

W = total moisture content, lb/lb

Period I^{o}:

This period is unobserved and may be neglected.

Period I:

$$F_1 = 2.20 \text{ lb/lb}$$

$$F_2 = 0.95 + 2X_o + 0.50R_I \text{ lb/lb} \qquad (42)$$

$$R_1 = R_2 = \frac{h_c}{\lambda_{T_s'}}(T_g - T_s') \text{ lb/ft}^2\text{hr} \qquad (43)$$

where

F_1 = initial moisture content, lb/lb

F_2 = first critical moisture content, lb/lb

X_o = thickness of stock drying from one side, ft

= one-half thickness of stock drying from both sides, ft

$R_1 = R_2 = \left(\frac{dF}{Ad\theta}\right)_I$ = rate of drying in the constant rate period, lbs/ft^{2}hr

T_s' = adiabatic saturation temperature of drying gases, °F (See Figure 1)

$\lambda_{T_s'}$ = latent heat of water, BTU/lb, at T_s'

h_c = film coefficient of heat transfer for combined conduction and convection, BTU/ft^{2}hr°F

Period II:

The relationship between drying rate, $\frac{dF}{Ad\theta}$, and free moisture content, F, may be assumed linear within this period.

$$F_2 = 0.95 + 2X_o + 0.50R_2 \quad \text{lb/lb} \qquad (42)$$

$$F_3 = \frac{F_2}{1.70} = 0.59\ F_2 \quad \text{lb/lb} \qquad (44)$$

$$R_2 = \frac{h_c}{\lambda_{T_s'}} (T_g - T_s') \quad \text{lb/ft}^2\text{hr} \qquad (43)$$

$$R_3 = \frac{h_c}{\lambda_{T_s''}} (T_g - T''_{s,r}) \quad \text{lb/ft}^2\text{hr} \qquad (45)$$

$$T''_{s,r} = \frac{1}{f_E} (P_g + \Delta P_r) \quad {}^\circ\text{F} \qquad (46)$$

where

F_3 = second critical moisture content, lb/lb

$R_3 = \left(\frac{dF}{Ad\theta}\right)_3$ = rate of drying at F_3, lb/ft^{2}hr

ΔP_r = vapor pressure depression due to curvature of exposed menisci, psi.abs.

$\frac{1}{f_E}$ denotes equilibrium relationship

$T''_{s,r}$ = temperature of surface at F_3, °F

Period III:

The relationship between drying rate, $\frac{dF}{Ad\theta}$, and free moisture content, F, may be assumed linear within this period down to values of F of the order of 0.10.

F_3 = .59 F_2 lb/lb

F_4 = 0.10 lb/lb

$$R_3 = \frac{h_c}{\lambda_{T_s''}} (T_g - T_x'') \quad lb/ft^2hr \qquad (45)$$

$$R_4 = \frac{1}{\lambda_{T_x'''} \left(\frac{1}{h_c} + \frac{0.65X_o}{K_E} \right)} (T_g - T_x''') \quad lb/ft^2hr \qquad (47)$$

where

$$T_x''' = \frac{1}{f_E} (P_g + \Delta P_r) = T_{s,r}'' \quad °F \qquad (48)$$

F_4 = final moisture content, arbitrarily assumed

= 0.10 lb/lb

R_4 = $\left(\frac{dF}{Ad\theta}\right)_4$ = rate of drying at $F = \frac{F_3}{8}$ which is assumed close to $F_4 = 0.10$, lbs/ft^{2}hr

K_E = average thermal conductivity of the stock at its equilibrium moisture content, BTU/ft hr°F

These equations embody a large number of assumptions and must therefore be used with extreme caution. They represent satisfactorily the bulk of the present experimental data, but may be in error when applied to other conditions. They are not applicable to stock densities other than 20 pcf, although similar equations may be expected to hold for other densities. It is believed that these "practical" relations are sufficiently sound for use in a study of the relative merits of different drying media and conditions of operation.

V. EFFECT OF TEMPERATURE, PRESSURE, AND NATURE OF DRYING MEDIUM ON COST-DETERMINING FACTORS

As was discussed earlier, the relative merits of humid air and superheated steam as drying media depend upon those factors which influence the ultimate cost and quality of the product. Since the factors influencing cost are subject to a more quantitative treatment, they will be analyzed first. It will be recalled that the cost-determining elements which are influenced by the drying agent and its conditions are: Temperature (T_g), Partial Pressure of Water Vapor (P_g), Drying Time (θ), Cost of Heat (c_h), and Cost of Electrical Energy (c_p). The first two factors, Temperature and Pressure, not only have their own direct influence on cost, but also exert an indirect effect through the remaining items of Time, Heat, and Electrical Energy. It therefore seems advisable to select Temperature and Pressure as the independent variables and to determine the effect of these factors on Time, Heat and Electrical Energy.

Now in a study of this sort, it is necessary to select some common basis for comparison; i.e., if drying times are to be compared at two different temperatures, it is necessary that the values of the other important variables be known, and preferably, that they be the same for each case, in order that the effect of temperature alone upon drying time is determined. For this analysis the following conditions are chosen to define the constant basis of the comparison:

1. Stock density: 20 lbs/ft^3
2. Stock to be dried from both sides
3. Stock thickness: 1-3/4 in
4. Constant temperature of drying gases throughout all periods of the drying process
5. Constant partial pressure of water vapor in the drying gases throughout all periods of the drying process
6. Constant expenditure of power for circulation of the drying gases. This power expenditure is assumed constant, not only throughout all periods of drying, but <u>in all conditions under consideration</u>.*
7. Drying gases flowing in turbulent motion

On this basis, then, the influence of the temperature of the drying gases and the partial pressure of water vapor will be determined for the drying time, the total heat consumption, and the total electrical energy consumption.

An analysis of the influence of change in stock thickness and power expenditure is then presented, in order, partially, to remove the restrictions imposed by the arbitrarily-selected basis for comparison.

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*The power expenditure over the drying surface is arbitrarily taken as that required to realize a film coefficient of heat transfer, for steam at 30 psi.abs. and 400°F, of $h_c = 3.15$ BTU/ft^{2}hr°F. With steam at these conditions, this coefficient corresponds with a lineal velocity of approximately 550 fpm and a rate of drying in the constant rate period of 0.500 lbs/ft^{2}hr. The actual value of the power requirement, per unit of drying surface, is dependent upon spatial arrangement, surface roughness, and other factors not considered in this report; its order of magnitude, however, is 0.06 watts per square foot.

The Drying Time:

The influence of temperature, T_g, and pressure, P_g, upon the drying time is of course complicated by the several periods of drying. A detailed analysis of this problem has been carried out, based upon the equations of the previous section. This analysis involves, as a preliminary step, the expression of the film coefficient for heat transfer, h_c, in terms of the power expenditure per unit of drying surface and the properties of the drying medium. The method of Parsons and Gaffney (17) was used in order to arrive at the equation:

$$h_c = B\phi_t E_s^{0.286} \qquad (49)$$

where

B = constant depending upon width and spacing of the stock

$\phi_t = \dfrac{C^{0.4} \; k_g^{0.6} \rho^{0.571}}{\mu^{0.457}}$, the Parsons and Gaffney (17) "ϕ_t function" dependent upon fluid properties.

C = specific heat of drying gas, BTU/lb°F

k_g = thermal conductivity of drying gas, BTU/ft hr°F

ρ = density of drying gas, $\mathrm{lbs/ft^3}$

μ = viscosity of drying gas, lbs/ft hr

E_s = power expenditure per unit drying surface, ft lbs/hr $\mathrm{ft^2}$

With E_s then chosen as a constant (and equal to the value necessary to realize an h_c of 3.15 $\mathrm{BTU/ft^2 hr°F}$ with steam at 30.0 psi.abs. and 400°F), the influence of temperature, T_g, and pressure, P_g, upon the drying rate and drying time may be determined with the aid of the practical equations of the previous section.

Period I - Constant Rate: The results of the calculations for this period are summarized in Figures 3 and 4.

Figure 3 shows the net influence of temperature, T_g, and partial pressure of water vapor, P_g, upon the drying rate in this period. The following points are to be noted:

1. There is a continuous graduation in effect ranging from humid air (P_g = 1.5, 3.0 and 6.0) to superheated steam (P_g = 14.7, 30, 60 and 125).
2. At temperatures below about 375°F, the use of humid air results in higher rates, whereas above about 400°F the use of superheated steam gives higher rates.
3. The effect of temperature upon drying rate is more marked at the higher P_g values and is particularly marked in the superheated steam region when P_g is greater than 30 psi. abs.
4. In the case of superheated steam there is a reversal in the effect of pressure at temperatures in the general neighborhood of 350°F. At the lower temperatures, increase in pressure results in a decrease in drying rate; whereas, at the higher temperature levels, increase in pressure results in an increase in drying rate.

Figure 4 shows the effect of temperature, T_g, and pressure P_g, upon the drying time in Period I. This reflects the influence of T_g and P_g upon both the drying rate (as in Figure 3) and the first critical moisture content. The curves indicate several interesting features:

1. At 300°F humid air results in decidedly shorter drying times than superheated steam; at 400°F superheated steam is slightly superior; at 500°F superheated steam is markedly superior.
2. At each temperature, the drying time increases sharply as the pressure is increased to a point approaching the adiabatic saturation value. Actually these times

become infinite at $P_g = 67$ psi.abs. for $T_g = 300°F$, $P_g = 247$ psi.abs. for $T_g = 400°F$, and $P_g \cong 681$ psi.abs. for $T_g = 500°F$.

3. A point of inflection (a maximum) exists at each temperature at a P_g value of 14.7 psi.abs. This reflects a slight discontinuity in the change of physical properties through humid air to superheated steam.

4. A second point of inflection (a mimimum) exists at each temperature at pressures beyond 14.7 psi.abs. This point indicates, at each temperature, the pressure corresponding with the shortest drying time in the superheated steam region. These pressures may be considered the optimum values at the corresponding temperatures. The dotted line reflects the trend of these optimum pressures toward higher values at higher temperatures.

