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THE DETECTION OF RANDOMLY OCCURRING EVENTS WITH RANDOM DURATIONS

by

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ABSTRACT

This report considers the detection of events for which the occurrence times and durations are random variables. In this context, two special cases are considered: (1) the continuous-action detectors that seek to respond at each time when an event is present and (2) the discrete-action detectors that seek to respond at only a single time during which each event is present. For both cases, the optimum detectors are specified in terms of recursion relations and examples of the numerical solutions of these relations are presented and compared.

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1. INTRODUCTION

Classical detection theory assumes that an observation extends over an interval of time at the end of which a decision is made to either accept or reject the hypothesis that a signal was present during that observation. This theory has found great application in modeling the decision process for those systems where it is possible to consider the occurrence of signals as being confined to well-defined observation intervals (e.g., active radar systems, active sonar systems, and digital communication systems). In many detection systems, however, both the times at which the signals begin and the durations of the signals are unknown. In these cases, the classical detection theory paradigm may no longer be applicable, and the decision theoretic model for the detection problem becomes more complicated.

As a concrete example, consider a simple passive surveillance system for which the observation $x(t)$ is given by

$$x(t) = n(t) \qquad \text{if no target is present at time } t$$

$$x(t) = n(t) + s(t) \quad \text{if a target is present at time } t$$

and the arrival time of each target and the length of time each target spends in the systems field of view are random variables. In these systems, it is often the case that an action must be taken each time a target appears and while the target is still present. For example, the function of the system may be to take a continuous action, such as to sound an alarm when targets are present. Alternatively, its function may be to take a discrete action such as to allocate some resource to investigate the nature of the target further. In both of these cases, the system must operate in real time to detect the presence of randomly occurring events (target appearances) as they occur. In the continuous action system, the detector must decide at each point in time whether or not a target is present in order to control the alarm, whereas, in the discrete action system the detector must designate only a single time when each target is present in order to take the appropriate action. This distinction in the purpose of the systems detectors defines two different detection problems. In this report, the optimum Bayes detectors for each problem are determined.

2. CONTINUOUS ACTION AND DISCRETE ACTION DETECTORS

In this section the continuous action detector (CAD) and the discrete action detector (DAD) are defined by specifying in terms of loss functions how the performance of each is to be measured. It is assumed that the observation x(t) is available at the input of the detector for $t \in [0, T]$. The detector is to be a casual device that produces at its output the response function a(t) that extracts from x(t) the information about each event that is necessary to take the appropriate action. The loss function for each detector, therefore, depends on both the response function a(t) and the randomly occurring events present at the input. We consider the CAD case first.

The objective of the continuous action detector is to take some action at each instant of time that an event is present. Thus, when no noise is present in the observation, the CAD response function may be defined by

$$a(t) = \begin{cases} 1 & \text{if an event is present at time } t \\ 0 & \text{otherwise} \end{cases}$$

The effect of noise on the performance of the CAD can be seen in Fig. 1. Figures 1(a) to 1(e) illustrate the times at which events are present in a typical event sequence, the corresponding noisy

