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Technical Report

SOME INTERPOLATION THEOREMS FOR PARTITIONS OF GRAPHS

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In this paper we consider certain partitions of the set of points and of the set of lines of a graph and we define for each such partition a corresponding factor graph. The concepts of a complete P-partition and a complete P-line partition of order m are then defined for an arbitrary property P of a graph G. Two results are then obtained which answer the following questions: for what properties P of a graph G does it follow that if G has complete P-partitions (P-line partitions) or orders m and n, then G has complete P-partitions (P-line partitions) of orders k for any k, $m < k < n$.

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SOME INTERPOLATION THEOREMS FOR PARTITIONS OF GRAPHS

In this paper we consider certain partitions of the set of points and of the set of lines of a graph and we define for each such partition a corresponding factor graph. The concepts of a complete P-partition and a complete P-line partition of order m are then defined for an arbitrary property P of a graph G. Two results are then obtained which answer the following questions: for what properties P of a graph G does it follow that if G has complete P-partitions (P-line partitions) or orders m and n, then G has complete P-partitions (P-line partitions) of orders k for any k, $m < k < n$.

The author wishes to acknowledge that the proof techniques used here were originally developed by Geert Prins in an unpublished manuscript in which an Interpolation Theorem was proved for the class of partitions which correspond to homomorphisms of graphs.

By a graph G we mean a set $V = V(G)$ of points together with a set $E = E(G)$ of unordered pairs (u,v) of distinct elements of $V(G)$, called lines of G. A graph G' is a subgraph of G, $G' \subset G$, if $V' \subset V$ and $E' \subset E$; G' is an induced subgraph if for every pair of points $u, v \in V'$, $(u,v) \in E$ implies $(u,v) \in E'$. The subgraph induced by a set of points S, $\langle S \rangle$, is the induced subgraph G' for which $V(G') = S$. The subgraph induced by a set of lines T, $\langle T \rangle$, is the minimal subgraph of G containing all the lines of T. A graph G is called complete if for every pair of distinct points $u,v \in V(G)$, $(u,v) \in E(G)$.

Given two subgraphs G_1 and G_2 of G, $G_1 \cup G_2$ is the subgraph of G for which $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. Two subgraphs G_1 and G_2 of G are said to be line adjacent if there exist points $u \in V(G_1)$, $v \in V(G_2)$ such that $(u,v) \in E(G)$; G_1 and G_2 are point adjacent if there exist points u,v,w such that $(u,v) \in E(G_1)$ and $(u,w) \in E(G_2)$. Any definitions not given here can be found in [2].

Let $\pi = \{V_1, V_2, \ldots, V_m\}$ be a partition of the set of points of a graph G. By the factor graph G/π we mean the graph for which $V(G/\pi) = \{V_1, V_2, \ldots, V_m\}$ and $(V_i,V_j) \in E(G/\pi)$ if and only if $\langle V_i \rangle$ and $\langle V_j \rangle$ are line adjacent. A partition π of G is complete if G/π is complete.

Let P denote any property of a graph G. A subset $S \subset V(G)$ is a P-set if $\langle S \rangle$ has property P. A P-partition of G of order m is a partition $\pi = \{V_1, V_2, \ldots, V_m\}$ of V(G) such that for every i, $1 \leq i \leq m$, V_i is a P-set. The P-chromatic number of a graph G, $\chi_P(G)$, is the minimum order of all complete P-partitions of G. Similarly, the P-achromatic number of G, $\psi_P(G)$, is the maximum order of all complete P-partitions of G.

We will be interested in properties P of a graph G which satisfy the following two conditions. For any graph G and any subsets $V_i, V_j \subset V(G)$: (i) if V_j is a P-set and $V_i \subset V_j$, then V_i is a P-set, and (ii) if V_i and V_j are disjoint P-sets and $\langle V_i \rangle$ and $\langle V_j \rangle$ are not line adjacent, then $V_i \cup V_j$ is also a P-set. We will say that any property P which satisfies these two conditions is homogeneous. Examples of homogeneous properties are numerous. We will mention two examples in particular because of their connections with other established concepts in graph theory.

The property P_1 of being a planar graph can be seen to be homogeneous. The P_1-chromatic number of a graph G then is the minimum number of planar, point disjoint, full subgraphs into which G can be decomposed. It would seem that this parameter is closely related to the thickness of a graph, $\theta(G)$, i.e., the minimum number of planar subgraphs of G whose union equals G.

