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ELLIPTIC FUNCTIONS WITH COMPLEX ARGUMENTS

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ABSTRACT

Values of the elliptic functions sn w, cn w, dn w, and E(w) are tabulated for complex values of the argument $w = u + iv$. Results are not only tabulated but are also plotted in the form of charts which illustrate the behavior of these functions in the w plane. Contours of x and y, the real and imaginary parts of the elliptic function, are drawn on the w plane, giving a representation of the conformal mapping problem to which elliptic functions provide the solution. Advantages claimed for this arrangement are compactness, convenience, and the simple illustration of the general behavior of the functions.

OBJECTIVE

The objective of this report is the computation of values of the elliptic functions sn w, cn w, dn w, and E(w) for a full range of complex values of the argument, $w = u + iv$. Values of the modulus $k = \sin \theta$ are such that θ occurs at 5° intervals, except for values of 1° and 89° at the extreme ends of the range.

1.0 INTRODUCTION

The results contained in the Appendices to this report are intended to fill the need for complex values of the elliptic functions that arises in connection with plane harmonic, or "potential" problems, i.e., the mapping problems of hydrodynamics and electromagnetic theory. When such problems require the mapping of the interior of a rectangle, or of some other figures consisting of mutually perpendicular straight lines, on to the upper half of another plane, then use of the Schwarz-Cristoffel transformation almost always introduces one or another of the elliptic integrals.

The form of the problem has suggested the method of presentation; as will be shown in more detail below, the charts in the Appendices are drawn up so as to display the solution of the typical mapping problem, and hence to show in compact and convenient form the (complex) values of the elliptic functions for the whole range of (complex) values of the argument. Conversely we may, in the case of the sn-function, obtain the value of the elliptic integral of the first kind for any complex argument without the interpolation usually necessary in the use of tables of elliptic functions.

Following this mode of presentation has of course meant a sacrifice of precision. However, the charts can be read directly to two significant figures and by estimation to three figures; this, after all, corresponds closely to the "slide-rule accuracy" which governs, in practice, the solution of many problems in physics and engineering. More precise tables could easily be drawn up by modern high-speed computing machines, but the resulting volume would be extremely bulky. Furthermore, the trend seems to be away from the formal publication of such tables since on the rare occasions when precise values are needed they can be easily obtained on high-speed computers. There is still, however, a strong case for the type of display embodied in this report, from which quick rough values, and a general picture of the function's behavior, can be obtained.

Opposite each chart is a table giving values to four significant figures, with u and v taken at intervals of 1/10 K and 1/10 K', respectively. These tables may be useful to those who need rather more precision than the charts can offer, although the intervals of u and v are rather too large for satisfactory linear interpolation. Space limitations, however, precluded taking u and v at smaller intervals.

The functions dealt with are $z = \mathrm{sn}\ w$, $z = \mathrm{cn}\ w$, $z = \mathrm{dn}\ w$, and $E = E(w)$. This last is plotted as a function of w, since it is most easily dealt with as an explicit function of w; it would be possible, although more difficult, to treat it as a function of z, i.e., as the elliptic integral of the second kind,

and in this form the results would be more directly useful. This point will be amplified in the discussion below; in the meantime it may be remarked that the tabulation and plotting of $E = E(z)$ suggests itself as a desirable addition to the work embodied in this report.

The following paragraphs are intended to summarize enough of the theory of elliptic integrals and elliptic functions to make the tables and charts understandable. They are essentially a statement of certain equations and propositions without any pretension to completeness in the manner of treatment. There are many useful tests available on the subject. Many texts on advanced calculus, e.g., Franklin's,[1] have a chapter giving a brief basic treatment of the subject; Bowman[2] goes further and gives a thorough treatment of complex values. Milne-Thomson[3] gives a remarkably complete, although brief, discussion of the equations necessary for manipulating the elliptic functions and calculating all real and complex values, together with a basic set of tables; Neville[4] has an attractive account of the underlying theory. Byrd and Friedman[5] make a very complete job of the more modest task of showing how to reduce any integral basically elliptic in form (see Section 2.0), to the standard forms of elliptic integral. The principal authority for the advanced general theory is Whittaker and Watson's[6] comprehensive treatment of the subject.

Where the text goes beyond what is strictly necessary to introduce the tables and charts, it has been in the attempt to summarize information on how the elliptic integrals may be evaluated for all real and imaginary values of the argument. This is a necessary background to the problem of coping with complex arguments.

2.0 THE ELLIPTIC INTEGRALS

The integrals

$$F(k,x) = \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \quad , \quad 0 < k < 1 \tag{1}$$

$$E(k,x) = \int_0^x \sqrt{\frac{1-k^2x^2}{1-x^2}}\, dx \quad , \quad 0 < k < 1 \tag{2}$$

and

$$\pi(n,k,x) = \int_0^x \frac{dx}{(1+nx^2)\sqrt{(1-x^2)(1-k^2x^2)}} \quad , \quad 0 < k < 1 \tag{3}$$

are defined as elliptic integrals of the first, second, and third kinds, respec-

tively. The integral of the first kind may be transformed by the substitution $x = \sin \phi$ into:

$$F(k,\phi) = \int_0^{\phi} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} \tag{4a}$$

and the integral of the second kind transforms under the same substitution into:

$$E(k,\phi) = \int_0^{\phi} \sqrt{1-k^2\sin^2\phi}\ d\phi\ . \tag{4b}$$

These are known as Legendre's normal forms. ϕ is termed the amplitude and k the modulus. We shall refer to x, whether real, imaginary, or complex, as the argument.

The integral of the third kind will not be discussed further: since it is a function of three variables, it is usually regarded as being beyond the scope of any systematic tabulation.

Since k may take any value between 0 and 1, it has become customary to regard k itself as the sine of an angle, usually termed α or θ. In most tables the integral is tabulated as a function of α and ϕ, with values for both angles taken at 1° or 5° intervals from 0° to 90°.

It is shown in standard texts on the subject that if X is any quartic function of x, it is always possible to reduce the integral

$$\int \frac{dx}{\sqrt{X}}$$

to an elliptic integral of the first kind. Depending on the form of X it is sometimes, but not always, possible to express the integral

$$\int \frac{x^n dx}{\sqrt{X}}$$

in terms of elliptic integrals of the first and second kind. In many cases, however, this is more easily said than done, but Byrd and Friedman[5] provide a very useful guide.

2.1 THE BEHAVIOR OF THE ELLIPTIC INTEGRALS

Considering the form of the integral given in Eq. (1), it will be seen that for all values of x in the range $0 < x < 1$ the integrand is real; hence the integral is real. For $1 < x < 1/k$ the integrand is imaginary; hence the integral is complex with a constant real component equal to the integral taken between the limits 0 and 1. For $x > 1/k$ the integrand is once more real; hence the integral is complex with constant imaginary component equal to the integral taken between the limits 1 and $1/k$.

When x is an imaginary number equal to, say, $i\xi$, where ξ is a real number, we have:

$$F(k,i\xi) = \int_0^{i\xi} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} .$$

The substitution $x = iy$ leads to the integral

$$i\int_0^{\xi} \frac{dy}{\sqrt{(1+y^2)(1+k^2y^2)}} ,$$

and the further substitution $y = \tan \phi$ leads to

$$i\int_0^{\tan^{-1}\xi} \frac{d\phi}{\sqrt{1 - (1-k^2)\sin^2\phi}} ,$$

which can be written as

$$i\, F\left(\sqrt{1-k^2}, \frac{\xi}{\sqrt{1+\xi^2}}\right) .$$

Hence we have, finally,

$$F(k,i\xi) = i\, F\left(\sqrt{1-k^2}, \frac{\xi}{\sqrt{1+\xi^2}}\right) . \qquad (5)$$

The positive root $\sqrt{1-k^2}$ is usually written as k', and is called the <u>complementary modulus</u>.

<u>The complete elliptic integral</u> is the integral of Eq. (1) taken between the limits 0 and 1. It is thus a function of the modulus k alone and is given the symbol K. The complete integral corresponding to the complementary modulus k' is denoted by K'. It is possible to evaluate the integral

$$F(k,1/k) = \int_0^{1/k} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

in terms of the complete integrals K and K', by means of suitable substitutions. We have

$$F(k,1/k) = K + \int_1^{1/k} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} .$$

The substitution $1 - k^2x^2 = (1-k^2)\sin^2\phi$ reduces the integral to the form

$$i\int_0^{\pi/2} \frac{d\phi}{\sqrt{1-(1-k^2)\sin^2\phi}} \quad \text{i.e., } iK', \text{ and we have:}$$

$$F(k,1/k) = K + iK' . \tag{6}$$

It is possible to generalize this result further to cover all real values of x. For $x = x_o$, $1 < x_o < 1/k$ the above substitution leads to the integral

$$i\int_{\sin^{-1}\sqrt{\frac{1-k^2x_o^2}{1-k^2}}}^{\pi/2} \frac{d\phi}{\sqrt{1-(1-k^2)\sin^2\phi}}$$

which is equal to

$$i\left[K' - F\left(k', \sqrt{\frac{1-k^2x_o^2}{1-k^2}}\right)\right] .$$

Hence

$$F(k,x_o) = K + iK' - iF\left(k', \sqrt{\frac{1-k^2x_o^2}{1-k^2}}\right) . \tag{7}$$

For $x = x_o$, $> 1/k$ we observe that where there is no likelihood of confusion, the notation for the elliptic integral may be shortened bo the form F(x), and introduce the further notation

$$F\begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = F(x_2) - F(x_1) = \int_{x_1}^{x_2} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} .$$

Considering the integral

$$F\begin{pmatrix} x_o \\ 1 \end{pmatrix} = \int_1^{x_o} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} ,$$

the substitution $x = 1/k\xi$ leads to the integral

$$\int_{1/kx_o}^{1/k} \frac{d\xi}{(1-\xi^2)(1-k^2\xi^2)} ,$$

whence we have

$$F\begin{pmatrix} x_o \\ 1 \end{pmatrix} = F\begin{pmatrix} 1/k \\ 1/kx_o \end{pmatrix} .$$

Subtracting $F\left(\begin{matrix}1/k\\1\end{matrix}\right)$ from both sides of the equation we obtain finally

$$F\left(\begin{matrix}x_o\\1/k\end{matrix}\right) = F\left(\begin{matrix}1\\1/kx_o\end{matrix}\right) \tag{8}$$

Equations (5) - (8) make it clear that working from tabulated values of elliptic integrals for $0 < x < 1$, we may deduce values for all real and imaginary values of x. However when we consider complex values of x (or, more properly, of $z = x + iy$) the elliptic integral proves to be intractable whatever path of integration we select between the limits $z = 0$ and $z = x + iy$. The basis of the difficulty is that there is no true addition formula for elliptic integrals, so that there is no means of evaluating $F(x+iy)$ in terms of $F(x)$ and $F(iy)$.

The only way out of the difficulty is to invert the elliptic integral and to discuss the properties of the inverse function so obtained, called the elliptic function.

With the elliptic integral of the second kind, it is better to postpone the kind of discussion given above until after consideration of the elliptic functions. However, it may be noted here that the complete integrals corresponding to K and K' are written as E and E', and that $E(k,1/k)$ can be evaluated by exactly the same substitutions as are used above to obtain $F(k,1/k)$. In this case, however, they lead to the result

$$E(k,1/k) = E + i(K'-E') \quad . \tag{9}$$

3.0 JACOBI'S ELLIPTIC FUNCTIONS

We have seen that Eqs. (1) and (4)—

$$u = F(k,\phi) = \int_o^{\phi} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} = \int_o^{x} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \tag{1),(4}$$

—define u as a function of ϕ or x, for a given fixed value of k. They also define ϕ as a function of u, $\phi = \text{am } u$ (i.e., amplitude of u). Since $x = \sin\phi$, we may also write $x = \sin(\text{am } u)$, or more briefly sn u. We may in fact define three related functions thus:

$$\text{sn } u = \sin(\text{am } u) = \sin\phi = x \tag{10a}$$

$$\text{cn } u = \cos(\text{am } u) = \cos\phi = \sqrt{1-x^2} \tag{10b}$$

$$\text{dn } u = \Delta(\text{am } u) = \Delta\phi = \sqrt{1-k^2\sin^2\phi} = \sqrt{1-k^2x^2} \tag{10c}$$

These functions were introduced by Jacobi, Abel, and Gauss, who were the first to realize that the elliptic function, rather than the elliptic integral, should be regarded as fundamental in the general theory of functions. The elliptic function as defined above can be developed and discussed at some length (as a function of a complex variable) through its relationship with certain other functions known as Theta-functions. In particular the very fast convergence of the series which define the Theta functions provides a ready means of computing and tabulating the elliptic functions, whereas there is no direct method (other than numerical integration) of computing values of elliptic integrals. Most important of all, for the purposes of the present discussion, addition formulae can be derived for the elliptic functions, so that the values of the function of a complex number may be readily computed. The addition formulae are:

$$\mathrm{sn}\,(u \pm v) = \frac{\mathrm{sn}\,u\,\mathrm{cn}\,v\,\mathrm{dn}\,v \pm \mathrm{sn}\,v\,\mathrm{cn}\,u\,\mathrm{dn}\,u}{1 - k^2\mathrm{sn}^2u\,\mathrm{sn}^2v} \tag{11a}$$

$$\mathrm{cn}\,(u \pm v) = \frac{\mathrm{cn}\,u\,\mathrm{cn}\,v \mp \mathrm{sn}\,u\,\mathrm{sn}\,v\,\mathrm{dn}\,u\,\mathrm{dn}\,v}{1 - k^2\mathrm{sn}^2u\,\mathrm{sn}^2v} \tag{11b}$$

$$\mathrm{dn}\,(u \pm v) = \frac{\mathrm{dn}\,u\,\mathrm{dn}\,v \mp k^2\,\mathrm{sn}\,u\,\mathrm{sn}\,v\,\mathrm{cn}\,u\,\mathrm{cn}\,v}{1 - k^2\mathrm{sn}^2u\,\mathrm{sn}^2v} \tag{11c}$$

The function sn u is odd, while cn u and dn u are even. That is to say

$$\mathrm{sn}(-u) = -\mathrm{sn}\,u, \quad \mathrm{cn}(-u) = \mathrm{cn}\,u, \quad \mathrm{dn}(-u) = \mathrm{dn}\,u \tag{12}$$

Where it is necessary in the interests of clarity, the modulus k is included in the notation by writing sn(u,k) instead of sn u. In general, however, we shall simply write sn u for sn(u,k), and for sn(u,k') we shall write sn'u.

We may note here the elementary results

$$\left.\begin{array}{l} \mathrm{sn}(0) = 0,\ \mathrm{sn}\,K = 1;\quad \mathrm{cn}\,0 = 1,\ \mathrm{cn}\,K = 0;\quad \mathrm{dn}(0) = 1,\ \mathrm{dn}\,K = k' \\ \mathrm{sn}'(0) = 0,\ \mathrm{sn}'K' = 1;\quad \mathrm{cn}'(0) = 1,\ \mathrm{cn}'K' = 0;\quad \mathrm{dn}'(0) = 1,\ \mathrm{dn}'K' = k \end{array}\right\} \tag{13}$$

Equations (5) - (8) can now be rewritten

$$\mathrm{sn}\,i\,u = i\,\frac{\mathrm{sn}'u}{\mathrm{cn}'u} \tag{5a}$$

$$\mathrm{sn}\,(K + iK') = 1/k \tag{6a}$$

$$\mathrm{sn}\,(K + iv) = \frac{1}{\sqrt{\mathrm{cn}'^2u + k^2\mathrm{sn}'^2v}} \tag{7a}$$

$$\mathrm{sn}\,(K + iK' - u) = \frac{\mathrm{dn}\,u}{k\,\mathrm{cn}\,u} \tag{8a}$$

Equation (5a) can readily be deduced from Eq. (5), and Eq. (6a) is a trivial restatement of Eq. (6). Equations (7a), and (8a) are most readily obtained by working from Eqs. (11) and (13), but with some manipulation they could be derived directly from Eqs. (7) and (8).

A further condensation of the notation is possible by writing

$$\frac{\text{sn } u}{\text{cn } u} = \text{sc } u\,, \qquad \frac{\text{cn } u}{\text{dn } u} = \text{cd } u\,, \qquad \text{etc.}$$

There will, however, be very little occasion to use this notation in this report.

3.1 THE PERIODICITY OF THE ELLIPTIC FUNCTIONS

From Eqs. (11a) and (13) it is easy to derive the equations

$$\text{sn } (u + 2K) = - \text{sn } u \tag{14a}$$

$$\text{sn } (u + 4K) = \text{sn } u \tag{14b}$$

$$\text{sn } (2K - u) = \text{sn } u \tag{14c}$$

These equations make it clear that the function sn u has a period 4K and in general if u is a real number sn u behaves in similar fashion to the sine function, with the period 4K corresponding to the period 2π, as shown in Fig. 1.

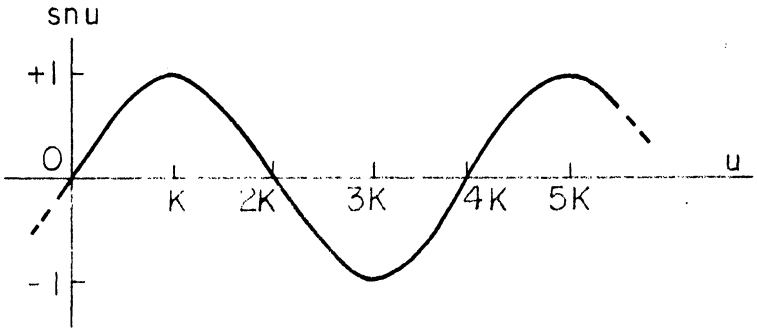


Fig. 1

From Eqs. (11a) and (13) we can further derive the equation

$$\text{sn } (u + 2iK') = \text{sn } u \tag{15a}$$

and from (5) and (13) the further equation

$$\text{sn } i\,K' = \infty \quad . \tag{15b}$$

These equations, together with the knowledge that sn u is an odd function, show that the function sn u has a second period of 2iK', and in general if u is an imaginary number equal to, say, iv, where v is real, sn iv, as a function of v, behaves in similar fashion to the tangent function, with the period 2iK' corresponding to the period π, as shown in Fig. 2.

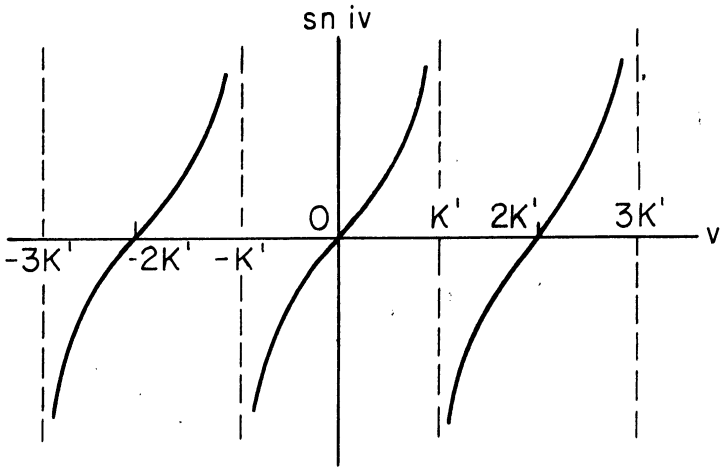


Fig. 2

Considering now the elliptic function of a complex number, sn w = sn (u + iv), we see that, just as the variation of the sine and the tangent of an angle θ can be compactly shown by describing their behavior in the range $0 < \theta < \pi/2$, so sn w can be completely defined by describing its behavior inside, and on, the rectangle bounded by the lines u = 0, u = K, v = 0, v = iK'. Consideration of the exact mode of its behavior within the rectangle can conveniently be deferred until the Schwarz-Cristoffel transformation is discussed in Section 3.3.

The doubly periodic property deduced above is peculiar to elliptic functions; in fact, from the viewpoint of the theory of functions it is the defining property of the generalized elliptic function. The rectangle having a side of 4K parallel to the real axis and a side of 2K' parallel to the imaginary axis is called a period-parallelogram.

3.2 THE ELLIPTIC FUNCTIONS AND THE INTEGRAL OF THE SECOND KIND

To obtain an addition formula for this integral, it is necessary to transpose it from a function of x into a function of u. Since x = sn u, we have:

$$\sqrt{\frac{1-k^2x^2}{1-x^2}} = \frac{\text{dn } u}{\text{cn } u}$$

and

$$du = \frac{dx}{\sqrt{(1-k^2)(1-k^2x^2)}}$$

$$= \frac{dx}{cn\ u\ dn\ u}$$

Hence

$$E = \int \sqrt{\frac{1-k^2x^2}{1-k^2}}\ dx = \int dn^2u\ du \qquad (16)$$

E(u) is not periodic in either real or imaginary values of u, although it is possible to define a related function, the Zeta function, which is singly periodic. However, the main interest in this report attaches to E as a function of x: we consider it as a function of u simply to obtain an addition formula to work with. By using Zeta functions it is possible to derive the addition formula

$$E\ (u+v) = E(u) + E(v) - k^2\ sn\ u\ sn\ v\ sn\ (u+v)\ . \qquad (17)$$

An imaginary transformation can also be obtained:

$$E(iu) = iu + i\ dn'u\ sc'u - i\ E'(u)\ . \qquad (18)$$

It would have been possible, by an argument like those in Section 2.1, to obtain an imaginary transformation for E considered as a function of x. However, there is little point in doing so, as Eq. (18) is already in the correct form for application to the addition formula (17).

3.3 ELLIPTIC FUNCTIONS IN PLANE-POTENTIAL PROBLEMS

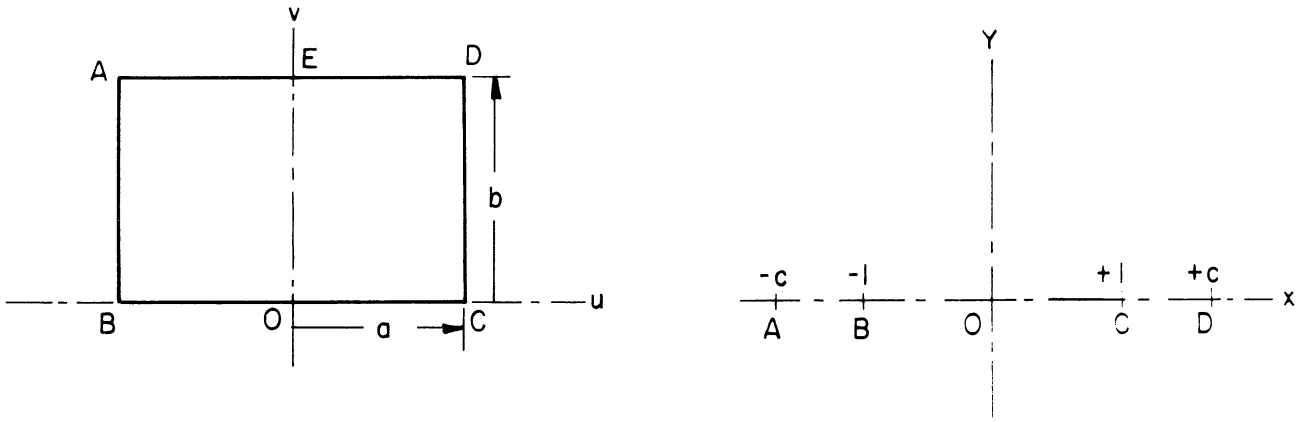
Many plane-potential (also called plane-harmonic) problems can be reduced to the problem of mapping the interior of a polygon in one plane onto the upper half of another plane. When this polygon is a rectangle, the Schwarz-Cristoffel transformation leads immediately to an elliptic integral of the first kind.

Consider, as shown in Fig. 3, the mapping of the interior of the rectangle ABCD on the w plane on to the upper half of the z plane.

The Schwarz-Cristoffel theorem states that such a mapping may be effected my means of the transformation

$$\frac{dw}{dz} = G.\prod(z - \alpha_k)^{\beta_k/\pi - 1} \qquad (19)$$

where α_k is the abscissa of the point on the real axis of the z-plane corresponding to the vertex of the polygon having an internal angle β_k, and G is an arbitrary constant.



w plane z plane

Fig. 3

In our case all the internal angles of the polygon are equal to $\pi/2$ and hence $\beta_k/\pi - 1 = -1/2$. If the lengths a, b, and c are as defined in Fig. 3, we have:

$$w = G\int \frac{dx}{\sqrt{(z^2-1)(z^2-c^2)}} + C$$

$$= \frac{G}{c}\int \frac{ds}{\sqrt{(1-z^2)[1(z^2/c^2)]}} + C$$

$$= Gk\int \frac{dz}{\sqrt{(1-z^2)(1-k^2z^2)}} + C, \text{ where } k = 1/c$$

$$= Gk\, F\,(k,z) + C \quad . \tag{20}$$

Considering the boundary values, we have:

Pt. O: $0 = 0 + C \quad \therefore C = 0$

Pt. C: $a = GkK$

Pt. D: $a + ib = GkF\,(k,1/k)$

$$= Gk\,(K + iK')$$

$$\therefore b = GkK' \quad .$$

We may without loss of generality select G so that $Gk = 1$, whence

$$a = K\,, \quad b = K' \tag{21}$$

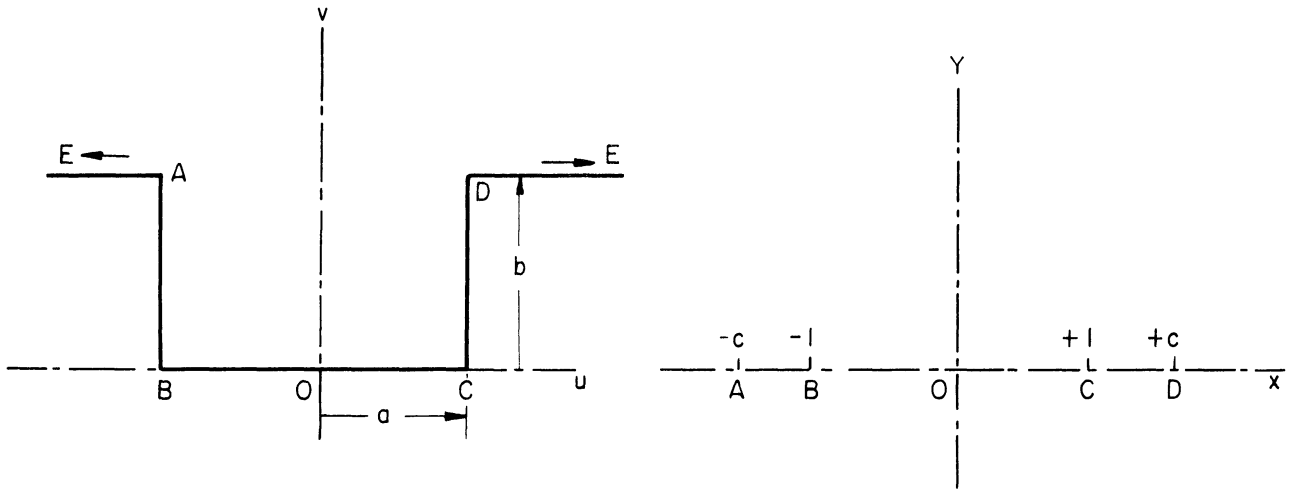
and Eq. (20) becomes simply

$$\text{or} \quad \left.\begin{aligned} w &= F(k,z) \\ z &= \operatorname{sn} w \end{aligned}\right\} \tag{22}$$

These results confirm those obtained in the preceding paragraph, while at the same time making it clear how the function sn w varies within the rectangle OCDE, which corresponds to the whole of the first quadrant on the z plane. The point E corresponds to the point at infinity, and the boundary OCDE corresponds of course to the real axis, confirming in general the points made at the beginning of Section 2.1 about the behavior of the elliptic integral, and the results given in Eqs. (7) and (8). The only modification to this equation which now appears to be necessary is that when z is real and greater than $1/k$, the negative sign must be taken for the integrand, so that the real part of F(z) returns to zero as z goes to (real) infinity.

The flow situation represented is that of flow from a source at E into the boundary of the rectangle, or alternatively (interchanging the roles of the two sets of lines in the flow net) flow about a doublet, or dipole, at E, with the boundary of the rectangle becoming the bounding streamline. If the rectangle is now cut at E, and the sides AE and DE folded back so that E goes to infinity on both sides, as in Fig. 4, we have the case of flow across a horizontal floor containing a rectangular slot. In this case two of the angles are equal to $\pi/2$, giving $\beta_k/\pi - 1 = -1/2$, and the other two are equal to $3\pi/2$, giving $\beta_k/\pi - 1 = +1/2$. Hence we obtain

$$w = Gk \int \sqrt{\frac{1-k^2z^2}{1-z^2}}\, dz + C$$



w plane x plane

Fig. 4

We can again set Gk = 1, and the boundary values lead this time to:

$$a = E \ , \quad b = K'-E' \tag{23}$$

from Eq. (9).

4.0 THE CONSTRUCTION OF THE CHARTS

It should now be clear that the charts presented in the Appendices contain all the information needed to solve the type of problems outlined in the preceding section. Considering the first problem, the first quadrant of the z plane is to be folded up inside the rectangle OCDE on the w plane, and this is precisely what is shown in the charts in Appendix I (x = sn w) where contours of **x** and y are drawn in on this rectangle on the w plane. Appendices II and III display in similar fashion the behavior of the functions z = cn w and z = dn w; the corresponding problems are simple extensions of the first problem. Again we may think of the point E as being a source; if the x-contours are considered as streamlines, then flow is from E to the boundaries OE, OC, for z = cn w, and to the boundaries OE, OC, CD, for z = dn w. Scales on the w plane have been selected so as to make the rectangle OCDE a unit square; in other words, the quantities measured along the axes of the w plane are not u and v, but u/K and v/K'. Although this arrangement distorts the **x**-y net so as to conceal its orthogonal property, it has a generality and a convenience that appear to outweigh this disadvantage.

Considering the second problem, the difficulty is that the E-function has to be computed as a function of w, and not as a function of z, where z = sn w. It has been plotted on the w plane (Appendix IV) because this was the simplest course to follow in the limited time available, but it would of course have been better to plot contours of x and y on the E plane, and hence to show directly the solution to the mapping problem. However, values of E can be obtained for a given value of z by first obtaining the appropriate value of w from Appendix I, and then entering this value into the charts in Appendix IV. The mapping problem has still been kept in mind by dividing E_R and E_I, the components of E, by E and K'-E', respectively, i.e., the lengths OC and CD in Fig. 4. Contours of the resulting normalized values,

$$E_R' = \frac{E_R}{E}$$

and

$$E_I' = \frac{E_I}{K'-E'}$$

are plotted on the w plane in the charts in Appendix IV.

The range of values of k selected is as follows:

$$\sin^{-1} 1°, \quad \sin^{-1} 5°, \quad \sin^{-1} 10°, \ . \ . \ . \ . \ \sin^{-1} 80°, \quad \sin^{-1} 85°, \quad \sin^{-1} 89°,$$

making 19 values in all. This should be sufficient for the purposes of most physical problems.

It remains only to give the equations from which the tables and charts have been computed. From Eq. (5) we have

$$\text{sn } iv = i \frac{\text{sn}'v}{\text{cn}'v} ,$$

whence

$$\text{cn } iv = \sqrt{1 + \left(\frac{\text{sn}'v}{\text{cn}'v}\right)^2}$$

$$= \frac{1}{\text{cn}'v} ,$$

and

$$\text{dn } iv = \sqrt{1 + k^2 \left(\frac{\text{sn}'v}{\text{cn}'v}\right)^2}$$

$$= \frac{\text{dn}'v}{\text{cn}'v} .$$

Hence from Eqs. (11) we have:

$$\text{sn } (u+iv) = \frac{\text{sn } u \text{ dn}'v + i \text{ cn } u \text{ dn } u \text{ sn}'v \text{ cn}'v}{\text{cn}'^2v + k^2 \text{ sn}^2u \text{ sn}'^2v} \tag{24}$$

$$\text{cn } (u+iv) = \frac{\text{cn } u \text{ cn}'v - i \text{ sn } u \text{ dn } u \text{ sn}'v \text{ dn}'v}{\text{cn}'^2v + k^2 \text{ sn}^2u \text{ sn}'^2u} \tag{25}$$

$$\text{dn } (u+iv) = \frac{\text{dn } u \text{ cn}'v \text{ dn}'v - i \, k^2 \text{ sn } u \text{ cn } u \text{ sn}'v}{\text{cn}'^2v + k^2 \text{ sn}^2u \text{ sn}'^2v} \tag{26}$$

For the E function, we use Eqs. (17), (18), and (24) to obtain:

$$E(u+iv) = E_R + iE_I' = E(u) + \frac{k^2 \text{ sn } u \text{ cn } u \text{ dn } u \text{ sn}'^2v}{\text{cn}'^2v + k^2 \text{ sn}^2u \text{ sn}'^2v}$$

$$+ i \left[v - E'(v) + \frac{\text{sn}'v \text{ cn}'v \text{ dn}'v \text{ dn}^2u}{\text{cn}'^2v + k^2 \text{ sn}^2u \text{ sn}'^2v} \right] . \tag{27}$$

The denominator in Eqs. (24) - (27) could be expressed in other ways, e.g., as $1 - dn^2u\ sn'^2v$, but the form given was selected as being better for computing purposes since it is a sum rather than a difference of two terms.

The computing was done on the University's IBM 650 digital computer, which handles numbers having a maximum of ten digits. Fixed-point arithmetic was used, the decimal point being kept fixed between the fourth and fifth digits in both input and output data. Since all the constants fed in (sn u, cn u, etc.) lay between 0 and 1, this meant that these numbers had at least five, and usually six, significant figures. However, for the lower values of k some of the results had only two significant figures because there were four zeros after the decimal point. This was not a serious source of error in plotting the charts, but for the four-figure tables such results, and any others of doubtful precision, were recomputed manually to at least three significant figures. In most cases, however, the results had six nonzero digits, which were rounded off to four before printing. The reader can take it, therefore, that whether three or four digits are given, the last digit is reliable.

The values of sn u, cn u, etc., fed into the machine were taken straight from Spenceley's[7] twelve-figure tables of elliptic functions.

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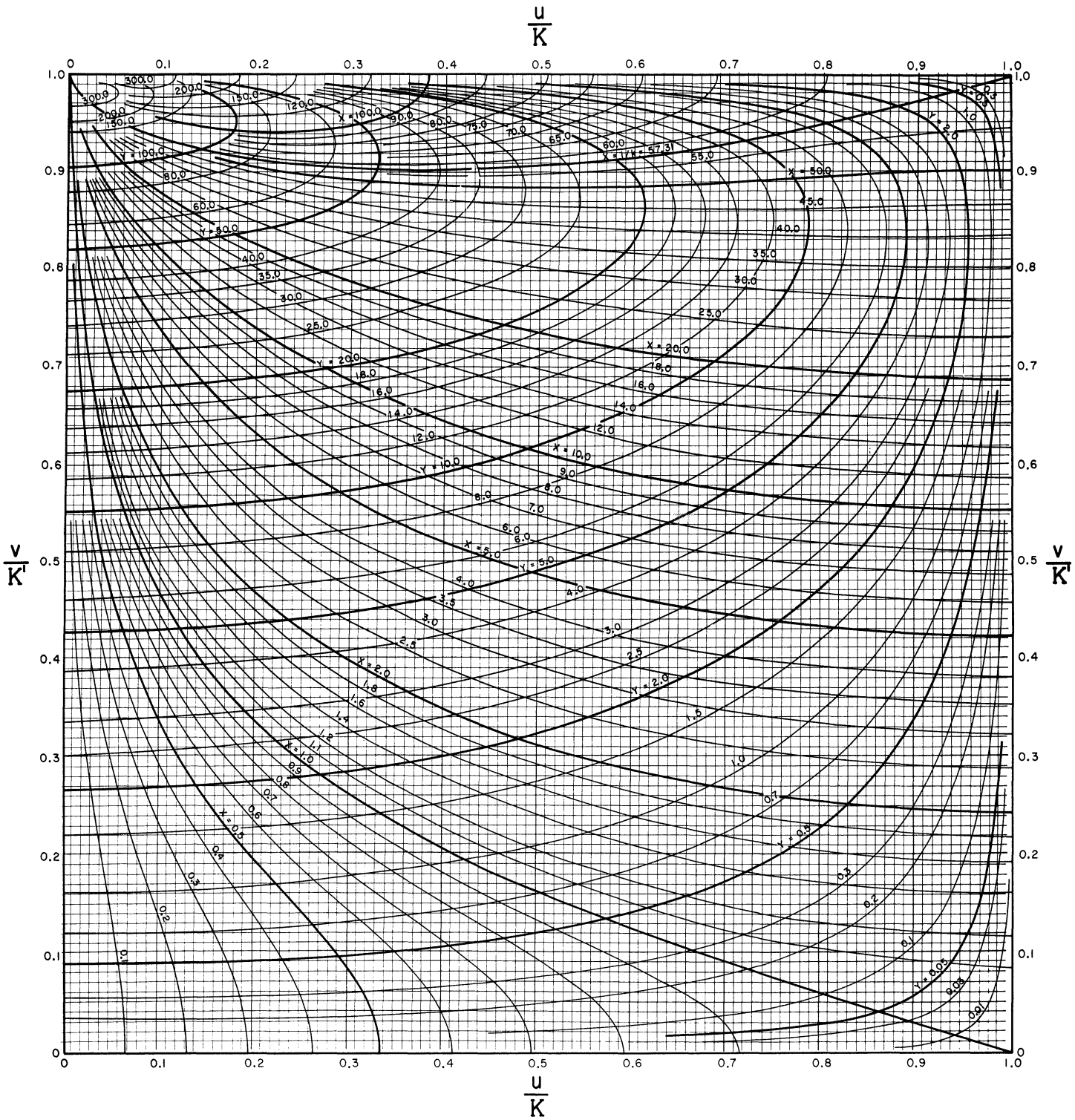
APPENDIX I

THE sn-FUNCTION

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 1° = 0.0174524; K = 1.570916, K' = 5.434910

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1801	0.3558	0.5228	0.6768	0.8142	0.9315	1.026	1.095	1.137	1.151
	y	0.5707	0.5636	0.5427	0.5084	0.4616	0.4034	0.3354	0.2590	0.1763	0.08924	0
0.2	x	0	0.2584	0.5104	0.7499	0.9708	1.168	1.336	1.471	1.570	1.631	1.651
	y	1.314	1.298	1.250	1.171	1.063	0.9289	0.7721	0.5963	0.4058	0.2054	0
0.3	x	0	0.4153	0.8202	1.205	1.559	1.875	2.145	2.362	2.520	2.617	2.650
	y	2.456	2.426	2.336	2.188	1.986	1.735	1.442	1.113	0.7576	0.3835	0
0.4	x	0	0.6995	1.381	2.028	2.623	3.153	3.604	3.967	4.231	4.392	4.446
	y	4.346	4.291	4.131	3.867	3.508	3.064	2.544	1.964	1.336	0.6758	0
0.5	x	0	1.204	2.376	3.484	4.500	5.399	6.161	6.769	7.211	7.480	7.570
	y	7.570	7.473	7.187	6.720	6.087	5.306	4.399	3.389	2.302	1.164	0
0.6	x	0	2.120	4.172	6.094	7.833	9.348	10.61	11.60	12.32	12.74	12.89
	y	13.19	13.01	12.48	11.62	10.48	9.082	7.490	5.744	3.888	1.961	0
0.7	x	0	3.921	7.670	11.07	14.02	16.46	18.40	19.85	20.85	21.43	21.63
	y	23.33	22.90	21.84	20.10	17.85	15.23	12.37	9.358	6.268	3.141	0
0.8	x	0	8.475	16.05	22.23	26.84	30.05	32.15	33.45	34.21	34.59	34.71
	y	43.60	42.56	39.29	34.70	29.39	23.91	18.58	13.56	8.840	4.358	0
0.9	x	0	29.45	48.41	56.96	57.80	56.52	54.45	52.51	51.01	50.08	49.77
	y	100.4	92.24	73.84	54.81	39.42	28.01	19.61	13.26	8.212	3.931	0
1.0	x	∞	366.3	185.4	126.2	97.48	81.03	70.82	64.31	60.24	58.01	57.30
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

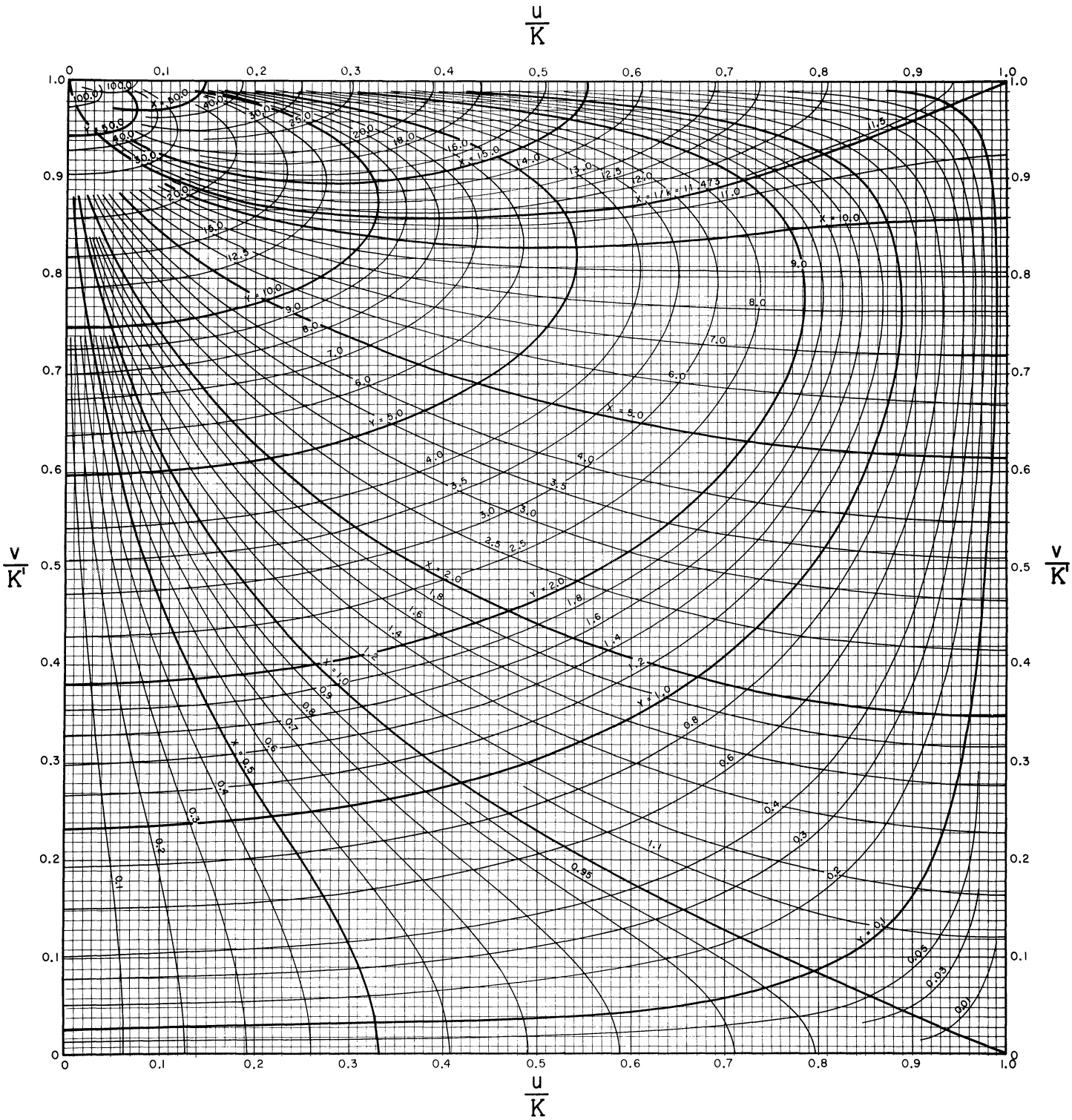
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 1° = 0.0175$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 5° = 0.0871557; K = 1.573792, K' = 3.831742

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1567	0.3096	0.4547	0.5885	0.7078	0.8095	0.8914	0.9512	0.9877	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1685	0.3327	0.4886	0.6324	0.7604	0.8696	0.9573	1.021	1.061	1.074
	y	0.3927	0.3878	0.3732	0.3494	0.3169	0.2767	0.2298	0.1773	0.1206	0.06102	0
0.2	x	0	0.2057	0.4060	0.5960	0.7709	0.9264	1.059	1.165	1.242	1.289	1.305
	y	0.8444	0.8338	0.8022	0.7507	0.6805	0.5938	0.4927	0.3800	0.2583	0.1307	0
0.3	x	0	0.2747	0.5420	0.7948	1.027	1.232	1.405	1.544	1.645	1.706	1.727
	y	1.424	1.406	1.352	1.264	1.144	0.9966	0.8257	0.6359	0.4318	0.2183	0
0.4	x	0	0.3891	0.7664	1.121	1.444	1.726	1.963	2.151	2.286	2.368	2.395
	y	2.225	2.195	2.107	1.965	1.773	1.540	1.272	0.9764	0.6615	0.3339	0
0.5	x	0	0.5759	1.131	1.645	2.104	2.497	2.820	3.070	3.247	3.352	3.387
	y	3.387	3.338	3.193	2.961	2.655	2.288	1.876	1.432	0.9652	0.4857	0
0.6	x	0	0.8981	1.749	2.514	3.168	3.702	4.117	4.424	4.632	4.752	4.791
	y	5.157	5.067	4.809	4.405	3.892	3.302	2.667	2.008	1.340	0.6702	0
0.7	x	0	1.536	2.932	4.093	4.987	5.630	6.070	6.354	6.525	6.616	6.644
	y	8.056	7.860	7.312	6.506	5.556	4.555	3.566	2.616	1.713	0.8463	0
0.8	x	0	3.199	5.762	7.443	8.365	8.779	8.908	8.902	8.853	8.808	8.790
	y	13.59	12.97	11.39	9.374	7.384	5.627	4.146	2.904	1.841	0.8925	0
0.9	x	0	10.81	15.27	15.53	14.50	13.33	12.34	11.59	11.08	10.78	10.69
	y	29.22	24.89	17.13	11.11	7.265	4.850	3.260	2.147	1.308	0.6205	0
1.0	x	∞	73.21	37.09	25.23	19.50	16.21	14.17	12.87	12.06	11.62	11.47
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

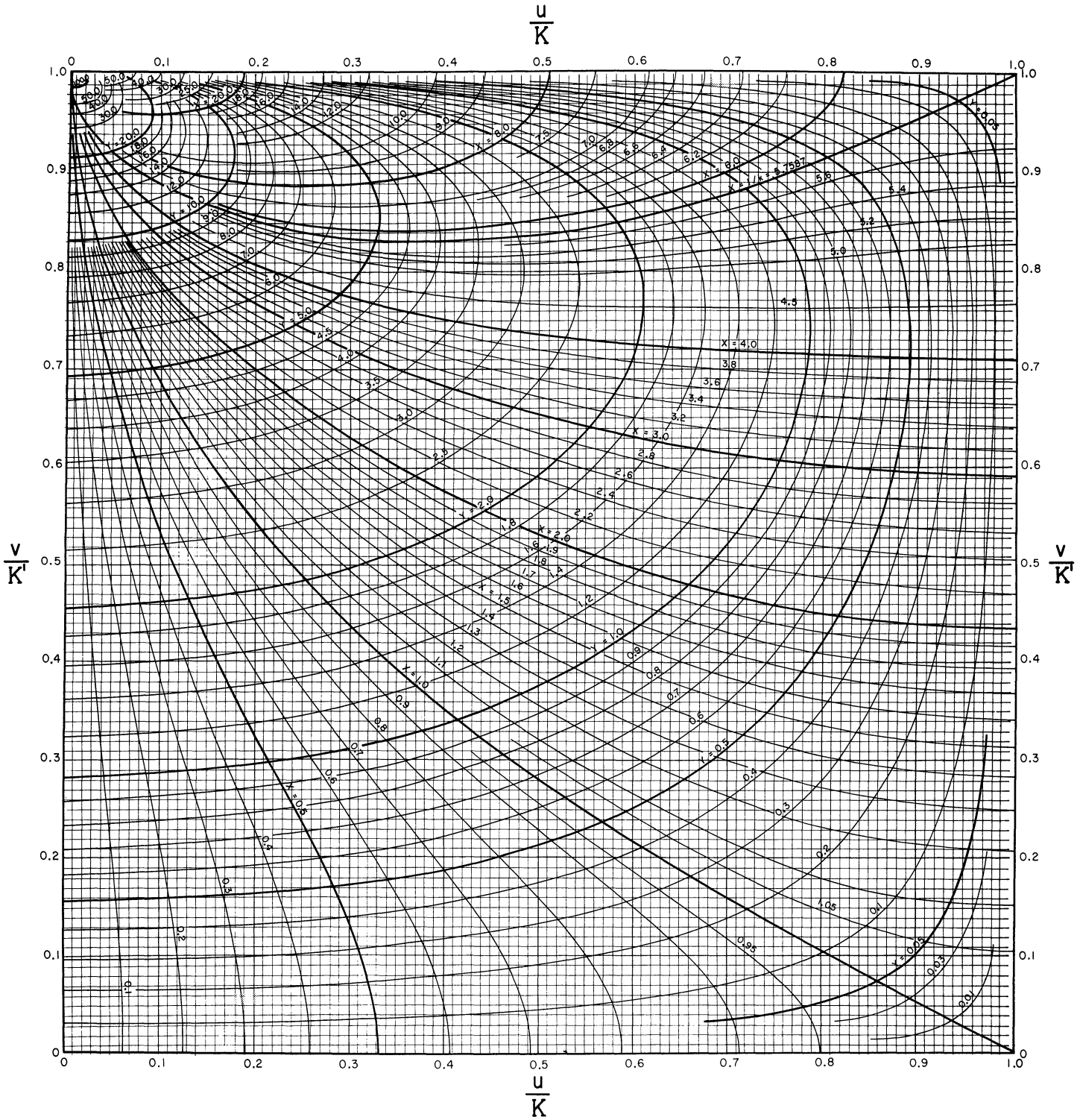
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 5° = 0.0872$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 10° = 0.173648; K = 1.582843, K' = 3.153385

