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THEORY OF REVERSIBLE MAGNETIC SUSCEPTIBILITY WITH APPLICATION TO FERRITES

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ABSTRACT

Formulas expressing the reversible susceptibility both parallel to and perpendicular to the biasing field are developed based on Brown's statistical theory. It is concluded that parallel fields give a larger frequency ratio change for a given change in biasing field than do the transverse fields. Expressions giving the temperature coefficient of the frequency ratio in terms of J/J_s, $J_s(T)$ and $\chi_o(T)$ are developed. A table is presented showing when each type of field has the lower temperature coefficient. The material under special consideration is the ferromagnetic spinels -- ferrites.

THEORY OF REVERSIBLE MAGNETIC SUSCEPTIBILITY WITH APPLICATION TO FERRITES

1. INTRODUCTION

If a small magnetic field ΔH is applied to a magnetic body the induced magnetization will be ΔJ. The magnetic flux $\Delta B = \Delta H + 4\pi\Delta J$. The incremental susceptibility χ_Δ, is defined by:

$$\chi_\Delta = \frac{\Delta J}{\Delta H} .$$

The incremental permeability, μ_Δ, is defined by:

$$\mu_\Delta = \frac{\Delta B}{\Delta H} .$$

The reversible susceptibility, χ_r, is defined by:

$$\lim_{\Delta H \to 0} \chi_\Delta = \chi_r .$$

If a biasing magnetic field, H_0, is applied to the specimen, χ_r will depend upon the magnetic history of the sample, the magnitude of H_0, and the angle α. α is defined as the angle between the vectors H_0 and ΔH.

We will designate χ_r when $\alpha = 0$ as χ_{rp}, when $\alpha = \pi/2$ as χ_{rt}. In general, χ_r can be expressed as:

$$\chi_r = \chi_{rp} \cos \alpha + \chi_{rt} \sin \alpha \qquad (1)$$

Equations are derived for both χ_{rp} and χ_{rt} in isotropic materials and in anisotropic materials magnetized in the [111] direction. Their temperature coefficient is expressed in terms of $\chi_o(T)$, $J_s(T)$ and J/J_s. On the basis of these equations the two types of magnetization are compared.

The results are applicable to most ferro-spinels, or ferrites since the [111] direction is the direction of easy magnetization in their crystalline structure. It is concluded that for the largest change in permeability ratio parallel fields should be used. This is of significance in the design of magnetic tuning units.

2. DERIVATION OF χ_{rp} AND χ_{rt}

2.1 General Remarks

Brown[1,2,3] derived a theoretical expression for χ_{rp} as a function of the magnetization J for various classes of ferromagnetic crystalline anisotropy. He assumed N domains per unit volume*, each with an equal and fixed volume. He assumed each domain was magnetized to saturation in some direction of "easy magnetization," that is, a direction requiring minimum free energy for magnetization. The partial derivative of the free energy with respect to magnetizing angle in the domain is the generalized anisotropy force.

Physically, the anisotropy forces are those forces which act to prevent the magnetization in a given domain from being magnetized in a direction other than the "easy" direction.

When Brown's assumption is used, if an external field is applied to the specimen the number of domains favorably oriented with respect to the field increases at the expense of the number of those domains not so favorably oriented.

*For readers unfamiliar with the theory of magnetic domains, reference No. 9 is recommended.

Although this model is highly artificial, it can be shown[3,4] that the model assumed does not significantly alter the macroscopic results. The microscopic forces are a sum of random and ordered forces both constant for a given domain. When integrated over a macroscopic volume of material, the results depend upon the energy magnitudes and not upon the exact nature of the forces. Thus, as long as one deals with purely reversible phenomena, the mechanism through which the forces act is unimportant.

Brown[1,2,3] utilizes these assumptions in a statistical approach to obtain the parallel reversible susceptibility and the magnetostriction in a ferromagnetic material. His results may be stated as follows:

$$J = \int \frac{S_o}{S} \frac{d\omega}{4\pi} \qquad (2)$$

where:

$$S = \int \exp \left[L_o H_r J_p (\theta) \right] \frac{d\omega}{4\pi}$$

$$S_o = \int J_p (\theta) \exp \left[L_o H_r J_p (\theta) \right] \frac{d\omega}{4\pi} .$$

$J_p (\theta)$ is the component of magnetization parallel to H_r when the domain is magnetized at an angle θ with respect to the field.

$d\omega$ is an increment of solid angle.

H_r is a multivalued function of the applied field and the history of the sample. It is the value of the applied field that would be necessary to produce a magnetization J in the sample if there were no irreversible internal processes, i.e., no hysteresis. It can be defined by the following equation:

$$H_r = \int_0^J \frac{1}{\chi_r} \, dJ \; .$$

Note that H_r, J, and $J_p(\theta)$ are parallel by these definitions.

L_o is a constant for a given material at a given temperature. It is a measure of the internal energy of the material and has the dimension (energy per unit volume)$^{-1}$.

J_s is the saturation magnetization. It is the value of magnetization in the material when all domains are aligned. It is also at all times the magnetization in a single domain.