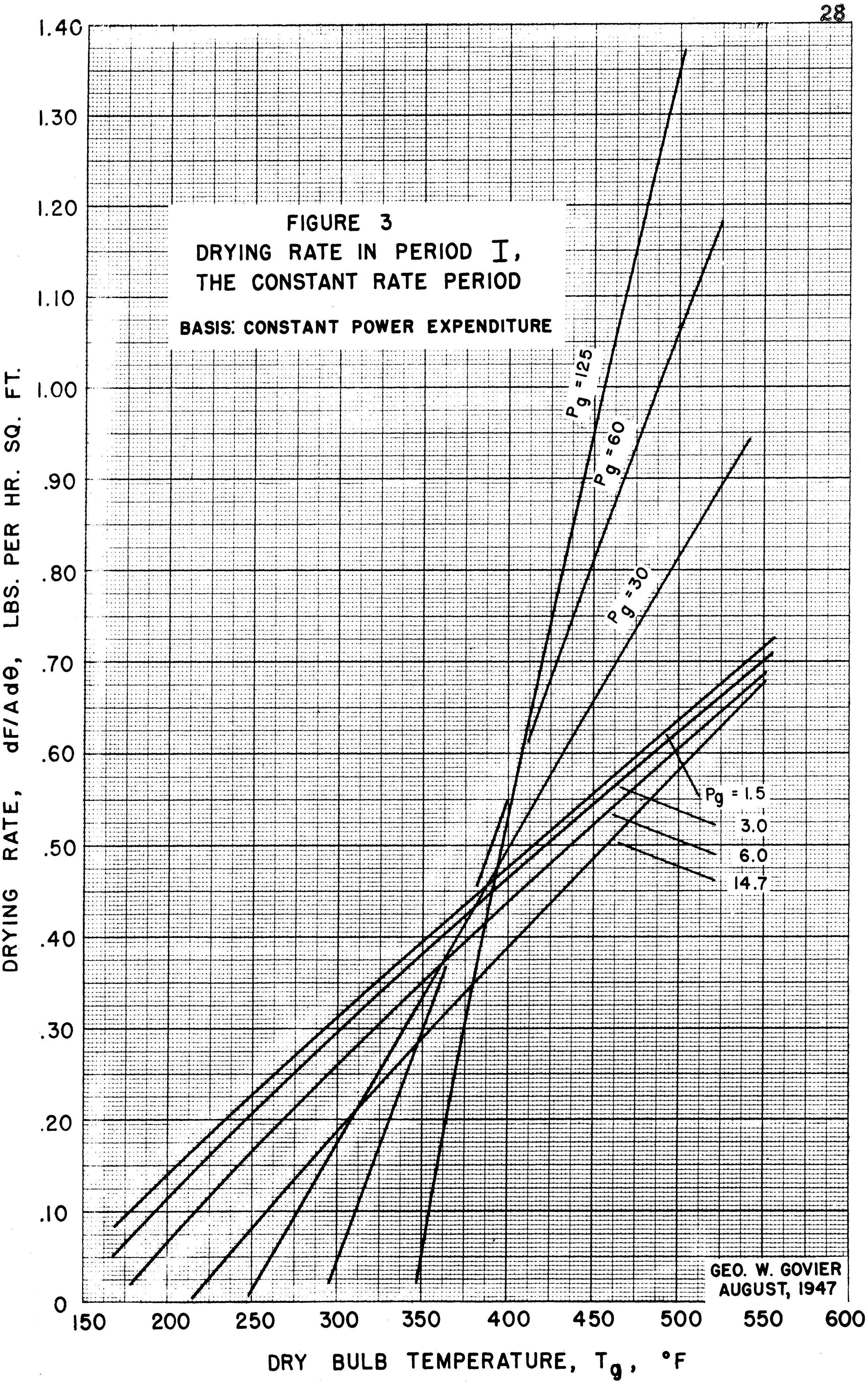

FIGURE 3
DRYING RATE IN PERIOD I,
THE CONSTANT RATE PERIOD

BASIS: CONSTANT POWER EXPENDITURE

GEO. W. GOVIER
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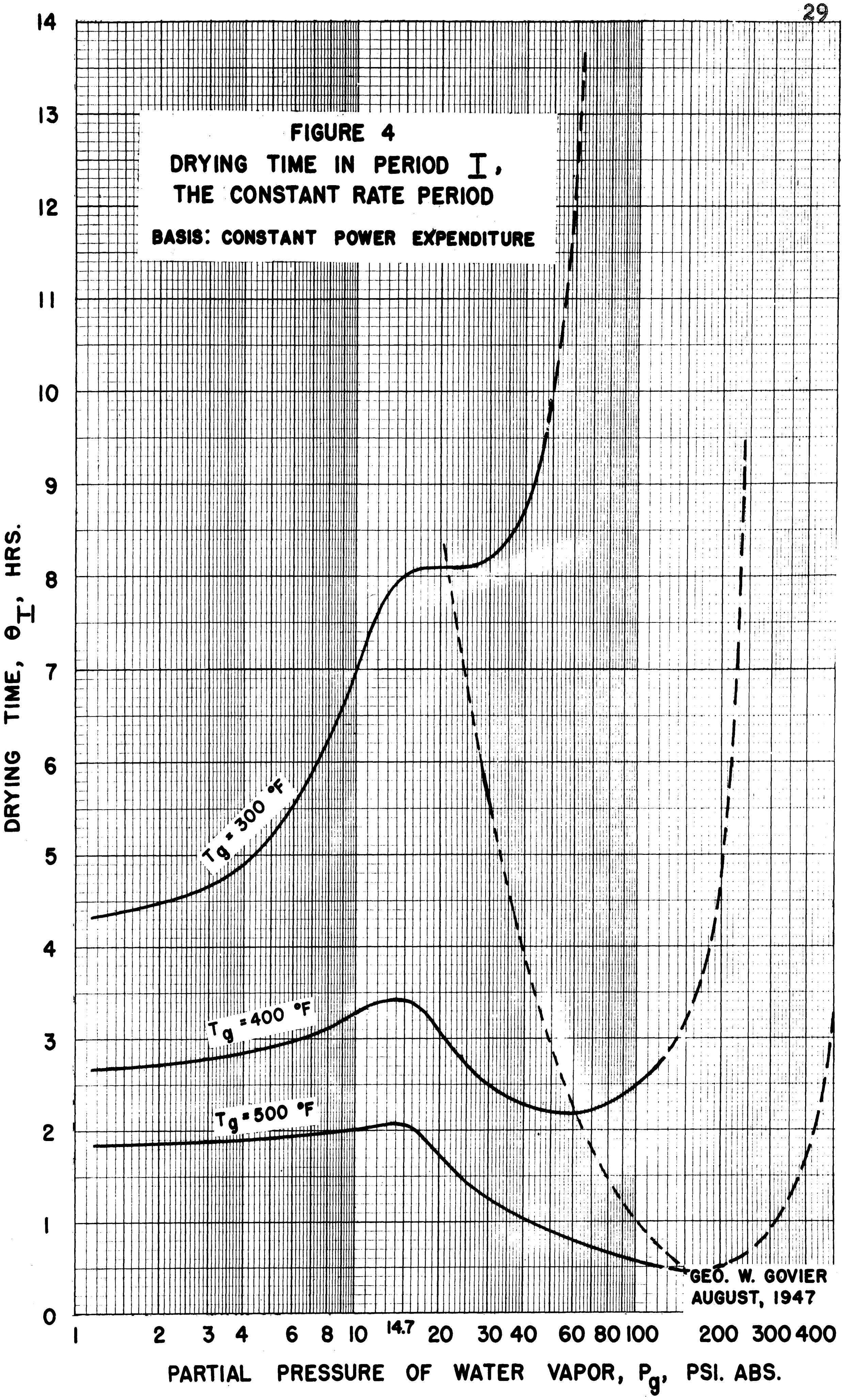

FIGURE 4
DRYING TIME IN PERIOD I, THE CONSTANT RATE PERIOD

BASIS: CONSTANT POWER EXPENDITURE

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Period II - First Falling Rate: The calculations for this period may be summarized by Figures 5 and 6, showing the effect of temperature, T_g, and pressure, P_g, upon the drying rates and times, respectively.

Since the drying rate in Period II decreases constantly throughout the period, the logarithmic mean rate* for the period is plotted in Figure 5. The general trend of this mean rate with change in T_g and P_g is seen to be similar to that of Period I, although the rates are of course much smaller. The extreme importance of temperature and the reversal in the effect of pressure are again evident.

Figure 6 indicates the influence of temperature, T_g, and pressure, P_g, upon drying time; the effect upon both drying rate (as in Figure 5) and first and second critical moisture contents being considered. Features similar to those of Figure 4 are exhibited: the superiority of humid air at low temperatures, the rapid increase in drying time as the pressure is increased to the equilibrium value**, and the two points of inflection. As in Figure 4 the dotted line indicates the trend of the optimum pressure toward higher values at higher temperatures.

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* It may be shown that this is the proper mean when the rate is linear with F.

**This value is less than the adiabatic saturation value by the amount of the depression due to the curvature of the menisci.

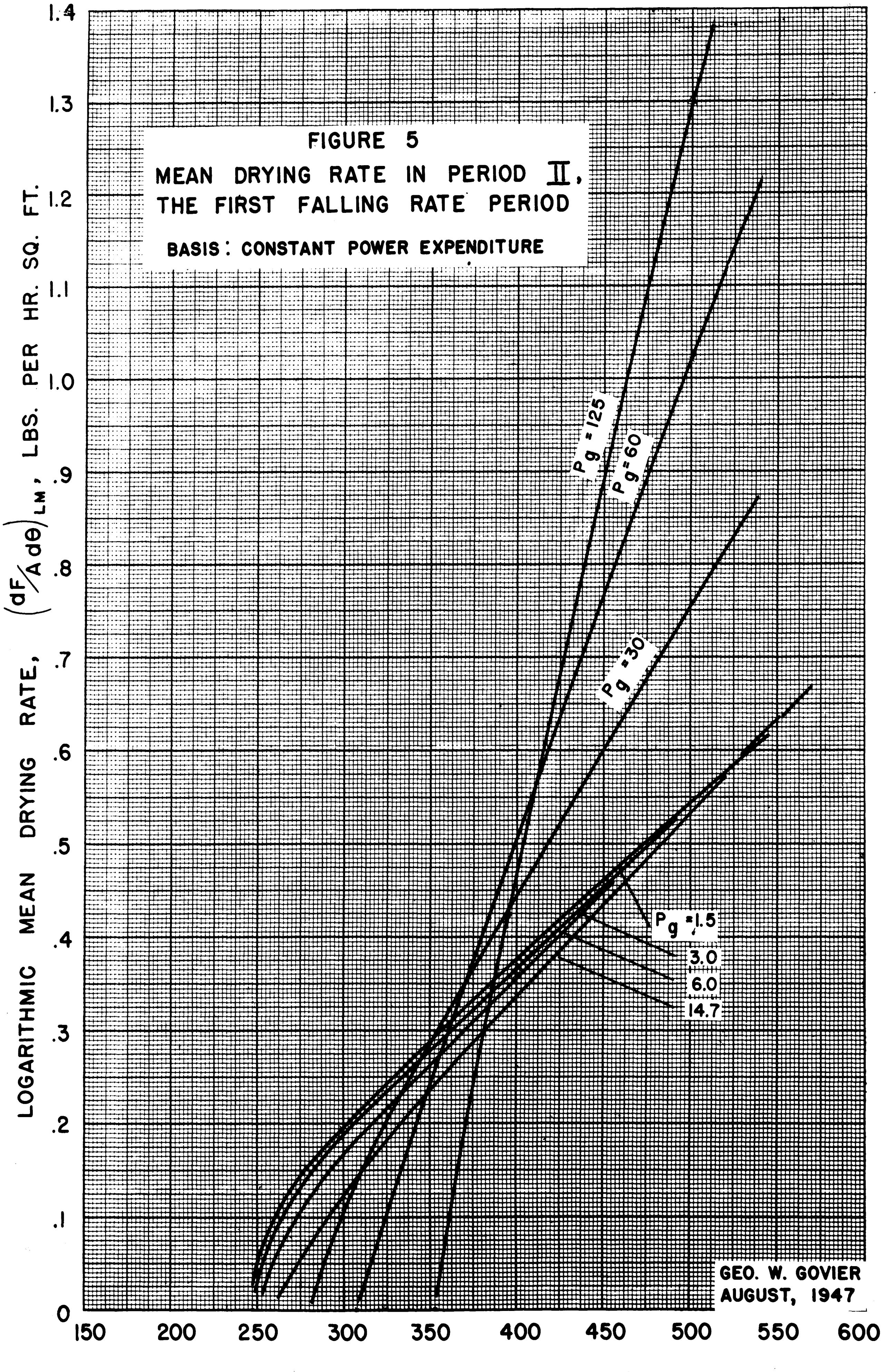

FIGURE 5

MEAN DRYING RATE IN PERIOD II, THE FIRST FALLING RATE PERIOD

BASIS: CONSTANT POWER EXPENDITURE

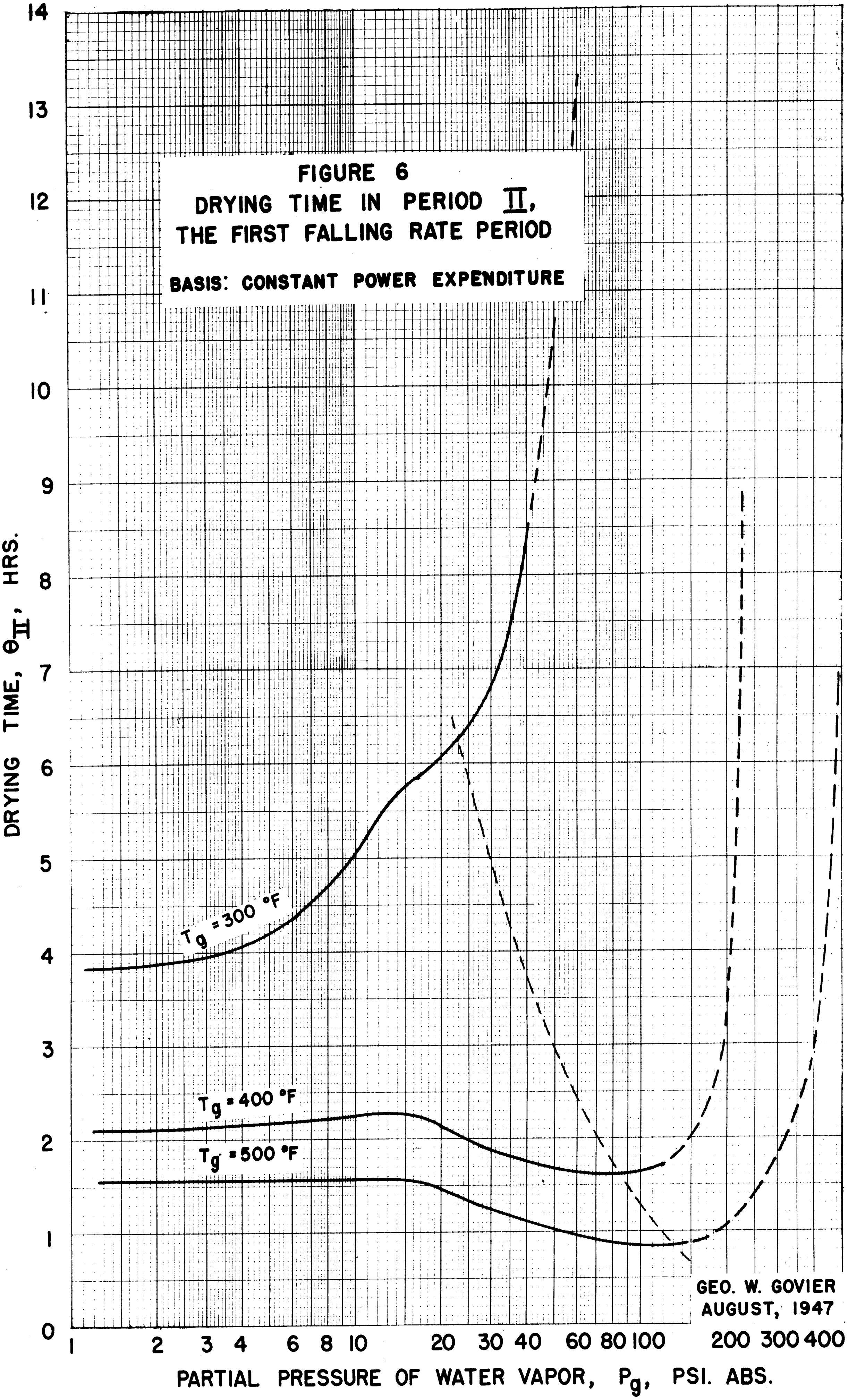

FIGURE 6
DRYING TIME IN PERIOD II,
THE FIRST FALLING RATE PERIOD

BASIS: CONSTANT POWER EXPENDITURE

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Period III - Second Falling Rate: Figures 7 and 8 summarize the calculations of drying rate and time in Period III.

