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REVERSIBLE PROPERTIES OF FERROMAGNETS

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REVERSIBLE PROPERTIES OF FERROMAGNETS

The magnetic moment of a piece of ferromagnetic material is equal to the weighted-averaged value of the cosine of the angle between the direction of magnetization of the sample and that of each atomic magnetic moment. If it be assumed that the atomic moments, even though grouped into domains, remain oriented according to a Boltzmann distribution,[1,2,3] quantitative relationships between the magnetization and the reversible properties of ferromagnets can be given. Let η be a dimensionless parameter proportional to the applied magnetic field H plus a history dependent constant. Under these circumstances,

$$\frac{M}{M_s} = \widehat{\cos\theta} = \operatorname{ctnh}\eta - \frac{1}{\eta} = L(\eta) \tag{1}$$

The reversible susceptibility parallel to the biasing field is then given by:

$$\chi^w_{rp} = M_s \frac{\partial \widehat{\cos\theta}}{\partial H} = 3\chi^w_o \frac{dL(\eta)}{d\eta} \tag{2}$$

The parametric relationship involving the η was given as early as 1911 by Gans.[4]

The derivation of the Boltzmann distribution was put on a more quantitative basis by Brown,[1,5] in the late 30's. Although reasonable agreement with experiment was noted in many cases, as was pointed out by Tebble and Corner[6] in 1950, the susceptibility is not a single valued function of the magnetization particularly when going from one hysteresis loop to another. In 1954 Grimes and Martin[7] extended the work of Brown[5] to obtain the expression for low frequencies that:

$$\chi^w_{rt} = \frac{M_s \widehat{\cos\theta}}{(H + D)} = 3\chi^w_o \frac{L(\eta)}{\eta} \tag{3}$$

where D is a history dependent term with the dimensions of H. Early this year[2,3] it was shown that equations 2 and 3 would be approximated for the case where the reversible susceptibility has its origin in domain-wall motion. It was also shown that for the case where the susceptibility is due to the rotation of all the moments of a domain that the susceptibilities were given by:

$$\chi^r_{rp} = \frac{3}{2}\chi^r_o (1 - \widehat{\cos^2\theta}) = 3\chi^r_o \frac{L(\eta)}{\eta} \tag{4}$$

and for low frequencies or infinite material:

$$\chi^{r}_{rt} = \frac{3}{4}\chi^{r}_{o}\,(1 + \widehat{\cos^2\theta}) = \frac{3}{2}\chi^{r}_{o}\,\left(1 - \frac{L(\eta)}{\eta}\right) \tag{5}$$

The exact form of the parametric equations depends upon the material remaining always oriented in "easy" crystallographic directions, thus they can be considered to have quantitative significance only for $M < \sim 0.5\ M_s$.

In a forthcoming paper[3] it is shown that the variation of the susceptibility matrix with magnetization when it results from domain rotation is given by:

$$\chi = \begin{bmatrix} \frac{3\chi_o}{4}\,(1 + \widehat{\cos^2\theta}), & K\,\widehat{\cos\theta}, & 0 \\ -K\,\widehat{\cos\theta}, & \frac{3}{4}\chi_o\,(1 + \widehat{\cos^2\theta}), & 0 \\ 0, & 0, & \frac{3}{2}\chi_o\,(1 - \widehat{\cos^2\theta}) \end{bmatrix} = \begin{bmatrix} \frac{3\chi_o}{2}\,\left(1 - \frac{L(\eta)}{\eta}\right), & K\,L(\eta), & 0 \\ -KL(\eta), & \frac{3\chi_o}{2}\,\left(1 - \frac{L(\eta)}{\eta}\right), & 0 \\ 0, & 0, & 3\chi_o\,\frac{L(\eta)}{\eta} \end{bmatrix} \tag{6}$$

In the usual form for which the matrix is written for microwave application the susceptibility term $\chi_{11} = \chi_{22} = \frac{3}{2}\chi_o$. The off-diagonal terms are proportional to M, a fact previously pointed out by Rado.[8]

In the same paper,[3] the differential magnetostriction "d" for each of the four cases where equations 2, 3, 4, and 5 are applicable is described. The results are based upon the equation:

$$\lambda = \frac{3}{2}\lambda_s\,\left(\widehat{\cos^2\theta} - \frac{1}{3}\right) \tag{7}$$

where $\lambda_s = \lambda_{100} = \lambda_{111}$ is the saturation magnetostriction and θ is the angle between the applied magnetic field and the magnetization. The results are that "d" for the case of domain-wall motion and parallel fields is given by:

$$d^{w}_{p} = \frac{3}{2}\lambda_s\,\frac{\partial\,\widehat{\cos^2\theta}}{\partial H} = \frac{3\,d^{w}_{o}}{\eta}\left(\frac{L(\eta)}{\eta} - \frac{dL(\eta)}{d\eta}\right) \tag{8}$$

for domain-wall motion and transverse fields by:

$$d_t^w = \frac{3\,\lambda_s}{2\,(H + D)}\left(\widehat{\cos^2\theta} - \frac{1}{3}\right) = \frac{d_o^w}{\eta}\left(1 - \frac{3\,L(\eta)}{\eta}\right) \tag{9}$$

for domain rotation and parallel fields by:

$$d_p^r = \frac{9\,\lambda_s\,\chi_o^r}{2\,M_s}\left(\widehat{\cos\theta} - \widehat{\cos^3\theta}\right) = \frac{3\,d_o^r}{\eta}\left(1 - \frac{3\,L(\eta)}{\eta}\right) \tag{10}$$

and for domain rotation and transverse fields by:

$$d_t^r = \frac{9\,\lambda_s\,\chi_o^r}{4\,M_s}\left(\widehat{\cos\theta} + \widehat{\cos^3\theta}\right) = \frac{3\,d_o^r}{2}\left[L(\eta)\left(1 + \frac{3}{\eta^2}\right) - \frac{1}{\eta}\right] \tag{11}$$

where:

$$d_o = \frac{3\,\lambda_s\,\chi_o}{M_s} \quad . \tag{12}$$

Independent of the exact form of the distribution, so long as the effective magnetic field can be considered as the sum of the applied field and a history dependent field, the wall-motion susceptiblities are related by:

$$\chi_{rp}^w = \chi_{rt}^w + \frac{1}{\eta}\,\frac{d\,\chi_{rt}^w}{d\eta} \quad . \tag{13}$$

Thus if χ_{rp}^w is a monotonic decreasing function of M, then χ_{rt}^w must also be monotonic decreasing if the two susceptibilities are equal when M = 0.