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1576	0.3112	0.4568	0.5907	0.7098	0.8112	0.8924	0.9517	0.9879	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1658	0.3272	0.4801	0.6207	0.7454	0.8514	0.9363	0.9983	1.036	1.049
	y	0.3208	0.3166	0.3043	0.2843	0.2572	0.2239	0.1853	0.1426	0.09681	0.04892	0
0.2	x	0	0.1914	0.3775	0.5532	0.7142	0.8563	0.9765	1.072	1.142	1.184	1.198
	y	0.6750	0.6661	0.6397	0.5968	0.5392	0.4686	0.3873	0.2976	0.2018	0.1019	0
0.3	x	0	0.2385	0.4696	0.6865	0.8833	1.055	1.200	1.313	1.395	1.444	1.461
	y	1.101	1.086	1.041	0.9691	0.8727	0.7558	0.6226	0.4770	0.3226	0.1626	0
0.4	x	0	0.3157	0.6196	0.9013	1.153	1.368	1.545	1.681	1.778	1.836	1.855
	y	1.650	1.626	1.554	1.439	1.288	1.108	0.9065	0.6905	0.4649	0.2337	0
0.5	x	0	0.4420	0.8619	1.241	1.569	1.838	2.050	2.208	2.316	2.379	2.400
	y	2.400	2.359	2.240	2.054	1.816	1.543	1.247	0.9398	0.6276	0.3139	0
0.6	x	0	0.6630	1.276	1.801	2.223	2.543	2.773	2.931	3.032	3.087	3.105
	y	3.490	3.414	3.198	2.875	2.483	2.059	1.627	1.204	0.7926	0.3930	0
0.7	x	0	1.111	2.073	2.803	3.299	3.607	3.782	3.874	3.918	3.937	3.942
	y	5.230	5.059	4.596	3.957	3.260	2.583	1.963	1.407	0.9060	0.4433	0
0.8	x	0	2.295	3.940	4.810	5.136	5.176	5.096	4.987	4.891	4.828	4.806
	y	8.532	7.987	6.677	5.190	3.878	2.832	2.021	1.384	0.8642	0.4154	0
0.9	x	0	7.474	9.437	8.881	7.919	7.088	6.458	6.012	5.716	5.547	5.492
	y	17.95	14.28	8.777	5.258	3.281	2.129	1.406	0.9158	0.5543	0.2619	0
1.0	x	∞	36.54	18.51	12.61	9.749	8.113	7.100	6.453	6.051	5.830	5.759
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

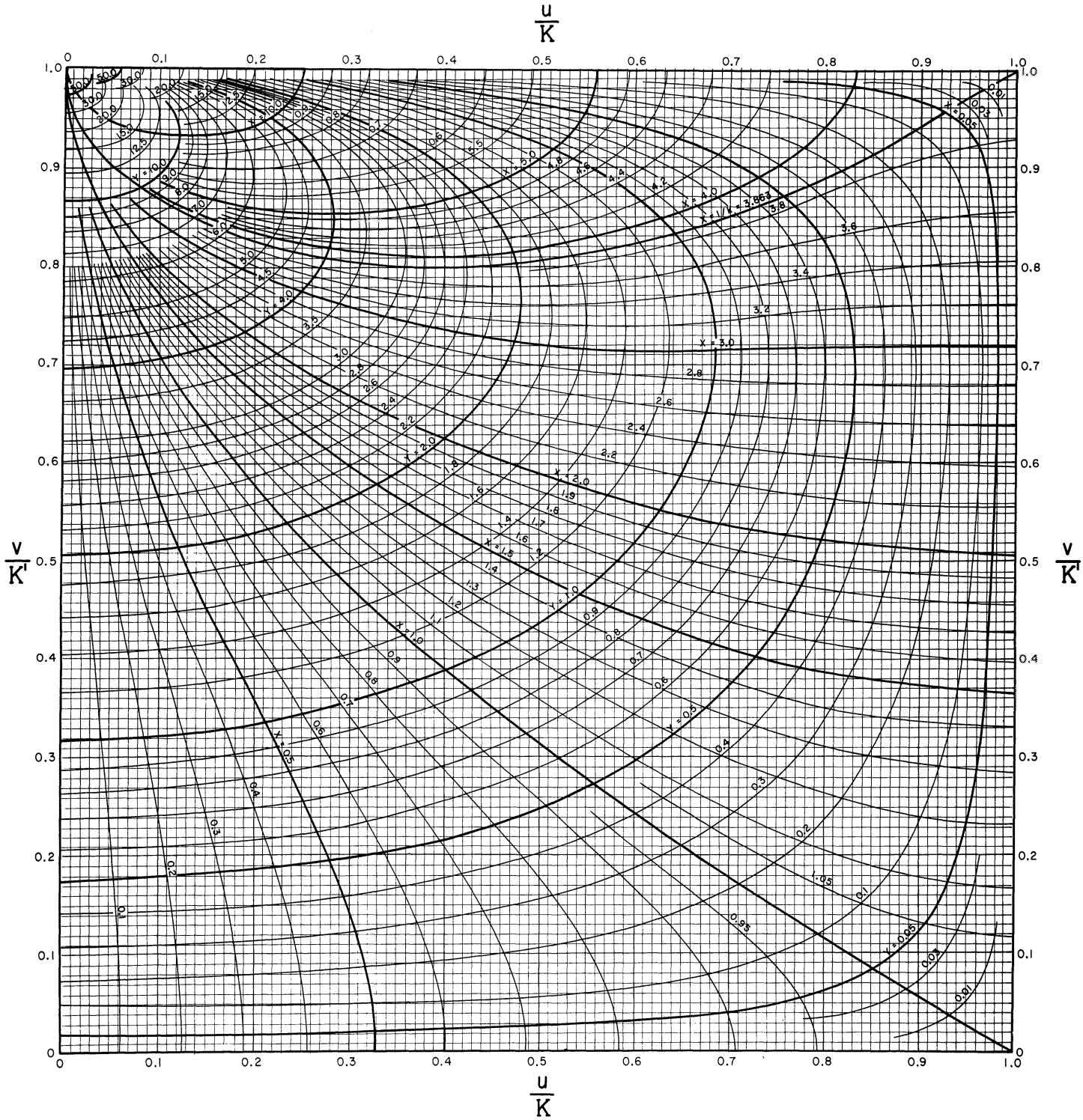
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 10° = 0.1737$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 15° = 0.258819; K = 1.598142, K' = 2.768063

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1591	0.3139	0.4602	0.5944	0.7132	0.8138	0.8942	0.9526	0.9881	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1656	0.3267	0.4787	0.6179	0.7407	0.8445	0.9272	0.9873	1.024	1.036
	y	0.2806	0.2768	0.2654	0.2471	0.2225	0.1928	0.1588	0.1217	0.08233	0.04151	0
0.2	x	0	0.1863	0.3669	0.5366	0.6908	0.8260	0.9392	1.029	1.093	1.132	1.145
	y	0.5847	0.5764	0.5520	0.5129	0.4608	0.3981	0.3272	0.2501	0.1688	0.08503	0
0.3	x	0	0.2243	0.4406	0.6418	0.8221	0.9776	1.106	1.206	1.277	1.319	1.333
	y	0.9393	0.9252	0.8837	0.8177	0.7311	0.6282	0.5135	0.3907	0.2628	0.1320	0
0.4	x	0	0.2871	0.5612	0.8113	1.030	1.212	1.359	1.469	1.546	1.591	1.605
	y	1.381	1.358	1.290	1.185	1.050	0.8932	0.7232	0.5457	0.3648	0.1825	0
0.5	x	0	0.3911	0.7573	1.080	1.348	1.560	1.719	1.833	1.909	1.952	1.966
	y	1.966	1.926	1.814	1.643	1.431	1.197	0.9533	0.7095	0.4693	0.2333	0
0.6	x	0	0.5761	1.095	1.520	1.840	2.066	2.216	2.312	2.368	2.398	2.407
	y	2.798	2.724	2.518	2.220	1.876	1.522	1.180	0.8588	0.5589	0.2752	0
0.7	x	0	0.9563	1.746	2.295	2.625	2.798	2.875	2.901	2.904	2.901	2.899
	y	4.114	3.945	3.502	2.924	2.334	1.798	1.335	0.9401	0.5979	0.2904	0
0.8	x	0	1.962	3.227	3.763	3.871	3.797	3.669	3.547	3.452	3.394	3.374
	y	6.609	6.070	4.855	3.597	2.582	1.830	1.278	0.8623	0.5332	0.2549	0
0.9	x	0	6.154	7.127	6.375	5.537	4.887	4.420	4.097	3.888	3.769	3.731
	y	13.77	10.28	5.790	3.290	1.994	1.272	0.8313	0.5379	0.3242	0.1528	0
1.0	x	∞	24.29	12.31	8.395	6.500	5.417	4.748	4.321	4.056	3.910	3.864
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

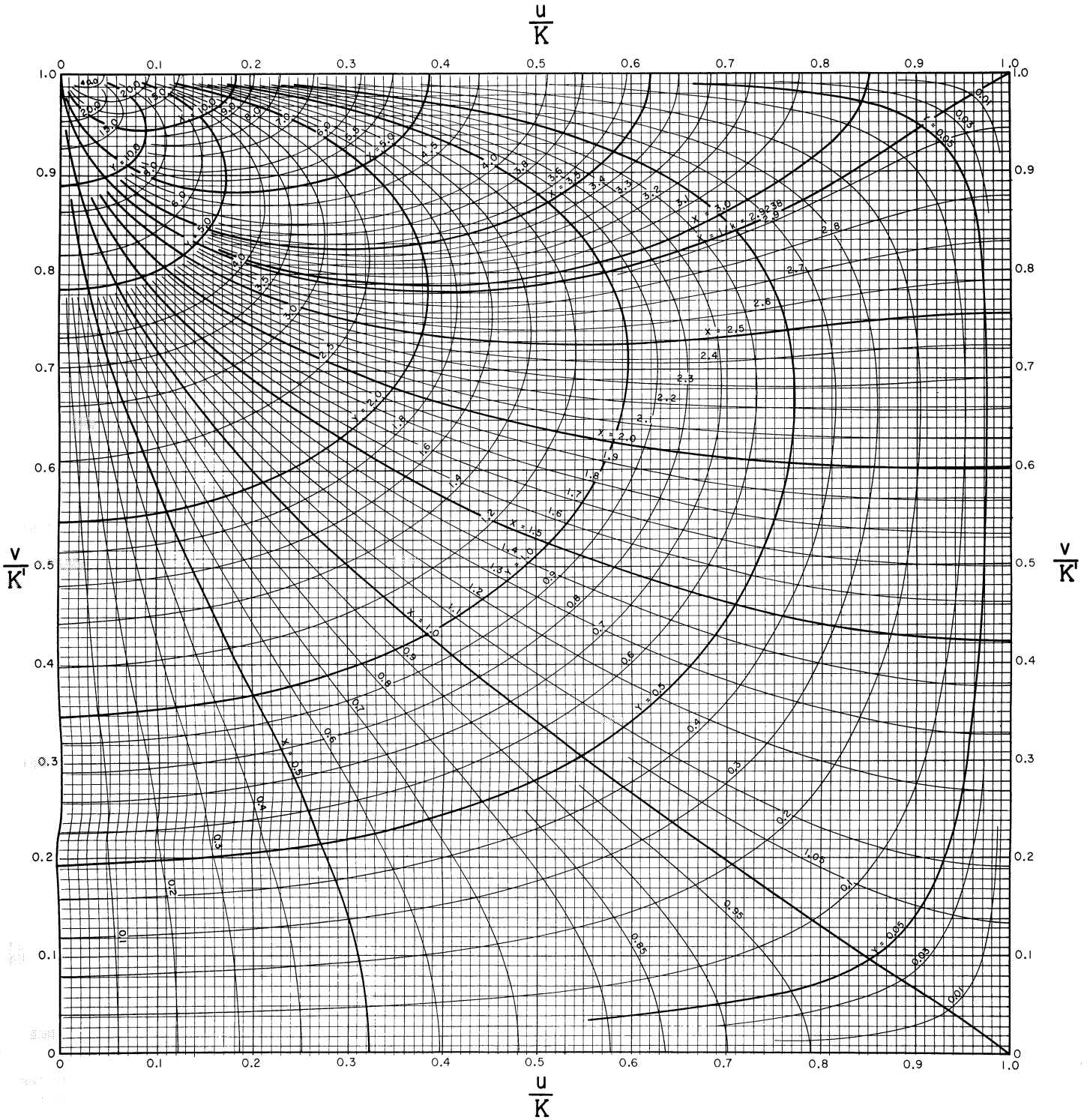
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 15° = 0.2588$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 20° = 0.342020; K = 1.620026, K' = 2.504550

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1612	0.3178	0.4652	0.5997	0.7180	0.8176	0.8966	0.9538	0.9884	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1669	0.3288	0.4809	0.6193	0.7406	0.8424	0.9229	0.9810	1.016	1.028
	y	0.2534	0.2497	0.2387	0.2211	0.1979	0.1703	0.1394	0.1062	0.07146	0.03592	0
0.2	x	0	0.1848	0.3634	0.5301	0.6803	0.8105	0.9184	1.003	1.063	1.100	1.112
	y	0.5250	0.5169	0.4933	0.4557	0.4065	0.3485	0.2841	0.2157	0.1448	0.07268	0
0.3	x	0	0.2181	0.4273	0.6198	0.7899	0.9341	1.051	1.141	1.204	1.241	1.253
	y	0.8362	0.8222	0.7817	0.7180	0.6362	0.5414	0.4383	0.3307	0.2210	0.1106	0
0.4	x	0	0.2736	0.5324	0.7645	0.9626	1.124	1.249	1.342	1.405	1.442	1.454
	y	1.216	1.193	1.126	1.024	0.8965	0.7532	0.6025	0.4499	0.2983	0.1485	0
0.5	x	0	0.3667	0.7048	0.9940	1.225	1.401	1.527	1.614	1.669	1.700	1.710
	y	1.710	1.670	1.558	1.391	1.193	0.8908	0.7694	0.5652	0.3703	0.1830	0
0.6	x	0	0.5343	1.003	1.369	1.627	1.795	1.899	1.959	1.991	2.007	2.012
	y	2.405	2.329	2.122	1.833	1.515	1.203	0.9155	0.6565	0.4227	0.2067	0
0.7	x	0	0.8813	1.574	2.014	2.245	2.343	2.370	2.365	2.350	2.338	2.333
	y	3.497	3.322	2.880	2.334	1.808	1.358	0.9882	0.6855	0.4314	0.2083	0
0.8	x	0	1.793	2.831	3.172	3.167	3.045	2.906	2.787	2.700	2.647	2.630
	y	5.569	5.015	3.842	2.726	1.892	1.309	0.8989	0.5993	0.3677	0.1750	0
0.9	x	0	5.411	5.825	5.019	4.284	3.750	3.379	3.127	2.965	2.874	2.845
	y	11.54	8.095	4.228	2.307	1.369	0.8623	0.5590	0.3597	0.2160	0.1016	0
1.0	x	∞	18.14	9.202	6.285	4.875	4.072	3.576	3.261	3.065	2.958	2.924
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

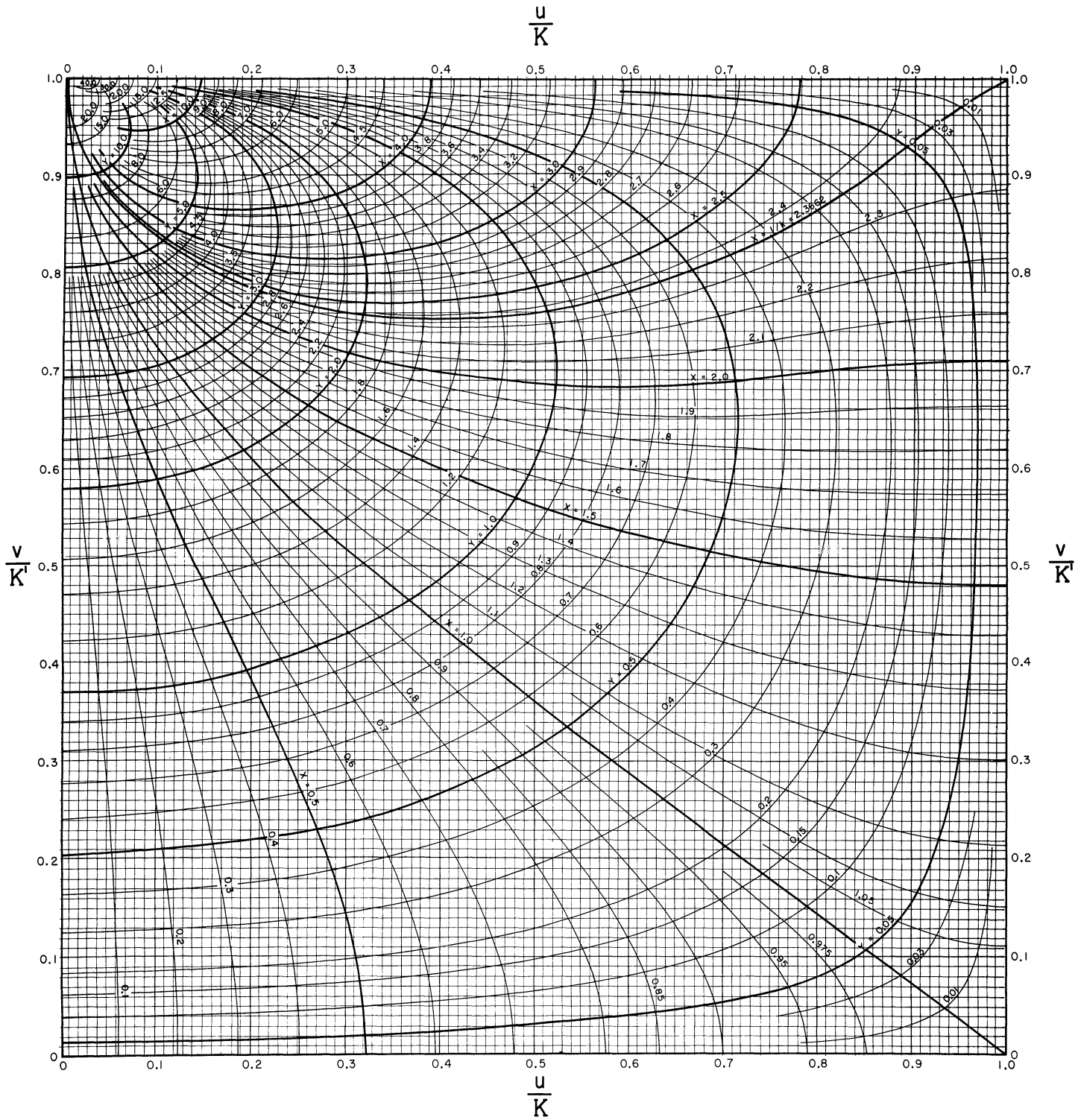
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 20° = 0.3420$

THE sn-FUNCTION, $x + iy = sn(u + iv)$

$k = \sin 25° = 0.422618$; $K = 1.648995$, $K' = 2.308787$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1640	0.3229	0.4717	0.6066	0.7243	0.8225	0.8998	0.9554	0.9888	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1692	0.3328	0.4857	0.6237	0.7435	0.8431	0.9212	0.9771	1.011	1.022
	y	0.2333	0.2295	0.2185	0.2012	0.1786	0.1524	0.1236	0.09341	0.06249	0.03129	0
0.2	x	0	0.1856	0.3642	0.5295	0.6768	0.8030	0.9063	0.9862	1.043	1.076	1.088
	y	0.4817	0.4735	0.4497	0.4124	0.3646	0.3095	0.2499	0.1881	0.1254	0.06268	0
0.3	x	0	0.2162	0.4222	0.6094	0.7722	0.9077	1.016	1.097	1.153	1.186	1.197
	y	0.7631	0.7489	0.7078	0.6444	0.5648	0.4750	0.3802	0.2840	0.1883	0.09377	0
0.4	x	0	0.2677	0.5183	0.7387	0.9222	1.067	1.177	1.256	1.309	1.339	1.349
	y	1.102	1.078	1.010	0.9083	0.7842	0.6495	0.5125	0.3783	0.2486	0.1231	0
0.5	x	0	0.3549	0.6767	0.9436	1.149	1.297	1.400	1.467	1.509	1.531	1.538
	y	1.538	1.497	1.381	1.215	1.023	0.8265	0.6380	0.4625	0.3000	0.1474	0
0.6	x	0	0.5133	0.9511	1.275	1.489	1.618	1.690	1.727	1.745	1.752	1.754
	y	2.147	2.067	1.854	1.568	1.266	0.9843	0.7354	0.5199	0.3313	0.1610	0
0.7	x	0	0.8420	1.471	1.833	1.996	2.046	2.043	2.022	1.998	1.982	1.976
	y	3.101	2.917	2.466	1.938	1.460	1.071	0.7650	0.5235	0.3264	0.1567	0
0.8	x	0	1.698	2.573	2.781	2.710	2.566	2.426	2.315	2.237	2.191	2.175
	y	4.912	4.332	3.178	2.166	1.460	0.9889	0.6691	0.4416	0.2691	0.1275	0
0.9	x	0	4.924	4.968	4.158	3.506	3.054	2.747	2.543	2.412	2.339	2.315
	y	10.14	6.680	3.262	1.722	1.004	0.6259	0.4028	0.2578	0.1542	0.07240	0
1.0	x	∞	14.43	7.329	5.016	3.901	3.267	2.877	2.630	2.477	2.393	2.366
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

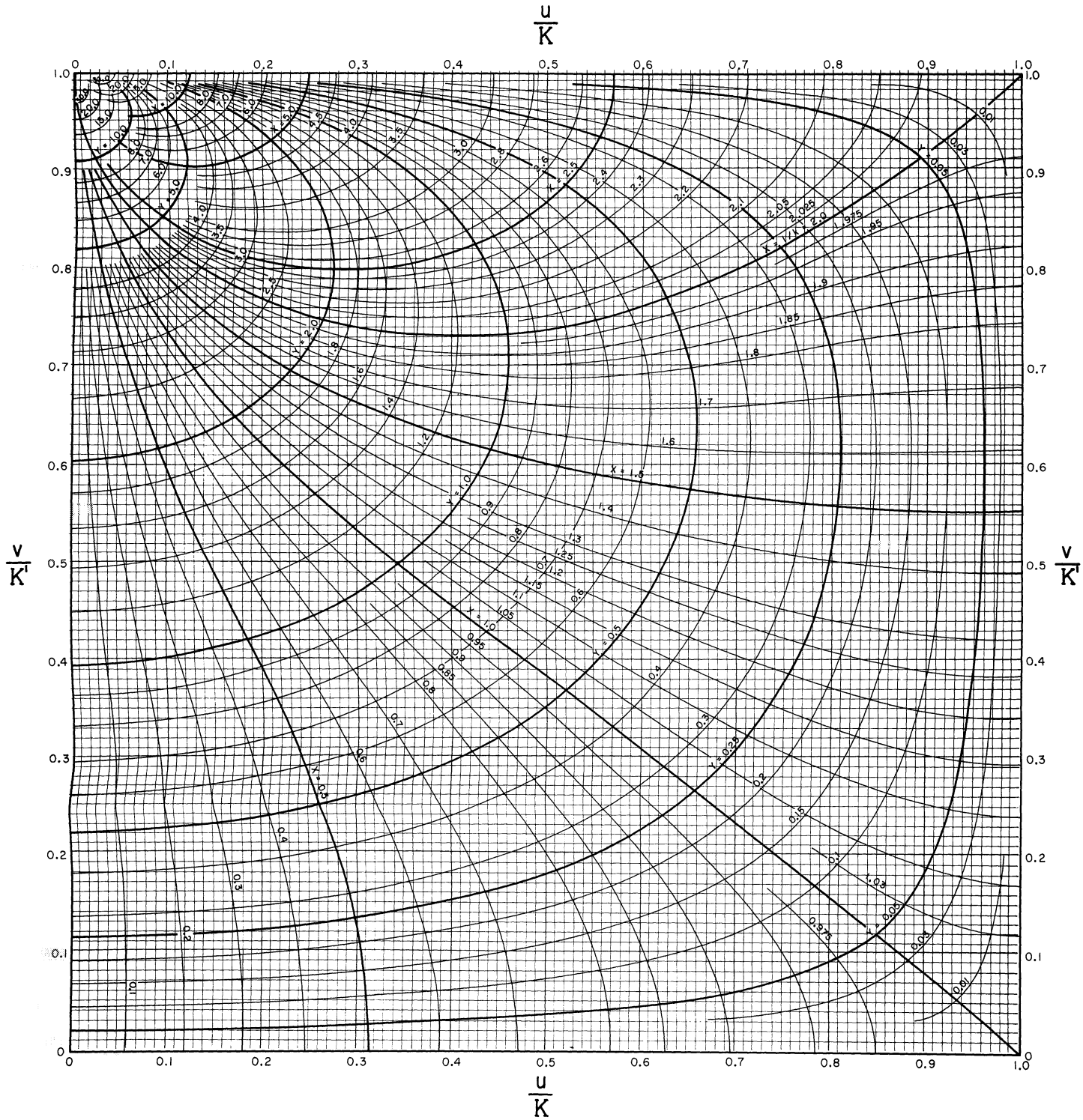
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 25° = 0.4226$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 30° = 0.500000; K = 1.685750, K' = 2.156516

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1676	0.3293	0.4800	0.6152	0.7321	0.8285	0.9037	0.9573	0.9893	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1725	0.3386	0.4928	0.6305	0.7489	0.8461	0.9214	0.9749	1.007	1.017
	y	0.2178	0.2139	0.2025	0.1849	0.1626	0.1372	0.1101	0.08238	0.05469	0.02725	0
0.2	x	0	0.1880	0.3679	0.5329	0.6781	0.8007	0.8996	0.9750	1.028	1.059	1.069
	y	0.4486	0.4401	0.4155	0.3777	0.3302	0.2769	0.2210	0.1646	0.1089	0.05412	0
0.3	x	0	0.2171	0.4223	0.6064	0.7635	0.8917	0.9916	1.066	1.116	1.145	1.155
	y	0.7081	0.6932	0.6506	0.5861	0.5071	0.4207	0.3323	0.2454	0.1613	0.07984	0
0.4	x	0	0.2664	0.5130	0.7253	0.8972	1.029	1.126	1.194	1.239	1.264	1.272
	y	1.018	0.9930	0.9223	0.8182	0.6955	0.5668	0.4405	0.3210	0.2089	0.1028	0
0.5	x	0	0.3506	0.6627	0.9130	1.097	1.225	1.308	1.361	1.393	1.409	1.414
	y	1.414	1.370	1.249	1.080	0.8920	0.7071	0.5366	0.3836	0.2462	0.1202	0
0.6	x	0	0.5041	0.9212	1.213	1.392	1.491	1.540	1.562	1.570	1.572	1.573
	y	1.964	1.879	1.656	1.369	1.079	0.8210	0.6023	0.4197	0.2647	0.1279	0
0.7	x	0	0.8228	1.404	1.705	1.818	1.835	1.813	1.782	1.755	1.738	1.732
	y	2.824	2.627	2.163	1.648	1.207	0.8656	0.6076	0.4104	0.2536	0.1211	0
0.8	x	0	1.642	2.389	2.498	2.384	2.231	2.097	1.994	1.924	1.883	1.870
	y	4.458	3.844	2.698	1.770	1.161	0.7716	0.5151	0.3367	0.2038	0.09624	0
0.9	x	0	4.571	4.350	3.558	2.974	2.584	2.325	2.153	2.045	1.985	1.966
	y	9.185	5.667	2.602	1.335	0.7670	0.4733	0.3025	0.1925	0.1147	0.05372	0
1.0	x	∞	11.94	6.073	4.167	3.251	2.732	2.414	2.213	2.089	2.022	2.000
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

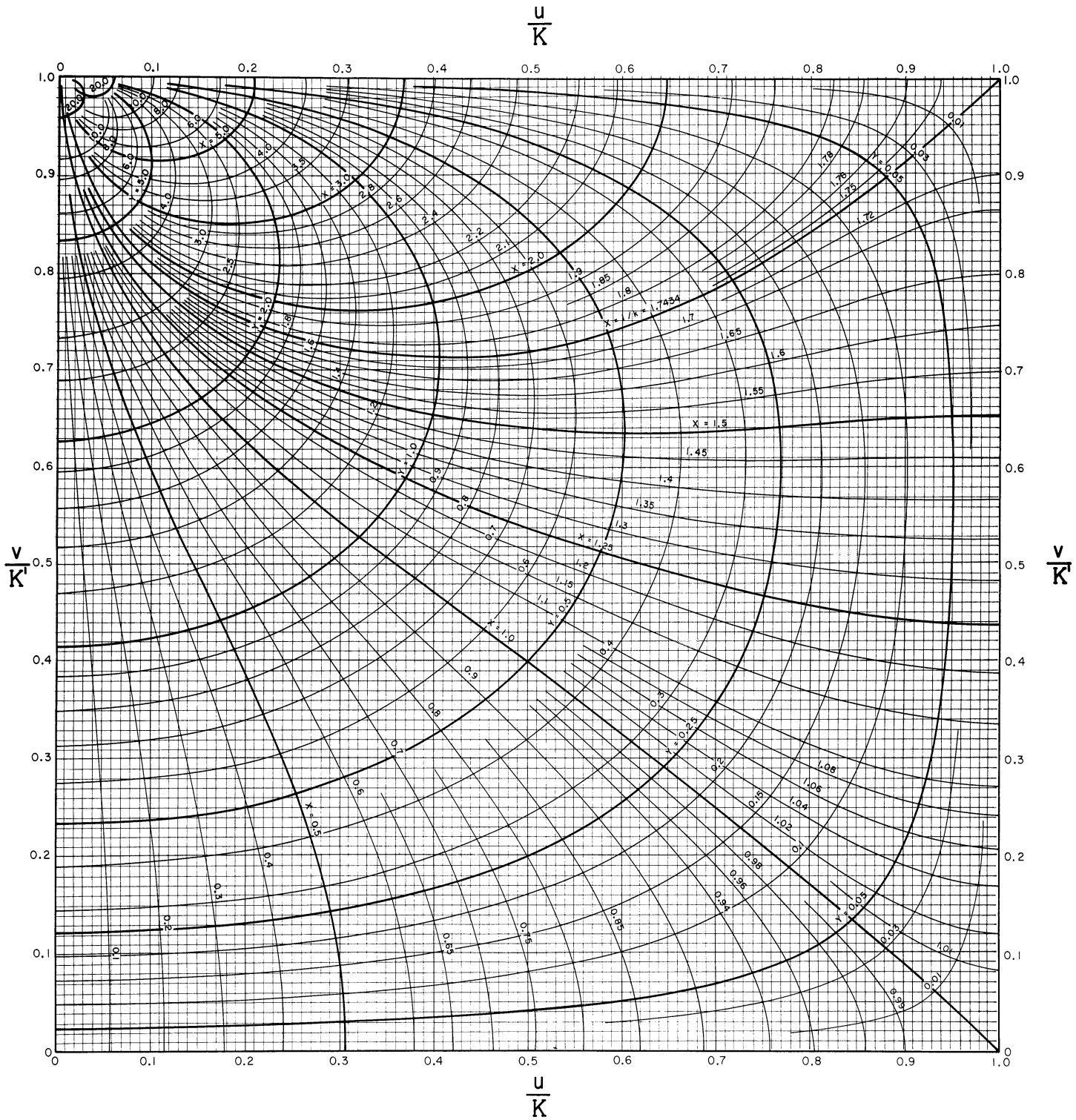
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 30° = 0.5000$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 35° = 0.573576; K = 1.731245, K' = 2.034715

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1720	0.3373	0.4900	0.6257	0.7414	0.8357	0.9083	0.9595	0.9899	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1767	0.3462	0.5020	0.6397	0.7564	0.8508	0.9231	0.9739	1.004	1.014
	y	0.2054	0.2012	0.1894	0.1712	0.1487	0.1238	0.09802	0.07249	0.04767	0.02361	0
0.2	x	0	0.1918	0.3742	0.5397	0.6832	0.8023	0.8967	0.9677	1.017	1.046	1.055
	y	0.4224	0.4133	0.3876	0.3485	0.3007	0.2485	0.1956	0.1438	0.09423	0.04655	0
0.3	x	0	0.2201	0.4265	0.6087	0.7610	0.8825	0.9752	1.042	1.088	1.114	1.122
	y	0.6652	0.6493	0.6044	0.5377	0.4582	0.3741	0.2910	0.2120	0.1379	0.06784	0
0.4	x	0	0.2685	0.5138	0.7201	0.8822	1.003	1.089	1.147	1.185	1.206	1.212
	y	0.9537	0.9266	0.8518	0.7443	0.6215	0.4974	0.3802	0.2731	0.1758	0.08594	0
0.5	x	0	0.3514	0.6579	0.8949	1.062	1.171	1.240	1.281	1.305	1.317	1.320
	y	1.320	1.273	1.145	0.9709	0.7851	0.6097	0.4543	0.3200	0.2032	0.09851	0
0.6	x	0	0.5029	0.9052	1.171	1.321	1.395	1.427	1.438	1.440	1.439	1.438
	y	1.828	1.736	1.501	1.210	0.9305	0.6920	0.4983	0.3422	0.2136	0.1025	0
0.7	x	0	0.8164	1.359	1.609	1.681	1.675	1.642	1.606	1.578	1.560	1.554
	y	2.621	2.408	1.926	1.421	1.012	0.7099	0.4899	0.3267	0.2000	0.09502	0
0.8	x	0	1.610	2.248	2.280	2.138	1.983	1.856	1.763	1.700	1.664	1.653
	y	4.127	3.470	2.328	1.472	0.9408	0.6143	0.4048	0.2620	0.1575	0.07409	0
0.9	x	0	4.296	3.877	3.111	2.586	2.245	2.022	1.877	1.786	1.736	1.720
	y	8.490	4.892	2.121	1.061	0.6011	0.3674	0.2330	0.1474	0.08743	0.04082	0
1.0	x	∞	10.14	5.169	3.558	2.786	2.352	2.086	1.919	1.817	1.761	1.743
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

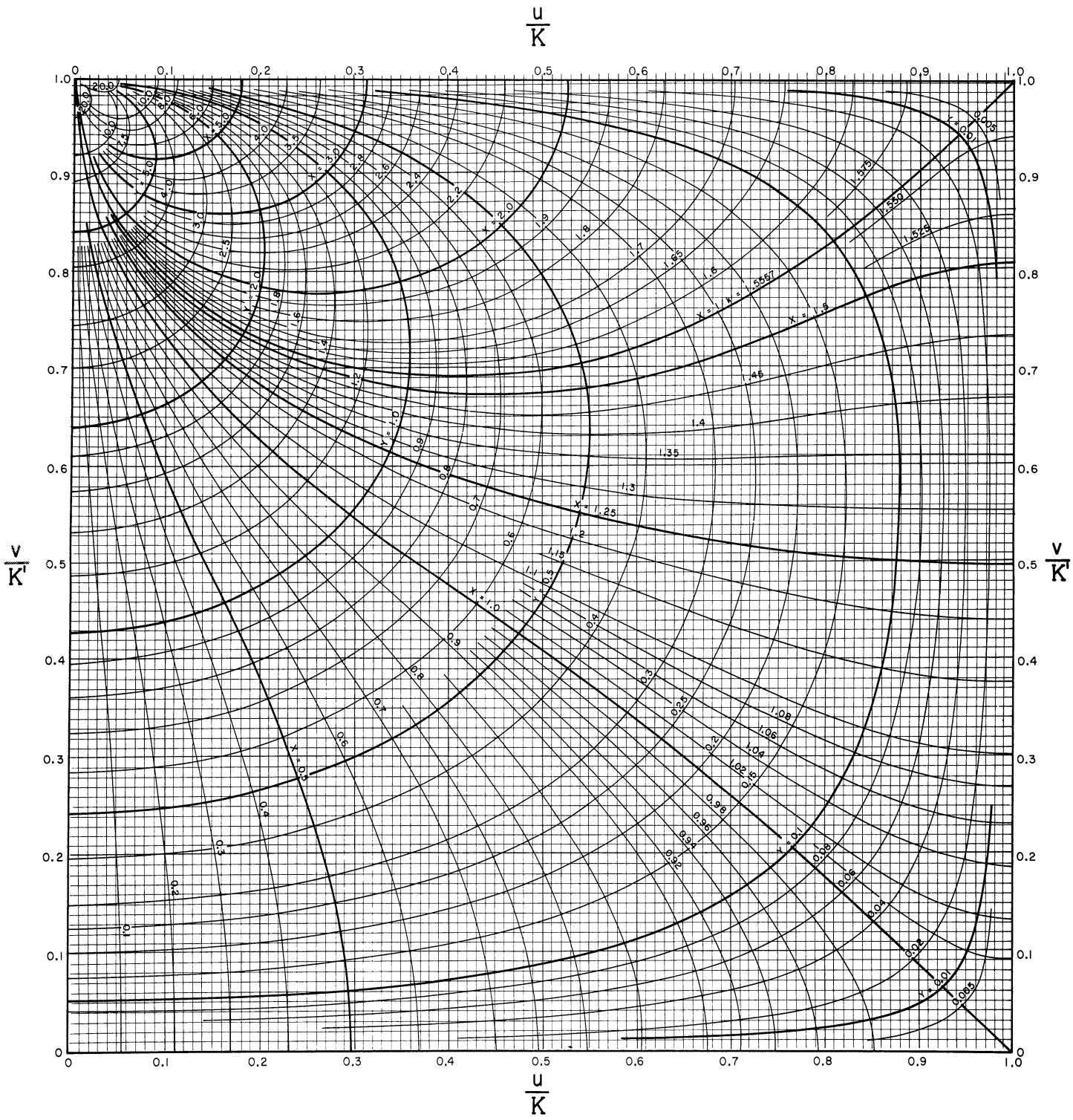
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 35° = 0.5736$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 40° = 0.642788; K = 1.786769, K' = 1.935581

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1773	0.3469	0.5021	0.6382	0.7525	0.8442	0.9137	0.9621	0.9906	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1820	0.3556	0.5136	0.6512	0.7659	0.8572	0.9261	0.9738	1.002	1.011
	y	0.1953	0.1908	0.1782	0.1592	0.1362	0.1116	0.08696	0.06340	0.04124	0.02028	0
0.2	x	0	0.1969	0.3830	0.5495	0.6914	0.8070	0.8969	0.9632	1.009	1.035	1.043
	y	0.4013	0.3916	0.3640	0.3231	0.2743	0.2229	0.1725	0.1251	0.08099	0.03972	0
0.3	x	0	0.2251	0.4341	0.6152	0.7632	0.8784	0.9641	1.025	1.065	1.088	1.096
	y	0.6310	0.6138	0.5657	0.4959	0.4152	0.3327	0.2543	0.1825	0.1173	0.05729	0
0.4	x	0	0.2733	0.5194	0.7209	0.8742	0.9844	1.060	1.111	1.143	1.160	1.165
	y	0.9028	0.8735	0.7933	0.6811	0.5574	0.4371	0.3278	0.2318	0.1475	0.07155	0
0.5	x	0	0.3562	0.6598	0.8854	1.036	1.130	1.186	1.218	1.236	1.245	1.247
	y	1.247	1.195	1.058	0.8785	0.6940	0.5271	0.3852	0.2670	0.1675	0.08065	0
0.6	x	0	0.5077	0.8988	1.140	1.265	1.320	1.339	1.342	1.339	1.336	1.335
	y	1.723	1.622	1.373	1.077	0.8067	0.5861	0.4140	0.2800	0.1729	0.08244	0
0.7	x	0	0.8195	1.328	1.533	1.573	1.549	1.509	1.471	1.443	1.426	1.420
	y	2.465	2.234	1.731	1.236	0.8557	0.5868	0.3979	0.2619	0.1589	0.07503	0
0.8	x	0	1.595	2.134	2.104	1.944	1.791	1.673	1.588	1.533	1.501	1.491
	y	3.877	3.171	2.028	1.237	0.7713	0.4948	0.3218	0.2062	0.1231	0.05763	0
0.9	x	0	4.072	2.497	2.764	2.289	1.989	1.796	1.672	1.595	1.552	1.539
	y	7.967	4.269	1.753	0.8567	0.4789	0.2898	0.1822	0.1145	0.06753	0.03142	0
1.0	x	∞	8.773	4.484	3.098	2.438	2.067	1.843	1.703	1.617	1.570	1.556
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

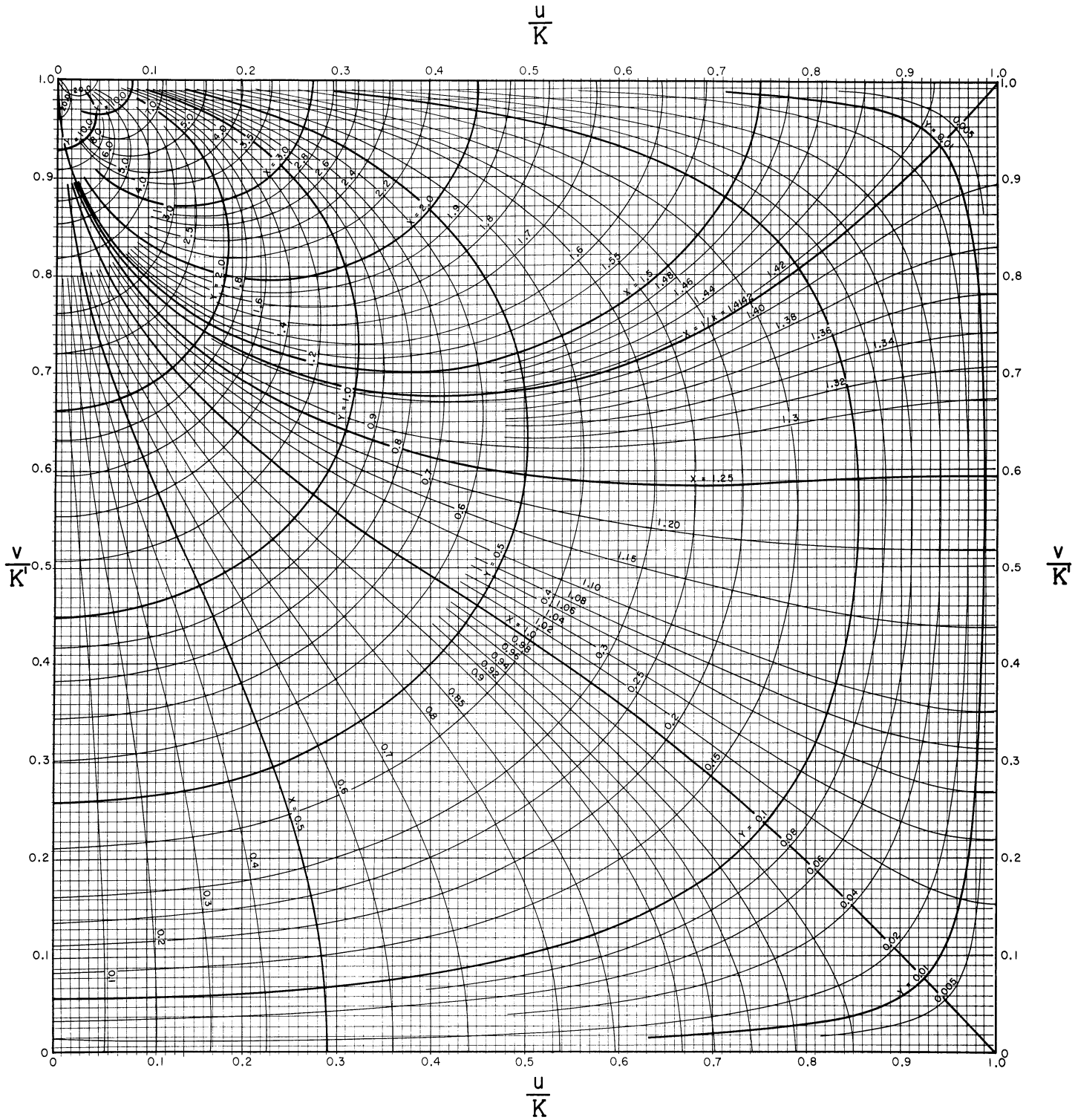
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 40° = 0.6428$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 45° = 0.707107; K = K' = 1.854075

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1838	0.3585	0.5166	0.6529	0.7654	0.8539	0.9198	0.9650	0.9914	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1885	0.3671	0.5276	0.6651	0.7775	0.8652	0.9301	0.9745	1.000	1.009
	y	0.1870	0.1822	0.1685	0.1484	0.1247	0.1002	0.07661	0.05493	0.03525	0.01719	0
0.2	x	0	0.2035	0.3942	0.5623	0.7027	0.8144	0.8995	0.9610	1.002	1.026	1.034
	y	0.3841	0.3734	0.3436	0.3002	0.2502	0.1992	0.1512	0.1077	0.06885	0.03348	0
0.3	x	0	0.2320	0.4448	0.6255	0.7693	0.8782	0.9572	1.012	1.048	1.068	1.074
	y	0.6033	0.5844	0.5323	0.4586	0.3761	0.2950	0.2210	0.1558	0.09883	0.03785	0
0.4	x	0	0.2807	0.5291	0.7266	0.8715	0.9719	1.039	1.082	1.109	1.123	1.127
	y	0.8620	0.8297	0.7429	0.6251	0.5000	0.3831	0.2814	0.1955	0.1227	0.05904	0
0.5	x	0	0.3645	0.6672	0.8822	1.018	1.099	1.144	1.168	1.181	1.187	1.189
	y	1.189	1.132	0.9844	0.7974	0.6139	0.4551	0.3256	0.2218	0.1374	0.06560	0
0.6	x	0	0.5174	0.8995	1.118	1.221	1.259	1.268	1.266	1.260	1.256	1.254
	y	1.641	1.529	1.263	0.9622	0.7004	0.4965	0.3436	0.2286	0.1395	0.06604	0
0.7	x	0	0.8299	1.307	1.471	1.484	1.447	1.403	1.365	1.338	1.322	1.317
	y	2.344	2.091	1.564	1.078	0.7254	0.4862	0.3238	0.2101	0.1262	0.05923	0
0.8	x	0	1.592	2.039	1.957	1.786	1.638	1.529	1.453	1.404	1.377	1.368
	y	3.682	2.920	1.777	1.045	0.6360	0.4009	0.2571	0.1629	0.09645	0.04493	0
0.9	x	0	3.880	3.182	2.484	2.054	1.789	1.622	1.515	1.449	1.414	1.402
	y	7.562	3.749	1.461	0.6985	0.3852	0.2306	0.1436	0.08947	0.05244	0.02430	0
1.0	x	∞	7.693	3.944	2.738	2.166	1.848	1.656	1.538	1.465	1.427	1.414
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