Figure 7, which shows the effect of temperature, T_g, and pressure, P_g, upon the logarithmic mean drying rate, shows features similar to those of the two previous periods, although again at lower rates. The effect of partial pressure of water vapor, P_g, in the humid air range is seen to be of some importance at temperatures below 400°F, but of little above.

Figure 8 indicates the net effect of T_g and P_g upon the time required for the removal of all the water from the second critical moisture content, F_3, down to $F = 0.10$ lbs/lb. The tremendous influence of temperature, particularly between 300 and 400° F, is immediately apparent. A shift in optimum P_g similar to that of the previous periods is evident, although less pronounced.

It should be noted at this point that since

$$\begin{aligned} W &= F + E \\ &= F + f\,(P_g, T_g) \end{aligned}$$

the reduction of the water content to $F = 0.10$ does not mean to an absolute water content of 0.10 lbs/lb, but to this plus the prevailing equilibrium moisture.

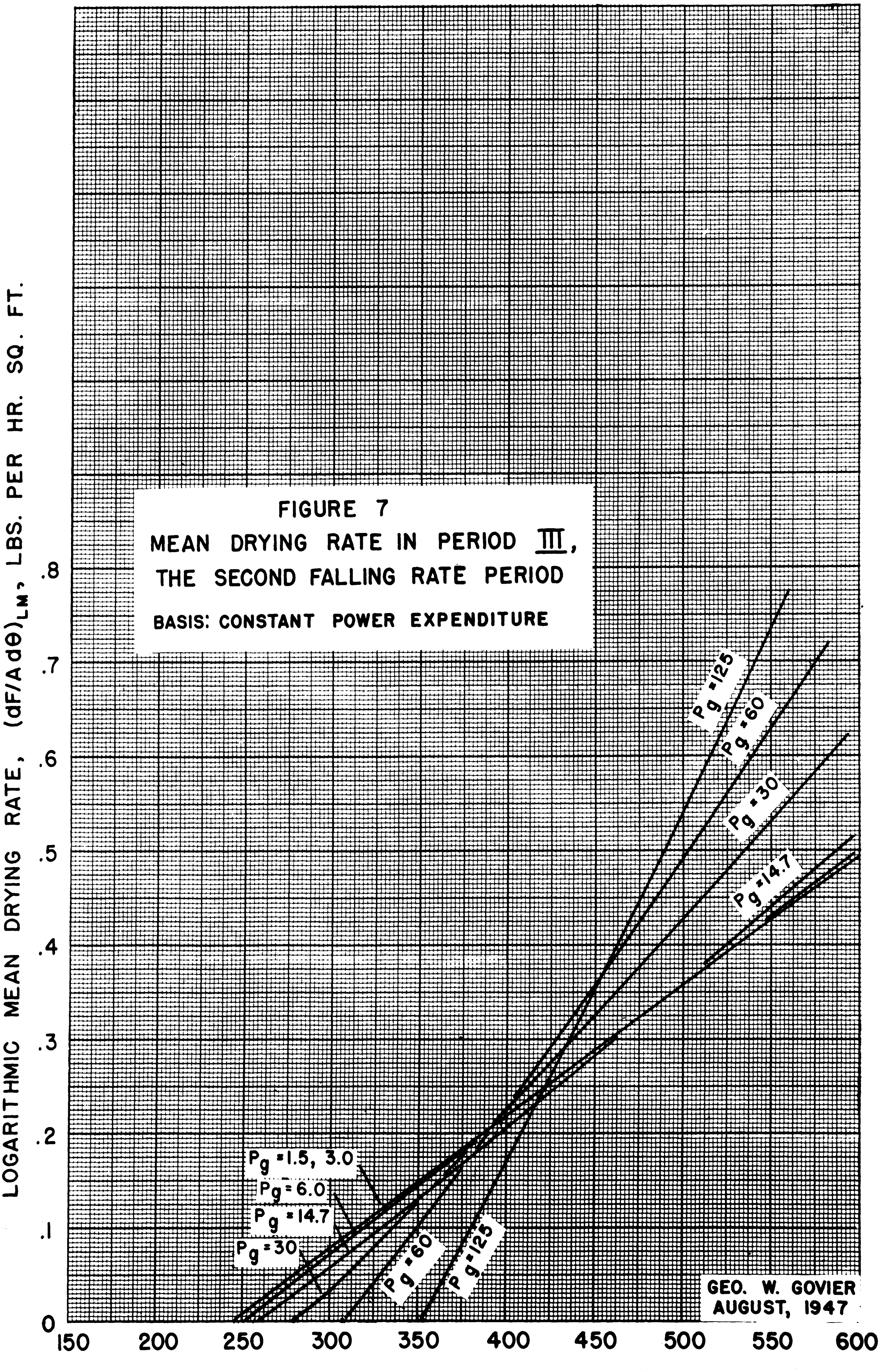

FIGURE 7
MEAN DRYING RATE IN PERIOD III,
THE SECOND FALLING RATE PERIOD

BASIS: CONSTANT POWER EXPENDITURE

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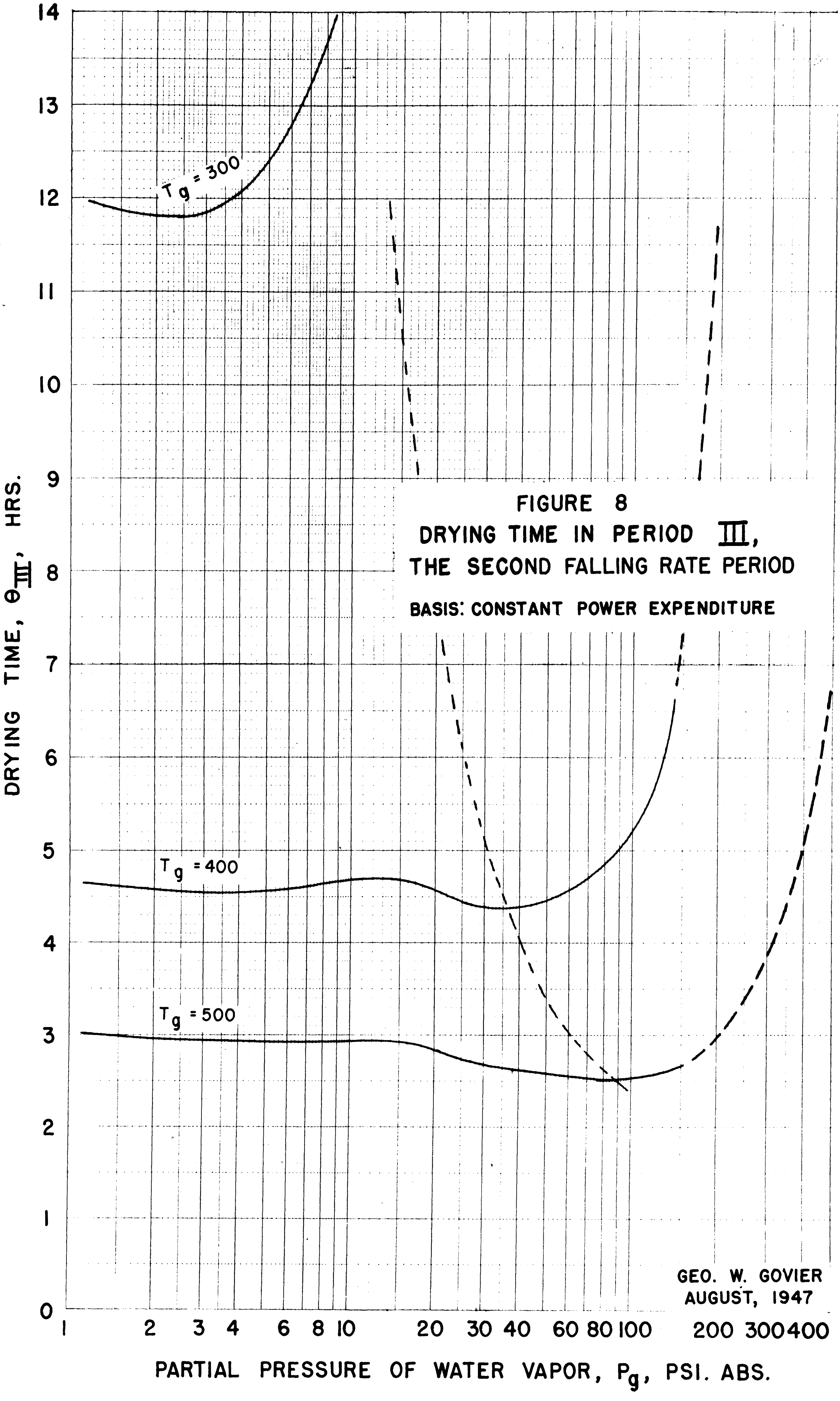

FIGURE 8
DRYING TIME IN PERIOD III,
THE SECOND FALLING RATE PERIOD
BASIS: CONSTANT POWER EXPENDITURE

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Total Drying Time: The influence of temperature, T_g, and partial pressure of water vapor, P_g, upon the total time required for drying is of course the factor of most interest. Figure 9 shows this relationship in full detail. Below the critical partial pressure values at which the drying times increase asymptotically to infinity, temperature is the important variable and particularly so in the region 300-400°F. At temperatures of 350°F and less, humid air offers shorter drying times than does superheated steam. Above 400°F superheated steam is to be favored, especially in the optimum partial pressure range. This is considered to be that range of P_g in which the drying time is within five per cent of the minimum. The situation is summarized in Table 1.

The Total Heat Consumption:

The cost of heat consumed in the drying operation is of course directly proportional to the quantity of heat used, and dependent also upon the temperature level at which the heat is required. The quantity of heat required for the evaporation of one pound of water* has been determined under the full range of conditions of temperature (T_g) and pressure (P_g).

In the case of humid air drying ($P_g < 14.7$ psi.abs.) the important factors determining the heat requirement are:

1. The weight of equipment per pound of dry stock, and its specific heat

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*This is the amount of heat required for the drying of one pound of bone dry stock divided by 2.10, the pounds of water removed per pound of stock.

TABLE I

SUMMARY - EFFECT OF TEMPERATURE (T_g) UPON PARTIAL PRESSURE OF WATER VAPOR (P_g)

CORRESPONDING WITH MINIMUM TOTAL DRYING TIME

Temperature (T_g) - °F	Optimum Pressure (P_g)opt.- psi.abs.	Drying Time (θ) - hrs.	Optimum Drying Medium and Partial Pressure Range
300	Below 2	20.0	Humid air up to $P_g = 3$
325	About 2	15.4	Humid air up to $P_g = 4.8$
350	About 2	14.1	Humid air up to $P_g = 4.7$
375	35	10.5	Humid air up to $P_g = 5.3$ Steam from $P_g = 23$ to $P_g = 30$
400	48	8.4	Steam from $P_g = 28$ to $P_g = 86$
450	100	5.3	Steam from $P_g = 46$ to $P_g = 190$ (?)
500	150 (?)	3.9	Steam from $P_g = 82$ to $P_g = 250$ (?)

