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THE EFFECT OF ATMOSPHERIC TEMPERATURE VARIATIONS ON COSMIC-RAYS UNDERGROUND

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ABSTRACT

The variations in the intensity of cosmic-rays observed underground, in a salt mine 1100 ft. below the surface (846 mwe), are correlated with variations in temperatures in the stratosphere and at sea-level, over the period from April 1, 1951 to November 30, 1951. Two trays of Geiger counters were operated in coincidence as a vertical telescope and this apparatus with associated eletronic equipment were set up in the mine (in Detroit, Michigan) to record the hourly rate of cosmic-rays. The atmospheric temperatures at the 100 mb, 150 mb, 200 mb, 300 mb pressure levels, and sea-level were measured with radiosonde equipment sent aloft in balloons by the personnel of the weather observation stations located at Selfridge Field, Michigan and Toledo, Ohio. The cosmic-ray intensities and the atmospheric temperatures recorded at times corresponding to those of the cosmic-ray observations are employed in the study of the effect of

temperature variations on the intensity of cosmic-rays underground.

The theoretical model, describing the production of mesons, is developed on the basic assumption that μ-mesons originate only in the decay products of π-mesons and this model is shown to indicate an expected temperature coefficient of ~ 0.4 percent per degree for μ-mesons of energy $> 10^{11}$ ev. The cosmic-rays observed in the salt-mine are μ-mesons, with essentially this average energy, and their secondary products. The correlation analysis of the atmospheric temperatures and observed cosmic-ray intensity is performed for each pressure level separately and the following experimental temperature coefficients are obtained: at 100 mb - 0.062 ± 0.042 percent per degree, at 150 mb - 0.056 ± 0.039 percent per degree, at 200 mb - (-0.012 ± 0.034) percent per degree, at 300 mb - 0.014 ± 0.029 percent per degree, and at sea-level - (-0.011 ± 0.017) percent per degree. These results are considerably smaller than the predicted theoretical value and lie well outside the uncertainty inherent in the mathematical treatment of the π-μ decay model.

The accuracy of the experimental procedure is established and the significance of the disagreement between experimental and theoretical results is discussed. It is concluded that the π-μ decay model does not describe the origin of μ-mesons of energy $\geq 10^{11}$ ev. Consideration is given to the pertinent properties of hypothetical particles which could replace π-mesons as the progenitors of μ-mesons. In view of the recently discovered χ-meson, which has been observed to decay into a μ-meson, this particle is suggested as a possible parent of high-energy μ-mesons. Using the observed mass of the χ-meson ($\sim$ 1200 electron masses) and a mathematical model similar to that developed for π-mesons, it is found that if χ-mesons are the parents of μ-mesons of energy $\geq 10^{11}$ ev, then a maximum value of 4×10^{-10} sec for the mean lifetime of χ-mesons would be associated with a maximum temperature coefficient of 0.08 percent per degree.

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CHAPTER I

INTRODUCTION

1.1 Historical Introduction

In an effort to resolve the difficulties associated with the theoretical treatment of nuclear forces, Yukawa assumed that these forces were transmitted by a particle of several hundred electron masses. The discovery of such a particle, associated with the penetrating components of cosmic-rays, (in which particles of the highest known energies can be observed) gave promise of new insight into this theory of nuclear forces.

Subsequent experiments revealed more than one type of particle with mass intermediate between the masses of the electron and the proton (now well known as mesons) and that the particle first discovered in cosmic-rays, the μ-meson, does not possess the properties required of the Yukawa meson. It is now known that the μ-meson is the most abundant particle in the penetrating component of cosmic-rays but does not interact strongly with nuclei[1], which is contrary to the expected behaviour of the meson associated with nuclear forces. The π-meson, which is more rarely observed in the lower atmosphere, frequently produces nuclear disintegrations[2]. When not absorbed, the π-meson is known to decay, with a mean lifetime of approximately 10^{-8} sec[3], into a μ-meson and a neutral particle. Photographic plate experiments reveal that the primary cosmic-rays consist of some 75 percent protons and 25 percent stripped nuclei of $Z \geq 2$[4]. Hence it appears reasonable to deduce that primary cosmic-rays, on penetrating the earth's atmosphere, produce other particles (in interactions with atmospheric matter), among which are π-mesons, and that these latter particles may decay into μ-mesons or themselves experience nuclear interactions. These deductions are supported by increasing experimental studies of the behaviour of mesons and nucleons.

1.2 The Present Investigation

Among the features of the model proposed above for the origin of μ-mesons is the competition process which determines the ultimate fate of their π-meson progenitors; i.e. π-mesons can either decay into μ-mesons or interact with nuclei in the atmosphere. At least three reasons are suggested for investigating this process.

i) The decay-interaction competition must depend on the properties of π-mesons (mean lifetime, interaction cross-section, energy, etc.) and on the properties of the atmosphere traversed by these mesons (density and composition). Experimental investigation may show the competition process to be strongly dependent on some of these properties and thus provide quantitative information concerning them.

ii) The competition process presents a possible explanation for the behaviour of the penetrating component of cosmic-rays observed underground (see 1.3).

iii) The wide acceptance of the π-μ decay model is supported, in part, by observations of the decay of π-mesons into μ-mesons in photographic emulsions and cloud-chambers. Independent support would be provided by verifying the existence of the competition process. Moreover, the competition process can be verified at higher π-meson energies than those observable in emulsions and cloud-chambers (total energies of ~ 2-3×10^8 ev).

The decay-interaction competition can be verified by measuring the effects of changing atmospheric density on the relative probability of nuclear interaction for π-mesons compared with the probability of their decay into μ-mesons. The variation of density, in the region of the atmosphere where most π-mesons spend their lives, can be obtained from observations of atmospheric temperature made with radiosonde apparatus sent aloft in free balloons. These temperature variations can be compared with variations in μ-meson intensity, as observed in the penetrating component of cosmic-rays, for possible agreement with the temperature effect expected from the known properties of π-mesons and the assumed π-μ decay model.

Such experiments have been performed. One of the most recent was conducted by Duperier[5], who investigated the combined effect of variations in surface barometric pressure, mean altitude for π-meson production, and temperature at the mean production level, on the intensity of cosmic-rays observed under 25 cm. of lead. This experiment was performed at sea-level, where the average energy of the penetrating component is lower than those energies for which the theoretical model (chap.IV,4.2) predicts the largest temperature coefficient. Moreover, a linear dependence of intensity on the three parameters was assumed with no indication of the degree of approximation implied by such an assumption and a crude description of the competition process was used to interpret experimental results.

In the present experiment, the variations in cosmic-ray intensity, observed in a salt-mine 1100 ft. underground, are compared with the temperature variations at four pressure levels in the upper atmosphere.

(The theoretical model indicates that very little correlation, between intensity and temperature, should exist at atmospheric levels deeper than 300 mb. This was confirmed by investigating the correlation at 450 mb. and 1000 mb. (sea-level).) There are three distinct advantages in performing the experiment underground.

i) The average energy of mesons observed in the mine[6] is believed to be approximately the energy for which the theoretical meson-production model predicts a temperature coefficient near its constant maximum value (see Chap. IV, 4.2).

ii) The effects of changes in barometric pressure and changes in altitude of π-meson production are negligible, since they affect only low energy μ-mesons (through absorption and decay).

iii) The proposed scheme of meson genesis is tested for mesons with higher average energies than those involved in sea-level investigations. Finally, a more detailed treatment of the production scheme than has been attempted previously will be considered in this investigation.

1.3 Related Problems

This experiment is intimately related to the general problems associated with observations of cosmic-rays underground, as described by Randall[6] in the introduction to the report of his investigations in the same salt mine. In particular, it is known that the underground depth-intensity relationship[7,8] is best fitted by

$$I_v = \frac{\text{const.}}{h^{1.9}}$$

where I_v is the vertical intensity at depth $\underline{h}$ below the top of the atmosphere, for $20 < h < 250$ meters water equivalent.* For $h > 250$ mwe, however, the intensity decreases more rapidly, the exponent increasing slowly with the depth up to values ~ 3.5 at the greatest depths where measurements were performed [7,9]. Randall [6] describes several explanations offered for this increase in absorption, among which are those of Greisen [10,11] and Hayakawa[12]. They suggest that the "bend in the underground depth-intensity curve can be explained by the existence of π-mesons, with a power-law energy spectrum, which are subject to a collision-decay process. This explanation is based on a meson production model which is similar to that used in calculating the temperature effect,

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* This is a frequently used unit obtained by multiplying the depth in meters by a suitable average density of the medium traversed.

and is discussed in the following paragraph.

It is strongly believed that cosmic-ray energy underground is carried to all depths by μ-mesons.[13] The mechanism by which the number of μ-mesons, observed at depths > 250 mwe, is depleted more rapidly than those observed higher up, can be seen by considering the energies of the mesons involved and their proposed origin. In the probability for decay-in-flight of the π-meson, which has the form $\sim \exp(-t/\tau)$ (where t is the time after it has been produced and τ is the mean lifetime in the laboratory system), it is clear that τ depends on the energy of the π-meson through the relativistic time contraction. If the mean collision path, λ, is taken to correspond (approximately) to the geometric cross-section for collision, then the collision probability is independent of energy. Thus the probability of decay will decrease, with increasing π-meson energy, until collision becomes the dominant process at high energies. In the event that collision occurs, the energy of the π-meson is distributed among its progeny, thereby producing radiation degraded in energy compared with the parent π-meson. If a π-meson, which would otherwise decay into a μ-meson barely capable of reaching a given depth, collides, then the collision cannot produce ionizing radiation capable of reaching the same depth. In this way the increased probability of collision for high energy π-mesons serves to increase the depletion of μ-mesons, observed at great depths, beyond that due to absorption.

This scheme may be a tenable one, but since it assumes that π-mesons are produced with a power-law energy spectrum this spectrum must be verified. The explanation also assumes that μ-mesons originate from the decay of parent particles with a mean lifetime (in the parents' rest system) $\sim 10^{-8}$sec. and a mean collision length $<$ 1000 gm cm^{-2}. Since the temperature effect is based on a model in which such assumptions are made, the present investigation may contribute toward understanding the "bend" in the depth-intensity curve. In particular, the temperature effect depends more critically on the mean collision length, than the "bend" does, so that the present experiment may provide an estimate of this parameter.

Apart from underground phenomena, study of the very energetic constituents of cosmic-rays can perhaps contribute to considerations of cosmic-ray origin. It will be seen, (Chap.II) that the apparatus used for measurement of the temperature effect provides hourly recording of cosmic-ray intensities over an extended period of time. This feature makes it easy to observe time variations in the intensity of high energy cosmic-rays, which may be associated with possible diurnal or seasonal variations of primary particles. On the basis of observations gathered over the period of the present experiment, estimates of diurnal variation are given in Chapter III.