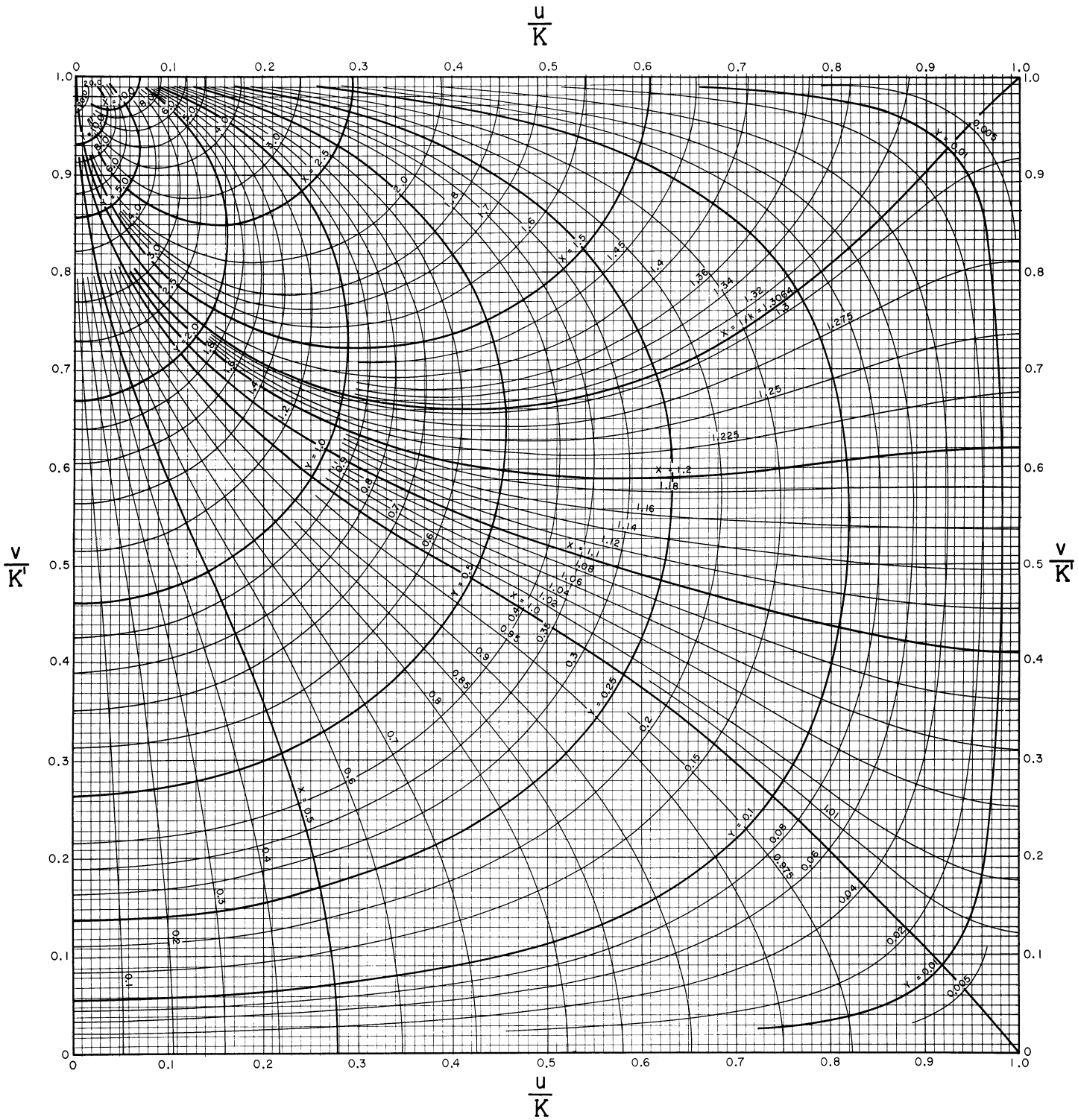
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 45° = 0.7071$

THE sn-FUNCTION, $x + iy = sn(u + iv)$

$k = \sin 50° = 0.766044$; $K = 1.935581$, $K' = 1.786769$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.1917	0.3724	0.5336	0.6701	0.7802	0.8649	0.9267	0.9683	0.9922	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.1965	0.3810	0.5444	0.6815	0.7911	0.8747	0.9352	0.9758	0.9990	1.007
	y	0.1802	0.1748	0.1599	0.1383	0.1138	0.08933	0.06679	0.04694	0.02966	0.01432	0
0.2	x	0	0.2118	0.4082	0.5782	0.7168	0.8243	0.9042	0.9607	0.9979	1.019	1.026
	y	0.3699	0.3581	0.3254	0.2791	0.2274	0.1769	0.1312	0.09161	0.05763	0.02774	0
0.3	x	0	0.2408	0.4588	0.6393	0.7789	0.8813	0.9535	1.002	1.034	1.051	1.057
	y	0.5806	0.5596	0.5027	0.4242	0.3397	0.2599	0.1901	0.1314	0.08205	0.03933	0
0.4	x	0	0.2906	0.5427	0.7365	0.8730	0.9639	1.022	1.059	1.081	1.093	1.097
	y	0.8290	0.7930	0.6983	0.5739	0.4471	0.3338	0.2394	0.1630	0.1008	0.04803	0
0.5	x	0	0.3761	0.6793	0.8839	1.006	1.074	1.109	1.128	1.137	1.141	1.143
	y	1.143	1.079	0.9187	0.7239	0.5415	0.3908	0.2731	0.1824	0.1114	0.05272	0
0.6	x	0	0.5318	0.9058	1.103	1.185	1.209	1.210	1.204	1.197	1.192	1.191
	y	1.575	1.451	1.165	0.8593	0.6067	0.4188	0.2835	0.1853	0.1116	0.05239	0
0.7	x	0	0.8469	1.293	1.418	1.408	1.363	1.317	1.280	1.255	1.240	1.236
	y	2.248	1.968	1.417	0.9407	0.6141	0.4019	0.2626	0.1678	0.09961	0.04642	0
0.8	x	0	1.598	1.955	1.831	1.655	1.514	1.414	1.347	1.304	1.280	1.273
	y	3.529	2.701	1.559	0.8840	0.5250	0.3248	0.2051	0.1284	0.07529	0.03485	0
0.9	x	0	3.708	2.913	2.252	1.863	1.629	1.484	1.392	1.337	1.306	1.297
	y	7.244	3.300	1.222	0.5723	0.3112	0.1840	0.1133	0.06988	0.04062	0.01872	0
1.0	x	∞	6.811	3.505	2.446	1.948	1.673	1.509	1.409	1.348	1.316	1.305
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

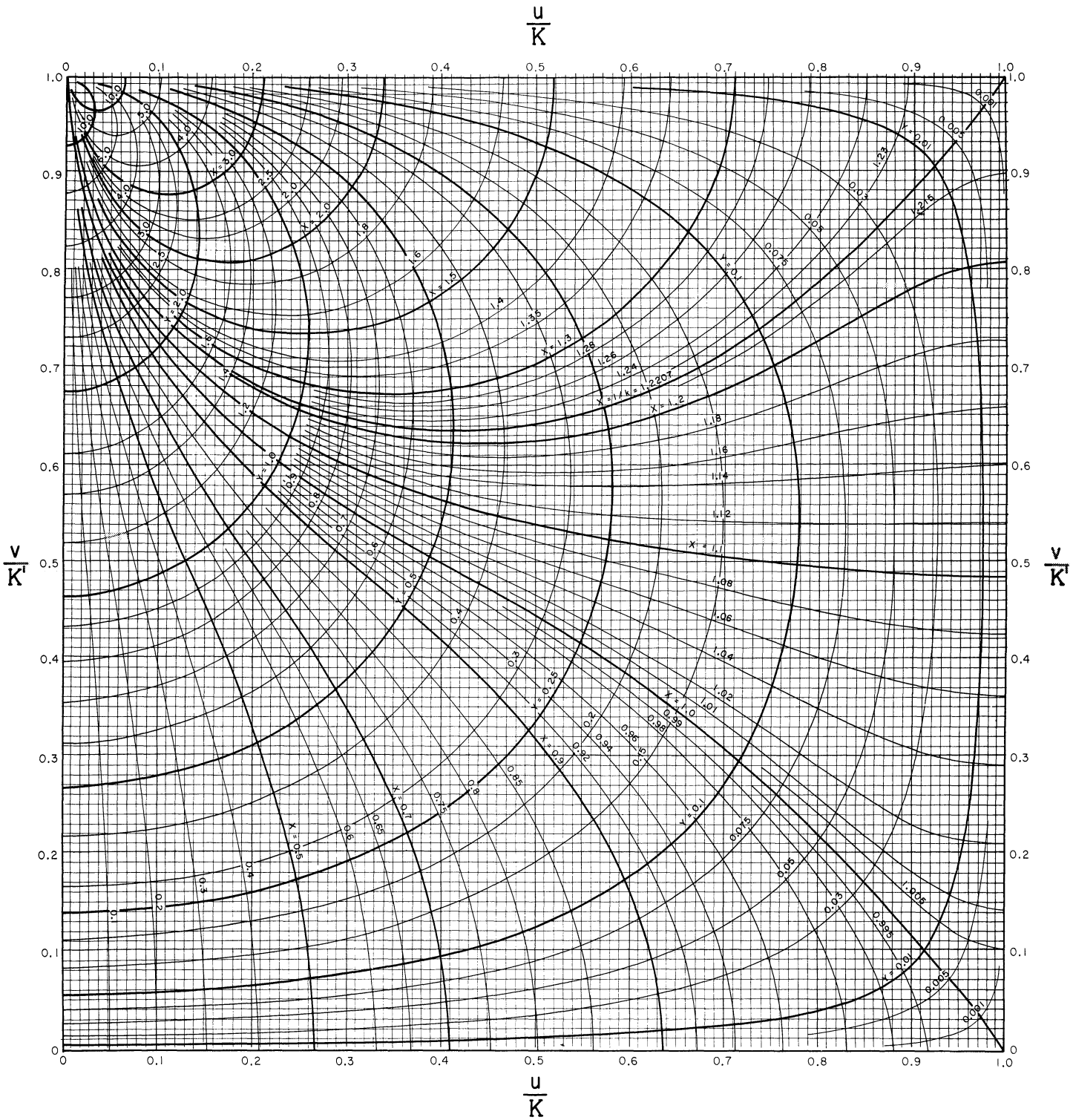
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 50° = 0.7660$

THE sn-FUNCTION, $x + iy = sn(u + iv)$

$k = \sin 55° = 0.819152$; $K = 2.034715$, $K' = 1.731245$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.2012	0.3891	0.5539	0.6901	0.7972	0.8773	0.9343	0.9719	0.9931	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.2061	0.3979	0.5645	0.7010	0.8071	0.8859	0.9414	0.9779	0.9985	1.005
	y	0.1746	0.1686	0.1520	0.1287	0.1032	0.07881	0.05736	0.03936	0.02441	0.01164	0
0.2	x	0	0.2219	0.4252	0.5974	0.7339	0.8367	0.9109	0.9620	0.9950	1.013	1.019
	y	0.3583	0.3450	0.3088	0.2590	0.2054	0.1553	0.1121	0.07646	0.04721	0.02246	0
0.3	x	0	0.2519	0.4762	0.6568	0.7917	0.8872	0.9526	0.9957	1.023	1.037	1.042
	y	0.5621	0.5385	0.4756	0.3916	0.3048	0.2265	0.1613	0.1088	0.06674	0.03163	0
0.4	x	0	0.3033	0.5603	0.7502	0.8781	0.9596	1.010	1.041	1.059	1.068	1.071
	y	0.8021	0.7617	0.6574	0.5255	0.3972	0.2878	0.2009	0.1337	0.08119	0.03827	0
0.5	x	0	0.3913	0.6958	0.8897	0.9978	1.054	1.082	1.095	1.101	1.104	1.105
	y	1.105	1.033	0.8583	0.6652	0.4744	0.3322	0.2261	0.1478	0.08876	0.04157	0
0.6	x	0	0.5508	0.9168	1.092	1.154	1.167	1.163	1.154	1.146	1.141	1.139
	y	1.522	1.383	1.076	0.7646	0.5220	0.3501	0.2312	0.1482	0.08787	0.04087	0
0.7	x	0	0.8700	1.284	1.371	1.343	1.292	1.246	1.212	1.189	1.176	1.172
	y	2.172	1.860	1.282	0.8176	0.5170	0.3298	0.2109	0.1324	0.07758	0.03585	0
0.8	x	0	1.610	1.880	1.720	1.543	1.410	1.320	1.261	1.224	1.204	1.198
	y	3.407	2.503	1.365	0.7458	0.4318	0.2618	0.1625	0.1002	0.05809	0.02669	0
0.9	x	0	3.550	2.678	2.056	1.705	1.498	1.373	1.295	1.248	1.223	1.215
	y	6.993	2.903	1.023	0.4689	0.2510	0.1463	0.08888	0.05413	0.03115	0.01426	0
1.0	x	∞	6.069	3.137	2.204	1.769	1.531	1.391	1.307	1.256	1.229	1.221
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

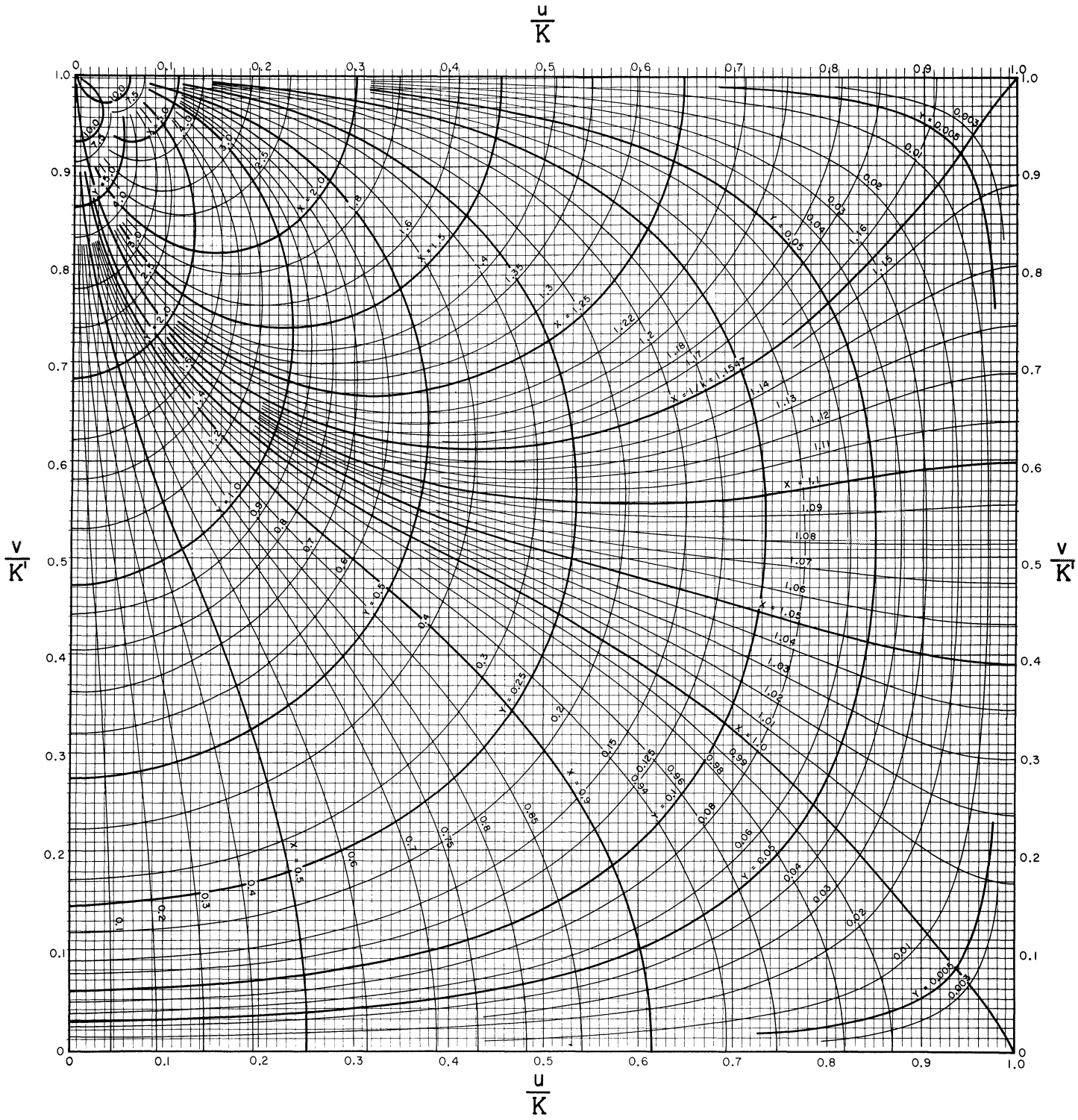
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 55° = 0.8192$

THE sn-FUNCTION, $x + iy = sn(u + iv)$

$k = \sin 60° = 0.866025$; $K = 2.156516$, $K' = 1.685750$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.2128	0.4093	0.5779	0.7134	0.8165	0.8912	0.9427	0.9758	0.9941	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.2179	0.4181	0.5883	0.7235	0.8252	0.8982	0.9482	0.9802	0.9979	1.004
	y	0.1700	0.1631	0.1445	0.1192	0.09264	0.06841	0.04821	0.03215	0.01949	0.009163	0
0.2	x	0	0.2344	0.4460	0.6205	0.7543	0.8515	0.9193	0.9647	0.9932	1.009	1.014
	y	0.3488	0.3336	0.2931	0.2391	0.1836	0.1342	0.09382	0.06219	0.03756	0.01762	0
0.3	x	0	0.2657	0.4976	0.6781	0.8076	0.8958	0.9540	0.9913	1.014	1.026	1.030
	y	0.5471	0.5201	0.4498	0.3596	0.2705	0.1943	0.1339	0.08792	0.05275	0.02465	0
0.4	x	0	0.3192	0.5821	0.7677	0.8862	0.9583	1.001	1.027	1.041	1.049	1.051
	y	0.7804	0.7343	0.6185	0.4784	0.3489	0.2442	0.1652	0.1070	0.06365	0.02961	0
0.5	x	0	0.4106	0.7167	0.8988	0.9929	1.038	1.059	1.068	1.072	1.074	1.075
	y	1.075	0.9931	0.8006	0.5889	0.4109	0.2781	0.1837	0.1171	0.06893	0.03188	0
0.6	x	0	0.5749	0.9318	1.083	1.128	1.132	1.123	1.113	1.105	1.100	1.099
	y	1.480	1.323	0.9900	0.6752	0.4441	0.2884	0.1853	0.1160	0.06755	0.03106	0
0.7	x	0	0.8993	1.277	1.329	1.286	1.231	1.187	1.156	1.136	1.125	1.121
	y	2.111	1.761	1.154	0.7048	0.4306	0.2670	0.1667	0.1025	0.05908	0.02702	0
0.8	x	0	1.628	1.808	1.620	1.445	1.323	1.243	1.192	1.161	1.144	1.139
	y	3.311	2.317	1.188	0.6245	0.3519	0.2086	0.1269	0.07684	0.04390	0.01998	0
0.9	x	0	3.396	2.467	1.885	1.570	1.390	1.282	1.216	1.178	1.157	1.151
	y	6.793	2.542	0.8525	0.3822	0.2011	0.1152	0.06879	0.04124	0.02342	0.01063	0
1.0	x	∞	5.427	2.821	1.998	1.619	1.414	1.296	1.225	1.183	1.162	1.155
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

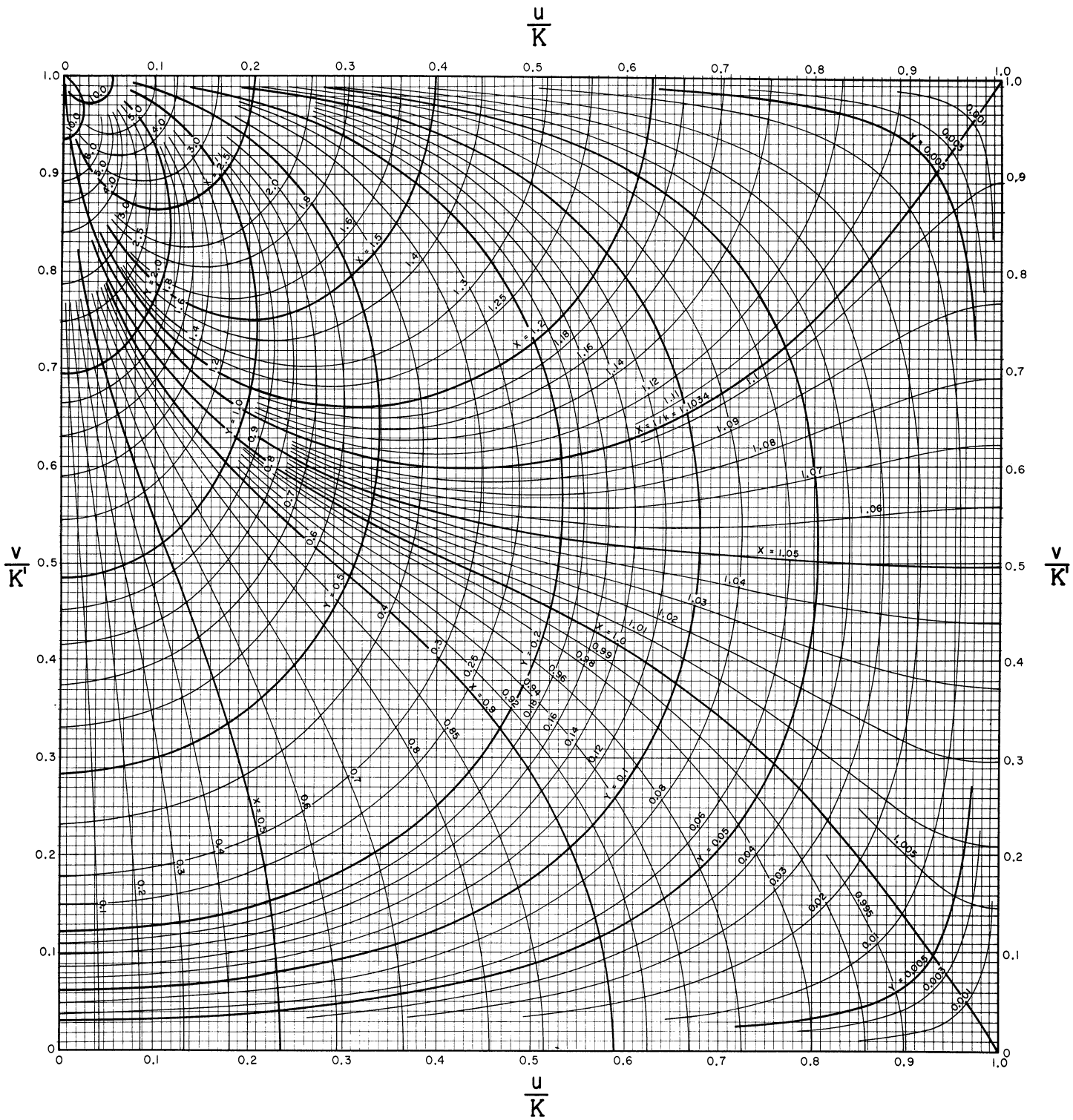
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 60° = 0.8660$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 65° = 0.906308; K = 2.308787, K' = 1.648995

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.2272	0.4340	0.6066	0.7405	0.8384	0.9065	0.9517	0.9799	0.9952	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.2327	0.4430	0.6168	0.7499	0.8460	0.9123	0.9560	0.9831	0.9978	1.002
	y	0.1663	0.1583	0.1371	0.1095	0.08181	0.05799	0.03929	0.02530	0.01492	0.006891	0
0.2	x	0	0.2500	0.4715	0.6481	0.7782	0.8688	0.9295	0.9687	0.9926	1.005	1.009
	y	0.3411	0.3235	0.2776	0.2188	0.1615	0.1133	0.07613	0.04875	0.02865	0.01321	0
0.3	x	0	0.2830	0.5238	0.7037	0.8268	0.9069	0.9576	0.9890	1.008	1.017	1.020
	y	0.5350	0.5037	0.4244	0.3270	0.2361	0.1627	0.1079	0.06850	0.04003	0.01839	0
0.4	x	0	0.3391	0.6089	0.7889	0.8972	0.9596	0.9952	1.016	1.027	1.033	1.035
	y	0.7630	0.7096	0.5798	0.4309	0.3011	0.2023	0.1318	0.08268	0.04795	0.02194	0
0.5	x	0	0.4346	0.7423	0.9109	0.9906	1.026	1.040	1.047	1.049	1.050	1.050
	y	1.050	0.9563	0.7432	0.5231	0.3495	0.2274	0.1449	0.08958	0.05151	0.02346	0
0.6	x	0	0.6049	0.9504	1.077	1.105	1.101	1.090	1.079	1.072	1.068	1.067
	y	1.446	1.266	0.9050	0.5883	0.3710	0.2321	0.1443	0.08786	0.05006	0.02269	0
0.7	x	0	0.9352	1.272	1.290	1.234	1.179	1.138	1.110	1.093	1.084	1.081
	y	2.062	1.665	1.030	0.5992	0.3523	0.2115	0.1282	0.07691	0.04343	0.01960	0
0.8	x	0	1.650	1.738	1.528	1.359	1.249	1.179	1.136	1.111	1.097	1.093
	y	3.234	2.135	1.023	0.5160	0.2821	0.1628	0.09657	0.05718	0.03206	0.01441	0
0.9	x	0	3.242	2.273	1.734	1.454	1.298	1.207	1.153	1.122	1.106	1.101
	y	6.636	2.206	0.7035	0.3079	0.1587	0.08898	0.05199	0.03052	0.01703	0.007637	0
1.0	x	∞	4.856	2.542	1.819	1.490	1.316	1.217	1.159	1.126	1.109	1.103
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

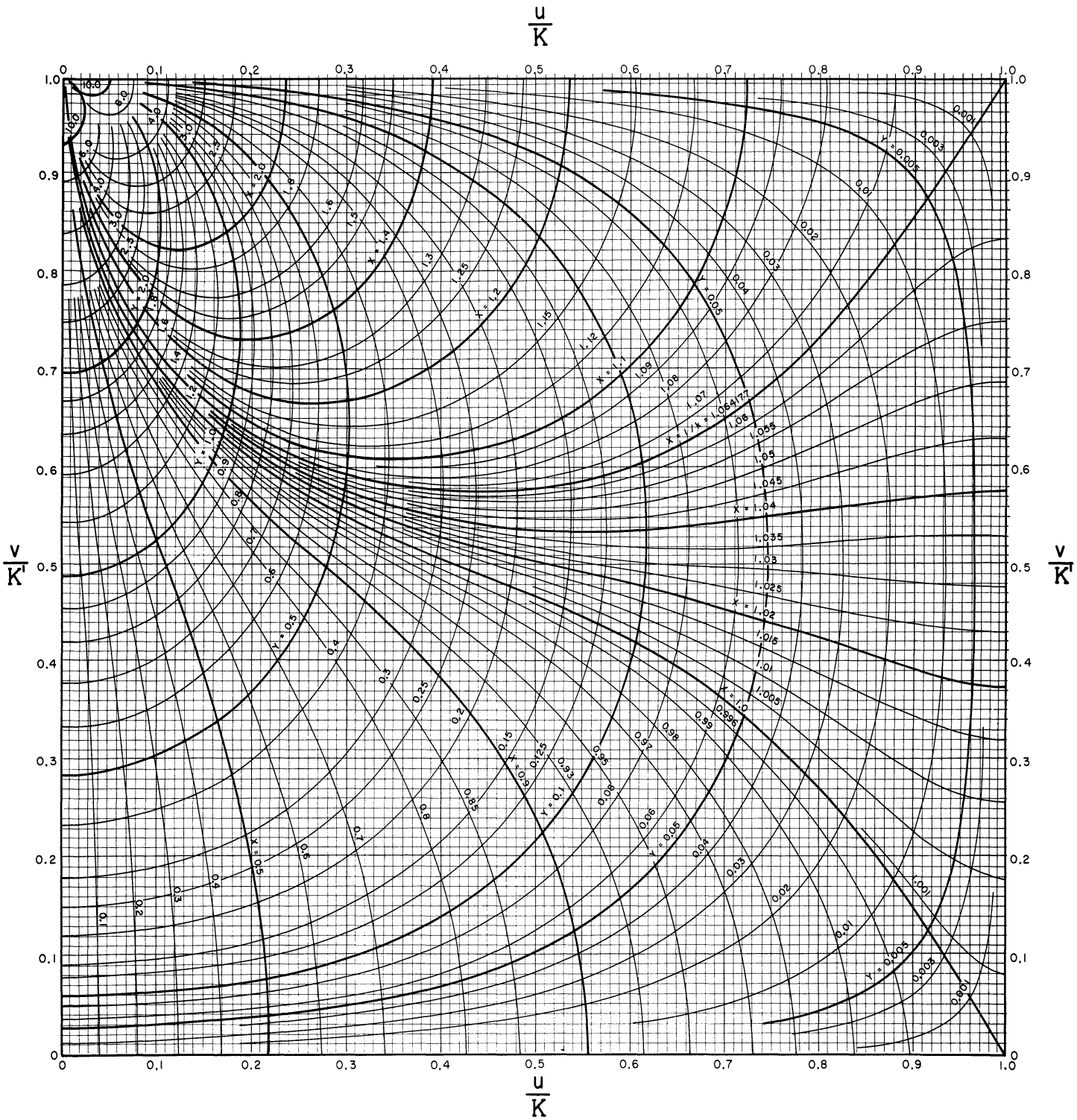
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 65° = 0.9063$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 70° = 0.939693; K = 2.504550, K' = 1.620026

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.2456	0.4649	0.6415	0.7723	0.8632	0.9234	0.9615	0.9843	0.9963	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.2515	0.4741	0.6513	0.7807	0.8696	0.9279	0.9646	0.9865	0.9980	1.002
	y	0.1634	0.1539	0.1294	0.09903	0.07040	0.04739	0.03057	0.01884	0.01073	0.004844	0
0.2	x	0	0.2700	0.5032	0.6815	0.8063	0.8888	0.9414	0.9738	0.9929	1.003	1.006
	y	0.3351	0.3142	0.2613	0.1971	0.1383	0.09213	0.05898	0.03618	0.02055	0.009259	0
0.3	x	0	0.3050	0.5564	0.7345	0.8495	0.9203	0.9631	0.9885	1.003	1.011	1.013
	y	0.5255	0.4885	0.3976	0.2924	0.2005	0.1313	0.08305	0.05055	0.02857	0.01284	0
0.4	x	0	0.3645	0.6417	0.8143	0.9110	0.9634	0.9919	1.008	1.016	1.020	1.022
	y	0.7494	0.6863	0.5391	0.3810	0.2528	0.1616	0.1005	0.06056	0.03402	0.01524	0
0.5	x	0	0.4651	0.7732	0.9256	0.9903	1.016	1.026	1.030	1.031	1.031	1.032
	y	1.032	0.9208	0.6829	0.4554	0.2889	0.1791	0.1093	0.06507	0.03629	0.01620	0
0.6	x	0	0.6424	0.9722	1.072	1.085	1.074	1.062	1.052	1.046	1.043	1.042
	y	1.420	1.209	0.8167	0.5017	0.3010	0.1802	0.1076	0.06325	0.03502	0.01557	0
0.7	x	0	0.9787	1.266	1.251	1.187	1.133	1.097	1.074	1.060	1.053	1.051
	y	2.025	1.567	0.9048	0.4978	0.2801	0.1617	0.09457	0.05490	0.03019	0.01338	0
0.8	x	0	1.674	1.666	1.441	1.282	1.185	1.126	1.091	1.071	1.061	1.058
	y	3.176	1.948	0.8649	0.4168	0.2199	0.1228	0.07055	0.04055	0.02217	0.009796	0
0.9	x	0	3.079	2.089	1.597	1.352	1.220	1.146	1.103	1.079	1.066	1.063
	y	6.515	1.884	0.5703	0.2428	0.1219	0.06649	0.03774	0.02154	0.01174	0.005176	0
1.0	x	∞	4.332	2.289	1.659	1.378	1.233	1.153	1.107	1.081	1.068	1.064
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

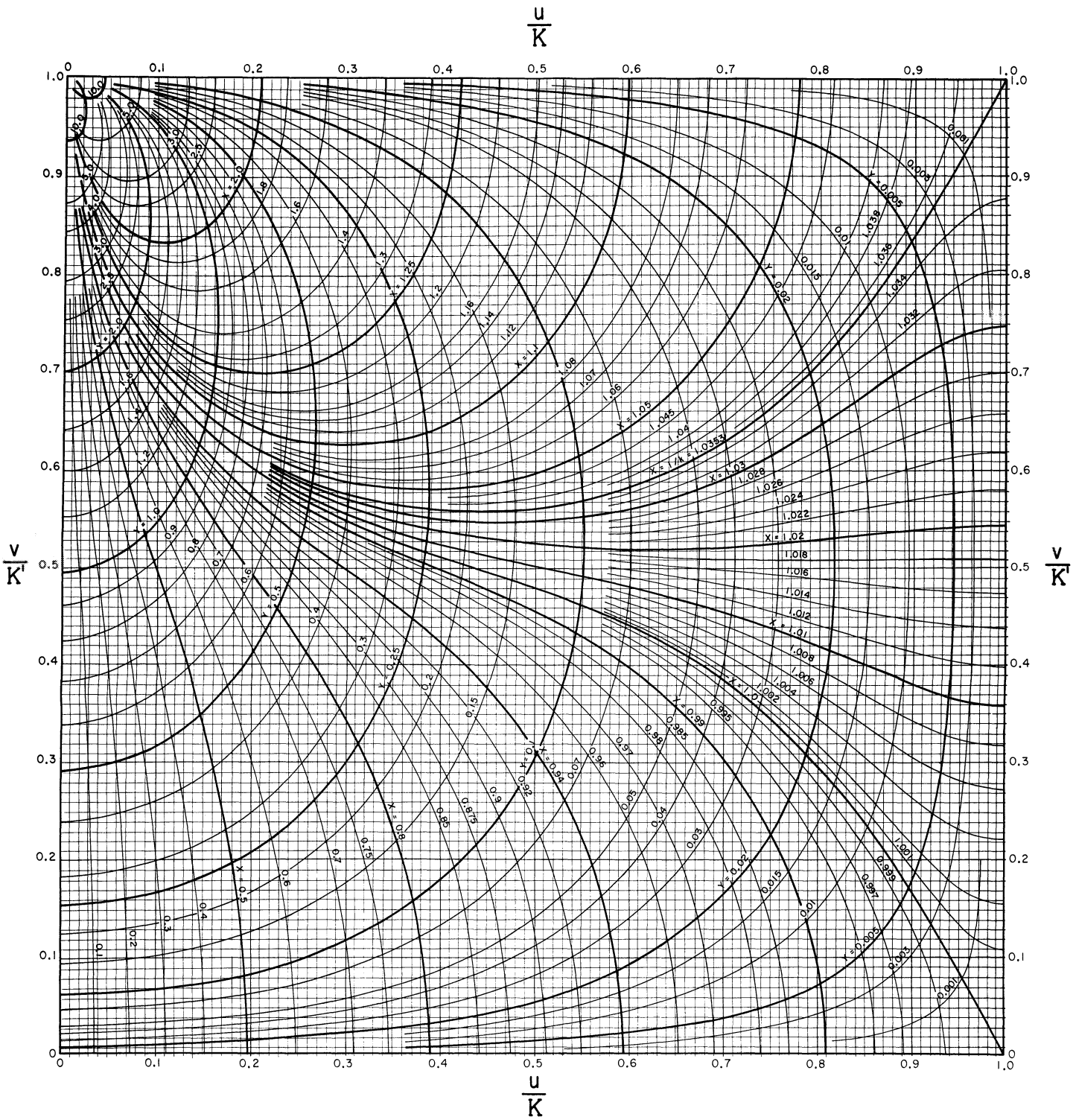
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 70° = 0.9397$

THE sn-FUNCTION, $x + iy = sn(u + iv)$

$k = \sin 75° = 0.965926$; $K = 2.768063$, $K' = 1.598142$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.2702	0.5047	0.6846	0.8099	0.8913	0.9417	0.9717	0.9887	0.9974	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.2765	0.5142	0.6939	0.8172	0.8964	0.9450	0.9738	0.9901	0.9983	1.001
	y	0.1611	0.1495	0.1207	0.08712	0.05796	0.03647	0.02206	0.01284	0.006981	0.003055	0
0.2	x	0	0.2965	0.5439	0.7224	0.8393	0.9116	0.9548	0.9800	0.9941	1.001	1.003
	y	0.3306	0.3050	0.2429	0.1725	0.1132	0.07055	0.04239	0.02458	0.01333	0.005828	0
0.3	x	0	0.3342	0.5976	0.7717	0.8761	0.9363	0.9705	0.9897	1.000	1.006	1.007
	y	0.5184	0.4732	0.3673	0.2536	0.1627	0.09972	0.05930	0.03416	0.01847	0.008057	0
0.4	x	0	0.3980	0.6826	0.8444	0.9274	0.9693	0.9909	1.002	1.008	1.011	1.012
	y	0.7392	0.6624	0.4931	0.3262	0.2024	0.1214	0.07118	0.04067	0.02188	0.009524	0
0.5	x	0	0.5048	0.8102	0.9427	0.9917	1.009	1.015	1.017	1.017	1.017	1.017
	y	1.017	0.8834	0.6155	0.3829	0.2276	0.1328	0.07663	0.04337	0.02321	0.01008	0
0.6	x	0	0.6900	0.9966	1.067	1.066	1.052	1.039	1.031	1.026	1.024	1.023
	y	1.400	1.148	0.7200	0.4122	0.2326	0.1317	0.07466	0.04185	0.02227	0.009643	0
0.7	x	0	1.031	1.257	1.211	1.142	1.093	1.063	1.045	1.035	1.029	1.028
	y	1.997	1.460	0.7727	0.3979	0.2121	0.1165	0.06495	0.03606	0.01910	0.008249	0
0.8	x	0	1.697	1.587	1.356	1.211	1.129	1.082	1.056	1.041	1.034	1.032
	y	3.132	1.745	0.7086	0.3237	0.1634	0.08736	0.04804	0.02648	0.01397	0.006020	0
0.9	x	0	2.897	1.908	1.469	1.260	1.153	1.095	1.063	1.046	1.037	1.034
	y	6.425	1.567	0.4479	0.1844	0.08939	0.04691	0.02556	0.01402	0.007372	0.003174	0
1.0	x	∞	3.832	2.051	1.512	1.278	1.162	1.099	1.065	1.047	1.038	1.035
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

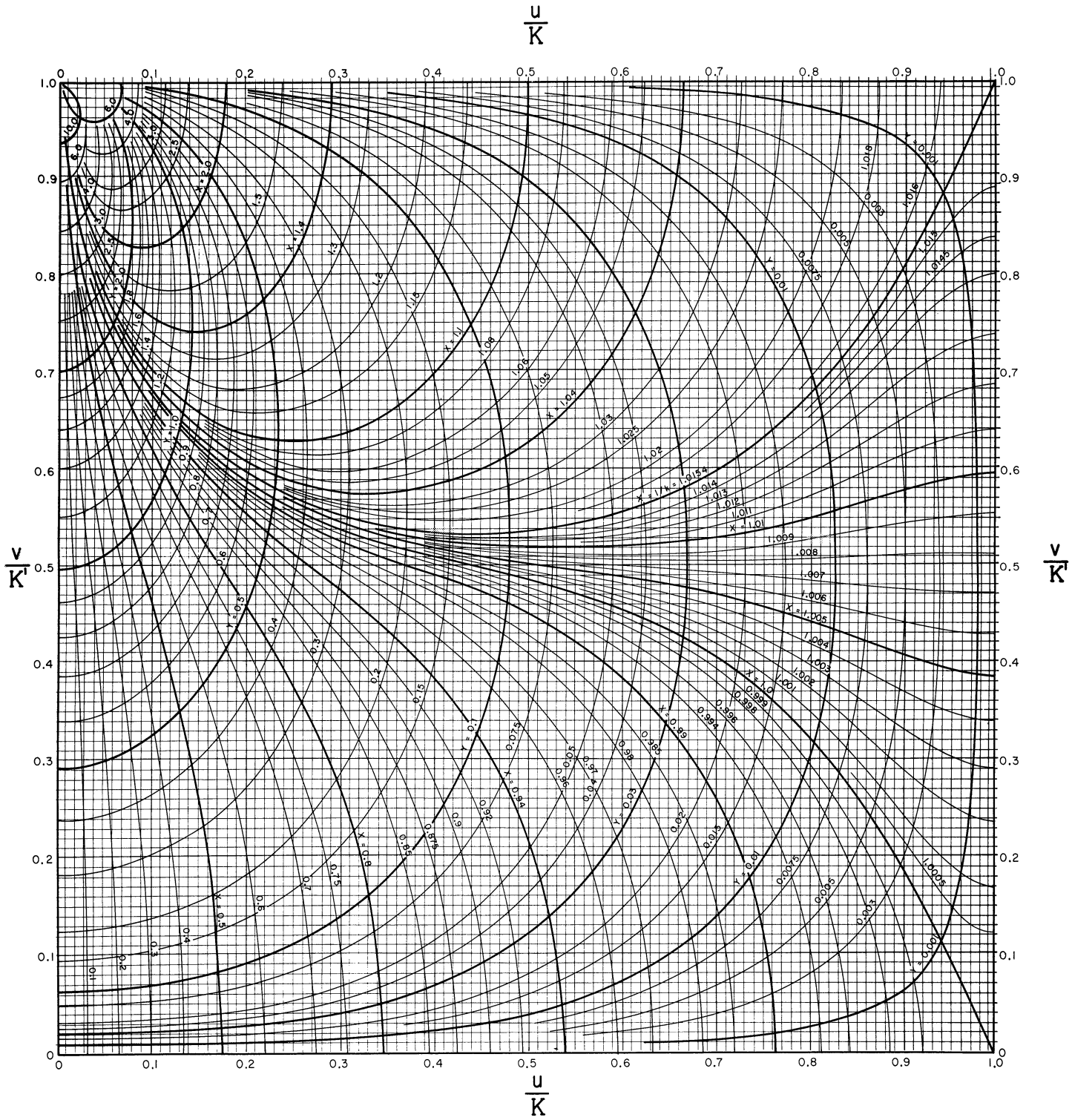
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 75° = 0.9659$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 80° = 0.984808; K = 3.153385, K' = 1.582843

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.3054	0.5595	0.7403	0.8552	0.9231	0.9613	0.9822	0.9932	0.9985	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.3124	0.5691	0.7487	0.8611	0.9267	0.9634	0.9834	0.9939	0.9989	1.000
	y	0.1596	0.1446	0.1095	0.07246	0.04380	0.02505	0.01384	0.007427	0.003773	0.001577	0
0.2	x	0	0.3344	0.5990	0.7743	0.8787	0.9375	0.9696	0.9869	0.9959	1.000	1.001
	y	0.3274	0.2945	0.2193	0.1425	0.08502	0.04821	0.02650	0.01418	0.007192	0.003004	0
0.3	x	0	0.3758	0.6525	0.8179	0.9076	0.9547	0.9795	0.9924	0.9991	1.002	1.003
	y	0.5134	0.4554	0.3288	0.2072	0.1209	0.06758	0.03685	0.01962	0.009931	0.004143	0
0.4	x	0	0.4451	0.7352	0.8803	0.9467	0.9774	0.9922	0.9995	1.003	1.005	1.005
	y	0.7321	0.6341	0.4356	0.2622	0.1482	0.08133	0.04388	0.02323	0.01172	0.004882	0
0.5	x	0	0.5595	0.8553	0.9616	0.9943	1.004	1.007	1.007	1.008	1.008	1.008
	y	1.008	0.8381	0.5328	0.3012	0.1637	0.08782	0.04681	0.02462	0.01238	0.005148	0
0.6	x	0	0.7530	1.022	1.060	1.047	1.032	1.021	1.015	1.012	1.011	1.010
	y	1.387	1.073	0.6057	0.3156	0.1639	0.08585	0.04517	0.02360	0.01183	0.004910	0
0.7	x	0	1.095	1.241	1.167	1.099	1.058	1.035	1.023	1.016	1.013	1.012
	y	1.978	1.327	0.6254	0.2956	0.1464	0.07490	0.03895	0.02022	0.01010	0.004188	0
0.8	x	0	1.710	1.495	1.268	1.145	1.080	1.046	1.029	1.020	1.015	1.014
	y	3.101	1.506	0.5473	0.2335	0.1108	0.05555	0.02860	0.01478	0.007363	0.003049	0
0.9	x	0	2.677	1.721	1.344	1.176	1.095	1.054	1.033	1.022	1.017	1.015
	y	6.362	1.239	0.3312	0.1300	0.05983	0.02960	0.01514	0.007797	0.003879	0.001605	0
1.0	x	∞	3.325	1.815	1.372	1.187	1.100	1.056	1.034	1.022	1.017	1.015
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

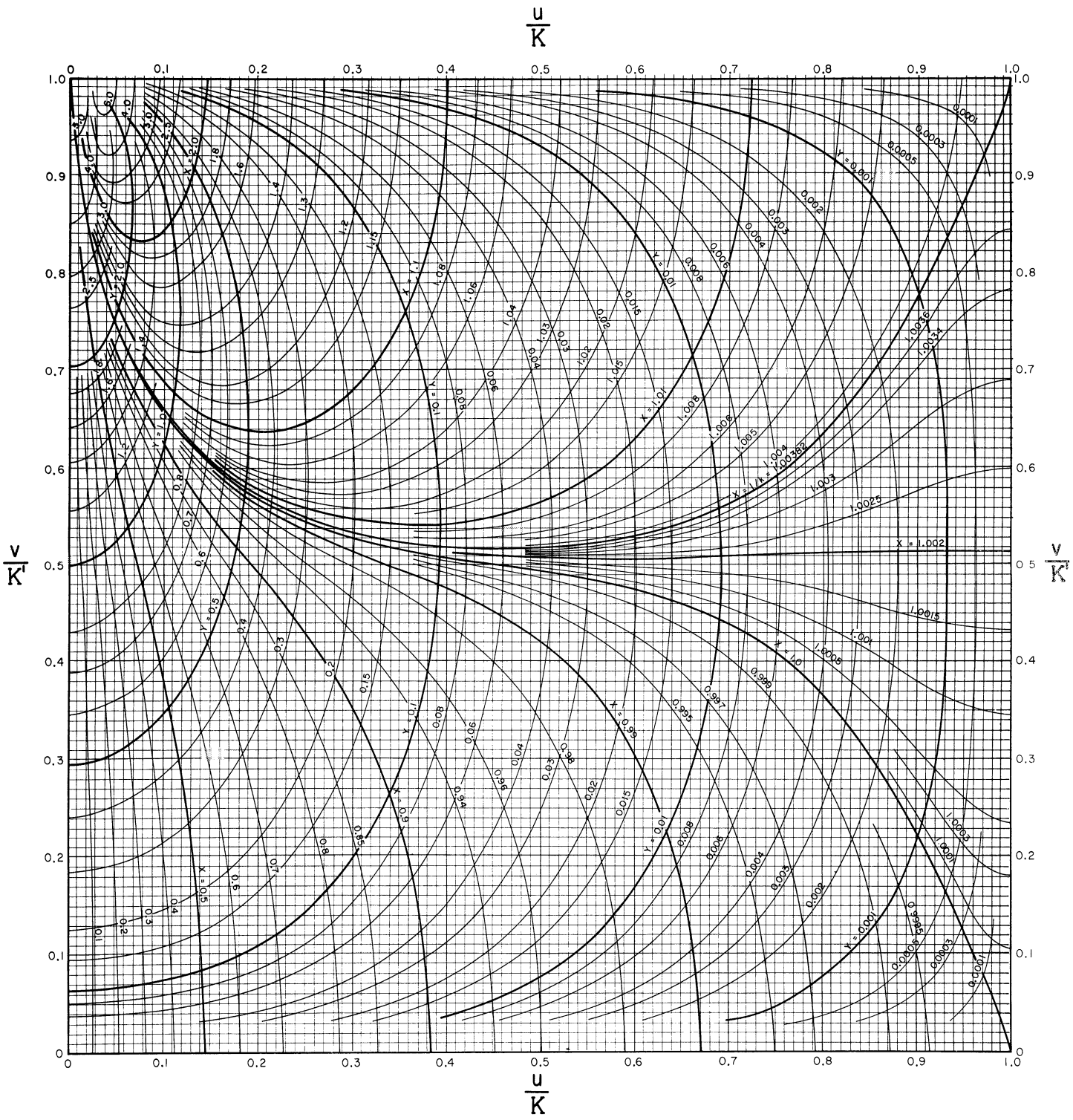
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 80° = 0.9848$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 85° = 0.996195; K = 3.831742, K' = 1.573792