The property P_2 that a graph can be totally disconnected, i.e., contain no lines, is also homogeneous. It can be shown without much difficulty that the P_2-chromatic number is what is traditionally called the chromatic number, $\chi(G)$.

Consider the following algorithm for obtaining a P-partition of a given graph G. Let V_1 be any maximal P-set of G, i.e., for any point $u \notin V_1$, $\langle V_1 \cup \{u\}\rangle$ does not have property P. Remove the set V_1 from G and select a second maximal P-set V_2 of $G_1 = G - V_1 = \langle V(G) - V_1\rangle$. Next, select a third maximal P-set V_3 of $G_2 = G_1 - V_2$, etc. until we obtain a partition $V_1, V_2, \ldots, V_m$ of V(G) such that for every i, $1 \le i \le m$, V_i is a P-set. We will say that any P-partition of G which can be obtained in this way is of <u>type-1</u>.

<u>Lemma 1</u>. If P is a homogeneous property of a graph G, then every P-partition of G of type-1 is complete.

<u>Proof</u>. Let $\pi = \{V_1, V_2, \ldots, V_m\}$ be a P-partition of G of type-1 which is not complete; then there exist two subsets V_i and V_j, $i < j$, such that V_i and V_j are P-sets and $\langle V_i\rangle$ and $\langle V_j\rangle$ are not line adjacent. But since P is homogeneous, $V_i \cup V_j$ is also a P-set, and hence V_i is not a maximal P-set of $G - \bigcup_{k=1}^{i-1} V_k$. Therefore π is not of type-1; a contradiction.||

If P is a homogeneous property of a graph G and $\pi = \{V_1, V_2, \ldots, V_m\}$ is a P-partition of G which is not complete, it readily follows that we can obtain from π a P-partition π' of order $m' < m$ by adding together two sets V_i and V_j of π which are not line adjacent. Thus we obtain the following.

<u>Lemma 2</u>. For any homogeneous property P and any graph G, if $\chi_P(G) = m$

then every P-partition of G of order m is complete.

Theorem 1. For any homogeneous property P, if a graph G has a P-partition of order m, then G has a P-partition of type-1 of order $m' \le m$.

Proof. Let $\pi = \{V_1, V_2, \ldots, V_m\}$ be a P-partition of G of order m, and let $u_1, u_2, \ldots, u_p$ be an ordering of the points of G such that the points in V_i precede the points in V_j for $i < j$. Define the set $W_1^1 = \{u_1\}$; then define $W_1^{i+1} = W_1^i \cup \{u_{i+1}\}$ if $W_1^i \cup \{u_{i+1}\}$ is a P-set, otherwise define $W_1^{i+1} = W_1^i$. Define $W_1 = W_1^p$; clearly $V_1 \subseteq W_1$, since P is homogeneous. Now form W_2 in the same way from the set of points $V - W_1$, i.e., set $W_2^0 = \phi$, and set $W_2^{i+1} = W_2^i \cup \{u_{i+1}\}$ if $u_{i+1} \notin W_1$ and $W_2^i \cup \{u_{i+1}\}$ is a P-set: otherwise set $W_2^{i+1} = W_2^i$. Continuing in this way we will obtain partition $\pi' = \{W_1, W_2, \ldots, W_m\}$, with the property that $V_i \subset \bigcup_{j=1}^{i} W_i$, which corresponds to a P-partition of type-1 of order $m' \lesssim m$.

Theorem 2. If property P is homogeneous and a graph G has a (complete) P-partition of type-1 of orders k and m, then for all ℓ, $k < \ell < m$, G has a (complete) P-partition of type-1 of order ℓ.

