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CUMULATIVE FATIGUE DAMAGE DUE TO SPECTRAL LOADING

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CUMULATIVE FATIGUE DAMAGE DUE TO SPECTRAL LOADING

The fatigue life of a metal part subjected to a spectrum of load intensities cannot be adequately evaluated from the results of a constant stress vs. cycles (S - N curve) experiment alone.

Laboratory tests have been conducted by several investigators to determine an appropriate means of calculating the number of stress cycles to failure with a known applied stress spectrum. There have been many papers written which relate to this basic problem and some of them give a new insight to previous data.[6 - 11]

In 1945 M. A. Miner[1] proposed the relation:

$$\sum_{i=1}^{M} \frac{n_i}{N_i} = A \tag{1}$$

where: M = the number of different stress levels

n_i = the actual number of stress cycles applied at stress 'i'

N_i = the limiting life at the stress level 'i'

Figure 1 illustrates the meaning of these symbols on S - N coordinates. The S - N curve can be constructed empirically as a straight line on a log-log cycle from a point at $.9\ S_u$, and 10^3 cycles to a point at a stress equal to the endurance strength at 10^6 cycles.

This relationship was orginally evaluated by Miner using Alclad 24 S - T aluminum sheet specimens with up to 4 applied stress levels, ranging between .32 and .72 S_u (ultimate strength) for which