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.3655	0.6452	0.8184	0.9121	0.9591	0.9817	0.9924	0.9973	0.9994	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.3734	0.6546	0.8251	0.9160	0.9610	0.9827	0.9928	0.9975	0.9995	1.000
	y	0.1587	0.1371	0.09193	0.05193	0.02661	0.01297	0.006157	0.002872	0.001293	$0.0^{3}4956$	0
0.2	x	0	0.3985	0.6834	0.8453	0.9273	0.9668	0.9855	0.9942	0.9982	0.9999	1.000
	y	0.3255	0.2783	0.1826	0.1012	0.05124	0.02481	0.01174	0.005470	0.002461	$0.0^{3}9430$	0
0.3	x	0	0.4450	0.7337	0.8787	0.9455	0.9759	0.9899	0.9963	0.9992	1.000	1.001
	y	0.5105	0.4278	0.2698	0.1448	0.07191	0.03447	0.01624	0.007544	0.003391	0.001299	0
0.4	x	0	0.5218	0.8087	0.9247	0.9695	0.9876	0.9954	0.9989	1.000	1.001	1.001
	y	0.7279	0.5896	0.3496	0.1791	0.08668	0.04101	0.01920	0.008893	0.003991	0.001528	0
0.5	x	0	0.6452	0.9121	0.9818	0.9975	1.001	1.002	1.002	1.002	1.002	1.002
	y	1.002	0.7665	0.4145	0.1999	0.09378	0.04370	0.02031	0.009378	0.004202	0.001608	0
0.6	x	0	0.8449	1.046	1.046	1.027	1.015	1.008	1.005	1.003	1.003	1.002
	y	1.379	0.9547	0.4521	0.2027	0.09185	0.04213	0.01944	0.008945	0.004002	0.001530	0
0.7	x	0	1.172	1.205	1.112	1.056	1.027	1.014	1.008	1.005	1.003	1.003
	y	1.966	1.127	0.4432	0.1832	0.08029	0.03629	0.01663	0.007630	0.003409	0.001302	0
0.8	x	0	1.693	1.371	1.171	1.079	1.038	1.018	1.010	1.006	1.004	1.003
	y	3.084	1.182	0.3663	0.1401	0.05964	0.02663	0.01214	0.005555	0.002479	$0.0^{3}9468$	0
0.9	x	0	2.369	1.504	1.212	1.095	1.044	1.021	1.011	1.006	1.004	1.004
	y	6.326	0.8697	0.2112	0.07626	0.03181	0.01409	0.006400	0.002924	0.001304	$0.0^{3}4979$	0
1.0	x	∞	2.746	1.556	1.227	1.101	1.047	1.023	1.012	1.007	1.004	1.004
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = \mathrm{sn}(u + iv)$

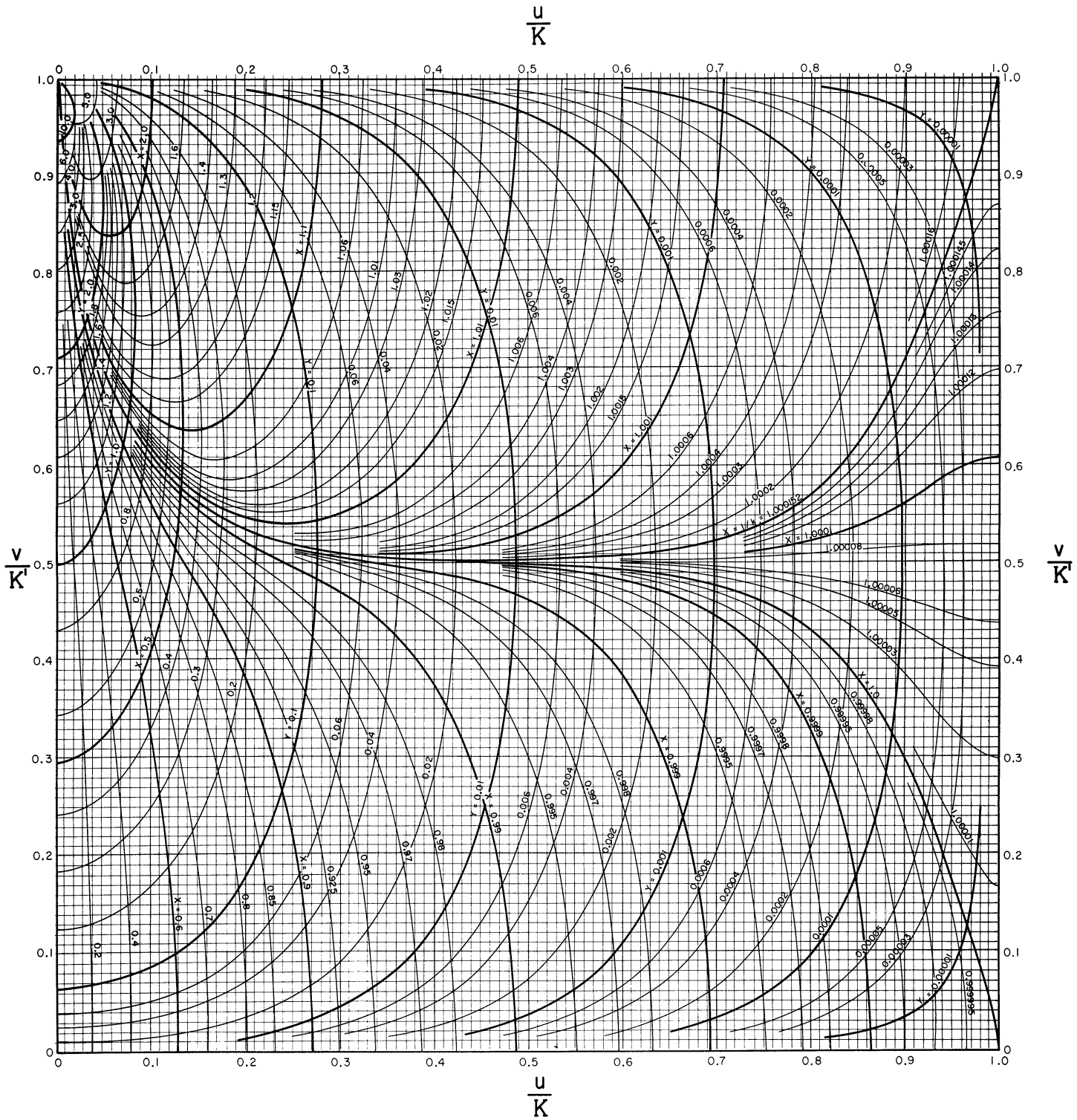
$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 85° = 0.9962$

THE sn-FUNCTION, x + iy = sn(u + iv)

k = sin 89° = 0.999848; K = 5.434910, K' = 1.570916

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	0	0.4956	0.7958	0.9262	0.9745	0.9914	0.9971	0.9991	0.9997	1.000	1.000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	0	0.5050	0.8030	0.9294	0.9757	0.9918	0.9973	0.9991	0.9998	1.000	1.000
	y	0.1584	0.1188	0.05719	0.02207	0.007803	0.002674	$0.0^{3}9066$	$0.0^{3}5060$	$0.0^{3}1021$	$0.0^{4}3092$	0
0.2	x	0	0.5341	0.8247	0.9389	0.9792	0.9930	0.9977	0.9993	0.9998	1.000	1.000
	y	0.3249	0.2389	0.1117	0.04241	0.01490	0.005093	0.001726	$0.0^{3}5822$	$0.0^{3}1942$	$0.0^{4}5882$	0
0.3	x	0	0.5869	0.8609	0.9542	0.9848	0.9949	0.9983	0.9995	0.9999	1.000	1.000
	y	0.5096	0.3614	0.1605	0.05931	0.02062	0.007024	0.002377	$0.0^{3}8015$	$0.0^{3}2673$	$0.0^{4}8096$	0
0.4	x	0	0.6703	0.9113	0.9741	0.9918	0.9973	0.9992	0.9998	1.000	1.000	1.000
	y	0.7266	0.4852	0.1997	0.07118	0.02441	0.008277	0.002796	$0.0^{3}9424$	$0.0^{3}3143$	$0.0^{4}9517$	0
0.5	x	0	0.7958	0.9745	0.9971	0.9997	1.000	1.000	1.000	1.000	1.000	1.000
	y	1.000	0.6057	0.2246	0.07662	0.02587	0.008727	0.002943	$0.0^{3}9912$	$0.0^{3}3305$	$0.0^{3}1001$	0
0.6	x	0	0.9790	1.047	1.021	1.008	1.003	1.001	1.000	1.000	1.000	1.000
	y	1.376	0.7087	0.2295	0.07463	0.02480	0.008322	0.002801	$0.0^{3}9430$	$0.0^{3}3143$	$0.0^{4}9518$	0
0.7	x	0	1.236	1.123	1.044	1.015	1.005	1.002	1.001	1.000	1.000	1.000
	y	1.963	0.7609	0.2093	0.06491	0.02125	0.007096	0.002385	$0.0^{3}8024$	$0.0^{3}2674$	$0.0^{4}8097$	0
0.8	x	0	1.560	1.191	1.063	1.021	1.007	1.002	1.001	1.000	1.000	1.000
	y	3.078	0.6980	0.1613	0.04801	0.01553	0.005166	0.001734	$0.0^{3}5831$	$0.0^{3}1943$	$0.0^{4}5883$	0
0.9	x	0	1.877	1.239	1.076	1.025	1.008	1.003	1.001	1.000	1.000	1.000
	y	6.314	0.4414	0.08826	0.02554	0.008196	0.002719	$0.0^{3}9120$	$0.0^{3}3066$	$0.0^{3}1022$	$0.0^{4}3093$	0
1.0	x	∞	2.018	1.257	1.080	1.026	1.009	1.003	1.001	1.000	1.000	1.000
	y	∞	0	0	0	0	0	0	0	0	0	0



The sn-function, $z = x + iy = sn(u + iv)$

$$u + iv = F(z) = \int_0^z \frac{dz}{\sqrt{(1 - z^2)(1 - k^2 z^2)}}$$

$k = \sin 89° = 0.9999$

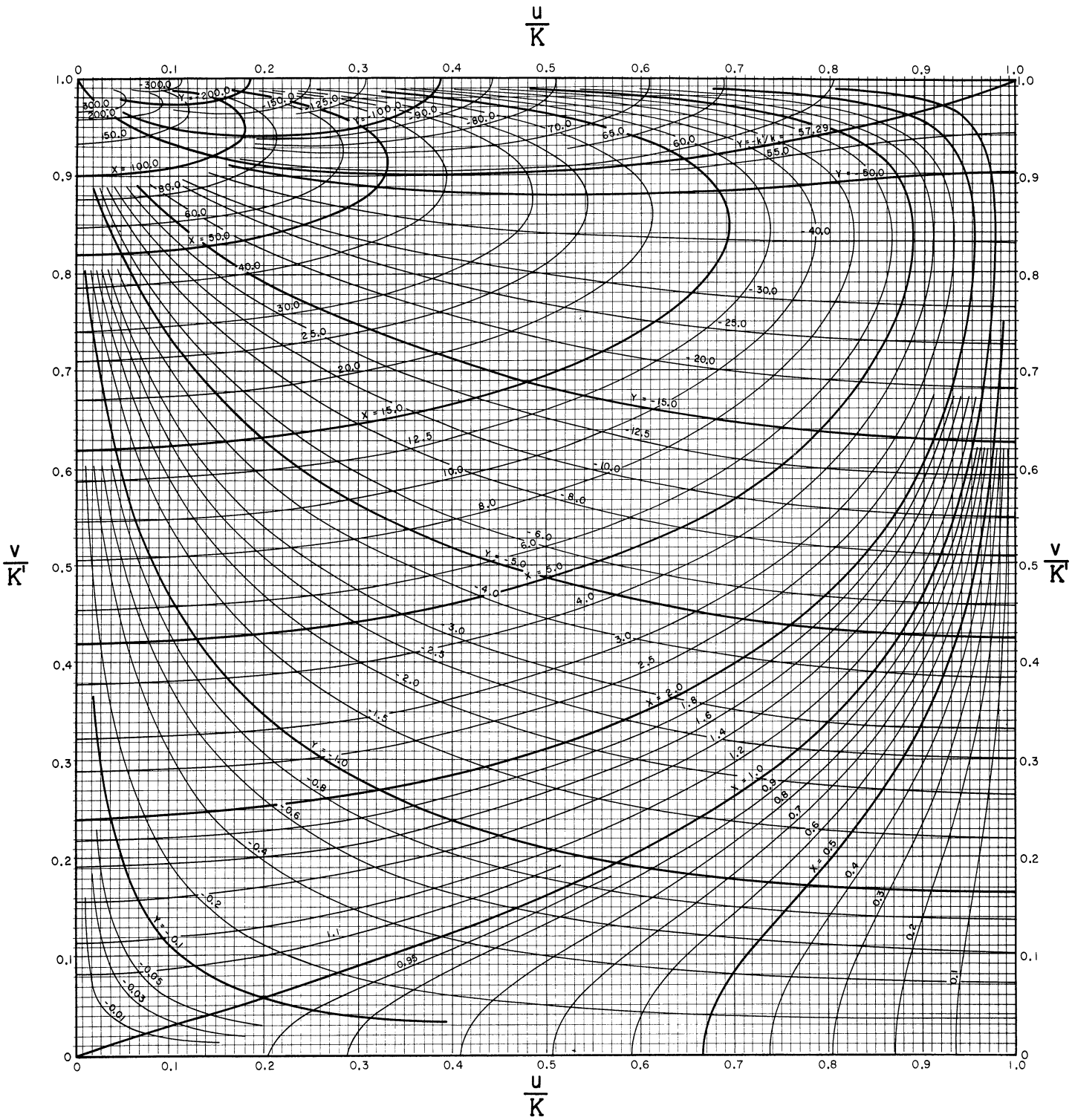
APPENDIX II

THE cn-FUNCTION

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 1° = 0.0174524$; $K = 1.570916$, $K' = 5.434910$

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9877	0.9511	0.8910	0.8090	0.7071	0.5878	0.4540	0.3090	0.1564	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.151	1.137	1.095	1.026	0.9314	0.8141	0.6767	0.5226	0.3557	0.1801	0
	y	0	0.08928	0.1764	0.2591	0.3354	0.4035	0.4616	0.5084	0.5426	0.5635	0.5705
0.2	x	1.651	1.631	1.570	1.471	1.336	1.167	0.9703	0.7494	0.5100	0.2582	0
	y	0	0.2056	0.4062	0.5967	0.7725	0.9292	1.063	1.171	1.249	1.297	1.314
0.3	x	2.652	2.619	2.522	2.362	2.144	1.873	1.557	1.202	0.8181	0.4141	0
	y	0	0.3846	0.7596	1.116	1.444	1.737	1.986	2.187	2.334	2.424	2.454
0.4	x	4.459	4.404	4.239	3.968	3.600	3.144	2.611	2.015	1.371	0.6936	0
	y	0	0.6817	1.346	1.976	2.557	3.073	3.512	3.865	4.123	4.280	4.332
0.5	x	7.635	7.538	7.250	6.779	6.140	5.352	4.437	3.419	2.323	1.174	0
	y	0	1.194	2.356	3.454	4.461	5.352	6.107	6.710	7.148	7.414	7.503
0.6	x	13.22	1304	12.51	11.65	10.51	9.109	7.512	5.761	3.899	1.967	0
	y	0	2.114	4.160	6.077	7.810	9.320	10.58	11.57	12.28	12.71	12.85
0.7	x	23.35	22.93	21.86	20.12	17.87	15.25	12.38	9.367	6.274	3.144	0
	y	0	3.917	7.663	11.06	14.00	16.45	18.38	19.83	20.83	21.41	21.61
0.8	x	43.61	42.58	39.28	34.72	29.40	23.91	18.59	13.56	8.844	4.359	0
	y	0	8.472	16.04	22.27	26.84	30.04	32.14	33.44	34.19	34.58	34.70
0.9	x	100.4	92.24	73.84	54.81	39.43	28.01	19.60	13.26	8.211	3.930	0
	y	0	29.48	48.41	56.35	57.79	56.51	54.45	52.50	51.00	50.07	49.76
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	366.3	185.4	126.2	97.47	81.02	70.81	64.30	60.23	58.01	57.29



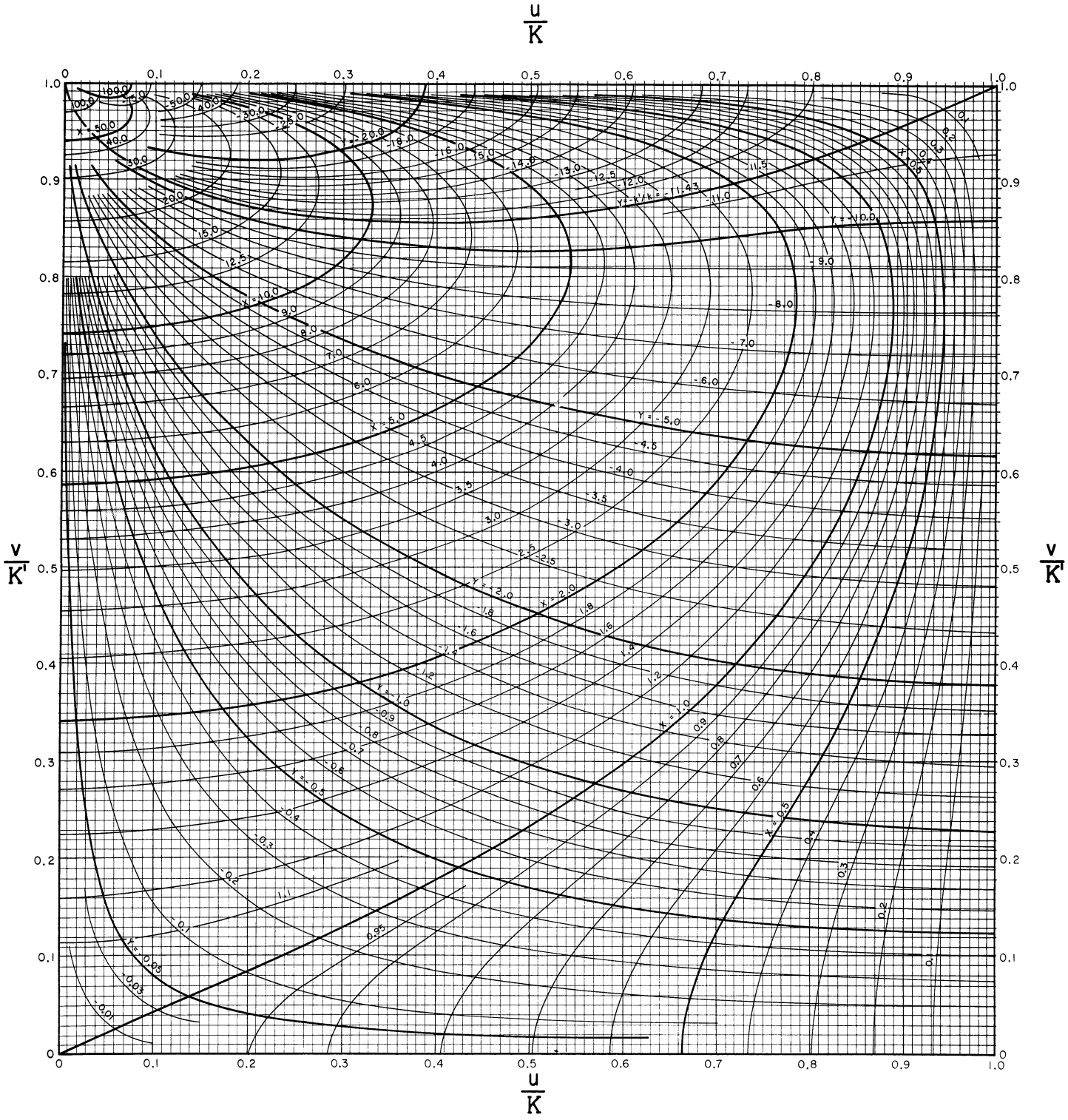
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 1^\circ = 0.0175$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 5° = 0.0871557$; $K = 1.573792$, $K' = 3.831742$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9876	0.9509	0.8907	0.8085	0.7064	0.5871	0.4533	0.3085	0.1561	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.074	1.061	1.021	0.9566	0.8682	0.7585	0.6302	0.4866	0.3311	0.1676	0
	y	0	0.06157	0.1216	0.1785	0.2308	0.2774	0.3171	0.3489	0.3721	0.3862	0.3910
0.2	x	1.309	1.292	1.244	1.164	1.056	0.9221	0.7656	0.5908	0.4018	0.2033	0
	y	0	0.1327	0.2619	0.3843	0.4967	0.5965	0.6813	0.7492	0.7987	0.8288	0.8390
0.3	x	1.740	1.718	1.652	1.545	1.400	1.220	1.011	0.7793	0.5295	0.2677	0
	y	0	0.2248	0.4435	0.6500	0.8391	1.006	1.147	1.260	1.342	1.391	1.408
0.4	x	2.439	2.407	2.311	2.156	1.947	1.691	1.398	1.074	0.7278	0.3674	0
	y	0	0.3549	0.6988	1.022	1.315	1.572	1.786	1.956	2.078	2.151	2.176
0.5	x	3.532	3.481	3.331	3.090	2.772	2.391	1.961	1.497	1.010	0.5083	0
	y	0	0.5523	1.084	1.576	2.015	2.391	2.698	2.935	3.103	3.203	3.236
0.6	x	5.253	5.162	4.900	4.491	3.969	3.370	2.723	2.052	1.370	0.6852	0
	y	0	0.8816	1.716	2.466	3.106	3.627	4.032	4.329	4.531	4.648	4.686
0.7	x	8.118	7.921	7.371	6.561	5.606	4.599	3.602	2.644	1.732	0.8560	0
	y	0	1.524	2.908	4.058	4.942	5.577	6.008	6.286	6.453	6.542	6.568
0.8	x	13.61	13.01	11.42	9.408	7.414	5.653	4.168	2.921	1.852	0.8983	0
	y	0	3.190	5.745	7.417	8.331	8.738	8.862	8.851	8.799	8.751	8.733
0.9	x	29.24	24.90	17.14	11.12	7.279	4.862	3.270	2.154	1.313	0.6232	0
	y	0	10.80	15.25	15.51	14.47	13.29	12.30	11.55	11.04	10.74	10.64
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	73.20	37.05	25.21	19.47	16.18	14.14	12.84	12.02	11.57	11.43



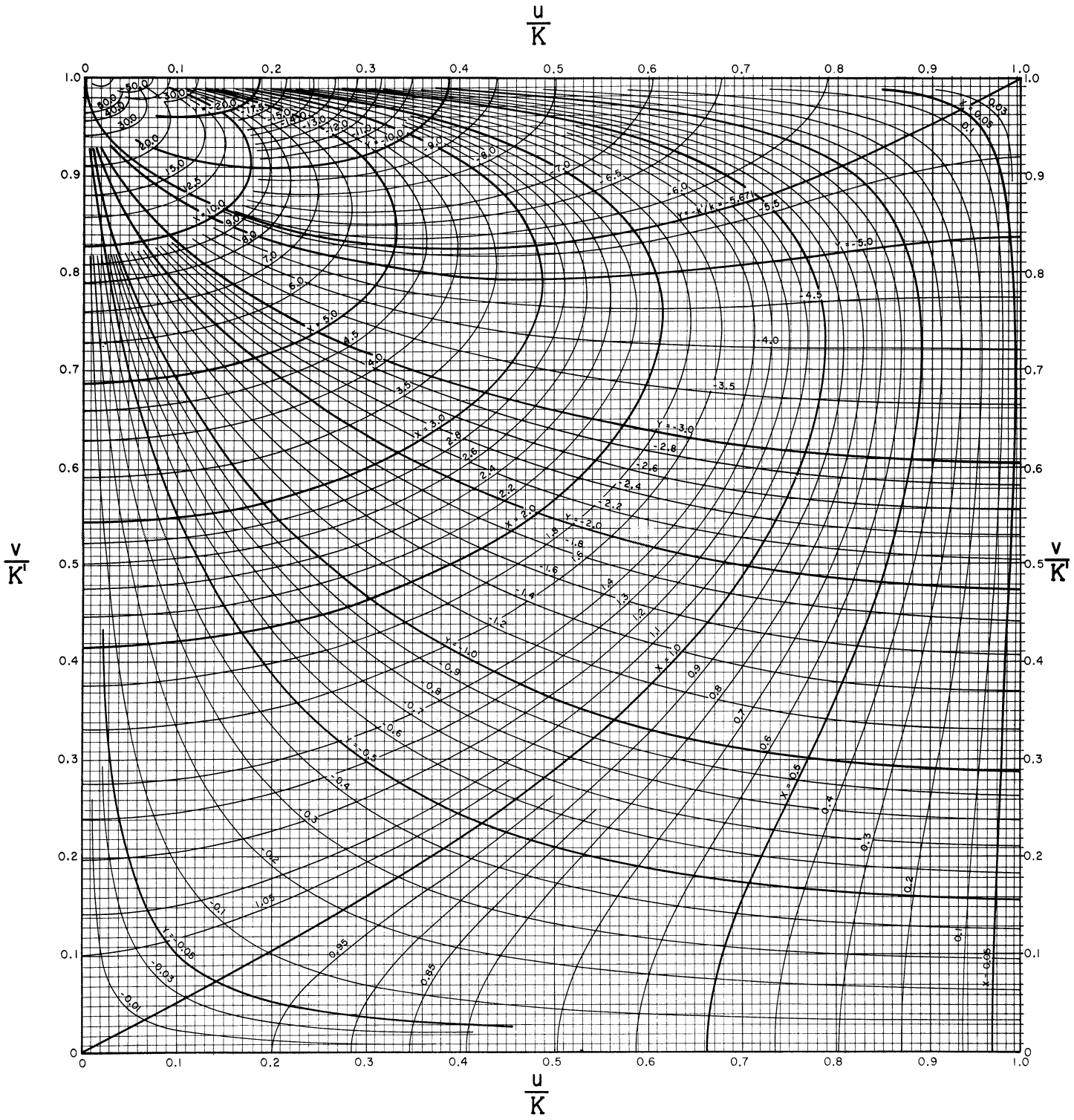
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 5° = 0.0872$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 10° = 0.173648; K = 1.582843, K' = 3.153385

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9875	0.9504	0.8896	0.8069	0.7044	0.5848	0.4512	0.3069	0.1553	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.050	1.037	0.9978	0.9336	0.8464	0.7386	0.6129	0.4727	0.3214	0.1626	0
	y	0	0.05061	0.09979	0.1462	0.1886	0.2259	0.2575	0.2825	0.3007	0.3117	0.3154
0.2	x	1.206	1.191	1.145	1.070	0.9688	0.8440	0.6993	0.5385	0.3657	0.1848	0
	y	0	0.1070	0.2109	0.3085	0.3974	0.4754	0.5409	0.5927	0.6301	0.6527	0.6602
0.3	x	1.487	1.468	1.409	1.313	1.185	1.029	0.8495	0.6522	0.4418	0.2230	0
	y	0	0.1765	0.3471	0.5066	0.6505	0.7754	0.8792	0.9604	1.019	1.053	1.065
0.4	x	1.930	1.902	1.819	1.688	1.514	1.305	1.071	0.8172	0.5511	0.2774	0
	y	0	0.2699	0.5291	0.7684	0.9806	1.161	1.308	1.420	1.500	1.547	1.562
0.5	x	2.600	2.556	2.430	2.232	1.978	1.684	1.365	1.031	0.6894	0.3452	0
	y	0	0.4078	0.7944	1.142	1.440	1.684	1.874	2.014	2.109	2.164	2.181
0.6	x	3.630	3.553	3.332	3.000	2.596	2.158	1.710	1.267	0.8360	0.4150	0
	y	0	0.6371	1.224	1.726	2.125	2.426	2.639	2.784	2.874	2.924	2.939
0.7	x	5.325	5.153	4.686	4.042	3.336	2.650	2.019	1.450	0.9353	0.4581	0
	y	0	1.091	2.033	2.745	3.224	3.516	3.678	3.759	3.796	3.810	3.813
0.8	x	8.590	8.044	6.733	5.242	3.925	2.874	2.056	1.411	0.8822	0.4245	0
	y	0	2.278	3.908	4.763	5.074	5.101	5.011	4.893	4.791	4.724	4.701
0.9	x	17.98	14.30	8.804	5.283	3.304	2.148	1.422	0.9284	0.5629	0.2663	0
	y	0	7.460	9.409	8.840	7.865	7.023	6.384	5.930	5.628	5.457	5.401
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	36.53	18.48	12.57	9.697	8.051	7.029	6.375	5.968	5.743	5.671



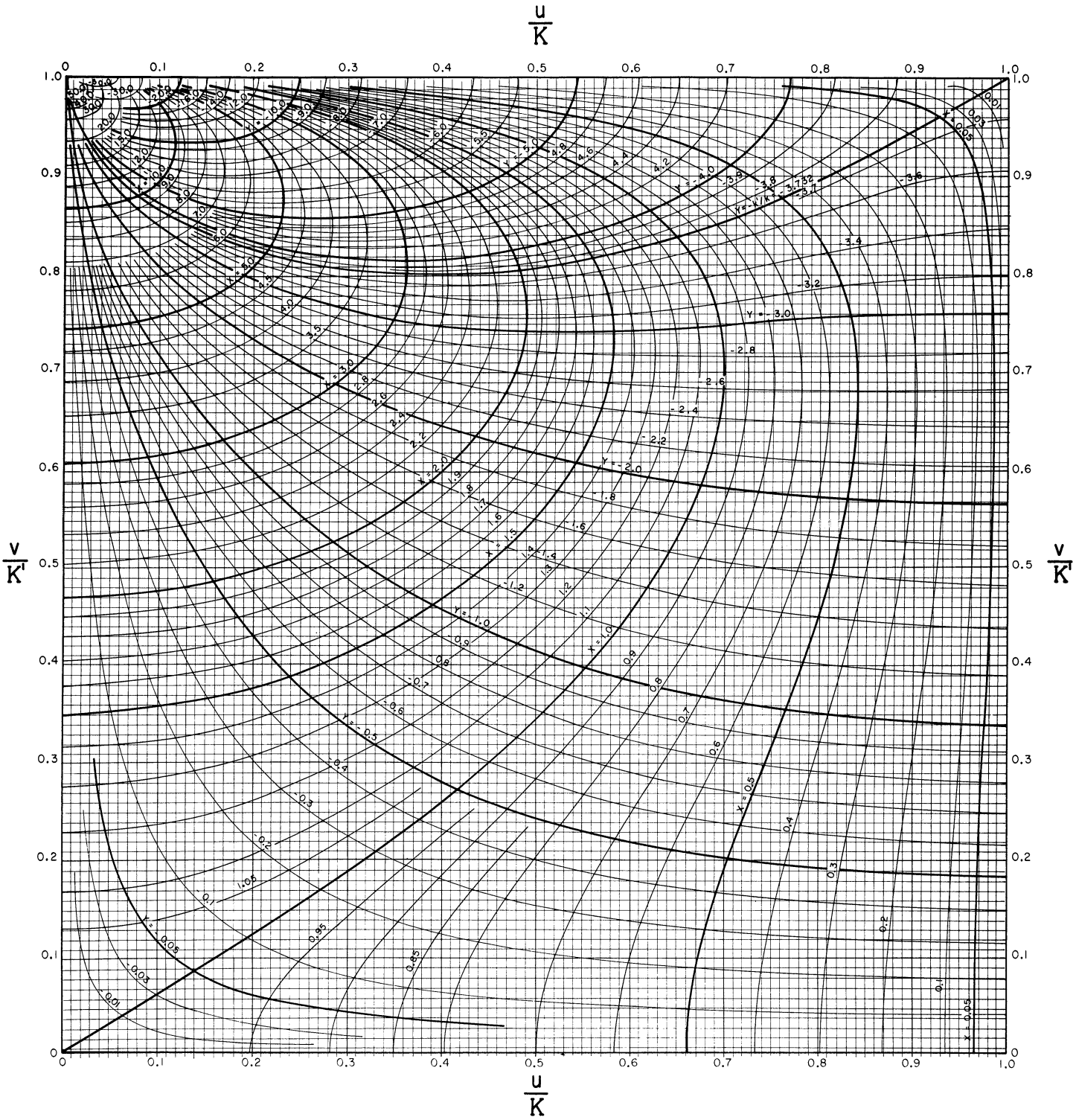
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 10° = 0.1737$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 15° = 0.258819; K = 1.598142, K' = 2.768063

$\frac{v}{K'}$	$\frac{u}{K}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0 7	0.8	0.9	1.0
0	x	1.000	0.9873	0.9495	0.8878	0.8041	0.7010	0.5811	0.4477	0.3042	0.1538	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.039	1.025	0.9856	0.9211	0.8336	0.7261	0.6015	0.4631	0.3144	0.1589	0
	y	0	0.04472	0.08797	0.1284	0.1649	0.1967	0.2230	0.2437	0.2585	0.2674	0.2703
0.2	x	1.158	1.143	1.097	1.023	0.9240	0.8026	0.6631	0.5093	0.3452	0.1743	0
	y	0	0.09394	0.1846	0.2689	0.3445	0.4097	0.4634	0.5051	0.5348	0.5525	0.5584
0.3	x	1.372	1.352	1.295	1.203	1.081	0.9336	0.7672	0.5866	0.3961	0.1995	0
	y	0	0.1534	0.3007	0.4363	0.5562	0.6578	0.7401	0.8029	0.8470	0.8730	0.8816
0.4	x	1.705	1.678	1.599	1.474	1.312	1.122	0.9135	0.6926	0.4647	0.2331	0
	y	0	0.2323	0.4530	0.6524	0.8241	0.9651	1.076	1.157	1.213	1.245	1.256
0.5	x	2.205	2.163	2.042	1.856	1.625	1.366	1.094	0.8182	0.5433	0.2708	0
	y	0	0.3483	0.6728	0.9554	1.187	1.366	1.498	1.590	1.649	1.682	1.692
0.6	x	2.972	2.895	2.683	2.374	2.016	1.644	1.282	0.9375	0.6125	0.3023	0
	y	0	0.5420	1.028	1.421	1.712	1.912	2.040	2.118	2.161	2.183	2.189
0.7	x	4.233	4.063	3.616	3.031	2.431	1.882	1.405	0.9945	0.6349	0.3091	0
	y	0	0.9285	1.691	2.214	2.520	2.672	2.731	2.742	2.735	2.725	2.721
0.8	x	6.684	6.144	4.927	3.664	2.643	1.883	1.322	0.8965	0.5564	0.2666	0
	y	0	1.938	3.180	3.695	3.782	3.689	3.547	3.412	3.308	3.244	3.223
0.9	x	31.81	10.32	5.824	3.322	2.024	1.298	0.8527	0.5543	0.3354	0.1585	0
	y	0	6.133	7.085	6.313	5.457	4.790	4.309	3.976	3.758	3.634	3.594
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	24.27	12.27	8.335	6.422	5.324	4.641	4.204	3.931	3.780	3.732



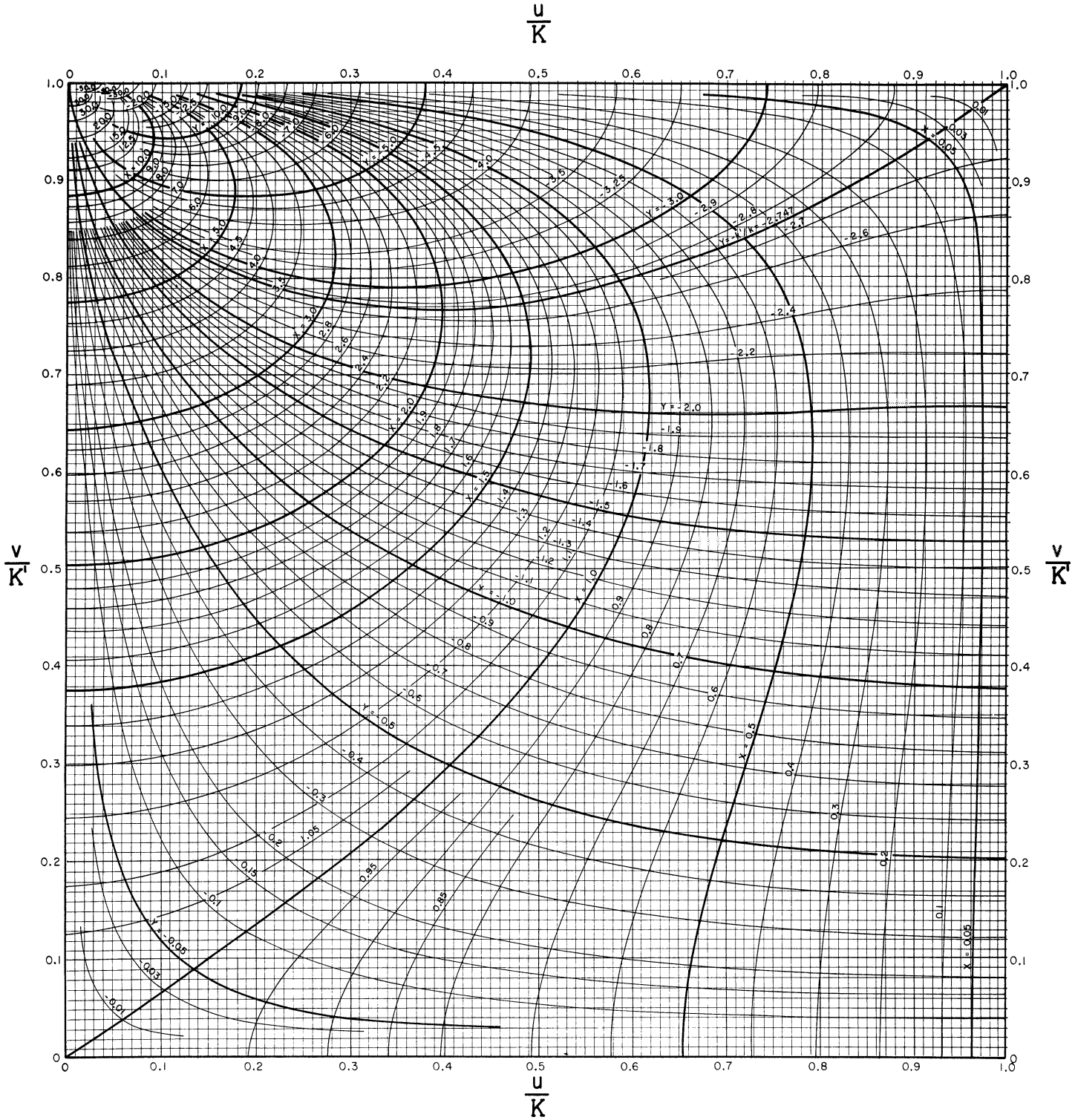
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 15° = 0.2588$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 20° = 0.342020$; $K = 1.620026$, $K' = 2.504550$

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9869	0.9482	0.8852	0.8002	0.6960	0.5758	0.4428	0.3004	0.1517	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.032	1.018	0.9774	0.9117	0.8233	0.7153	0.5910	0.4541	0.3078	0.1554	0
	y	0	0.04093	0.08028	0.1166	0.1489	0.1764	0.1987	0.2158	0.2278	0.2349	0.2372
0.2	x	1.129	1.114	1.067	0.9929	0.8934	0.7733	0.6366	0.4875	0.3296	0.1661	0
	y	0	0.08579	0.1679	0.2433	0.3095	0.3652	0.4099	0.4437	0.4673	0.4811	0.4856
0.3	x	1.304	1.284	1.226	1.134	1.013	0.8706	0.7116	0.5416	0.3644	0.1831	0
	y	0	0.1397	0.2725	0.3925	0.4959	0.5808	0.6473	0.6964	0.7298	0.7492	0.7555
0.4	x	1.574	1.547	1.467	1.343	1.186	1.006	0.8125	0.6120	0.4086	0.2043	0
	y	0	0.2110	0.4088	0.5829	0.7276	0.8415	0.9265	0.9866	1.026	1.048	1.055
0.5	x	1.981	1.938	1.816	1.633	1.411	1.172	0.9283	0.6880	0.4538	0.2253	0
	y	0	0.3160	0.6048	0.8471	1.035	1.172	1.266	1.326	1.362	1.381	1.387
0.6	x	2.605	2.526	2.312	2.011	1.676	1.344	1.033	0.7470	0.4843	0.2379	0
	y	0	0.4926	0.9207	1.248	1.470	1.607	1.683	1.721	1.738	1.744	1.745
0.7	x	3.637	3.461	3.013	2.458	1.922	1.457	1.071	0.7491	0.4747	0.2302	0
	y	0	0.8460	1.505	1.912	2.113	2.184	2.188	2.164	2.136	2.115	2.108
0.8	x	5.658	5.103	3.927	2.806	1.965	1.372	0.9512	0.6397	0.3952	0.1889	0
	y	0	1.762	2.770	3.082	3.051	2.905	2.746	2.611	2.512	2.452	2.433
0.9	x	11.58	8.137	4.269	2.346	1.404	0.8928	0.5844	0.3793	0.2293	0.1084	0
	y	0	5.383	5.769	4.937	4.177	3.621	3.232	2.965	2.792	2.695	2.663
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	18.11	9.148	6.205	4.772	3.947	3.433	3.104	2.898	2.784	2.747



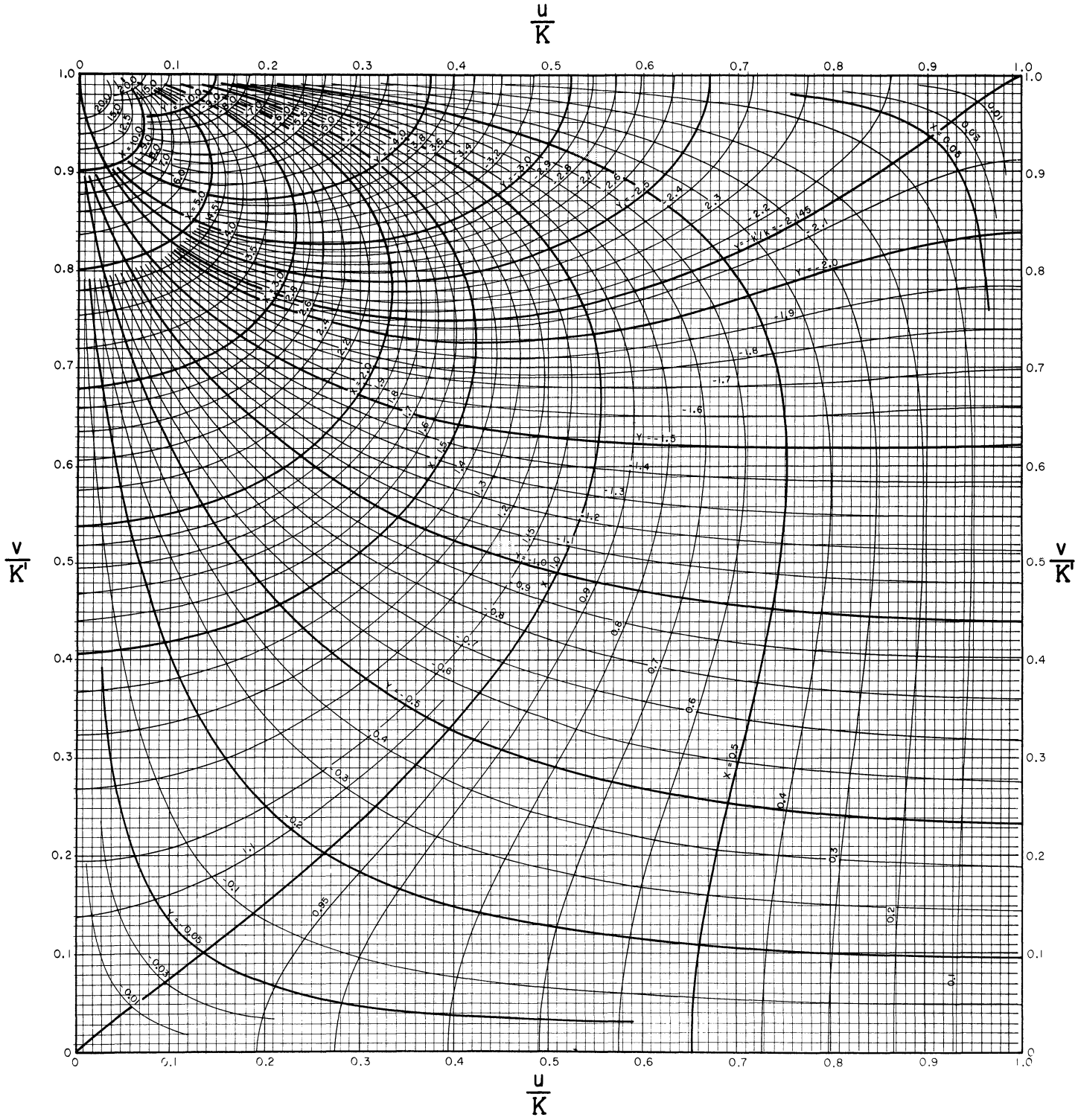
The cn-function, $z = x + iy = \mathrm{cn}(u+iv) = \sqrt{1 - \mathrm{sn}^2(u+iv)}$

$k = \sin 20^\circ = 0.3420$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 25° = 0.422618; K = 1.648995, K' = 2.308787

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9865	0.9464	0.8817	0.7950	0.6895	0.5688	0.4363	0.2954	0.1490	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.027	1.013	0.9709	0.9035	0.8134	0.7044	0.5802	0.4445	0.3006	0.1516	0
	y	0	0.03835	0.07491	0.1081	0.1370	0.1608	0.1796	0.1936	0.2031	0.2086	0.2104
0.2	x	1.110	1.094	1.046	0.9698	0.8692	0.7491	0.6141	0.4686	0.3159	0.1590	0
	y	0	0.08035	0.1566	0.2252	0.2839	0.3318	0.3688	0.3958	0.4140	0.4244	0.4278
0.3	x	1.258	1.237	1.178	1.084	0.9632	0.8225	0.6684	0.5062	0.3393	0.1701	0
	y	0	0.1308	0.2537	0.3623	0.4528	0.5242	0.5777	0.6154	0.6400	0.6538	0.6582
0.4	x	1.488	1.459	1.377	1.252	1.096	0.9212	0.7381	0.5523	0.3669	0.1829	0
	y	0	0.1978	0.3802	0.5361	0.6601	0.7526	0.8176	0.8605	0.8869	0.9010	0.9054
0.5	x	1.835	1.790	1.663	1.479	1.262	1.036	0.8115	0.5964	0.3911	0.1935	0
	y	0	0.2968	0.5621	0.7752	0.9310	1.036	1.100	1.138	1.157	1.166	1.169
0.6	x	2.369	2.286	2.065	1.765	1.445	1.141	0.8653	0.6201	0.3994	0.1955	0
	y	0	0.4642	0.8541	1.133	1.305	1.396	1.436	1.448	1.447	1.443	1.441
0.7	x	3.258	3.072	2.615	2.078	1.587	1.182	0.8573	0.5947	0.3748	0.1812	0
	y	0	0.7994	1.387	1.710	1.837	1.854	1.824	1.780	1.740	1.714	1.705
0.8	x	5.013	4.431	3.274	2.256	1.541	1.060	0.7283	0.4872	0.3001	0.1433	0
	y	0	1.659	2.498	2.671	2.566	2.393	2.229	2.098	2.005	1.950	1.932
0.9	x	10.19	6.728	3.309	1.766	1.044	0.6606	0.4317	0.2801	0.1694	0.08008	0
	y	0	4.888	4.898	4.055	3.373	2.894	2.564	2.340	2.196	2.115	2.088
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	14.39	7.260	4.915	3.770	3.110	2.698	2.432	2.266	2.174	2.145