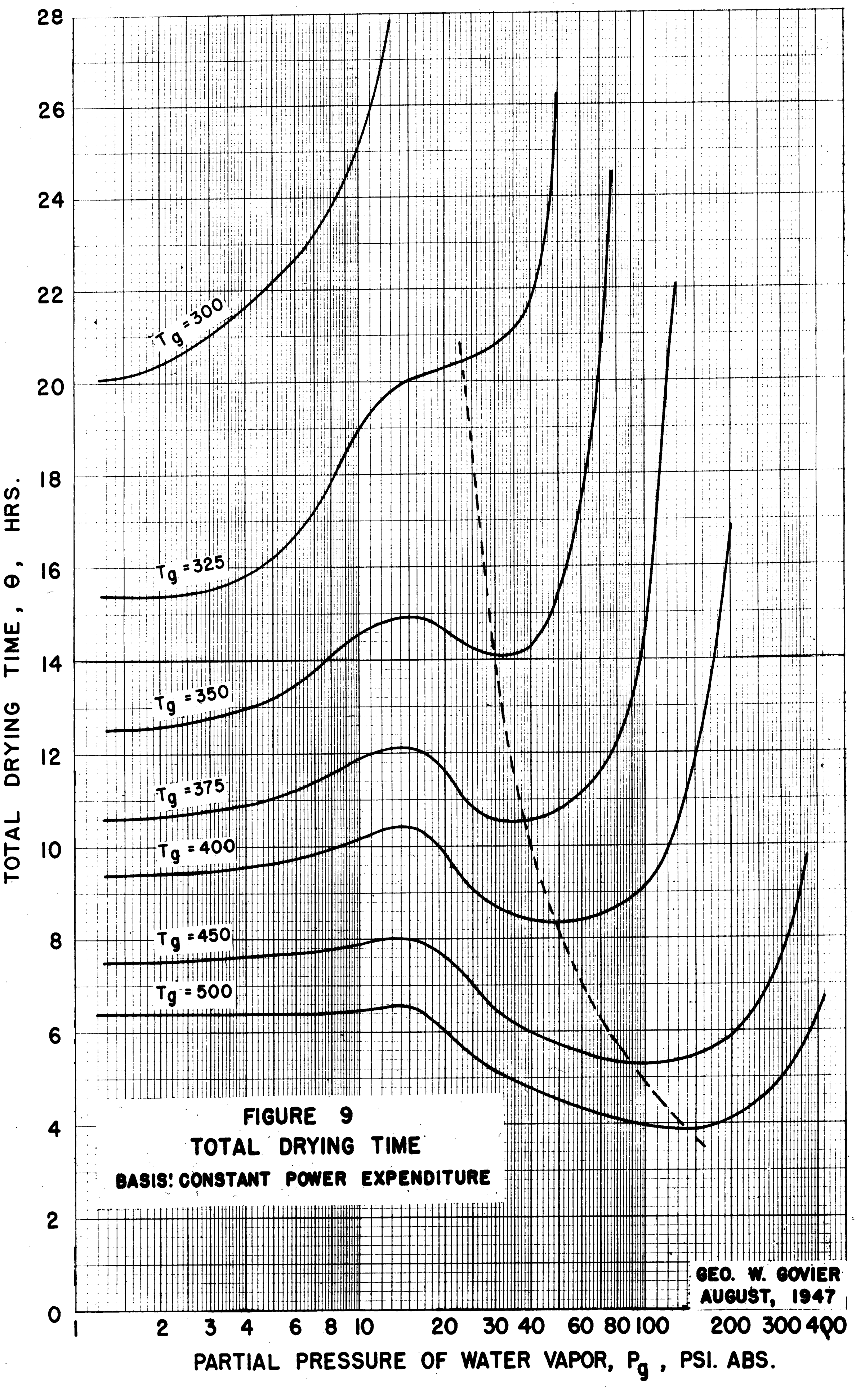

FIGURE 9
TOTAL DRYING TIME
BASIS: CONSTANT POWER EXPENDITURE

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2. The specific heat of the dry stock
3. The temperature of the drying gases (T_g)
4. The average rate of fresh air supply, m, (lbs dry air)/(lb stock)(hr)
5. The supply air temperature and relative humidity (taken arbitrarily as 70°F and 65%)
6. The drying time, θ
7. The exposed heat-dissipating surface of the equipment in ft^2/lb of dry stock

Item 4 of this list may be shown to be related to the partial pressure of water vapor (P_g) and the drying time (θ) through the equation:

$$m = \frac{4.20 \left(\frac{4080}{1+H}\right)\rho^{0.715}}{2.10 + 2H\theta \left(\frac{4080}{1+H}\right)\rho^{0.715} - .02\theta \left(\frac{4080}{1+H}\right)\rho^{0.715}} \qquad (50)$$

where

$$H = \frac{P_g}{14.7 - P_g}\left(\frac{18.06}{28.97}\right) = \text{humidity of drying gas, lb/lb} \qquad (6)$$

ρ = density of humid air, lbs/ft^3

θ = drying time, hrs

It is possible to estimate Items 1 and 7, and then to combine the several items of heat consumption to give the total heat requirement per pound of dry stock as:

$$2.10\,(\Sigma Q) = 30 + (.058 + .260m)(T_g - 70)\,\theta + 2.32\,(T_g - 350) + 2.10\,\lambda_{T_g} \text{ BTU/lb stock} \qquad (51)$$

and the heat requirement per pound of water evaporated as:

$$\Sigma Q = 14.3 + (.0276 + .124m)(T_g - 70)\,\theta + 1.105\,(T_g - 350) + \lambda_{T_g} \quad \text{BTU/lb water} \qquad (52)$$

Figure 10 is a plot showing the influence of temperature (T_g) and pressure (P_g) upon the heat consumption. The importance of P_g, being a factor in determining m, is particularly to be noted.

In the case of superheated steam drying ($P_g > 14.7$ psi. abs.) the important factors influencing heat requirement are:

1. The weight of equipment per pound of dry stock and its specific heat
2. The specific heat of the dry stock
3. The temperature of the drying gases, T_g
4. The drying time, θ
5. The exposed heat-dissipating surface of the equipment in ft2/lb of dry stock
6. The operating pressure, P_g

These factors may be combined to give:

$$2.10\Sigma Q = 2.10H - 708 + T_g\,(.058\theta + .22) - 4.06\theta + \frac{0.10}{v}\left(H_{P_g, T_g} - 38\right) \quad \text{BTU/lb stock} \qquad (53)$$

or

$$\Sigma Q = H_{P_g, T_g} - 337 + T_g\,(.0276\theta + .105) - 1.935\theta + \frac{0.0476}{v}\left(H_{P_g, T_g} - 38\right) \quad \text{BTU/lb water evaporated} \qquad (54)$$

where H_{P_g,T_g} = enthalpy of steam at P_g, T_g BTU/lb

v = specific volume of steam at P_g, T_g, ft^3/lb

The effect of P_g and T_g upon the heat requirement for this case is also shown in Figure 10, where it will be noted that the influence of P_g is small.

The situation covering the full range of humid air and superheated steam is also shown in Figure 11, which is a plot of ΣQ vs. P_g with separate lines for the various temperatures. This figure brings out several important points:

1. Low partial pressure values (i.e., low humidities) lead to excessive heat requirements.*
2. At low P_g values, temperature (T_g) is an important factor in determining the heat requirement.
3. At high P_g values (say $P_g > 6.0$ psi.abs.), T_g is an unimportant factor in determining the heat requirement.
4. At high P_g values the heat requirement becomes almost independent of P_g until a certain limiting P_g is reached, after which there is a rapid increase in heat requirement.

Table II summarizes the effect of temperature (T_g) upon the optimum range of pressure (P_g) corresponding with the minimum heat requirement. The optimum pressure range is considered to be that in which the heat requirement is within five per cent of the minimum.

---

*So far as can be determined in the Shuman report, the Owens-Illinois Glass Company now considers a wet bulb temperature of about 145°F as "normal" and one of 180°F as "high humidity" when drying at 330°F (Dec.1946 Monthly Report). These wet bulb temperatures correspond respectively with partial pressures of water vapor of 2.2 psi.abs. and 7.1 psi.abs. (Figure 1). From Figure 11 it is seen that at 330°F and these partial pressures, the heat requirement is respectively 1620 and 1110 BTU/lb of water evaporated. These figures represent 154 per cent and 106 per cent of the minimum heat requirement of 1050 BTU/lb at 330°F.

TABLE II

SUMMARY - EFFECT OF TEMPERATURE (T_g) UPON PARTIAL PRESSURE OF WATER VAPOR (P_g) RANGE CORRESPONDING WITH MINIMUM HEAT REQUIREMENT

Temperature (T_g) - °F	Optimum Pressure $(P_g)_{opt.}$ - psi.abs.	Minimum Heat Requirement, $\Sigma Q_{min.}$ @T_g BTU/lb water	Optimum Drying Medium and Optimum Pressure Range (P_g range) - psi.abs.
300	23	1070	Humid air from P_g = 5 to 14.7 Steam from P_g = 14.7 to 44
350	29	1067	Humid air from P_g = 7 to 14.7 Steam from P_g = 14.7 to 90
400	35	1055	Humid air from P_g = 10 to 14.7 Steam from P_g = 14.7 to 140
450	90	1035	Humid air from P_g = 12 to 14.7 Steam from P_g = 14.7 to 250 (?)
500	150 (?)	1015	Steam from P_g = 14.7 to 370 (?)

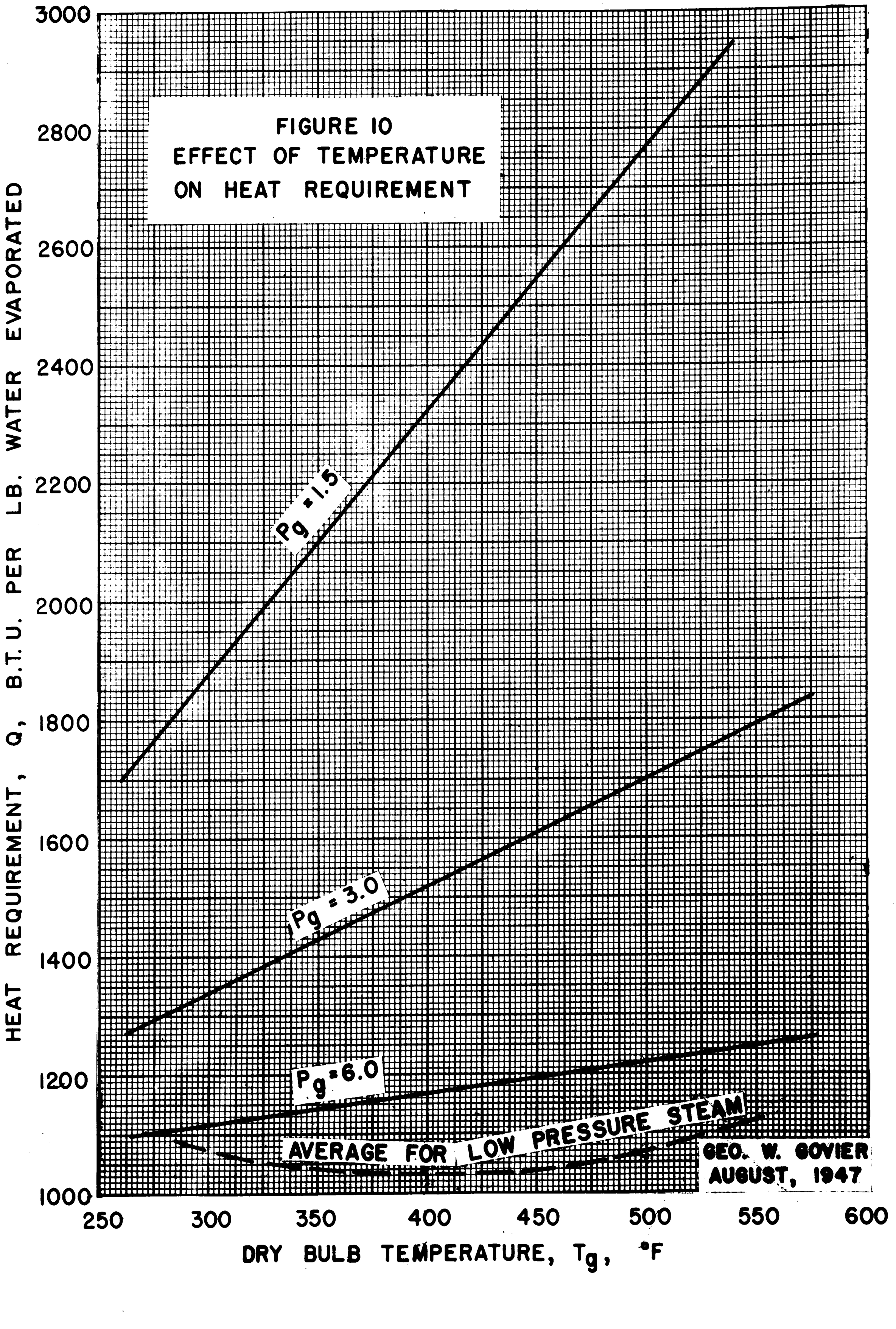
FIGURE 10
EFFECT OF TEMPERATURE
ON HEAT REQUIREMENT
HEAT REQUIREMENT, Q, B.T.U. PER LB. WATER EVAPORATED
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
250
300
350
400
450
500
550
600
DRY BULB TEMPERATURE, T_g, °F
Pg = 1.5
Pg = 3.0
P_g = 6.0
AVERAGE FOR LOW PRESSURE STEAM
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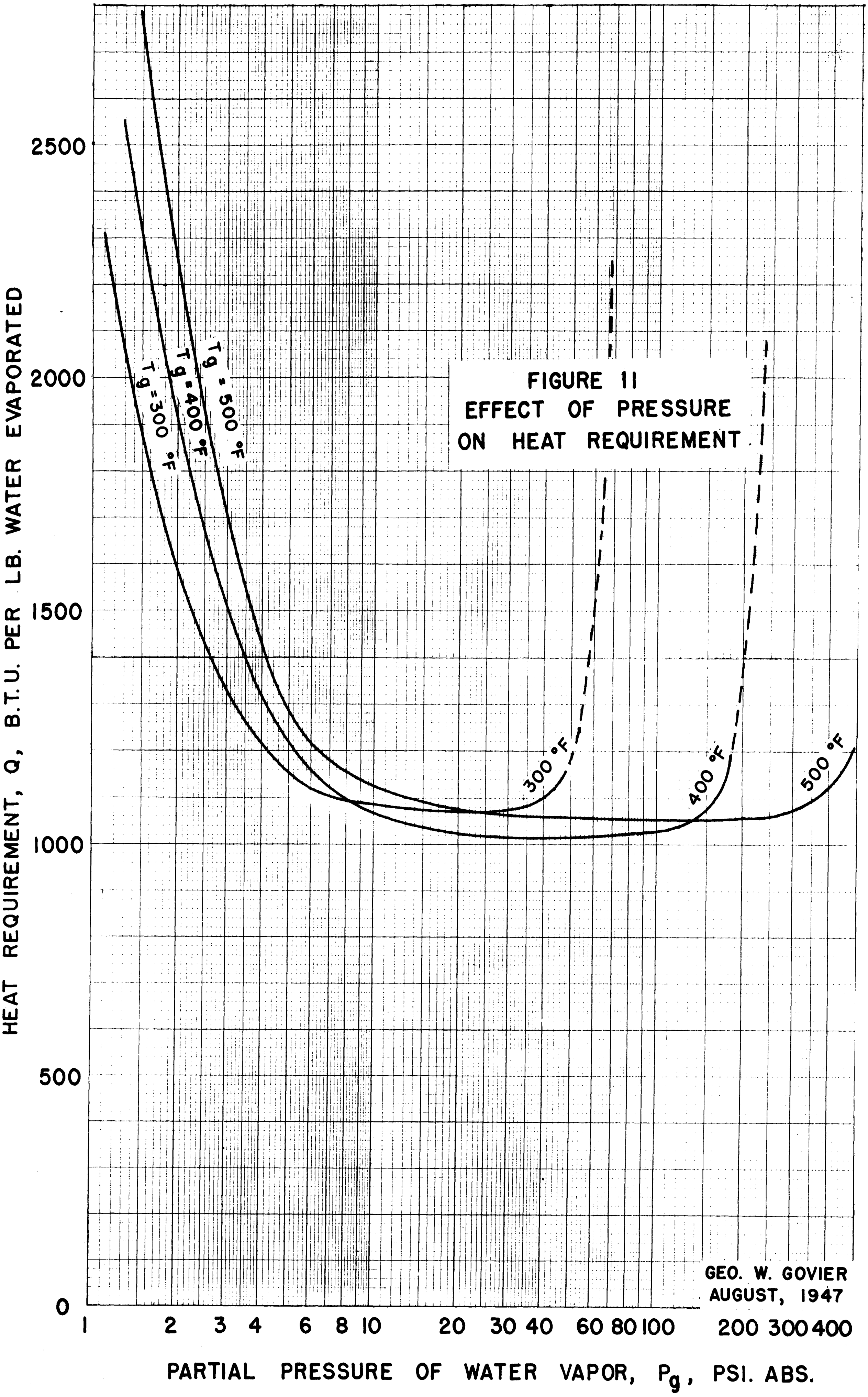