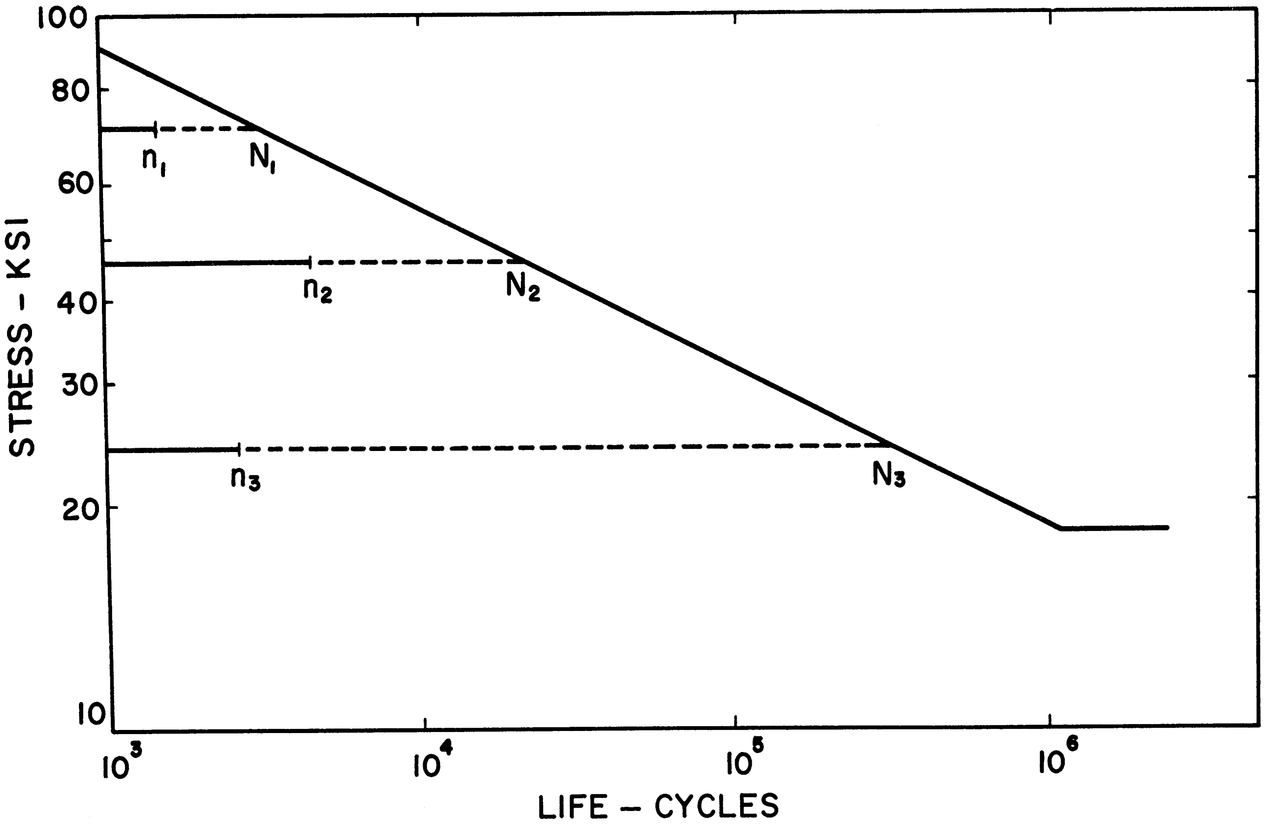


Figure 1. S - N Curve and Miners Line.

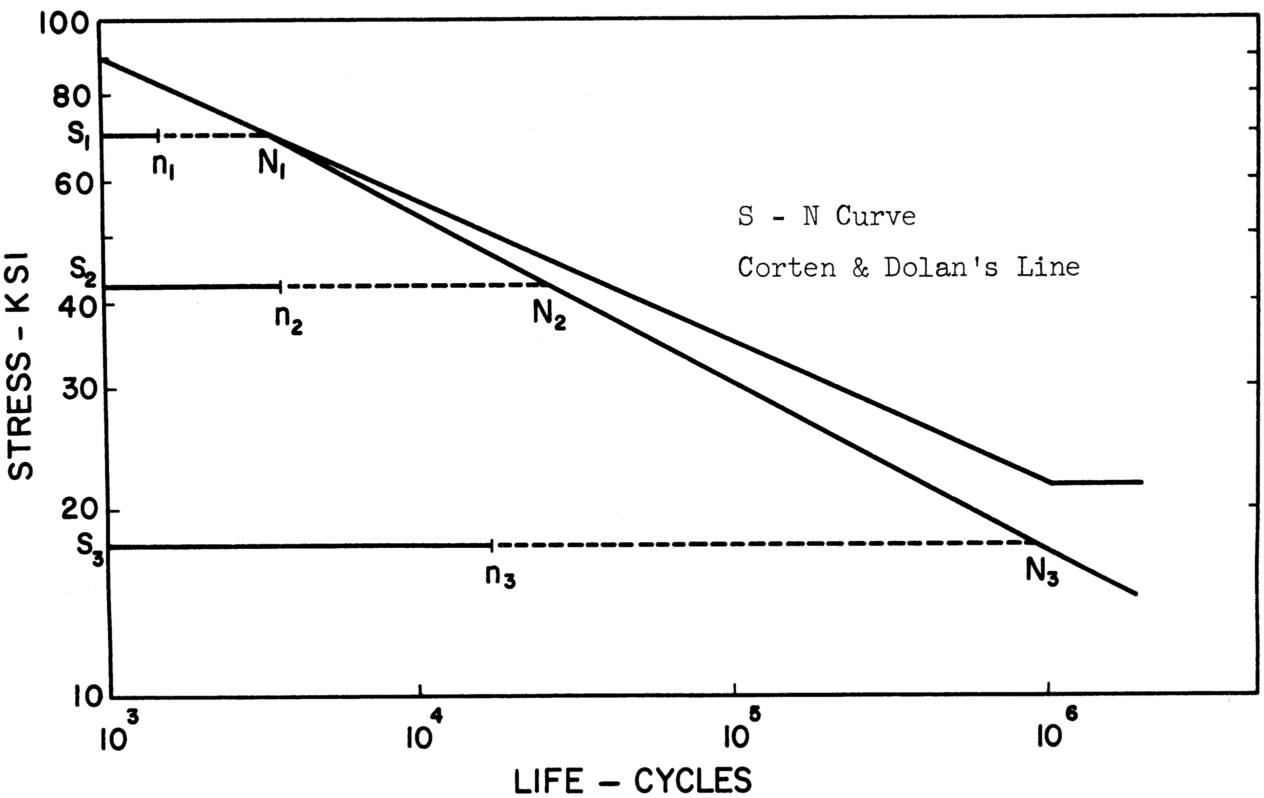


Figure 2. S - N Curve Compared to Corten and Dolan's Line.

he obtained values of A between .161 and 1.49, with an average value based on 22 items of 1.02 and a standard deviation of .24. He thus concludes that A equals 1.0 will yield a good estimate of fatigue life.