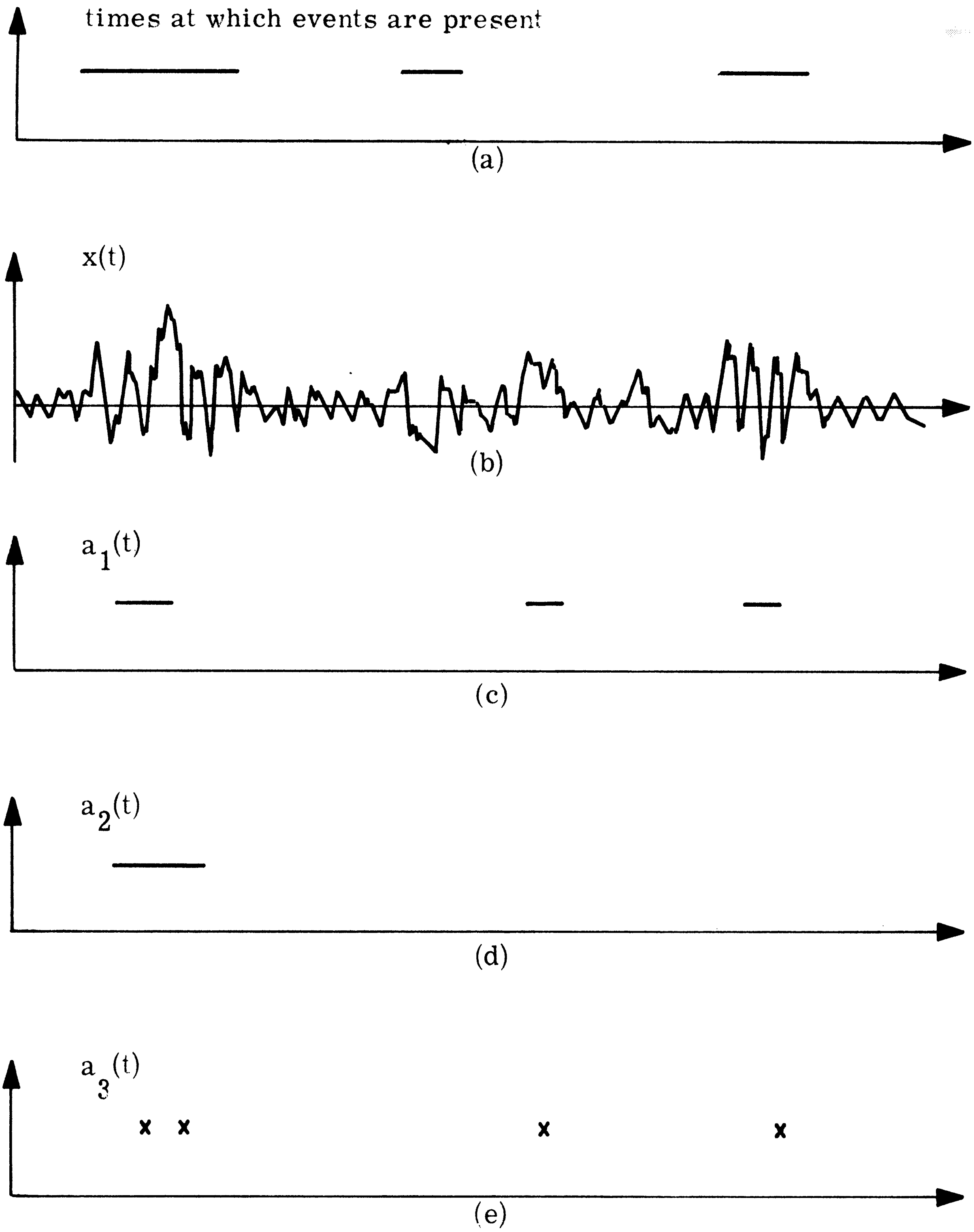

Fig. 1. Illustration of response functions

observation, and a typical response function. It is seen by comparing Fig. 1(a) and Fig. 1(c) that the time axis can be divided into four sets: detection times [a(t) = 1 and an event present] , false alarm times [a(t) = 1 and an event present] , miss times [a(t) = 0 and no event present] , and correct rejection times [a(t) = 0 and no event present] . If we let T_D , T_F , T_M , and T_C denote the total amount of detection times, false alarm times, miss time, and correct rejection time, respectively, then a reasonable choice for the CAD loss function is

$$L = L_D^T T_D + L_F^T T_F + L_M^T T_M + L_C^T T_C \tag{1}$$

where L_D^T , L_F^T , L_M^T , and L_C^T are losses per unit detection time, etc.

A more general definition for the CAD loss function is needed when it is necessary to show special concern for the possibility of missing an event altogether. For example in Fig. 1(d) we have illustrated a second response function that results in the same values of T_D , T_F , T_M , and T_C as the response function in Fig. 1(c), but that detects only one of the three events. This distinction may be incorporated into the loss function by adding in the term

$$L_D N_D + L_M N_M$$

where N_D and N_M are the total number of detected pulses and missed pulses respectively, and L_D and L_M are the corresponding losses. The resulting general CAD loss function is

$$L = L_D^T T_D + L_F^T T_F + L_M^T T_M + L_C^T T_C + L_D N_D + L_M N_M \quad (2)$$

Hereafter we shall refer to the CAD with the loss function in Eq. 1 as the "classical" CAD for reasons which will be apparent later.

Next we define the loss function for the discrete action detector. The objective of the DAD is to take a single discrete action for each event that occurs. When no noise is present in the observation, this task can be accomplished if the response function a(t) is defined as

$$a(t) = \begin{cases} 1 & \text{if an event begins at time } t \\ 0 & \text{otherwise} \end{cases}$$

The effect of noise on the DAD performance can be seen by examining the typical response function in Fig. 1(e). It is seen that some false alarms are made and some events are missed altogether. When an event is detected, the initial detection occurs some time after the arrival of the event, and finally some events are detected more than once. Let N_F and N_X denote the number of false alarms and extra detections respectively, and let L_F and L_X be the corresponding losses. Let T_{LD} be the total amount of "late detection time."

Specifically, T_{LD} is the total amount of elapsed time until each event is detected or the duration of the event if it is not detected. Let L_{LD} be the loss per unit late detection time. Then the performance of the DAD can be characterized in terms of the loss function

$$L = L_D N_D + L_M N_M + L_X N_X + L_F N_F + L_{LD} T_{LD} \qquad (3)$$

where N_D, N_M, L_D, and L_M are as defined above.