FIGURE 11
EFFECT OF PRESSURE
ON HEAT REQUIREMENT

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The possibility of heat recovery must not be overlooked in the case of superheated steam, where a high proportion of the total heat requirement is available for recovery in the form of latent heat in steam vented from the dryer. The efficient recovery of this heat is contingent upon a large multi-unit installation permitting scheduling and load levelling, and upon additional capital outlay for a recovery system. The attractiveness of this is not to be denied, but it is believed wise in this preliminary analysis to assume merely that the possible savings of "multiple-effect" or other steam re-use operation will cancel the higher unit cost of heat at the pressure where re-use could be effected.

The Total Electrical Energy Consumption:

Electrical power is required for the purpose of circulating the drying gases over the stock, through any system of baffles which may be desirable, and over heating coils for supplying the heat requirement. It is convenient to consider the power for these three purposes separately, although in practice all may be supplied together.

Taking one pound of stock as a basis, the power over the stock is 1.46 E_s*, where E_s is the unit power discussed earlier and taken as constant throughout the comparison. The actual numerical value of E_s is dependent upon the spacing and arrangement of the stock and its effective surface roughness -- factors which are not analyzed in this study.

---

*Since one pound of 1-3/4 inch thick, 20 pcf stock exposes a surface of 1.46 square feet.

The power expenditure through the baffle system, etc., is also largely dependent upon the specific arrangement and is incapable of actual evaluation at this time. It may, however, be represented as:

$$E_b = k_b\,(1.46\,E_s) \qquad (55)$$

where k_b is a proportionality factor.

Similarly the power expenditure over the heating coils is dependent upon their spatial arrangement and, in addition, upon the ratio of heating coil surface to drying surface and the ratio of heating coil temperature differential to drying temperature differential. Regardless of these complications, however, the power expenditure over the heaters may be represented by:

$$E_h = k_h\,(1.46\ E_s) \qquad (56)$$

where k_h is a proportionality factor.

The total power consumption is therefore:

$$\Sigma E = 1.46E_s(1 + k_b + k_h) \qquad (57)$$

However, E_s has been chosen as one of the constant factors in the comparison, which means that ΣE, the total power consumption, is also constant, although unknown in actual magnitude. Consequently, the total electrical energy consumption may be represented by:

$$KWH = 1.46E_s(1 + k_b + k_h)\ \theta \qquad (58)$$

where

KWH = kilowatt hours of electrical energy
θ = drying time, hrs

and is directly proportional to the drying time, θ.

The effect of temperature (T_g) and pressure (P_g) upon the total electrical energy requirement is therefore precisely the same as their effect upon the drying time (θ). Figure 9 may be used as a direct indication of the relative electrical energy requirement at various temperatures and pressures.

Influence of Change in Stock Thickness and Power Consumption:

Although it has been stated that a constant stock thickness of $2X_o$ = 1-3/4 in and a constant power expenditure are parts of the chosen basis for the evaluation, it is nonetheless of interest to determine the influence of changes in these quantities on the drying rates.

This situation has been given careful study, with results which are summarized below.

Period I - Constant Rate: In this period the drying rate is given by:

$$\frac{dF}{Ad\theta} = \frac{h_c}{\lambda_{T_s'}} (T_g - T_s') \quad \text{lbs/ft}^2\text{hr} \qquad (43)$$

Substituting the expression for the film coefficient, h_c, in terms of the energy consumption,

$$\frac{dF}{Ad\theta} = B\phi_t \, E_s^{0.286} \left(\frac{T_g - T_s'}{\lambda_{T_s'}} \right) \qquad (59)$$

from which it may be seen that the drying rate, $\frac{dF}{Ad\theta}$, is independent of stock thickness, $2X_o$, but directly dependent upon $E_s^{0.286}$.

Period II - First Falling Rate: Here the logarithmic mean drying rate may be shown equal to:

$$\left(\frac{dF}{Ad\theta}\right)_{lm} = B\phi_t\, E_s^{0.286}\, \frac{(T_g - T_s'')_{lm}}{\lambda_{T_s''}} \qquad (60)$$

and again, the logarithmic mean drying rate, $\left(\frac{dF}{Ad\theta}\right)_{lm}$, is independent of stock thickness, $2X_o$, but dependent directly upon $E_s^{0.286}$.

Period III - Second Falling Rate: In this period the situation is more complicated, and the logarithmic mean drying rate may be shown equal to:

$$\left(\frac{dF}{Ad\theta}\right)_{lm} = \frac{.65X_oh_c}{(.65X_oh_c + K_E)\ \ln\frac{.65X_oh_c + K_E}{K_E}}\ B\phi_t E_s^{0.286}\left(\frac{T_g - T_s''}{\lambda_{T_s''}}\right) \qquad (61)$$

Designating the first part of the right-hand expression as ψ, the equation becomes:

$$\left(\frac{dF}{Ad\theta}\right)_{lm} = \psi B\phi_t\, E_s^{0.286}\left(\frac{T_g - T_s''}{\lambda_{T_s''}}\right) \qquad (62)$$

which is similar in form to the equations of the previous periods. The expression ψ, however, depends upon the film coefficient, $h_c = B\phi_t\, E_s^{0.286}$, and upon the stock thickness, $2X_o$. For convenience, values of ψ have been calculated for various values of h_c and X_o, and the relationship is plotted in Figure 12. In this period, then, the influence of X_o and E_s may be determined as follows:

1. Evaluate the influence of the change on $\left(\frac{dF}{Ad\theta}\right)_{lm}$ as in Period II.

2. Evaluate the influence of the change on ψ (from Figure 12).

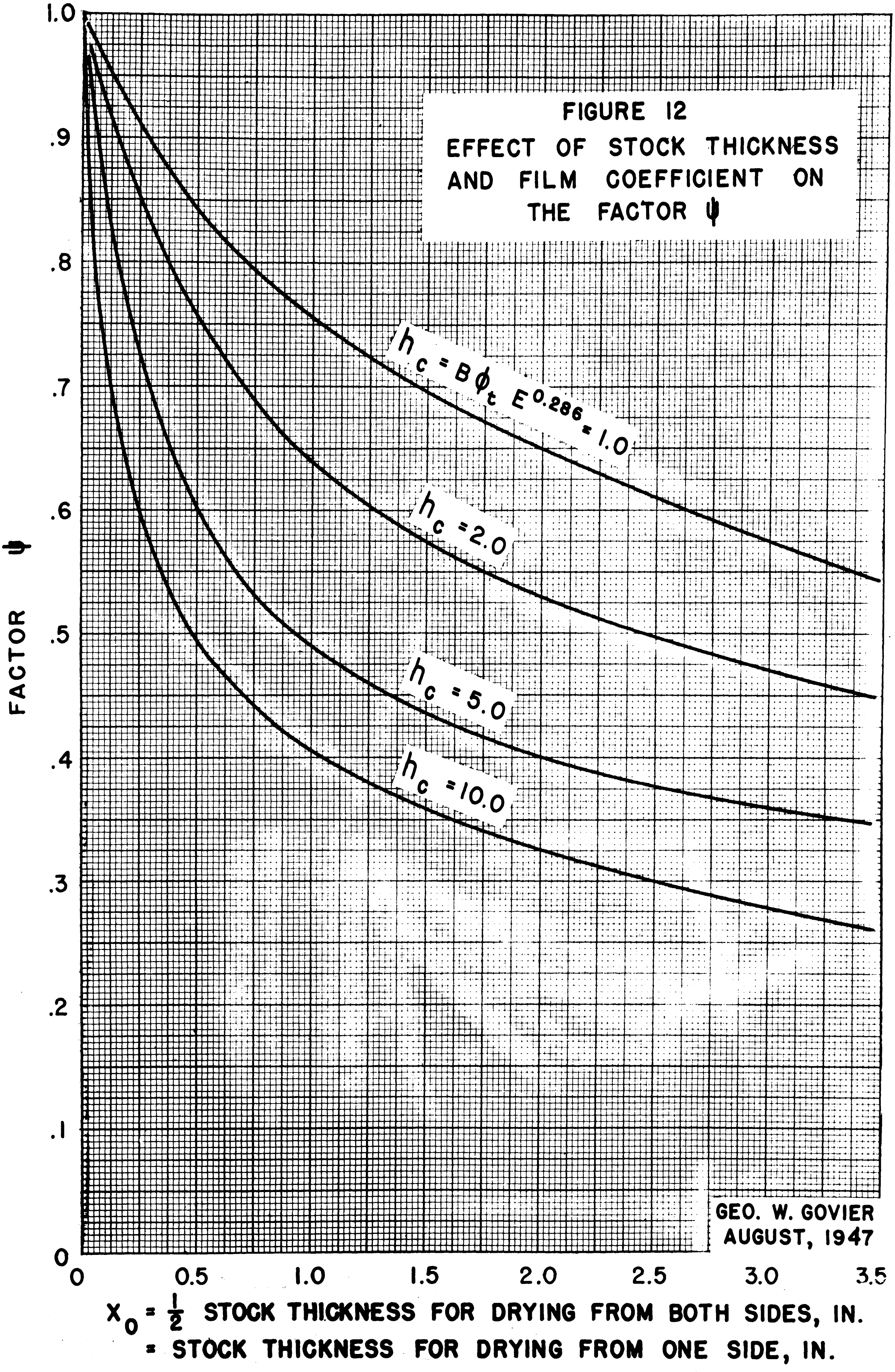

FIGURE 12

EFFECT OF STOCK THICKNESS AND FILM COEFFICIENT ON THE FACTOR ψ

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3. Multiply the factors of (1) and (2) to obtain the net influence of the change on $\left(\frac{dF}{Ad\theta}\right)_{lm}$ in Period III.