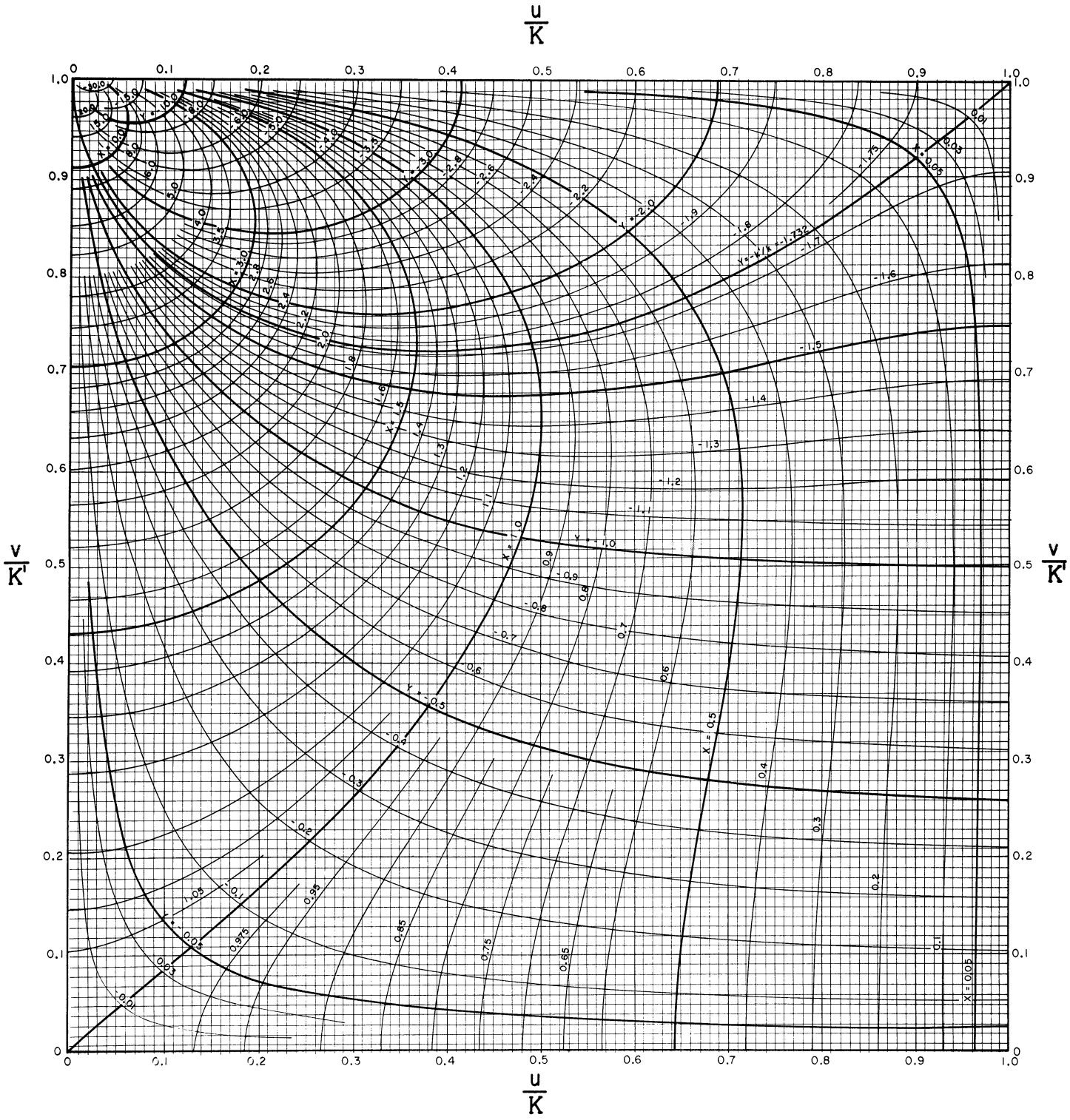
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 25° = 0.4226$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 30° = 0.500000; K = 1.685750, K' = 2.156516

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9859	0.9442	0.8773	0.7883	0.6813	0.5600	0.4282	0.2892	0.1456	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.023	1.009	0.9651	0.8954	0.8032	0.6928	0.5685	0.4340	0.2927	0.1473	0
	y	0	0.03657	0.07106	0.1018	0.1277	0.1483	0.1638	0.1749	0.1821	0.1862	0.1875
0.2	x	1.096	1.079	1.029	0.9505	0.8479	0.7271	0.5932	0.4508	0.3029	0.1521	0
	y	0	0.07667	0.1485	0.2118	0.2641	0.3050	0.3351	0.3560	0.3694	0.3768	0.3791
0.3	x	1.225	1.204	1.141	1.045	0.9222	0.7822	0.6318	0.4759	0.3178	0.1590	0
	y	0	0.1250	0.2407	0.3402	0.4198	0.4795	0.5216	0.5494	0.5663	0.5753	0.5781
0.4	x	1.427	1.397	1.311	1.181	1.025	0.8537	0.6784	0.5043	0.3335	0.1658	0
	y	0	0.1894	0.3610	0.5023	0.6090	0.6834	0.7314	0.7601	0.7760	0.7835	0.7858
0.5	x	1.732	1.684	1.551	1.363	1.148	0.9306	0.7221	0.5266	0.3435	0.1694	0
	y	0	0.2852	0.5337	0.7237	0.8526	0.9306	0.9724	0.9915	0.9984	0.9999	1.000
0.6	x	2.204	2.116	1.884	1.582	1.273	0.9899	0.7426	0.5279	0.3383	0.1651	0
	y	0	0.4477	0.8097	1.050	1.181	1.237	1.249	1.242	1.228	1.218	1.214
0.7	x	2.996	2.797	2.326	1.801	1.346	0.9867	0.7082	0.4880	0.3064	0.1478	0
	y	0	0.7729	1.305	1.560	1.630	1.609	1.556	1.499	1.453	1.424	1.414
0.8	x	4.569	3.953	2.803	1.869	1.250	0.8498	0.5800	0.3868	0.2379	0.1135	0
	y	0	0.1596	2.299	2.366	2.213	2.026	1.862	1.736	1.648	1.597	1.580
0.9	x	9.239	5.721	2.654	1.384	0.8109	0.5117	0.3343	2.171	0.1314	0.06218	0
	y	0	4.528	4.266	3.433	2.813	2.391	2.103	1.910	1.785	1.715	1.692
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	11.89	5.990	4.045	3.093	2.542	2.197	1.974	1.834	1.757	1.732



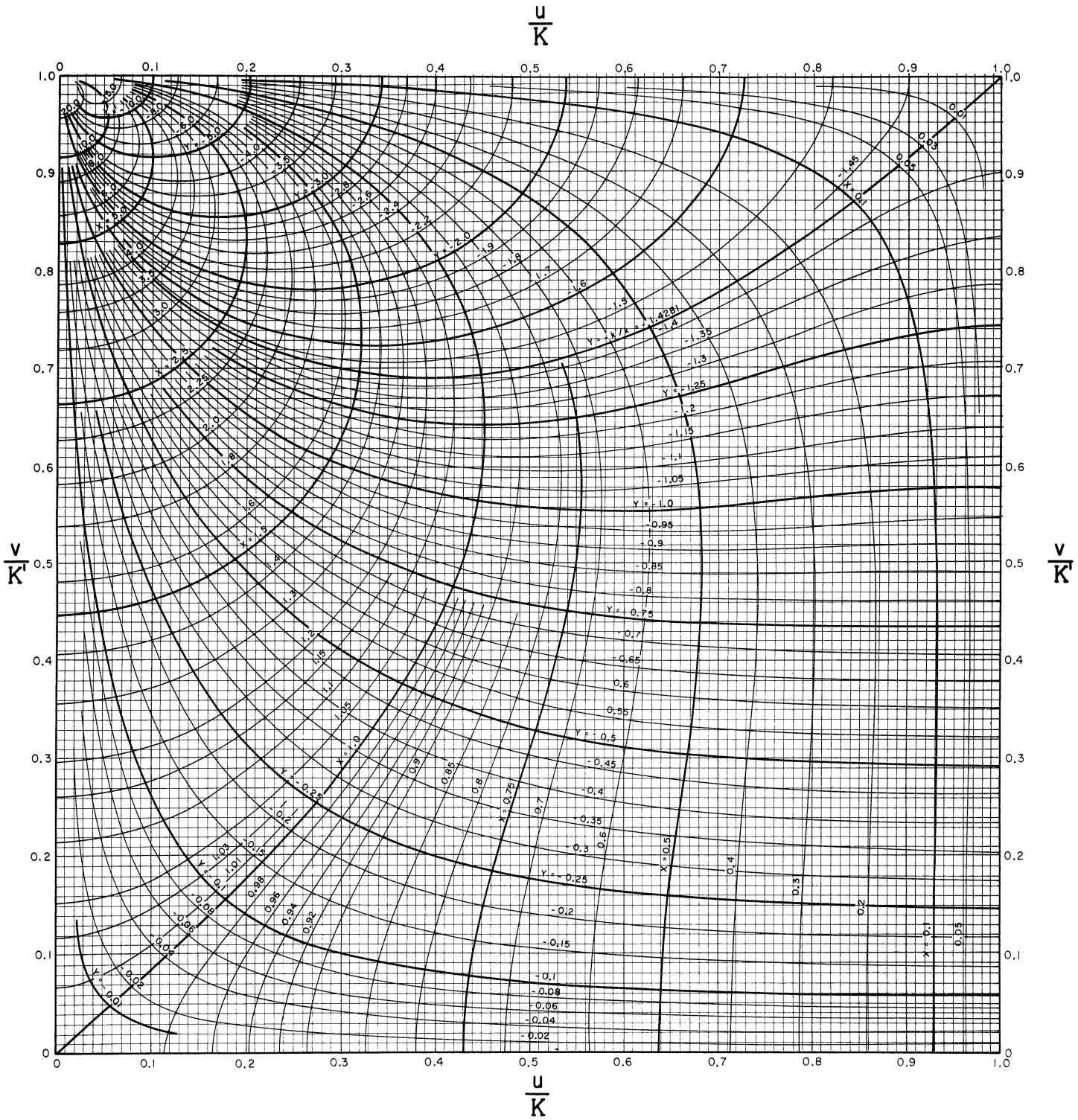
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 30° = 0.5000$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 35° = 0.573576$; $K = 1.731245$, $K' = 2.034715$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9850	0.9414	0.8717	0.7801	0.6710	0.5491	0.4183	0.2816	0.1416	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.021	1.005	0.9595	0.8869	0.7920	0.6799	0.5552	0.4222	0.2839	0.1426	0
	y	0	0.03537	0.06832	0.09692	0.1201	0.1377	0.1502	0.1585	0.1636	0.1662	0.1671
0.2	x	1.086	1.067	1.015	0.9331	0.8278	0.7057	0.5726	0.4331	0.2900	0.1453	0
	y	0	0.07426	0.1429	0.2016	0.2481	0.2825	0.3062	0.3214	0.3304	0.3349	0.3363
0.3	x	1.201	1.178	1.112	1.012	0.8864	0.7462	0.5987	0.4485	0.2982	0.1488	0
	y	0	0.1213	0.2318	0.3235	0.3934	0.4424	0.4740	0.4928	0.5030	0.5078	0.5091
0.4	x	1.382	1.349	1.258	1.124	0.9649	0.7963	0.6276	0.4635	0.3051	0.1513	0
	y	0	0.1844	0.3479	0.4769	0.5683	0.6264	0.6595	0.6759	0.6828	0.6849	0.6854
0.5	x	1.656	1.604	1.464	1.269	1.055	0.8450	0.6494	0.4703	0.3052	0.1501	0
	y	0	0.2787	0.5145	0.6846	0.7899	0.8450	0.8674	0.8717	0.8684	0.8640	0.8622
0.6	x	2.084	1.988	1.744	1.437	1.136	0.8715	0.6472	0.4570	0.2916	0.1420	0
	y	0	0.4391	0.7791	0.9855	1.081	1.108	1.099	1.077	1.055	1.039	1.034
0.7	x	2.805	2.590	2.101	1.585	1.161	0.8395	0.5974	0.4096	0.2564	0.1235	0
	y	0	0.7591	1.246	1.442	1.466	1.416	1.346	1.281	1.231	1.200	1.189
0.8	x	4.247	3.588	2.441	1.578	1.037	0.6984	0.4745	0.3159	0.1942	0.09261	0
	y	0	1.557	2.143	2.126	1.939	1.744	1.583	1.462	1.379	1.331	1.316
0.9	x	8.549	4.950	2.177	1.113	0.6484	0.4087	0.2673	0.1739	0.1054	0.04993	0
	y	0	4.246	3.777	2.965	2.397	2.018	1.763	1.591	1.481	1.419	1.399
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	10.09	5.072	3.414	2.601	2.128	1.831	1.638	1.517	1.450	1.428



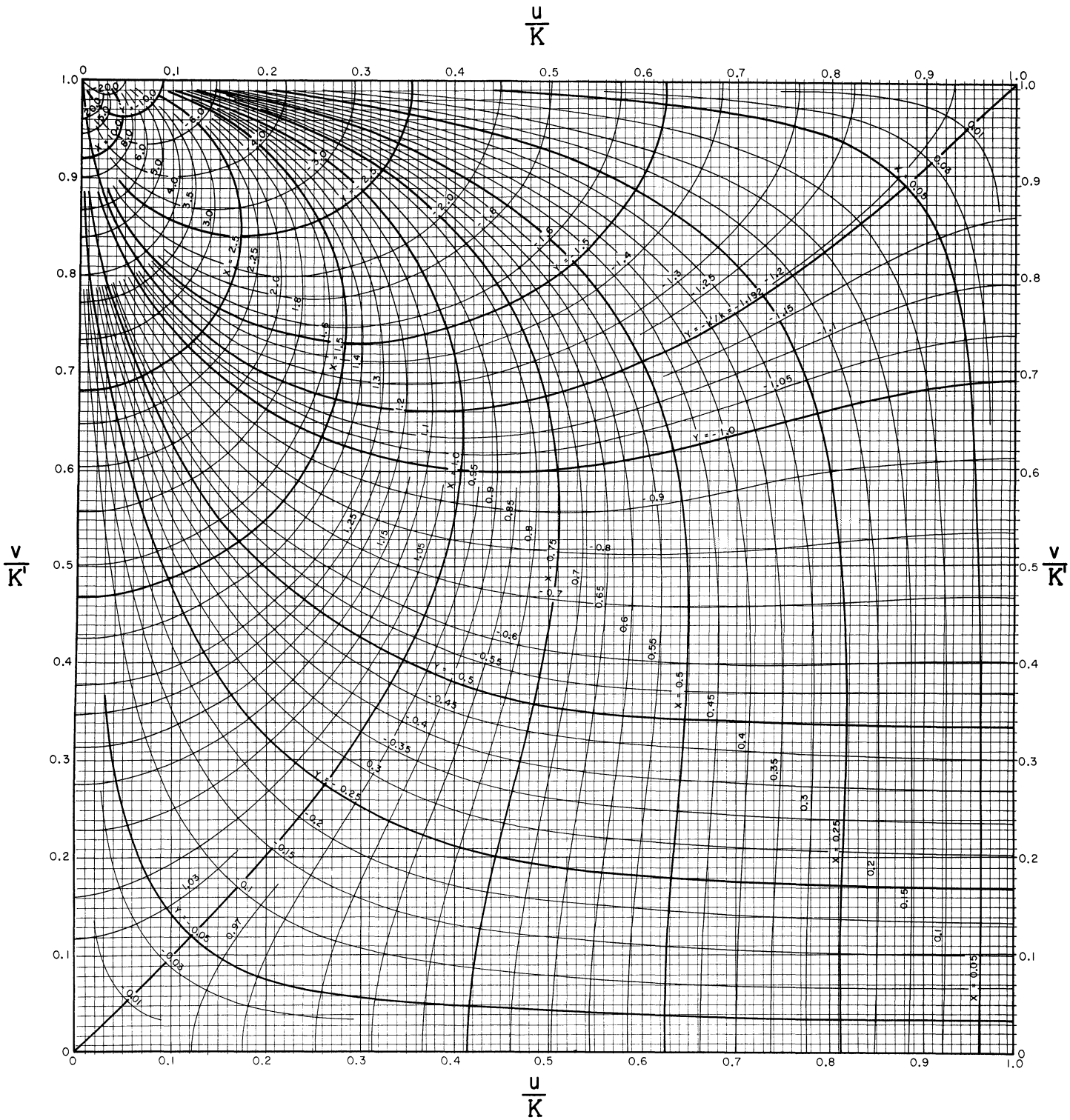
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 35° = 05736$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 40° = 0.642788; K = 1.786769; K' = 1.935581

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9841	0.9379	0.8648	0.7699	0.6586	0.5361	0.4064	0.2726	0.1367	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.019	1.002	0.9538	0.8776	0.7794	0.6651	0.5401	0.4087	0.2738	0.1372	0
	y	0	0.03466	0.06644	0.09317	0.1138	0.1285	0.1380	0.1437	0.1467	0.1480	0.1484
0.2	x	1.078	1.058	1.003	0.9165	0.8077	0.6839	0.5515	0.4149	0.2767	0.1383	0
	y	0	0.07287	0.1390	0.1937	0.2349	0.2630	0.2806	0.2904	0.2952	0.2972	0.2977
0.3	x	1.182	1.158	1.087	0.9818	0.8532	0.7124	0.5674	0.4225	0.2798	0.1392	0
	y	0	0.1194	0.2258	0.3107	0.3714	0.4103	0.4322	0.4427	0.4468	0.4478	0.4479
0.4	x	1.347	1.312	1.214	1.074	0.9121	0.7452	0.5824	0.4274	0.2800	0.1385	0
	y	0	0.1820	0.3393	0.4573	0.5343	0.5774	0.5969	0.6025	0.6016	0.5993	0.5982
0.5	x	1.599	1.542	1.392	1.190	0.9754	0.7719	0.5878	0.4228	0.2733	0.1341	0
	y	0	0.2761	0.5018	0.6538	0.7374	0.7719	0.7775	0.7694	0.7578	0.7488	0.7455
0.6	x	1.992	1.888	1.628	1.316	1.023	0.7743	0.5698	0.4000	0.2543	0.1236	0
	y	0	0.4362	0.7578	0.9334	0.9980	0.9993	0.9728	0.9394	0.9103	0.8912	0.8846
0.7	x	2.661	2.427	1.916	1.409	1.013	0.7234	0.5112	0.3492	0.2182	0.1050	0
	y	0	0.7544	1.200	1.344	1.329	1.256	1.174	1.103	1.051	1.019	1.008
0.8	x	4.004	3.296	2.149	1.349	0.8734	0.5839	0.3956	0.2631	0.1618	0.07718	0
	y	0	1.534	2.015	1.928	1.717	1.518	1.361	1.245	1.166	1.121	1.106
0.9	x	8.030	4.331	1.812	0.9122	0.5292	0.3337	0.2186	0.1425	0.08661	0.04107	0
	y	0	4.014	3.382	2.595	2.071	1.727	1.497	1.343	1.243	1.188	1.170
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	8.716	4.371	2.932	2.223	1.810	1.548	1.378	1.271	1.211	1.192



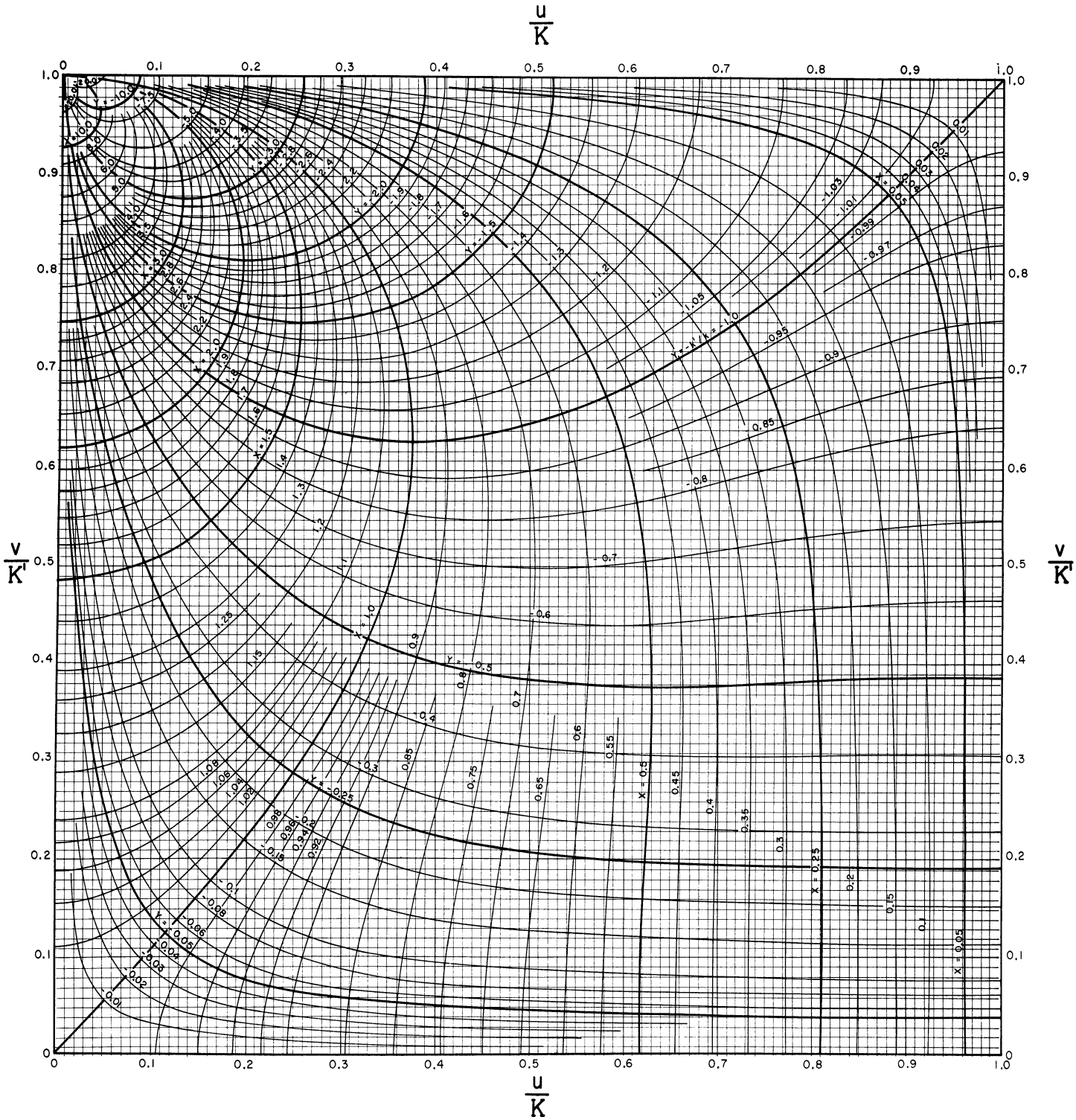
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 40° = 0.6428$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 45° = 0.707107; K = K' = 1.854075

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9830	0.9335	0.8563	0.7574	0.6436	0.5205	0.3924	0.2621	0.1311	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.017	0.9994	0.9476	0.8671	0.7649	0.6481	0.5228	0.3934	0.2624	0.1311	0
	y	0	0.03436	0.06528	0.09029	0.1084	0.1202	0.1268	0.1299	0.1309	0.1311	0.1311
0.2	x	1.071	1.050	0.9906	0.8995	0.7866	0.6609	0.5291	0.3956	0.2627	0.1309	0
	y	0	0.07236	0.1367	0.1877	0.2235	0.2455	0.2571	0.2617	0.2627	0.2624	0.2621
0.3	x	1.168	1.141	1.065	0.9537	0.8209	0.6792	0.5366	0.3971	0.2617	0.1299	0
	y	0	0.1188	0.2223	0.3008	0.3525	0.3815	0.3941	0.3971	0.3956	0.3934	0.3924
0.4	x	1.320	1.282	1.176	1.029	0.8633	0.6978	0.5407	0.3941	0.2571	0.1268	0
	y	0	0.1817	0.3342	0.4417	0.5047	0.5336	0.5407	0.5366	0.5291	0.5228	0.5205
0.5	x	1.554	1.492	1.330	1.119	0.9043	0.7071	0.5336	0.3815	0.2455	0.1202	0
	y	0	0.2766	0.4940	0.6286	0.6915	0.7071	0.6978	0.6792	0.6609	0.6481	0.6436
0.6	x	1.921	1.806	1.529	1.210	0.9248	0.6915	0.5047	0.3525	0.2235	0.1084	0
	y	0	0.4381	0.7429	0.8890	0.9248	0.9043	0.8633	0.8209	0.7866	0.7649	0.7574
0.7	x	2.549	2.292	1.758	1.259	0.8890	0.6286	0.4417	0.3008	0.1877	0.09029	0
	y	0	0.7569	1.163	1.259	1.210	1.119	1.029	0.9537	0.8995	0.8671	0.8563
0.8	x	3.816	3.052	1.903	1.163	0.7429	0.4940	0.3342	0.2223	0.1367	0.06528	0
	y	0	1.523	1.903	1.758	1.529	1.330	1.176	1.065	0.9906	0.9476	0.9335
0.9	x	7.628	3.814	1.523	0.7569	0.4381	0.2766	0.1817	0.1188	0.07236	0.03436	0
	y	0	3.814	3.052	2.292	1.806	1.492	1.282	1.141	1.050	0.9994	0.9830
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	7.628	3.816	2.549	1.921	1.554	1.320	1.168	1.071	1.017	1.000



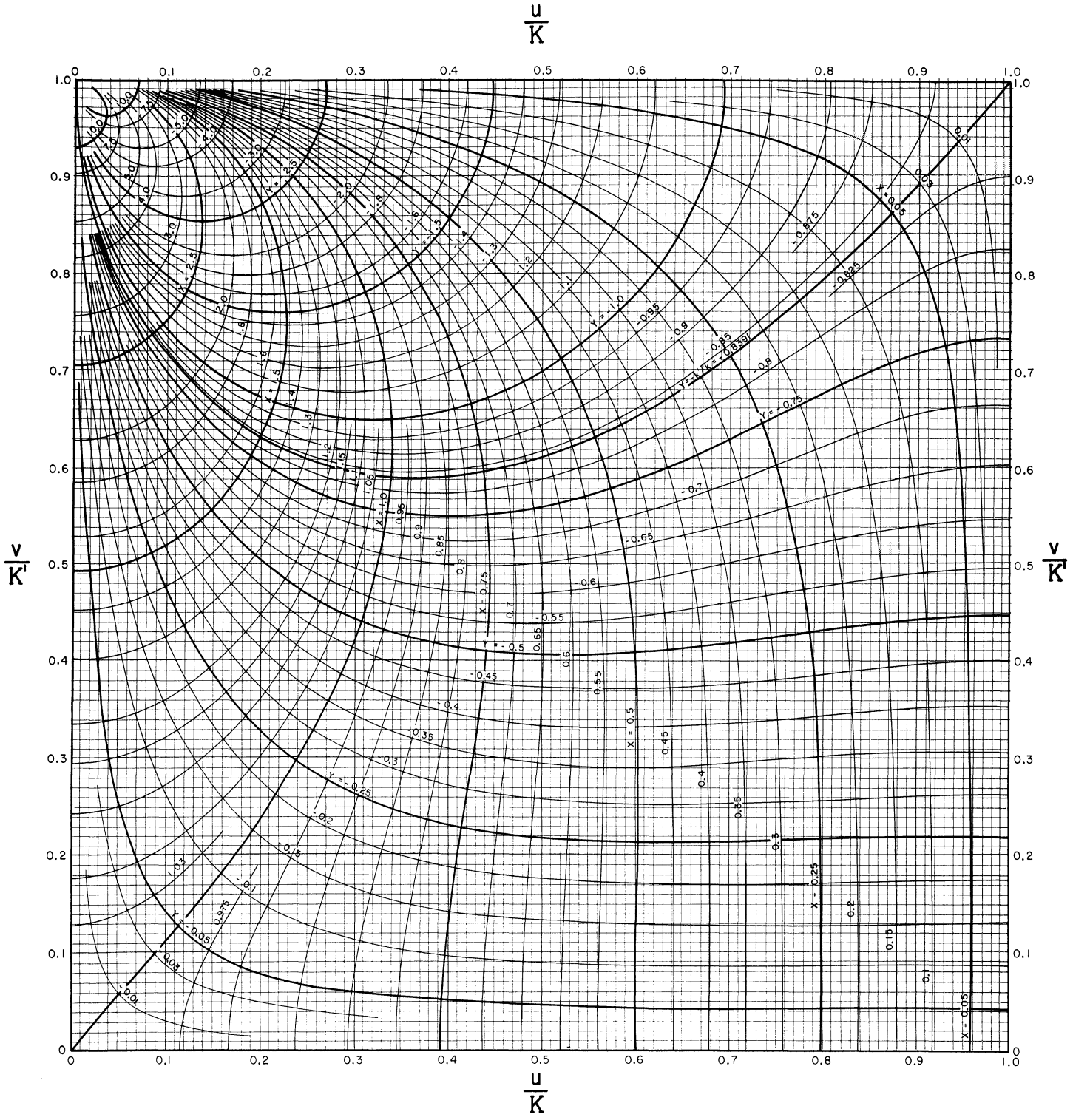
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 45° = 0.7071$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 50° = 0.766044$; $K = 1.935581$, $K' = 1.786769$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9815	0.9281	0.8457	0.7423	0.6255	0.5019	0.3759	0.2498	0.1245	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.016	0.9966	0.9405	0.8547	0.7478	0.6283	0.5028	0.3758	0.2493	0.1242	0
	y	0	0.03446	0.06476	0.08811	0.1037	0.1125	0.1162	0.1168	0.1161	0.1151	0.1147
0.2	x	1.066	1.043	0.9786	0.8816	0.7639	0.6359	0.5048	0.3749	0.2477	0.1231	0
	y	0	0.07268	0.1357	0.1831	0.2134	0.2293	0.2350	0.2348	0.2322	0.2297	0.2288
0.3	x	1.156	1.127	1.044	0.9258	0.7883	0.6456	0.5056	0.3715	0.2436	0.1205	0
	y	0	0.1196	0.2208	0.2930	0.3356	0.3548	0.3586	0.3545	0.3481	0.3429	0.3410
0.4	x	1.299	1.256	1.142	0.9854	0.8163	0.6524	0.5009	0.3626	0.2354	0.1158	0
	y	0	0.1835	0.3320	0.4290	0.4781	0.4932	0.4887	0.4761	0.4627	0.4532	0.4498
0.5	x	1.518	1.449	1.274	1.054	0.8386	0.6477	0.4845	0.3442	0.2207	0.1078	0
	y	0	0.2800	0.4899	0.6070	0.6497	0.6477	0.6253	0.5978	0.5739	0.5581	0.5526
0.6	x	1.865	1.738	1.440	1.115	0.8374	0.6188	0.4483	0.3117	0.1971	0.09551	0
	y	0	0.4441	0.7329	0.8496	0.8582	0.8185	0.7653	0.7159	0.6777	0.6540	0.6460
0.7	x	2.461	2.178	1.618	1.128	0.7832	0.5486	0.3837	0.2607	0.1625	0.07818	0
	y	0	0.7654	1.132	1.182	1.104	0.9983	0.9011	0.8239	0.7690	0.7364	0.7256
0.8	x	3.668	2.838	1.690	1.007	0.6358	0.4211	0.2847	0.1895	0.1167	0.05575	0
	y	0	1.520	1.803	1.608	1.366	1.168	1.019	0.9125	0.8413	0.8003	0.7870
0.9	x	7.313	3.368	1.287	0.6330	0.3660	0.2317	0.1527	0.1002	0.06114	0.02908	0
	y	0	3.634	2.766	2.036	1.584	1.294	1.101	0.9714	0.8880	0.8410	0.8258
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.737	3.359	2.232	1.672	1.341	1.130	0.9922	0.9041	0.8550	0.8391



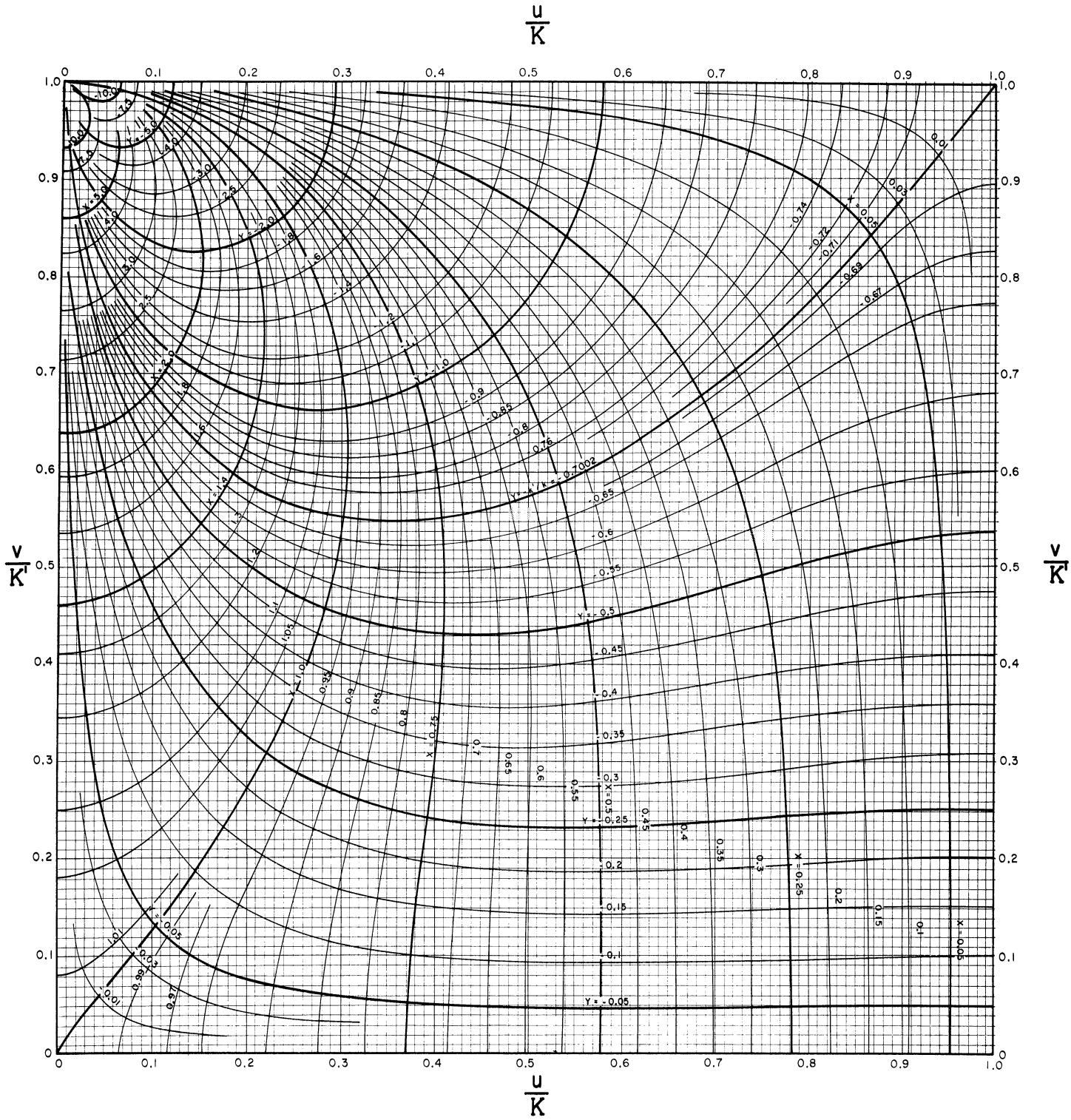
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 50° = 0.7660$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 55° = 0.819152; K = 2.034715, K' = 1.731245

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9796	0.9212	0.8326	0.7237	0.6037	0.4799	0.3565	0.2355	0.1170	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.015	0.9936	0.9323	0.8400	0.7276	0.6051	0.4796	0.3556	0.2345	0.1164	0
	y	0	0.03497	0.06486	0.08652	0.09945	0.1051	0.1059	0.1042	0.1018	0.09986	0.09915
0.2	x	1.062	1.037	0.9659	0.8617	0.7384	0.6080	0.4781	0.3522	0.2313	0.1145	0
	y	0	0.07383	0.1359	0.1796	0.2042	0.2137	0.2136	0.2088	0.2031	0.1988	0.1972
0.3	x	1.147	1.114	1.024	0.8968	0.7540	0.6104	0.4733	0.3451	0.2251	0.1110	0
	y	0	0.1217	0.2212	0.2868	0.3200	0.3293	0.3246	0.3140	0.3032	0.2956	0.2929
0.4	x	1.282	1.234	1.108	0.9425	0.7695	0.6073	0.4618	0.3319	0.2144	0.1052	0
	y	0	0.1872	0.3323	0.4183	0.4532	0.4547	0.4395	0.4192	0.4010	0.3888	0.3845
0.5	x	1.490	1.413	1.221	0.9916	0.7758	0.5917	0.4386	0.3098	0.1978	0.09642	0
	y	0	0.2862	0.4890	0.5878	0.6103	0.5917	0.5576	0.5225	0.4941	0.4760	0.4699
0.6	x	1.821	1.678	1.358	1.027	0.7573	0.5531	0.3979	0.2755	0.1738	0.08410	0
	y	0	0.4540	0.7262	0.8129	0.7957	0.7388	0.6756	0.6206	0.5796	0.5546	0.5462
0.7	x	2.391	2.076	1.489	1.010	0.6901	0.4794	0.3339	0.2265	0.1411	0.06786	0
	y	0	0.7795	1.105	1.110	1.006	0.8886	0.7869	0.7083	0.6534	0.6210	0.6104
0.8	x	3.551	2.645	1.501	0.8722	0.5455	0.3603	0.2436	0.1623	0.1000	0.04784	0
	y	0	1.524	1.709	1.471	1.221	1.025	0.8809	0.7788	0.7108	0.6719	0.6592
0.9	x	7.064	2.973	1.090	0.5315	0.3074	0.1952	0.1291	0.08496	0.05200	0.02477	0
	y	0	3.466	2.513	1.814	1.392	1.123	0.9448	0.8249	0.7475	0.7039	0.6898
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.986	2.974	1.964	1.459	1.160	0.9676	0.8410	0.7601	0.7148	0.7002



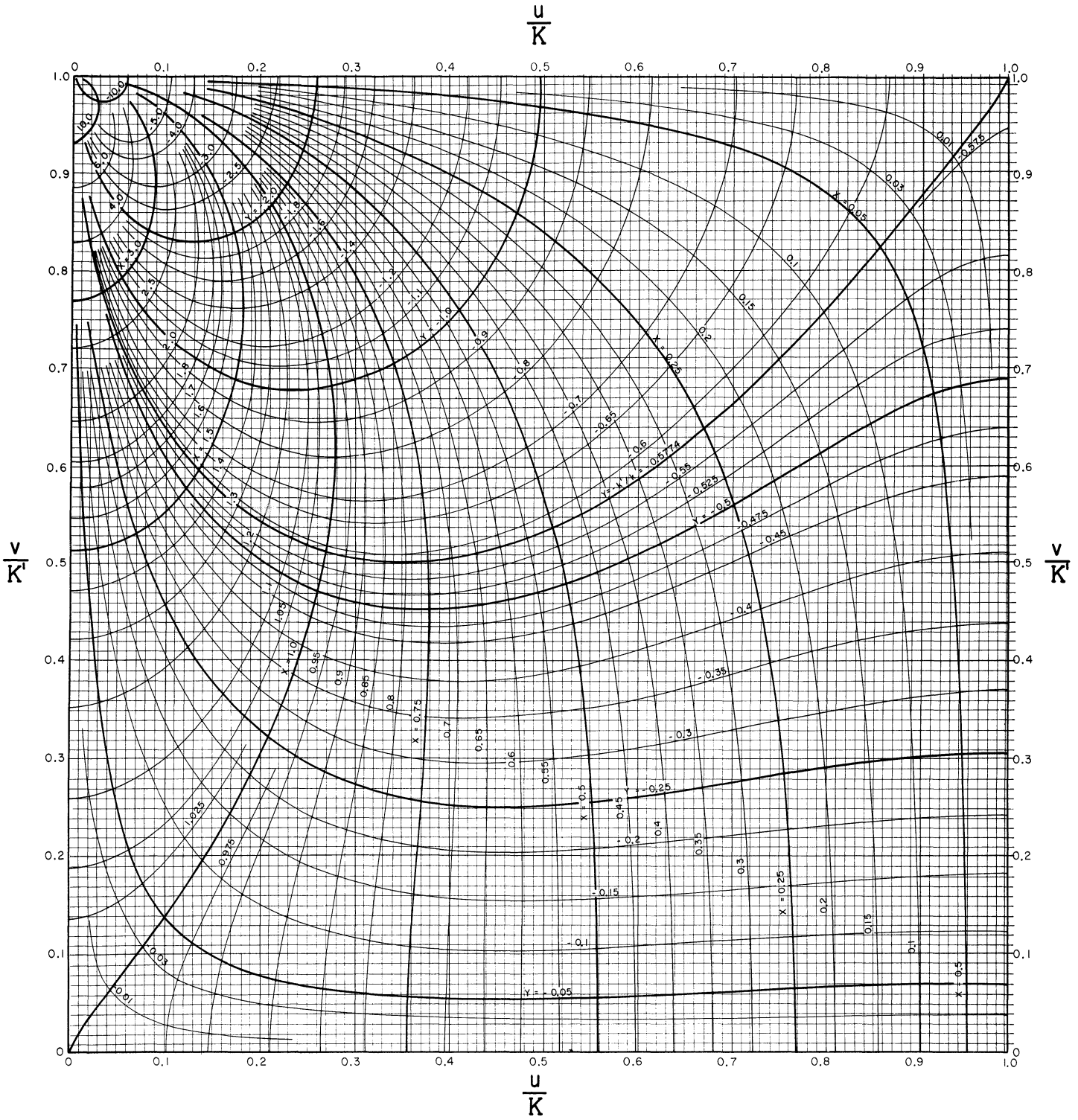
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 55° = 0.8192$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 60° = 0.866025; K = 2.156516, K' = 1.685750

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9771	0.9124	0.8161	0.7007	0.5774	0.4537	0.3338	0.2189	0.1082	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.014	0.9901	0.9221	0.8219	0.7030	0.5773	0.4524	0.3322	0.2175	0.1075	0
	y	0	0.03590	0.06552	0.08535	0.09533	0.09779	0.09572	0.09177	0.08783	0.08507	0.08409
0.2	x	1.059	1.031	0.9518	0.8388	0.7092	0.5764	0.4480	0.3270	0.2133	0.1052	0
	y	0	0.07588	0.1373	0.1769	0.1953	0.1983	0.1925	0.1835	0.1749	0.1690	0.1669
0.3	x	1.140	1.103	1.002	0.8654	0.7168	0.5724	0.4389	0.3172	0.2055	0.1010	0
	y	0	0.1253	0.2233	0.2818	0.3048	0.3040	0.2912	0.2748	0.2603	0.2506	0.2472
0.4	x	1.268	1.214	1.075	0.8982	0.7212	0.5614	0.4223	0.3011	0.1935	0.09460	0
	y	0	0.1930	0.3349	0.4089	0.4287	0.4169	0.3917	0.3648	0.3425	0.3282	0.3233
0.5	x	1.468	1.380	1.170	0.9292	0.7139	0.5373	0.3946	0.2769	0.1761	0.08561	0
	y	0	0.2954	0.4906	0.5697	0.5715	0.5373	0.4929	0.4516	0.4198	0.4000	0.3933
0.6	x	1.786	1.624	1.278	0.9413	0.6817	0.4922	0.3516	0.2424	0.1525	0.07370	0
	y	0	0.4681	0.7219	0.7771	0.7349	0.6629	0.5915	0.5324	0.4895	0.4637	0.4552
0.7	x	2.335	1.982	1.366	0.9009	0.6060	0.4178	0.2900	0.1964	0.1223	0.05877	0
	y	0	0.7988	1.079	1.040	0.9135	0.7867	0.6821	0.6032	0.5488	0.5170	0.5065
0.8	x	3.458	2.463	1.327	0.7536	0.4675	0.3081	0.2084	0.1390	0.08576	0.04103	0
	y	0	1.532	1.619	1.343	1.088	0.8956	0.7567	0.6590	0.5942	0.5572	0.5451
0.9	x	6.866	2.614	0.9216	0.4462	0.2585	0.1647	0.1094	0.07217	0.04426	0.02111	0
	y	0	3.303	2.282	1.615	1.222	0.9722	0.8064	0.6950	0.6230	0.5823	0.5692
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.334	2.638	1.730	1.273	1.000	0.8239	0.7074	0.6328	0.5909	0.5774



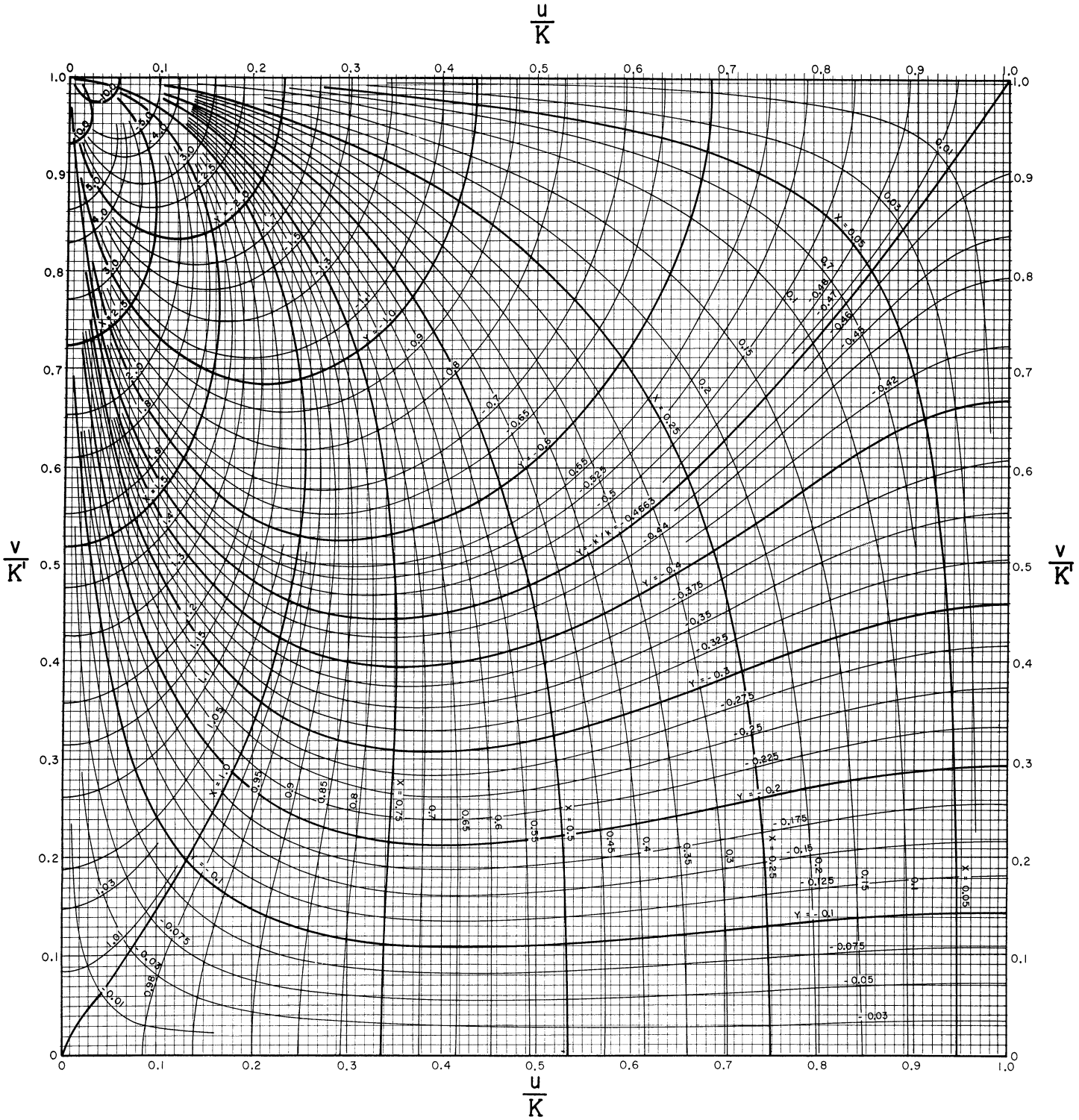
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 60° = 0.8660$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 65° = 0.906308; K = 2.308787, K' = 1.648995