3. THE OPTIMUM BAYES DETECTORS

In this section the optimum Bayes detectors are specified in terms of a recursion relationship. In order to minimize the mathematical difficulties involved in the derivation, the following assumptions are made.

(1) Both the continuous action and the discrete action detectors make decisions only at the discrete decision times $t \in \mathscr{T} = \{t_k = k\Delta,\ k = 1, 2, \ldots, N\}$ where $N = T/\Delta$.

This assumption states that the detectors determine the response functions $a(\cdot)$ only through the values $a(t_k)$. In order to interpret this assumption in the context of Section 2 we define the response function for the DAD by

$$a(t) = \begin{cases} a(t_k) & t_k \in \mathscr{T} \\ 0 & \text{otherwise} \end{cases}$$

and

$$a(t) = a(t_k) \qquad t \in [t_{k-1}, t_k)$$

for the CAD. Thus, assumption (1) restricts the DAD to respond only at the times $t_k \in \mathscr{T}$ and restricts the CAD to respond either at every time in the interval $[t_{k-1}, t_k)$ or at no time in the interval

$[t_{k-1}, t_k)$.

The decision separation time Δ can be interpreted as the shortest possible duration of an event as a result of the next assumption:

(2) Events begin and end at the decision times $t_k \in \mathscr{T}$ in the sense that if an event is present at any time in the interval $(t_{k-1}, t_k]$, then it is present at every time in that interval.

As a consequence of assumption (2), the occurrence of events may be completely specified by the sequence of random variables $\{H_k\}_{k=1}^{N}$ where

$$H_k = \begin{cases} 1 & \text{if an event is present in the interval } (t_{k-1}, t_k] \\ 0 & \text{otherwise} \end{cases}$$

The specification of the prior probability law on the occurrence of events constitutes the third assumption:

(3) The sequence $\{H_k\}$ is a stationary two-state Markov chain with

$$P[H_k = 1 \mid H_{k-1} = 0] = \nu$$

$$P[H_k = 0 \mid H_{k-1} = 1] = \delta$$

and

$$P[H_k = 1] = \nu/(\nu + \delta)$$

Assumption (3) asserts that the number of decision times between successive events is geometrically distributed with parameter ν , and the number of decision times that each event is present is geometrically distributed with parameter δ . It follows that the average duty (percentage of time when an event is present), D, is equal to $\nu/(\delta + \nu)$, and the rate (number of events per second) is $\delta D \Delta^{-1}$.

The final assumption specifies the probability law on the observation process. Specifically, let x_k denote the "current" observation pertaining to the interval $[t_{k-1}, t_k)$, and let X_k denote the "total past" observation pertaining to the interval $[0, t_k)$ so that

$$X_k = (X_{k-1}, x_k)$$

Then, it is assumed that

(4) The observation X_N is continually independent and the current observation x_k is described by the densities

$$p(x_k | S) = p(x_k | H_k = 1)$$

$$p(x_k | N) = p(x_k | H_k = 0)$$

It should be noted that as a consequence of assumption (4) the conditional density of x_k depends only on whether or not an event is present at time t_k and not on the arrival time of that event.

As a first consequence of the above assumptions, an updating formula for the a posteriori probability that an event is present at time t_k may be obtained. Denoting this probability by $P_k = \Pr[H_k = 1 \quad X_k]$, it is easily shown that,

$$P_{k+1} = P[P_k, x_{k+1}] \tag{4}$$
$$= [P_k(1 - \delta) + (1 - P_k)\nu] \, p(x_{k+1} \mid S) / p(x_{k+1} \mid X_k)$$

where

$$p(x_{k+1} \mid X_k) \tag{5}$$
$$= [P_k(1 - \delta) + (1 - P_k)\nu] \, p(x_{k+1} \mid S)$$
$$+ [P_k \delta + (1 - P_k)(1 - \nu)] \, p(x_{k+1} \mid N)$$

Next, we develop an updating formula for the a posteriori probability that any event present at time t_k has been previously detected. This is done as follows. Let $\tau_k \epsilon \{1, 2, \ldots\}$ denote the elapsed number of decision times since the most recent response as measured from time t_k . Let $\hat{d}_k(\tau_k)$ be the a posteriori probability that a previously detected event is present at time t_k . Then,

$$\hat{d}_k(\tau_k) = \Pr[H_k = 1, H_{k-1} = 1, \ldots, H_{k-\tau_k} = 1 \mid X_k]$$

and the a posteriori probability that any event present at time t_k has been previously detected is

$$d_k(\tau_k) = P_r[H_k = 1, H_{k-1} = 1, \ldots, H_{k-\tau_k} = 1 \mid X_k, H_k = 1]$$

$$= \hat{d}_k(\tau_k)/P_k$$

With these definitions, it is easy to show that

$$d_{k+1}(\tau_{k+1}) = d_{k+1}[P_k, d_k] \tag{6}$$

$$= \frac{(1-\delta)P_k\, d_k(\tau_{k+1}-1)}{\nu(1-P_k) + (1-\delta)P_k}$$

where

$$d_k(0) \triangleq 1$$

With the above results at hand, it is possible to characterize the optimum Bayes detectors. The basic result is contained in the following theorem.