Dependent only upon the applied magnetic field being much smaller than the anisotropy field, the following relationship holds:

$$3\,\chi_o^r = 2\,\chi_{rt}^r + \chi_{rp}^r \quad . \tag{14}$$

So if χ_{rp}^r is a monotonic decreasing function of M, χ_{rt}^r must be monotonic increasing function. These differences point out a new technique for the separation of the magnetization mechanisms. Namely, the measurement and comparison of the parallel and transverse reversible susceptibilities.

Figure 1 shows the expected variation of the wall-motional susceptibilities according to equations 1, 2, and 3; Figure 2 shows the expected variation of the domain-rotational susceptibilities according to equations 1, 4, and 5; Figure 3 shows the expected wall-motional differential magnetostrictions according to equations 8 and 9; Figure 4 shows the expected domain-rotational differential magnetostrictions according to equations 1, 10, and 11. Note that the largest value of d is expected from d_t^r, and remember no quantitative correlation is expected for $M \gtrsim 0.5\ M_s$.

Experimental data have been taken on several ferrite samples to compare with the theoretical curves. Figure 5 shows susceptibility data from specimen F-6-2, which was made in our laboratory by mixing to the composition $Ni_{.1682}Co_{.2992}Zn_{.5326}Fe_2O_4$, firing at 1375°C for 1/2 hour, 1200°C for 2 hours and then slowly cooling---all in a N_2 atmosphere. The shaded area of Figure 5 represents the area between the susceptibility curves calculated from equations 2 and 3 and from assuming all moments to be either parallel or antiparallel with the applied magnetic field. From the data of Figure 5 it is concluded that the susceptibility of F-6-2 is very predominantly due to domain rotation. To check this point the relaxation frequency was calculated from the equation:

$$f = \frac{\gamma}{2\pi} \cdot \frac{2\ M_s}{3\chi_o} \qquad (15)$$

and found to be on the order of 150 mc/sec. Experimentally, the peak in the loss portion of the susceptibility was found to be 140 mc/sec,9 and the frequency at which the susceptibility was one-half of the initial value was 160 mc/sec, verifying the rotational susceptibility mechanism.

Figure 6 shows a similar curve for core I-15-1. It is from a batch of material prepared by Dr. D. Fresh at the U.S. Bureau of Mines, College Park, Maryland. The frequency dependence of this sample has been measured by Rado[10] et.al., and will be analysed by him at the forthcoming London conference. The composition is assumed to be the same as that of their "Ferrite F": $Mg_{.97}Fe_{.03}Fe_2O_4$. The measurement of the reversible susceptibility of this particular material was initiated by request of Dr. Rado. The conclusion from Figure 6 is that one-half or more of the initial susceptibility is due to domain wall motion.

Figure 7 shows still another similar set of data on core F-10-2. F-10-2 was fired with F-6-2, but is believed to be of the composition $Ni_{.3800}Fe_{.1282}Co_{.0330}Zn_{.4588}Fe_2O_4$.[13] From the data of Figure 7 it is inferred that over one half of the susceptibility is due to domain-wall motion.

Figure 8 shows a comparison of the experimental differential magnetostriction data of Bozorth and Williams[11] with the results of equations 1, 8, and 12. d_o was taken from their values for B_s and λ_s while χ_o was taken from Bozorth.[12] There have been no arbitrary scale corrections. Equation 8 is deemed superior to their corresponding equation for (a) as they pointed out, they used a domain-rotation model where the susceptibility was due to domain wall motion, and (b) they substituted

$$\widehat{\cos\,\theta^3} \quad \text{for} \quad \widehat{\cos^3\,\theta}.$$

In conclusion, I wish to thank Dr. Rado for making their manuscript available to me before the conference.