For the case of isotropic materials $J_p(\theta) = J_s \cos \theta$. Using this, Eq 2 becomes:

$$J = \frac{S_o}{S} \; .$$

Let $\eta = L_o H_r J_s$ and $\gamma = \cos \theta$. Then:

$$S = \frac{1}{2} \int_{-1}^{1} e^{\eta\gamma} \, d\gamma = \frac{\sinh\eta}{\eta} \; ,$$

$$S_o = J_s \frac{dS}{d\eta} \; .$$

Upon solving for J/J_s:

$$\frac{J}{J_s} = \operatorname{ctnh}\eta - \frac{1}{\eta} \; . \qquad (3)$$

This can be expanded, for small values of η, into:

$$\frac{J}{J_s} = \frac{\eta}{3} - \frac{\eta^3}{45} + \frac{2\eta^5}{945} + \; . \; . \; . \; . \; . \qquad (3a)$$

If the magnetic material is assumed to have domains magnetized in the [111] direction,

$$J_p(\ell) = \frac{J_s}{\sqrt{3}} \sum_i \ell_i \tanh\left(\frac{\eta\,\ell_i}{3}\right).$$

The ℓ_i's are the direction cosines of the directions of easy magnetization with respect to the crystalline axis. (In this case there are four such directions.) For this condition:

$$\frac{J}{J_s} = \frac{3}{\eta^2} \int_0^{\frac{\eta}{\sqrt{3}}} u \tanh u \, d u \quad . \tag{4}$$

If one puts this in the form of a series solution:

$$\frac{J}{J_s} = \frac{\eta}{3} - \frac{\eta^3}{45} + \frac{2\eta^5}{945} + \ldots\ldots \tag{4a}$$

For either the isotropic case or the case of [111] domains J/J_s can be expressed as:

$$\frac{J}{J_s} = f(\eta) \quad . \tag{5}$$

From Eqs 3a and 4a it is observed that the series expansion is identical to the seventh order in η. Upon differentiating with respect to H_r, both equations give:

$$\chi_o = \frac{1}{3} L_o J_s^2 \tag{6}$$

where

$$\lim_{H_r \to 0} \chi_r = \chi_o \quad .$$

2.2 Derivation of χ_{rp}

From the definition of χ_{rp},

$$\chi_{rp} = \frac{dJ_p}{dH_r} \quad .$$

From Eqs 5 and 6 and the definition of η ,

$$\frac{\chi_{rp}}{\chi_o} = 3 f'(\eta) . \qquad (7)$$

The prime indicates $d/d\eta$. Figure 1 shows a plot of χ_{rp}/χ_o (Eq 7) for both the isotropic case (Eq 3) and the case of magnetization in the [111] direction (Eq 4) against the normalized magnetization (Eq 5). From the curve it is seen that the two curves are superimposed over the range $0 < J/J_s < 0.6$.

At this point it becomes necessary to examine the mechanism through which the magnetic field exerts itself in order to obtain a better idea of the validity of the assumptions made. In the isotropic material we are in effect assuming that rotations can and do occur at any value of magnetization. In the case of the anisotropic material we assume no rotation. In a typical B-H loop, when $J/J_s \simeq 0.6$, rotations begin to occur. As J/J_s increases, rotations become increasingly more important. This is the criteria for isotropic materials. Therefore even though the above anisotropic assumptions are made, at high values of applied field the equations for isotropic material must be used.

From Eqs 3a and 4a (see Fig. 1), the two curves are identical at low fields. At high fields Eq 3 must be used. Therefore it is assumed that Eq 3 can be used throughout the entire region. (Fig. 1, Curve (1)).

In at least the ferrites of iron and nickel, the magnetization direction requiring the least energy expenditure are the [111] directions.[5,6,7] Therefore, we will use throughout $f(\eta) = \text{ctnh}\ \eta - 1/\eta$.

2.3 Derivation of χ_{rt}

In a multicrystalline specimen when H_r is rotated through a small angle[8] J remains parallel to H_r and thus is rotated through the same small angle.