Theorem I. The optimum Bayes detector determines $a(t_k)$ according to:

$$a(t_k) = \begin{cases} 1 & \text{if } R_k(P_k, d_k) \leq W_k(P_k, d_k) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where

$$R_k(P_k, d_k) = (1 - P_k) - [A d_k + B(1 - d_k)] P_k + W_k(P_k, d_k = 1) \tag{9}$$

$$W_k(P_k, d_k) = \int V_{k+1}[P_{k+1}(P_k, x_{k+1}), d_{k+1}(P_k, d_k)] \, p(x_{k+1} | X_k) \, dx_{k+1} \tag{10}$$

and

$$V_{k+1}(P_{k+1}, d_{k+1}) = \begin{cases} 0 & k = N \\ \min[R_{k+1}(P_k, d_k), W_k(P_k, d_k)] \end{cases} \tag{11}$$

The quantities A and B in Eq. 9 are "normalized losses" given by

$$A = \frac{(L_M^T - L_M^T)\,\Delta}{(L_F^T - L_C^T)\,\Delta} \tag{12}$$

$$B = A + \frac{L_M - L_D}{(L_F^T - L_C^T)\,\Delta}, \qquad 0 < A \leq B < \infty \tag{13}$$

for the continuous-action detectors, and

$$A = -\frac{L_X}{L_F} \tag{14}$$

$$B = \frac{L_M - L_D}{L_F} + \frac{L_{LD}\Delta}{L_F} \qquad -\infty < A \leq 0 < B < \infty \tag{15}$$

for the discrete-action detectors. Note from Eqs. 12 and 13 that for the classical CAD, $A = B$.

The proof of the above theorem is obtained by interpreting the loss functions from the preceding sections in terms of the loss at a particular decision time and then applying standard dynamic programming techniques to obtain the recursion relation for $V_k(P_k, d_k)$. The details of the argument are found in the appendix.

The use of Theorem I to obtain an explicit expression for the Bayes detectors involves first computing the decision rule at the terminal time t_N and then proceeding recursively through decreasing indices to determine the decision rule at time $t_k < t_N$. In the case of the classical CAD, the solution can be obtained immediately by setting $B = A$ and noting that the dependence on d_k in Eqs. 8-11 vanishes. The resulting decision rule is,

$$a(t_k) = \begin{cases} 1 & \text{if } P_k \geq \dfrac{1}{1+A} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

A more familiar expression for the response condition is obtained in terms of the a posteriori odds ration,

$$O_k = P_k/(1 - P_k)$$

The updating equations for O_k and d_k are obtained immediately from applying the definition to Eqs. 4 and 6. The results are

$$O_k = \ell_k \left[\frac{O_{k-1}(1 - \delta) + \nu}{O_{k-1}\delta + (1 - \nu)} \right]$$

and

$$d_k = \frac{O_{k-1}(1 - \delta) d_{k-1}}{O_{k-1}\delta + (1 - \nu)}$$

In terms of O_k , the response condition for the classical CAD is

$$a(t_k) = 1 \quad \text{iff} \quad O_k \geq \frac{1}{A} = \frac{L_F^T - L_C^T}{L_M^T - L_D^T}$$

It is noted that this is precisely the same response condition as would be obtained in the problem of testing the hypothesis that an event is present in the interval $[t_{k-1}, t_k)$ against the alternative that no event is present during $[t_{k-1}, t_k)$.

The decision rules for the other detectors have been obtained using a digital computer to solve the equations in Theorem I. The major results of these computations are stated as two properties. The first of the two properties states that, as in the classical CAD case, the optimum detector responds iff P_k exceeds a threshold. The threshold, however, depends on the current values of d_k, the a posteriori probability that any event that is present has been previosly detected. Stated formally:

<u>Property I (The Threshold Property).</u> For each optimum detector there exists a sequence of functions $K_k(\cdot)$ $k = 1, \ldots, N$, such that

$$a(t_k) = \quad \text{iff} \quad P_k \geq K_k(d_k)$$

or, stated in terms of the odds ration,

$$a(t_k) = 1 \quad \text{iff} \quad O_k \geq T_k(d_k)$$

$$T_k(d_k) = \begin{cases} \dfrac{1}{1 - K_k(d_k)} \\ \\ +\infty \end{cases}$$

It follows from Property I that the general form of the optimum Bayes detectors is as indicated in Fig. 2.