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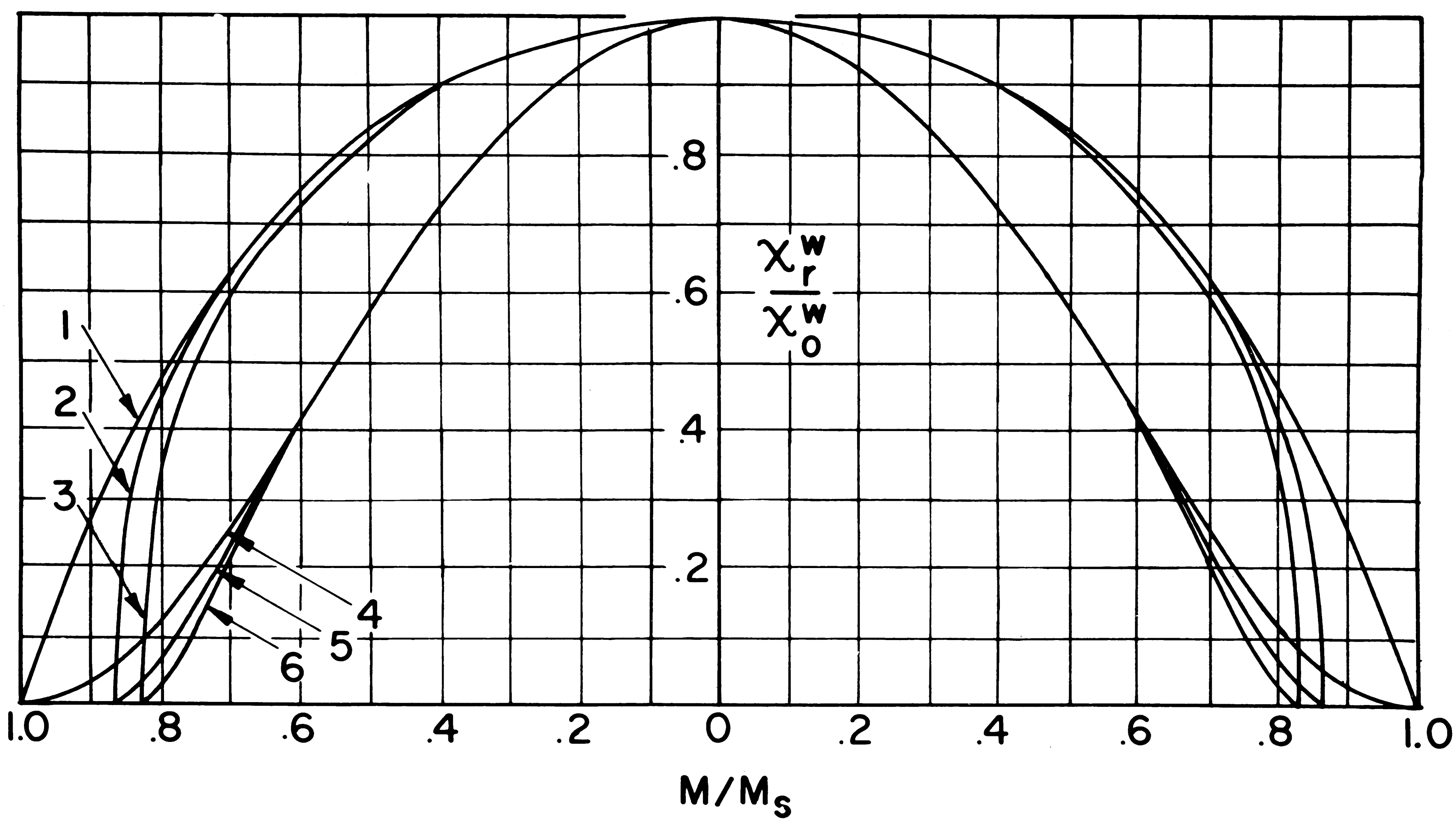
Figure 1.

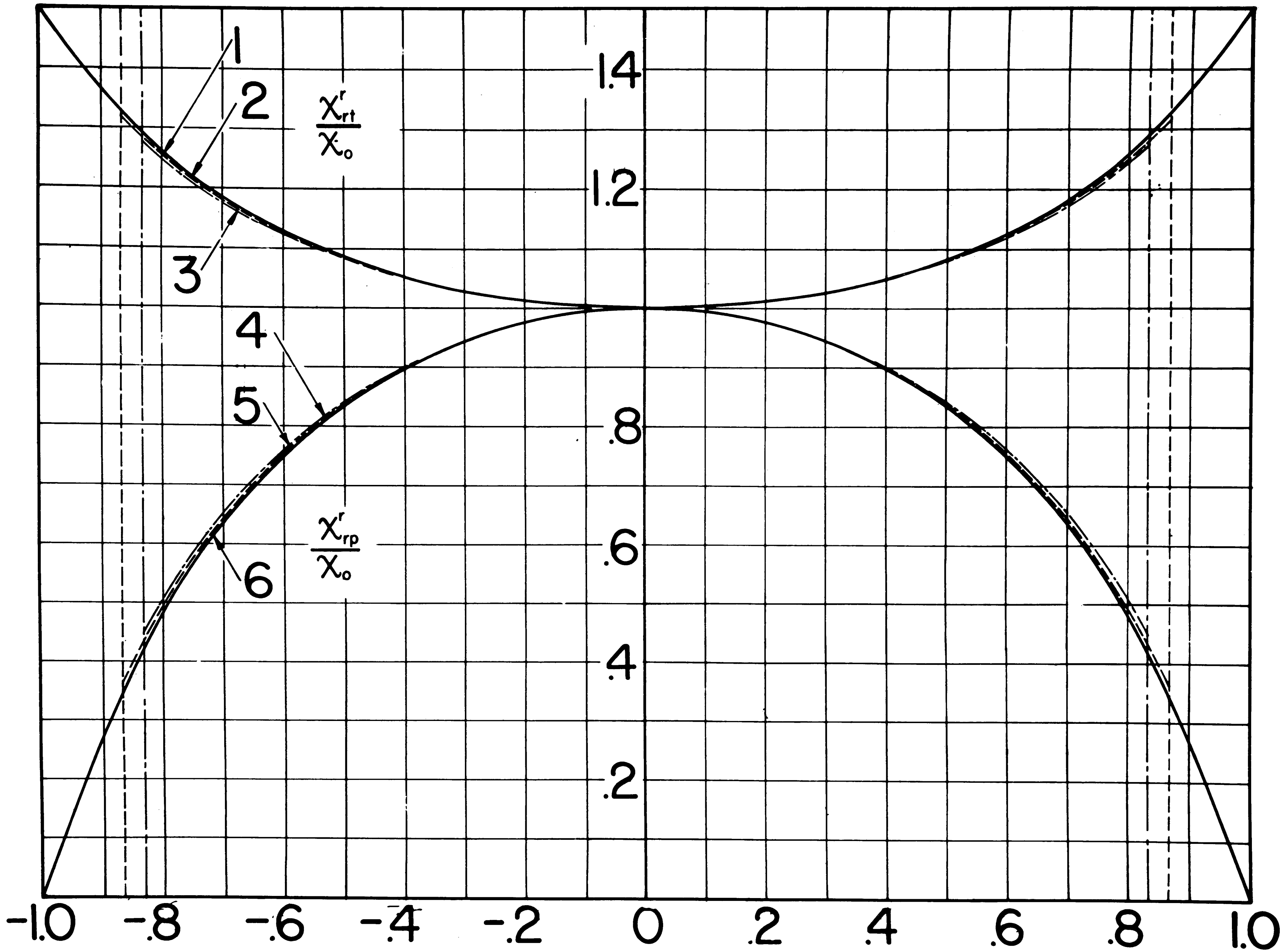
Theoretical reversible susceptibility due to domain-wall motion. Curves 1, 2, and 3 are for transverse fields, curves 4, 5, and 6 are for parallel fields. Curves 3 and 6 are for six possible directions of orientation of the magnetic moment in the single crystal. Curves 2 and 5 are for eight possible directions, and curves 1 and 4 are for an infinite number of such directions.