Several points should be noted with regard to these tests. They were made with plates containing stress concentrations with a net effect of K_f (fatigue strength reduction factor due to geometrical notch in specimen) equal to 2.5 combined with mode of loading effects which reduce the endurance limit to .11 S_u .[5] Miner's experiments used reversed axial loading combined with static axial loading. The applied loads were never less than the experimentally found endurance limit, and only aluminum alloys were tested.

In 1956 A. M. Freudenthal[2] tested 3/16 inch diameter aluminum specimens of 2024 and 7075 aluminum alloys in bending. He obtained values of A in Equation (1) between .13 and .99. He concluded that the test results did not support the cumulative damage based on linearity of accumulation theory (Miner's rule with $A = 1$), but that his data would approximately fit a relation similar to Miner's, except with a value of A equal to .58.

It will be noted that these tests were also based on the performance of aluminum alloys. Freudenthal tested at 6 stress levels applied for a known percentage of the time for each specimen. The stress levels ranged from .33 to .91 of the ultimate strength.

He does not explain that S - N curve was used to obtain his values of N_i , nor does he give any listing of his results, but it is clear that the specimens had no initial stress concentration, and that they were loaded in a combination of shear and bending. As soon as the specimen is cracked, a stress concentration factor will appear and reduce the magnitudes of the locus of N_i appreciably. It is this effect which is important in explaining the value of $A = .58$ in Freudenthal's experiments.

In 1956 Corten and Dolan(3) developed a new theory as the result of tests that they conducted using hardened steel wire of .050 inch diameter on a total of 497 specimens. Each specimen was tested at two stress levels with a known percentage of the cycles at each level. The stresses ranged from .32 to .67 S_u. Rather than evaluating their data by Miner's linear cumulative damage rule, Corten and Dolan related the total number of stress cycles to the portion of total cycles at each stress level as follows (see Figure 2):

$$N_G = N_1 / \sum_{i=1}^{M} \alpha_i (S_i/S_1)^d \qquad (2)$$

where: N_G = the total number of stress cycles to failure for spectrum loading

N_1 = the number of cycles to failure at S_1

M = the number of different stress levels

S_i = the stress at level 'i'

S_1 = the maximum stress applied

α_1 = the fraction of the total cycles at stress S_i , $\frac{n_i}{N_G} = \alpha_i$

$d = .87\ m$ ($d = 6.57$ for steel wire)

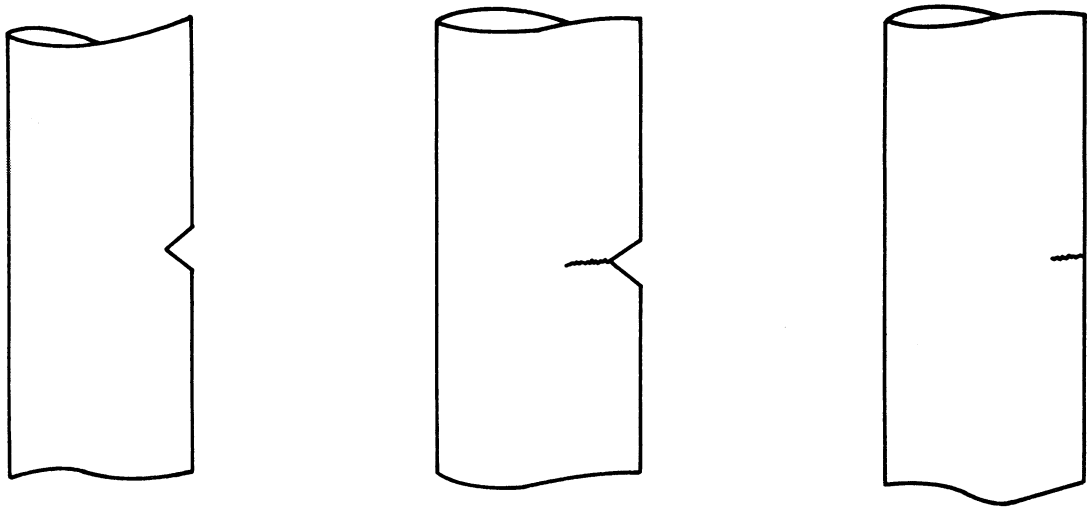
m = the inverse slope of the S - N curve

n_i = actual stress cycles at S_i

This relation is in many ways similar to Miner's rule except that it does not use the S - N curve as a reference for the locus of N_i, but rather a line of slope d which is .87 of the inverse slope of the S - N curve, and extends as a straight line to the lowest stress level. Corten and Dolan's specimens fit this curve, shown in Figure 2, very well.