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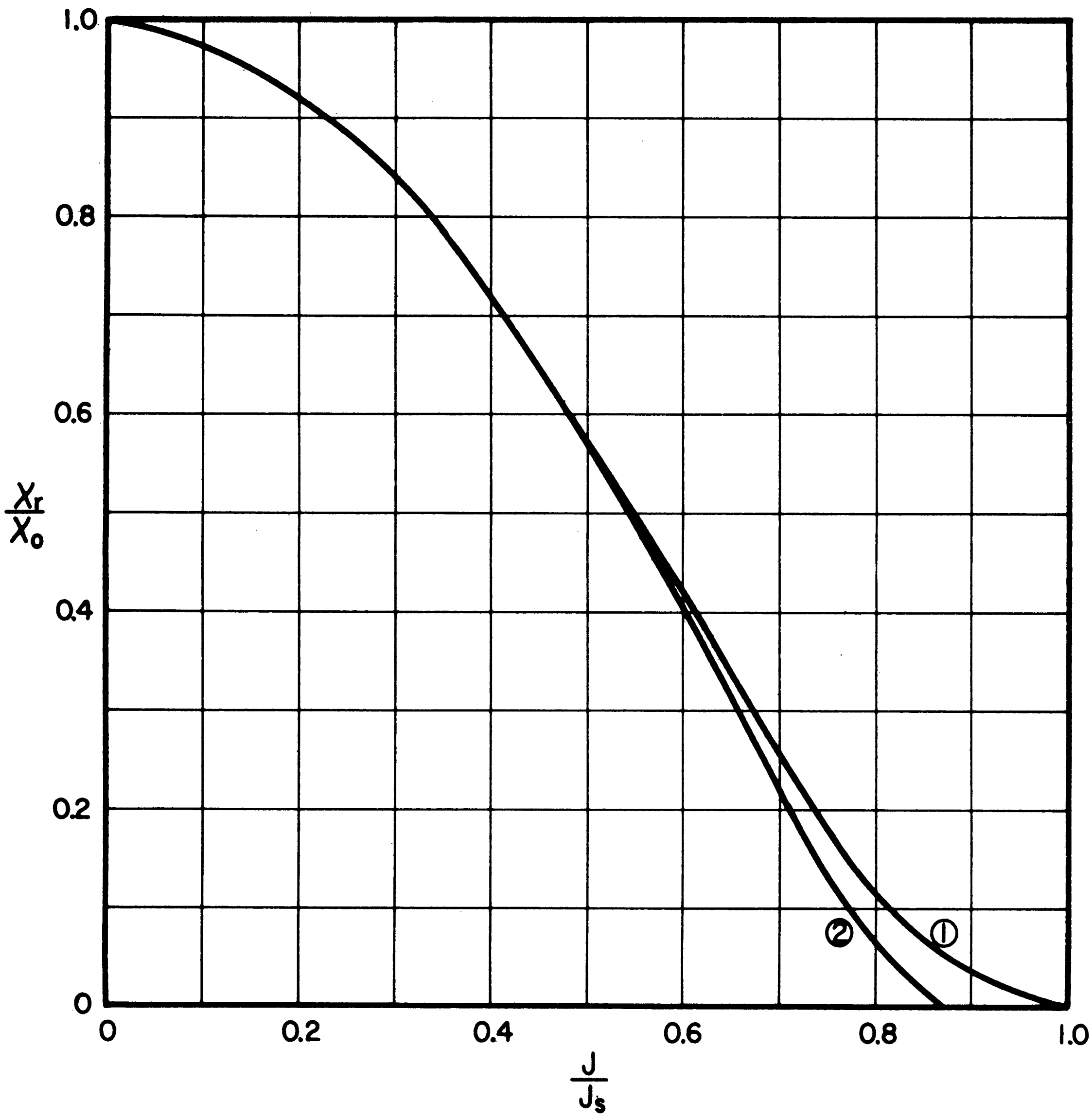


FIG. 1
REVERSIBLE SUSCEPTIBILITY
CURVE NO. 1 ISOTROPIC MATERIAL
CURVE NO. 2 MATERIAL MAGNETIZED
IN [111] DIRECTION

Therefore the transverse reversible susceptibility, χ_{rt}, is given by: (see Fig. 2)

$$\chi_{rt} = \frac{dJ}{dH_r} = \frac{J}{H_r} .$$

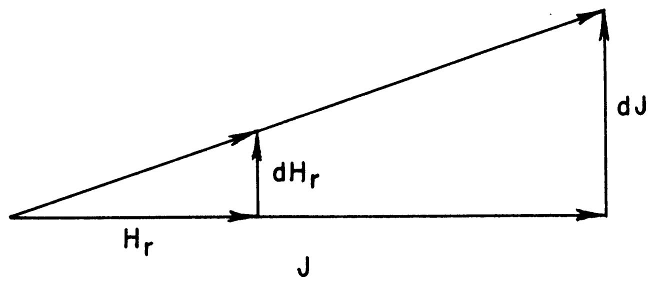


Fig. 2

The Relationship Between dJ_t/dH_r and J/H_r

In order to relate χ_{rt} to χ_{rp}:

$$\frac{dH_r}{dJ} = \frac{d}{dJ}\left[\frac{J}{\chi_{rt}}\right] = \frac{1}{\chi_{rp}} .$$

Upon combining with Eqs 5 and 7,

$$\frac{\chi_{rt}}{\chi_o} = 3\,\frac{f(\eta)}{\eta} . \qquad (8)$$

A plot of χ_{rp}/χ_o (Eq 7) and χ_{rt}/χ_o (Eq 8) against J/J_s (Eq 5) using the Langevin function for $f(\eta)$ is given in Fig. 3.