Proof. Let $\pi = \{V_1, V_2, \ldots, V_k\}$ and $\tau = \{W_1, W_2, \ldots, W_m\}$ be the P-partitions of type-1 of orders k and m, respectively. Form the new partition $\pi_1^* = \{W_1, V_1 - W_1, \ldots, V_k - W_1\}$. Since P is homogeneous, π_1^* is also a P-partition. Applying the construction used in Theorem 1 we can obtain a partition $\pi_1 = \{W_1, V_{1,1}, V_{1,2}, \ldots, V_{1,k(1)}\}$ from π_1^* which corresponds to a P-partition of type-1 of order $k(1) \lesssim k + 1$. Note that the first member of π_1 is W_1 because τ is of type-1. Next we form the partition $\pi_2^* = \{W_1, W_2, V_{1,1} - W_2, V_{1,2} - W_2, \ldots, V_{1k(1)} - W_2\}$, and then the partition $\pi_2 = \{W_1, W_2, V_{2,3}, \ldots, V_{2k(2)}\}$ which corresponds to a P-partition of type-1 of order $k(2) \lesssim k + 2$. Continuing in this way we obtain a series of P-partitions of type-1, $\pi, \pi_1, \pi_2, \ldots, \pi_{m-k}$ in which $\pi_{m-k} = \tau$ and the order of π_{i+1} is at most one greater than the order of π_i.

Theorem 3. For any homogeneous property P and any graph G, if G has a complete P-partition of order m which is not of type-1, then G has either a complete P-partition of type-1 of order m or a complete P-partition of order m - 1.

Proof. Let $\pi = \{V_1, V_2, \ldots, V_m\}$ be a complete P-partition of G of order m. If π is not of type-1, let V_i be the first set for which there exists a point $u \in V_j$, $i < j$, such that $V_i \cup \{u\}$ is a P-set. Consider the partition $\pi' = \{V_1, \ldots, V_i \cup \{u\}, \ldots, V_j - \{u\}, \ldots, V_m\}$ and note that if $V_j - \{u\}$ is empty then we have obtained a complete P-partition of order m - 1. If on the other hand, $V_j - \{u\}$ is not empty, then $V_j - \{u\}$ is still a P-set, since P is homogeneous. Thus π' is also a P-partition. If π' is not complete then there exists a set V_k such that $\langle V_j - \{u\}\rangle$ and $\langle V_k' \rangle$ not line adjacent. Then since P is homogeneous, the partition $\pi'' = \{V_1, \ldots, V_i \cup \{u\}, \ldots, V_j - \{u\} \cup V_k, \ldots, V_m\}$ is a complete P-partition of order m - 1. If on the other hand π' is complete, then either it is of type-1, and the theorem is proved, or it is not of type-1 and we repeat the above process. Eventually we must find either a P-partition of type-1 of order m or a complete P-partition of order m - 1.||

Finally, we obtain as in immediate consequence of Lemma 2 and Theorems 2 and 3 the following general Interpolation Theorem for complete partitions of graphs.

Theorem 4. For any homogeneous property P, any graph G, and any integer k, $\chi_P(G) \leq k \leq \psi_P(G)$, G has a complete P-partition of order k.

Since homomorphisms correspond one-to-one with P_2-partitions, where for each subset V_i, $\langle V_i \rangle$ is totally disconnected, and since the property P_2 that a graph be totally disconnected is homogeneous, the Homomorphism Interpolation Theorem of [3] is an immediate corollary of Theorem 4.

We now develop for each of the above concepts and results for partitions of the set of points of a graph, corresponding concepts and results for partitions of the set of lines of a graph. Since proofs of the corresponding line-results are nearly identical to those given already, they are omitted.

Let $\tau = \{E_1, E_2, \ldots, E_m\}$ be a partition of the set of lines of a graph G. By the <u>factor graph</u> G/τ we mean the graph for which $V(G/\tau) = \{E_1, E_2, \ldots, E_m\}$, and $(E_i, E_j) \in E(G/\tau)$ if and only if the subgraphs $\langle E_i \rangle$ and $\langle E_j \rangle$ are point adjacent. A partition τ of the lines of G is <u>complete</u> if G/τ is a complete graph.

Let P denote any property of a graph G. A set of lines $E' \subset E(G)$ is a <u>P-line set</u> if the subgraph $\langle E' \rangle$ has property P. A <u>P-line partition of G of order m</u> is a partition $\tau = \{E_1, E_2, \ldots, E_m\}$ of E(G) such that for every i, $1 \leq i \leq m$, E_i is a P-line set.

The <u>P-line chromatic number</u> of a graph G, $\chi'_P(G)$, is the minimum order of all complete P-line partitions of G. The <u>P-line achromatic number</u>, $\psi'_P(G)$, is the maximum order of all complete P-line partitions of G.