Figure 13 has been prepared to facilitate determining the influence of E upon the drying rates in Periods I and II.

It is seen, therefore, that the influence of a change in stock thickness, $2X_o$, or unit power expenditure, E_s, upon the drying rate may readily be determined. This, of course, is drying rate in terms of lbs/ft^{2}hr, which, except in Period III, is independent of stock thickness. Obviously however, the drying times in the various periods are not independent of thickness, because the quantity of water to be evaporated is directly dependent upon the thickness. Changes in the two critical moisture contents with drying rate and thickness of stock render an exact analysis of the net effect of $2X_o$ and E_s upon drying times difficult, and in any case result in unwieldy expressions. As a rough approximation, the drying times in the various periods will be inversely proportional to the new drying rate and directly proportional to the new thickness.

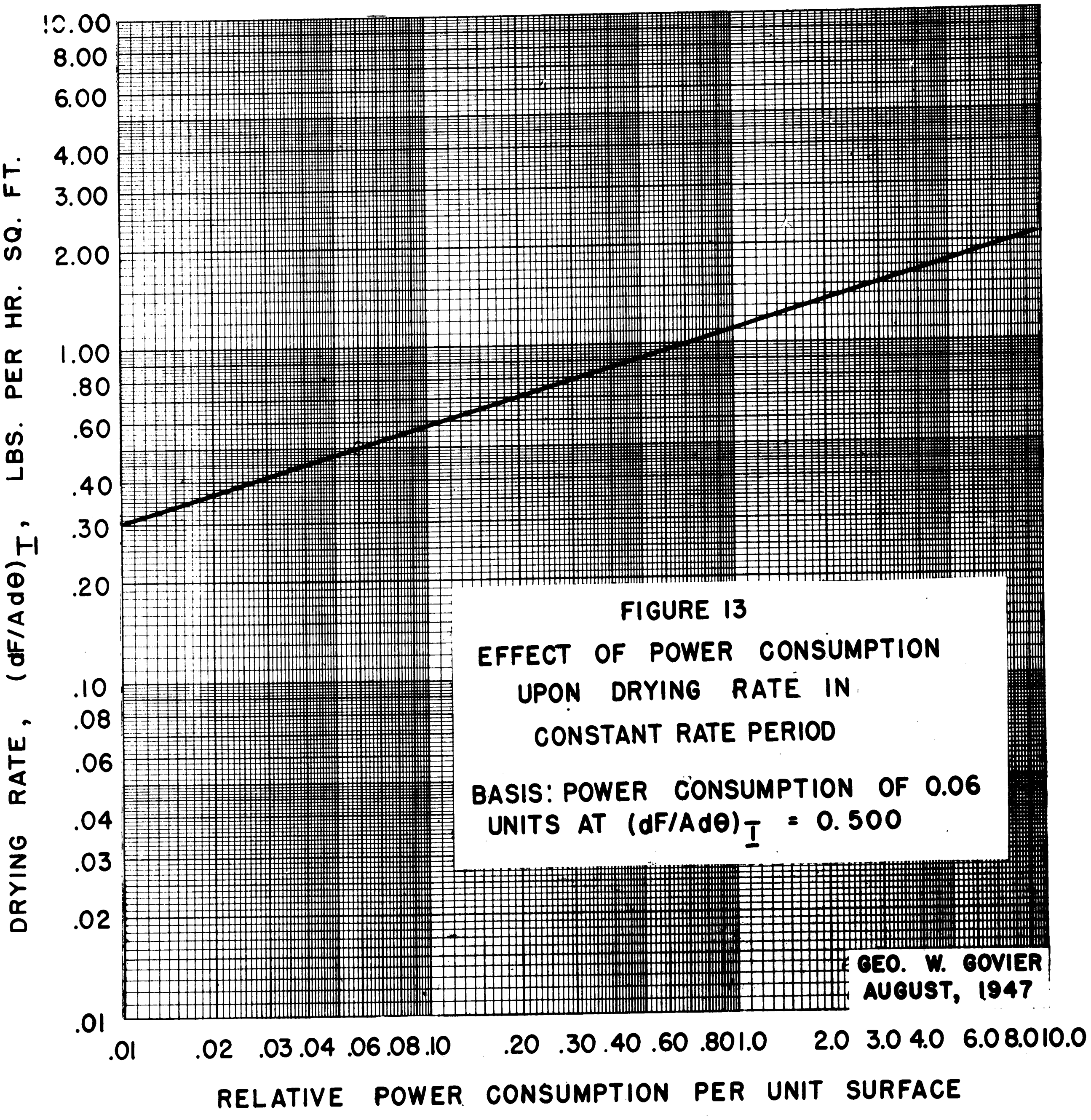

FIGURE 13
EFFECT OF POWER CONSUMPTION
UPON DRYING RATE IN
CONSTANT RATE PERIOD

BASIS: POWER CONSUMPTION OF 0.06 UNITS AT $(dF/Ad\theta)_{\underline{I}} = 0.500$

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VI. INFLUENCE OF TEMPERATURE, PRESSURE, AND NATURE OF DRYING MEDIUM ON QUALITY-DETERMINING FACTORS

At the present time it is impossible, unfortunately, to give other than the most preliminary qualitative treatment to the influence of temperature, T_g, and partial pressure, P_g, upon the quality of finished product. Earlier mention was made of the fact that the absolute moisture content W, the rate of drying $\frac{dF}{Ad\theta}$, the moisture and temperature gradients $\frac{dW}{dX}$ and $\frac{dT}{dX}$, and the temperature gradient of the drying gases $\frac{dTg}{dL}$ are the factors most probably of importance in determining quality. Each of these will be discussed separately.

Absolute Moisture Content (W)

It is believed that if data on strength and rigidity versus moisture content of the finished material were available, they would show that the ability of the stock to withstand the shrinkage and thermal stresses encountered during drying is greater when the stock is above some "critical" moisture content.* On this assumption it may be argued that the quality of the finished stock will depend upon the approach of the local moisture concentration to this "critical" value. If the local concentration at any point should fall below the critical, there is increased danger of the shrinkage and thermal stresses causing actual failure of the material with the resultant "shrinkage cracks." Now, the numerical

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*This contention is not supported by actual data, but represents the point of view of Mr. J. K. Selden and other workers in the field.

value of the critical moisture content is unknown. Also, the minimum local moisture content, (which is E, the equilibrium moisture content for the particular T_g, P_g conditions) is unknown except by the functional equation:

$$E = f\ (\text{stock}, T_g, P_g).$$

All that may be said, therefore, is that probably low temperature, T_g, and high pressure, P_g, values, which favor high values of E, will favor good quality.

Rate of Drying $\left(\frac{dW}{Ad\theta} = \frac{dF}{Ad\theta}\right)$:

It is usually assumed that rate of drying, per se, may have an influence upon quality. One cannot be sure whether this is the case, or whether it is merely that factors favoring high rates of drying, such as those discussed below, also favor poor quality. In any case, however, it is safer to assume that high drying rates may have deleterious effects upon quality and that good quality is favored by those factors which favor low values of drying rate, $\frac{dF}{Ad\theta}$, or high values of drying time, θ.

Moisture and Temperature Gradients $\left(\frac{dW}{dX}, \frac{dT}{dX}\right)$:

Moisture and temperature gradients through the stock are responsible for the stresses which arise during the latter stages of drying. Naturally, the larger these gradients, the greater the stresses and the more serious the possible damage to the stock may be. The previous analysis of the mechanism of drying indicates $\frac{dW}{dX}$ and $\frac{dT}{dX}$ remain small until the second falling rate period.

Moreover, during this period $\frac{dW}{dX}$ will depend almost directly upon the drying rate and $\frac{dT}{dX}$ almost directly upon the drying rate and inversely upon K_E, the thermal conductivity of the stock. Thus, low gradients are dependent upon low drying rates and a high K_E. The thermal conductivity K_E is known to be dependent upon both temperature and equilibrium moisture content; i.e.,

$$\begin{aligned} K_E &= f''(\text{stock}, T_g, E) \\ &= f'(\text{stock}, T_g, P_g) \end{aligned}$$

It may therefore be concluded that low gradients and good quality will be favored by low rates of drying, low temperatures and high pressures.

Temperature Gradient of the Drying Gases $\left(\frac{dT_g}{dL}\right)$:

The temperature gradient in the drying gases in the direction of their flow is an important factor in determining uniformity of drying conditions and, therefore, the uniformity of quality in the finished product. The permissible variation in T_g without giving rise to appreciable non-uniformity must be determined by actual test. Nonetheless, it may be seen from Figures 4, 6, 8 and 9 that, in the neighborhood of $T_g = 400°F$ and $P_g = 30$ psi.abs., a variation in T_g of 5°F ($\Delta T_g = 5$) will cause a variation in drying time of:

Period I	(Figure 4)	0.063 hrs	2.5%
Period II	(Figure 6)	0.033 hrs	1.8%
Period III	(Figure 8)	0.086 hrs	1.9%
Over-all	(Figure 9)	0.182 hrs	2.1%

This means that stock exposed to the higher T_g will be dried to the desired point approximately 0.18 hours before that exposed to the lower T_g. It is therefore inevitable that a portion of the charge will be overdried, while another portion will be underdried. Storage after drying will tend to alleviate this effect, but still it must be considered undesirable.

In practice, the solution to the problem is to locate the heaters at such intervals along the path of the gases that the maximum ΔT_g will not cause serious difficulty. If the distance between heaters, measured along the path of the gases, is taken as ΔL, then:

$$\Delta L = \frac{(\Delta T_g)_{max}}{\frac{dT_g}{dL}}$$

$$= (\Delta T_g)_{max} \left(\frac{dL}{dT_g}\right) \qquad (63)$$

and the significance of the gradient $\frac{dT_g}{dL}$ and its reciprocal $\frac{dL}{dT_g}$ becomes apparent.

An analysis of the effect of temperature, T_g; pressure, P_g; and the properties of the drying medium upon $\frac{dT_g}{dL}$ has been carried out. It may be shown that the maximum value of the gradient is that obtained in Period I.

$$\left(\frac{dT_g}{dL}\right)_{max} = \left(\frac{dT_g}{dL}\right)_I = \frac{N}{h^{1.214}\ E_s^{.071}} \nabla_t\ (T_g - T_s') \qquad (64)$$

where

E_s = unit power expenditure, ft lbs/ft^{2}hr

N = a numerical constant

h = the "thickness" of the flowing gas stream, ft

∇_t = a combination of fluid properties

$$= \frac{k_g^{0.6}\, v^{0.144}}{c_p^{0.6}\, \mu^{0.385}}$$ and dependent upon T_g and P_g

The relative $\left(\frac{dT_g}{dL}\right)_{max}$, for constant values of gas stream "thickness," h, and power expenditure, E_g, may therefore be expressed in terms of ∇_t and $(T_g - T_s')$. The influence of T_g and P_g upon this relative $\left(\frac{dT_g}{dL}\right)_{max}$ is shown in Figure 14, where the gradient at T_g = 400°F and P_g = 30 psi.abs. is taken arbitrarily as unity.

The figure shows that low gradients are favored by low temperatures and high pressures, superheated steam giving lower values than humid air at the same temperatures.

Since low gradients mean greater uniformity with the same heater spacing, or, the same uniformity at greater heater spacing, it is obvious that low temperatures and high pressures favor uniformity of product quality.