Next we should examine the influence of using a specimen with a initial stress concentration such as a notch or corner. A stress concentration of this type will generally be the source of fatigue failure. Shown in Figure 3a is the member with a stress concentration of 2.5. When this member is damaged by fatigue the physical structure at the notch is rearranged, producing a microscopic crack, and the stress concentration increases toward $K_f = 3.0$, a relatively small change from 2.5. A member which has no notch or corner to act as an initial stress concentration will be designed on the basis of $K_f = 1.0$, but when damage begins the fatigue strength reduction factor K_f increases to $K_f = 3.0$, and the part will then approach failure at a greatly increased rate.(12)

Considering the differences between the test conditions and material properties used in the experiments of Miner, Freudenthal, and Corten and Dolan, it is possible to postulate a logical homogeneous



(a) Bar with Notch, $K_f = 2.5$

(b) Bar with Notch and Crack, $K_f = 3.0$

(c) Bar with Crack $K_f = 3.0$

Figure 3. Effect of Incipient Fatigue Failure on a Specimen with and without and Initial Stress Concentration.

relationship which is consistent with the results obtained by each. The three theories each take the summation of $\frac{n_i}{N_i}$; the difference between them is in the line they choose to represent the locus of values of N_i (see Figure 4). That is:

1. Miner chose the S - N curve using a part with an initial stress concentration of 2.5 for the locus of his values of N_i .
2. Freudenthal chose a line of .58 of the S - N curve using a specimen with no initial stress concentration.
3. Corten and Dolan chose a line of slope d on S - N co-ordinates which is equivalent to a line of 87% of the inverse slope of their S - N curve using a specimen with no stress concentration. In Figure 4 the line is drawn for an arbitrary value of $S_1 = .8\,S_u$.

As can be seen from Figure 4, these lines are not as deviant as the theories would seem.

In order to unify the above three relations the following equation is suggested:

$$N_G = N_1 / \sum_{i=1}^{M} \alpha_i (S_i/S_1)^{d'} \tag{3}$$

where the symbols have the same meaning as in the Corten and Dolan Equation (2) except that:

$$d' = d(.79 + .08\,K_f) \tag{4}$$

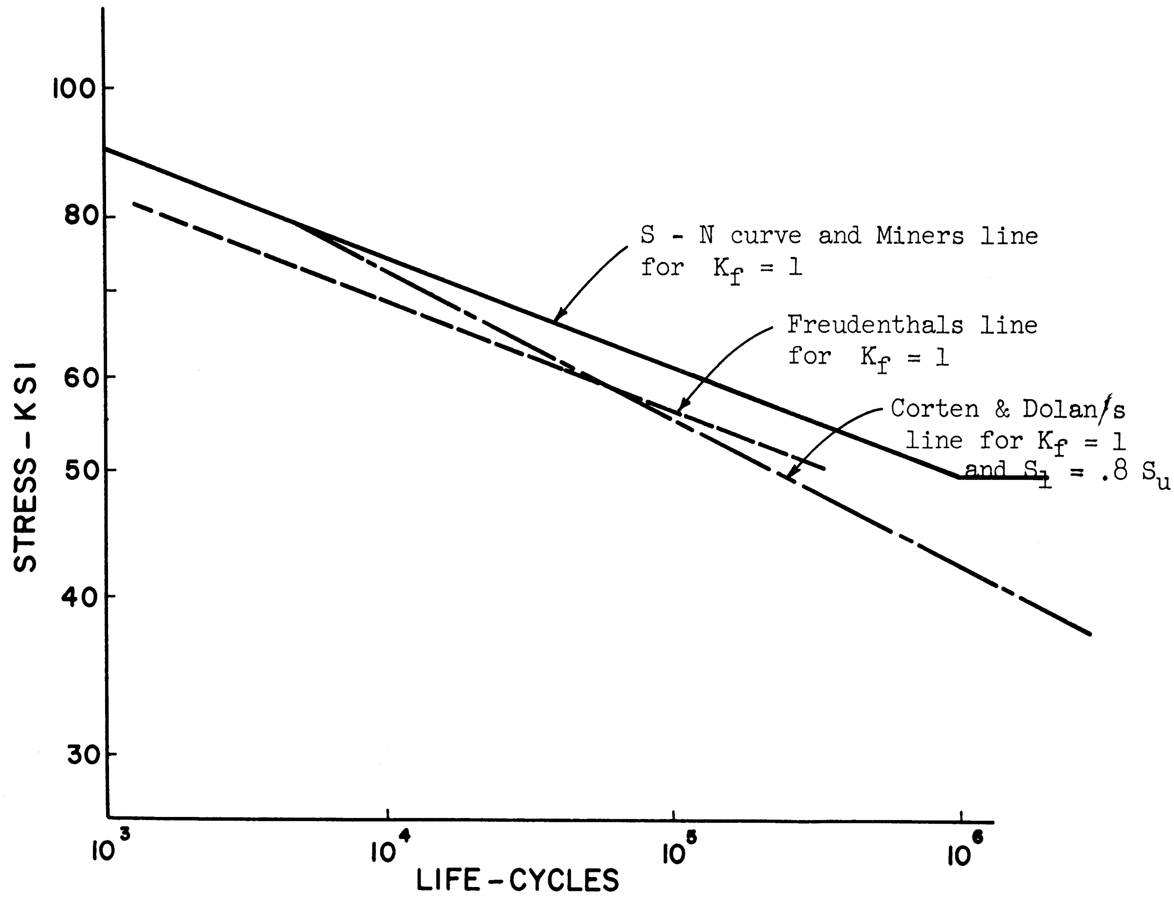


Figure 4. S - N Curve Comparing Miners', Freudenthals' and Corten and Dolan's Line.