The operation of the optimum detector is clear from Fig. 2. At time t_{k-1}, O_{k-1}, d_{k-1} and a_{k-1} are all stored in memory.

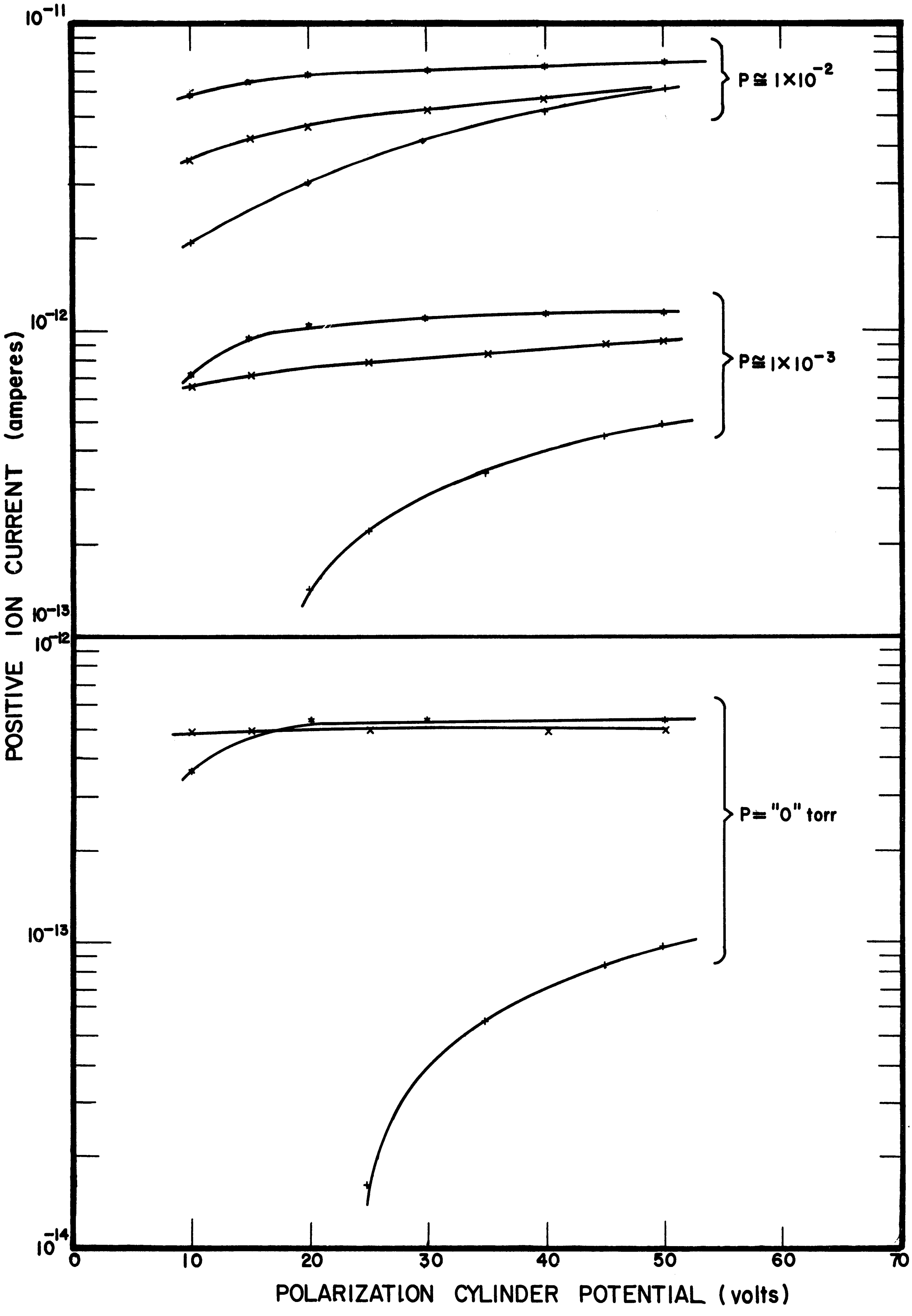

Figure 9. Cylindrical chamber characteristics, P = 10^{-2}, 10^{-3}, and "0" torr.

Then at time t_k, the current likelihood ratio ℓ_k along with O_{k-1} are used to determine O_k. Simultaneously, d_k is calculated from O_{k-1} and d'_{k-1}, where d'_{k-1} can be considered as the result of setting d_{k-1} equal to one whenever the detector responds. The statistic d_k is then used to determine the value of the threshold $T_k(d_k)$. Finally the detector responds iff O_k exceeds $T_k(O_k)$ and O_k, d_k and a_k are stored for use at the next decision time t_{k+1}.