.8
.6
.4
.2
$\frac{\chi_r^w}{\chi_o^w}$
1
2
3
4
5
6
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
M/M$_s$

Figure 2.

Theoretical reversible susceptibility due to domain rotation. Curves 1, 2, and 3 are for transverse fields, curves 4, 5, and 6 are for parallel fields. Curves 3 and 4 are for six possible directions of static orientation of the magnetic moment in the single crystal. Curves 2 and 5 are for eight possible directions and curves 1 and 6 are for an infinite number of such directions.


1
2
3
4
5
6
$\frac{\chi^r_{rt}}{\chi_o}$
$\frac{\chi^r_{rp}}{\chi_o}$
1.4
1.2
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0

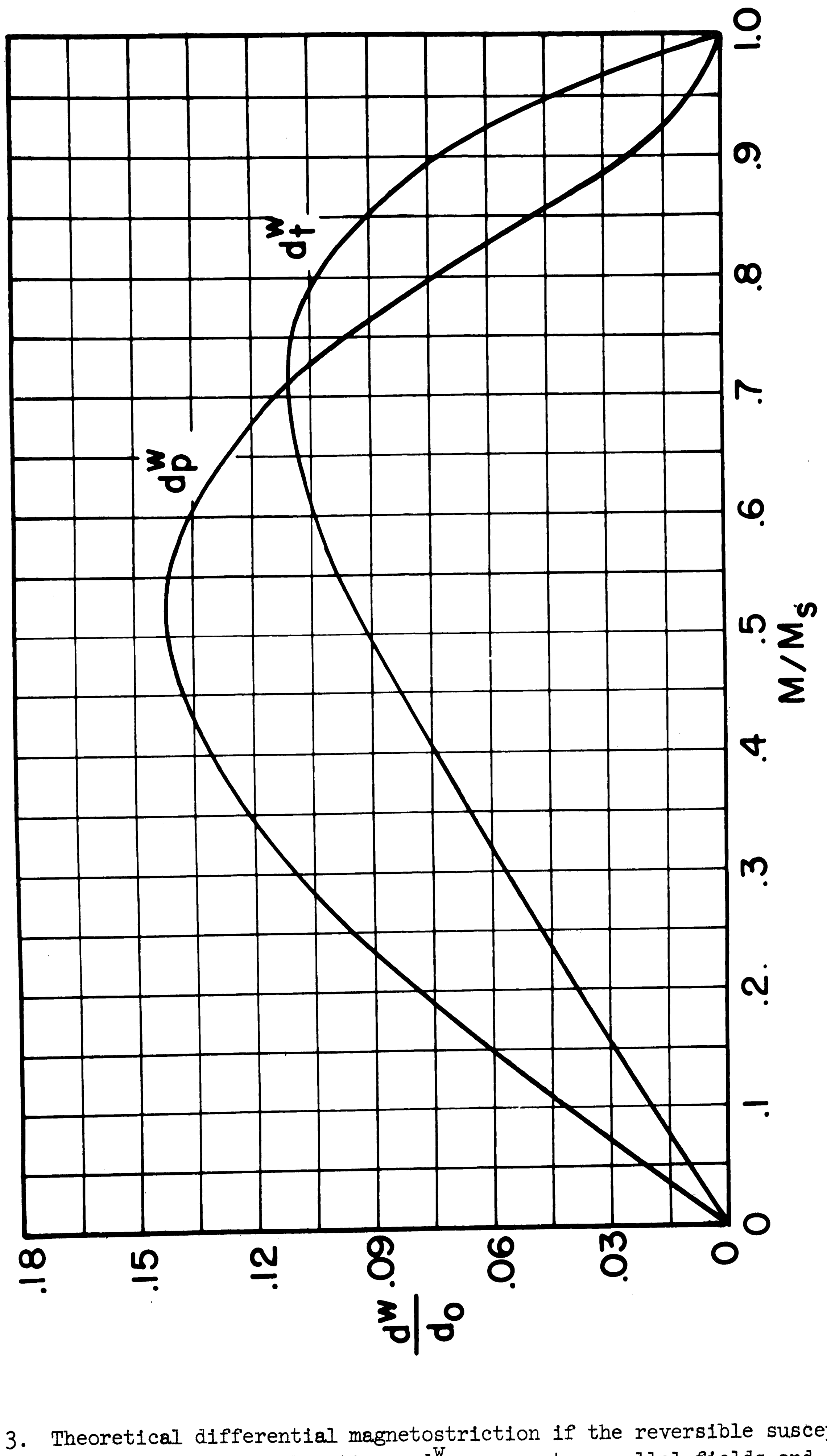


Figure 3. Theoretical differential magnetostriction if the reversible susceptibility is due to domain-wall motion. d_p^W represents parallel fields and d_t^W represents transverse fields