3. SPECIFIC APPLICATIONS

3.1 Tuned Circuit Frequency Range

These formulas apply to electronic circuits employing variable

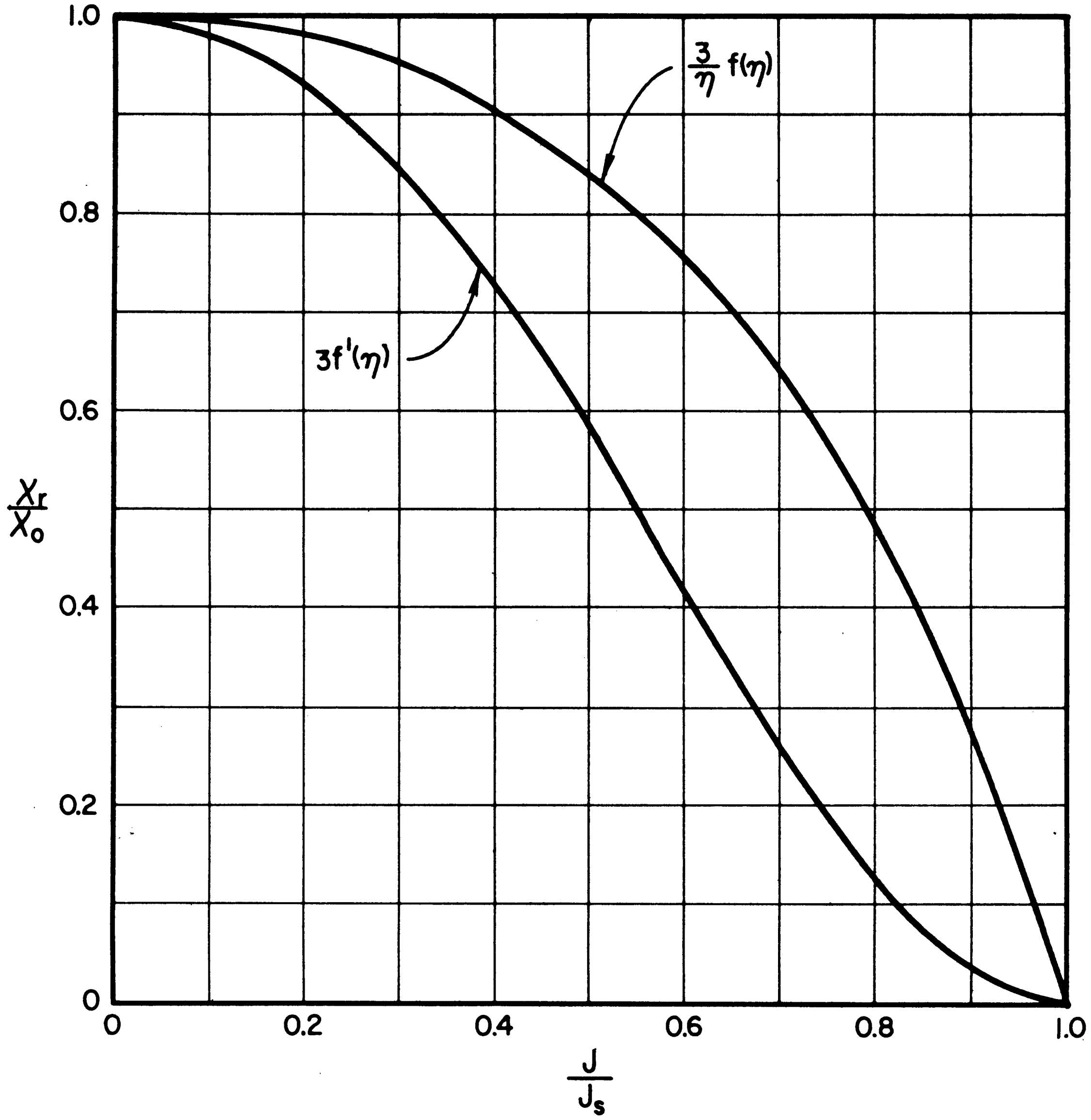


FIG. 3
THEORETICAL SUSCEPTIBILITY

permeability inductive elements where the resonance frequency is varied by the application of a biasing magnetic field.

In particular, they apply to radio frequency circuits using ferrite cores with an electronically controlled magnetic biasing field.

It is desirable to be able to predict the resultant frequency variation when a given biasing field is applied to a ferrite core whose B-H loop and whose history are known. As a special case it is desirable to know whether the parallel or the transverse fields would give the larger frequency ratio shift. The internal flux in the tuning unit at no time returns to zero. The resonance frequency minimum will be different for the two cases. Thus not only $df/d\eta$, but $1/f \cdot df/d\eta$ must be considered to compare the relative value of the two cases.

For the case of parallel fields:

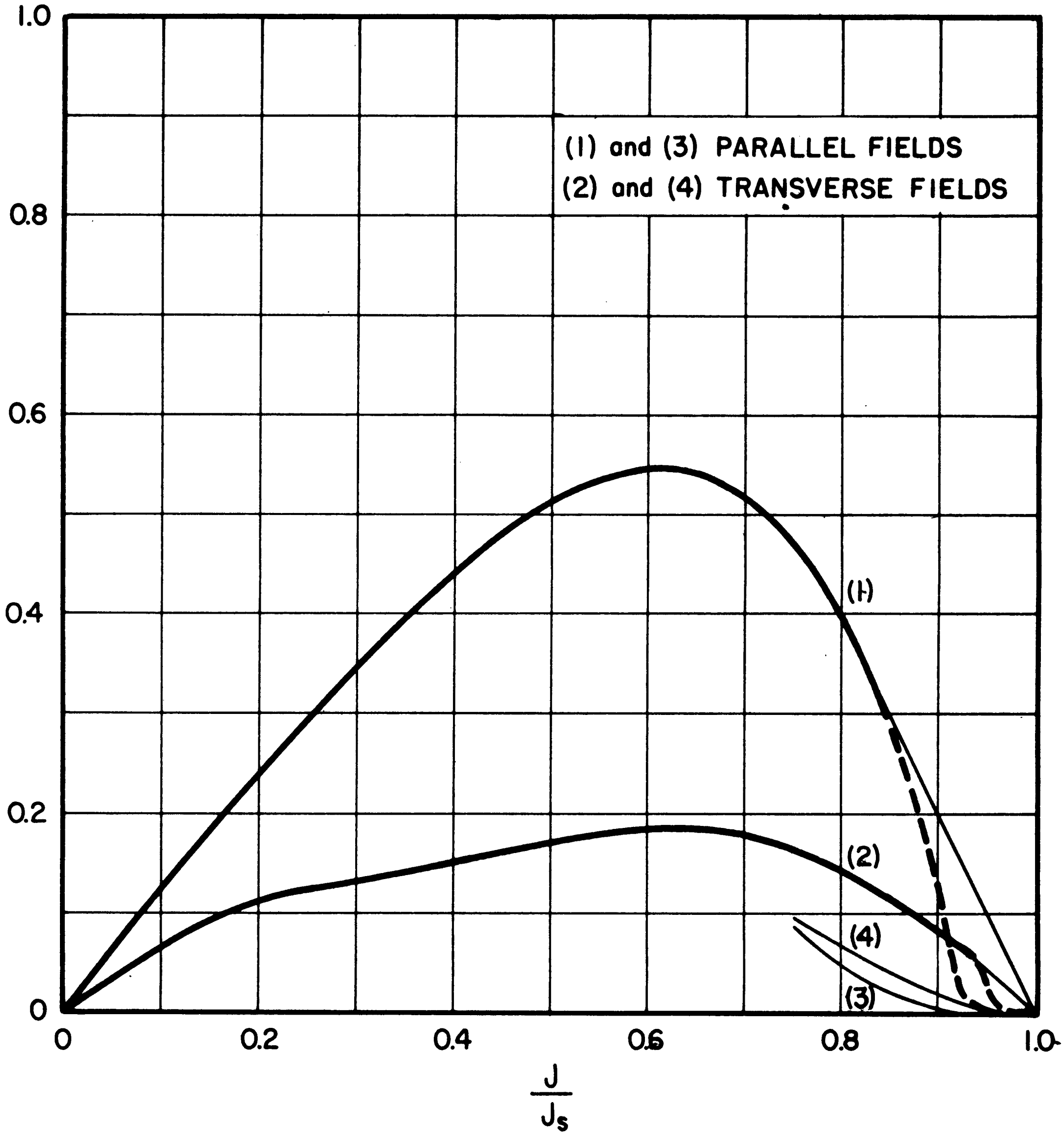
$$\frac{1}{f_p}\frac{df_p}{d\eta} = -2\pi \frac{3\chi_o}{\left[1 + 4\pi(3f'(\eta))\right]} f''(\eta). \qquad (9)$$

For the case of transverse fields:

$$\frac{1}{f_t}\frac{df_t}{d\eta} = -2\pi \frac{3\chi_o}{\left[1 + 4\pi \frac{3f(\eta)}{\eta}\right]} \left[\frac{f'(\eta)}{\eta} - \frac{f(\eta)}{\eta^2}\right]. \qquad (10)$$

$\left[\frac{-f''(\eta)}{f'(\eta)}\right]$ and $\left[\frac{\frac{f(\eta)}{\eta^2} - \frac{f'(\eta)}{\eta}}{\frac{f(\eta)}{\eta}}\right]$ are plotted against J/J_s in Fig. 4.

(Curves (1) and (2) respectively.) These equations are applicable when $\mu_r \gg 1$ in which case the functions of η in the denominator are large compared to unity. For large values of η, μ_r becomes small, approaching unity. For this circumstance curves (3) and (4), Fig. 4, show plots of $\left[-f''(\eta)\right]$ and $\left[\frac{f(\eta)}{\eta^2} - \frac{f'(\eta)}{\eta}\right]$ respectively.



(1) $\left[-\frac{f''(\eta)}{f'(\eta)}\right]$ (3) $\left[-f''(\eta)\right]$

(2) $\left[\frac{\frac{f(\eta)}{\eta} - f'(\eta)}{f(\eta)}\right]$ (4) $\left[\frac{f(\eta)}{\eta^2} - \frac{f'(\eta)}{\eta}\right]$

FIG. 4
FREQUENCY RATIO COEFFICIENT

Since μ_o for a typical ferrite would probably be between 100 and 800, curves (1) and (2) of Fig. 4 should be used until J/J_s is about .85. For $J/J_s > 0.95$, curves (3) and (4) should be used. The dashed lines connecting curves (1) and (3), and (2) and (4) represent a possible transition path between the two curves. It is to be noted that the heavy curves cross when J/J_s is about 0.9.

Since for $J/J_s < 0.9$, the curve for the parallel case lies above that for the transverse case,

$$\left| \frac{1}{f_p} \frac{df_p}{d\eta} \right| > \left| \frac{1}{f_t} \frac{df_t}{d\eta} \right| \quad \text{until } J = 0.9\, J_s .$$

For most magnetic bodies, this represents a biasing field of at least 10 oersteds. Therefore, for practical applications, the maximum frequency change ratio is obtained using parallel fields.

3.2 Temperature Dependence of the Tuned Circuit Resonant Frequency

The difference between this section and that of Section 3.1 is that χ_o and J_s are assumed to be temperature dependent phenomena, although independent of η .

