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A STATISTICALLY VERIFIED MODEL FOR CORRELATING VOLUME
LOSS DUE TO CAVITATION OR LIQUID IMPINGEMENT

(Submitted for presentation at ASTM Symposium on Characterization and Determination of Erosion Resistance)

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ABSTRACT

Rocket sled data has been used to evaluate statistically best values for threshold velocity and velocity exponent as well as the coefficient n in the relation:

$$\text{MDPR} = K(V \sin \Theta - V_o)^a / \sin^n \Theta$$

It is found that a reasonable assumption for most materials is that threshold velocity is zero, $n=1$, and that there is a rough correlation between K and a . The correlations with the data are relatively poor. However, within the basic precision attained, a single figure of merit for the material in terms of either K or a could be easily derived.

A more basic relation for the evaluation of MDPR has been assumed in terms of the energy flux model suggested by Hoff et al (5). In this relation the material properties are described in terms of an energy per volume term describing material failure. It has been found that the material energy term is best described as proportional to ultimate resilience. No correlation with any efficiency term relating liquid and material properties alone has been found.

TABLE OF CONTENTS

I. INTRODUCTION	1
II. ROCKET SLED DATA CORRELATION	2
A. General Considerations	2
B. Computer Correlation Results	4
III. GENERALIZED EROSION MODEL	5
A. General Considerations	6
B. Basic Equation Selected	7
C. Evaluation of Energy Parameter	8
1. General Remarks	8
2. Selection of Data for Evaluation	10
3. Best Fit Results Attained	11
IV. CONCLUSIONS	16
NOMENCLATURE	19
BIBLIOGRAPHY	20
TABLES	22
FIGURES	30
APPENDIX	33

I. INTRODUCTION

One of the major objectives of much past and present erosion research, either cavitation or impingement, is to establish a mathematical model with fluid-flow, and material parameters as input data which would allow the engineering prediction of erosion rates for given, as yet untested, materials. A precise model of this sort has so far eluded investigators. This appears to be inevitable in view of the highly complex and varied natures of the erosion processes, even though produced by droplet impingement or cavitation, for example, alone. Nevertheless, it is desirable, using a large and diverse group of data, to attempt to determine optimum correlation relationships, and also to determine roughly what degree of precision can be expected from correlation models using easily measured standard engineering parameters as input data. The development of an optimum model and the examination of other possible models, particularly for droplet impingement in the range of interest of aircraft rain erosion, is the objective of the present paper. A fairly complex set of data, including both impingement and cavitation data has been used. This combination of data seems reasonable due to the presumed basic similarity of the erosion phenomena in impingement and cavitation. The model chosen for further investigation has been made dimensionally-consistent and as simple as possible, in hopes of obtaining a maximum generality and applicability for the results. This objective is also enhanced by the use of a diverse data set including items generated in different impact and cavitation type tests. It is expected that additional data items as they become available (or are reduced to the form here required) will be incorporated into the overall analysis, thus increasing its generality still further.

Prior to the investigation of an overall erosion model, attempts were made to correlate data obtained on rain erosion materials in the

Holloman AFB rocket sled facility, using previously published semi-empirical relations between erosion rate, velocity, and angle of impact. The relatively poor fit achieved in this instance emphasized the necessity for a model more closely based on the details of the physical processes involved. For this reason it was decided to attempt a step-by-step development, relating as closely as possible to measurable data at all times, of a more basic model along the lines suggested by Hoff et al (1), relating the rate of erosion (MDPR = mean depth of penetration rate) to the kinetic energy impacting the target, the efficiency of energy transfer between drop and target (η), and a material parameter (\mathcal{E}) with dimensions of energy per unit target material volume. The equation adopted, explained in detail later, relates MDPR to the impinging kinetic energy and the energy necessary to remove material:

$$\text{MDPR} = \left(\frac{\eta}{\mathcal{E}} \right) \left(\frac{A_p}{A_e} \right) \left(\frac{P_{\text{eff}}}{2} \right) (V^3) \quad - - - (1)$$

Our analysis to date, carried out within the framework of this equation and utilizing a data set which includes both impingement and cavitation data, has concentrated on the optimum evaluation of the material parameter \mathcal{E} in terms of mechanical material properties, and has also included the contribution to the energy transfer efficiency term η due to the material and fluid, rather than geometrical and flow, parameters.

II. ROCKET SLED DATA CORRELATION

A. General Considerations

As a portion of the overall program aimed at the evaluation of potential rain erosion resistant materials, we have examined some of the data generated by the 1967 rocket sled tests at

Holloman AFB to determine the suitability of certain semi-empirical damage-predicting equations. The portion of the rocket sled data selected for this analysis comprised ten groups of materials including ceramics, plastics, and metals. They had been tested in the 6000 ft. rain field at Holloman AFB at Mach numbers ranging from 1.5 to 3.0, at various angles of impact ranging from 13.5° to 90° . The full details of this analysis have been reported previously (2). However, certain salient features will be repeated here for convenience.

An earlier report by Tatnall, et al. (3), based upon an experimental fit of rocket sled data suggests the velocity appears in an exponential form:

$$\text{MDPR} = C_1 e^{aV \sin \Theta} + C_2 e^{bV} \quad - - - - - (2)$$

where C_1 , C_2 , a and b are constants depending on material properties.

Baker, et al. (4), proposed a relation based on their impact data, which includes the concept of a threshold velocity below which damage is essentially zero:

$$\begin{aligned} \text{MDPR} &= K(V \sin \Theta - V_o)^a / \sin \Theta \quad \text{for } V \sin \Theta > V_o \\ &= 0 \quad \text{for } V \sin \Theta \leq V_o \quad - - - (3) \end{aligned}$$

More recently Hoff and Langbein (5) have suggested a modification of eq.(3) whereby the denominator is squared:

$$\begin{aligned} \text{MDPR} &= K(V \sin \Theta - V_o)^a / \sin^2 \Theta \quad \text{for } V \sin \Theta > V_o \\ &= 0 \quad \text{for } V \sin \Theta \leq V_o \quad - - - (4) \end{aligned}$$

Eq. (2) is simply a curve-fitting expression, not based on any physical model. Eq. (3) on the other hand assumed basically

that MDPR is proportional to the difference between the normal component of the impact velocity and some "threshold velocity", all raised to some power, α . A similar assumption has often been made in the cavitation literature (6, e.g.) where damage was assumed proportional to the 6th power of the flow velocity. In eq. (3), $\sin\Theta$ has been added to the denominator to take some account of the damage due to shear from the high radial velocity after impact, which increases for oblique collisions. Actually, since in the rocket sled type test the specimen impacts a reduced number of raindrops if the impact is not normal, it might be argued that an additional $\sin\Theta$ is required in the numerator, cancelling that in the denominator. This latter variation was not tried in the present analysis.

Eq. (4) is identical to eq. (3) except that $\sin^2\Theta$ appears in the denominator. This term can be derived logically from a model assuming energy flux on the target to be the predominant mechanism (5), if it is also assumed that the efficiency of energy transfer between impacting drop and target is a function of $V\sin\Theta$ only. However, it seems unlikely that this is strictly the case, so that eq. (3) and (4) remain semi-empirical in nature, and to be tested only in terms of a data fit.

B. Computer Correlation Results

The most comprehensive analysis of the rocket sled materials was made using Baker's proposed eq. (3). For each material a least mean square fit regression analysis was made to determine the best value of threshold velocity V_0 , and of the amplitude constant K and the velocity exponent α . Full details are given elsewhere (2), but the important general results are included here. Table I shows the comparison between predicted and measured MDPR for Pyroceram, along with the best values of V_0 , K , and α . It is noted that these differ by factors in excess of 10 in many cases. This is typical for most of the results. It

is also noted that the best value for the threshold velocity in this particular case is zero, which is also fairly typical.

Table II shows the effect of choice of V_o on the best values for the exponent α and the amplitude constant, K . The effect on K of varying V_o between 0 and 2000 f/s is small but α varies over this range from 6.44 to 2.28. A plot of MDPR vs. V shows a small or zero MDPR for small velocities and then a rapidly increasing MDPR for larger velocities. Such a curve can be fit almost equally well by various combinations of V_o (including 0) and velocity exponents α , as the present calculations show. Since, strictly, it is unlikely that there will be zero damage for repeated impacts at any velocity, it may be permissible to avoid the concept of threshold velocity entirely. If it is used, it is obviously a function of number of impacts per second as well as velocity, and it may be necessary to define an arbitrarily small but finite limit for MDPR which will then define the threshold velocity. Fig. 1 shows two typical sketches for the relation between probable error and choice of threshold velocity for this data. For those materials exhibiting behavior of the type shown in sketch 1-a, the optimum choice for threshold velocity is zero. For other materials, as in sketch 1-b, a definite optimum V_o appears.