CHAPTER II

APPARATUS AND EXPERIMENTAL CONDITIONS

2.1 Experimental Problems

The experimental problem is that of obtaining information on variations in the intensity of cosmic-rays observed underground and comparing them with variations in temperature of the atmosphere traversed by the progenitors of the underground cosmic-rays. Theoretical analysis (Chap. IV) predicts variations in intensity of approximately 4 to 8 percent corresponding to the maximum variation in observed stratospheric temperatures ($\sim$10-20°C). The low intensity of cosmic-rays observed at the depth of the experiment ($\sim 2 \times 10^{-6} cm^{-2} sec^{-1}$ which is 10^{-4} times sea-level intensity) presents several problems associated with measuring small variations in this quantity.

Since the statistical error (in percent) connected with cosmic-ray counting rates is proportional to $n^{-1/2}$, where n is the number of cosmic-rays observed, a desired accuracy of 0.5 percent in the intensity corresponding to a given atmospheric temperature requires that at least 4×10^{4} counts be recorded during the period in which the temperature has its given value. Stratospheric temperatures are rarely observed to remain within a three degree interval for more than two or three days. These considerations indicate that the counting equipment employed should be large enough to count several thousand cosmic-rays per day in order that the time required for the experiment does not become prohibitive. The counting rate with the equipment to be described in this chapter is $\sim$ 100 hr^{-1}.

Since the Geiger counters employed in this experiment (and similar radiation detectors in general) do not distinguish between ionizing particles of different kinds or different origin (μ-mesons, Compton electrons, radioactive contaminants, etc.) it is necessary to determine the number of observed counts which are due to cosmic-rays. In order that the statistical error associated with the number of counts caused by events other than cosmic-rays be much smaller than the corresponding error in cosmic-ray intensity, it is also important that these extraneous counting-rates be a very small fraction of the total rate. The methods used to treat this problem are discussed in 2.3.

The long period of time required for collecting cosmic-ray data corresponding to the temperatures in the range observed, makes it necessary to insure that possible variations in the operating characteristics of the equipment or in the contributions to the counting rates from events other than cosmic-rays do not produce effects which can be confused with variations in cosmic-ray intensity. The precautions taken to provide this insurance are discussed in 2.3.

The information reflecting temperature variations in the atmosphere above the cosmic-ray apparatus is also subject to precautionary treatment in order that the small expected correlation ($\sim$0.4 percent per degree) be accurately represented. The temperature measurements were performed by the personnel of two weather observation stations (as part of their usual duties) located less than 50 miles southeast and north of the apparatus, respectively. The accuracy of the temperature measurements and the fidelity with which they describe atmospheric conditions above the apparatus are affirmed by scientists at the Air Force Cambridge Research Center where atmospheric properties and measurements are studied. The fact that atmospheric temperature varies with altitude raises the question of how the temperatures recorded at different pressure levels are to be employed in characterizing the atmosphere. In the absence of practical a priori weights which might be assigned to the various levels in obtaining a suitable mean temperature, in this investigation the temperature effect is sought for at each of four pressure-levels separately. Finally the large variations ($\sim$10 degrees) sometimes observed in successive temperature observations do not allow a dependable temperature value to be associated with cosmic-rays observed over periods in which such fluctuations occur. The selection of useful data and the methods used in determining the temperature-intensity correlation are described in detail in Chap. III.

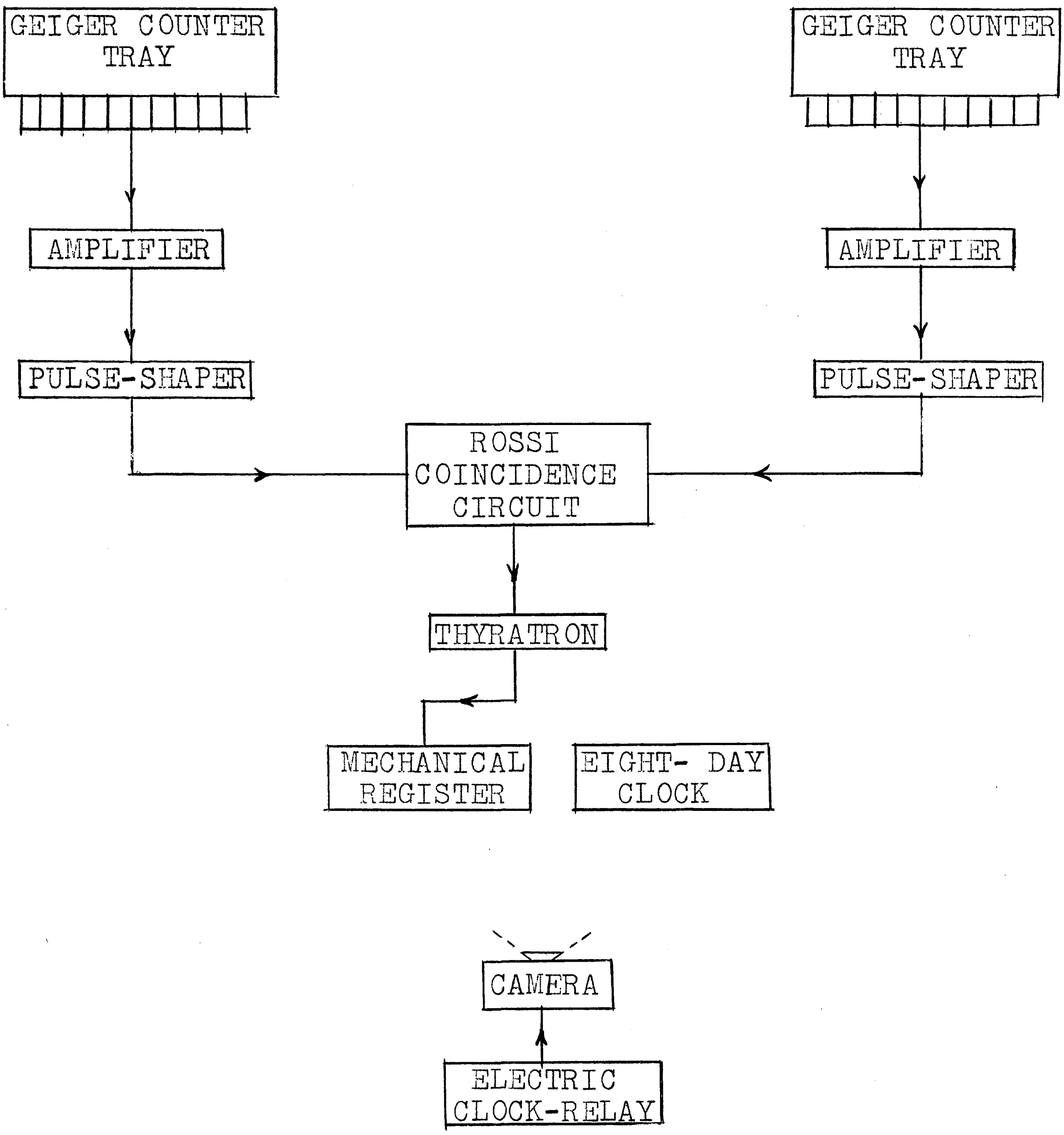
2.2 Description of Experimental Apparatus

Before describing the components in detail the apparatus will be described in outline. Twenty Geiger counters of conveniently large size were arranged in two trays of ten counters each where the counters in separate trays operated in parallel. The two trays were placed one above the other and were operated in coincidence. The passage of one or more ionizing particles through one or more counters in one tray was detected by a pulse produced at the output of the tray. The pulses from the two trays were amplified separately and then introduced to a coincidence circuit. The coincidence circuit produced pulses whenever two amplified pulses from different trays were introduced within the resolving time of 4×10^{-6} sec. (In this way the passage of a cosmic-ray through one or more counters in both trays appeared as a pulse at the output of the coincidence circuit.) The coincidence pulses were used to activate a mechanical register which counted the number of these pulses. The number appearing on the register was automatically photographed each hour. This provided a record of cosmic-ray intensity for later comparison with temperatures observed at corresponding times. A block diagram of the components used in collecting data is given in Fig. 1 and a more detailed description of all the components follows.

Each component was adapted for its function by revising the design of conventional devices. The description will then consist of whatever special features may be involved and references will be given for the details describing fundamental characteristics.

i) The twenty Geiger-counters were unusually large but similar in construction to those described in reference [14]. Each counter consisted of a tungsten central-wire anode 0.006 in. in diameter and a cylindrical brass cathode 90 in. long, 2 in. in diameter,

FIGURE 1



BLOCK DIAGRAM OF APPARATUS

and 1/32 in. thick. The counters were filled with 10 parts argon and 1 part alcohol to a total pressure of 11 cm. Hg. They were operated about 40 volts above the beginning of the plateau which occurred at about 1100 volts.

ii) The electronic apparatus was designed to count the number of pulses produced by the simultaneous discharge of one or more counters in each tray. This was accomplished in four steps. First the pulses from the counter trays were amplified separately by a two-stage resistance-coupled amplifier. The amplified pulses were used to trigger a univibrator pulse-shaping circuit. The shaped pulses from the separate trays were then introduced to a Rossi coincidence circuit. Finally, a thyratron-activated mechanical register counted the coincidences. These components are essentially the same as those described in Elmore and Sands [15]. (In this reference, see Fig. 2.14(b)-amplifiers, Fig. 2.34(a)-pulse-shaper, Fig. 2.46(a)-coincidence circuit, and Fig. 2.32(a)-thyratron. The mechanical register is the Mercury brand discussed in 4.1 of the reference.)

iii) The power supplies consisted of dry-cells for counter operation and the voltage-regulated

Model 50 (described in section 4.1 of reference [15]) for the apparatus described in ii) above.

iv) The recording equipment consisted of a 16 mm. movie camera that was operated to take one exposure each hour by the action of an electric clock and relay. The camera photographed the mechanical register and the face of an eight-day mechanical clock. The clock was used to detect the infrequent power-failures which occurred in the salt-mine. The first photograph taken after a power-failure showed the elapsed time between photographs to be more than one hour.

Laboratory equipment consisting of a synchroscope, a pulse-generator, and a scaler were employed in making periodic tests on the apparatus above.

2.3 Relations Between Observed Counting Rate and Cosmic-Ray Coincidences

In order to justify confidence in the data collected, all necessary precautions were taken and periodic tests were made, which will be discussed here. First, the plateau characteristics of the Geiger counters were considered. In order to insure that changes in these characteristics did not produce effects which could be confused with variations in cosmic-ray intensity, the plateaus were recorded for each counter and frequently checked. The tests conducted over the period of the experiment indicated that the plateaus remained stable with few exceptions. These tests were performed twice weekly and revealed either that all counters had plateaus longer than 40 volts about the operating point or that one counter had a greatly increased counting rate and had no plateau at all. The regular testing and the behaviour of the counters provided assurance that counter failures occurred only in the short intervals between tests. On the infrequent occasion of such a failure, only the data accumulated since the preceding test were discarded.