From Appendix I --

For the case of parallel fields:

$$-2\frac{1}{f_p}\frac{df_p}{dT} = \frac{1-\zeta(\eta)}{\chi_o}\frac{d\chi_o}{dT} + \frac{\zeta(\eta)}{J_s}\frac{dJ_s}{dT} \qquad (11)$$

$$\zeta(\eta) = -\frac{\eta f''(\eta)}{f'(\eta)} .$$

For the case of transverse fields:

$$-2\frac{1}{f_t}\frac{df_t}{dT} = \frac{1-\xi(\eta)}{\chi_o}\frac{d\chi_o}{dT} + \frac{\xi(\eta)}{J_s}\frac{dJ_s}{dT} \qquad (12)$$

$$\xi(\eta) = 1 - \frac{\eta f'(\eta)}{f(\eta)} .$$

The above equations, Eqs 11 and 12, are based on the assumption that $4\pi\ \chi_r >> 1$. Plots of $\zeta(\eta)$ and $\xi(\eta)$ are shown in Fig. 5. From Fig. 5 it is apparent that:

$$0 < \zeta(\eta) < 2$$

$$0 < \xi(\eta) < 1.$$

Using Eqs 11 and 12, together with Fig. 5, it is possible to calculate the temperature coefficient of the frequency change ratio if $J_s(T)$ and $\chi_o(T)$, both readily measured experimentally, are known.

In order to design a magnetic tuning unit with the lowest temperature coefficient, it is desirable to know which type of biasing field will give the lowest coefficient. Table I shows the values of the ratio y/x for which the parallel biasing fields give the lowest temperature coefficient using $y = 1/J_s \frac{dJ_s}{dT}$ and $x = 1/\chi_o\ d\chi_o/dT$.

A plot of $\frac{-2+\zeta(\eta)+\xi(\eta)}{\zeta(\eta)+\xi(\eta)}$ against J/J_s is given in Fig. 6.

4. THEORETICAL ASPECTS OF $\chi_o(T)$ AND $J_s(T)$

It remains to predict $J_s(T)$ and $\chi_o(T)$ as a function of the chemical composition, the heat treatment, and the impurities. The reversible susceptibilities are impurity dependent magnetic properties.[9] Our knowledge of the