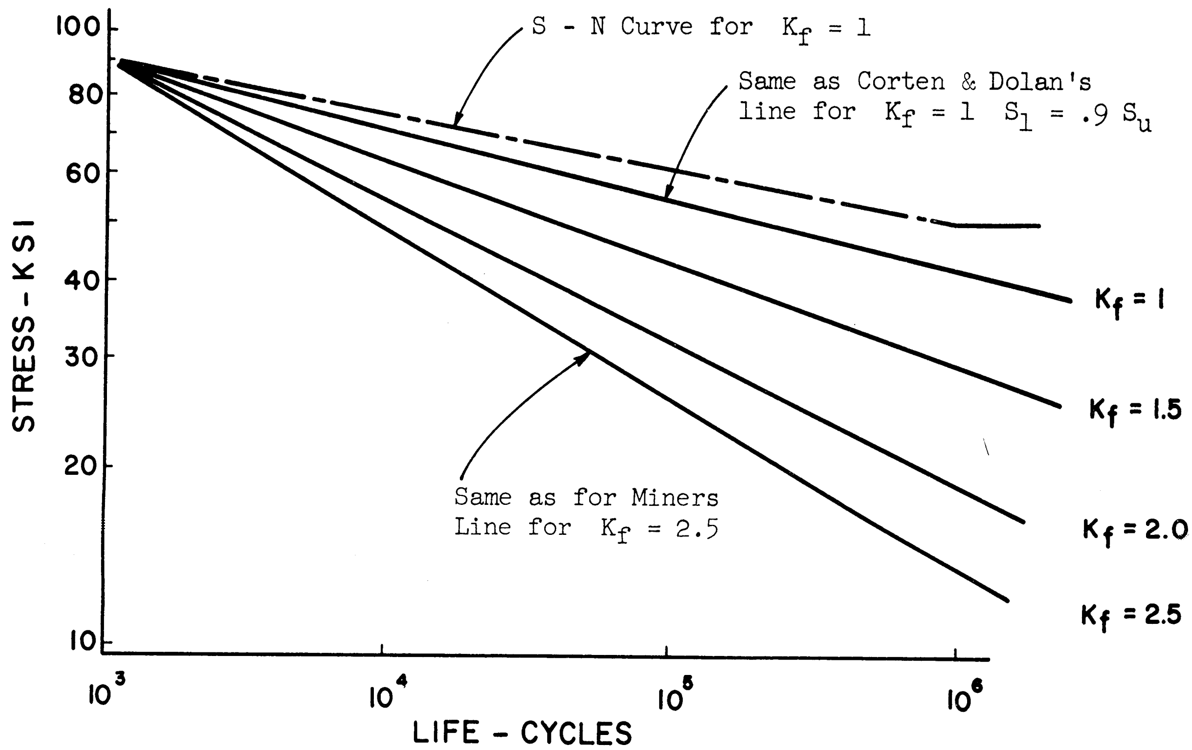


Figure 5. N_i Locus Lines for Proposed Relation for Various K_f.

where K_f = the fatigue strength reduction factor.[5] This equation results in N_i locus lines as shown in Figure 5.

As an example of the above approach, consider the following problem.

Select a steel with a tensile strength adequate to withstand the following spectrum of loading for 10^5 cycles with a reliability of 99.9%. The applied load is symmetrical bending with a random loading sequence as shown in Table 1 and an estimated fatigue strength reduction factor, K_f = 2.0.

TABLE 1

APPLIED LOAD DISTRIBUTION

S_i Load-KSI	n_t Spectrum Test Cycles
70	300
60	400
40	1000
20	1000
10	2000
	4700 = N_t = total number of cycles in spectrum test

Solution:

1. The maximum value of $S_i = S_1 = 70$ KSI

2. The value of d in Equation (4) can be calculated by either of two methods. If the S - N curve has been found by previous testing d is equal to .87 of the inverse slope of this curve. If the S - N curve is not known the values at the end points of the straight line are sufficient to determine the line and the value of d . The left end point of the S - N curve at 10^3 cycles is equal to .9 S_u . The right end point at 10^6 cycles is calculated with the following empirical correlations for this example.*

   $S_{@10^6} = S_u \times K_1 \times K_2 \times 1/K_f$

   $K_1 = .5$, endurance limit correction for bending

   $K_2 = .8$, reliability correction for 99.9% reliability from the S - N curve value of 50%

   $K_f = 2.0$, fatigue strength reduction factor due to stress concentration

   $S_{@10^6} = .5 \times .8 \times 1/2.0 \times S_u = .2\ S_u$

   Taking the ratios $10^6/10^3 = (.9\ S_u/.2\ S_u)^m = 4.5^m$; solving yields $m = 4.6$, and since $d = .87m$, $d = 4.0$.

3. Using Equation (4); $d' = 4.0\ (.79 + .08 \times 2.0) = 3.8$

4. To find $\Sigma\, \alpha_i (S_i/S_1)^{d'}$ (refer to Table 1) recalling that $N_t = 4700$, $S_1 = 70$ KSI, $N_G = 10^5$, and $d' = 3.8$.

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*For further information on obtaining the S - N curve empirically see Reference 5, Chapter 11.

TABLE 2

CALCULATION OF $\Sigma \alpha_i (S_i/S_1)^{d'}$

S_i Load-KSI	$\alpha_i = \frac{n_t}{N_t}$	$\alpha_i (S_i/S_1)^{d'}$
70	.064	.064
60	.085	.047
40	.212	.027
20	.212	.002
10	.414	.000
		.140 = $\Sigma \alpha_i (S_i/S_1)^{d'}$

5. Transposing Equation (3) to

$$N_1 = N_G \cdot \Sigma \alpha_i (S_i/S_1)^{d'}$$

$$N_1 = 10^5(.140) = 1.40 \times 10^4$$

6. Referring to Figure 6, drawing a line of slope d' from point N_1S_1 to the right gives the locus of values of N_i .

7. Drawing a line of slope $m = 4.6$ from N_1S_1 to the left yields the value of $.9\ S_u$ at $N = 10^3$ cycles which yields $S_{@10^3} = .9\ S_u = 124$ KSI.