Another characteristic of Geiger counters which required attention is the distribution in size of the pulses that occur on discharge. This made it necessary to design the electronic circuits so that very few of the pulses escaped detection and that variations in the number of such pulses could be easily detected. The requirements were met by amplifying almost all of the counter pulses to heights well above the threshold for triggering the pulse-shaping univibrator. The percentage of counter pulses which failed to trigger the univibrator was checked twice weekly and was found to remain below 0.01 percent.

The main precaution required by the electronic apparatus was that the resolving time of the coincidence circuit must not vary significantly. (As will be seen in the discussion which follows the contribution to the counting rate due to accidental coincidences depends partly on the resolving time.) The purpose of the pulse-shaper was to provide the coincidence circuit with pulses of approximately rectangular shape and uniform size in an effort to minimize possible variations in the resolving time. Monthly checks showed variations of < 2 percent.

Finally, the nature of the recorded events must be considered in order to determine the reliability of cosmic-ray variations deduced therefrom. Among the coincidences recorded were true coincidences from cosmic-rays and from γ-rays (emitted from nearby radioactive contaminants) which produced Compton electrons in the walls of counters in both trays. The remaining coincidences were accidental and due to incoherent particles passing through each tray during the resolving time of the coincidence circuit.

If the average rates of γ -ray and accidental coincidences are known, the cosmic-ray rate can be obtained by subtraction. To reduce the statistical error involved in such a procedure, precautions were taken to reduce the accidental and γ-ray coincidences to a minimum. A layer of lead, 1 in. thick, was placed between the two counter trays to virtually eliminate the effects of γ -ray coincidences. Less than one percent of the γ-rays ($E \leq 2.5$ mev) can traverse this absorber, and those that do traverse it produce coincidences with an efficiency of only 0.007 [6]. Since 1/3 of the counts registered in a single tray are produced by γ-rays[6], even if all of these represented quanta moving in the direction of the other tray, the maximum contribution to the coincidence rate would be less than 1 percent. (This can be seen as follows: the total single tray rates were ~ 800 min^{-1}; of these ~ 270 min^{-1} are γ -ray induced, therefore less than 2.7 min^{-1} traverse the absorber and less than $0.007 \times 2.7 \cong 0.019$ min^{-1} produce coincidences. In contrast the total coincidence rate is ~ 2 min^{-1}.) Thus statistical variations of true γ-coincidences have a negligible effect on the accuracy of the cosmic-ray coincidence rate.

Systematic time variations due to variation in γ-ray intensity also have a negligible effect since the γ-ray intensity (essentially determined by the singles rates) remained constant.

The accidental coincidences can be determined from the resolving time of the coincidence circuit and the counting rates of the separate counter trays by the equation

$$A = 2\tau N_1 N_2 ,$$

where A is the accidental rate, τ is the resolving time, and N_1 and N_2 are the counting rates of the separate trays. The extreme values observed in monthly measurements of the resolving time were $(4.16 \pm 0.002) \times 10^{-6}$ sec. and $(4.09 \pm 0.002) \times 10^{-6}$ sec. From the single counting rates (320 min^{-1} and 512 min^{-1} with negligible statistical errors) and the resolving times the accidental rate is seen to be 1.3 hr^{-1}. Variations in this accidental rate were primarily due to the small changes in the resolving time (less than 2 percent). The variations in accidental rate are approximately 0.02 percent of the cosmic-ray rate ($\sim$ 110 hr^{-1}) and are small compared with the cosmic-ray variations expected from the temperature effect ($\sim$1 percent). The cosmic-ray counting rate was corrected for the contribution due to accidental coincidences. As in the case of γ-ray coincidences, both statistical and and also systematic variations in the accidental rate have a negligible effect on the accuracy of derived cosmic-ray coincidences.

The remaining events recorded by the apparatus which constitute ~ 98 percent of the total counting rate, where presumably μ-mesons and their energetic secondary products. Correlation between variations in this component and variations in stratospheric temperatures are taken to reflect the validity of the π-μ decay model.

The detailed precautions described above are believed adequate to insure the elimination of significant systematic variations which are not due to a temperature effect. The absence of random experimental errors is reflected in the comparison between the variance [16] of the observed counting rates and the variance of the distribution of cosmic-ray counting rates given by the mean rate. (Random experimental errors would be expected to cause a broader distribution in counting rates than that due to cosmic-rays alone.) The variance of the observed hourly rates was 110.1 ± 2.7 (i.e. the probability that the variance has some value in this interval is 0.67) and the theoretical variance of counting rates as observed from the mean rate was 107.3. These values are consistent with one another indicating that experimental errors caused no significant broadening of the cosmic-ray distribution. Since the presence of systematic errors which may have occurred over periods of weeks or months but which compensated one another would not be detected by the foregoing comparison, another test for consistency can be made. The variances of the data collected each month can be calculated and the variance of these quantities can be compared with that expected from monthly sampling from the theoretical distribution of cosmic-rays with a mean rate equal to that observed. This comparison can reveal a spread of monthly variances reflecting errors that would not be seen from the average of these variances. This procedure was followed and the variances were also found to be consistent with each other.

CHAPTER III

REDUCTION OF DATA AND EXPERIMENTAL RESULTS

3.1 Selection of Data

The experimental data consist of the underground coincidence rates and atmospheric temperatures collected over the period April 1, 1951 - November 30, 1951. The atmospheric temperatures that were considered were those recorded at the 100 mb., 150 mb, 200 mb, and 300 mb. pressure levels by radiosonde apparatus sent aloft on free balloons.* The balloons were released and the temperatures collected by the weather station personnel located at Selfridge Field, Michigan, and Toledo, Ohio. The Selfridge Field observations were made four times daily (at 4 AM, 10 AM, 4 PM, and 10 PM) and the Toledo observations twice daily (at 10 AM and 10 PM). Both of these stations provided temperature data which are considered to reflect the atmospheric conditions above the experimental equipment, located at Detroit, Michigan, which is approximately equidistant from these stations ($\sim$ 50 miles). This was confirmed by

* The temperatures at sea-level were also noted but these data were not subjected to the same selection process as described below for the stratospheric temperatures. The reason for this distinction is that diurnal temperature variations at sea-level would render the selection process useless. The relatively stable temperatures at this level, however, make a straightforward correlation analysis feasible without necessity for careful selection.

analysis of the correlation between temperatures, at the 150 mb pressure level, reported by the two stations. A correlation coefficient (see Appendices III and IV) of $0.92 \, {}^{+0.015}_{-0.032}$ was obtained which is believed to be a conservative estimate of the correlation.

In order that cosmic-ray data taken only during periods when atmospheric temperatures are relatively steady be used in evaluating the temperature effect, the temperature data were employed in the following manner. 1)Each radiosonde observation at Selfridge Field was considered to reflect atmospheric conditions over the period from three hours before to three hours after the observation. 2)From these Selfridge Field data, only those temperatures which did not vary by more than 3°C for at least two consecutive readings were used and the corresponding temperatures for the twelve hour (or longer) periods which were recorded at Toledo were noted. 3)The hourly cosmic-ray coincidence rates were averaged over six-hour periods centered about the hours when balloons were released at Selfridge Field and the averages corresponding to the selected Toledo temperatures were noted. 4)The Toledo temperatures (for each pressure level separately) and the averaged coincidence rates were analyzed for possible correlation.

3.2 Statistical Treatment of Data

The theoretical model of meson production treated in Chapter IV indicates that the dependence of cosmic-ray intensity on stratospheric temperature should be linear (at least in first order approximation). Since the stratosphere is not exactly uniform in temperature and the atmospheric depth at which π-mesons are most abundant is not precisely known, a temperature coefficient was sought for separately at each of the four pressure levels listed above. The linear dependence was analyzed in accordance with the usual statistical methods of least-squares and linear regression, as found in reference [16] and described in Appendices I-III of this dissertation. In brief, the problem is divided into three parts.

1) For a known linear dependence of intensity on temperature, the coefficient of proportionality can be represented as the slope of the line that would result

if one variable were plotted against the other. The first part of the analysis is, then, to obtain this slope for the line which "best fits" the pairs of experimental data. The method of least-squares is used for this purpose, and is described in Appendix I.

2) Having obtained this "best fit", one must estimate "how well" it fits the data. Such an estimate is indicated by the correlation coefficient, which is defined and discussed in Appendix II.

3) Finally, one must estimate the statistical error associated with the value of the correlation coefficient. This can be accomplished by considering the statistical significance of the correlation coefficient and this procedure is described in Appendix III.

3.3 Values of Temperature and Correlation Coefficients

The foregoing procedure was followed and the results obtained are listed below in Table I.

PRESSURE LEVEL (mb.)	TEMPERATURE COEFFICIENT (%deg^{-1})	CORRELATION COEFFICIENT	SIGNIFICANCE LEVEL
100	0.062 ± 0.042	0.0645	0.14
150	0.056 ± 0.039	0.0618	0.14
200	-0.012 ± 0.034	-0.0150	0.72
300	0.014 ± 0.029	0.0193	0.63
1000	-0.011 ± 0.017	-0.0250	0.54