For some materials the best values of V_o , K and α were computed from both eq. (3) and (4). Table III shows the comparison for an inorganic laminate, D-2 and a thermal plastic, I-2. While eq. (4) calls for an exponent 2 for the $\sin\Theta$ term, the effect of exponents ranging from 1.0 to 2.5 was examined ($n=1$ corresponds to eq. (3)). It is noted that for these materials, the choice of n affects the best choice of threshold velocity (and of course α , which is not listed), but affects the minimum probable error

only slightly. From this data it appears that a choice of $n=1$, desirable for the sake of simplicity, would not significantly reduce the "goodness" of the correlation. The effect on probable error of assuming zero threshold velocity (also desirable for simplicity) is shown in the last column. It is noted that the additional error so introduced is not particularly large.

For the best-fit values of V_0 for the different materials as analyzed under eq. (3), it has been noted that there is a rough correlation between K and α (Fig. 2). If a sufficiently precise correlation of this type existed, it might be possible to characterize a given material by a single figure of merit, which could be either K or α .

III. GENERALIZED EROSION MODEL

A. General Considerations

The limited success achieved in correlating the rocket sled test data using eq. (3) or (4), leads to the general conclusion that a more basic mathematical model is required. However, in the present instance the lack of good correlation is partly due to the type of data used. It is not permissible to compare damage attained after a fixed exposure period for materials of widely differing resistances, since only a mean MDPR can then be computed for materials in very different portions of their MDP vs. time (or number of impacts) curve. It is thus necessary to use data wherein the total MDP vs. exposure curve is available so that only comparable portions of this curve will be compared. After further understanding and the verification of predicting equations has been achieved, it may then become possible to interpret data from a test such as that of the rocket sled in a more suitable manner. However, this data is not adequate for

the generation and verification of a basic model. For these reasons, data from various types of facilities, both impact and cavitation, have been compiled together and used for the remainder of the present investigation.

B. Basic Equation Selected

The best hope of achieving a relationship of the generality necessary to allow possible applicability over the broad range of rain erosion materials, lies in a relation which is directly related to a physical model of the erosion process, is dimensionally consistent, and is as simple as possible. While it will be possible often to achieve a better fit for a given data set with more complex mathematical expressions, the likelihood of fitting other data sets with the same relation is reduced. Following this line of reasoning, we have elected to use the basic energy flux model suggested by Hoff et al. (1). However, we have not carried this beyond the stage where verification from our available data was possible, and hence have not introduced some of the assumptions used in the Hoff paper (1). We assume simply that the product of the rate of volume loss per unit exposed area (MDPR) times the exposed area (A_e) is proportional to the product of the impacting kinetic energy per unit projected area times the projected area. The constant of proportionality is the quotient of the efficiency of energy transfer between impacting drop and material damage processes (η) and a material parameter (ξ) describing the energy per unit material volume absorbed in the material in such a way as to cause damage. This relation is expressed by eq. (1).

To utilize this equation it is of course necessary to evaluate \mathcal{E} and η . Our analysis to the present has produced a simple best fit expression for \mathcal{E} , as will be explained later. We have not as yet, however, fully evaluated η .

For the moment it appears that the efficiency will be influenced by several factors, and may perhaps be considered as a product of several separate terms reflecting each of these mechanisms. Considering the details of the collision process between a liquid drop and a material surface, it seems likely that η will be a function of (a) material and liquid properties perhaps as reflected by the acoustic impedance ratio (7), (b) geometrical factors involved in the collision, i.e., shape of impacting drop, angle of impact, surface roughness, etc., and (c) velocity of impact which will affect the pressure applied to the surface and hence the degree of surface deformation and the departure from the concept of an elastic material. Since (a) material and liquid properties involves no other parameters of the collision, we have lumped its consideration into that of the energy parameter \mathcal{E} , assuming as a first approximation that this portion of the efficiency term η_a may be some function of the acoustic impedance ratio. No attempt has yet been made to evaluate the remaining portions of

C. Evaluation of Energy Parameter, \mathcal{E}

1. General Remarks

It is desired to find a material mechanical property with units of energy per unit volume having the characteristic that for a given test (impingement or cavitation) with fixed test

parameters (velocity, fluid conditions, geometry, etc.), the product of MDPR $\times \mathcal{E}$ will be nearly constant as possible over a broad range of test materials. The material property must appear only to the first power, i.e., a polynomial expression of an energy term will not do, since this would destroy the dimensional consistency of eq. (1). To be of use in a predicting equation, the energy term must be measurable in a simple mechanical test, and hopefully already available in the literature for most standard materials. The only parameters meeting these conditions are those energy terms which can be computed from the standard stress-strain curve. Our own previous work (8, e.g.) as well as that of Hobbs (9), and Rao (10) (all, incidently for cavitation tests) and Heymann (11,12) for a combined data set suggests that the best single parameter correlation is to be found with ultimate resilience = $T.S.^2 / 2E$, i.e. the area under the elastic portion of a stress-strain curve if elastic strain were continued up to the full tensile strength (T.S.). Thiruvengadem (13, e.g.), on the other hand, has reported that the best fit is in terms of strain energy (area under the complete stress-strain curve of a material). This latter can be evaluated either as the "engineering strain energy" (SE), i.e., area under the conventional stress-strain curve where tensile strength is computed from the observed machine breaking load without consideration of reduced area) or "true strain energy" (TSE) where actual breaking stress is used. We have used approximations of both in this analysis. In the case of ultimate resilience, for simplicity we have used the observed breaking load only, since for many rain erosion materials, reduction of area data is not available. Also, our previous work with other materials indicated this to be preferable (8).

2. Selection of Data for Evaluation

In the interest of maximum applicability and generality of results we have elected to use as broad a group of data as possible in the evaluation of ϵ/η_a , including some from cavitation tests and some from impingement tests. However, for incorporation in the analysis it is necessary that the stress-strain curves for the materials be accurately known to us, and that the damage data exist in such a fashion that the entire MDP vs. time curve is available so that a comparable portion of this curve can be used in all cases. Consistent with our previous practice (8), and that of Hobbs (9), we have selected the maximum MDPR as the characteristic value for the material. The largest single portion of the data we have used is that generated by our own vibratory cavitation facility in water (20 kHz, 2 mil nominal operating condition at 75° F). Other data has been incorporated into the analysis only when tests were available on at least one common material (i.e. identical material, from same bar stock, etc., if at all possible). In these cases, a ratio between maximum MDPR for the common material in the differing tests or facilities was established, and the additional materials tested in the other facility (or test condition) normalized to the common material in our vibratory facility. Thus values of the amplitude constants apply to this particular vibratory facility. In this manner it is possible to incorporate data from various types of tests since the efficiency factors involving test geometry and velocity are thus removed from consideration. Data from the following sources, in addition to our standard vibratory cavitation data has been used.

- a. Impact tests by King of RAE (14) in Dornier rotating arm facility
- b. Impact tests by Electricite de France (15,16) on rotating wheel
- c. Venturi tests by Rao et al (10)
- d. Vibratory cavitation tests in our laboratory (17) using stationary specimen arrangement in close proximity to vibrating horn (same unit also used in standard set-up).

As tests on additional common materials become available, it may be possible to include further data sets, hopefully including some from other impact facilities.

The materials and their mechanical properties are listed in Table IV. Test data on Stellite 6-B was not included in the actual data fits since much previous data has indicated that its resistance is much greater than expected on the basis of its mechanical properties (a factor of about 10 in this case). Other exotic alloys for which we have data also were not included since these are very far removed in their properties from any rain erosion materials.