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9738	0.9009	0.7950	0.6720	0.5450	0.4222	0.3069	0.1995	0.09813	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.014	0.9861	0.9094	0.7992	0.6729	0.5438	0.4201	0.3049	0.1979	0.09729	0
	y	0	0.03734	0.06681	0.08450	0.09117	0.09021	0.08531	0.07934	0.07412	0.07068	0.06949
0.2	x	1.057	1.024	0.9351	0.8114	0.6747	0.5396	0.4136	0.2985	0.1931	0.09471	0
	y	0	0.07900	0.1400	0.1748	0.1863	0.1824	0.1711	0.1582	0.1473	0.1402	0.1377
0.3	x	1.134	1.091	0.9784	0.8298	0.6751	0.5305	0.4013	0.2870	0.1846	0.09027	0
	y	0	0.1306	0.2272	0.2773	0.2891	0.2781	0.2575	0.2360	0.2185	0.2073	0.2035
0.4	x	1.258	1.195	1.040	0.8503	0.6697	0.5131	0.3812	0.2694	0.1720	0.08376	0
	y	0	0.2013	0.3396	0.3997	0.4035	0.3784	0.3442	0.3117	0.2864	0.2706	0.2652
0.5	x	1.450	1.349	1.116	0.8646	0.6510	0.4829	0.3509	0.2445	0.1547	0.07500	0
	y	0	0.3081	0.4943	0.5511	0.5318	0.4829	0.4296	0.3835	0.3493	0.3285	0.3215
0.6	x	1.758	1.573	1.197	0.8564	0.6084	0.4341	0.3078	0.2111	0.1324	0.06387	0
	y	0	0.4869	0.7187	0.7401	0.6739	0.5886	0.5109	0.4491	0.4053	0.3793	0.3707
0.7	x	2.292	1.891	1.245	0.7972	0.5282	0.3615	0.2500	0.1689	0.1050	0.05043	0
	y	0	0.8235	1.052	0.9692	0.8231	0.6895	0.5837	0.5055	0.4522	0.4213	0.4112
0.8	x	3.385	2.284	1.165	0.6466	0.3983	0.2621	0.1773	0.1183	0.07301	0.03493	0
	y	0	1.543	1.527	1.220	0.9627	0.7756	0.6422	0.5492	0.4878	0.4527	0.4413
0.9	x	6.711	2.279	0.7738	0.3728	0.2164	0.1384	0.09222	0.06101	0.03747	0.01788	0
	y	0	3.137	2.066	1.433	1.066	0.8345	0.6805	0.5770	0.5100	0.4722	0.4600
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	4.752	2.337	1.519	1.105	0.8555	0.6939	0.5866	0.5176	0.4788	0.4663



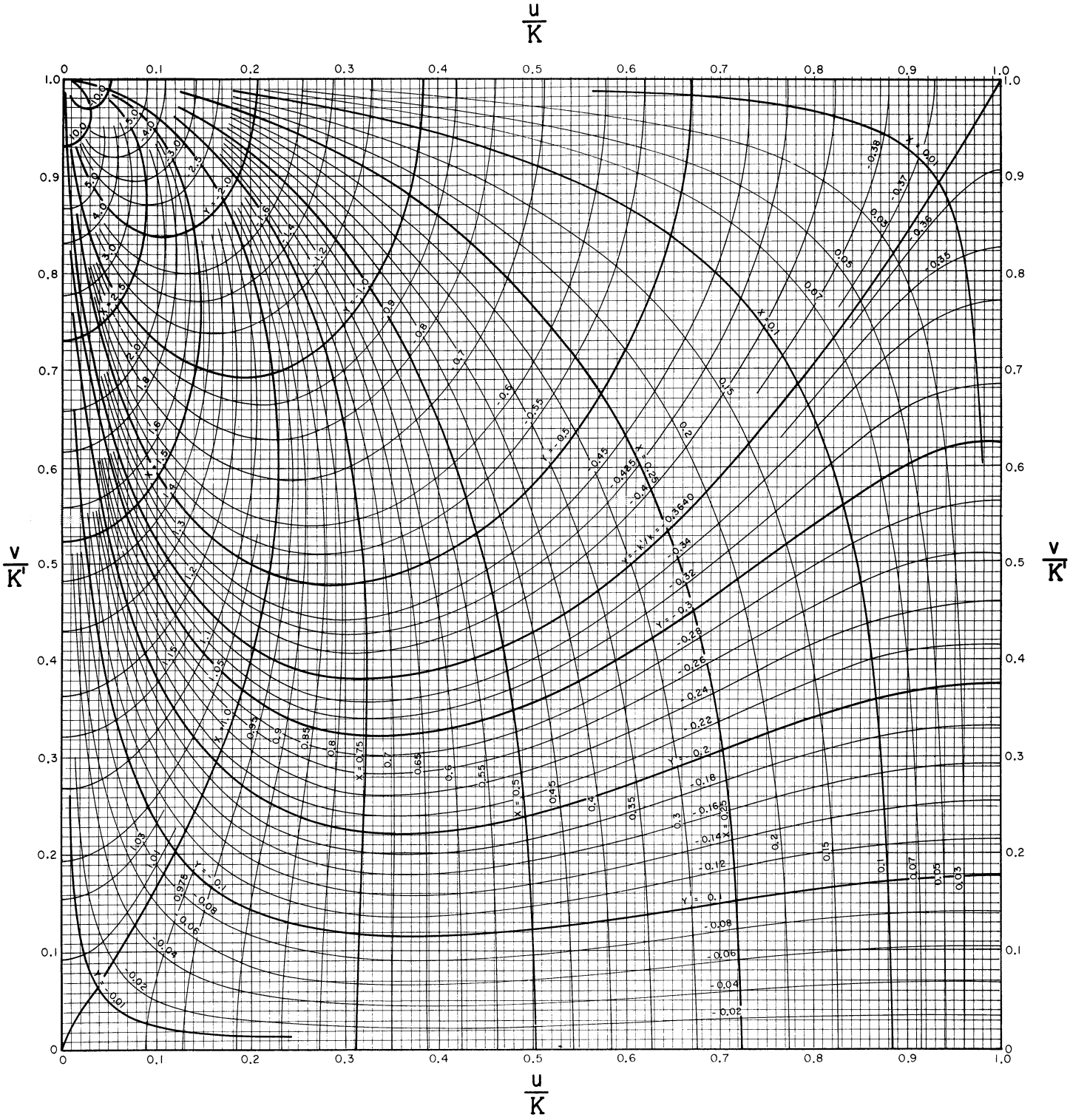
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 65° = 0.9063$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 70° = 0.939693$; $K = 2.504550$, $K' = 1.620026$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9694	0.8854	0.7671	0.6352	0.5048	0.3840	0.2750	0.1767	0.08635	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.013	0.9808	0.8926	0.7699	0.6347	0.5027	0.3814	0.2727	0.1751	0.08549	0
	y	0	0.03944	0.06876	0.08377	0.08659	0.08199	0.07437	0.06666	0.06047	0.05655	0.05522
0.2	x	1.055	1.016	0.9142	0.7774	0.6325	0.4958	0.3734	0.2656	0.1701	0.08290	0
	y	0	0.08347	0.1438	0.1728	0.1763	0.1652	0.1487	0.1326	0.1200	0.1120	0.1093
0.3	x	1.130	1.079	0.9501	0.7876	0.6265	0.4826	0.3591	0.2535	0.1615	0.07853	0
	y	0	0.1381	0.2328	0.2726	0.2719	0.2504	0.2227	0.1971	0.1774	0.1653	0.1612
0.4	x	1.250	1.176	0.9993	0.7962	0.6126	0.4606	0.3372	0.2356	0.1492	0.07231	0
	y	0	0.2127	0.3462	0.3896	0.3758	0.3379	0.2957	0.2590	0.2317	0.2151	0.2096
0.5	x	1.437	1.318	1.057	0.7948	0.5849	0.4266	0.3063	0.2114	0.1329	0.06419	0
	y	0	0.3250	0.4994	0.5303	0.4892	0.4266	0.3662	0.3169	0.2814	0.2603	0.2533
0.6	x	1.737	1.520	1.110	0.7690	0.5350	0.3768	0.2648	0.1805	0.1127	0.05419	0
	y	0	0.5110	0.7150	0.6995	0.6101	0.5137	0.4317	0.3688	0.3252	0.2997	0.2913
0.7	x	2.258	1.797	1.122	0.6958	0.4541	0.3083	0.2122	0.1429	0.08855	0.04245	0
	y	0	0.8538	1.021	0.8948	0.7320	0.5942	0.4889	0.4127	0.3614	0.3318	0.3222
0.8	x	3.329	2.099	1.008	0.5476	0.3351	0.2201	0.1488	0.09917	0.06113	0.02922	0
	y	0	1.554	1.429	1.097	0.8414	0.6608	0.5339	0.4462	0.3885	0.3557	0.3451
0.9	x	6.591	1.959	0.6414	0.3079	0.1793	0.1150	0.07680	0.05085	0.03123	0.01490	0
	y	0	2.962	1.857	1.260	0.9194	0.7053	0.5629	0.4672	0.4054	0.3705	0.3592
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	4.215	2.059	1.324	0.9480	0.7210	0.5730	0.4744	0.4111	0.3755	0.3640



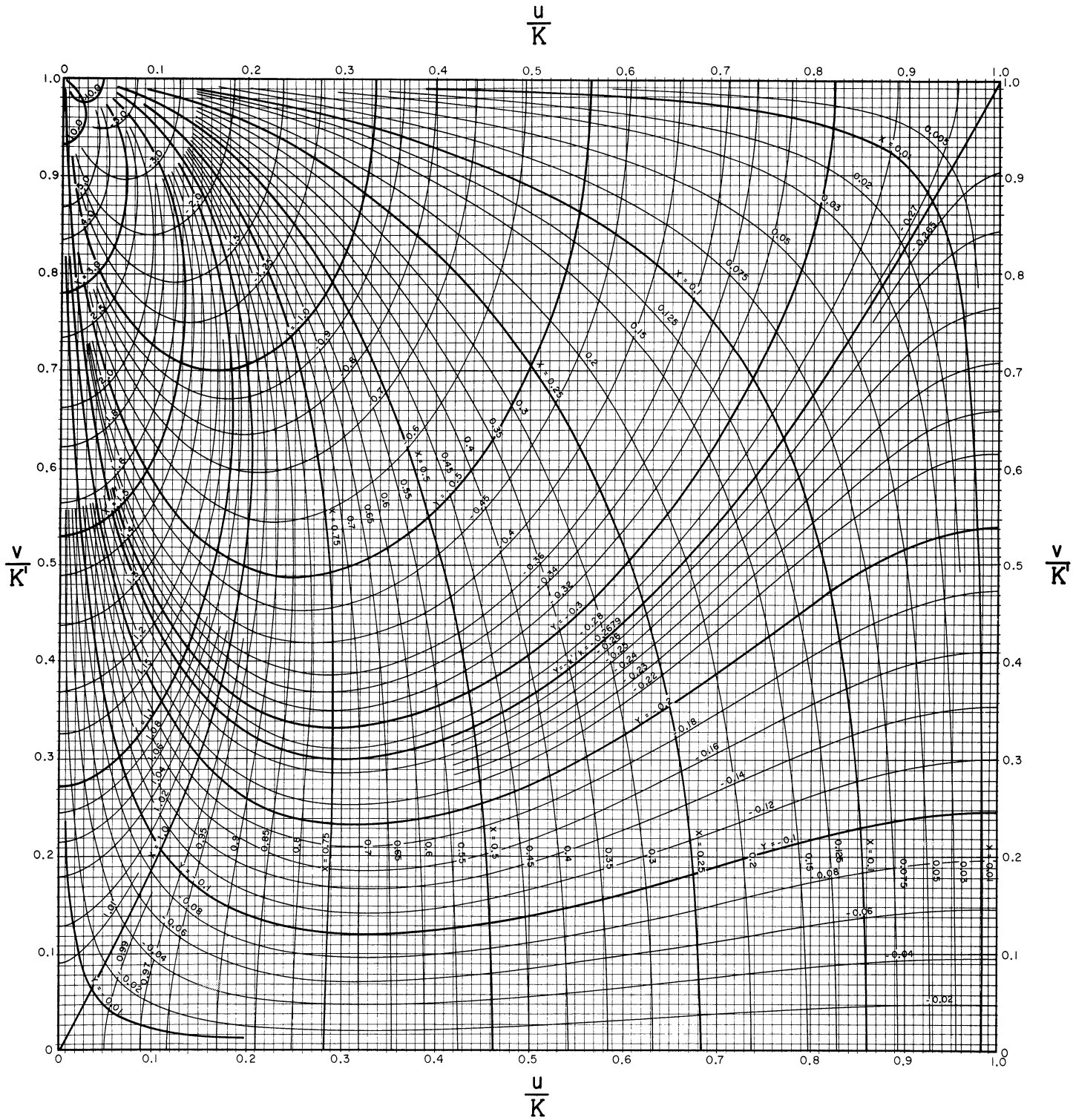
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 70° = 0.9397$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 75° = 0.965926; K = 2.768063, K' = 1.598142

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9628	0.8633	0.7289	0.5866	0.4534	0.3365	0.2362	0.1496	0.07244	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.013	0.9735	0.8691	0.7300	0.5848	0.4506	0.3337	0.2339	0.1480	0.07164	0
	y	0	0.04246	0.07143	0.08281	0.08099	0.07255	0.06247	0.05345	0.04669	0.04258	0.04121
0.2	x	1.053	1.007	0.8862	0.7327	0.5791	0.4418	0.3250	0.2269	0.1433	0.06927	0
	y	0	0.08984	0.1491	0.1701	0.1641	0.1456	0.1245	0.1061	0.09249	0.08425	0.08151
0.3	x	1.126	1.065	0.9140	0.7347	0.5674	0.4259	0.3101	0.2151	0.1354	0.06530	0
	y	0	0.1485	0.2402	0.2664	0.2512	0.2192	0.1856	0.1572	0.1365	0.1241	0.1200
0.4	x	1.244	1.154	0.9501	0.7316	0.5466	0.4013	0.2882	0.1983	0.1242	0.05977	0
	y	0	0.2284	0.3543	0.3765	0.3434	0.2931	0.2448	0.2056	0.1777	0.1612	0.1557
0.5	x	1.427	1.283	0.9884	0.7158	0.5123	0.3660	0.2586	0.1763	0.1098	0.05270	0
	y	0	0.3476	0.5045	0.5043	0.4406	0.3660	0.3007	0.2502	0.2151	0.1946	0.1878
0.6	x	1.721	1.462	1.013	0.6752	0.4587	0.3180	0.2208	0.1490	0.09232	0.04420	0
	y	0	0.5420	0.7081	0.6513	0.5402	0.4354	0.3515	0.2896	0.2476	0.2234	0.2155
0.7	x	2.233	1.691	0.9898	0.5932	0.3807	0.2560	0.1748	0.1169	0.07205	0.03441	0
	y	0	0.8900	0.9816	0.8121	0.6362	0.4974	0.3949	0.3223	0.2742	0.2468	0.2379
0.8	x	3.287	1.898	0.8520	0.4531	0.2754	0.1803	0.1214	0.08056	0.04946	0.02357	0
	y	0	1.560	1.320	0.9688	0.7189	0.5471	0.4283	0.3470	0.2940	0.2641	0.2544
0.9	x	6.502	1.643	0.5192	0.2488	0.1452	0.09332	0.06225	0.04111	0.02517	0.01198	0
	y	0	2.764	1.646	1.089	0.7758	0.5796	0.4495	0.3624	0.3063	0.2747	0.2645
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	3.699	1.791	1.134	0.7962	0.5909	0.4568	0.3676	0.3104	0.2783	0.2679



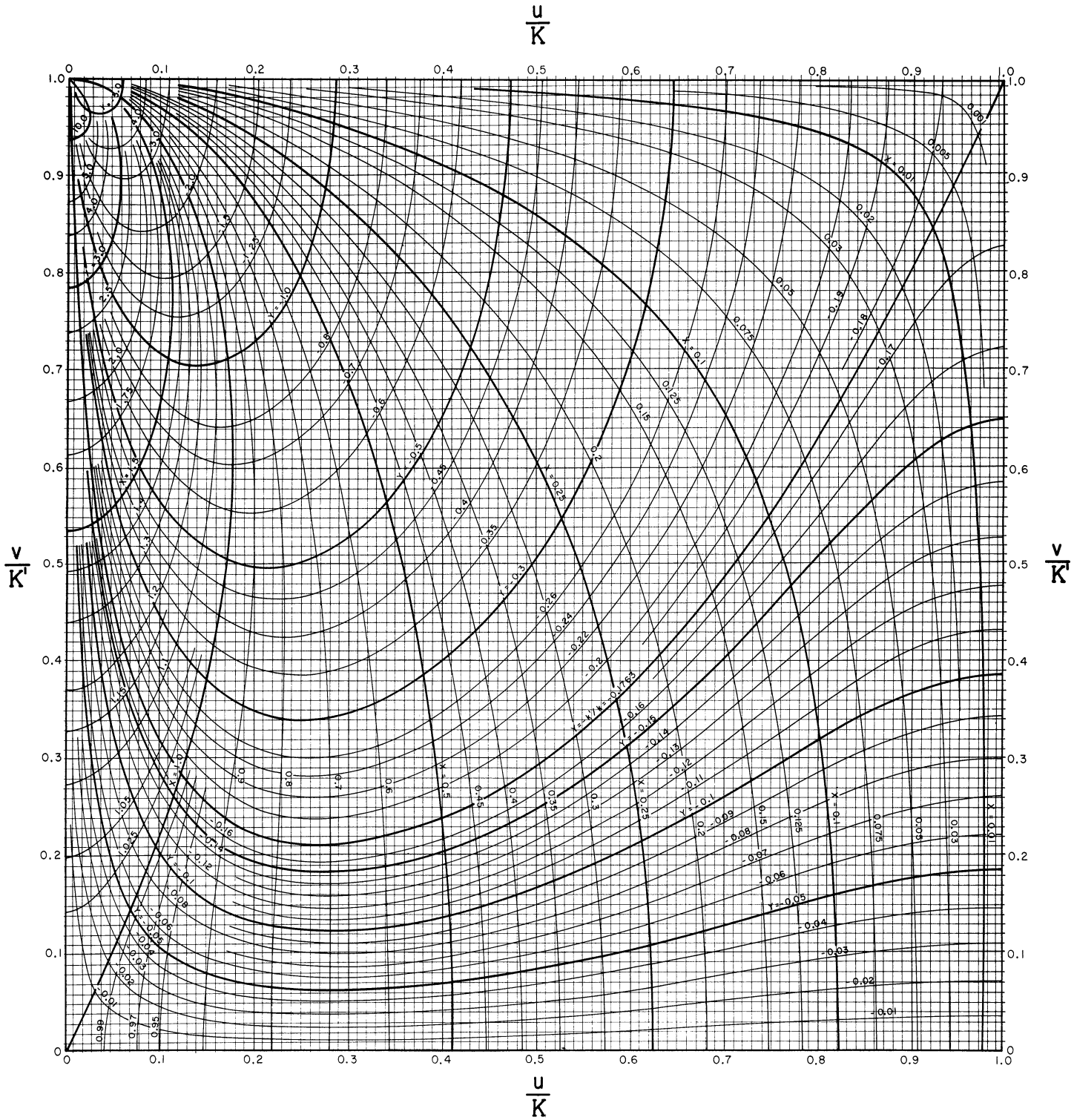
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 75° = 0.9659$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 80° = 0.984808; K = 3.153385, K' = 1.582843

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9522	0.8289	0.6723	0.5183	0.3847	0.2755	0.1878	0.1164	0.05561	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.013	0.9620	0.8329	0.6717	0.5155	0.3815	0.2727	0.1858	0.1151	0.05496	0
	y	0	0.04695	0.07485	0.08077	0.07317	0.06086	0.04891	0.03932	0.03259	0.02867	0.02738
0.2	x	1.052	0.9923	0.8447	0.6693	0.5068	0.3718	0.2644	0.1796	0.1111	0.05302	0
	y	0	0.09924	0.1556	0.1649	0.1474	0.1216	0.09718	0.07791	0.06448	0.05667	0.05411
0.3	x	1.124	1.045	0.8627	0.6629	0.4908	0.3550	0.2505	0.1693	0.1045	0.04982	0
	y	0	0.1637	0.2487	0.2557	0.2235	0.1817	0.1441	0.1150	0.09495	0.08335	0.07956
0.4	x	1.239	1.126	0.8835	0.6485	0.4654	0.3304	0.2306	0.1550	0.09537	0.04540	0
	y	0	0.2507	0.3624	0.3559	0.3015	0.2406	0.1888	0.1498	0.1233	0.1081	0.1031
0.5	x	1.420	1.238	0.8995	0.6199	0.4277	0.2969	0.2047	0.1367	0.08383	0.03984	0
	y	0	0.3787	0.5066	0.4672	0.3805	0.2969	0.2301	0.1814	0.1488	0.1302	0.1242
0.6	x	1.710	1.387	0.8948	0.5684	0.3748	0.2540	0.1729	0.1147	0.07008	0.03325	0
	y	0	0.5825	0.6921	0.5882	0.4578	0.3487	0.2669	0.2090	0.1708	0.1493	0.1423
0.7	x	2.216	1.559	0.8398	0.4839	0.3043	0.2014	0.1355	0.08932	0.05440	0.02578	0
	y	0	0.9316	0.9242	0.7126	0.5289	0.3935	0.2976	0.2316	0.1887	0.1646	0.1569
0.8	x	3.259	1.659	0.5891	0.3584	0.2159	0.1401	0.09330	0.06120	0.03718	0.01759	0
	y	0	1.552	1.187	0.8263	0.5875	0.4284	0.3208	0.2484	0.2019	0.1759	0.1676
0.9	x	6.441	1.315	0.4017	0.1923	0.1123	0.07191	0.04759	0.03111	0.01887	0.008923	0
	y	0	2.522	1.418	0.9085	0.6264	0.4507	0.3353	0.2588	0.2100	0.1828	0.1741
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	3.171	1.515	0.9389	0.6401	0.4584	0.3402	0.2623	0.2127	0.1852	0.1763



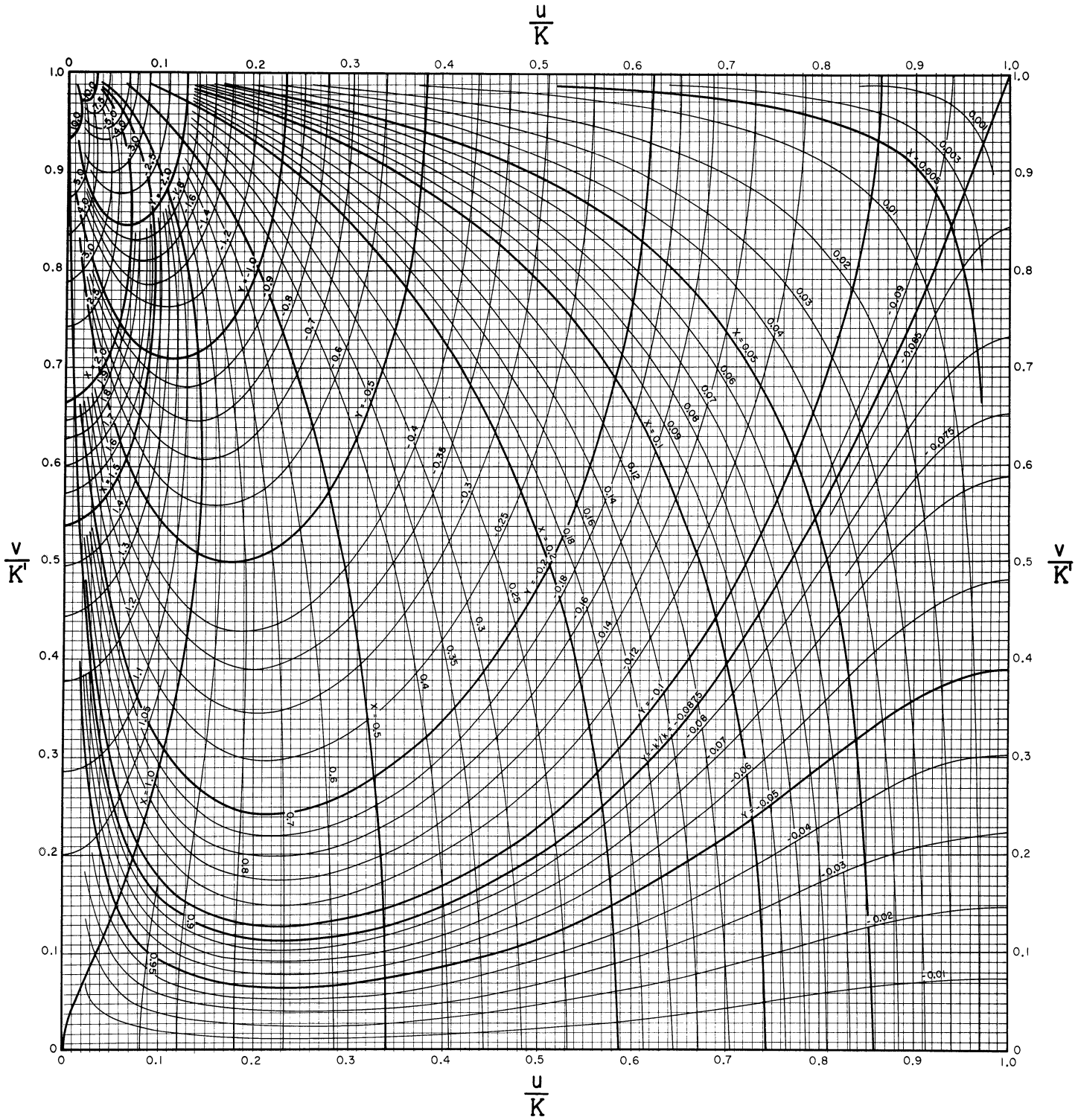
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 80° = 0.9848$

THE cn-FUNCTION, $x + iy = cn(u + iv)$

$k = \sin 85° = 0.996195$; $K = 3.831742$, $K' = 1.573792$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9308	0.7640	0.5746	0.4099	0.2831	0.1904	0.1232	0.07340	0.03421	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.013	0.9393	0.7656	0.5722	0.4066	0.2802	0.1882	0.1217	0.07252	0.03379	0
	y	0	0.05451	0.07859	0.07488	0.05944	0.04447	0.03215	0.02342	0.01779	0.01466	0.01336
0.2	x	1.052	0.9653	0.7698	0.5645	0.3964	0.2715	0.1818	0.1174	0.06988	0.03255	0
	y	0	0.1149	0.1621	0.1515	0.1199	0.08835	0.06367	0.04632	0.03515	0.02896	0.02698
0.3	x	1.123	1.010	0.7745	0.5499	0.3788	0.2568	0.1711	0.1102	0.06555	0.03052	0
	y	0	0.1885	0.2556	0.2313	0.1795	0.1310	0.09394	0.06818	0.05168	0.04257	0.03966
0.4	x	1.237	1.076	0.7753	0.5256	0.3527	0.2360	0.1563	0.1004	0.05961	0.02774	0
	y	0	0.2860	0.3646	0.3151	0.2382	0.1716	0.1223	0.08849	0.06698	0.05514	0.05136
0.5	x	1.416	1.163	0.7645	0.4879	0.3173	0.2092	0.1375	0.08802	0.05219	0.02427	0
	y	0	0.4253	0.4946	0.4023	0.2948	0.2092	0.1480	0.1067	0.08067	0.06636	0.06180
0.6	x	1.703	1.266	0.7289	0.4323	0.2717	0.1763	0.1150	0.07341	0.04346	0.02020	0
	y	0	0.6371	0.6486	0.4904	0.3472	0.2425	0.1703	0.1224	0.09240	0.07596	0.07073
0.7	x	2.206	1.357	0.6490	0.3551	0.2157	0.1379	0.08939	0.05687	0.03362	0.01561	0
	y	0	0.9733	0.8230	0.5740	0.3929	0.2703	0.1886	0.1352	0.1019	0.08369	0.07792
0.8	x	3.242	1.335	0.5026	0.2545	0.1501	0.09482	0.06114	0.03880	0.02291	0.01064	0
	y	0	1.500	0.9992	0.6448	0.4287	0.2914	0.2022	0.1446	0.1088	0.08937	0.08176
0.9	x	6.404	0.9454	0.2791	0.1333	0.07714	0.04832	0.03105	0.01967	0.01161	0.005387	0
	y	0	2.179	1.138	0.6930	0.4516	0.3045	0.2106	0.1503	0.1131	0.09283	0.08641
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	2.558	1.192	0.7102	0.4596	0.3090	0.2134	0.1523	0.1145	0.09399	0.08749



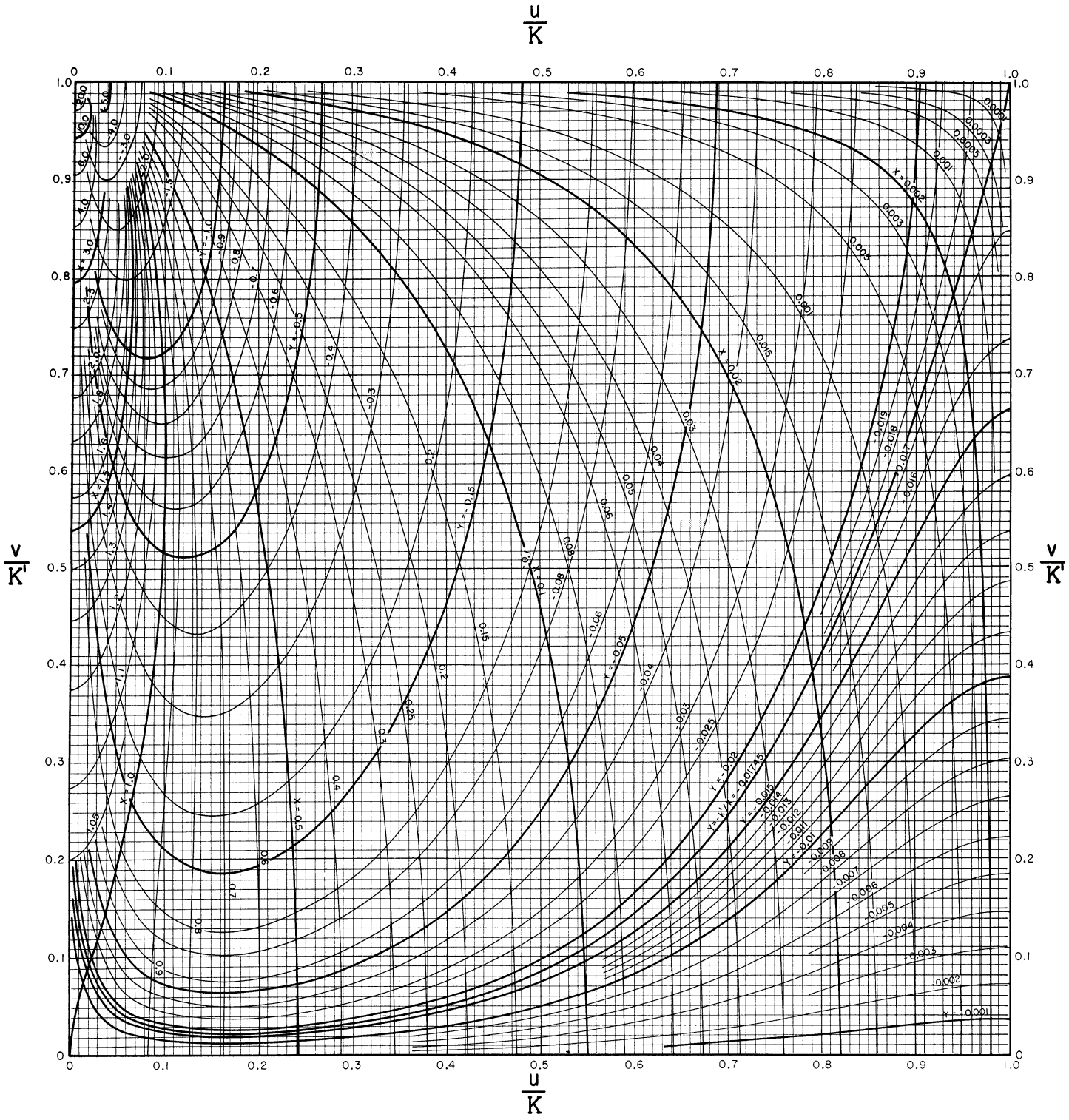
The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 85° = 0.9962$

THE cn-FUNCTION, x + iy = cn(u + iv)

k = sin 89° = 0.999848; K = 5.434910, K' = 1.570916

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.8685	0.6056	0.3771	0.2243	0.1310	0.07562	0.04283	0.02293	0.009959	0
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.8740	0.6035	0.3737	0.2218	0.1294	0.07470	0.04230	0.02265	0.009836	0
	y	0	0.06862	0.07609	0.05488	0.03433	0.02050	0.01211	0.007227	0.004506	0.003143	0.002730
0.2	x	1.051	0.8902	0.5968	0.3636	0.2143	0.1248	0.07196	0.04074	0.02181	0.009471	0
	y	0	0.1434	0.1544	0.1095	0.06806	0.04054	0.02393	0.01428	0.008902	0.006209	0.005393
0.3	x	1.122	0.9164	0.5837	0.3461	0.2019	0.1171	0.06746	0.03818	0.02043	0.008874	0
	y	0	0.2314	0.2367	0.1635	0.1006	0.05968	0.03517	0.02098	0.01308	0.009122	0.007923
0.4	x	1.236	0.9504	0.5610	0.3208	0.1847	0.1066	0.06131	0.03467	0.01855	0.008057	0
	y	0	0.3423	0.3245	0.2161	0.1311	0.07745	0.04558	0.02717	0.01694	0.01181	0.01026
0.5	x	1.414	0.9861	0.5244	0.2871	0.1627	0.09342	0.05363	0.03031	0.01622	0.007043	0
	y	0	0.4888	0.4174	0.2661	0.1590	0.09342	0.05488	0.03270	0.02038	0.01421	0.01234
0.6	x	1.701	1.008	0.4684	0.2444	0.1363	0.07787	0.04462	0.02521	0.01348	0.005855	0
	y	0	0.6880	0.5132	0.3119	0.1834	0.1072	0.06284	0.03742	0.02331	0.01626	0.01412
0.7	x	2.203	0.9831	0.3879	0.1930	0.1061	0.06029	0.03449	0.01947	0.01042	0.004522	0
	y	0	0.9563	0.6059	0.3511	0.2034	0.1183	0.06927	0.04123	0.02568	0.01791	0.01555
0.8	x	3.236	0.8450	0.2801	0.1337	0.07262	0.04112	0.02349	0.01326	0.007090	0.003078	0
	y	0	1.289	0.6860	0.3816	0.2184	0.1265	0.07398	0.04402	0.02741	0.01911	0.01660
0.9	x	6.393	0.5145	0.1475	0.06850	0.03690	0.02084	0.01190	0.006713	0.003589	0.001558	0
	y	0	1.610	0.7414	0.4009	0.2277	0.1316	0.07686	0.04572	0.02847	0.01985	0.01724
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	1.753	0.7612	0.4076	0.2308	0.1333	0.07783	0.04629	0.02882	0.02010	0.01745



The cn-function, $z = x + iy = cn(u+iv) = \sqrt{1 - sn^2(u+iv)}$

$k = \sin 89° = 0.9999$

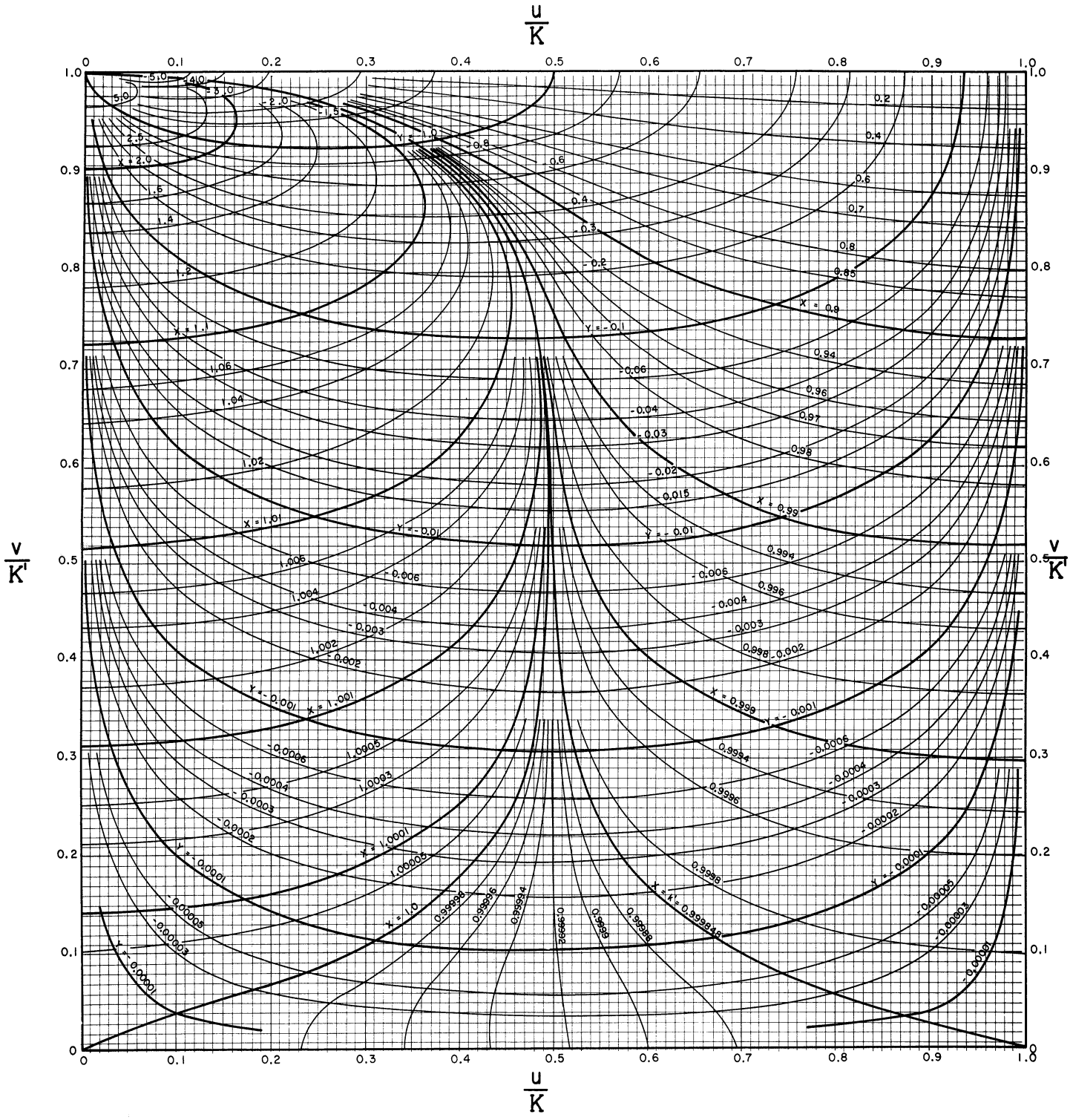
APPENDIX III

THE dn-FUNCTION

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 1° = 0.0174524; K = 1.570916, K' = 5.434910

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	1.000	1.000	1.000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
	y	0	0	0	0	0	0	0	0	0	0	0
.1	x	1.000	1.000	1.000	1.000	1.000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998
	y	0	$0.0^4$3092	$0.0^4$5882	$0.0^4$8095	$0.0^4$9399	$0.0^4$1001	$0.0^4$9516	$0.0^4$8094	$0.0^4$5881	$0.0^4$3092	0
.2	x	1.000	1.000	1.000	1.000	1.000	0.9999	0.9998	0.9997	0.9996	0.9996	0.9996
	y	0	$0.0^3$1021	$0.0^3$1943	$0.0^3$2674	$0.0^3$3104	$0.0^3$3304	$0.0^3$3142	$0.0^3$2673	$0.0^3$1942	$0.0^3$1021	0
0.3	x	1.001	1.001	1.001	1.001	1.000	0.9999	0.9996	0.9993	0.9991	0.9990	0.9989
	y	0	$0.0^3$3066	$0.0^3$5830	$0.0^3$8023	$0.0^3$9313	$0.0^3$9911	$0.0^3$9423	$0.0^3$8014	$0.0^3$5821	$0.0^3$3060	0
0.4	x	1.003	1.003	1.002	1.002	1.001	0.9999	0.9990	0.9982	0.9975	0.9972	0.9970
	y	0	$0.0^3$9118	0.001734	0.002385	0.002767	0.002943	0.002796	0.002376	0.001725	$0.0^3$9067	0
0.5	x	1.009	1.008	1.007	1.005	1.003	0.9999	0.9972	0.9949	0.9929	0.9917	0.9913
	y	0	0.002719	0.005165	0.007096	0.008219	0.008726	0.008276	0.007023	0.005093	0.002674	0
0.6	x	1.026	1.025	1.021	1.015	1.008	0.9996	0.9917	0.9846	0.9791	0.9756	0.9744
	y	0	0.008196	0.01553	0.02125	0.02450	0.02587	0.02441	0.02062	0.01489	0.007802	0
0.7	x	1.080	1.073	1.063	1.044	1.021	0.9970	0.9739	0.9540	0.9388	0.9293	0.9260
	y	0	0.02548	0.04800	0.06490	0.07370	0.07661	0.07117	0.05931	0.04240	0.02206	0
0.8	x	1.257	1.242	1.191	1.123	1.047	0.9744	0.9111	0.8607	0.8246	0.8029	0.7957
	y	0	0.08846	0.1613	0.2093	0.2267	0.2246	0.1997	0.1605	0.1117	0.05718	0
0.9	x	2.018	1.877	1.560	1.236	0.9792	0.7959	0.6703	0.5868	0.5340	0.5049	0.4956
	y	0	0.4413	0.6979	0.7610	0.7000	0.6054	0.4851	0.3613	0.2389	0.1187	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.313	3.076	1.963	1.376	0.9999	0.7265	0.5095	0.3249	0.1584	0



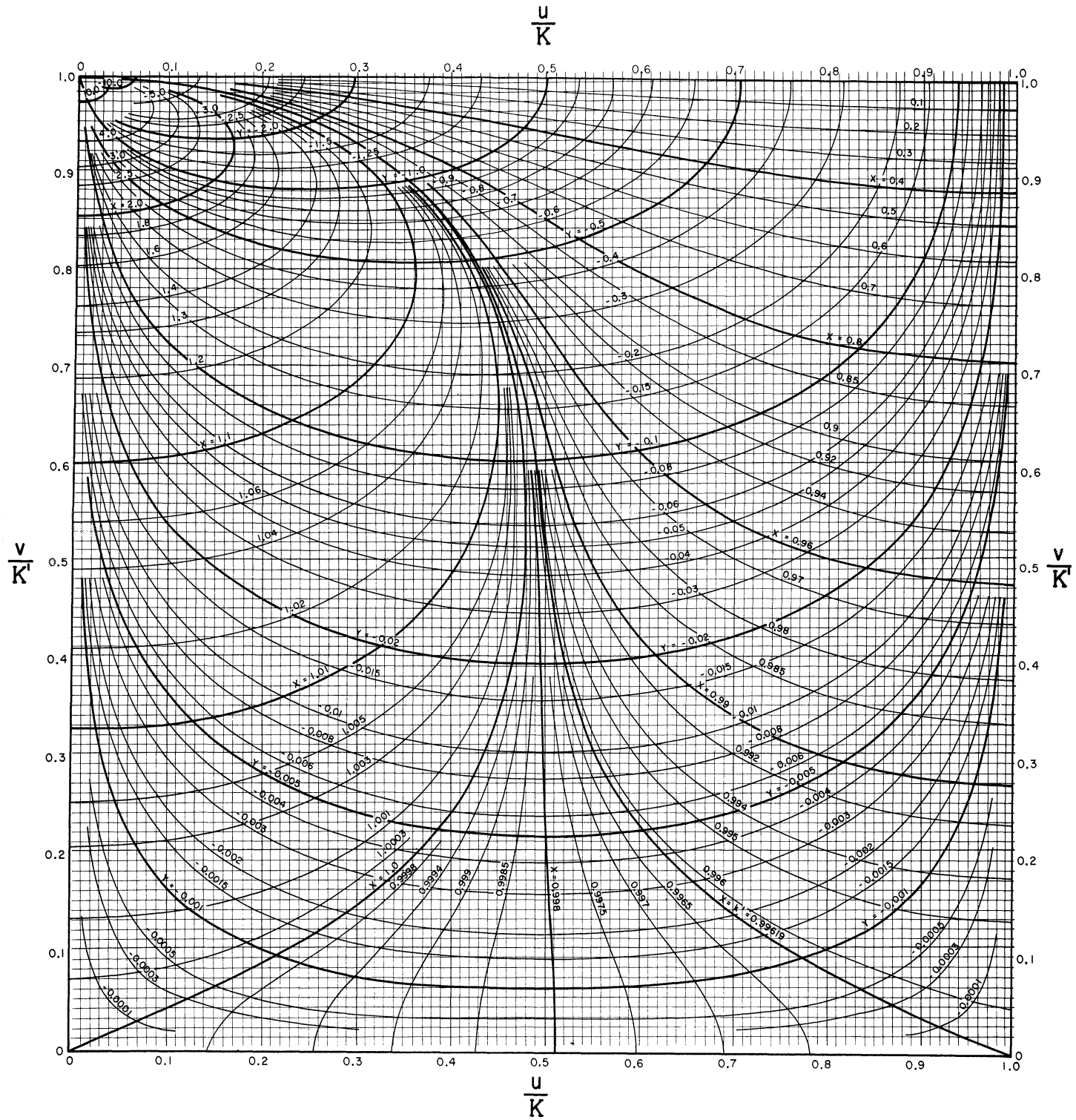
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 1° = 0.0175$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 5° = 0.0871557; K = 1.573792, K' = 3.831742

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9999	0.9996	0.9992	0.9987	0.9981	0.9975	0.9970	0.9966	0.9963	0.9962
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.001	1.000	1.000	0.9996	0.9989	0.9981	0.9973	0.9966	0.9961	0.9957	0.9956
	y	0	$0.0^{3}4960$	$0.0^{3}9432$	0.001297	0.001524	0.001601	0.001522	0.001294	$0.0^{3}9394$	$0.0^{3}4937$	0
0.2	x	1.003	1.002	1.002	1.001	0.9995	0.9981	0.9967	0.9954	0.9944	0.9937	0.9935
	y	0	0.001299	0.002469	0.003395	0.003986	0.004186	0.003975	0.003377	0.002451	0.001288	0
0.3	x	1.008	1.007	1.006	1.004	1.001	0.9981	0.9951	0.9925	0.9904	0.9891	0.9886
	y	0	0.002912	0.005532	0.007598	0.008909	0.009340	0.008857	0.007513	0.005447	0.002860	0
0.4	x	1.019	1.018	1.015	1.010	1.004	0.9979	0.9917	0.9861	0.9817	0.9789	0.9780
	y	0	0.006372	0.01210	0.01657	0.01937	0.02024	0.01913	0.01618	0.01171	0.006132	0
0.5	x	1.043	1.040	1.034	1.023	1.011	0.9972	0.9839	0.9722	0.9631	0.9574	0.9554
	y	0	0.01404	0.02653	0.03615	0.04197	0.04354	0.04085	0.03434	0.02472	0.01292	0
0.6	x	1.096	1.091	1.075	1.052	1.023	0.9937	0.9658	0.9419	0.9238	0.9125	0.9087
	y	0	0.03169	0.05941	0.07998	0.09150	0.09342	0.08635	0.07164	0.005105	0.02651	0
0.7	x	1.222	1.207	1.166	1.108	1.042	0.9781	0.9213	0.8754	0.8421	0.8221	0.8153
	y	0	0.07597	0.1396	0.1826	0.2019	0.1992	0.1784	0.1442	0.1008	0.05173	0
0.8	x	1.550	1.498	1.366	1.201	1.042	0.9087	0.8056	0.7308	0.6808	0.6521	0.6427
	y	0	0.2104	0.3649	0.4415	0.4503	0.4130	0.3482	0.2687	0.1819	0.09158	0
0.9	x	2.736	2.359	1.686	1.168	0.8414	0.6428	0.5199	0.4434	0.3971	0.3721	0.3642
	y	0	0.8663	1.178	1.122	0.9507	0.7637	0.5875	0.4262	0.2773	0.1366	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.302	3.072	1.959	1.374	0.9981	0.7252	0.5086	0.3243	0.1581	0