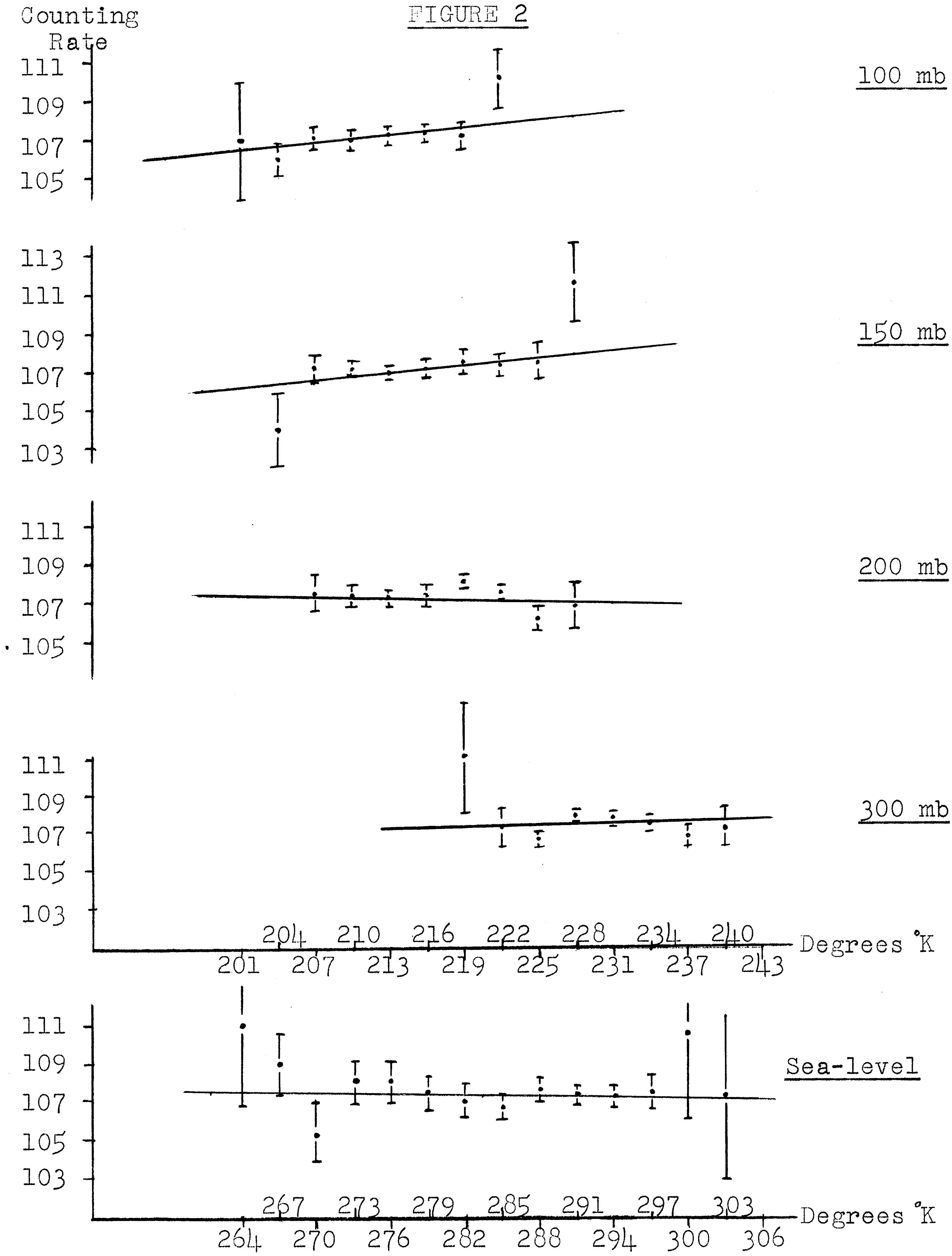
The correlation coefficient represents the square root of the fraction of the variance in cosmic-ray intensity which can be accounted for by the recorded temperature variations (App.II). The significance level represents the probability that the result considered could be obtained from completely uncorrelated pairs of data (App.III).

For a more graphic, but statistically less accurate representation of the data the range of observed temperatures was divided into three degree intervals and the cosmic-ray intensities, corresponding to temperatures in these intervals, were averaged in each interval and plotted against the mean temperatures. This was done for each pressure level, and these graphs appear in Fig. 2. On each graph there also appears the straight line which best fits the data, as determined from the more accurate treatment described above.

3.4 The Diurnal Effect

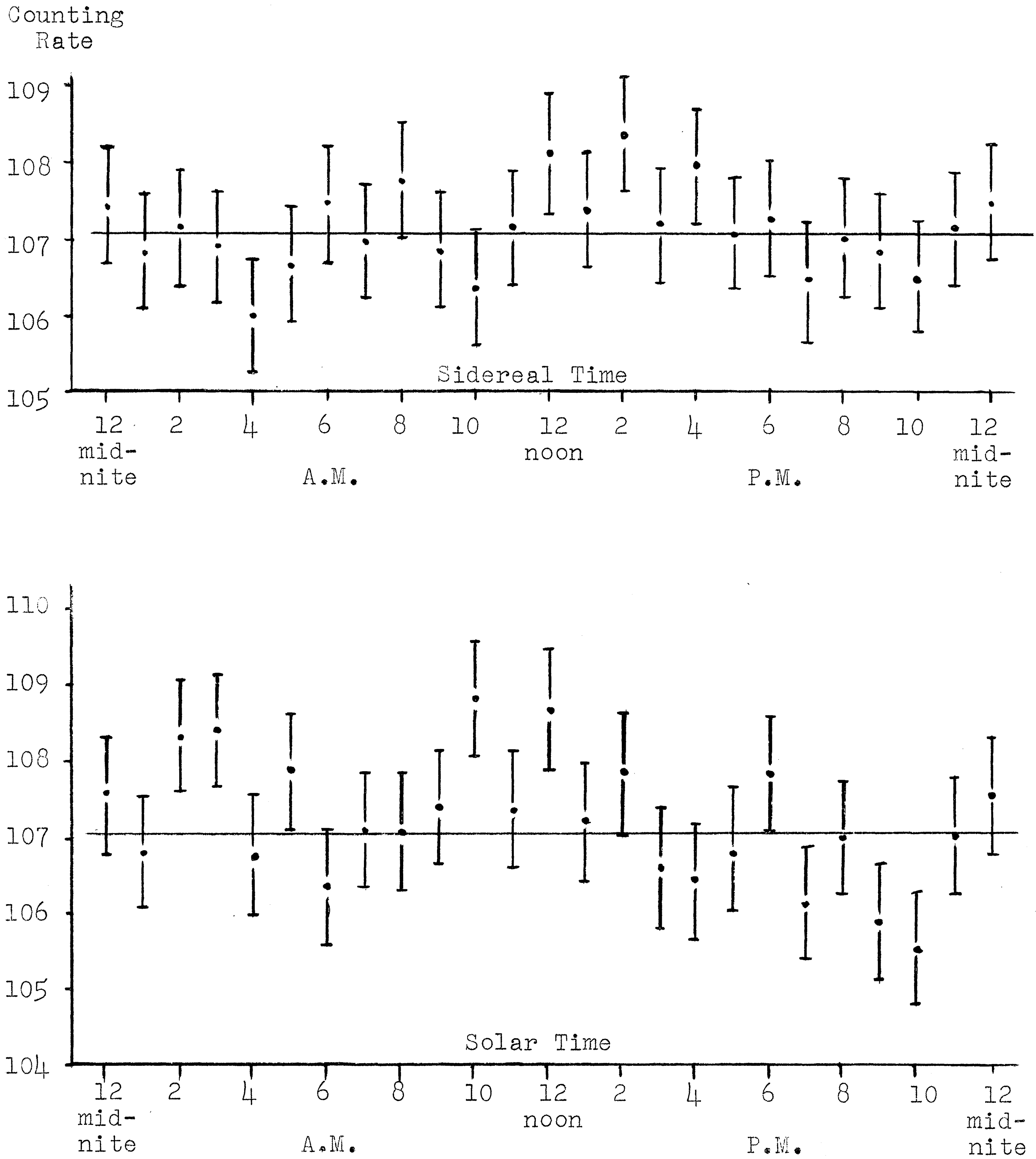
As mentioned in Chap. I, 1.3, the apparatus is well suited for determining possible diurnal variations. The hourly rates of cosmic-rays were averaged for each hour, and plotted against solar and sidereal time. These graphs appear in Fig. 3, and indicate that any diurnal effect for high energy μ-mesons must be less than 2 percent.

FIGURE 2



CORRELATION GRAPHS

FIGURE 3



HOURLY COUNTING RATES

CHAPTER IV

INTERPRETATION OF RESULTS AND CONCLUSION

4.1 Physical Assumptions in the Meson Production Model

The detailed mathematical treatment of the meson production model, described in Chap. I, is based on several physical assumptions which may be important in the interpretation of experimental results. In order that the roles played by these assumptions in the mathematical development be clear, the explicit assumptions will be listed and discussed. The result of the detailed development will be an expression relating the variations in the intensity of μ-mesons which penetrate to a given depth underground with the variations in the temperature of the atmosphere traversed by their π-meson progenitors.

First, the differential energy spectrum of π-mesons as a function of atmospheric depth will be expressed in a differential equation which describes the rates of production and loss of π-mesons. In solving the differential equation three assumptions will be made:

i) The atmosphere is an ideal gas whose temperature varies with depth in a manner specified later.

ii) The component of cosmic-rays which is capable of producing mesons on interacting with atmospheric matter is absorbed exponentially in the atmosphere with a mean absorption length which is independent of energy.

iii) The attenuation of π-mesons which is due to their interaction with atmospheric matter is characterized by the same energy-independent mean absorption length as the meson-producing component.

Next, the spectrum of μ-mesons as a function of atmospheric depth will be obtained by integrating the intensity of the decay products of π-mesons over the appropriate π-meson energy range. This procedure is based on three more assumptions.

iv) μ-mesons originate only as the decay products of π-mesons.

v) A π-meson of energy greater than the minimum required by its decay product to reach the depth of the experiment ($\sim 10^{11}$ ev) decays into one μ-meson and one neutral particle of negligible rest mass.

vi) Absorption of μ-mesons in the atmosphere may be neglected for mesons with sufficient energy to reach the depth of the experiment.

The differential energy spectrum of μ-mesons at sea-level will then be obtained directly from the expression describing its variation with depth. The intensity of μ-mesons underground will then be obtained by integrating the sea-level spectrum over all energies with which μ-mesons are capable of reaching the depth of the experiment. This implies a final assumption:

vii) The production of μ-mesons underground may be neglected compared with those produced in the atmosphere.

When the underground intensity is obtained, the temperature effect will be discussed. The mathematical treatment, with minor revisions, is essentially that of Hayakawa et al [17].

The first assumption is easily validated. First order corrections to the ideal gas law, as given by the Van der Waals equation, indicate that the error in the ideal gas expressions for air density and its temperature derivative are less than 0.2 percent at standard conditions [18] (ice temperature at atmospheric pressure). Since this assumption will be applied to air which is even more rarified than it is at standard conditions, use of the ideal gas law will not produce significant error.