3. Best Fit Results Attained

a. Predominant Mechanical Property

Previous work here and elsewhere led us to the conclusion that the most likely form for the energy parameter \mathcal{E} would be a combination of ultimate resilience and strain energy so arranged that the resultant term would have the units of energy/volume. To attain reasonable flexibility within this limitation, the following relation was postulated:

$$\mathcal{E} = C_1 \left(\frac{UR}{ESE} \right)^a UR + C_2 \left(\frac{UR}{ESE} \right)^b SE \quad - - - - - (5)$$

where C_1 , C_2 , a , and b are constants to be computed by a least square fit regression analysis of the data. Investigation of this relation showed that the best values for a and b were close to zero, so that the simpler relation of eq. (6) was indicated. An additive constant, C_0 , was used since this improved the data fit. The physical interpretation of C_0 is that of a threshold energy necessary to cause measurable damage, i.e., a concept analogous to that of threshold velocity.

$$\mathcal{E} = C_0 + C_1 UR + C_2 SE \text{ - - - - - (6)}$$

Using the least mean square fit analysis with eq. (6), or the following special case versions of it:

$$\begin{aligned} \mathcal{E} &= C_0 + C_1 UR \text{ - - - - - (a)} \\ \mathcal{E} &= C_0 + C_1 SE \text{ - - - - - (b)} \\ \mathcal{E} &= C_0 + C_1 TSE \text{ - - - - - (c)} \\ \mathcal{E} &= C_0 + C_1 UR + C_2 SE \text{ - - (d)} \\ \mathcal{E} &= C_1 UR \text{ - - - - - (e)} \end{aligned} \quad \text{ - - - - - (7)}$$

it was found that the best correlation coefficient and the smallest percent standard error of estimate resulted from eq. (7-d), although in general (7-a) was in all cases nearly as good, indicating that ultimate resilience was the material parameter of major importance. This was further verified by the dominance of the second term over the third in eq. (7-d). The statistics of the correlation with either eq. (7-b) or (7-c) were relatively very poor with TSE worse than SE. Hence SE is used in eq. (7-d). This data is summarized in Tables V and VIII. While the correlation with eq. (7-d) is better than that with (7-a), it is only slightly so. Hence for the present data set it is permissible to use eq. (7-a) in preference to (7-d) in the

interests of increased simplicity, so that the only mechanical property involved in the correlation becomes ultimate resilience, which is much more easily measured for materials such as those used for rain erosion than is engineering strain energy. Since the best value of C_o in eq. (7-a) is relatively very small, it is justified to use the form of (7-e) where this threshold energy term is neglected.

The standard error of estimate has been computed in such a way that it is always approximately proportional to give equal weight to both weak and strong materials in the correlation, and allow the reasonably accurate prediction of MDPR for materials of low \mathcal{E} . The applicable relations are shown in the Appendix.

b. Determination of Efficiency Factor, η

As previously discussed, it has been assumed that one factor of the overall energy-transfer efficiency term in the basic eq. (5), i.e. η_a , may be represented to a first approximation as a function of the acoustic impedance ratio (AI) between liquid and material ($AI = \rho_L^c / \rho_S^c$)

A consideration of the "water-hammer equation" for materials of finite elasticity, usually assumed to give a reasonable approximation of the pressure applied to the material surface under droplet impact (18, e.g.), indicated the importance of AI in determining this pressure, and in fact suggests a functional form of AI,

$f(AI)$, which might be tried.

$$\Delta p = \frac{\rho_L c V}{AI + 1} \quad - - - - - (8)$$

$$\text{so } f(AI) = AI + 1$$

Here $f(AI)$ is taken as a direct factor in the relation describing the pressure generated at the point of impact. Since pressure has units of energy per volume, the consideration of pressure is dimensionally consistent with the general model assumed.

Another possible form of $f(AI)$ is the "transmission coefficient" giving the ratio of absorbed to reflected energy for the case of a shock wave impinging upon a solid surface in a continuous medium (which is not identical to the present case).

$$\text{Then } f(AI) = \frac{(AI + 1)^2}{4AI} \quad - - - - - (9)$$

The best fit correlations have been investigated for both forms. It was assumed that:

$$\eta_a = f(AI)^n, \text{ where } n = -1, -2, -3 \quad - - - - - (10)$$

Table VI summarizes the results. It appears that there is no substantial improvement in the correlation to be attained by the use of $f(AI)$ in any of these forms. This is surprising in the light of Heymann's result (11, 12) that the fit with UR was improved by using $UR \times E$, since $E^2 \cong \rho E = \rho^2 c^2$ for the metals used. As also suggested by Heymann's discussion (12) it seems necessary that η_a differ substantially between materials, since the ratio between

the extreme material erosion resistances is orders of magnitude greater than that between the corresponding material energy properties. Nevertheless, in light of the present results η_a has been assumed unity, and omitted from subsequent relations.

c. Non-Linear Parameter Fits

i. Polynomial Energy Parameter Fit

Our postulated basic equation (1) requires a first power energy term for dimensional consistency. In order to verify that the assumption of such a linear relationship with energy is reasonable, polynomial data fits of the type

$$\mathcal{E} = C_0 + C_1(UR) + C_2(UR)^2 + C_3(UR)^3 - - - - - (11)$$

were investigated. An earlier incomplete data set was used, but it is felt that the values shown in Table VII are typical.

As expected there was some improvement over the linear fit, but it was not great, Table VIII indicating that the linear relationship is physically reasonable and suitable for the present purpose where the maintenance of dimensional consistency is important.

ii. Fit with $UR \times E^2$

Heymann's correlation (11, 12) was improved by using $UR \times E^2$ rather than E . However, this statistical fit for our present data is not as good as that with UR alone, and of course is dimensionally inconsistent with the assumed model (Table VII).

d. Recommended Relation

Based on the foregoing, the following relations are recommended for common metals and alloys at this time. As additional data is incorporated, it is anticipated that the best values of the constants may change slightly, but it is believed that such a change will be small, and that the form of the equation will remain unchanged.

$$\mathcal{E} = C_o + C_1 UR \quad - - - - - (12-a)$$

$$\mathcal{E} = C_1 UR \quad - - - - - (12-b)$$

where $C_o =$	$\frac{(12-a)}{0.463}$	$\frac{(12-b)}{-}$
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$C_1 =$	1.999	2.330
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Coefficient of Correlation =	0.808	0.808
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Standard Error of Estimate =	1.981	2.007
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Since the improvement due to the inclusion of C_o is small, the form (12-b) is recommended.

Table VIII lists the full data set used along with measured and predicted values of \mathcal{E} (which is equivalent to MDPR for data normalized in the fashion here used), according to eq. (12) and the coefficient of correlation and standard error of estimate, computed as shown in the Appendix. The predicted and measured values are tabulated for both eq. (12-a) and (12-b) along with the deviations for each material. Fig. 3. presents the same information graphically for the recommended eq. (12-b) where the "conical" standard error of estimate band is shown.

The amplitude constants apply to the UM vibratory facility only. Constants for other facilities are found by multiplying the given constants by the ratio between maximum damage rates in the other facility and the UM facility.

IV. CONCLUSIONS

It is postulated that the most likely form for an equation relating material, liquid, and test parameters with impingement or cavitation erosion rates with good hope for general applicability, is one which is based on a clear physical model with dimensional consistency. For the evaluation of impingement erosion rates, consistent with the previous suggestion of Hoff, et al (1) the equation

$$MDPR = \left(\frac{\eta_i \eta_a}{\mathcal{E}} \right) \left(\frac{A_p}{A_e} \right) \left(\frac{\rho_{eff} V^3}{2} \right) \quad - - - - (1-a)$$

has been chosen.

A statistical evaluation of \mathcal{E} , which must have units of energy per volume, has shown the best fit with a comprehensive data set including both impingement and cavitation data, in the form:

$$\mathcal{E} = C_1 UR \quad \text{--- (12-b)}$$

Neither higher power terms in UR or terms in SE improved the statistics of the fit substantially, and the fit in terms only of SE was relatively very poor. It is thus concluded that for the large group of metals here used the best linear energy per volume mechanical property correlation for volume loss rate under droplet impingement or cavitation attack is the expression eq. (10-b) in ultimate resilience alone.