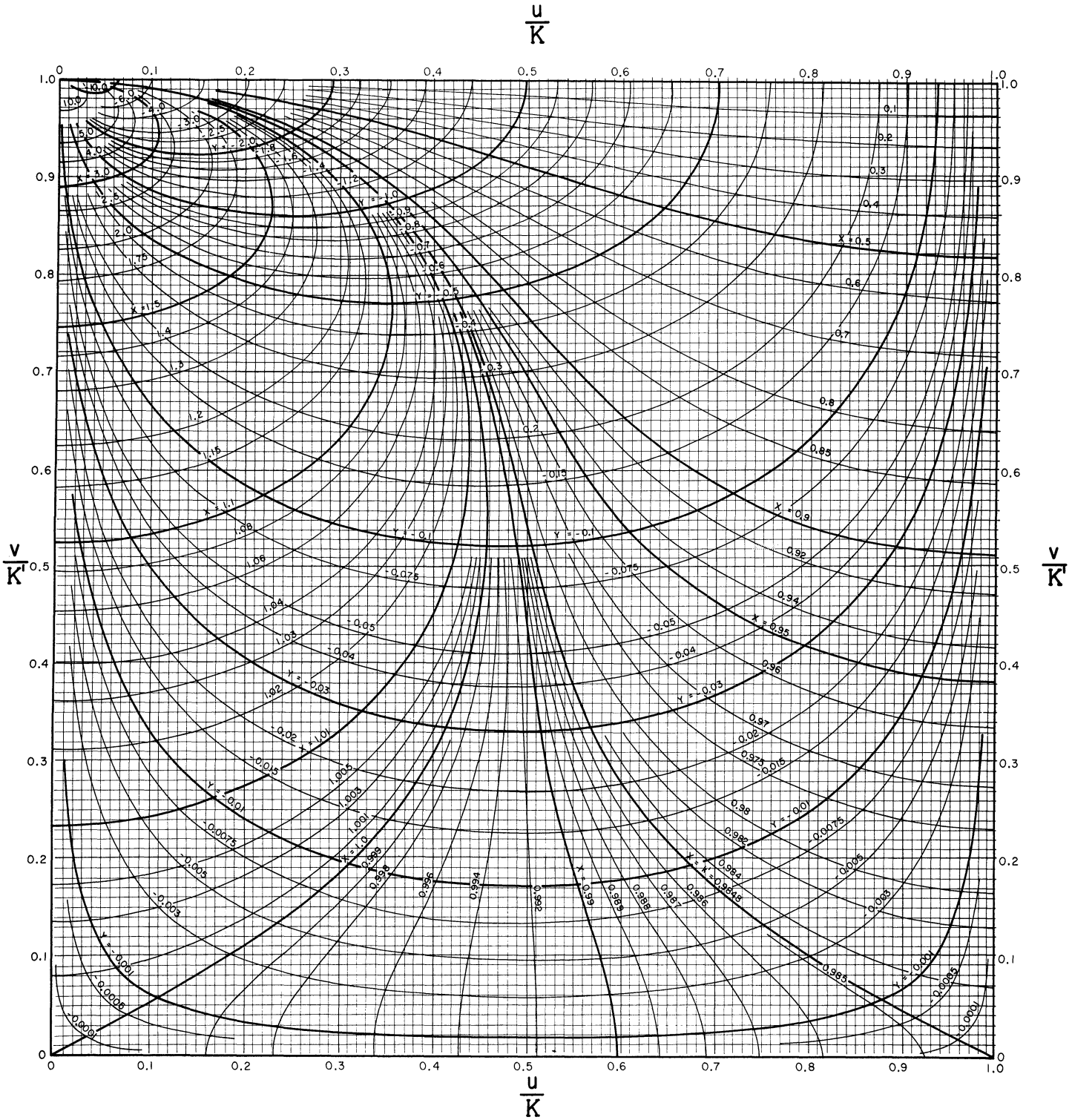
The dn - function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 5° = 0.0872$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 10° = 0.173648; K = 1.582843, K' = 3.153385

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9996	0.9985	0.9969	0.9947	0.9924	0.9900	0.9879	0.9862	0.9852	0.9848
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.002	1.001	0.9998	0.9977	0.9952	0.9924	0.9895	0.9870	0.9850	0.9837	0.9833
	y	0	0.001580	0.003002	0.004124	0.004835	0.005070	0.004808	0.004079	0.002958	0.001552	0
0.2	x	1.007	1.006	1.004	1.001	0.9968	0.9923	0.9879	0.9839	0.9808	0.9788	0.9781
	y	0	0.003819	0.007251	0.009947	0.01165	0.01219	0.01154	0.009779	0.007082	0.003715	0
0.3	x	1.018	1.017	1.013	1.007	1.000	0.9921	0.9843	0.9774	0.9719	0.9685	0.9673
	y	0	0.007678	0.01455	0.01991	0.02324	0.02424	0.02288	0.01932	0.01396	0.007312	0
0.4	x	1.040	1.038	1.031	1.020	1.006	0.9913	0.9771	0.9646	0.9549	0.9488	0.9467
	y	0	0.01491	0.02816	0.03835	0.04448	0.04609	0.04321	0.03628	0.02610	0.01363	0
0.5	x	1.083	1.078	1.064	1.042	1.016	0.9886	0.9626	0.9402	0.9232	0.9126	0.9090
	y	0	0.02915	0.05471	0.07376	0.08454	0.08648	0.08008	0.06654	0.04747	0.02467	0
0.6	x	1.169	1.158	1.128	1.083	1.031	0.9792	0.9324	0.8938	0.8654	0.8481	0.8422
	y	0	0.05891	0.1091	0.1442	0.1614	0.1612	0.1459	0.1190	0.08372	0.04313	0
0.7	x	1.351	1.323	1.249	1.149	1.043	0.9471	0.8670	0.8055	0.7626	0.7374	0.7291
	y	0	0.1280	0.2299	0.2911	0.3108	0.2966	0.2582	0.2041	0.1404	0.07135	0
0.8	x	1.787	1.694	1.472	1.222	1.007	0.8424	0.7240	0.6426	0.5900	0.5605	0.5510
	y	0	0.3261	0.5388	0.6158	0.5963	0.5247	0.4289	0.3238	0.2160	0.1079	0
0.9	x	3.274	2.636	1.684	1.078	0.7416	0.5511	0.4384	0.3701	0.3294	0.3077	0.3008
	y	0	1.220	1.483	1.306	1.057	0.8254	0.6245	0.4485	0.2900	0.1424	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.266	3.054	1.948	1.366	0.9924	0.7210	0.5056	0.3224	0.1572	0



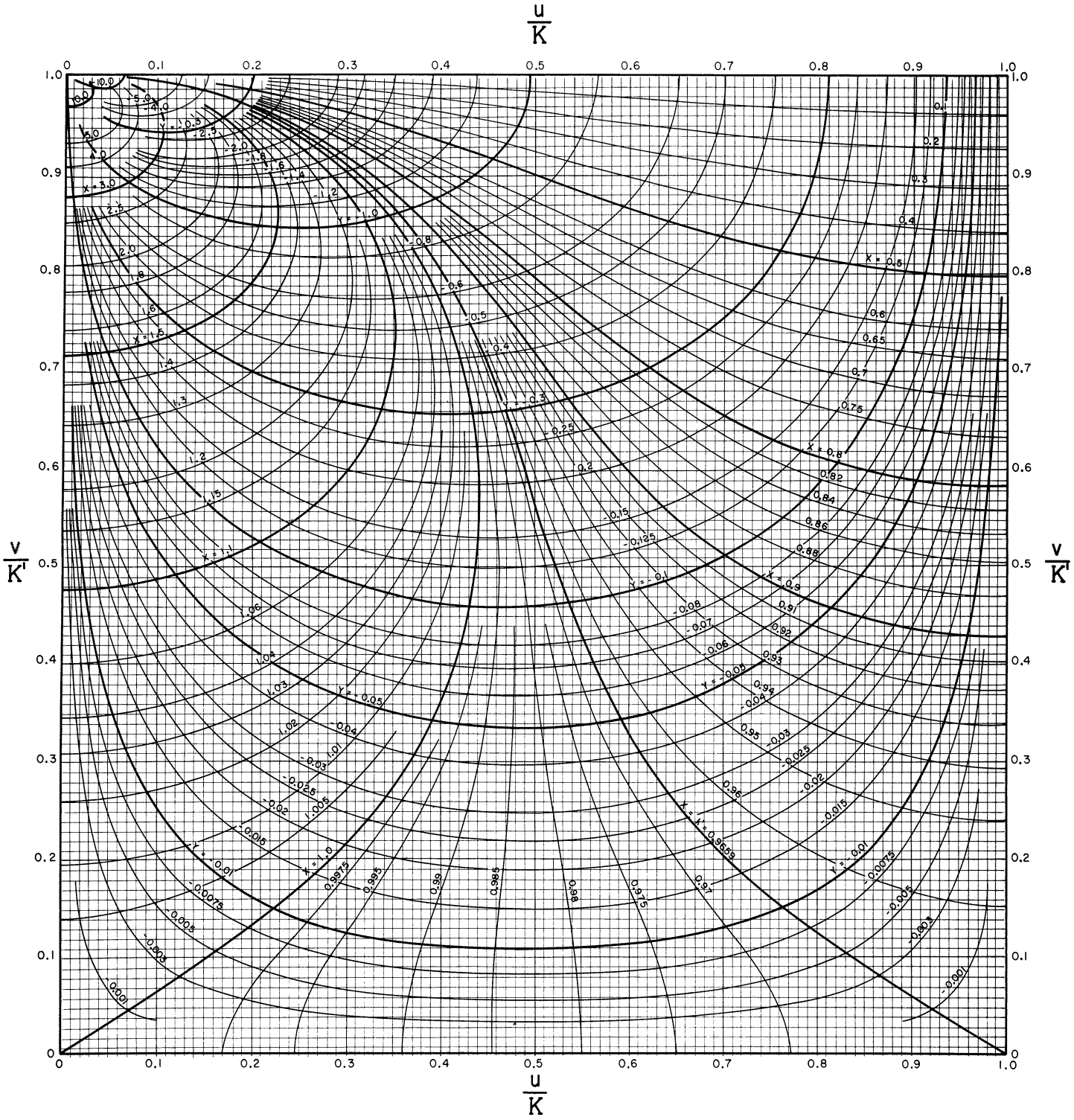
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 10° = 0.1737$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 15° = 0.258819; K = 1.598142, K' = 2.768063

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9992	0.9967	0.9929	0.9881	0.9828	0.9776	0.9729	0.9691	0.9667	0.9659
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.003	1.002	0.9988	0.9944	0.9889	0.9828	0.9767	0.9713	0.9671	0.9643	0.9634
	y	0	0.003064	0.005813	0.007965	0.009312	0.009730	0.009197	0.007781	0.005628	0.002950	0
0.2	x	1.011	1.010	1.006	0.9993	0.9913	0.9826	0.9739	0.9662	0.9602	0.9564	0.9551
	y	0	0.007118	0.01348	0.01844	0.02151	0.02241	0.02113	0.01783	0.01287	0.006741	0
0.3	x	1.029	1.027	1.020	1.009	0.9961	0.9819	0.9682	0.9560	0.9466	0.9406	0.9386
	y	0	0.01353	0.02557	0.03482	0.04041	0.04188	0.03927	0.03299	0.02373	0.01240	0
0.4	x	1.062	1.058	1.045	1.027	1.004	0.9800	0.9572	0.9374	0.9223	0.9128	0.9096
	y	0	0.02468	0.04639	0.06271	0.07210	0.07400	0.06874	0.05727	0.04094	0.02130	0
0.5	x	1.122	1.114	1.090	1.056	1.016	0.9744	0.9363	0.9044	0.8806	0.8659	0.8610
	y	0	0.04529	0.08436	0.1125	0.1271	0.1282	0.1172	0.09630	0.06812	0.03522	0
0.6	x	1.235	1.218	1.170	1.103	1.029	0.9580	0.8959	0.8464	0.8108	0.7895	0.7824
	y	0	0.08631	0.1578	0.2048	0.2246	0.2198	0.1955	0.1571	0.1093	0.05597	0
0.7	x	1.461	1.419	1.310	1.170	1.031	0.9107	0.8157	0.7456	0.6979	0.6704	0.6614
	y	0	0.1780	0.3126	0.3842	0.3981	0.3698	0.3150	0.2450	0.1666	0.08414	0
0.8	x	1.981	1.843	1.533	1.215	0.9628	0.7828	0.6595	0.5774	0.5255	0.4968	0.4876
	y	0	0.4325	0.6843	0.7463	0.6953	0.5944	0.4763	0.3547	0.2346	0.1166	0
0.9	x	3.702	2.800	1.639	0.9962	0.6666	0.4878	0.3846	0.3229	0.2865	0.2671	0.2610
	y	0	1.513	1.686	1.410	1.109	0.8533	0.6398	0.4570	0.2946	0.1444	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.206	3.025	1.929	1.353	0.9828	0.7140	0.5007	0.3193	0.1557	0



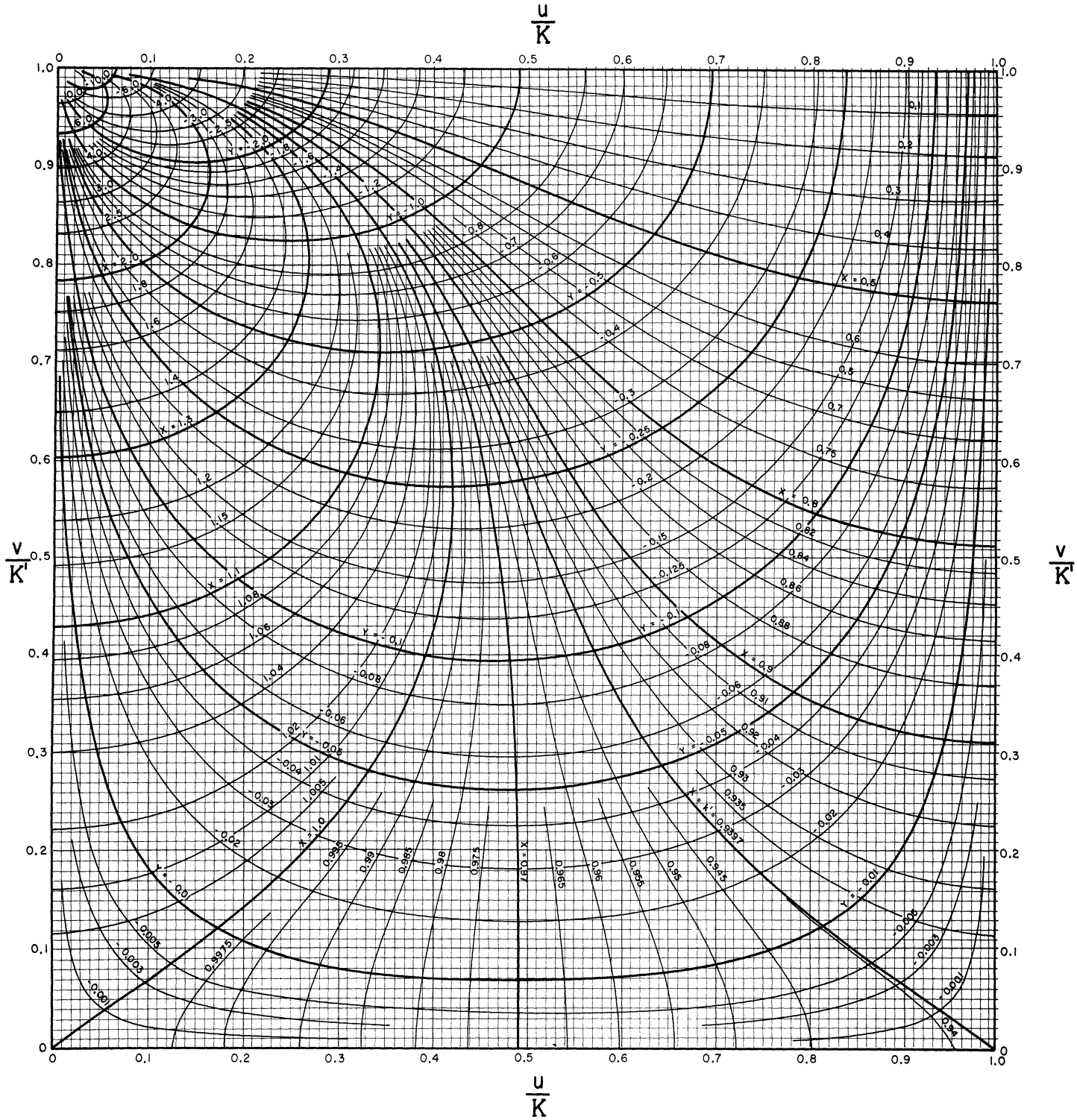
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 15° = 0.2588$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 20° = 0.342020; K = 1.620026, K' = 2.504550

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9985	0.9941	0.9873	0.9787	0.9694	0.9601	0.9518	0.9453	0.9411	0.9397
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.004	1.002	0.9970	0.9894	0.9798	0.9693	0.9589	0.9497	0.9424	0.9378	0.9362
	y	0	0.00486	0.009205	0.01257	0.01464	0.01522	0.01432	0.01207	0.008701	0.004552	0
0.2	x	1.016	1.014	1.007	0.9961	0.9830	0.9688	0.9549	0.9426	0.9330	0.9270	0.9249
	y	0	0.01103	0.02083	0.02837	0.03291	0.03410	0.03197	0.02684	0.01931	0.01008	0
0.3	x	1.040	1.036	1.025	1.009	0.9889	0.9674	0.9468	0.9289	0.9151	0.9064	0.9035
	y	0	0.02024	0.03810	0.05159	0.05944	0.06114	0.05691	0.04750	0.03400	0.01771	0
0.4	x	1.083	1.077	1.058	1.031	0.9979	0.9639	0.9321	0.9050	0.8846	0.8720	0.8677
	y	0	0.03546	0.06629	0.08887	0.1011	0.1027	0.09447	0.07804	0.05542	0.02872	0
0.5	x	1.158	1.146	1.113	1.065	1.010	0.9547	0.9053	0.8648	0.8352	0.8172	0.8112
	y	0	0.06248	0.1154	0.1519	0.1693	0.1683	0.1518	0.1234	0.08657	0.04454	0
0.6	x	1.295	1.271	1.205	1.115	1.019	0.9306	0.8560	0.7983	0.7577	0.7337	0.7257
	y	0	0.1146	0.2066	0.2632	0.2828	0.2715	0.2375	0.1884	0.1299	0.06615	0
0.7	x	1.559	1.501	1.354	1.175	1.008	0.8698	0.7652	0.6902	0.6404	0.6120	0.6028
	y	0	0.2282	0.3916	0.4678	0.4714	0.4279	0.3580	0.2747	0.1852	0.09305	0
0.8	x	2.151	1.963	1.565	1.190	0.9136	0.7265	0.6030	0.5228	0.4729	0.4455	0.4368
	y	0	0.5359	0.8127	0.8503	0.7675	0.6418	0.5066	0.3736	0.2455	0.1216	0
0.9	x	4.071	2.894	1.573	0.9197	0.6036	0.4371	0.3426	0.2866	0.2537	0.2363	0.2308
	y	0	1.771	1.831	1.473	1.136	0.8653	0.6449	0.4590	0.2952	0.1446	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.122	2.984	1.903	1.334	0.9694	0.7042	0.4938	0.3149	0.1535	0



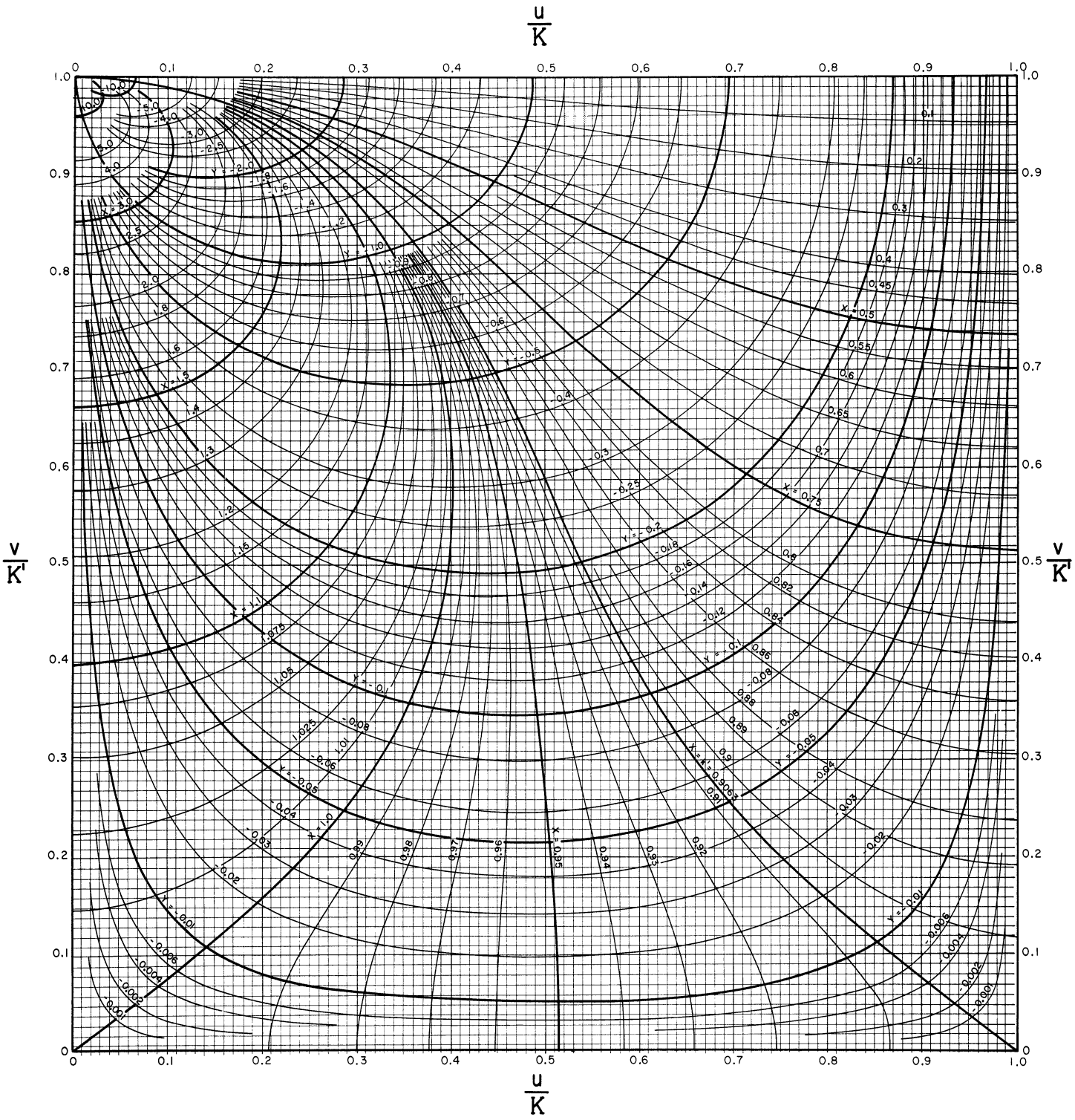
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 20° = 0.3420$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 25° = 0.422618; K = 1.648995, K' = 2.308787

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9976	0.9906	0.9799	0.9666	0.9520	0.9376	0.9249	0.9149	0.9085	0.9063
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.005	1.002	0.9944	0.9826	0.9678	0.9518	0.9360	0.9221	0.9112	0.9043	0.9019
	y	0	0.006921	0.01306	0.01776	0.02056	0.02126	0.01989	0.01667	0.01197	0.006245	0
0.2	x	1.021	1.017	1.007	0.9909	0.9716	0.9509	0.9308	0.9132	0.8996	0.8910	0.8881
	y	0	0.01544	0.02906	0.03936	0.04537	0.04668	0.04346	0.03628	0.02597	0.01352	0
0.3	x	1.051	1.045	1.030	1.006	0.9782	0.9485	0.9204	0.8963	0.8779	0.8665	0.8626
	y	0	0.02766	0.05183	0.06970	0.07963	0.08119	0.07493	0.06208	0.04419	0.02293	0
0.4	x	1.103	1.094	1.069	1.031	0.9875	0.9429	0.9020	0.8679	0.8424	0.8268	0.8216
	y	0	0.04711	0.08752	0.1162	0.1308	0.1313	0.1195	0.09781	0.06899	0.03561	0
0.5	x	1.193	1.176	1.132	1.068	0.9977	0.9294	0.8697	0.8219	0.7874	0.7667	0.7599
	y	0	0.08064	0.1475	0.1916	0.2104	0.2060	0.1834	0.1474	0.1027	0.05256	0
0.6	x	1.350	1.318	1.232	1.118	1.002	0.8977	0.8132	0.7493	0.7053	0.6796	0.6712
	y	0	0.1438	0.2556	0.3193	0.3362	0.3168	0.2729	0.2140	0.1463	0.07415	0
0.7	x	1.648	1.572	1.385	1.169	0.9764	0.8256	0.7150	0.6378	0.5874	0.5590	0.5498
	y	0	0.2791	0.4676	0.5430	0.5332	0.4741	0.3905	0.2963	0.1983	0.09924	0
0.8	x	2.304	2.060	1.575	1.153	0.8614	0.6728	0.5519	0.4748	0.4273	0.4015	0.3933
	y	0	0.6376	0.9272	0.9337	0.8202	0.6735	0.5255	0.3846	0.2516	0.1243	0
0.9	x	4.401	2.938	1.496	0.8476	0.5482	0.3939	0.3073	0.2564	0.2266	0.2109	0.2059
	y	0	1.999	1.935	1.509	1.147	0.8667	0.6431	0.4566	0.2932	0.1434	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	6.014	2.931	1.869	1.311	0.9520	0.6915	0.4849	0.3092	0.1507	0



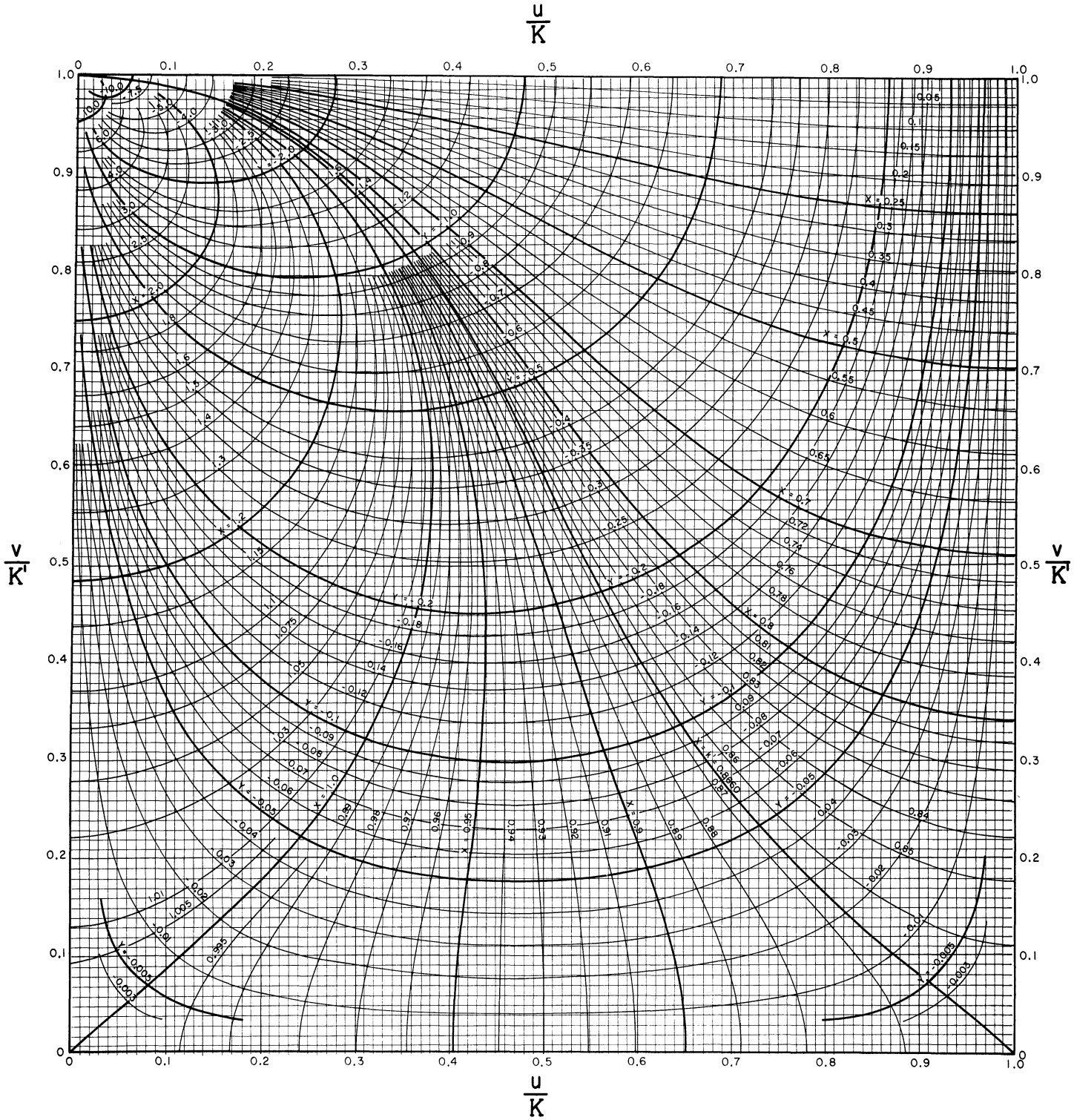
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 25° = 0.4226$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 30° = 0.500000; K = 1.685750, K' = 2.156516

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9965	0.9864	0.9708	0.9515	0.9306	0.9102	0.8921	0.8780	0.8691	0.8660
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.006	1.002	0.9909	0.9739	0.9529	0.9302	0.9081	0.8888	0.8737	0.8642	0.8609
	y	0	0.009201	0.01730	0.02340	0.02690	0.02761	0.02564	0.02135	0.01526	0.007935	0
0.2	x	1.025	1.020	1.005	0.9835	0.9569	0.9287	0.9016	0.8782	0.8602	0.8489	0.8450
	y	0	0.02028	0.03802	0.05117	0.05850	0.05969	0.05512	0.04569	0.03253	0.01688	0
0.3	x	1.061	1.053	1.032	1.001	0.9637	0.9251	0.8890	0.8585	0.8354	0.8212	0.8164
	y	0	0.03571	0.06655	0.08878	0.1004	0.1014	0.09266	0.07614	0.05386	0.02784	0
0.4	x	1.122	1.110	1.077	1.028	0.9723	0.9169	0.8671	0.8262	0.7962	0.7779	0.7718
	y	0	0.05958	0.1099	0.1443	0.1604	0.1591	0.1431	0.1160	0.08125	0.04175	0
0.5	x	1.225	1.204	1.146	1.066	0.9799	0.8989	0.8299	0.7758	0.7374	0.7147	0.7071
	y	0	0.09977	0.1806	0.2312	0.2497	0.2409	0.2115	0.1683	0.1162	0.05924	0
0.6	x	1.402	1.360	1.252	1.113	0.9770	0.8599	0.7675	0.6994	0.6532	0.6266	0.6178
	y	0	0.1741	0.3047	0.3729	0.3846	0.3559	0.3021	0.2343	0.1590	0.08023	0
0.7	x	1.730	1.633	1.403	1.151	0.9383	0.7784	0.6648	0.5873	0.5373	0.5094	0.5005
	y	0	0.3310	0.5409	0.6104	0.5847	0.5100	0.4143	0.3114	0.2071	0.1033	0
0.8	x	2.443	2.136	1.566	1.106	0.8070	0.6207	0.5041	0.4309	0.3863	0.3621	0.3545
	y	0	0.7383	1.029	0.9998	0.8574	0.6933	0.5356	0.3896	0.2538	0.1251	0
0.9	x	4.700	2.942	1.410	0.7788	0.4979	0.3555	0.2764	0.2301	0.2030	0.1887	0.1843
	y	0	2.201	2.007	1.525	1.145	0.8600	0.6359	0.4505	0.2889	0.1412	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.883	2.867	1.828	1.281	0.9306	0.6759	0.4738	0.3021	0.1472	0



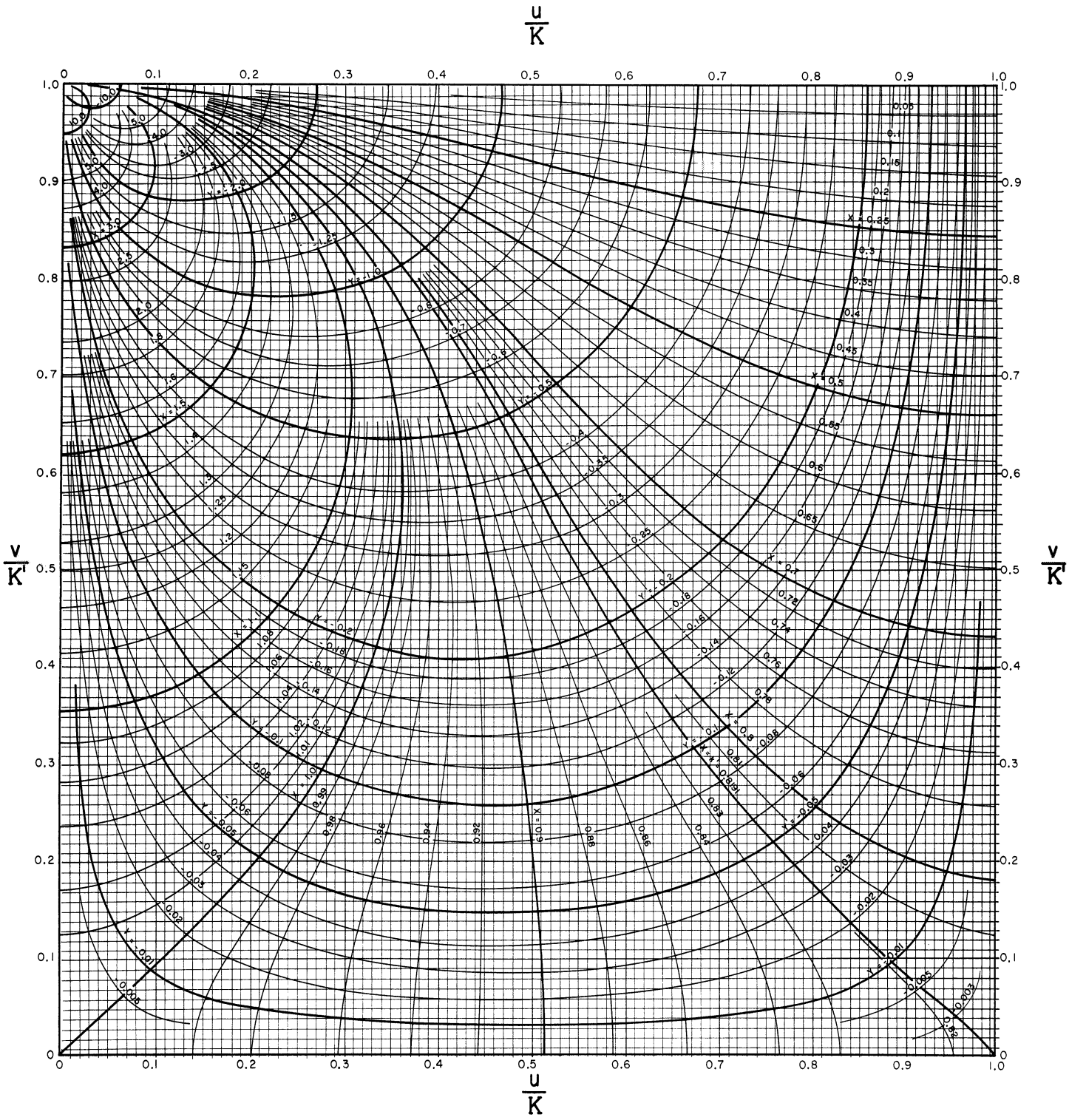
The dn - function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 30° = 0.5000$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 35° = 0.573576; K = 1.731245, K' = 2.034715

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9951	0.9811	0.9597	0.9334	0.9051	0.8776	0.8536	0.8349	0.8232	0.8192
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.001	1.002	0.9863	0.9631	0.9348	0.9044	0.8752	0.8497	0.8301	0.8177	0.8135
	y	0	0.01168	0.02186	0.02936	0.03348	0.03405	0.03135	0.02591	0.01840	0.009535	0
0.2	x	1.029	1.022	1.003	0.9737	0.9388	0.9021	0.8674	0.8377	0.8150	0.8009	0.7961
	y	0	0.02551	0.04758	0.06355	0.07198	0.07271	0.06651	0.05467	0.03867	0.01999	0
0.3	x	1.070	1.061	1.033	0.9925	0.9451	0.8969	0.8526	0.8156	0.7880	0.7710	0.7653
	y	0	0.04434	0.08210	0.1085	0.1214	0.1211	0.1095	0.08916	0.06263	0.03224	0
0.4	x	1.140	1.124	1.081	1.021	0.9523	0.8859	0.8274	0.7803	0.7461	0.7255	0.7187
	y	0	0.07280	0.1331	0.1728	0.1894	0.1852	0.1646	0.1321	0.09185	0.04698	0
0.5	x	1.254	1.227	1.155	1.058	0.9561	0.8632	0.7860	0.7268	0.6854	0.6610	0.6530
	y	0	0.1199	0.2145	0.2701	0.2868	0.2722	0.2358	0.1855	0.1272	0.06455	0
0.6	x	1.449	1.396	1.264	1.100	0.9455	0.8174	0.7193	0.6485	0.6012	0.5741	0.5653
	y	0	0.2056	0.3537	0.4235	0.4276	0.3886	0.3253	0.2497	0.1683	0.08454	0
0.7	x	1.805	1.684	1.409	1.123	0.8941	0.7288	0.6146	0.5380	0.4894	0.4624	0.4537
	y	0	0.3840	0.6109	0.6697	0.6264	0.5367	0.4305	0.3208	0.2122	0.1054	0
0.8	x	2.570	2.194	1.540	1.051	0.7510	0.5700	0.4589	0.3901	0.3483	0.3259	0.3188
	y	0	0.8379	1.118	1.050	0.8811	0.7030	0.5385	0.3896	0.2530	0.1245	0
0.9	x	4.971	2.908	1.319	0.7127	0.4512	0.3206	0.2484	0.2063	0.1818	0.1688	0.1648
	y	0	2.378	2.050	1.524	1.133	0.8464	0.6239	0.4411	0.2826	0.1381	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.728	2.791	1.779	1.247	0.9051	0.6571	0.4605	0.2935	0.1430	0



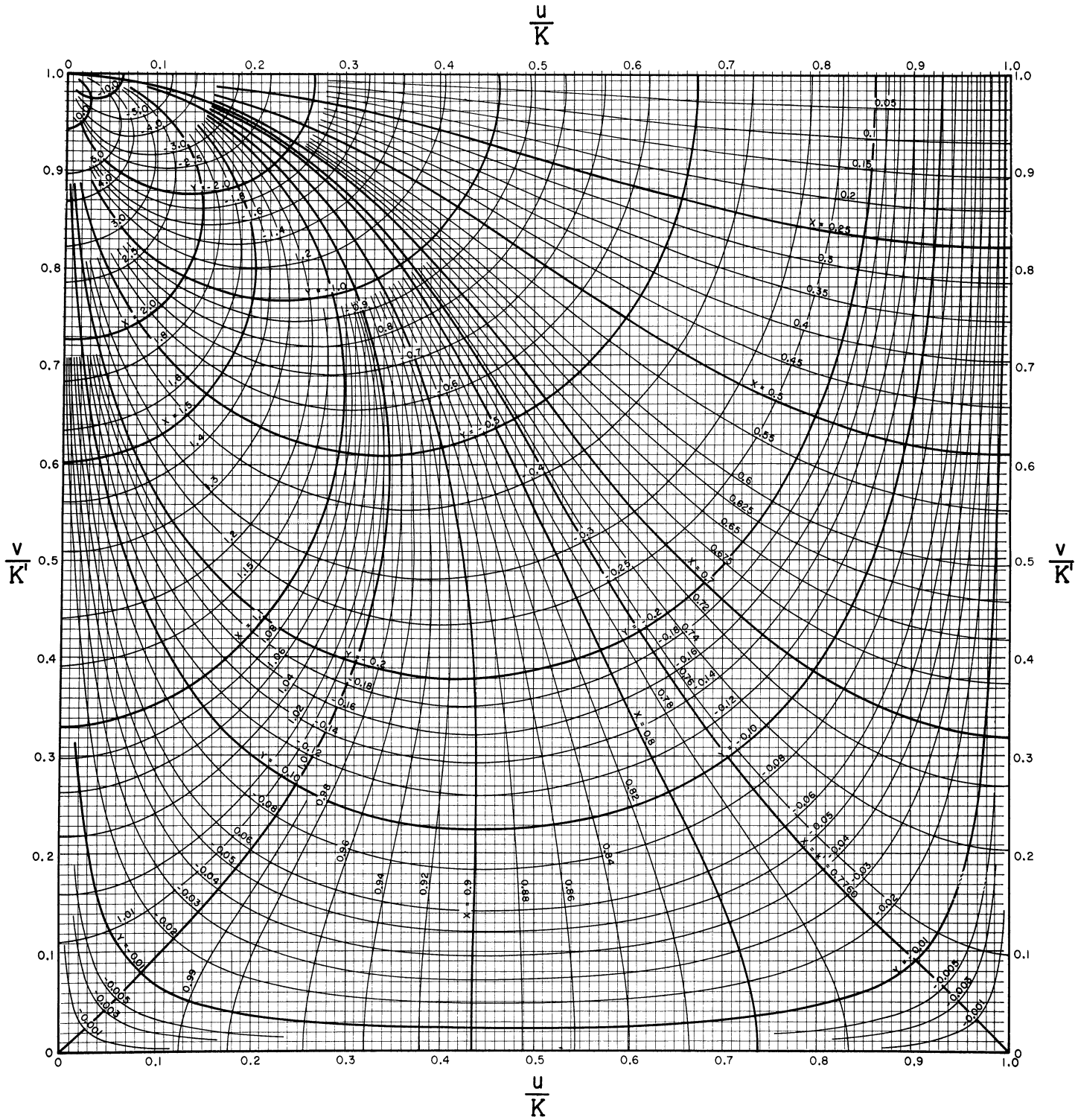
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 35° = 05736$

THE dn-FUNCTION, $x + iy = dn(u + iv)$

$k = \sin 40° = 0.642788$; $K = 1.786769$, $K' = 1.935581$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9935	0.9748	0.9465	0.9120	0.8752	0.8400	0.8094	0.7858	0.7711	0.7660
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.008	1.001	0.9806	0.9501	0.9133	0.8743	0.8372	0.8051	0.7806	0.7653	0.7601
	y	0	0.01434	0.02670	0.03556	0.04013	0.04038	0.03679	0.03013	0.02125	0.01097	0
0.2	x	1.033	1.024	0.9987	0.9613	0.9170	0.8711	0.8282	0.7919	0.7645	0.7475	0.7418
	y	0	0.03112	0.05767	0.07630	0.08547	0.08533	0.07719	0.06285	0.04414	0.02272	0
0.3	x	1.079	1.067	1.032	0.9807	0.9223	0.8640	0.8113	0.7679	0.4359	0.7164	0.7099
	y	0	0.05353	0.09836	0.1285	0.1420	0.1398	0.1249	0.1007	0.07018	0.03595	0
0.4	x	1.156	1.137	1.083	1.009	0.9272	0.8499	0.7832	0.7304	0.6926	0.6701	0.6626
	y	0	0.08679	0.1572	0.2011	0.2171	0.2092	0.1834	0.1457	0.1005	0.05117	0
0.5	x	1.282	1.248	1.160	1.044	0.9264	0.8225	0.7384	0.6751	0.6315	0.6060	0.5977
	y	0	0.1409	0.2488	0.3079	0.3208	0.2993	0.2557	0.1991	0.1355	0.06843	0
0.6	x	1.492	1.427	1.268	1.079	0.9075	0.7707	0.6688	0.5967	0.5491	0.5221	0.5133
	y	0	0.2384	0.4022	0.4704	0.4647	0.4148	0.3425	0.2602	0.1742	0.08718	0
0.7	x	1.874	1.725	1.403	1.086	0.8448	0.6771	0.5642	0.4898	0.4429	0.4171	0.4088
	y	0	0.4384	0.6772	0.7206	0.6583	0.5545	0.4396	0.3250	0.2138	0.1060	0
0.8	x	2.685	2.232	1.498	0.9905	0.6939	0.5204	0.4157	0.3514	0.3127	0.2919	0.2853
	y	0	0.9364	1.194	1.085	0.8928	0.7037	0.5349	0.3851	0.2493	0.1225	0
0.9	x	5.218	2.841	1.224	0.6488	0.4074	0.2881	0.2226	0.1845	0.1622	0.1505	0.1468
	y	0	2.528	2.069	1.508	1.112	0.8265	0.6075	0.4287	0.2743	0.1339	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.550	2.703	1.722	1.206	0.8752	0.6350	0.4448	0.2834	0.1380	0



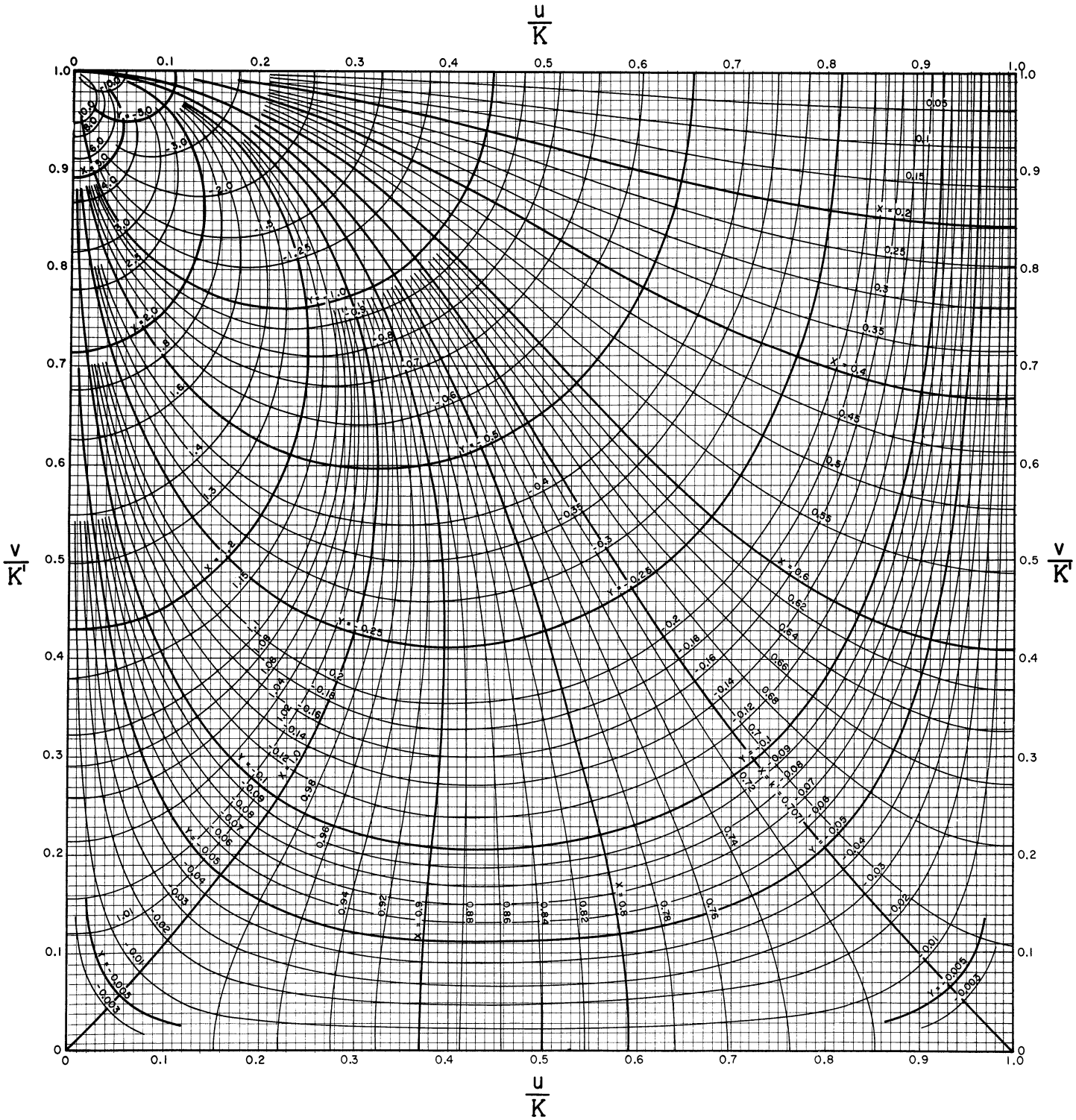
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 40° = 0.6428$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 45° = 0.707107; K = K' = 1.854075