Assumption ii) is supported by considerable experimental observation of the meson-producing component [19] but these do not directly validate the assumption. The experiments were limited to energies below 10^{11} ev whereas this investigation is concerned only with those meson-producing particles with energies above 10^{11} ev. An estimate can be made, however, concerning the order of magnitude of the error involved in assuming exponential absorption for particles with energy $\geqslant 10^{11}$ ev. Consider the differential energy spectrum for energies above 10^{11} ev at the top of the atmosphere. The number of primaries which will be removed from a given differential energy interval due to interactions with atmospheric matter (neglecting ionization loss) in a depth dl (in gm cm^{-2}) will be written as

$$f(E_o)dE_o \frac{dl}{\lambda(E_o)} \quad , \qquad (1)$$

where $f(E_o)dE_o$ is the number of primary cosmic-rays with energies in the interval dE_o and $\lambda(E_o)$ is the interaction mean free path. The number of particles entering this energy interval is determined by the energies of the secondary products produced by primaries of energy $> E_o$ in their interactions with the same atmospheric matter. If this number entering dE_o is much less than the number leaving dE_o, then the net loss corresponds approximately to absorption alone and the mean free paths for interaction and for absorption are essentially equal. For an estimate of the number of meson-producing secondaries entering dE_o, we take an oversimplified model in which all of the primary energy is used to produce secondaries and in which this energy is evenly divided among the secondaries. On this model it is seen that the number of secondaries entering dE_o is

$$f(nE_o)\, d(nE_o) \quad n \; \frac{dl}{\lambda(nE_o)} \quad , \qquad (2)$$

where n is the number of secondaries produced by each primary, $f(nE_o)d(nE_o)$ is the number of primaries which can produce secondaries in the interval dE_o, and $\lambda(nE_o)$ is the interaction mean free path for primaries of energy nE_o. For $E_o = 10^{11}$ ev, the Fermi theory [20] of high energy nuclear interactions predicts an average multiplicity of 6 and the experimental results of Camerini et al [21] (see Fig. 11 (b) of their extensive report) are consistent with this theory. Then equation (2) can be written as

$$f(6E_o)\, dE_o \; \frac{dl}{\lambda(6E_o)} \times 6^2 \quad .$$

For a primary spectrum described by a power-law

$$f(E_o) = \text{const.}\; E_o^{-\gamma} \; ,$$

the ratio between (2) and (1) becomes

$$\frac{\lambda(E_o)}{\lambda\,(6E_o)} \times 6^{-(\gamma-2)} \quad . \qquad (3)$$

Considerable evidence over a wide range of energies[24] (10^8ev - 10^{10}ev) indicates that the interaction length $\lambda(E_o)$ corresponds approximately to the geometrical cross section at high energies. The exponent in the power-law can be estimated in the region of 10^{11} ev as follows: a The integral spectrum of primaries with energies up to 15×10^9 ev can be obtained from rocket and balloon measurements performed at various latitudes (the integral intensity at 15×10^9 ev is $1.0 \times 10^6 m^{-2} hr^{-1} sterad^{-1}$).[22] b At energies from 10^{14} to 10^{16} ev the integral intensity of extensive air showers as a function of shower energy is given by Williams [23] as $F(E_s) = 2 \times 10^{-3}(10^{15}/E_s)^{1.9}$ m^{-2} $hr^{-1} sterad^{-1}$, where E_s(in ev) is the energy in the shower. For $E_s = 10^{14}$ ev the primary intensity can be found by estimating the fraction, $1/m$, of primary energy which goes into extensive showers (i.e. $F(10^{14}) = 0.16$ m^{-2} hr^{-1} $sterad^{-1}$ gives the intensity of primaries with $E_o \geqslant m \times 10^{14}$ ev). c The primary intensities at energies of 15×10^9 ev and $m \times 10^{14}$ ev can then be fitted by a power-law to obtain the exponent ($\gamma = 1 + 15.6/[8.83 + \ln m]$). For $m = 1$, $\gamma = 2.8$ and for $m = 10$, $\gamma = 2.4$. Taking the latter value (which is least favorable to the exponential absorption approximation) and equating mean free paths, we can finally write (3) as

$$\frac{\text{number of particles entering } dE_o}{\text{number of particles leaving } dE_o} \sim \frac{1}{6^{0.4}} \sim 0.5 \quad .$$

Therefore, if the number of particles entering dE_o were neglected (in the approximation of exponential absorption), the intensity after passing through the absorber would be about two times too small. To the extent that these data and the simplified scheme represent particle interactions at energies $> 10^{11}$ ev, this is an estimate of the error involved in the intensity of the meson-producing component. It is believed that the approximation may be better than indicated by the estimate since γ was taken to be the smallest reasonable value and the energy interval dE_o was considered to be replenished by all the secondaries produced by a primary of energy nE_o. If the collision processes at energies $> 10^{11}$ ev were better known, a more realistic analysis would probably reveal that the primaries with energies slightly greater than E_o produce more secondaries with energies $< E_o$ than with energies $\geqslant E_o$. This would result in a smaller replenishment of dE_o by contributions from these primaries. For primaries with energies much greater than E_o, it is believed [23] that $\gamma > 2.4$

so that contributions from these primaries would be reduced by virtue of their reduced number. These arguments apply more strongly for larger values of E_o and we may, therefore, expect that the estimate of the approximation is pessimistic. In any case assumption ii) of exponential absorption is believed to determine the limit of accuracy of the meson intensities developed in the model, by producing intensities ~ 2 times too small. (The temperature coefficient ultimately obtained will involve a ratio of two terms both of which contain the results of this approximation and the consequent error at that point is much smaller.) The results of further approximations will be seen to stay within the limit imposed by assumption ii). In accord with this assumption the production rate of π-mesons at a depth l will be assumed proportional to the integral spectrum of meson-producing particles at that depth and will be written in the form

$$f\ (E_o > E_\pi)\ e^{\frac{-l}{\lambda}} \quad .$$

The third assumption is also supported by experiments (involving π-mesons) with energies less than 10^{11} ev.[19] Since there is no reason to suspect that the π-meson absorption length should depend strongly on energy, the low energy parameter is assumed to describe the attenuation due to nuclear interaction of π-mesons at high energies.

The validity of assumption iv) is the main concern of this investigation. It is regarded as well established that particles identified as μ-mesons are the decay products of π-mesons, where these particles possess energies up to $\sim 2\text{-}3 \times 10^8$ ev [24,25]. The recent discovery [26] of what appears to be a μ-meson among the decay products of a heavy charged particle (of mass $\sim$ 1200 times that of the electron) suggests the possibility that μ-mesons of energy $> 10^{11}$ ev originate in a decay process (which would be extremely difficult to observe) involving progenitors other than π-mesons. It is expected that investigation of the temperature effect may shed some light on this interesting possibility.

The assumption of two-particle decay, which is made in v), is regarded as well verified for π-mesons observed at energies up to $\sim 2\text{-}3 \times 10^8$ ev [24] and there appears to be no reason for suspecting this process to be different at higher energies.

The assumption in vi), that atmospheric absorption of μ-mesons is negligible, is verified directly by comparing the energy loss in the atmosphere ($\sim 10^9$ev) with that experienced in reaching the depth of observation ($\sim 2 \times 10^{11}$ ev). Loss through μ-decay is negligible at energies $> 10^{11}$ ev due to the Lorentz contraction on the mean lifetime.

Assumption vii), that a negligible fraction of the number of observed μ-mesons is produced underground, is consistent with assumptions iii) and iv) and the known properties of μ-mesons. Since μ-mesons are assumed to originate from π-mesons and those π-mesons which survive absorption in the atmosphere are almost entirely absorbed on penetrating the earth, any μ-mesons produced underground must be decay products of π-mesons also produced underground. George and Evans[27] investigated nuclear interactions underground and found that underground nuclear events are initiated by μ-mesons with the extremely small cross-section $\sim 10^{-29}$cm^2 at 10^{10} ev. This cross-section indicates that less than two percent of the μ-mesons incident at the surface will experience nuclear interactions in reaching the depth of the experiment ($\sim$ 800 mwe). These interaction rates and the low probability for decay of π-mesons underground justify neglecting the consequent number of μ-mesons produced.

With these assumptions, in the mathematical development which follows, the μ-meson intensity at the experimental depth will be obtained. This expression will contain the atmospheric temperature and the temperature effect will be calculated therefrom.

4.2 Detailed Treatment of Model

To find the intensity of π-mesons with energy E_π in dE_π moving at an angle θ with the vertical at the atmospheric depth l in gm cm^{-2}, we write the diffusion equation describing the production and absorption processes (neglecting ionization) as

$$d(dn_\pi) = dE_\pi \frac{dl}{\lambda \cos\theta} \int_{E_\pi}^{\infty} n(E_0 > E_\pi, l) P(E_0 \rightarrow E_\pi) dE_0 - dn_\pi \left(\frac{dt}{\tau} + \frac{dl}{\lambda \cos\theta} \right) \qquad (4)$$

where $d(dn_\pi)$ is the increment in the differential energy spectrum dn_π (E_π, θ, l) due to production and loss in dl, λ is the mean absorption path for π-mesons and the particles which produce them (see assumptions ii) and iii) of 4.1), dt/τ is the probability of decay of a π-meson in the time required to traverse $dl/\cos\theta$, $n(E_0 > E_\pi, l)dE_0$ is the differential spectrum of meson-producing particles, and $P(E_0 \rightarrow E_\pi)dE_\pi$ is the probability that such particles will produce π-mesons in dE_π. In accord with assumption ii) of exponential absorbtion $n(E_0 > E_\pi, l)$ can be written as

$n(E_o > E_\pi)\exp\{-l/\lambda\cos\theta\}$ and the integral (which represents the production rate of π-mesons) is written $f(E_o > E_\pi)\exp\{-l/\lambda\cos\theta\}$

Now dt/τ can be expressed as

$$dt/\tau = dx/\beta c\tau \quad , \qquad (5)$$

where dx is the distance in cm. traversed by the π-meson in the time dt, β is the ratio of the π-meson velocity, v, to c, the velocity of light, and τ is the mean lifetime of a π-meson of energy E_π in the laboratory system. Next, τ can be written as

$$\tau = \tau_\pi / \sqrt{1-\beta^2} = \frac{E_\pi}{m_\pi c^2}\tau_\pi \quad ,$$

where τ_π is the mean lifetime of the π-meson in its rest system ($\sim 2\times10^{-8}$ sec)[28] and m_π is the rest mass of the π-meson (286 electron masses)[3]. We also express

$$dl = dp/g = \rho\, dx \cos\theta \quad ,$$

where dp is the increase in atmospheric pressure with depth dl, g is the acceleration of gravity and ρ is the atmospheric density at the depth l. In accord with assumption i) of 4.1, ρ is written as

$$\rho = \frac{pM}{RT} = \frac{lMg}{RT} \quad ,$$

where M is the molecular weight of air ($\sim$29 gm/mole), R is the gas constant (8.3×10^7 erg deg^{-1}mole^{-1}), and T is the atmospheric temperature at the depth l. Making all these substitutions in (5), we have

$$\frac{dt}{\tau} = \frac{1}{l}\cdot\frac{dl}{\cos\theta}\cdot\frac{Rm_\pi c^2}{Mg\beta c\tau_\pi}\cdot\frac{T}{E_\pi} = \frac{\alpha T}{E_\pi\cos\theta}\cdot\frac{dl}{l} \quad , \tag{6}$$

where $\alpha = Rm_\pi c^2/Mg\beta c\tau_\pi$. Substituting (6) in (4) we obtain

$$\frac{d(dn_\pi)}{dl} = \frac{dE_\pi}{\lambda\cos\theta} f(E_0>E_\pi)\, e^{-l/\lambda\cos\theta} - dn_\pi\left(\frac{\alpha T}{E_\pi\cos\theta}\cdot\frac{1}{l} + \frac{1}{\lambda\cos\theta}\right) \tag{7}$$

As can easily be verified by differentiation, this equation has the solution

$$dn_\pi = \frac{dE_\pi}{\lambda\cos\theta} f(E_0>E_\pi)\, e^{-l/\lambda\cos\theta}\cdot\exp\left\{\frac{-\alpha}{E_\pi\cos\theta}\int_\varepsilon^l \frac{T}{u}\,du\right\}\cdot\int_\varepsilon^l \exp\left\{\frac{\alpha}{E_\pi\cos\theta}\int_\varepsilon^v \frac{T}{w}\,dw\right\} dv \,,$$

where ε is a constant of integration. (ε represents the top of the atmosphere as indicated by dn_π vanishing at $l = \varepsilon$. After the integral expressions are evaluated, ε will be set equal to zero.)