Rocket sled rain erosion data has been statistically evaluated to find best values for threshold velocity and velocity exponent, as well as the coefficient n in the expression (4,5):

$$\text{MDPR} = K(V \sin \Theta - V_o)^a / \sin^n \Theta \quad \text{--- (4-a)}$$

It was found that the statistical fit is relatively insensitive to n so that n=1 is a suitable value. It was also found that for many materials, the statistical fit is also insensitive to the choice of a threshold velocity V_o , so that only slight reduction

in the "goodness" of the fit occurs for most materials if it is assumed that $V_o = 0$. However, the best fit values for K and α are sensitive to the choice of V_o and n . It was found that there is a rough correlation between best fit K and α , with K decreasing approximately linearly with increasing α . Thus it might be possible to characterize a material by a single figure of merit in terms of eq. (4-a), if a best fit relation between K and α is determined, so that either may be eliminated in terms of the other in eq. (4-a). However, the statistical precision of the correlations is relatively very poor for the rocket sled data.

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NOMENCLATURE

MDPR	= Mean depth of penetration rate (= volume loss rate/ exposed area).
K	= Amplitude constant eq. (3)
α	= Velocity exponent eq. (2)
V	= Impact velocity
V_o	= Threshold velocity
θ	= Angle between tangent to surface and direction of impact
η	= Efficiency of energy transfer between impacting drop or jet and surface - - eq. (5)
\mathcal{E}	= Removal energy (= energy/volume to remove given volume from surface).
A_p	= Projected target area in flight direction - - eq. (5)
A_e	= Exposed target area - - eq. (5)
ρ_{eff}	= Effective liquid density, mass of liquid per unit volume of gas-liquid mixture - - eq. (5)
C_o, C_1, C_2, C_3	= Constants, eq. (6), (7), (9), (10)
Δp	= Pressure differential due to liquid drop impacting surface - - eq. (8)
ρ	= density
c	= Sonic velocity, or velocity of propagation of shock wave.
AI	= Acoustic impedance ratio between impacting liquid and target material =
UR	= Ultimate resilience
SE	= Engineering strain energy
TSE	= True strain energy

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TABLE I

Comparison of Actual and Predicted MDPR's for Material A-1, Pyroceram, using $MDPR = (K(V\sin\theta - V_0))^a / \sin\theta$ as a Predicting Equation with $a = 6.27$, $V_0 = 0$, $K = 5.34 \times 10^{-5}$. (Standard Deviation of Predicting Equation = $428 \mu/s$.)

V (f/s)	θ ($^\circ$)	(MDPR) _{predicted} (μ/s)	(MDPR) _{actual} (μ/s)
1580	30	.9	7.9
1580	30	.9	0
1580	45	5.5	10.5
1580	45	5.5	10.5
1580	60	16.1	0
1580	60	16.1	5.3
2197	30	6.8	0
2197	30	6.8	0
2197	45	43.7	0
2197	45	43.7	3.6
2197	60	127.3	7.3
2197	60	127.3	80.6
2594	30	9.6	0
2594	30	9.6	0
2594	45	124.1	0
2594	45	124.1	4.3
2594	60	361.4	3,849.
2594	60	361.4	2,240.
2905	30	40.6	0
2905	30	40.6	14.5
2905	45	252.4	2,189.
2905	45	252.4	179.
2905	60	735.3	4,465.

TABLE II

Effect of V_O on Values of K and α for Material C-2, an Epoxy Laminate.

V_O (f/s)	α	K ($\times 10^5$)
0	6.44	25.7
200	6.36	27.9
400	6.24	29.7
600	6.08	32.3
800	5.87	34.8
1000	5.59	36.7
1200	5.22	39.6
1400	4.73	43.4
1600	4.09	41.3
1800	3.28	40.1
2000	2.28	28.4

TABLE III

Results of Evaluation of Equation $\text{MDPR} \sin^n \theta = \left[K(V - V_o) \sin \theta \right]^a$ for Various Values of n .

Material D-2.

n	Threshold Velocity (f/s)	Minimum Probable Error (μ/s)	Probable Error for $V_o = 0$ (μ/s)
1.0	1100	82	146
1.5	1000	88	143
2.0	900	95	141
2.5	800	101	140

Material I-2.

1.0	350	7.3	7.7
1.5	200	7.3	7.4
2.0	100	7.2	7.3
2.5	0	7.2	7.2

TABLE IV Mechanical Properties of Materials in Data Set

	YS	TS	Y	EL	HARD	MDPR	UR	SE
BS1433 COPPER	0.300E 05	0.360E 05	0.180E 08	0.180E 00	0.900E 02	0.647E 01	0.360E 02	0.648E 04
STAINLESS STEEL 316	0.310E 05	0.813E 05	0.260E 08	0.690E 00	0.748E 02	0.301E 00	0.127E 03	0.561E 05
NICKLE 270	0.800E 04	0.488E 05	0.277E 08	0.610E 00	0.249E 02	0.128E 01	0.430E 02	0.298E 05
AL 6061	0.407E 05	0.475E 05	0.910E 07	0.220E 00	0.600E 02	0.436E 01	0.124E 03	0.104E 05
STAINLESS STEEL 304	0.647E 05	0.945E 05	0.290E 08	0.638E 00	0.237E 03	0.330E 00	0.154E 03	0.603E 05
BRONZE #1	0.243E 05	0.452E 05	0.128E 08	0.230E 00	0.189E 03	0.189E 01	0.798E 02	0.104E 05
BRONZE #2	0.790E 05	0.112E 06	0.147E 08	0.205E 00	0.304E 03	0.163E 00	0.426E 03	0.229E 05
BRONZE #3	0.880E 05	0.119E 06	0.172E 08	0.150E 00	0.225E 03	0.220E 00	0.411E 03	0.178E 05
BRONZE #4	0.190E 05	0.282E 05	0.121E 08	0.600E-01	0.152E 03	0.176E 01	0.329E 02	0.169E 04
BRONZE #5	0.105E 05	0.189E 05	0.558E 07	0.130E 00	0.974E 02	0.330E 01	0.320E 02	0.246E 04
BRONZE #6	0.162E 05	0.193E 05	0.711E 07	0.300E-01	0.152E 03	0.257E 01	0.262E 02	0.579E 03
STAINLESS STEEL #1	0.115E 06	0.157E 06	0.263E 08	0.220E 00	0.290E 03	0.252E 00	0.470E 03	0.346E 05
STAINLESS STEEL #2	0.186E 06	0.188E 06	0.257E 08	0.750E-01	0.418E 03	0.270E 00	0.691E 03	0.141E 05
STAINLESS STEEL #3	0.104E 06	0.126E 06	0.251E 08	0.195E 00	0.264E 03	0.430E 00	0.319E 03	0.247E 05
COPPER	0.282E 05	0.333E 05	0.160E 08	0.543E 00	0.968E 02	0.671E 01	0.347E 02	0.181E 05
BRASS(65-35)	0.489E 05	0.605E 05	0.157E 08	0.393E 00	0.146E 03	0.170E 01	0.117E 03	0.238E 05
MILD STEEL 1020	0.897E 05	0.965E 05	0.300E 08	0.259E 00	0.277E 03	0.808E 00	0.155E 03	0.250E 05
STAINLESS STEEL 304	0.410E 05	0.994E 05	0.290E 08	0.168E 00	0.315E 03	0.332E 00	0.170E 03	0.167E 05
ASTM B144(SAE660)	0.175E 05	0.225E 05	0.140E 08	0.173E 00	0.174E 03	0.147E 01	0.181E 02	0.389E 04
MAGNESIUM	0.241E 05	0.392E 05	0.650E 07	0.255E 00	0.885E 02	0.434E 01	0.118E 03	0.100E 05
ALUMINUM 3003-O	0.680E 04	0.159E 05	0.900E 07	0.541E 00	0.512E 02	0.304E 02	0.140E 02	0.860E 04
COPPER	0.300E 05	0.360E 05	0.180E 08	0.180E 00	0.900E 02	0.647E 01	0.360E 02	0.648E 04
CR-130 STEEL	0.290E 05	0.780E 05	0.290E 08	0.280E 00	0.255E 03	0.465E 01	0.105E 03	0.218E 05
AL ALLOY	0.450E 05	0.560E 05	0.100E 08	0.100E 00	0.114E 03	0.802E 01	0.157E 03	0.560E 04
ALUMINUM	0.150E 05	0.160E 05	0.900E 07	0.500E-01	0.270E 02	0.255E 02	0.142E 02	0.800E 03
COPPER	0.142E 05	0.310E 05	0.170E 08	0.500E 00	0.600E 02	0.824E 01	0.283E 02	0.155E 05
PHOSPHOR BRONZE	0.394E 05	0.416E 05	0.150E 08	0.110E 00	0.950E 02	0.440E 01	0.577E 02	0.458E 04
BRASS	0.157E 05	0.260E 05	0.160E 08	0.530E 00	0.190E 03	0.200E 01	0.211E 02	0.138E 05
MILD STEEL	0.484E 05	0.650E 05	0.280E 08	0.600E-01	0.950E 02	0.236E 01	0.754E 02	0.390E 04
STAINLESS STEEL	0.354E 05	0.930E 05	0.280E 08	0.570E 00	0.170E 03	0.653E 00	0.154E 03	0.530E 05
STAINLESS STEEL 316	0.310E 05	0.813E 05	0.260E 08	0.690E 00	0.748E 02	0.713E 00	0.127E 03	0.561E 05
NICKLE 270	0.800E 04	0.488E 05	0.277E 08	0.610E 00	0.249E 02	0.128E 01	0.430E 02	0.298E 05
AL 6061	0.407E 05	0.475E 05	0.910E 07	0.220E 00	0.600E 02	0.436E 01	0.124E 03	0.104E 05
STELLITE 6-B	0.710E 05	0.138E 06	0.304E 08	0.210E 00	0.322E 03	0.180E-01	0.313E 03	0.290E 05
TOOL STEEL #1	0.540E 05	0.110E 06	0.275E 08	0.175E-01	0.235E 03	0.730E-01	0.220E 03	0.193E 04