It should be pointed out that only that portion of the general optimum detector enclosed in dotted lines is necessary in the classical CAD.

The second of the two computational properties states that, except for decision times t_k that are close to the terminal decision time t_N, the structure of the optimum detector does not depend on time. More precisely:

Property II (Asymptotic Stationarity Property). For each optimum detector, the sequence of threshold functions $\{K_k(\cdot)\}$ converges through decreasing indices k to a limiting threshold function $\hat{K}(\cdot)$.

As a consequence of Property II, the threshold functions $T_k(\cdot)$ in the general system model can be replaced by the limiting threshold function

$$\hat{T}(d) = \begin{cases} \hat{K}(d)/[1 - \hat{K}(d)] & \hat{K}(d) < 1 \\ +\infty & \hat{K}(d) = +\infty \end{cases} \tag{17}$$

except for decision times near the terminal decision times.

We complete the description of the optimum Bayes detectors by illustrating some specific examples of the threshold functions for both the continuous action and the discrete action detectors. For simplicity, we illustrate the thresholds $\hat{K}(\cdot)$ with the understanding that $\hat{T}(d)$ can be obtained by the monotonic relationship in Eq. 17. The CAD case is considered first.

Figure 3(a) illustrates three different threshold functions for continuous-action detectors. [The values of the losses $A = (L_M^T - L_D^T)/(L_F^T - L_C^T)$ and $B = A + (L_M - L_D)/(L_F^T - L_C^T)\Delta$ corresponding to each threshold function appear next to the curve as the couple (A, B) .] The three different threshold functions in Fig. 3(a) were computed for the same value of A . The threshold function for (A, B) = (0. 5, 0. 5), corresponding to the classical CAD, is constant at the value $1/(1+A) = 2/3$ as expected from Eq. 16. The other two threshold functions both lie below the classical CAD threshold except at the point $d = 1$ where they are all equal. [It can be shown that $\hat{K}(d = 1) = 1/(1 + A)$ for all continuous-action detectors.] It is noted that the threshold function for the larger value of B is smaller than the threshold function for the intermediate value of B . This is reasonable since a large value of B reflects a large value of the net loss of a missed pulse, $(L_M - L_D)$, and thus the optimum detector