Next we obtain the differential energy spectrum of μ-mesons. In accord with assumption iv) of 4.1, we first integrate the π-meson spectrum over those energies with which a π-meson can decay into a μ-meson with energy E in dE, in the time dt, required to traverse dl. We write this as

$$d(dn_\mu) = \int_{E_{min}}^{E_{max}} dn_\pi\,\frac{dt}{\tau}\,P(E_\pi\to E)\,dE_\pi$$

where dn_π and $\frac{dt}{\tau}$ have the same meanings as previously, $P(E_\pi \rightarrow E)$ is the probability that a π-meson with energy E_π in dE_π will decay into a μ-meson with energy E in dE, and the limits of integration will be discussed in what follows. Since we shall consider only those μ-mesons which result from the decay in flight of π-mesons with $E_\pi > 10^{11}$ev, the decay products will move essentially along the path of the parent π-meson thus eliminating integration over angles of decay. Then in accord with assumption v) of 4.1 and App. IV we write $P(E_\pi \rightarrow E)$ as

$$\frac{dE}{\beta E_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)} ,$$

and the integral is performed between the limits

$$(E_{min} = E) \leq E_\pi \leq \left[E_{max} = \frac{2E}{(1-\beta) + \left(\frac{m_\mu}{m_\pi}\right)^2 (1+\beta)}\right].$$

Making the proper substitutions, we have

$$\frac{d(dn_\mu)}{d\ell} = \frac{\alpha T dE e^{-\ell/\lambda\cos\theta}}{\lambda\cos^2\theta\ \ell} \int_{E_{min}}^{E_{max}} \left[\frac{F(E_0 > E_\pi)\exp\left\{\frac{-\alpha}{E_\pi\cos\theta}\int_\varepsilon^\ell \frac{T}{u}du\right\}}{\beta\left[1 - m_\mu^2/m_\pi^2\right] E_\pi^2} \int_\varepsilon^\ell \exp\left\{\frac{\alpha}{E_\pi\cos\theta}\int_\varepsilon^v \frac{T}{w}dw\right\}dv\right] dE_\pi$$

Integrating this expression from ε to l, we obtain the differential energy spectrum of μ-mesons as a function of depth (see assumption vi) of 4.1). Taking the upper limit of l to be 1000 gm cm^{-2}, we obtain the sea-level spectrum for μ-mesons incident at a zenith angle θ,

$$dn_\mu = \frac{\alpha\, dE}{\lambda\cos^2\theta}\int_\varepsilon^{1000}\left(e^{-l/\lambda\cos\theta}\,\frac{T}{l}\cdot\int_{E_{min}}^{E_{max}}\left[\frac{f(E_0>E_\pi)\exp\left\{\frac{-\alpha}{E_\pi\cos\theta}\int_\varepsilon^{l}\frac{T}{u}du\right\}}{\beta\left[1-m_\mu^2/m_\pi^2\right]E_\pi^2}\int_\varepsilon^{l}\exp\left\{\frac{\alpha}{E_\pi\cos\theta}\int_\varepsilon^{v}\frac{T}{w}dw\right\}dv\right]dE_\pi\right)dl$$

The μ-meson intensity underground is obtained by integrating the sea-level spectrum over all energies with which μ-mesons are capable of penetrating to the depth of the experiment (in accord with assumption vii) of 4.1). The minimum energy depends on θ, since the mesons reaching a given depth at inclined angles suffer greater energy loss (in traversing more matter) than those vertically incident. The expression for the intensity underground is then

$$n_\mu = \frac{\alpha}{\lambda\cos^2\theta}\int_{E(\theta)}^{\infty}dE\int_\varepsilon^{1000}\left(e^{-l/\lambda\cos\theta}\,\frac{T}{l}\cdot\right.$$

$$\left.\times\int_{E_{min}}^{E_{max}}\left[\frac{f(E_0>E_\pi)\exp\left\{\frac{-\alpha}{E_\pi\cos\theta}\int_\varepsilon^{l}\frac{T}{u}du\right\}}{\beta\left[1-m_\mu^2/m_\pi^2\right]E_\pi^2}\int_\varepsilon^{l}\exp\left\{\frac{\alpha}{E_\pi\cos\theta}\int_\varepsilon^{v}\frac{T}{w}dw\right\}dv\right]dE_\pi\right)dl \qquad (8)$$

We are now in a position to discuss the effect of variations in atmospheric temperature on the intensity of μ-mesons observed underground. Weather observations indicate that the atmospheric temperature variation with depth can be approximated by an isothermal atmosphere (the stratosphere) down to about 240 m b and then by a linear variation with depth (in metres) down to sea-level. (At 240 m b the temperature varies from 210 — 230°K and at sea-level from 270 - 310°K, depending on the season.) The analysis is then simplified by considering T to be constant, T_I, from $l = \mathcal{E}$ to l = 240 m b and then by taking some suitable average, $\bar{T}$, to represent the temperature below 240 m b. In equation (8) it is seen that the temperature appears only in the form $\frac{T}{l}$ so that contributions from integration over l, where l is large, play a much smaller role in the dependence of n_μ on T than do the values of T at small l. It is expected, then, that the error arising from use of a suitable average for the temperature in the lower three fourths of the atmosphere will be well within the limits of approximation already inherent in the model.

Using T_I and $\bar{T}$ as defined above, we can perform the integrations over l in (8) and write this as

$$n_\mu = n_I + n_V \quad ,$$

where n_I is the contribution to n_μ from integration over the isothermal portion of the atmosphere and n_V is the contribution from the non-isothermal portion. The expression for n_I is

$$n_I = \frac{\alpha T_I}{\cos\theta} \cdot \frac{1 - e^{-240/\lambda\cos\theta}}{\beta[1 - m_\mu^2/m_\pi^2]} \int_{E(\theta)}^{\infty} dE \int_{E_{min}}^{E_{max}} \frac{E_\pi^{-2} f(E_0 > E_\pi)\, dE_\pi}{(1 + \alpha T_I/E_\pi \cos\theta)} \tag{9}$$

and that for n_V is

$$n_V = \frac{\alpha \bar{T}}{\cos\theta} \cdot \frac{e^{-240/\lambda\cos\theta} - e^{-1000/\lambda\cos\theta}}{\beta[1 - m_\mu^2/m_\pi^2]} \int_{E(\theta)}^{\infty} dE \int_{E_{min}}^{E_{max}} \frac{E_\pi^{-2} f(E_0 > E_\pi)\, dE_\pi}{(1 + \alpha \bar{T}/E_\pi \cos\theta)} \tag{10}$$

where ε has been set equal to zero after integrating.

Next we consider the case of vertically incident μ-mesons and note that the minimum energy required by these particles in order to penetrate to the depth of the experiment is of the order 2×10^{11} ev,[6] so that $\beta \sim 1$ and

$$\frac{\alpha T_I}{E_\pi} = \frac{R m_\pi c^2 T_I}{M g c \tau_\pi E_\pi} < \frac{8.3\times10^7 \times 150\times10^6 \times 200}{29\times980\times3\times10^{10}\times2\times10^{-8}\times2\times10^{11}} \sim 0.7 \tag{11}$$

If we now set $\bar{T} = T_I$ in the integral appearing in (10), which introduces a maximum error $\sim$ 11 percent, we can easily compare (10) with (9).

$$\frac{n_V}{n_I} \cong \frac{\bar{T}}{T_I} \cdot \frac{e^{-240/\lambda} - e^{-1000/\lambda}}{1 - e^{-240/\lambda}} \sim \frac{250}{210} \cdot \frac{e^{-2} - e^{-8}}{1 - e^{-2}} \sim 0.2 \; .$$

With this approximation n_μ can be written as

$$n_\mu = n_I + n_V \sim 1.2\, n_I \quad .$$

The quantity we seek to compare with experimental results is the temperature coefficient

$$\sigma = \frac{1}{n_\mu}\cdot\frac{\partial n_\mu}{\partial T_I} \cong \frac{1}{1.2 n_I}\cdot\frac{\partial n_I}{\partial T_I} \quad , \tag{12}$$

where

$$\frac{\partial n_I}{\partial T_I} = \frac{\alpha(1-e^{-240/\lambda})}{[1-m_\mu^2/m_\pi^2]}\int_{2\times10^{11}}^{\infty} dE \int_{E}^{\left(\frac{m_\pi}{m_\mu}\right)^2 E} dE_\pi \frac{E_\pi^{-2} f(E_o > E_\pi)}{(1+\alpha T_I/E_\pi)}\left[1-\frac{\alpha T_I/E_\pi}{1+\alpha T_I/E_\pi}\right] \tag{13}$$

This expression can be evaluated numerically by using an integral spectrum for $f(E_o > E_\pi)$ in the usual form of a power-law (since $E_\pi > 10^{11}$ ev) and expanding the integrand of $\partial n_I/\partial T_I$ in a power series in $\alpha T_I/E_\pi$. This procedure can be circumvented by an approximation which is well within the accuracy of $\sim$ 100 percent prescribed by assumption ii) of 4.1. We note that the maximum value of $\alpha T_I/E_\pi$, as given by (11), is ~ 0.7 so that neglecting $(\alpha T_I/E_\pi)(\alpha T_I/E_\pi + 1)^{-1} = 0.4$ in comparison with one, in the integrand of (13), introduces a relative error of about 67 percent. Neglecting this term, then, we obtain very simply

$$\sigma \cong \frac{1}{1.2 T_I} = 0.4 \text{ percent per degree } ^\circ\mathrm{K}$$

The foregoing considerations apply to vertically incident μ-mesons and we now show that they apply equally well to mesons incident at all angles. From the range-energy relationship given by Hayakawa [17], we can calculate the energy required by μ-mesons incident on the surface at an angle θ with the vertical to reach the depth, x, of the experiment.

$$E(\theta) = \frac{a}{p + r}\left[e^{(p + r)\frac{x}{\cos\theta}} - 1\right] ,$$

where E(θ) is the minimum energy,

$a = 2.5 \times 10^{6}$ ev per gm cm^{-2},

$p = 1.6 \times 10^{-6}$ per gm cm^{-2} ,

$r = 10^{-6}$ per gm cm^{-2} ,

and $x = 8.46 \times 10^{4}$ gm cm^{-2} for the depth of the experiment.[6] This relationship indicates that although $\alpha T_I/E_\pi$, in the analysis above, must be replaced by $\alpha T_I/E_\pi \cos\theta$ the lower limit of integration, E(θ), increases more rapidly than $\sim 1/\cos\theta$. This means that $\alpha T_I/E_\pi \cos\theta < 0.7$ for all values of the integrand in the corresponding expression for σ and that the same value for the temperature coefficient is obtained. We have then as the quantity to be compared with experimental results

$$\underline{\sigma \cong 0.4 \text{ percent per degree } {}^{\circ}K}$$

and this is believed to be an accurate consequence of the assumed π-μ decay model to within a factor of two. This uncertainty arises from neglecting $(\alpha T_I/E_\pi)(1 + \alpha T_I/E_\pi)^{-1}$ in the integrand of (13) which introduces a maximum error of approximately 67 percent for E_π at the lower limit of integration, 10^{11} ev. If the remainder of the integrand is a rapidly decreasing function of energy (e.g. $f(E_0 > E_\pi)$ may be a power-law), then the maximum error may be approached more or less closely. It will be recalled that another uncertainty entered due to the assumption of exponential absorption and that this was approximately 100 percent. It can be seen, however, that, although the latter consideration effects the derived meson intensities, it appears in the expression for the temperature coefficient only in the form of a ratio between two quantities both of which involve the same approximation ($\sigma = \partial n_I/n_I \partial T_I$), thus partially cancelling this uncertainty. In any case the approximation of exponential absorption and that due to neglecting part of the integrand in (13) introduce errors acting in opposite directions (exponential absorption makes the integrand too small and neglecting the term to be subtracted makes the integrand too large) and the latter has the larger effect on the temperature coefficient. It is believed, then, that the π-μ decay model at energies $> 10^{11}$ ev will be subject to considerable doubt if the measured temperature coefficient is found to be less than one half the theoretical value of 0.4 percent per degree.