YS = Yield Strength (psi) MDPR = Maximum Mean Depth of Penetration Rate (mils/hr)

TS = Tensile Strength (psi) (All values are corrected to U. M. vibratory facility)

Y = Elastic Modulus (psi) UR = Ultimate Resilience = $\frac{TS^2}{2E}$ (psi)

EL = Elongation (%) SE = Strain Energy to Failure = $TS \times E$ (psi)

HARD = Brinell Hardness NUR = Ultimate Resilience normalized to BS 1433 Copper

NSE = Strain Energy normalized to BS 1433 Copper

TABLE V
Summary of Statistical Correlation Data for Eq. 7.

C ₁ = 0.914 C ₂ = 2.875 C ₁ = 1.633 C ₂ = 1.773 C ₁ = 1.897 C ₂ = 1.824 C ₁ = 0.445 C ₂ = 1.735 C ₁ = 0.889 C ₂ = 1.139											
MATERIAL & NORMALIZED	EPSILON	C1+C2*UR	C1+C2*SE	C1+C2*TSE	C1*UR+C2*SE	C0+C1*UR+C2*SE	SOURCES				
BS1433 COPPER	1.000	2.811	4.699	6.931	2.522	1.102	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
STAINLESS STEEL 316	21.482	7.611	18.664	12.474	13.464	14.218					
NICKLE 270	5.044	3.179	11.254	7.446	6.035	5.534					
AL 6061	1.482	7.446	5.817	7.103	7.057	6.040	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
STAINLESS STEEL 304	19.594	9.026	19.845	13.565	15.258	16.250					
BRONZE #1	3.421	5.119	5.801	6.924	5.046	3.902					
BRONZE #2	39.669	23.359	9.326	7.612	22.468	22.789	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
BRONZE #3	29.391	22.556	7.895	7.278	21.079	21.161					
BRONZE #4	3.674	2.645	3.352	6.553	1.723	0.109					
BRONZE #5	1.959	2.601	3.567	6.581	1.789	0.202	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
BRONZE #6	2.516	2.295	3.038	6.511	1.268	-0.408					
STAINLESS STEEL #1	25.659	25.666	12.609	8.452	26.055	26.951					
STAINLESS STEEL #2	23.948	37.310	6.850	7.230	33.271	34.005	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
STAINLESS STEEL #3	15.037	17.729	9.818	8.706	17.861	17.946					
COPPER	0.963	2.740	7.965	7.240	4.053	3.077					
BRASS (65-35)	3.801	7.056	9.567	7.455	8.550	8.026	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
MILD STEEL 1020	8.002	9.091	9.910	7.571	10.470	10.102					
STAINLESS STEEL 304	19.476	9.889	7.575	7.017	10.018	9.374					
ASTM B144 (SAE660)	4.384	1.867	3.971	6.643	1.354	-0.217	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
MAGNESIUM	1.490	7.142	5.689	6.849	6.733	5.682					
ALUMINUM 3003-O	0.213	1.654	5.296	6.764	1.818	0.417					
COPPER	1.000	2.811	4.699	6.931	2.522	1.102	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
CR-130 STEEL	1.391	6.441	9.022	7.654	7.755	7.123					
AL ALLOY	0.806	9.175	4.451	6.740	7.880	6.769					
ALUMINUM	0.254	1.663	3.100	6.523	0.755	-0.946	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
COPPER	0.785	2.403	7.238	7.320	3.409	2.315					
PHOSPHOR BRONZE	1.470	3.953	4.163	6.787	3.244	1.812					
BRASS	3.228	2.027	6.754	7.323	2.849	1.669	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
MILD STEEL	2.739	4.889	3.973	6.724	3.957	2.549					
STAINLESS STEEL	9.902	9.051	17.795	9.383	14.280	14.992					
STAINLESS STEEL 316	9.069	7.611	18.664	12.474	13.464	14.218	UM Vibratory Cavitation Facility	UM Vibratory Cavitation Facility with Stationary Specimen	RAE-Dornier Rotating Arm Facility	Venturi Facility	
NICKLE 270	5.111	3.179	11.254	14.510	6.035	5.534					
AL 6061	1.482	7.446	5.817	7.103	7.057	6.040					
CORRELATION COEFFICIENT											
STANDARD ERROR OF ESTIMATE											
0.808 0.466 0.236 0.854 0.856 0.856 0.856 0.856 0.856 0.856 0.856											

TABLE VI
Acoustic Impedance Correction

$f(AI)^n$	AI +1		$\frac{(AI +1)^2}{4AI}$	
n	Correlation Coefficient	Standard Error of Estimate	Correlation Coefficient	Standard Error of Estimate
0	0.808	2.007	0.808	2.007
1	0.807	2.005	0.807	2.101
2	0.807	2.003	0.782	2.324
3	0.806	2.001	0.743	2.668
-1	0.808	2.009	0.781	2.070
-2	0.808	2.011	0.721	2.431
-3	0.809	2.014	0.582	3.745

TABLE VII
Equations Using Non-linear Parameters

<u>Equation</u>	<u>Correlation Coefficient</u>	<u>Standard Error of Estimate</u>
$\Sigma = 2.330 \text{ UR}$	0.808	2.007
$\Sigma = -2.681 + 3.343 \text{ UR} - 0.087 \text{ UR}^2$	0.870	5.616*
$\Sigma = 0.266 \text{ UR} + 0.412 \text{ UR}^2 - 0.019 \text{ UR}^3$	0.919	4.459*
$\Sigma = 3.685 \text{ UR} \times E^2$	0.678	5.714
$\Sigma = 1.147 + 1.444 \text{ UR} \times E^2$	0.678	4.271