We will be interested in properties P of a graph G which satisfy the following two conditions. For any graph G and any subsets E_i, $E_j \subset E(G)$:
(i) if E_j is a P-line set and $E_i \subset E_j$ then E_i is also a P-line set, and
(ii) if E_i and E_j are disjoint P-line sets and the subgraphs $\langle E_i \rangle$ and $\langle E_j \rangle$ are not point adjacent, then $E_i \cup E_j$ is also a P-line set.

We will say that any property P which satisfies these two conditions is <u>line-homogeneous</u>. It is easy to find line homogeneous properties of graphs.

The property P_1 of being a planar graph is also line-homogeneous. The P_1-line chromatic number of a graph G is then the minimum number of planar line disjoint subgraphs whose union is G. This parameter $\chi'_{P_1}(G)$ is traditionally called the <u>thickness</u> of G, $\theta(G)$ (cf. Beineke and Harary [1]).

The property P_3 that a graph contain no cycles is homogeneous and line homogeneous. The P_3-line chromatic number of a graph G is the minimum number of line disjoint graphs, each of which contains no cycles, whose union is G. This parameter $\chi'_{P_3}(G)$ is traditionally called the <u>arboricity of G, arb(G)</u> (cf. Nash-Williams [ ]).

The property P_4 that a graph consist of a set of independent lines, i.e., no two lines have a point in common, is also line-homogeneous. The P_4-line chromatic number is traditionally called simply the <u>line-chromatic number</u>, $\chi'(G)$.

Consider the following algorithm for obtaining a P-line partition of a given graph G. Let E_1 be any maximal P-line set of G, i.e., for any line $[u,v] \notin E_1$, the subgraph $\langle E_1 \cup \{[u,v]\}\rangle$ does not have property P. Remove the set E_1 from G and select a second maximal P-line set E_2 from $G_1 = G - E_1$. Next select a third maximal P-line set E_3 from $G_2 = G_1 - E_2$, etc., until we finally obtain a partition $E_1, E_2, \ldots, E_m$ of E(G) such that for every i, $1 \leq i \leq m$, E_i is a P-line set. We will say that any P-line partition of G which can be obtained in this way is of type-1.

<u>Lemma 1'</u>. If P is a line-homogeneous property of a graph G, then every P-line partition of G of type-1 is complete.

If P is a line-homogeneous property of a graph G and $\tau = \{E_1, E_2, \ldots, E_m\}$ is a P-line partition of G which is not complete, it readily follows that we can obtain from τ a P-line partition τ' of order m - 1. Simply select two sets E_i and E_j for which the subgraphs E_i and E_j are not point adjacent and form the partition $\tau' = \{E_1, \ldots, E_i \cup E_j, \ldots, E_m\}$ of order m - 1; τ' is still a P-line partition since P is line-homogeneous and $E_i \cup E_j$ is still a P-line set. Thus we obtain

Lemma 2'. For any line-homogeneous property P and any graph G, if $\chi_P'(G) = m$ then every P-line partition of G of order m is complete.

It follows from this lemma, for example, that if the thickness of a graph G, $\theta(G)$, is m, then the factor graph formed from G by any set of m line disjoint planar subgraphs, whose union equals G, is a complete graph.

Theorem 1'. For any line-homogeneous property P, if a graph G has a P-line partition of order m, then G has a P-line partition of type-1 of order $m' \leq m$.

It follows from Theorem 1' that if the thickness $\theta(G)$ of a given graph G is m then there will exist a type-1 partition of the set of lines of G which will produce m planar subgraphs into which G can be decomposed.

Theorem 2'. If property P is line-homogeneous and a graph G has (complete) P-line partitions of type-1 of orders k and m, then for all integers ℓ, $k < \ell < m$, G has a (complete) P-line partition of type-1 of order ℓ.

Theorem 3'. For any line-homogeneous property P and any graph G, if G has a complete P-line partition of order m which is not of type-1, then G has either a complete P-line partition of type-1 of order m or a complete P-line partition of order m - 1.

Finally, we obtain as an immediate consequence of Lemma 2' and Theorems 2' and 3' the following general Interpolation Theorem for complete line partitions of graphs.

Theorem 4'. For any line-homogeneous property P, any graph G, and any integer k, $\chi'_P(G) \le k \le \psi'_P(G)$, G has a complete P-line partition of order k.

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14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

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