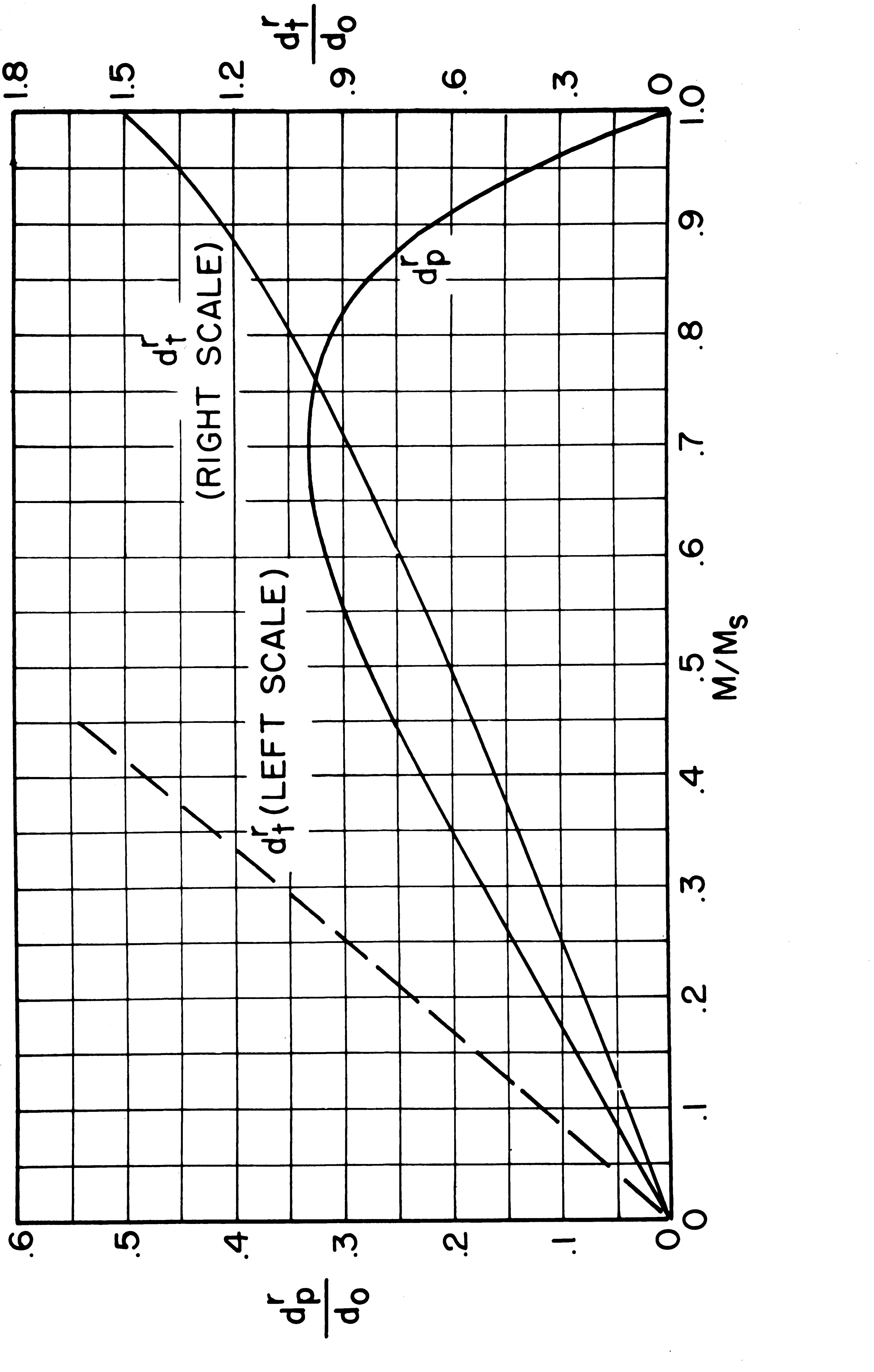
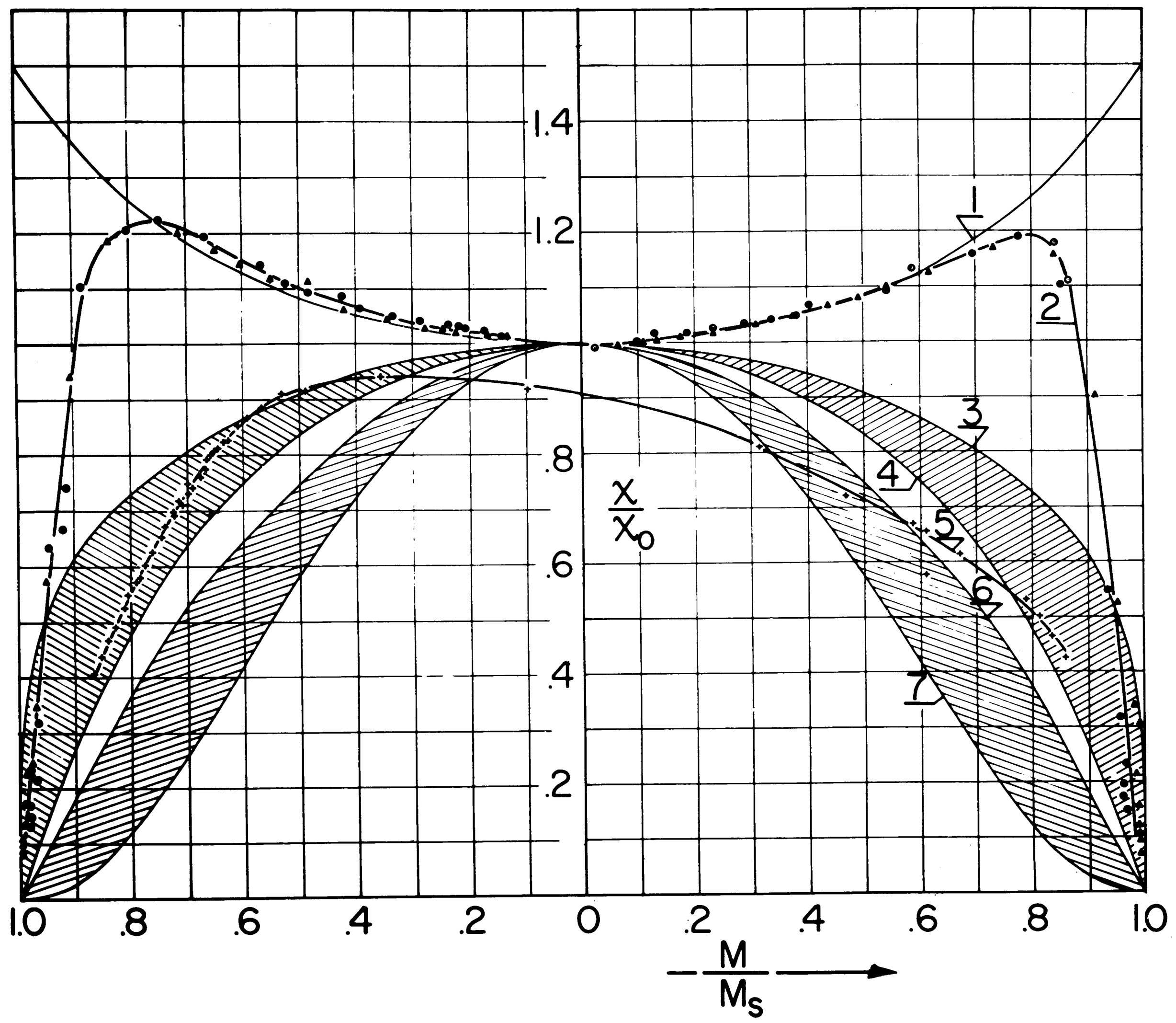