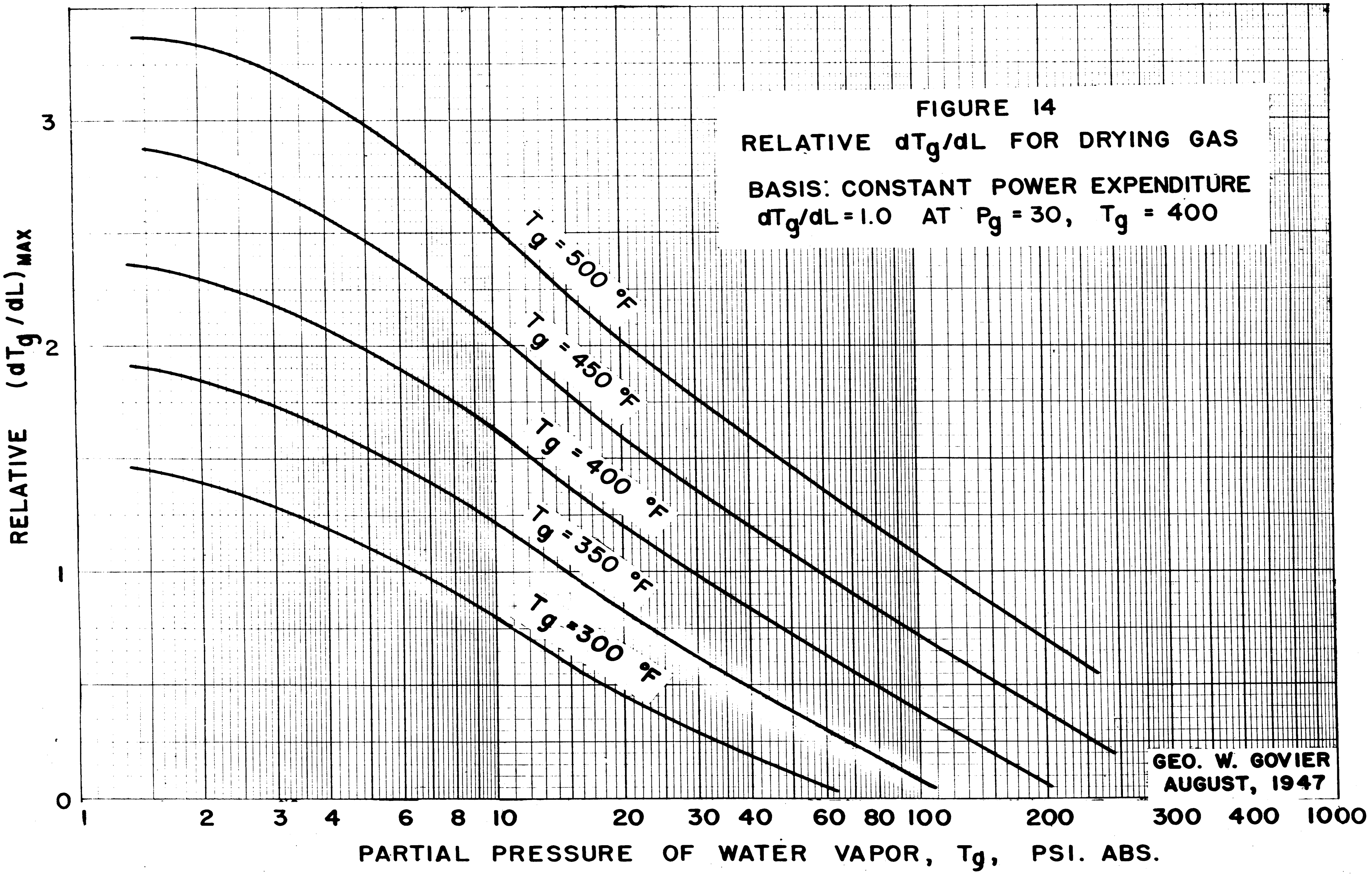

FIGURE 14

RELATIVE dT_g/dL FOR DRYING GAS

BASIS: CONSTANT POWER EXPENDITURE
$dT_g/dL = 1.0$ AT $P_g = 30$, $T_g = 400$

VII. CONCLUSION: THE COMPOSITE EFFECT OF TEMPERATURE, PRESSURE, AND NATURE OF DRYING MEDIUM ON UNIT COST AND QUALITY

It has been stated that the relative merits of different conditions of operation and of humid air versus steam depend upon their influence on unit cost and quality. These influences have now been determined separately, and it remains to explore the combined effect and to summarize the conclusions. The task is made difficult by the fact that the relative importance of the various cost-determining and quality-determining factors may, at this time, only be assumed.

Among the cost-determining factors, T_g, P_g, θ, c_h and c_p,* it will be assumed that the direct importance of temperature, T_g, and pressure, P_g, is slight compared with θ, c_h and c_p. Also, since the cost of electrical energy, c_p, has been shown to be directly dependent upon the drying time, θ, these three factors are reflected in two: drying time, θ, and cost of heat, c_h; or, drying time and heat requirement. Now, the relative importance of drying time, reflecting as it does the costs c_c, c_d and c_p, is considered to be greater than that of the heat requirement, which reflects only the single cost c_h. Thus, in attempting to determine from Tables I and II the range of conditions favoring minimum cost, it is believed desirable to weigh the conditions of Table I slightly more than those of Table II. Moreover, an examination of Table I indicates that within the optimum pressure range, the influence of temperature

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*See Pages 2 and 3.

upon the heat requirement is slight. This means that within the optimum pressure range for both drying time and heat requirement, the variable part of the total unit cost will be about proportional to the optimum drying time, $\theta_{\text{min.} @ T_g}$.

The influence of temperature upon the composite optimum pressure range (i.e., the range covering the optimum pressures for both drying time and heat requirement) is summarized in Table III. This table represents, in effect, a summary of those findings of this report which are related to cost-determining factors. The total unit cost of drying may be considered as equal to a constant plus a figure which is proportional to the drying time, $\theta_{\text{min.} @ T_g}$, reported in Table III.

To complete the picture, consideration will now be given to the results which have been obtained for the effect of those factors controlling product quality. Here, even more difficulty is encountered in establishing the relative importance of the factors E, $\frac{dF}{Ad\theta}$, $\frac{dW}{dX}$, $\frac{dT}{dX}$, and $\frac{dT_g}{dL}$. Fortunately however, except for the drying rate, $\frac{dF}{Ad\theta}$, low values of all these factors, and consequently good quality, are favored by high pressure and low temperature values. Low values of the drying rate require high values of the drying time. Thus, good quality is favored generally by low temperatures, high pressures and long drying times.

A comparison of these latter requirements with Table III indicates the need for a compromise between low total unit cost and good quality. Operation should, of course, be carried out at the maximum temperature consistent with satisfactory quality and at an operating pressure within the range of optimum pressures corresponding (in Table III) with this temperature. Regarding, now the relative merits of humid air and superheated steam from the over-all viewpoint, it is evident that:

1. Below about 375°F humid air is superior to steam.
2. Above about 375°F steam is superior to humid air.

The conclusion concerning the relative merits of the two media is thus seen to be dependent upon whether or not good quality is attainable at temperatures in excess of 375°F. If this is the case, as is likely, then superheated steam becomes an attractive medium for drying and offers promise of considerable economy.

At this point it is well to recall that this is a preliminary study based upon a minimum of experimental data. Thus, although the indications favor superheated steam over air as a drying medium, an experimental study is required to determine the effects on quality and to substantiate or refute the theory which has been developed.

TABLE III

SUMMARY - EFFECT OF TEMPERATURE UPON COMPOSITE RANGE OF OPTIMUM PRESSURE AND UPON TOTAL UNIT COST*

Temperature (T_g) - °F	Composite Range of Optimum Medium and Pressure (P_g) - psi.abs.	$\theta_{min.}$ @ T_g - hrs. Approximately Proportional to Total Unit Cost*
300	Humid air: P_g near 3-4	20.0
350	Humid air: P_g near 4-6	14.1
400	Steam: P_g 30-85	8.4
450	Steam: P_g 45-190 (?)	5.3
500	Steam: P_g 80-250 (?)	3.9

*Exclusive of overhead, c_o, and labor, c_l.

NOMENCLATURE

1 as a subscript, refers to initial condition

2 as a subscript, refers to condition at the first critical moisture content

3 as a subscript, refers to condition at the second critical moisture content

4 as a subscript, refers to final condition

5 as a subscript, refers to condition corresponding with the equilibrium moisture content

A = total exposed area of 1 lb of stock, ft^2

a = a constant

B = proportionality factor

b = a constant

C = number of components

c = specific heat of drying gas, BTU/lb°F

C_c = total capital investment in dryer

c_c = interest on capital investment, per unit product

c_d = insurance, depreciation, obsolescence, taxes, etc., all assumed proportional to the capital investment, per unit product

c_h = cost of heat, per unit product

c_l = direct labor cost, per unit product

c_o = fixed overhead, as management, etc., assumed independent of the capital investment, per unit product

c_p = cost of electrical energy, per unit product

d = a constant

E = equilibrium moisture content of stock, lbs water/lb dry stock

e = a constant

E_b = power expenditure through the baffle system per lb of stock, ft lbs/hr lb

E_h = power expenditure over the heater per lb of stock, ft lbs/hr lb

E_s = power expenditure per unit drying surface, ft lbs/hr ft^2

$F = W - E$ = free moisture content of stock, lbs water/lb dry stock

f = functional relationship

f_E = equilibrium functional relationship

$\frac{1}{f_E}$ = equilibrium relationship

H = humidity of drying gas, lb/lb

h = the "thickness" of the flowing gas stream, ft

h_c = film coefficient of heat transfer for combined conduction and convection, BTU/ft^{2}hr°F

H_{P_g,T_g} = enthalpy of steam at pressure P_g, temperature T_g; BTU/lb

KWH = electrical energy requirement per lb of dry stock, kilowatt hrs

K_E = thermal conductivity of stock at mean temperature and at equilibrium moisture content, BTU/ft hr°F

k_b = proportionality factor

k_c = proportionality factor

k_c' = a constant

k_c'' = a constant $= k_c k_c'$.

k_g = thermal conductivity of drying gas, BTU/ft hr°F

k_h = proportionality factor

L = distance measured along the path of the gases, ft lm as a subscript, denotes logarithmic mean value

ln denotes natural logarithm of

m = average rate of fresh air supply, lbs dry air/lb stock hr

N = a numerical constant

P = number of phases

P_g = partial pressure of water vapor in drying gas, psi.abs.

$(P_g)_{sat.}$ = saturation vapor pressure of water at temperature T_g

$R = \frac{dF}{Ad\theta} = \frac{dW}{Ad\theta}$ = drying rate, lbs water/ft^{2}hr

T_g = dry bulb temperature of drying gas

T_s' = surface temperature of stock during Period I, °F

= adiabatic saturation temperature of drying gases, °F

$T_{s,r}''$ = surface temperature of stock at moisture content F_3, °F

T_x''' = temperature of stock at receded plane of evaporation X ft below surface, °F

V = variance of system

v = specific volume of steam, ft^3/lb

V_p = variance of system at constant pressure

W = water content of stock, lbs water/lb dry stock

X_o = one-half thickness of stock drying both sides, ft

= thickness of stock drying from one side, ft

ΔL = distance between heaters, measured along the path of the gases, ft

ΔP_r = vapor pressure depression due to curvature of exposed menisci, psi.abs.

θ = drying time, hrs

λ_T = latent heat of vaporization of water at temperature T

μ = viscosity of drying gas, lbs/ft hr

ρ = density of drying gas, lbs/ft^3

ΣQ = total heat requirement of drying operation, BTU/lb water evaporated

$\phi_t = \dfrac{c^{0.4}\, k_g^{\,0.6}\, \rho^{0.571}}{\mu^{0.457}}$ = Parsons and Gaffney "ϕ_t function," dependent upon fluid properties

$\psi = \dfrac{.65X_o h_c}{(.65X_o h_c + K_E)\ln \dfrac{.65X_o h_c + K_E}{K_E}}$, a combination of the factors X_o, h_c and K_E encountered in the equation for $\left(\dfrac{dF}{Ad\theta}\right)_{lm}$ for Period III.

∇_t = a combination of fluid properties

$= \dfrac{k_g^{\,0.6}\, v^{0.144}}{c_p^{\,0.6}\, \mu^{0.385}}$ and dependent upon T_g and P_g

$<$ = less than

$>$ = greater than

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APPENDIX

MATHEMATICAL RELATIONS DESCRIPTIVE OF THE DRYING MECHANISM

The following mathematical relations are descriptive of the important physical phenomena and heat and water balances during the various periods of drying. These relations supplement the word description of the mechanism of drying as given in Section III, "Basic Mechanism and Theory."

PERIOD I° - INITIAL PERIOD

Heat Transfer to Stock Surface (Reference 22):

$$\frac{dQ}{Ad\theta} = (h_c + h_r)\ (T_g - T_s^o)$$

$$= \alpha h_c\ (T_g - T_s^o) \qquad (8)$$

where

$\frac{dQ}{Ad\theta}$ = heat supply from gas, BTU/ft^{2}hr

h_c = coefficient of heat transfer by combined conduction and convection, BTU/ft^{2}hr°F

h_r = effective coefficient of heat transfer by radiation, BTU/ft^{2}hr°F

T_s^o = surface temperature of stock at an instant during Period I°, °F

T_g = temperature of drying gases, °F

α = $\frac{(h_c + h_r)}{h_c}$, dimensionless factor

Heat Transfer to Stock Interior (References 13 and 11):

$$\frac{\partial T_i}{\partial \theta} = \frac{\rho_{Av}\, C_{Av}}{K_{Av}} \frac{\partial^2 T_i}{\partial X_i^2} = a_{Av} \frac{\partial^2 T_i}{\partial X_i^2} \qquad (9)$$

where

ρ_{Av} = average density of stock, lbs/ft^3

C_{Av} = average specific heat of stock, BTU/lb°F

K_{Av} = average thermal conductivity of stock, BTU ft/ft^2 hr°F

T_i = interior temperature at time θ and at X_i feet from the surface

$a_{Av} = \frac{\rho_{Av}\, C_{Av}}{K_{Av}}$ = average thermal diffusivity of stock, hr/ft^2

and the boundary counditions are:

$\theta = 0;\quad T_i = T_o$ for all values of X

$\theta = \theta_1;\quad T_i = T_s';\quad X_i = 0$

$\theta = \theta_1;\quad T_i \simeq T_s';\quad X_i = X_o$

Water Transfer from Stock Surface (Reference 22):

$$\frac{dF}{A d\theta} = k\,(P_s^o - P_g) \qquad (10)$$

where

k = coefficient of mass transfer, lbs/ft^2hr (psi.)