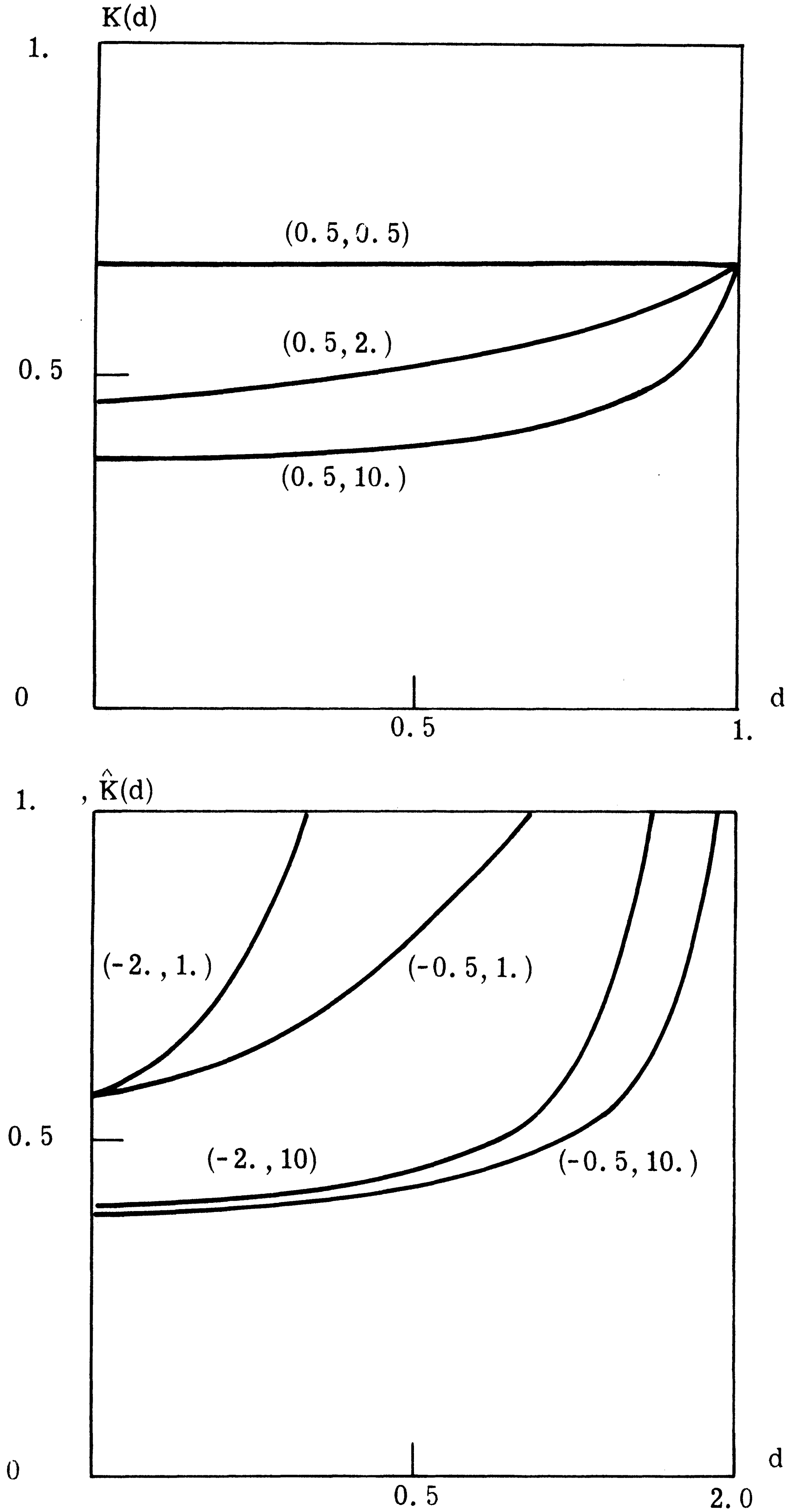

Fig. 3. Illustrations of threshold functions (a) CAD (b) DAD

would be expected to respond more often.

Figure 3(b) illustrates four different threshold functions for discrete-action detectors. It is noted that for each threshold function there is a d^* such that when $d_k \geq d^*$, $\hat{K}(d_k)$ equals 1, and it is impossible for the detector to respond. The value of the "inhibit threshold" d^* depends on both the losses $A = -L_X/L_F$ and $B = (L_M + L_{LD}\Delta)/L_F$. For constant values of B, an increase in the loss for an extraneous detection L_X results in a decrease in d^* [compare the threshold functions for (-2, 1) and (-0.5, 1)]. On the other hand, for fixed A, an increase in either the loss for a missed pulse L_M or the loss per unit detection delay L_{LD} causes an increase in d^* [compare the threshold functions for (-0.5, 1) and (-0.5, 10)].

The threshold functions for the CAD and the DAD depend not only on the normalized losses A and B, but on the parameters of the prior probability law and on the signal-to-noise as well. The threshold functions in Fig. 3 were calculated for a pulse rate $R = 0.05$ and a duty $D = 0.5$ and a signal-to-noise ratio $S/N = 1$. Other calculations indicate that although the basic form of the threshold functions do not vary with changes in R, D and SN, increases in either R, D or SN result in a decrease in the threshold functions.

4. SUMMARY AND CONCLUSIONS

In the previous sections we have considered two classes of detectors both of which function to detect randomly occurring events as they occur. One class, the continuous-action detectors, seek to respond at each point in time that an event is present whereas the other class, the discrete-action detectors, seek to respond at only one time while each event is present. It has been seen that in both cases, the optimum detectors have the same canonical form (Fig. 2). Both calculate the odds ratio O_k and the previous detection probability d_k at each decision time t_k and respond iff

$$O_k \geq T(d_k)$$

The difference in the optimum for the two cases appears in the form of the threshold function $T(\cdot)$. For the continuous action detectors, $T(\cdot)$, is everywhere finite. In the special case of the classical CAD, $T(\cdot)$ is equal to a constant $(L_F^T - L_c^T)/(L_m^T - L_D^T)$, independent of d_k , a result that could have been derived using classical detection theory. For the other continuous action detectors $T(\cdot)$ is an increasing function of d_k , bounded above by $(L_F^T - L_c^T)/(L_m^T - L_D^T)$. The threshold function for the discrete action detector equals $+\infty$ for d_k greater than some inhibiting threshold d^* . For $d_k < d^*$, the discrete-action threshold function is an increasing function of d_k .

APPENDIX

Theorem I must be proved for the CAD and the DAD separately.