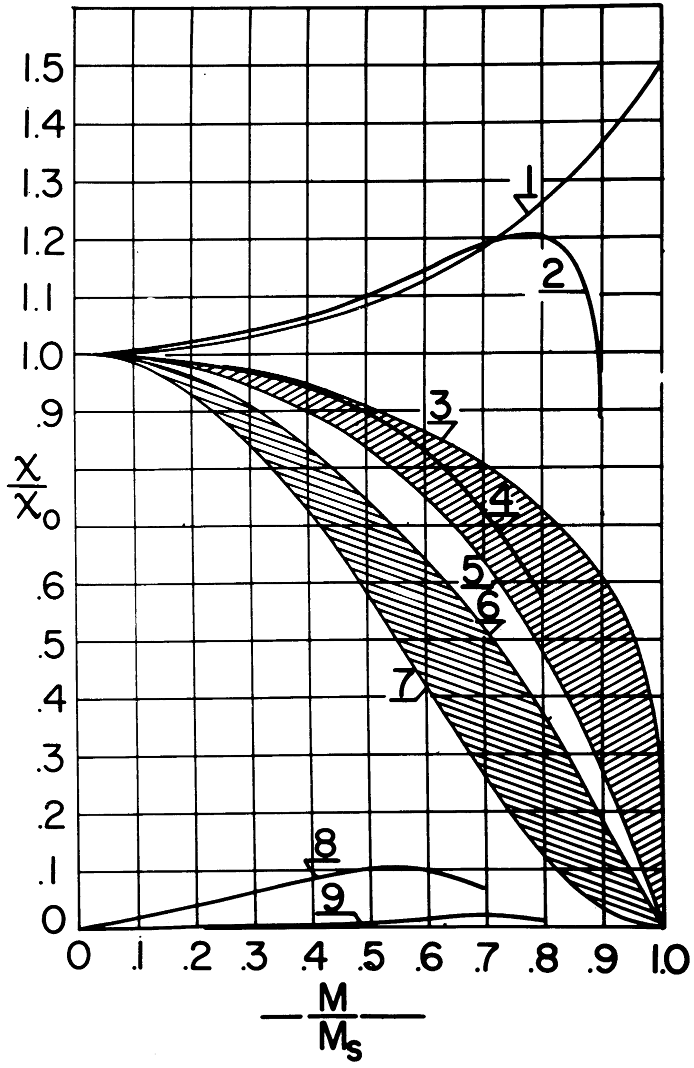
Figure 4. Theoretical differential magnetostriction if the reversible susceptibility is due to domain rotation. d_p^r represents parallel fields and d_t^r represents transverse fields.

Figure 5.

Figure 5a. represents the variation of the reversible susceptibility with magnetization. Curves 2 and 5 represent the experimental data taken on Core F-6-2 for transverse and parallel fields respectively at both 10 and 320 kc/sec. Curves 1 and 4 represent the theoretical curves for transverse fields for domain rotation and domain-wall motion respectively; curves 4 and 7 represent the variation for parallel fields, domain rotation and domain-wall motion respectively. Curves 3 and 6 represent transverse and parallel fields for all moments either parallel or antiparallel with the applied field when the susceptibility is due to domain-wall motion. The arrow indicates the direction of the change in magnetization.

Figure 5b. shows the corresponding theoretical curves, curve 2 represents the symmetrical part of the experimental transverse field data, curve 9 represents the antisymmetrical part; curve 4 represents the symmetrical part of the experimental parallel field data; curve 8 represents the antisymmetrical part.