P_s^o = vapor pressure of water at temperature T_s^o, psi.abs.

P_g = vapor pressure of water in drying gases, psi.abs.

Heat Balance (References 13, 11 and 22):

$$\frac{dQ}{Ad\theta} = \lambda_{T_s^o} \frac{dW}{Ad\theta} + (X_o \, \rho_{Av} \, c_{Av}) \frac{dT_{Av}}{d\theta} \qquad (10a)$$

where

$\lambda_{T_s^o}$ = latent heat of water at T_s^o

X_o = thickness of stock for stock drying from one face, ft

= one-half thickness of stock for stock drying from two faces, ft

$\frac{dT_{Av}}{d\theta}$ = rate of increase of average stock temperature at any instant, °F/hr

Equilibrium at the Drying Interface (Reference 22):

$$P_s^o = f_E \, (T_s^o) \qquad (11)$$

where

f_E denotes the equilibrium relationship between vapor pressure and temperature for water.

The simultaneous solution of the above equations is possible in principle, but extremely difficult in practice. It is apparent, however, that $\frac{dF}{Ad\theta}$, $\frac{dQ}{Ad\theta}$, T_s^o, and $\frac{dT}{dX}$ as a function of X, are all determined at any given time θ. When

$$\frac{dT_{Av}}{d\theta} = 0 = \frac{dT}{dX} \quad \text{at } \theta = \theta_1 \text{ and } F = F_1 \, ;$$

conditions become temporarily stabilized and Period I begins with the surface temperature of the stock at its equilibrium value of T_s'.

PERIOD I

Heat Transfer to Stock Surface (Reference 22):

$$\frac{dQ}{Ad\theta} = \alpha h_c \, (T_g - T_s') \tag{12}$$

Heat Transfer to Stock Interior (References 11 and 13):

$$\frac{\partial T_i}{\partial \theta} = a_{Av} \frac{\partial^2 T_i}{\partial X_i^2} = 0 \tag{13}$$

Heat Balance (References 11, 13 and 22):

$$\frac{dQ}{Ad\theta} = \lambda_{T_s'} \frac{dF}{Ad\theta} + 0 \tag{14}$$

Water Transfer from Stock Surface (Reference 22):

$$\left(\frac{dF}{Ad\theta}\right)_s = k \, (P_s' - P_g) \tag{15}$$

Equilibrium (Reference 22):

$$P_s' = f_E \, (T_s') \tag{16}$$

Water Transfer from Stock Interior (References 4, 6, 10 and 15):

$$\left(\frac{dF}{Ad\theta}\right)_{\substack{i \\ \text{Max.}}} = g\frac{K\rho}{\mu} \left[\frac{d}{dX} (\tau_s) - \frac{d}{dX} (\tau_i)\right] \tag{17}$$

where

τ_s = surface capillary pull, lbs/ft^2

$$= f\left(\frac{\sigma}{r_s}\right) \simeq a' \frac{\sigma}{r_s} \qquad \text{(Reference 6)} \tag{18}$$

τ_i = interior opposing capillary pull, lbs/ft^2

$\cong a' \frac{\sigma}{r_i}$

K = permeability of porous solid, ft^2

μ = viscosity of water, lbs/ft hr

ρ = density of water, lbs/ft^3

σ = surface tension of water, lbs/ft

r_s = minimum <u>effective</u> radius of surface menisci, ft

r_i = corresponding <u>effective</u> radius of interior menisci, ft

a' = a constant

g = gravitational constant, ft/hr^2

$$\left(\frac{dF}{Ad\theta}\right)_{i \atop \text{Max.}} = a'g\frac{\sigma K \rho}{\mu}\left[\frac{d}{dX}\left(\frac{1}{r_s}\right) - \frac{d}{dX}\left(\frac{1}{r_i}\right)\right] \qquad (19)$$

<u>Water Balance</u>:

$$\left(\frac{dF}{Ad\theta}\right)_s = \left(\frac{dF}{Ad\theta}\right)_i \qquad (20)$$

A study of these equations shows that within Period I the rate of drying is controlled entirely by Equations (12), (14), (15) and (16). Their combination leads to:

$$\frac{\propto h_c}{\lambda_{T_s'}}(T_g - T_s') = k\left[f_E(T_s') - P_g\right] \qquad (21)$$

which, in the case of air drying, is the equation of the "wet bulb" temperature (22). T_s' is the wet bulb temperature which for the air-water system is approximately equal to the adiabatic saturation temperature (22). In the case of drying with superheated steam, it may be shown that $P_s' = f_E (T_s')$ is very close to P_g. This arises from the fact that the $P_s' - P_g$ factor necessary for the transfer of $\frac{dF}{Ad\theta}$ (Equation 15) is of such a magnitude that the difference between the temperature in equilibrium with P_s' and the temperature in equilibrium with P_g is insignificantly small. Thus, in the case of steam T_s' becomes, to an excellent approximation, $\frac{1}{f_E} (P_g)$ or, the saturation temperature corresponding to P_g. The relationships of Equation (16), i.e., the wet bulb or adiabatic saturation lines, are given in Figure 1 for various values of T_s'. Data for these lines were obtained from References (16) and (22).

The importance of Equations (19) and (20) lies in the fact that they indicate the circumstances which cause Period I to be terminated. As drying proceeds r_i decreases, gradually approaching r_s. When the difference between r_i and r_s is such that the maximum capillary pull is just sufficient to maintain $\left(\frac{dF}{Ad\theta}\right)_s$ the period ends at $F = F_2$. The first critical moisture content F_2 is thus seen to be dependent primarily upon $\frac{dF}{Ad\theta}$, $\frac{\sigma \rho}{\mu}$, X_o and the nature of the stock as it influences a', r_s, r_i, and K. Since $\frac{\sigma \rho}{\mu}$ is dependent upon temperature, F_2 may be expressed functionally as:

$$F_2 = f_2 \left(\text{stock}, \frac{dF}{Ad\theta}, X_o, T_s'\right)$$

or, neglecting the probably small influence of T_s',

$$F_2 = f_2 \left(\text{stock}, \frac{dF}{Ad\theta}, X_o\right) \qquad (22)$$

PERIOD II

Heat Transfer to Stock Surface (Reference 22):

$$\frac{dQ}{Ad\theta} = \alpha h_c \left[f_w (T_g - T_s'') + (1 - f_w)(T_g - T_{s,r}'') \right] \qquad (23)$$

where

f_w = fraction of surface remaining wet at any instant; $f_w = 1.0$ when $F = F_2$; $f_w = 0$ when $F = F_3$.

T_s'' = temperature of "wetted" fraction of stock surface during Period II, °F

$T_{s,r}''$ = temperature of "dried" fraction of stock surface during Period II, °F

Heat Transfer to Stock Interior (References 11 and 13):

$$\frac{\partial T_i}{\partial \theta} = a_{Av} \frac{\partial^2 T_i}{\partial X_i^2} \qquad (24)$$

where the boundary conditions are:

$\theta = \theta_2;\quad T_i = T_s' = T_2$ for all values of X

$\theta = \theta_3;\quad X_i = 0;\quad T_i = T_3$

Heat Balance (References 11, 13, and 22):

$$\frac{dQ}{Ad\theta} = \lambda_{T_s''} \frac{dF}{Ad\theta} + X_o \rho_{Av} c_{Av} \frac{dT_{Av}}{d\theta} \qquad (25)$$

Water Transfer from Stock Surface (Reference 22):

$$\left(\frac{dF}{Ad\theta}\right)_s = k f_w (P_s'' - P_g) + k (1 - f_w)(P_{s,r}'' - P_g) \qquad (26)$$

where

P_s'' = vapor pressure of water at temperature T_s''

and $P_{s,r}''$ = reduced vapor pressure of water at temperature T_s'' and radius of curvature r

Water Transfer from Stock Interior (References 4, 6, 10 and 15):

$$\left(\frac{dF}{Ad\theta}\right)_i = \frac{a'\sigma K\rho}{\mu} f_w \left[\frac{d}{dX}\left(\frac{1}{r}\right) - \frac{d}{dX}\left(\frac{1}{r_i}\right)\right] \tag{27}$$

Water Balance:

$$\left(\frac{dF}{Ad\theta}\right)_i = \left(\frac{dF}{Ad\theta}\right)_s + \frac{df_w}{d\theta} X_r \rho_{Av} F_r \tag{28}$$

where

r = effective radius of receded menisci, ft

X_r = distance from surface to receded menisci, ft

F_r = local moisture content of wet fraction of thickness X_r, lb/lb

Equilibrium (Reference 22):

$$P_s'' = f_E(T_s'') \tag{29}$$

$$P_{s,r}'' = f_E(T_{s,r}'') \tag{30}$$

As before, these equations will serve, in principle, for the complete definition of the period. In practice, however, their exact simultaneous solution, even with all factors known, would prove extremely tedious, and certain simplifying assumptions are therefore justified. Equation (25) may be written:

$$\frac{dQ}{Ad\theta} = B\lambda_{T_s''}\left(\frac{dF}{Ad\theta}\right) \tag{31}$$

where B is a correction factor. Combining Equations (23) and (31):

$$\left(\frac{dF}{Ad\theta}\right)_s = \frac{\alpha h_c}{B \lambda_{T_s''}} \left[f_w (T_g - T_s'') + (1 - f_w)(T_g - T_{s,r}'') \right] \qquad (32)$$

Now, if it be assumed as before that $P_s'' - P_g \simeq P_{s,r}'' - P_g$ and that both quantities are numerically small,

$$P_s'' \simeq P_{s,r}'' \simeq P_g$$

$$T_s'' \simeq \frac{1}{f_E}(P_g) \qquad (33)$$

$$T_{s,r}'' \simeq \frac{1}{f_E}(P_{g,r}) \qquad (34)$$

Equations (27), (28), (32), (33), and (34) now give an approximate accounting of the process. It is seen that the termination of the period, i.e., $f_w = 0$, is determined by the recession of the last of the surface menisci to subsurface positions and the cessation of capillary flow. This corresponds to the "pendular" state of Ceaglske and Hougen (3). The moisture content at this second critical point, F_3, may be expected, therefore, to depend upon $\frac{dF}{Ad\theta}$, a', K, $\frac{\sigma \rho}{\mu}$, X and the nature of the stock as it influences the radii of the receded menisci. Thus, a functional equation similar to (22) may be written:

$$F_3 = f_3\left(\text{stock}, \frac{dF}{Ad\theta}, X_o\right) \qquad (35)$$

PERIOD III

Heat Transfer to Evaporating Surface (Reference 13):

$$\frac{dQ}{Ad\theta} = \frac{1}{\frac{1}{\alpha h_c} + \frac{X}{K_E}} (T_g - T_x''') \qquad (36)$$

(This equation neglects the effect of heat stored in the dried layer)

where

X = distance from the surface to the zone of evaporation, ft

K_E = thermal conductivity of the stock under conditions near its equilibrium moisture content, BTU ft/ft^2 hr°F

T_x''' = temperature at distance X from surface during Period III, °F

Heat Balance (References 11 and 13):

$$\frac{dQ}{Ad\theta} = \lambda_{T_x'''}\left(\frac{dF}{Ad\theta}\right) + \rho_{Av}\, C_{Av}\, X_o \left(\frac{dT_{Av}}{d\theta}\right) \qquad (37)$$

$$= B\, \lambda_{T_x'''}\left(\frac{dF}{Ad\theta}\right) \qquad (38)$$

Water Transfer from Evaporating Surface (References 15 and 22):

$$\frac{dF}{Ad\theta} = \frac{1}{\frac{\mu_v X}{K \rho_v} + \frac{1}{k}} \left(P_{x,r}''' - P_g\right) \qquad (39)$$

where

μ_v = viscosity of water vapor, lbs/ft hr

ρ_v = density of water vapor, lbs/ft hr

$P_{x,r}'''$ = vapor pressure of water at T_x''' and from radius r_x encountered at distance x, psi.abs.

Equilibrium (Reference 22):

$$P'''_{x,r} = f_E\,(T_x''', r_x) \qquad (40)$$

where

r_x = effective radius of menisci at distance X from surface, ft

Again, if the assumption of $P'''_{x,r} \simeq P_g$ be made, Equation (40) becomes:

$$T_x''' = \frac{1}{f_E}\,(P_g,\ r_x) \qquad (41)$$

The drying phenomena in Period III are therefore described, approximately, by the Equations (36), (38) and (41).