Continuous-Action Detector

We begin with the following observations:

(1) Since each event that occurs must be either detected or missed,

$$N_E = \text{total number of events}$$

$$= N_D + N_M$$

(2) Since at each instant that an event is present, either a detection or a miss occurs,

$$T_S = \text{total time when events are present}$$

$$= T_D + T_M$$

(3) Since at each instant that no event is present, either a false alarm or a correct rejection occurs,

$$T_N = \text{total time when no event is present}$$

$$= T_F + T_C$$

Substitute for, N_M , T_M and T_C from the above into Eq. 2 to obtain

$$L = [(L_D - L_M) N_D + (L_D^T - L_M^T) T_D + (L_F^T - L_C^T) T_F]$$

$$+ [L_M N_E + L_M^T T_S + L_C^T T_N]$$

The second bracketed term does not depend on the decision process and hence can be neglected. Denoting the first bracketed term by $\hat{L}$ and noting by assumptions (1) and (2) in Section 3, that

$$T_D = \Delta(N_D + N_X)$$

$$T_F = \Delta N_F$$

we may write

$$\hat{L} = [L_D - L_M + (L_D^T - L_M^T) \Delta] N_D + [(L_D^T - L_M^T) \Delta] N_X$$

$$+ [(L_F^T - L_C^T) \Delta] N_F$$

Now, since $\left(L_F^T - L_C^T\right)$ is assumed positive, it is sufficient to minimize the expected value of

$$\hat{L} = \tilde{L}/(L_F^T - L_C^T) \Delta$$

$$= -BN_D - AN_X + N_F$$

where

$$A = (L_M^T - L_D^T)/(L_F^T - L_C^T)$$

$$B = A + (L_M - L_D)/(L_F^T - L_C^T)$$

But $\hat{L}$ may also be written in the form

$$\hat{L} = \sum_{k=1}^{N} L_k$$

where

$$L_k = 0 \text{ if } a(t_k) = 0$$

and

$$L_k = \begin{cases} 1 & \text{if no event is present at time } t_k \text{ and } a(t_k) = 1 \\ -B & \text{if an undetected event is present at time } t_k \text{ and } a(t_k) = 1 \\ -A & \text{if a detected event is present at time } t_k \text{ and } a(t_k) = 1 \end{cases}$$

Equations 8 through 15 then follow immediately after noting that $1 - P_k$, $(1 - d_k)P_k$ and d_kP_k are the a posteriori probabilities that an event is present at time t_k, an undetected event is present at time t_k, and a detected event is present at time t_k, respectively.

Discrete-Action Detector

By using the first observation above to substitute for N_M in Eq. 3 and by dividing through by $L_F > 0$, we may reduce the problem of minimizing L to the problem of minimizing

$$\hat{L} = [(L_D - L_M)/L_F]\, N_D + [L_X/L_F]\, N_X$$

$$+ (L_{LD}\Delta/L_F)\, N_{LD} + N_F$$

where $N_{LD} = T_{LD}/\Delta$ is the number of times an undetected pulse is missed. Alternatively we may write,

$$\hat{L} = \sum_{k=1}^{N} L_k$$

where

$$L_k = \begin{cases} L_{LD}\Delta/L_F & \text{if } a(t_k) = 0 \text{ and undetected event present} \\ (L_D - L_M)/L_F & \text{if } a(t_k) = 1 \text{ and undetected event present} \\ L_X/L_F & \text{if } a(t_k) = 1 \text{ and previously detected event present} \\ 1 & \text{if } a(t_k) = 1 \text{ and no event present} \end{cases}$$

Then by taking expected values, we have

$$\tilde{V}(P_k,\ d_k) = \min\ \{\tilde{R}_k(P_k,\ d_k),\ \tilde{W}_k(P_k,\ d_k)\}$$

where

$$\tilde{R}_k(P_k, d_k) = 1 - P_k + \{[L_X/L_F]\ d_k + [(L_D - L_M)/L_F]\ (1 - d_k)\}\ P_k$$

$$+ \int \tilde{V}_{k+1}[P_{k+1},\ d_{k+1}(P_k,\ d_k = 1)]\ p(x_{k+1} |\ x_k)\, d_k$$

$$\tilde{W}_k(P_k, d_k) = L_{LD}\Delta/L_F(1 - d_k)\, P_k$$

$$+ \int \tilde{V}_{k+1}[P_{k+1},\ d_{k+1}(P_k, d_k)]\ p(x_{k+1})$$

from which Eqs. 8 through 15 follow with

$$R_k(P_k, d_k) = \tilde{R}_k(P_k, d_k) + (L_{LD}\Delta/\ L_F)\ (1 - d_k)\, P_k$$

$$W_k(P_k, d_k) = \tilde{W}_k(P_k, d_k) + (L_{LD}\Delta/\ L_F)\,(1 - d_k)\, P_k$$

and

$$A = -L_X/L_F$$

$$B = (L_M - L_D)/\ L_F + L_{LD}\Delta/L_F$$

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13. ABSTRACT

This report considers the detection of events whose occurrence times and durations are random variables. In this context, two special cases are considered: (1) the continuous-action detectors that seek to respond at each time when an event is present and (2) the discrete-action detectors that seek to respond at only a single time during which each event is present. For both cases, the optimum detectors are specified in terms of recursion relations and examples of the numerical solutions of these relations are presented and compared.

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