1.4
1.2
.8
.6
.4
.2
X/X_0
1
2
3
4
5
6
7
1.0 .8 .6 .4 .2 0 .2 .4 .6 .8 1.0
M/M_s


1.5
1.4
1.3
1.2
1.1
1.0
.9
X/X_0
.6
.5
.4
.3
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M/M_s

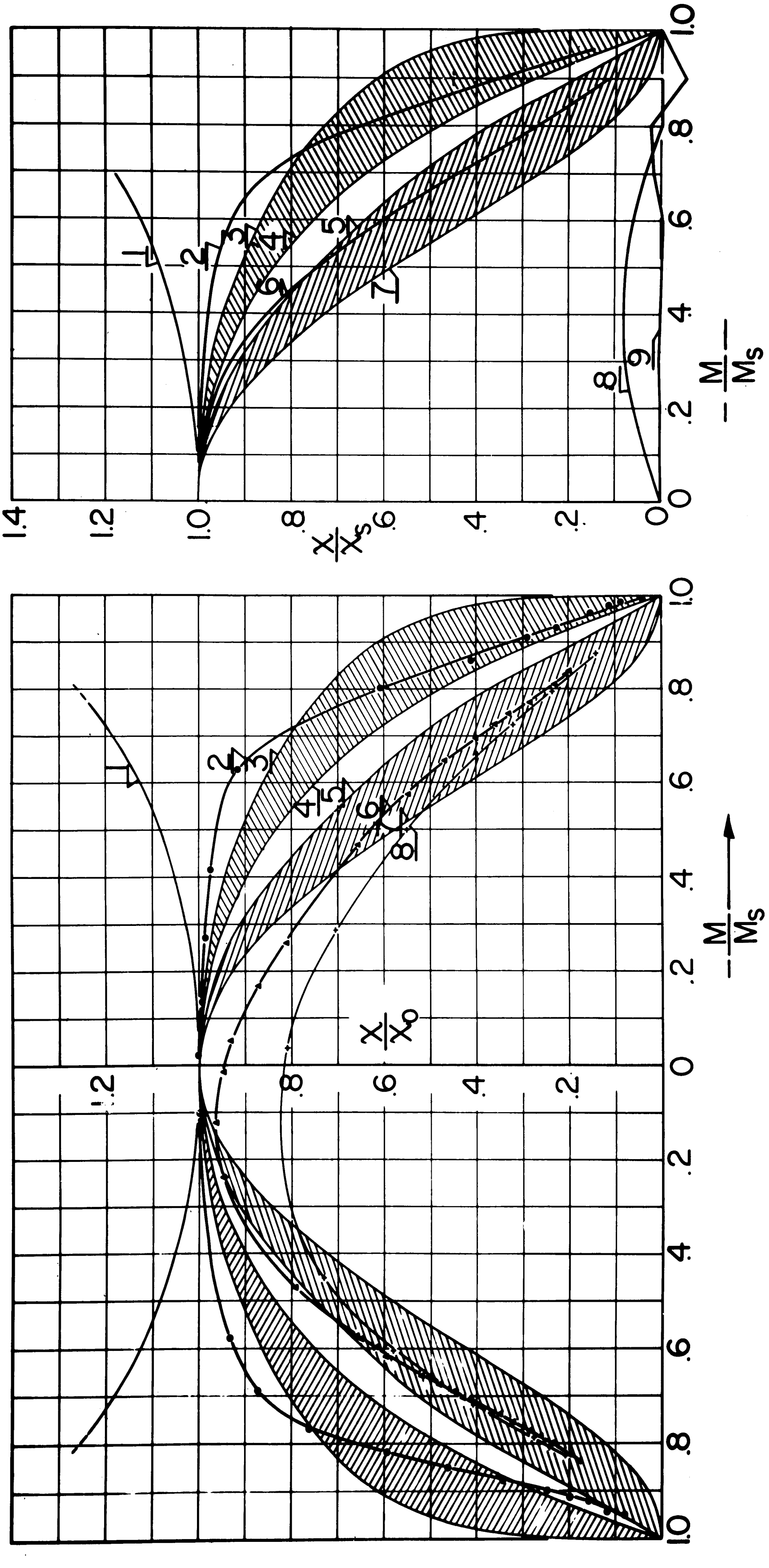

Figure 6. This figure represents the same data for specimen I-15-1 as that illustrated by Figure 5 for F-6-2.