* These values are more comparable to the results of Table V

Table VIII Recommended Correlating Equations

MATERIAL & NORMALIZED	EPSILON	$C_1 = 0.463$ $G_1 = 1.989$		$G_2 = 2.330$		SOURCES
		$C_1 + C_2 * UR$ C_1	$+ C_2 * UR$	C_2	$+ C_2 * UR$	
BS1433 COPPER	1.000	2.462	1.981	2.330	2.007	
STAINLESS STEEL 316	21.482	7.520	6.993	8.225	7.085	UM Vibratory Cavitation Facility
NICKLE 270	5.044	2.850	2.365	2.782	2.396	
AL 6061	1.482	7.346	6.820	8.022	6.911	
STAINLESS STEEL 304	19.594	9.011	8.471	9.964	8.584	
BRONZE #1	3.421	4.894	4.391	5.164	4.449	
BRONZE #2	39.669	24.115	23.438	27.568	23.750	
BRONZE #3	29.391	23.269	22.600	26.582	22.900	
BRONZE #4	3.674	2.288	1.808	2.127	1.832	
BRONZE #5	1.959	2.240	1.761	2.071	1.784	
BRONZE #6	2.516	1.918	1.442	1.696	1.461	
STAINLESS STEEL #1	25.659	26.547	25.847	30.402	26.191	
STAINLESS STEEL #2	23.948	38.817	38.007	44.704	38.513	
STAINLESS STEEL #3	15.037	18.182	17.559	20.653	17.792	
COPPER	0.963	2.387	1.906	2.242	1.932	
BRASS(65-35)	3.801	6.935	6.413	7.543	6.499	UM Vibratory Cavitation Facility with
MILD STEEL 1020	8.002	9.080	8.539	10.043	8.653	Stationary Specimen
STAINLESS STEEL 304	19.476	9.921	9.372	11.024	9.497	
ASTM B144(SAE660)	4.384	1.467	0.995	1.170	1.008	
MAGNESIUM	1.490	7.026	6.503	7.649	6.590	
ALUMINUM 3003-O	0.213	1.243	0.773	0.909	0.783	
COPPER	1.000	2.462	1.981	2.330	2.007	
CR-130 STEEL	1.391	6.287	5.771	6.788	5.848	RAE-Dornier Rotating Arm Facility
AL ALLOY	0.806	9.169	8.627	10.147	8.742	
ALUMINUM	0.254	1.253	0.782	0.920	0.793	
COPPER	0.785	2.032	1.555	1.829	1.576	
PHOSPHOR BRONZE	1.470	3.666	3.174	3.733	3.216	Venturi Facility
BRASS	3.228	1.636	1.162	1.367	1.178	
MILD STEEL	2.739	4.652	4.151	4.882	4.206	
STAINLESS STEEL	9.902	9.038	8.497	9.994	8.610	
STAINLESS STEEL 316	9.069	7.520	6.993	8.225	7.086	
NICKLE 270	5.111	2.850	2.365	2.782	2.396	Rotating Wheel Impact Facility
AL 6061	1.482	7.346	6.820	8.022	6.911	
CORRELATION COEFFICIENT						
PERCENTAGE STANDARD ERROR OF ESTIMATE		0.808		0.808		
		1.981		2.007		

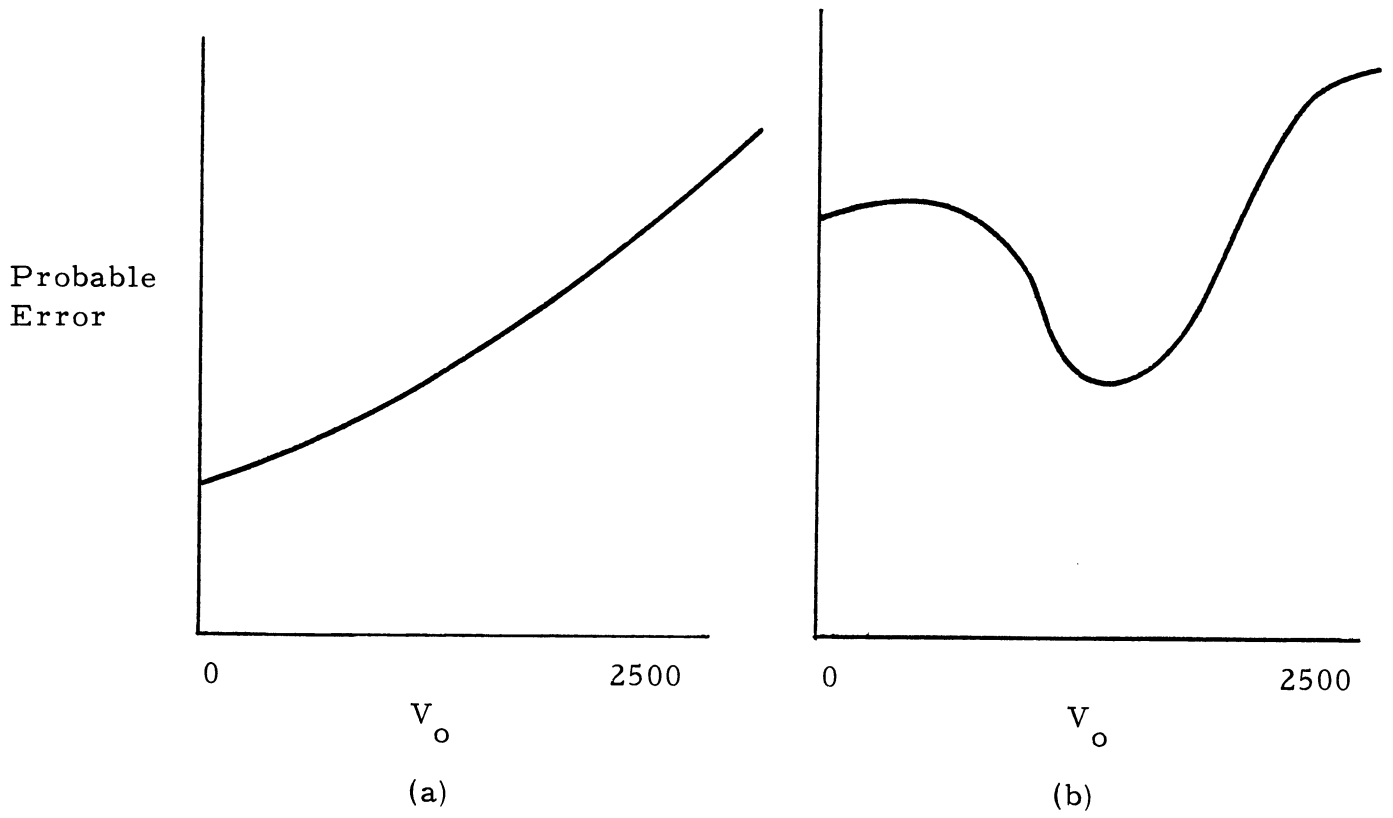


Figure 1. Typical Curves for Probable Error as a Function of Threshold Velocity.