4.3 Interpretation of Results

The temperature coefficients which were obtained in the present experiment (listed in Table I of Chap. III,3.2) are considerably smaller than those expected from the theory developed above. To make the most careful comparison we should calculate a weighted average of the measured coefficients, where the weights would be the relative intensities of high energy π-mesons which exist at the corresponding pressure levels, and compare this average coefficient with the value expected from the theory. This procedure would result in an experimental coefficient which could not exceed the maximum found at any one level and since the coefficients obtained at three of the four levels have approximately the same values, the results will be interpreted with sufficient accuracy by considering instead the maximum coefficient (~ 0.08 percent per degree) to be representative of an upper limit of the temperature effect.

Since the experimental procedure is believed to be sound and the quantitative model is believed to within 100 percent (in the direction of a smaller temperature coefficient), it is concluded that for μ-mesons of energy $> 10^{11}$ ev the π-μ decay model with its associated competition process is not supported by the temperature effect obtained experimentally. This conclusion is based on the following considerations of the experimental results. The upper limit on the temperature effect as represented by the largest experimental coefficient (~0.08 percent per degree) is less than the effect predicted by theory and lies outside the uncertainty inherent in the mathematical model. (The probability that the maximum observed temperature coefficient actually represents the smallest coefficient estimated from the model, namely 0.2 percent per degree, is less than .001. This corresponds to the probability with which an element of a Gaussian distribution will deviate from the mean by more than 2.6 times the standard deviation.) Consider also the statistical significance of the experimental results which indicates the probability ($\sim$10 percent) that no temperature effect exists. That is to say, the correlation coefficients obtained from the data may describe an apparent dependence of μ-meson intensity on stratospheric temperature (where this dependence does not actually exist) and the probability that these coefficients actually arise from a random sampling of unrelated quantities is given by the statistical significance level (see 3.2 and App. III). From the fluctuations in this probability in the course of analyzing successive groups of data it is believed that in order to gain real confidence in the physical existence of the temperature effect the correlation coefficients should be statistically significant at levels of 1 percent or even 0.1 percent. The lack of significance is supported by the calculated correlation between sea-level temperatures and underground cosmic-ray intensity. This calculation was made to confirm the absence of a significant temperature effect at sea-level for comparison with other experiments but the small coefficient ($\sim$0.01 percent per degree) and low significance level (.54) indicates the order of magnitude of results where no correlation is expected. Further doubt is cast on the existence of a temperature effect by the absence of even apparent correlation at the 200 mb. level (as indicated by the magnitude of the correlation coefficient for that level).

Comparison with other experiments reveals no violent contradictions to our conclusions. Although Forro[29] obtained a temperature coefficient of ~ 0.7 percent per degree, this was based on a small number of cosmic-ray events and these were correlated with sea-level temperatures where the number of π-mesons is much smaller than in the stratosphere. The poor statistics available and small correlation (expected from the model) between underground cosmic-rays and sea-level temperature allow us to regard this result as fortuitous. (The absence of sea-level correlation was experimentally confirmed in our investigation, with the result listed in Table I.) Duperier [5] investigated the correlation between cosmic-rays under 25 cm Pb at sea-level with stratospheric temperatures and obtained a temperature coefficient of 0.12 percent per degree with dependable statistics. In this case, however, the μ-mesons observed were those with an average energy less than 10^{10} ev and their origin may be compatible with a π-μ decay model. However, the very recent results of Cotton and Curtis [30], who performed essentially the same experiment as Duperier, indicate no significant temperature effect for the penetrating component at sea-level.

In view of the disagreement between experimental results and the π-μ decay scheme at energies greater than 10^{11} ev this model must be reexamined. The weakest assumption in the model is iv) of 4.1 in which μ-mesons are asserted to originate only from π-mesons. If we abandon this assumption on the origin of high-energy μ-mesons, then the properties of some alternative progenitor can be imagined which would result in a small or vanishing temperature coefficient. The pertinent properties are interaction length and mean lifetime in the laboratory reference frame; the latter, of course, depends also on the energy and rest-mass of the particle. If the mean lifetime is so short that the parent particles decay into μ-mesons before they have passed through enough matter to make the probability of interaction appreciable, then the change in this probability due to density variations will not cause appreciable variation in the number which decay. As an example the largest observed temperature coefficient was used to estimate the mean lifetime of the χ -meson. The same model, as was employed for the π-meson (which assumes a mean absorption path for progenitors of μ-mesons equal to that of the meson-producing component and two-particle decay) was applied to a particle of $\sim$ 1200 electron masses and it was found that such a particle would produce a maximum temperature coefficient of 0.08 percent per degree if the mean lifetime in its rest system were $\leqslant 4 \times 10^{-10}$ sec. Since there is no reason to suspect the validity of applying the model to the χ -meson (which has been observed to decay into a μ-meson) and if this process is actually the predominant mode of production of μ-mesons with energies greater than 10^{11} ev, then this value for the mean lifetime may be correct in order of magnitude. Another possibility may be that π-mesons are indeed the parents of μ-mesons but that their mean absorption paths are longer than those of their progenitors.

In that case the temperature effect should occur at depths below the stratosphere. The absence of correlation at sea-level and at 450 mb. and the believed π-meson absorption path serve to reject this possibility.

In view of the known properties of π-mesons of energy less than 10^{11} ev it is believed unlikely that their behaviour at higher energies is compatible with the assertion that they are the predominate source of μ-mesons observed over 800 mwe underground. It will, however, be fruitful to continue collecting data in an effort to establish more accurate and statistically significant estimates of the temperature effect. It will also be profitable to investigate this effect at depths smaller than 800 mwe in order to determine the energies at which the π-μ decay scheme may be dominated by some other production mechanism and to provide more information concerning cosmic-ray processes in the upper atmosphere.

4.4 Resumé of Results

i) The theoretical model, describing the production of mesons, is developed on the basic assumption that μ-mesons originate only in the decay products of π-mesons and this model is shown to indicate an expected temperature effect of approximately 0.4 percent per degree.

ii) The correlation analysis of the observed stratospheric temperatures and underground cosmic-ray intensity indicates a maximum temperature coefficient of 0.062 ± 0.042 percent per degree. The accuracy of the experimental procedure is established and the significance of the disagreement between experimental and theoretical results is discussed. It is concluded that the π-μ decay model does not describe the origin of μ-mesons of energy $> 10^{11}$ ev.

iii) The pertinent properties of particles involved in an alternate scheme of μ-meson production are discussed and χ-mesons are suggested as possible progenitors of high-energy μ-mesons. Using the observed mass of the χ-meson ($\sim$ 1200 electron masses) and a mathematical model similar to that developed for π-mesons, it is found that, if χ-mesons are the parents of μ-mesons of energy $> 10^{11}$ ev, then a maximum value of 4×10^{-10} sec for the mean lifetime of χ-mesons would be associated with a maximum temperature coefficient of 0.08 percent per degree.

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APPENDIX I

THE METHOD OF LEAST-SQUARES [16]

Consider the problem of determining the interdependence of two variables for which the interdependence is expected to be linear. If we plot pairs of these variables $(y_i\ x_i)$ as points in two dimensional space, then we may expect these points to be densest about the straight line, $y^* = m(x-\bar{x}) + a$, which best represents the linear dependence. The constants a and m are to be determined from the pairs of data and $\bar{x}$ is the average of the x_i values. The line which "best fits" the data is one which, in some manner, makes the "error" involved in representing the pairs of data by this line a minimum. Several ways of describing the "error" can be used, of which the most widely used is the sum of the squares of deviations of each point from the line. This is called the <u>method of least-squares</u>. The deviations from the line can be taken in any direction, but the most convenient is the deviation of the ordinate from the line when the abscissa is given. The results obtained in this way are easiest to interpret.

We wish, then to minimize

$$E = \sum_{i=1}^{N} (y_i - y_i^*)^2 = \sum_{i=1}^{N} (y_i - m(x_i - \bar{x}) - a)^2 ,$$

where E is the "error" as defined, (y_i, x_i) are the pairs of data, $(i = 1, 2, ----N)$, y_i^* is the ordinate of the best fit line corresponding to the abscissa x_i, $\bar{x}$ is the average of the x_i values, and m and a are the constants to be determined, which describe the best fit line. The conditions that E be a minimum are

$$\frac{\partial E}{\partial a} = 0 = -2\sum_{i=1}^{N}(y_i - m(x_i - \bar{x}) - a)$$

and

$$\frac{\partial E}{\partial m} = 0 = -2\sum_{i=1}^{N}(x_i - \bar{x})(y_i - m(x_i - \bar{x}) - a) .$$

These two equations determine a and m,

$$a = \frac{\sum_{i=1}^{N} y_i}{N} = \bar{y}$$

and (A1)

$$m = \frac{\sum_{i=1}^{N}(x_i - \bar{x})y_i}{\sum_{i=1}^{N}(x_i - \bar{x})^2} ,$$

which in turn describe the best fit line,

$$y^* = m(x - \bar{x}) + a \tag{A2}$$

APPENDIX II

THE CORRELATION COEFFICIENT [16]

To estimate "how well" the line obtained in App. I fits the pairs of data $(y_i x_i)$, consider the quantities

$$r^2 = 1 - \frac{E}{E^1}$$

and

$$E^1 = \sum_{i=1}^{N} (y_i - \bar{y})^2 \quad ,$$

where E and y are defined in App. I and $\left(\frac{E^1}{N}\right)$ is known as the <u>variance</u> of the variable y_i. The quantity r^2 can be used to measure the usefulness of the best fit line in estimating the strength of the dependence of y on x as indicated by the N pairs $(y_i x_i)$.