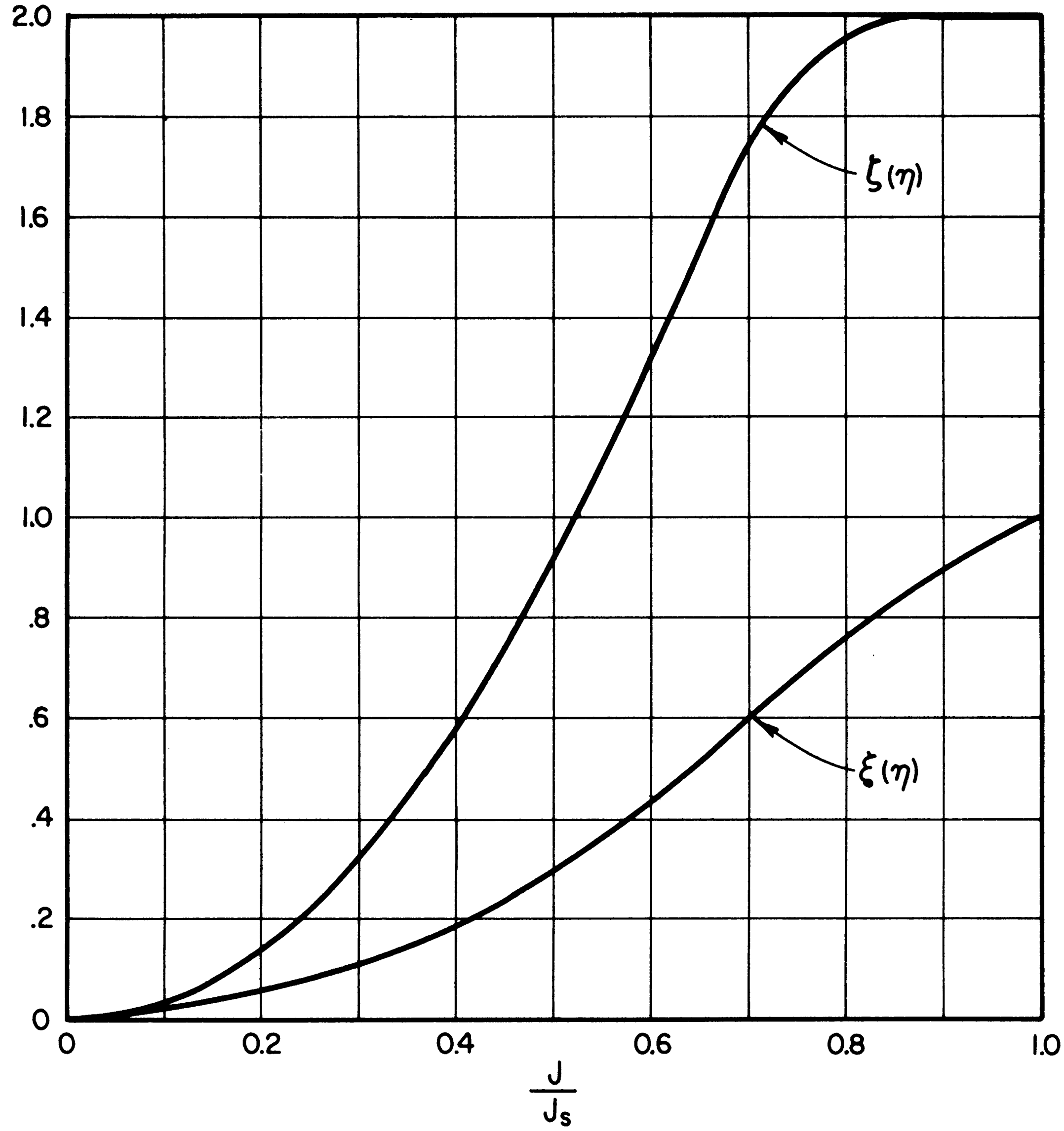


FIG. 5
TEMPERATURE COEFFICIENT
$\zeta(\eta)$ PARALLEL FIELDS
$\xi(\eta)$ TRANSVERSE FIELDS

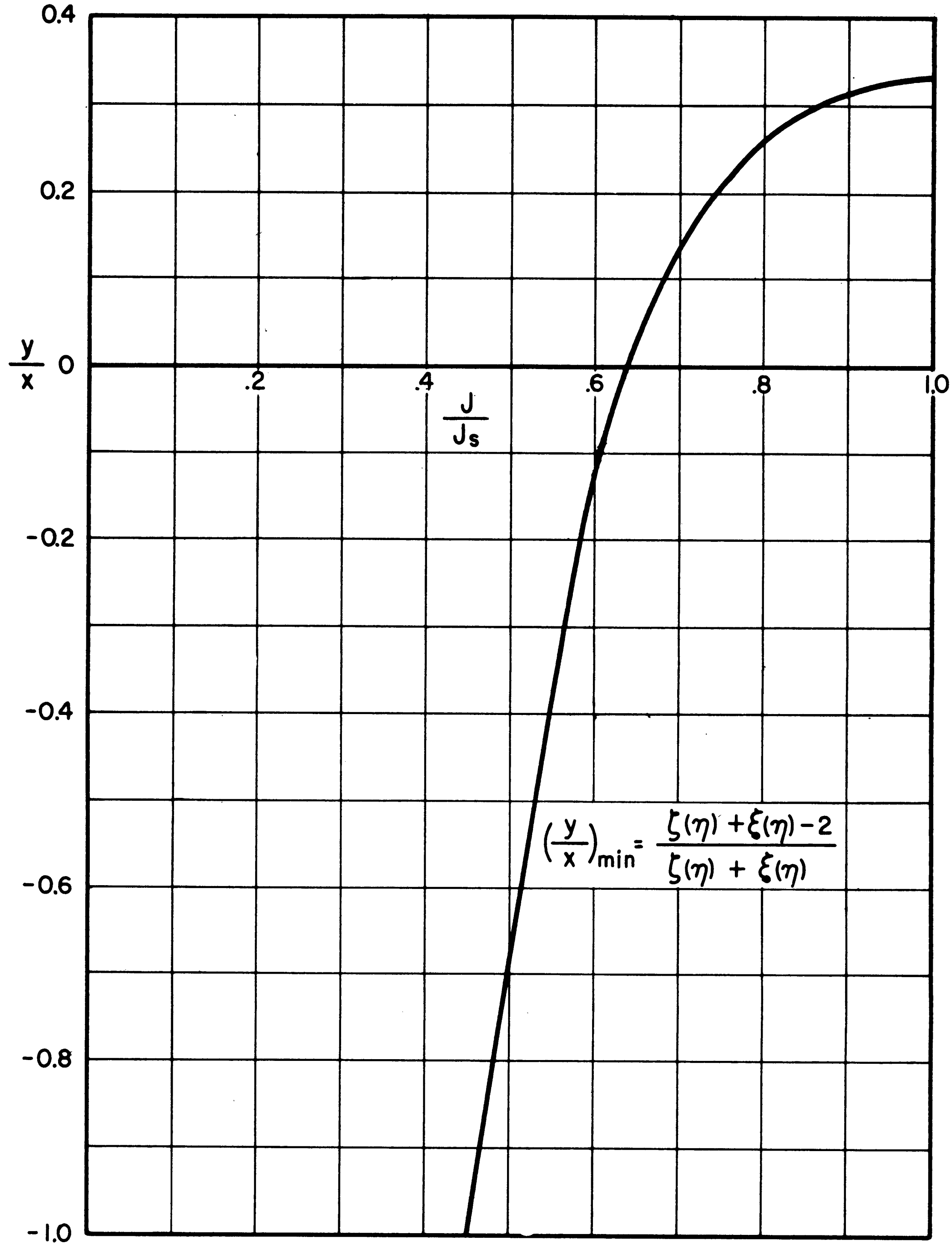


FIG. 6

MINIMUM VALUE OF y/x FOR THE PARALLEL FIELDS TO BE MORE ADVANTAGEOUS THAN TRANSVERSE FIELDS UNDER CERTAIN CONDITIONS. (SEE TABLE I)