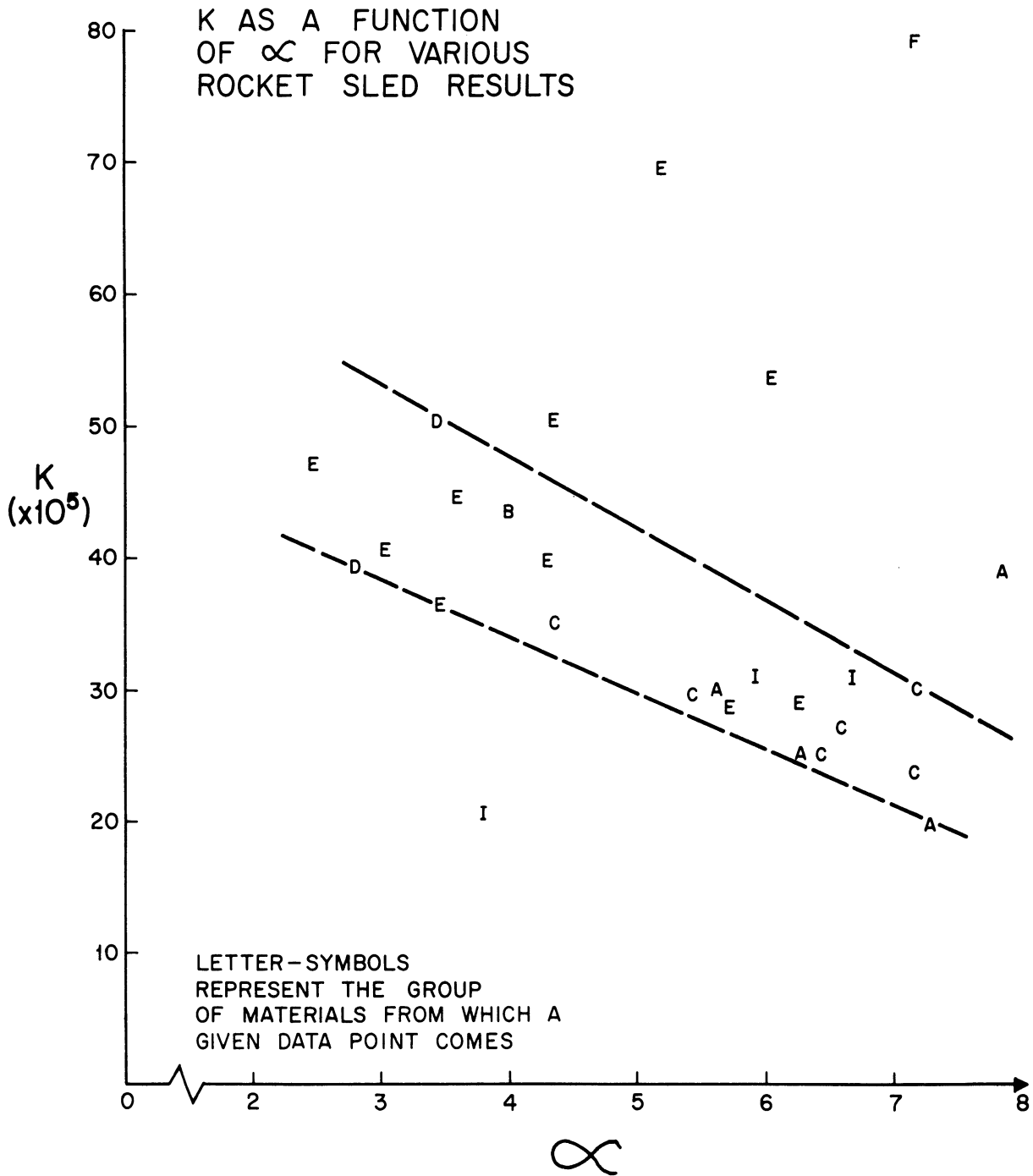


Figure 2

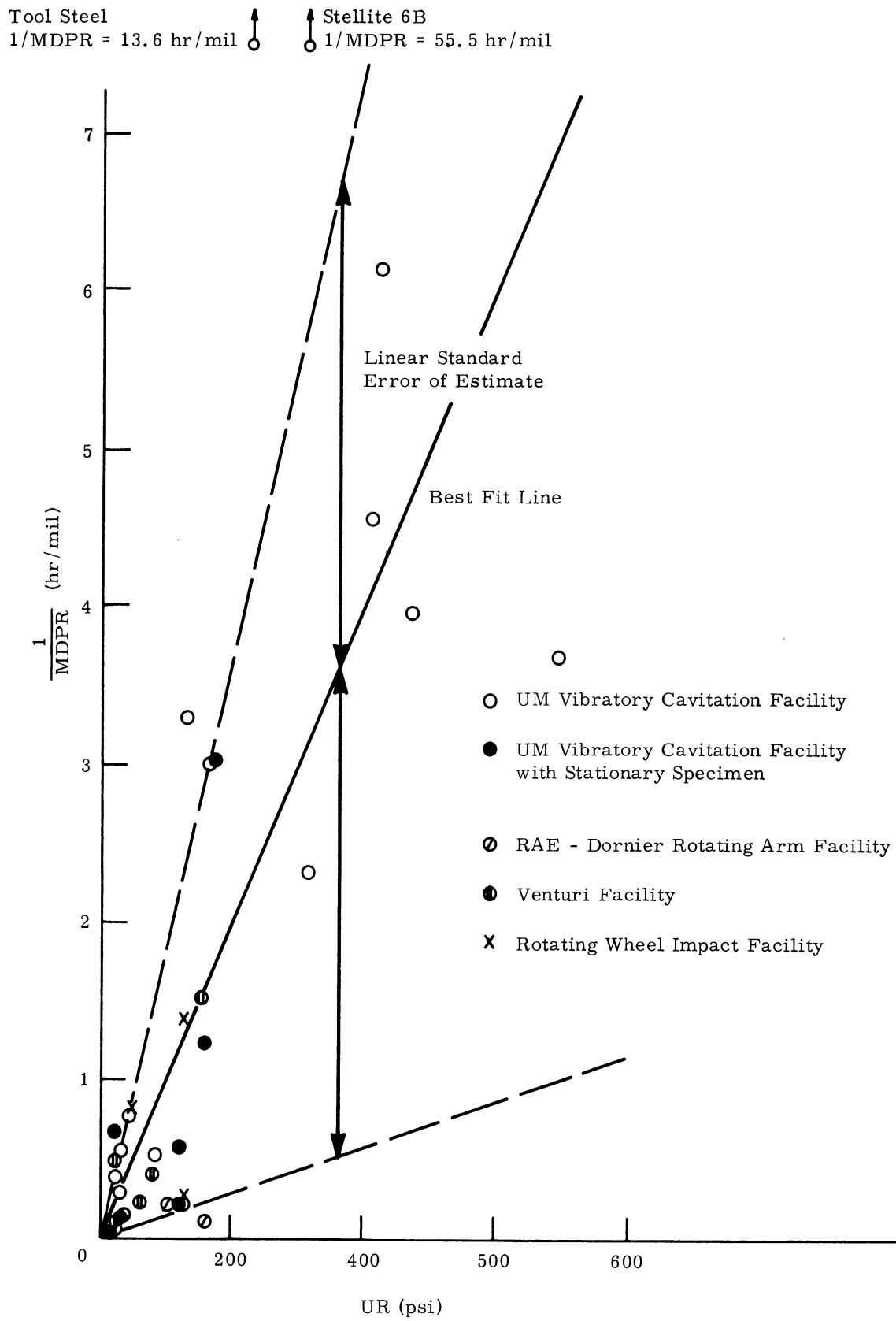


Figure 3.

V. APPENDIX

A. Correlation Coefficient r :

The correlation coefficient is defined as usual (19)
as

$$r = \frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}} \quad (1-A)$$

between two variables X and Y, if a linear relationship is assumed.

B. Standard error of estimate, s .

If we let Y_{est} represent the value of Y for a given value of X as estimated from a least square regression line of Y on X

$$Y = a + bX \quad (2-A)$$

the scatter about the regression line is measure by the so-called standard error of estimate of Y on X namely

$$s_{y,x} = \sqrt{\frac{\sum (Y - Y_{est})^2}{N}} \quad (3-A)$$

which is derived from the following equation

$$Y = a + bX + s_{Y,X} = Y_{est} + s_{Y,X} \quad (4-A)$$

if we assume the scatter is independent of X. However, it was found, from preliminary plotting of Y vs. X in our data, that the scatter is nearly linearly dependent on X so that

$$Y = a + bX + sX \quad (5-A)$$

was used. The constants a, b, and standard error of estimate were determined by least square regression analysis according to

$$\frac{Y}{X} = \left(\frac{a}{X} + b\right) + s \quad (6-A)$$

TABLE I-A

Least Square Fit Program

FORTRAN IV G COMPILER MAIN 05-14-69 23:20.17 PAGE 0001

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      C
      C  INVESTIGATION OF EROSION FAILURE ENERGY
0001      DIMENSION GROUP(60)
0002      DIMENSION NAME(200,5),DEN(200),WL(200),DPR(200),Z(200),AI(200)
0003      DIMENSION YS(200),TS(200),Y(200),EL(200),RDA(200),A(200)
0004      DIMENSION URO(200),SEO(200),TSEO(200),UR(200),SE(200),TSE(200)
0005      DIMENSION CON(200)
0006      DIMENSION EA(200), EB(200), EC(200), ED(200),EE(200)
0007      DIMENSION RA(200), RB(200), RC(200), RD(200), RE(200)
0008      DIMENSION CG(10), SG(10)
0009      DIMENSION H(100)
0010      DIMENSION EF(100),EG(100),RF(100),RG(100)
0011      COMMON N
0012      COMMON /S4/ CON,UR,SE,TSE
0013      COMMON /S5/ GROUP,NAME

      C
      C  INPUT
0014      DENW=1.0
0015      YW=0.28E6
0016      1 READ (5, 7) N, (GROUP(K), K=1,15)
0017      DO 20 I=1,N
0018      READ (5,10) (NAME(I,J), J=1,5), DPR(I), DEN(I)
0019      READ (5,12) YS(I), TS(I), Y(I), EL(I), RDA(I)
0020      READ (5,4) H(I)
0021      A(I)=1.0-RDA(I)
0022      CON(I)=1.
0023      20 CONTINUE
0024      7 FORMAT (I5, 5X, 15A4)
0025      10 FORMAT (5A4, 2F10.3)
0026      12 FORMAT (3E15.3, 2F10.3)
0027      4 FORMAT (F10.1)