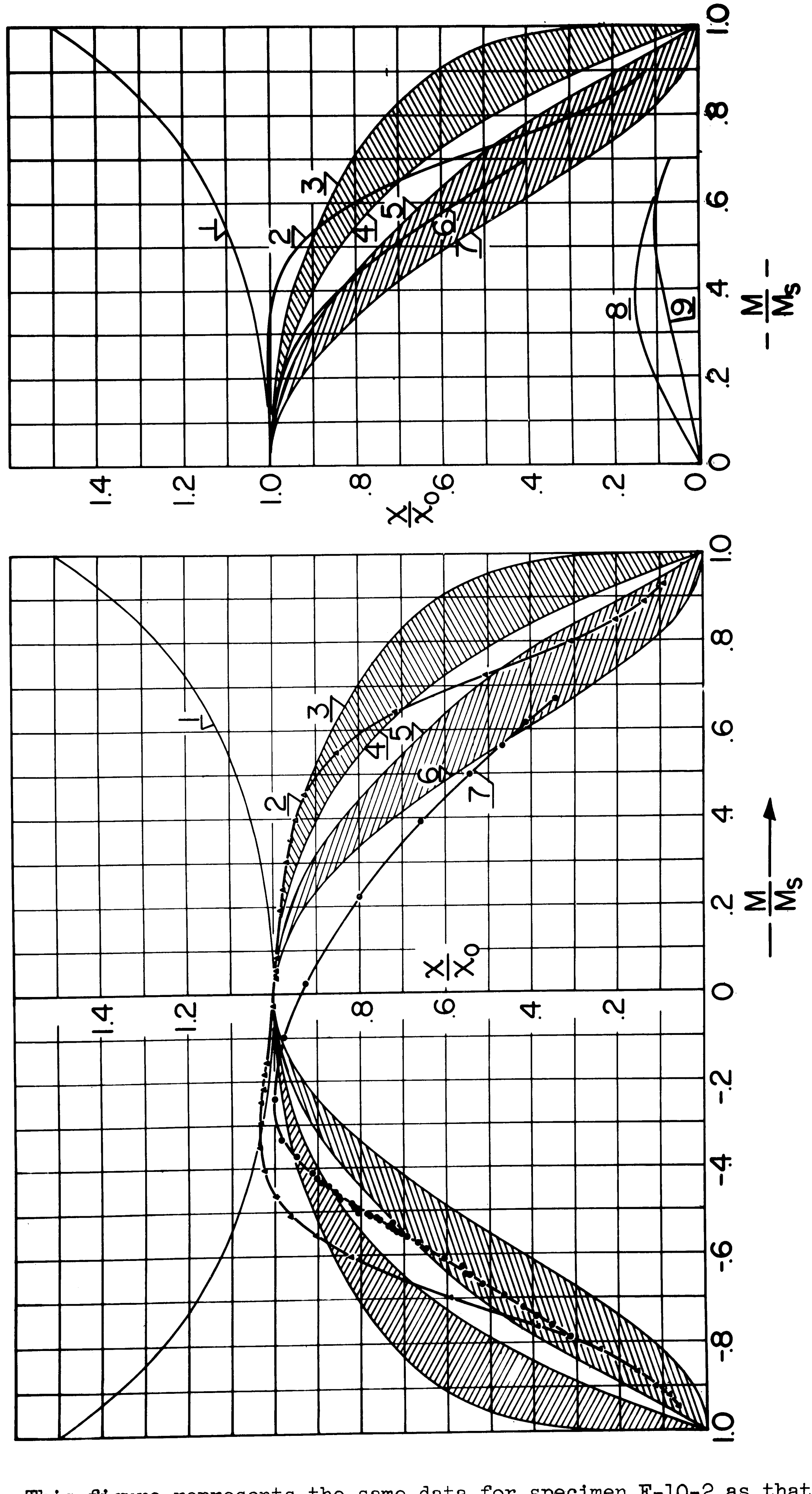

Figure 7. This figure represents the same data for specimen F-10-2 as that illustrated by Figure 5 for F-6-2 except that the data were taken only at 320 kc/sec.

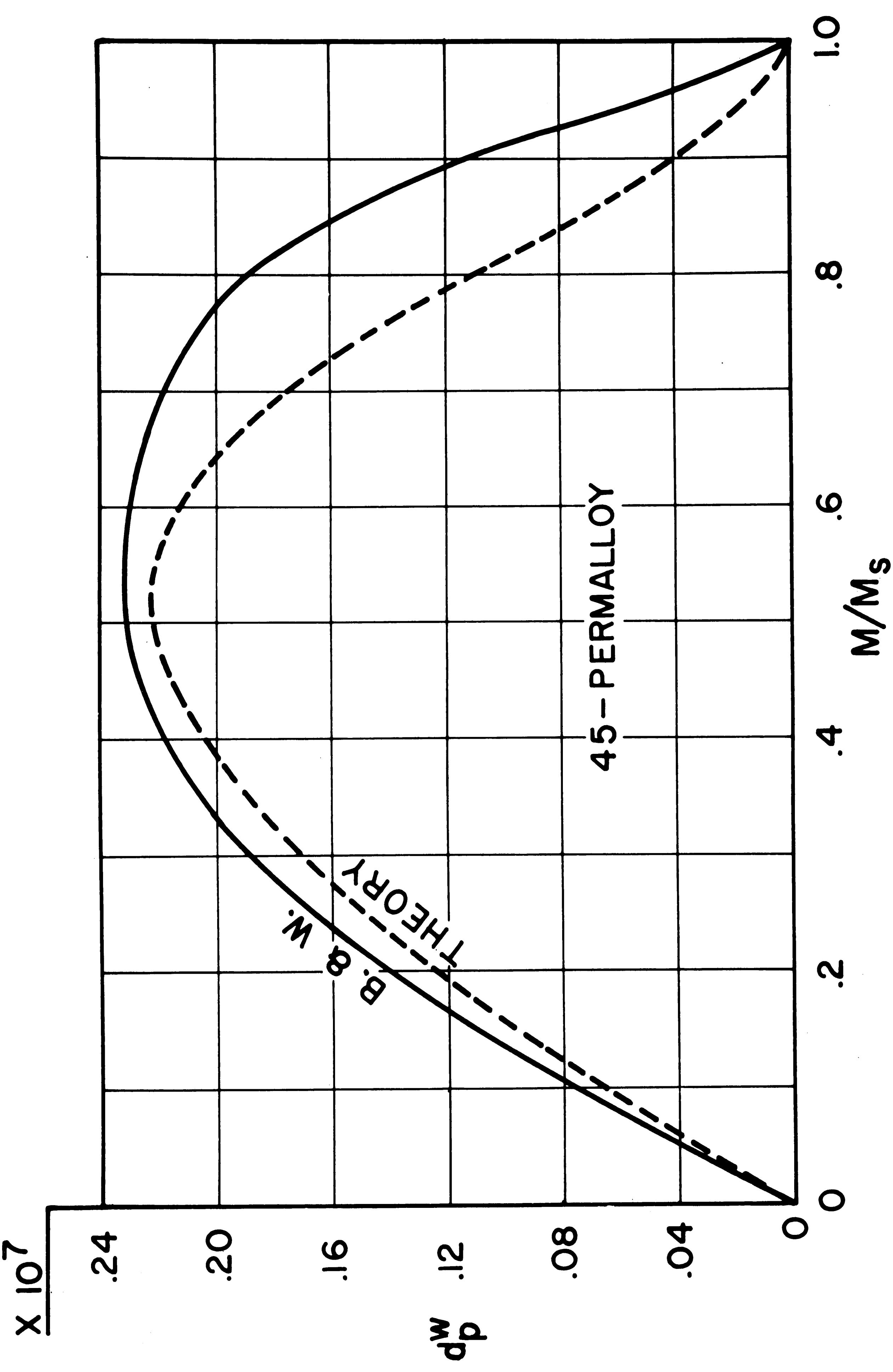

Figure 8. A comparison of the theoretical curve for the variation of the parallel differential magnetostriction due to domain-wall motion with that reported by Bozorth and Williams for Permalloy 45. There have been no arbitrary scale corrections.