TABLE I

x-y signs	$1 - \zeta(\eta)$	Special Conditions		y/x ratio
same	> 0	None		< 1
same	< 0	$\lvert(1-\zeta(\eta))x\rvert < \lvert\zeta(\eta)y\rvert$		< 1
same	< 0	$\lvert(1-\zeta(\eta))x\rvert > \lvert\zeta(\eta)y\rvert$		any
opposite	> 0	$\lvert(1-\zeta(\eta))x\rvert > \lvert\zeta(\eta)y\rvert$;	$\lvert(1-\xi(\eta))x\rvert > \lvert\xi(\eta)y\rvert$	any
opposite	> 0	$\lvert(1-\zeta(\eta))x\rvert > \lvert\zeta(\eta)y\rvert$;	$\lvert(1-\xi(\eta))x\rvert < \lvert\xi(\eta)y\rvert$	an incompatible system
opposite	> 0	$\lvert(1-\zeta(\eta))x\rvert < \lvert\zeta(\eta)y\rvert$;	$\lvert(1-\xi(\eta))x\rvert > \lvert\xi(\eta)y\rvert$	none
opposite	> 0	$\lvert(1-\zeta(\eta))x\rvert < \lvert\zeta(\eta)y\rvert$;	$\lvert(1-\xi(\eta))x\rvert < \lvert\xi(\eta)y\rvert$	none
opposite	< 0	$\lvert(1-\xi(\eta))x\rvert > \lvert\xi(\eta)y\rvert$		$\geq \dfrac{-2+\zeta(\eta)+\xi(\eta)}{\zeta(\eta)+\xi(\eta)}$
opposite	< 0	$\lvert(1-\xi(\eta))x\rvert < \lvert\xi(\eta)y\rvert$		none

physics of impurity sensitive materials is very small. Therefore any quantitative prediction of $\chi_o(T)$ would be extremely difficult.

$J_s(T)$ is not an impurity sensitive magnetic property, and thus better theoretical results can be expected. Néel[10,11] and Watanabe[12,13] have both considered the problem. Néel[10] showed the types of magnetization curves possible using different chemical composition. Watanabe[12] compares his theoretical results for various percentages of Ni-Zn ferrites with experimental results. The agreement is fair.

APPENDIX I

The Derivation of 1/f df/dT

Upon taking the derivative with respect to temperature of η and Eqs 6, 7, and 8:

$$\frac{dL_o}{dT} = \frac{3}{J_s^2} \frac{d\chi_o}{dT} - \frac{6\chi_o}{J_s^3} \frac{dJ_s}{dT}$$

$$\frac{d\eta}{dT} = H_r \left[J_s \frac{dL_o}{dT} + L_o \frac{dJ_s}{dT} \right]$$

$$\frac{d\chi_{rp}}{dT} = 3f'(\eta) \frac{d\chi_o}{dT} + 3\chi_o f''(\eta) \frac{d\eta}{dT}$$

$$\frac{d\chi_{rt}}{dT} = 3\chi_o \left(\frac{f'(\eta)}{\eta} - \frac{f(\eta)}{\eta^2} \right) \frac{d\eta}{dT} + 3 \frac{d\chi_o}{dT} \frac{f(\eta)}{\eta} .$$

Upon simplifying:

$$\frac{d\chi_{rp}}{dT} = \frac{d\chi_o}{dT} \left[3f'(\eta) + 3\eta f''(\eta) \right] - 3\frac{\chi_o}{J_s} \frac{dJ_s}{dT} \eta f''(\eta) \quad (13)$$

$$\frac{d\chi_{rt}}{dT} = \frac{d\chi_o}{dT} 3f'(\eta) + 3\frac{\chi_o}{J_s} \frac{dJ_s}{dT} \left[\frac{f(\eta)}{\eta} - f'(\eta) \right] . \quad (14)$$

The quantity of interest is 1/f df/dT, where f is the resonance frequency of the tuned circuit at any given temperature.

$$\frac{1}{f} \frac{df}{dT} = -2\pi \frac{d\chi_r/dT}{\left[1 + 4\pi \chi_r\right]} .$$

Using the assumption that $4\pi \ \chi_r >> 1$,

$$\frac{1}{f} \frac{df}{dT} = -\frac{1}{2} \frac{d\chi_r/dT}{\chi_r} .$$

Using Eqs 13 and 14:

For the parallel case,

$$-2 \frac{1}{f_p} \frac{df_p}{dT} = \frac{\left(1 + \frac{\eta f''(\eta)}{f'(\eta)}\right)}{\chi_o} \frac{d\chi_o}{dT} - \frac{\frac{\eta f''(\eta)}{f'(\eta)}}{J_s} \frac{dJ_s}{dT}$$

$$= \frac{1 - \zeta(\eta)}{\chi_o} \frac{d\chi_o}{dT} + \frac{\zeta(\eta)}{J_s} \frac{dJ_s}{dT} \qquad (15)$$

$$\text{if } \zeta(\eta) = -\frac{\eta f''(\eta)}{f'(\eta)} .$$

For the transverse case,

$$-2 \frac{1}{f_t} \frac{df_t}{dT} = \frac{\eta f'(\eta)}{f(\eta)} \frac{1}{\chi_o} \frac{d\chi_o}{dT} + \left[1 - \frac{\eta f'(\eta)}{f(\eta)}\right] \frac{1}{J_s} \frac{dJ_s}{dT}$$

$$= \frac{1 - \xi(\eta)}{\chi_o} \frac{d\chi_o}{dT} + \frac{\xi(\eta)}{J_s} \frac{dJ_s}{dT} \qquad (16)$$

$$\text{where } \xi(\eta) = 1 - \frac{\eta f'(\eta)}{f(\eta)} .$$

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