If we had no knowledge of the dependence of y on x, then b, that value of y which best fits the aggregate of observed y_i's, is found by minimizing

$$E^1 = \sum_{i=1}^{N} (y_i - b)^2 \quad ;$$

or

$$\frac{\partial E^1}{\partial b} = 0 = -2 \sum_{i=1}^{N} (y_i - b),$$

so that

$$b = \frac{\sum_{i=1}^{N} y_i}{N} = \bar{y}$$

as might be expected. The ratio $\left(\frac{E}{E^1}\right)$ is seen, then to be a measure of how the error in estimating y is reduced by knowing the linear dependence of y on x. We see, qualitatively, 1) that for a line which fits the data, y_i, no better than one for which no dependence is considered, $E = E^1$ and $r^2 = 0$; 2) for the special case in which a straight line can be passed through all N points, $E = 0$ and $r^2 = 1$.

For a more quantitative estimate, we rewrite

$$E' = \sum_{i=1}^{N} (y_i-\bar{y})^2 = \sum_{i=1}^{N} (y_i - y_i^* + y_i^* - \bar{y})^2$$
$$= \sum_{i=1}^{N} (y_i-y_i^*)^2 + 2\sum_{i=1}^{N} (y_i-y_i^*)(y_i^*-\bar{y}) + \sum_{i=1}^{N} (y_i^*-\bar{y})^2 , \qquad \text{(A3)}$$

where

$$y_i^* - \bar{y} = m(x_i-\bar{x})$$

and

$$m = \frac{\sum_{i=1}^{N} (x_i-\bar{x}) y_i}{\sum_{i=1}^{N} (x_i-\bar{x})^2} . \qquad \text{(A1)}$$

Substituting for y_i^* in the second sum in (A3),

$$E' = \sum_{i=1}^{N} (y_i-y_i^*)^2 + 2\sum_{i=1}^{N} [y_i-\bar{y}-m(x_i-\bar{x})][m(x_i-\bar{x})] + \sum_{i=1}^{N} (y_i^*-\bar{y})^2$$

and noting that

$$\sum_{i=1}^{N} (y_i^*-\bar{y}) = m\sum_{i=1}^{N} (x_i-\bar{x}) = 0 \quad \text{or}$$

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i}{N} = \overline{y^*} ,$$

we obtain

$$E' = E + 2m\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x}) - 2m^2\sum_{i=1}^{N}(x_i-\bar{x})^2 + \sum_{i=1}^{N}(y_i^*-\overline{y^*})^2. \qquad (A4)$$

Now

$$\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x}) = \sum_{i=1}^{N}y_i(x_i-\bar{x}) = m\sum_{i=1}^{N}(x_i-\bar{x})^2,$$

so that the second and third sums in (A4) vanish, and we can write

$$E^1 = E + Ns^{*2},$$

where

$$Ns^{*2} = \sum_{i=1}^{N}(y_i^*-\overline{y^*})^2.$$

Next we note that

$$r^2 = 1 - \frac{E}{E^1} = 1 - \left(1 - \frac{Ns^{*2}}{E^1}\right) = \frac{Ns^{*2}}{E^1}$$

and that $E^1 = N s_y^2$, (where s_y^2 is the variance of y_i), which finally gives

$$r^2 = \frac{s^{*2}}{s_y^2} \quad .$$

This formula can be interpreted quantitatively as saying that r² is the fraction of the variance, s_y^2, which is accounted for by the estimated dependence of y on x. Again if y* passed through every point, then $s^{*2} = s_y^2$ and the variance of y_i would be entirely accounted for as due to variations in x_i. The quantity r is usually referred to as the correlation coefficient. Thus a correlation coefficient of 0.8 between the variations in cosmic-ray intensity and stratospheric temperature means that 64 percent of the variance of observed intensity measurements may be explained by their dependence on temperature. The remaining 36 percent of s_y^2 would be due to the statistical properties of the population of cosmic-ray intensities and possible dependence on other variables which are independent of temperature.

The relationship between r and the slope of the best fit line can be seen from

$$r^2 = \frac{s^{*2}}{s_y^2} = \frac{\sum_{i=1}^{N}(y_i^* - \overline{y^*})^2}{N s_y^2} = \frac{\sum_{i=1}^{N}(y_i^* - \overline{y})^2}{N s_y^2}$$

$$= \frac{m^2 \sum_{i=1}^{N}(x_i - \overline{x})^2}{N s_y^2} = m^2 \frac{s_x^2}{s_y^2}$$

or

$$m = \frac{s_y}{s_x} r .$$

Here we see that for s_y and s_x of the same order of magnitude, a small slope indicates small correlation and vice-versa.

Finally it should be recognized that the qualitative and quantitative interpretations, discussed above, are purely mathematical and completely devoid of any cause-effect implications. The fact that 2 variables increase or decrease together, does not imply that one has any direct or indirect effect on the other. Both of them may be influenced by other variables which make them appear to have a strong relationship mathematically. The properties of variables must be known or assumed in order that cause-effect relationships may be evaluated for consistency by the correlation method outlined above.

APPENDIX III

STATISTICAL SIGNIFICANCE OF CORRELATION [16]

As a measure of the reliability of this method, we answer the question, "What is the probability that a group of N pairs of <u>independent</u> quantities show a correlation coefficient r, as the result of a particular sample chosen at random?" Now we may regard r as the first sample of the population from which a distribution in r is obtained, each time we draw a sample of N pairs $(x_i y_i)$. The dependence of y on x is considered to be described by a parameter ρ, and we use the sample r as an estimate of this parameter, in the same way that $\bar{x}$ is used to estimate the mean of the population from which x_i is drawn.

Since r is limited in range between 0 and 1, its distribution is asymmetrical and somewhat complicated. A simple transformation of variable, however, provides a new variable Z, which is nearly symmetrical and has an approximately normal distribution when x_i and y_i are members of a normal population [16]. It can be shown that under these circumstances

$$Z = \frac{1}{2} \ln \frac{1 + r}{1 - r} \tag{A5}$$

is approximately normally distributed about the mean

$$m_Z = \frac{1}{2} \ln \frac{1 + \rho}{1 - \rho}$$

with the variance

$$\sigma_Z^2 = \frac{1}{\sqrt{N-3}} \quad .$$

We use these relationships to answer our earlier question. We assume that x and y are independent, such that $\rho = 0$. Then $m_Z = 0$, and from tabulated values of the probability integral we find the probability that the variable Z, with variance σ_Z^2, deviates from a mean value of zero by the amount determined by inserting the sample value r in equation (A5). This procedure is called a significance test, and if the probability that r (or Z) can be obtained from uncorrelated data exceeds a given arbitrary level, r is said to be not significant. The arbitrary level is called a significance level; statisticians frequently use a level of 0.05 or 0.01. In Chap. III a slightly different meaning is attached to significance level, which is explained there.

The transformation from r to Z also provides a means of estimating the statistical error associated with the calculated value for r. The properties of a normal distribution (to which Z belongs) allow the assignment of a statistical error to Z. This is the so-called standard error and is given by σ_Z. This interpretation states that the probability is approximately 0.67 that m_Z lies in the interval $Z \pm \sigma_Z$. The values of r which correspond to the values $Z \pm \sigma_Z$ describe the standard error in r. This error will not be symmetrical about r, but the same probability statement applies to the interval about r.

APPENDIX IV

THE μ-MESON SPECTRUM OF π-MESON DECAY

Consider a π-meson, in its <u>rest system</u>, which decays into a neutral particle, ν , and a μ-meson which moves in the direction making an angle φ with the direction of motion of the π-meson in the <u>laboratory system</u>, (dotted line AB in Fig. A.1). In this system, the conservation of energy and momentum require

$$m_\pi c^2 = p_\nu' c + \sqrt{p_\mu'^2 c^2 + m_\mu^2 c^4}$$

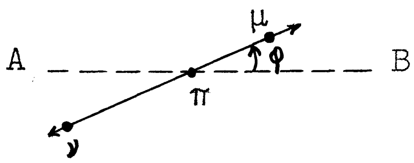

FIGURE A.1

and

$$p_\nu' = p_\mu'$$

where m_π, m_μ, c, have the same meanings as in Chap. IV 4.2, p_μ' is the momentum of the μ-meson in the rest system of the π-meson and p_ν' is the momentum of the neutral particle, of negligible rest-mass, in the rest system of the π-meson. Solving for p_μ', we obtain $p_\mu' = -\frac{m_\pi c}{2}\left(\frac{m_\mu^2}{m_\pi^2} - 1\right)$. Transforming p_μ' into the laboratory system by using the Lorentz transformations $p_{AB} = \gamma(p_\mu' \cos\varphi + \beta c E'/c)$ and

$E = \gamma(E' + \beta c p_\mu' \cos\varphi)$, where $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$,

p_{AB} is the component, in the direction AB, of the momentum of the μ-meson in the laboratory system,

$E' = (p_\mu'^2 c^2 + m_\mu^2 c^4)^{1/2}$ is the energy of the μ-meson in the rest system of the π-meson, and E is the energy of the μ-meson in the laboratory system, (and making the appropriate substitution for p'_μ) we obtain

$$E = \frac{E_0}{2}\left(1 + m_\mu^2/m_\pi^2 - \beta\left[m_\mu^2/m_\pi^2 - 1\right]\cos\varphi\right) \tag{A6}$$

where $E_0 = \gamma m_\pi c^2$ is the energy of the π-meson in the laboratory system.

First we note that the upper and lower limits on E are,

$$\frac{E_0}{2}\left([1-\beta] + m_\mu^2/m_\pi^2[1+\beta]\right) \leq E \leq \frac{E_0}{2}\left([1+\beta] + (m_\mu^2/m_\pi^2)[1-\beta]\right)$$

so that only those π-mesons that possess energies E_π, where

$$(E_{min} = E) \leq E_\pi \leq \left(E_{max} = \frac{2E}{[1-\beta] + (m_\mu^2/m_\pi^2)[1+\beta]}\right)$$

can decay into μ-mesons of energy E. Next we see that since the probability of decay, $dP_D(\varphi)$, at some angle, φ, in the solid angle $d\Omega$ (in the rest system of the π-meson,) is proportional to $d\Omega$, then $dP_D(\varphi) = d\Omega/4\pi = \frac{1}{2}\sin\varphi\, d\varphi$.

But from (A6) $dE = \frac{E_0\beta}{2}\left[1 - m_\mu^2/m_\pi^2\right]\sin\varphi\, d\varphi$ so that

$dP_D(\varphi) = dE/E_0\beta[1 - m_\mu^2/m_\pi^2]$ becomes the probability that a π-meson of energy E_π in dE_π will decay into a μ-meson with energy E in dE, (as previously denoted by $P(E_\pi \rightarrow E)$).

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