      C
      C  ANALYSIS & CALCULATIONS
0028      SE1 = TS(1)*EL(1)
0029      TSE1 = 0.5*((YS(1)+TS(1)/A(1))*EL(1)-TS(1)*YS(1)/(A(1)*Y(1)))
0030      UR1=0.5*TS(1)**2.0/Y(1)
0031      DO 11 I=1,N
0032      SEO(I) = TS(I)*EL(I)
0033      SE(I) = SEO(I)/SE1
0034      TSEO(I)=0.5*((YS(I)+TS(I)/A(I))*EL(I)-TS(I)*YS(I)/(A(I)*Y(I)))
0035      TSE(I)=TSEO(I)/TSE1
0036      URO(I)=0.5*TS(I)**2.0/Y(I)
0037      UR(I)=URO(I)/UR1
0038      11 CONTINUE
0039      Z1=SQRT(DENW*YW/(DEN(1)*Y(1)))
0040      AI1=1.0+Z1
0041      DO 30 I=1,N
0042      Z(I)=SQRT(DENW*YW/(DEN(I)*Y(I)))
0043      AI(I)=1.0+Z(I)
0044      AI(I)=AI(I)/AI1
0045      EA(I)=DPR(1)/DPR(I)
0046      EB(I)=EA(I)*AI(I)
0047      EC(I)=EA(I)*AI(I)**2.0
0048      ED(I)=EA(I)*AI(I)**3.0
0049      EE(I)=EA(I)/AI(I)

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FORTRAN IV G COMPILER		MAIN	05-14-69	23:20.17	PAGE 0002
0050		EF(I)=EA(I)/AI(I)**2.0			
0051		EG(I)=EA(I)/AI(I)**3.0			
0052	30	CONTINUE			
	C	DATA			
0053		WRITE (6,28) (GROUP(K), K=1,15)			
0054	28	FORMAT (1H1,15A4)			
0055		WRITE (6,25)			
0056	25	FORMAT (1H0,T28,'YS',T40,'TS',T52,'Y',T64,'EL',T76,'HARD', 1 T88,'MDPR',T100,'UR',T112,'SE')			
0057		WRITE (6,24) ((NAME(I,J),J=1,5),YS(I),TS(I),Y(I),EL(I),H(I), 1 DPR(I),URO(I),SEO(I),UR(I),SE(I),I=1,N)			
0058	24	FORMAT (5A4, 8E12.3,2F8.3)			
	C				
0059		CALL CAL(EA, RA, CG(1), SG(1))			
0060		CALL CAL(EB, RB, CG(2), SG(2))			
0061		CALL CAL(EC, RC, CG(3), SG(3))			
0062		CALL CAL(ED, RD, CG(4), SG(4))			
0063		CALL CAL(EE, RE, CG(5), SG(5))			
0064		CALL CAL(EF,RF,CG(6),SG(6))			
0065		CALL CAL(EG,RG,CG(7),SG(7))			
	C				
0066		GO TO 1			
0067		END			

FORTRAN IV G COMPILER CAL 04-24-69 15:55.10 PAGE 0001

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0001      SUBROUTINE CAL(EA,G ,OCC,OSD)
0002      DIMENSION URA(200),SEA(200),TSA(200),USA(200),AUS(200),EA(200)
0003      DIMENSION GROUP(60),NAME(200,5),CON(200)
0004      DIMENSION UR(200),SE(200),TSE(200)
0005      DIMENSION EU(100), PU(100)
0006      DIMENSION G(100),P(100),S(100)
0007      DIMENSION H(100),T(100)
0008      DIMENSION COC(10), SOD(10)
0009      DIMENSION CO(100)
0010      COMMON N /S1/ CC,SDEV /S2/ C1,C2 /S3/ AO,B,C
0011      COMMON /S4/ CON,UR,SE,TSE
0012      COMMON /S5/ GROUP,NAME
      C
      C      REGRESSION ANALYSIS
0013      WRITE (6,80)
0014      80 FORMAT (1H1)
      C
0015      SEU=0.
0016      DO 40 I=1,N
0017      EU(I)=EA(I)/UR(I)
0018      PU(I)=1.C/UR(I)
0019      SEU=SEU+EU(I)
0020      40 CONTINUE
0021      C=SEU/N
0022      CALL REG(EU, PU, CON)
0023      DO 41 I=1,N
0024      G(I)=C1+C2*UR(I)
0025      P(I)=G(I)/UR(I)
0026      H(I)=C*UR(I)
0027      CO(I)=C
0028      41 CONTINUE
      C
      C      CORRELATION ANALYSIS
0029      CALL COR(EA,G,COC(1),SOD(1))
0030      OCC=COC(1)
0031      CALL COR(EU,P,COC(2),SOD(2))
0032      OSD=SOD(2)
0033      DO 42 I=1,N
0034      S(I)=OSD*UR(I)
0035      42 CONTINUE
0036      CALL COR(EA,H,COC(3),SOD(3))
0037      CALL COR(EU,CO,COC(4),SOD(4))
0038      DO 43 I=1,N
0039      T(I)=SOD(4)*UR(I)
0040      43 CONTINUE
      C
      C      SOLUTION
0041      WRITE (6,8) (GROUP(K), K=1,15)
0042      8 FORMAT ( //,10X,15A4 )
0043      WRITE (6,21)
0044      21 FORMAT ( //, ' MATERIAL & NORMALIZED', T30, 'EPSILON',
1 T47,'A+B*UR',T62,'+-D*UR',T77,'C*UR',T92,'+-D*UR',/)
0045      WRITE (6,22) ((NAME(I,J),J=1,5),EA(I),G(I),S(I),H(I),T(I),I=1,N)
0046      22 FORMAT (5A4,5F15.3)
0047      WRITE (6,23) COC(1),COC(3),SOD(2),SOD(4)

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FORTTRAN IV G COMPILER	CAL	04-24-69	15:55.10	PAGE 0002
0048	23	FORMAT (/,16X,2F30.3,T1,'CORRELATION COEFFICIENT',/,16X,2F30.3,		
0049		1 T1,'PERCENTAGE STANDARD ERROR OF ESTIMATE')		
0050		RETURN		
		END		

```

      C
0001      SUBROUTINE REG(E,R,S)
0002      DIMENSION E(200), R(200), S(200)
0003      COMMON N /S2/ C1,C2
      C      REGRESSION ANALYSIS
0004      T1 = 0.
0005      T2 = 0.
0006      T3 = 0.
0007      T4 = 0.
0008      T5 = 0.
0009      T6 = 0.
0010      DO 102 I = 1, N
0011      T1=T1+R(I)*R(I)
0012      T2=T2+R(I)*S(I)
0013      T3=T2
0014      T4=T4+S(I)*S(I)
0015      T5=T5+E(I)*R(I)
0016      T6=T6+E(I)*S(I)
0017      102 CONTINUE
0018      D=T1*T4-T2*T3
0019      D1=T5*T4-T6*T3
0020      D2=T1*T6-T2*T5
0021      5 C1=D1/D
0022      4 C2=D2/D
0023      WRITE (6,1) C1, C2
0024      1 FORMAT (2F15.3)
0025      RETURN
0026      END

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      C
0001      SUBROUTINE COR(X,Y,CC,SDEV)
0002      DIMENSION X(200), Y(200), DEV(200)
0003      COMMON N
      C      CORRELATION ANALYSIS
0004      XN = 0.
0005      YN = 0.
0006      XY = 0.
0007      XX = 0.
0008      YY = 0.
0009      VAR=0.
0010      DO 6 I=1,N
0011      XN = XN+X(I)
0012      YN = YN+Y(I)
0013      XY = XY+X(I)*Y(I)
0014      XX = XX+X(I)*X(I)
0015      YY = YY+Y(I)*Y(I)
0016      DEV(I)=X(I)-Y(I)
0017      VAR=VAR+DEV(I)*DEV(I)
0018      6 CONTINUE
0019      XYM=XN*YN/N
0020      XXM=XN*XX/N
0021      YYM=YN*YY/N
0022      V1=XY-XYM
0023      V2=XX-XXM
0024      V3=YY-YYM
0025      WRITE (6,8) V1, V2, V3
0026      8 FORMAT (3E15.3)
0027      V4=V2*V3
0028      IF (V4) 2,2,1
0029      1 CONTINUE
0030      CD=(V1*V1)/(V2*V3)
0031      CC=SQRT(CD)
0032      GO TO 3
0033      2 CONTINUE
0034      CC=1.
0035      3 CONTINUE
0036      FN=N-2.
0037      SDEV=SQRT(VAR/FN)
0038      WRITE (6,7) CC, SDEV
0039      7 FORMAT (2F12.3)
0040      RETURN
0041      END

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