8. Thus the required ultimate strength, $S_u = 124/.9$ = 138 KSI to meet the given requirements.

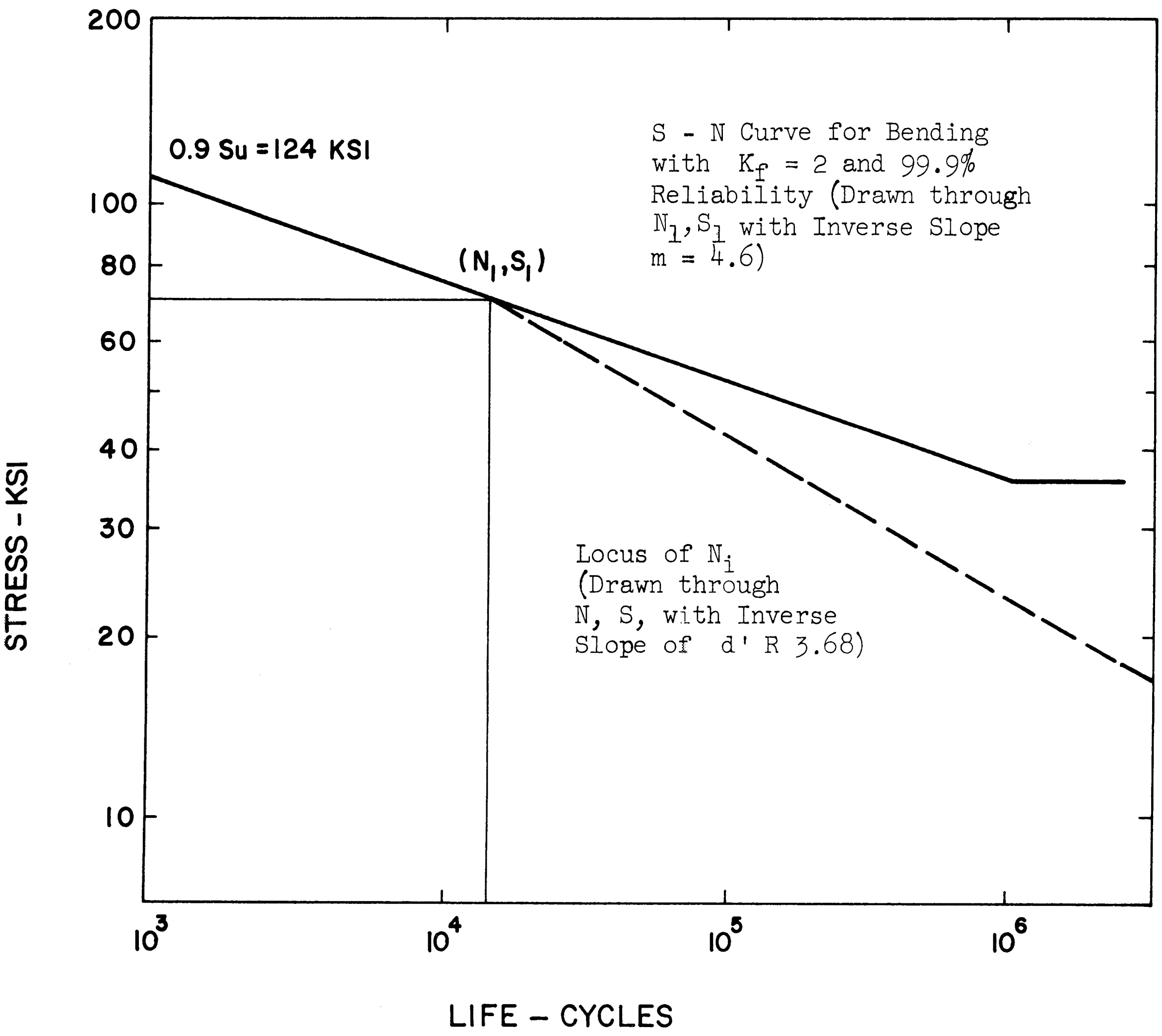

Figure 6. S - N Curve and N_i Locus Line for Sample Problem.

This paper has presented a method of modifying Corten and Dolan's original equation with the additional information of the fatigue strength reduction factor so that the resulting equation is capable of predicting the results obtained by Miner, Freudenthal, and Corten and Dolan.

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