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9915	0.9673	0.9309	0.8870	0.8409	0.7971	0.7596	0.7310	0.7132	0.7071
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.009	0.9996	0.9736	0.9346	0.8881	0.8396	0.7940	0.7550	0.7255	0.7072	0.7010
	y	0	0.01718	0.03177	0.04188	0.04670	0.04638	0.04174	0.03383	0.02367	0.01215	0
0.2	x	1.036	1.025	0.9930	0.9460	0.8912	0.8353	0.7839	0.7409	0.7088	0.6890	0.6824
	y	0	0.03708	0.06820	0.08923	0.09863	0.09713	0.08676	0.06988	0.04868	0.02493	0
0.3	x	1.087	1.071	1.028	0.9652	0.8950	0.8261	0.7651	0.7156	0.6796	0.6577	0.6504
	y	0	0.06326	0.1152	0.1486	0.1617	0.1568	0.1382	0.1102	0.07619	0.03884	0
0.4	x	1.171	1.147	1.081	0.9919	0.8968	0.8088	0.7346	0.6768	0.6360	0.6118	0.6038
	y	0	0.1015	0.1818	0.2290	0.2429	0.2302	0.1990	0.1563	0.1069	0.05417	0
0.5	x	1.307	1.265	1.159	1.023	0.8905	0.7769	0.6872	0.6210	0.5759	0.5497	0.5412
	y	0	0.1630	0.2835	0.3438	0.3511	0.3218	0.2709	0.2086	0.1409	0.07084	0
0.6	x	1.532	1.453	1.263	1.049	0.8633	0.7202	0.6162	0.5440	0.4969	0.4703	0.4617
	y	0	0.2724	0.4497	0.5129	0.4953	0.4341	0.3535	0.2660	0.1769	0.08819	0
0.7	x	1.936	1.756	1.384	1.040	0.7909	0.6238	0.5138	0.4423	0.3976	0.3731	0.3653
	y	0	0.4939	0.7389	0.7623	0.6804	0.5639	0.4420	0.3243	0.2123	0.1049	0
0.8	x	2.789	2.250	1.442	0.9244	0.6360	0.4718	0.3741	0.3145	0.2788	0.2596	0.2535
	y	0	1.033	1.256	1.106	0.8931	0.6961	0.5253	0.3764	0.2429	0.1191	0
0.9	x	5.440	2.743	1.126	0.5869	0.3659	0.2577	0.1985	0.1640	0.1439	0.1333	0.1300
	y	0	2.651	2.065	1.478	1.081	0.8004	0.5867	0.4132	0.2640	0.1288	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.347	2.604	1.658	1.160	0.8409	0.6095	0.4266	0.2716	0.1322	0



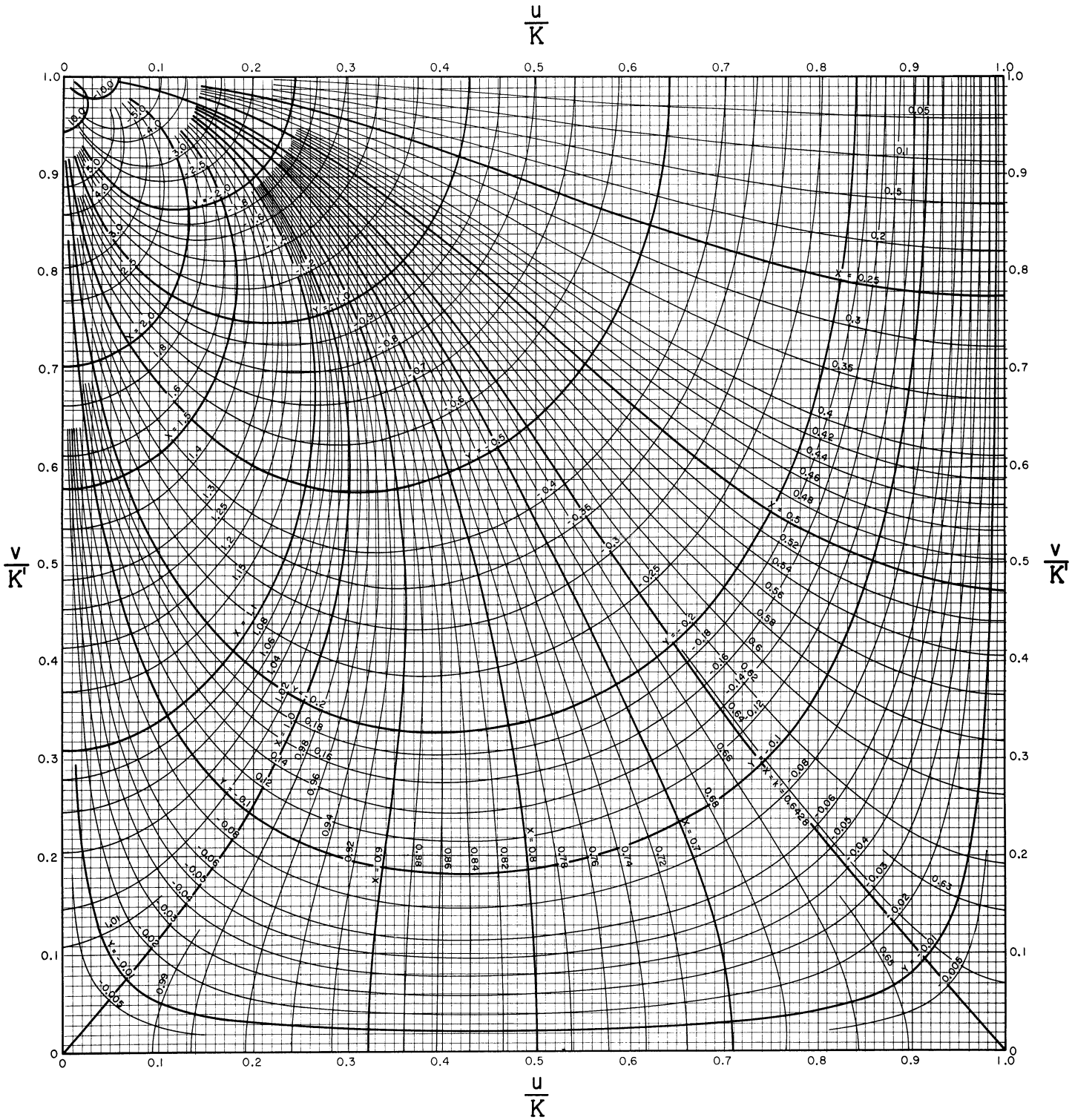
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 45° = 0.7071$

THE dn-FUNCTION, $x + iy = dn(u + iv)$

$k = \sin 50° = 0.766044$; $K = 1.935581$, $K' = 1.786769$

$\frac{v}{K'}$	$\frac{u}{K}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9892	0.9584	0.9126	0.8582	0.8017	0.7490	0.7043	0.6707	0.6498	0.6428
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.009	0.9978	0.9650	0.9163	0.8590	0.8001	0.7455	0.6996	0.6651	0.6439	0.6368
	y	0	0.02020	0.03704	0.04823	0.05299	0.05184	0.04599	0.03682	0.02553	0.01303	0
0.2	x	1.039	1.025	0.9852	0.9274	0.8609	0.7945	0.7344	0.6848	0.6483	0.6259	0.6184
	y	0	0.04341	0.07912	0.1021	0.1111	0.1077	0.09478	0.07541	0.05206	0.02651	0
0.3	x	1.094	1.075	1.021	0.9456	0.8626	0.7832	0.7141	0.6589	0.6192	0.5953	0.5873
	y	0	0.07358	0.1325	0.1683	0.1800	0.1716	0.1490	0.1173	0.08038	0.04075	0
0.4	x	1.185	1.155	1.075	0.9697	0.8607	0.7626	0.6816	0.6197	0.5765	0.5510	0.5426
	y	0	0.1171	0.2068	0.2558	0.2661	0.2476	0.2107	0.1635	0.1109	0.05589	0
0.5	x	1.329	1.279	1.151	0.9956	0.8486	0.7266	0.6328	0.5646	0.5187	0.4923	0.4837
	y	0	0.1863	0.3181	0.3772	0.3768	0.3388	0.2809	0.2139	0.1433	0.07172	0
0.6	x	1.567	1.471	1.250	1.011	0.8133	0.6662	0.5619	0.4906	0.4445	0.4186	0.4102
	y	0	0.3078	0.4958	0.5500	0.5186	0.4461	0.3583	0.2669	0.1763	0.08757	0
0.7	x	1.992	1.776	1.352	0.9853	0.7329	0.5691	0.4634	0.3954	0.3532	0.3301	0.3228
	y	0	0.5507	0.7950	0.7942	0.6924	0.5647	0.4378	0.3188	0.2077	0.1023	0
0.8	x	2.882	2.248	1.372	0.8537	0.5777	0.4241	0.3338	0.2790	0.2462	0.2286	0.2230
	y	0	1.126	1.304	1.113	0.8824	0.6804	0.5099	0.3636	0.2340	0.1145	0
0.9	x	5.639	2.617	1.025	0.5267	0.3263	0.2290	0.1757	0.1447	0.1266	0.1170	0.1140
	y	0	2.744	2.038	1.436	1.043	0.7684	0.5615	0.3945	0.2517	0.1227	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	5.121	2.492	1.585	1.108	0.8017	0.5803	0.4056	0.2580	0.1255	0



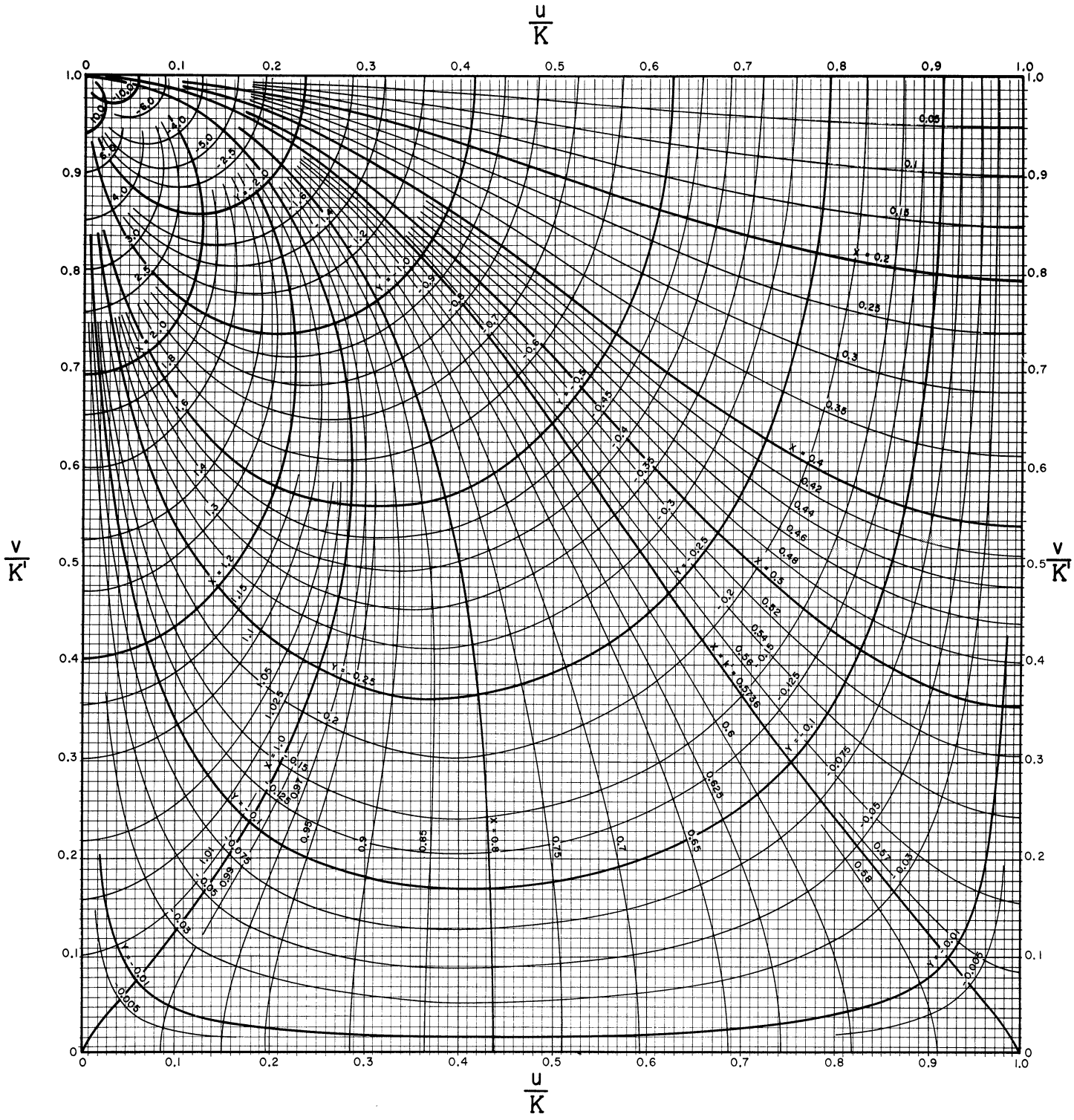
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 50° = 0.7660$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 55° = 0.819152; K = 2.034715, K' = 1.731245

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9863	0.9478	0.8911	0.8249	0.7573	0.6954	0.6436	0.6051	0.5815	0.5736
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.010	0.9956	0.9546	0.8947	0.8253	0.7553	0.6916	0.6388	0.5998	0.5759	0.5678
	y	0	0.02342	0.04250	0.05451	0.05883	0.05651	0.04930	0.03892	0.02670	0.01354	0
0.2	x	1.042	1.024	0.9751	0.9048	0.8258	0.7483	0.6796	0.6239	0.5832	0.5586	0.5504
	y	0	0.05014	0.09036	0.1147	0.1225	0.1165	0.1008	0.07911	0.05405	0.02734	0
0.3	x	1.101	1.077	1.011	0.9212	0.8248	0.7348	0.6581	0.5979	0.5551	0.5295	0.5210
	y	0	0.08454	0.1503	0.1874	0.1963	0.1835	0.1566	0.1216	0.08251	0.04158	0
0.4	x	1.197	1.160	1.065	0.9416	0.8187	0.7111	0.6245	0.5593	0.5143	0.4880	0.4794
	y	0	0.1336	0.2322	0.2810	0.2858	0.2606	0.2180	0.1669	0.1122	0.05622	0
0.5	x	1.349	1.288	1.137	0.9606	0.8003	0.6718	0.5752	0.5062	0.4602	0.4338	0.4253
	y	0	0.2107	0.3523	0.4072	0.3969	0.3497	0.2853	0.2146	0.1426	0.07099	0
0.6	x	1.598	1.483	1.226	0.9644	0.7575	0.6089	0.5060	0.4365	0.3919	0.3669	0.3589
	y	0	0.3448	0.5396	0.5807	0.5337	0.4503	0.3565	0.2628	0.1724	0.08530	0
0.7	x	2.041	1.784	1.308	0.9228	0.6714	0.5133	0.4130	0.3491	0.3096	0.2880	0.2811
	y	0	0.6086	0.8443	0.8152	0.6940	0.5569	0.4269	0.3084	0.1999	0.09821	0
0.8	x	2.965	2.223	1.289	0.7794	0.5192	0.3773	0.2947	0.2446	0.2147	0.1985	0.1935
	y	0	1.216	1.335	1.104	0.8608	0.6567	0.4886	0.3467	0.2223	0.1086	0
0.9	x	5.815	2.464	0.9235	0.4683	0.2885	0.2015	0.1540	0.1263	0.1100	0.1014	0.09864
	y	0	2.806	1.991	1.381	0.9956	0.7300	0.5315	0.3724	0.2371	0.1154	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	4.870	2.367	1.503	1.049	0.7573	0.5470	0.3816	0.2423	0.1178	0



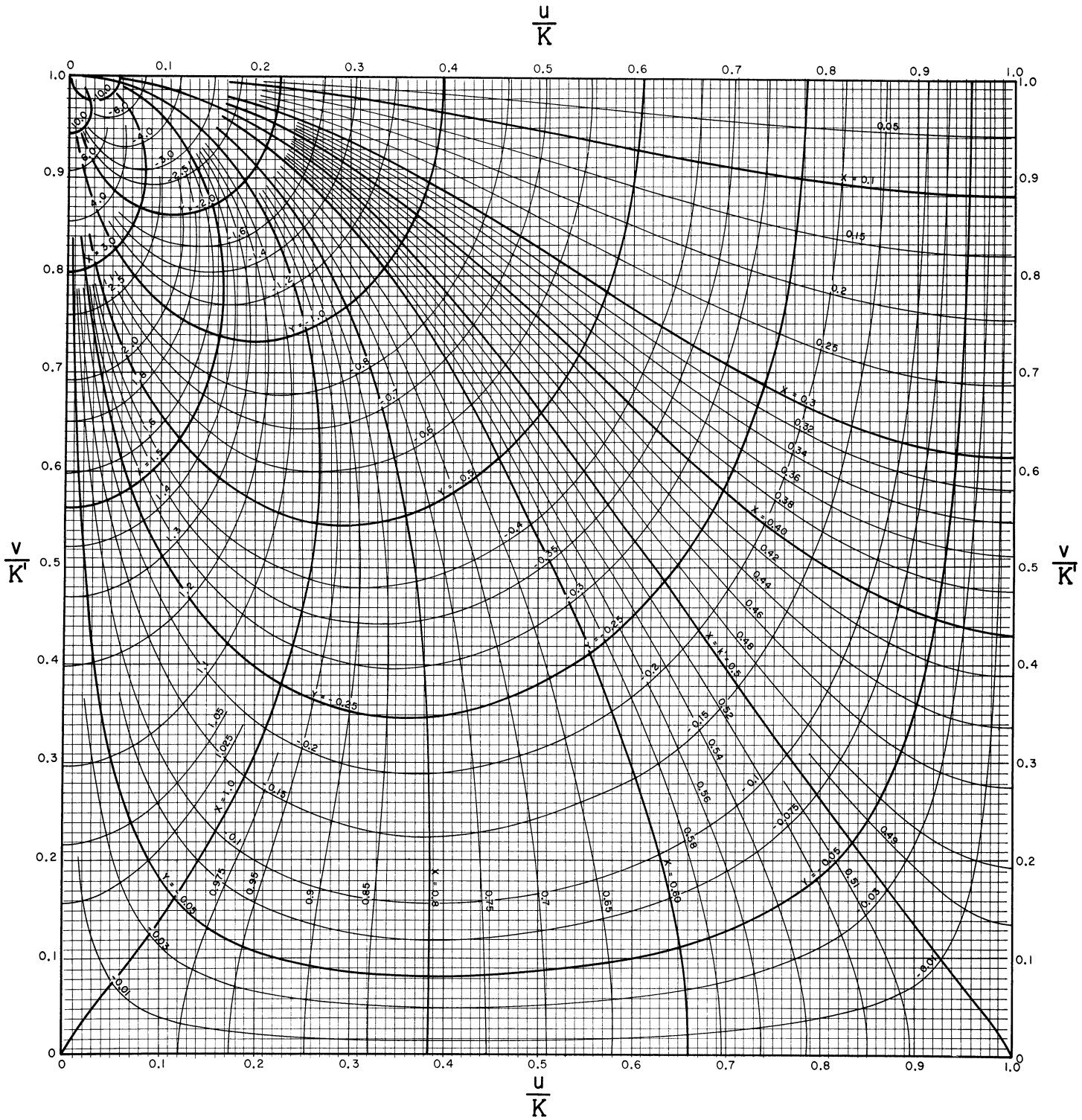
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 55° = 0.8192$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 60° = 0.866025; K = 2.156516, K' = 1.685750

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9829	0.9351	0.8657	0.7863	0.7071	0.6359	0.5775	0.5347	0.5087	0.5000
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.011	0.9925	0.9417	0.8688	0.7861	0.7045	0.6319	0.5727	0.5296	0.5034	0.4947
	y	0	0.02686	0.04812	0.06055	0.06394	0.06009	0.05140	0.03992	0.02706	0.01362	0
0.2	x	1.045	1.023	0.9621	0.8776	0.7849	0.6963	0.6194	0.5581	0.5139	0.4875	0.4786
	y	0	0.05736	0.1019	0.1268	0.1324	0.1231	0.1044	0.08063	0.05444	0.02734	0
0.3	x	1.107	1.077	0.9972	0.8912	0.7809	0.6806	0.5972	0.5328	0.4875	0.4607	0.4518
	y	0	0.09626	0.1683	0.2052	0.2098	0.1918	0.1605	0.1227	0.08230	0.04119	0
0.4	x	1.207	1.162	1.048	0.9066	0.7700	0.6542	0.5632	0.4958	0.4498	0.4230	0.4143
	y	0	0.1512	0.2576	0.3038	0.3012	0.2683	0.2203	0.1662	0.1105	0.05504	0
0.5	x	1.366	1.292	1.115	0.9173	0.7455	0.6124	0.5147	0.4458	0.4003	0.3744	0.3660
	y	0	0.2367	0.3858	0.4328	0.4105	0.3536	0.2834	0.2103	0.1385	0.06859	0
0.6	x	1.625	1.487	1.192	0.9088	0.6962	0.5487	0.4486	0.3817	0.3391	0.3153	0.3076
	y	0	0.3835	0.5804	0.6036	0.5397	0.4460	0.3477	0.2536	0.1651	0.08131	0
0.7	x	2.083	1.779	1.249	0.8525	0.6067	0.4565	0.3626	0.3032	0.2665	0.2464	0.2400
	y	0	0.6676	0.8852	0.8241	0.6844	0.5400	0.4091	0.2931	0.1888	0.09247	0
0.8	x	3.037	2.175	1.194	0.7019	0.4606	0.3313	0.2565	0.2112	0.1840	0.1693	0.1647
	y	0	1.301	1.349	1.081	0.8281	0.6247	0.4611	0.3253	0.2078	0.1013	0
0.9	x	5.967	2.285	0.8208	0.4114	0.2521	0.1753	0.1332	0.1085	0.09399	0.08623	0.08379
	y	0	2.833	1.922	1.314	0.9395	0.6850	0.4965	0.3466	0.2200	0.1069	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	4.592	2.229	1.412	0.9822	0.7071	0.5091	0.3541	0.2243	0.1089	0



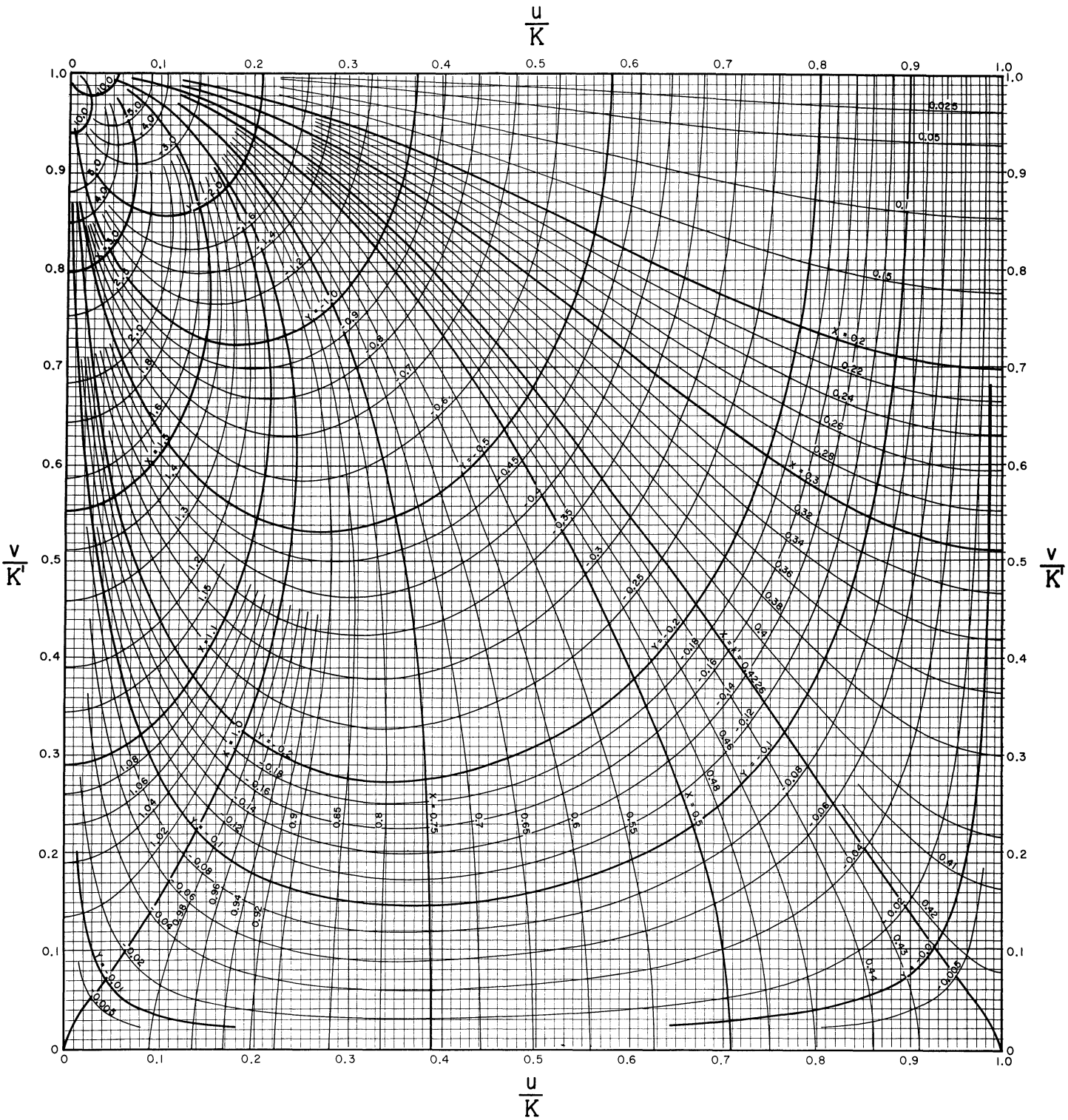
The dn-function, $z = x + iy = \mathrm{dn}(u+iv) = \sqrt{1 - k^2 \mathrm{sn}^2(u+iv)}$

$k = \sin 60° = 0.8660$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 65° = 0.906308; K = 2.308787, K' = 1.648995

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9786	0.9194	0.8353	0.7413	0.6501	0.5701	0.5060	0.4597	0.4319	0.4226
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.011	0.9885	0.9258	0.8377	0.7405	0.6471	0.5660	0.5014	0.4549	0.4271	0.4179
	y	0	0.03060	0.05390	0.06622	0.06805	0.06227	0.05202	0.03963	0.02649	0.01322	0
0.2	x	1.047	1.019	0.9453	0.8446	0.7373	0.6376	0.5533	0.4874	0.4407	0.4130	0.4038
	y	0	0.06519	0.1137	0.1379	0.1400	0.1268	0.1051	0.07959	0.05301	0.02641	0
0.3	x	1.111	1.075	0.9784	0.8544	0.7298	0.6200	0.5310	0.4636	0.4168	0.3893	0.3803
	y	0	0.1090	0.1866	0.2212	0.2197	0.1955	0.1599	0.1200	0.07949	0.03948	0
0.4	x	1.216	1.161	1.025	0.8636	0.7140	0.5915	0.4976	0.4292	0.3830	0.3563	0.3476
	y	0	0.1702	0.2828	0.3233	0.3108	0.2696	0.2166	0.1607	0.1056	0.05224	0
0.5	x	1.381	1.291	1.084	0.8648	0.6837	0.5483	0.4511	0.3836	0.3394	0.3142	0.3061
	y	0	0.2645	0.4179	0.4525	0.4160	0.3493	0.2745	0.2007	0.1308	0.06440	0
0.6	x	1.649	1.482	1.145	0.8437	0.6293	0.4855	0.3897	0.3263	0.2860	0.2636	0.2564
	y	0	0.4244	0.6169	0.6171	0.5352	0.4323	0.3314	0.2387	0.1541	0.07550	0
0.7	x	2.120	1.757	1.175	0.7747	0.5389	0.3988	0.3122	0.2575	0.2238	0.2053	0.1994
	y	0	0.7278	0.9156	0.8192	0.6626	0.5134	0.3839	0.2723	0.1743	0.08501	0
0.8	x	3.097	2.100	1.087	0.6216	0.4019	0.2860	0.2190	0.1784	0.1539	0.1406	0.1364
	y	0	1.379	1.343	1.042	0.7836	0.5838	0.4270	0.2991	0.1901	0.09236	0
0.9	x	6.097	2.081	0.7174	0.3558	0.2169	0.1500	0.1131	0.09136	0.07843	0.07151	0.06932
	y	0	2.823	1.831	1.233	0.8736	0.6326	0.4557	0.3165	0.2001	0.09701	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	4.286	0.2076	1.310	0.9074	0.6501	0.4657	0.3225	0.2036	0.09860	0



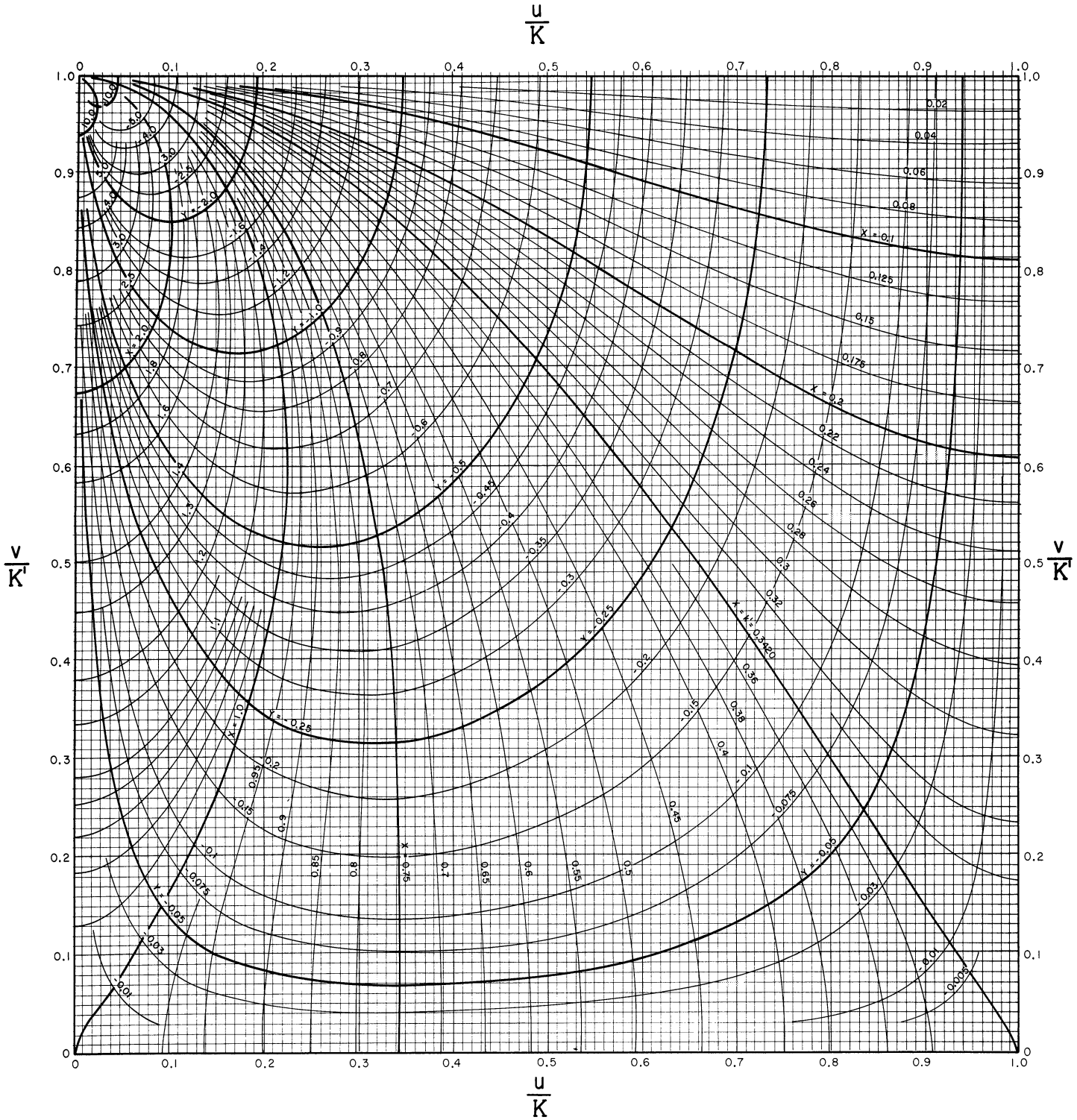
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 65° = 0.9063$

THE dn-FUNCTION, $x + iy = dn(u + iv)$

$k = \sin 70° = 0.939693$; $K = 2.504550$, $K' = 1.620026$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9730	0.8995	0.7979	0.6880	0.5848	0.4971	0.4286	0.3802	0.3515	0.3420
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.9830	0.9055	0.7995	0.6864	0.5815	0.4931	0.4244	0.3761	0.3475	0.3381
	y	0	0.03475	0.05985	0.07123	0.07071	0.06259	0.05079	0.03781	0.02486	0.01229	0
0.2	x	1.048	1.014	0.9233	0.8037	0.6810	0.5709	0.4806	0.4117	0.3637	0.3355	0.3262
	y	0	0.07387	0.1258	0.1476	0.1446	0.1266	0.1020	0.07558	0.04953	0.02444	0
0.3	x	1.115	1.069	0.9530	0.8087	0.6698	0.5519	0.4590	0.3901	0.3430	0.3156	0.3067
	y	0	0.1230	0.2050	0.2345	0.2245	0.1933	0.1539	0.1131	0.07377	0.03631	0
0.4	x	1.223	1.155	0.9937	0.8105	0.6494	0.5223	0.4274	0.3595	0.3141	0.2881	0.2796
	y	0	0.1912	0.3074	0.3380	0.3131	0.2631	0.2061	0.1499	0.09718	0.04767	0
0.5	x	1.393	1.282	1.041	0.8014	0.6140	0.4791	0.3844	0.3195	0.2772	0.2533	0.2456
	y	0	0.2949	0.4477	0.4644	0.4115	0.3354	0.2576	0.1852	0.1192	0.5825	0
0.6	x	1.667	1.465	1.083	0.7679	0.5563	0.4191	0.3292	0.2701	0.2327	0.2118	0.2051
	y	0	0.4682	0.6472	0.6185	0.5181	0.4079	0.3066	0.2176	0.1390	0.06770	0
0.7	x	2.150	1.717	1.085	0.6889	0.4681	0.3400	0.2615	0.2120	0.1813	0.1645	0.1591
	y	0	0.7891	0.9324	0.7981	0.6270	0.4759	0.3503	0.2456	0.1558	0.07562	0
0.8	x	3.147	1.992	0.9681	0.5384	0.3431	0.2411	0.1821	0.1461	0.1243	0.1124	0.1087
	y	0	1.446	1.314	0.9850	0.7257	0.5328	0.3852	0.2674	0.1687	0.08163	0
0.9	x	6.203	1.851	0.6133	0.3014	0.1827	0.1254	0.09358	0.07460	0.06322	0.05708	0.05514
	y	0	2.768	1.715	1.136	0.7965	0.5712	0.4079	0.2812	0.1768	0.08539	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	3.946	1.905	1.196	0.8225	0.5848	0.4158	0.2860	0.1796	0.08667	0



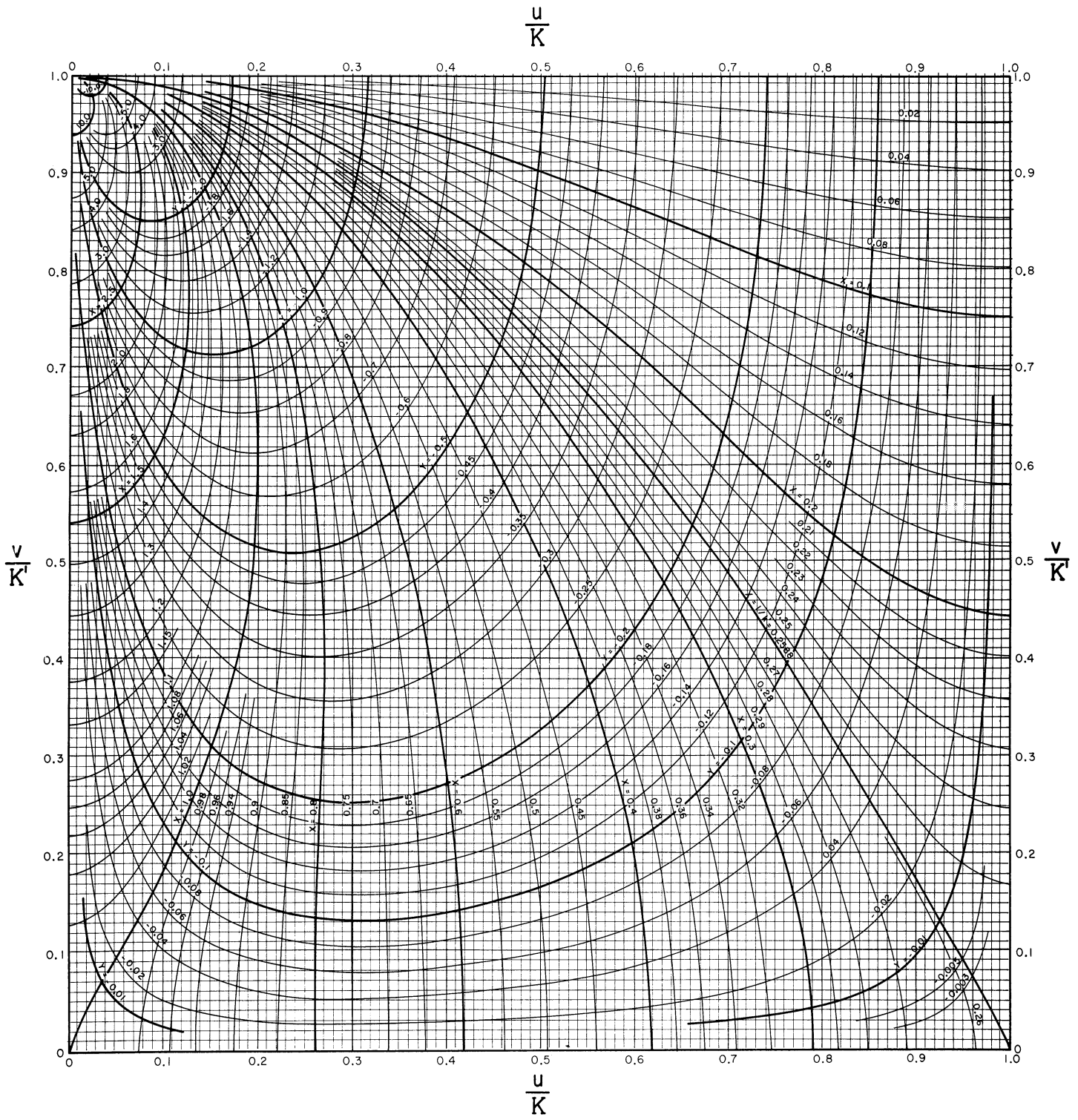
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 70° = 0.9397$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 75° = 0.965926; K = 2.768063, K' = 1.598142

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9653	0.8731	0.7501	0.6229	0.5087	0.4155	0.3450	0.2964	0.2681	0.2588
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.9752	0.8782	0.7506	0.6205	0.5051	0.4117	0.3414	0.2931	0.2650	0.2557
	y	0	0.03954	0.06595	0.07514	0.07121	0.06038	0.04724	0.03417	0.02201	0.01074	0
0.2	x	1.050	1.006	0.8933	0.7515	0.6129	0.4940	0.4000	0.3304	0.2830	0.2555	0.2466
	y	0	0.08388	0.1380	0.1547	0.1447	0.1214	0.09441	0.06802	0.04370	0.02131	0
0.3	x	1.118	1.060	0.9178	0.7507	0.5982	0.4745	0.3802	0.3120	0.2663	0.2400	0.2314
	y	0	0.1392	0.2231	0.2433	0.2223	0.1836	0.1412	0.1011	0.06473	0.03150	0
0.4	x	1.229	1.144	0.9495	0.7439	0.5736	0.4449	0.3516	0.2862	0.2431	0.2186	0.2106
	y	0	0.2151	0.3308	0.3455	0.3053	0.2467	0.1872	0.1329	0.08468	0.04111	0
0.5	x	1.402	1.264	0.9824	0.7240	0.5346	0.4036	0.3138	0.2530	0.2138	0.1917	0.1846
	y	0	0.3291	0.4736	0.4652	0.3939	0.3097	0.2312	0.1626	0.1030	0.04990	0
0.6	x	1.682	1.433	1.002	0.6793	0.4762	0.3488	0.2665	0.2128	0.1788	0.1599	0.1539
	y	0	0.5160	0.6682	0.6040	0.4855	0.3704	0.2717	0.1892	0.1193	0.05760	0
0.7	x	2.173	1.649	0.9739	0.5939	0.3934	0.2794	0.2±00	0.1661	0.1389	0.1239	0.1191
	y	0	0.8513	0.9308	0.7568	0.5746	0.4251	0.3067	0.2116	0.1327	0.06395	0
0.8	x	3.186	1.844	0.8351	0.4519	0.2834	0.1960	0.1452	0.1140	0.09496	0.08455	0.08124
	y	0	1.498	1.256	0.9063	0.6517	0.4695	0.3340	0.2287	0.1429	0.06869	0
0.9	x	6.286	1.592	0.5077	0.2475	0.1491	0.1012	0.07431	0.05805	0.04821	0.04287	0.04118
	y	0	2.660	1.571	1.021	0.7051	0.4986	0.3514	0.2394	0.1492	0.07163	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	3.564	1.710	1.065	0.7242	0.5087	0.3574	0.2431	0.1513	0.07263	0



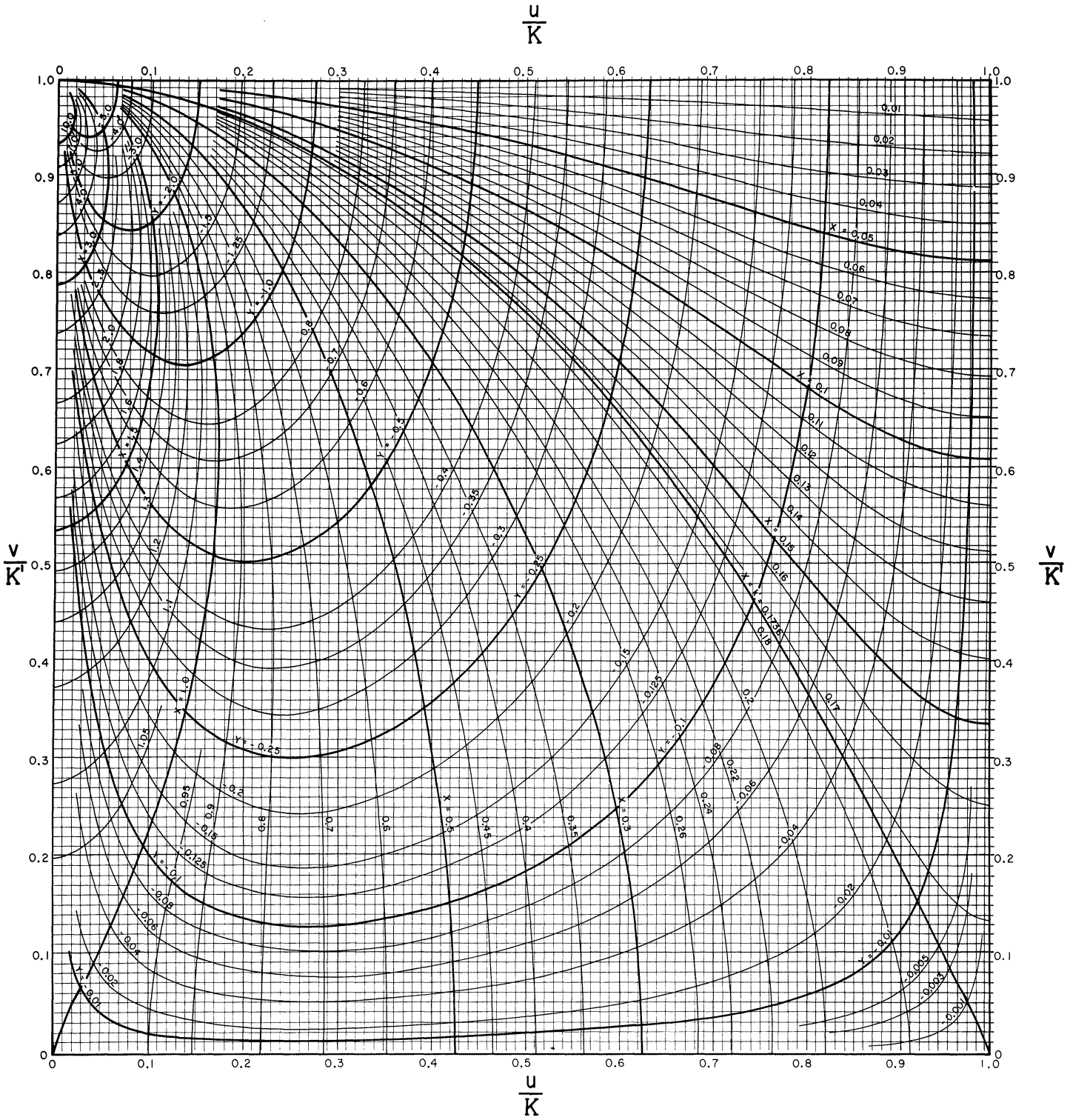
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 75° = 0.9659$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 80° = 0.984808; K = 3.153385, K' = 1.582843

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9537	0.8345	0.6845	0.5391	0.4167	0.3221	0.2537	0.2081	0.1821	0.1736
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.9632	0.8383	0.6836	0.5361	0.4131	0.3188	0.2508	0.2056	0.1799	0.1715
	y	0	0.04548	0.07213	0.07697	0.06824	0.05451	0.04058	0.02824	0.01769	0.008494	0
0.2	x	1.051	0.9924	0.8492	0.6804	0.5264	0.4022	0.3088	0.2423	0.1983	0.1733	0.1653
	y	0	0.09624	0.1501	0.1573	0.1376	0.1090	0.08072	0.05602	0.03504	0.01681	0
0.3	x	1.121	1.074	0.8659	0.6727	0.5090	0.3834	0.2919	0.2280	0.1862	0.1626	0.1550
	y	0	0.1590	0.2403	0.2443	0.2090	0.1632	0.1199	0.08283	0.05168	0.02477	0
0.4	x	1.233	1.121	0.8849	0.6567	0.4816	0.3560	0.2682	0.2083	0.1696	0.1479	0.1409
	y	0	0.2441	0.3510	0.3409	0.2826	0.2165	0.1574	0.1081	0.06726	0.03218	0
0.5	x	1.409	1.231	0.8987	0.6263	0.4415	0.3192	0.2376	0.1833	0.1487	0.1294	0.1233
	y	0	0.3696	0.4918	0.4485	0.3575	0.2679	0.1923	0.1312	0.08137	0.03887	0
0.6	x	1.693	1.375	0.8919	0.5729	0.3860	0.2724	0.2002	0.1534	0.1240	0.1078	0.1026
	y	0	0.5698	0.6734	0.5660	0.4311	0.3153	0.2236	0.1515	0.9362	0.04466	0
0.7	x	2.189	1.542	0.8353	0.4867	0.3127	0.2156	0.1565	0.1192	0.09607	0.08337	0.07931
	y	0	0.9131	0.9012	0.6871	0.4992	0.3566	0.2498	0.1683	0.1036	0.04936	0
0.8	x	3.214	1.639	0.6842	0.3599	0.2215	0.1497	0.1076	0.08154	0.06555	0.05681	0.05403
	y	0	1.524	1.160	0.7981	0.5553	0.3899	0.2698	0.1808	0.1111	0.05284	0
0.9	x	6.345	1.298	0.3985	0.1929	0.1151	0.07675	0.05482	0.04141	0.03323	0.02878	0.02737
	y	0	2.479	1.387	0.8785	0.5928	0.4096	0.2823	0.1886	0.1157	0.05498	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	3.118	1.482	0.9082	0.6060	0.4167	0.2865	0.1912	0.1172	0.05570	0



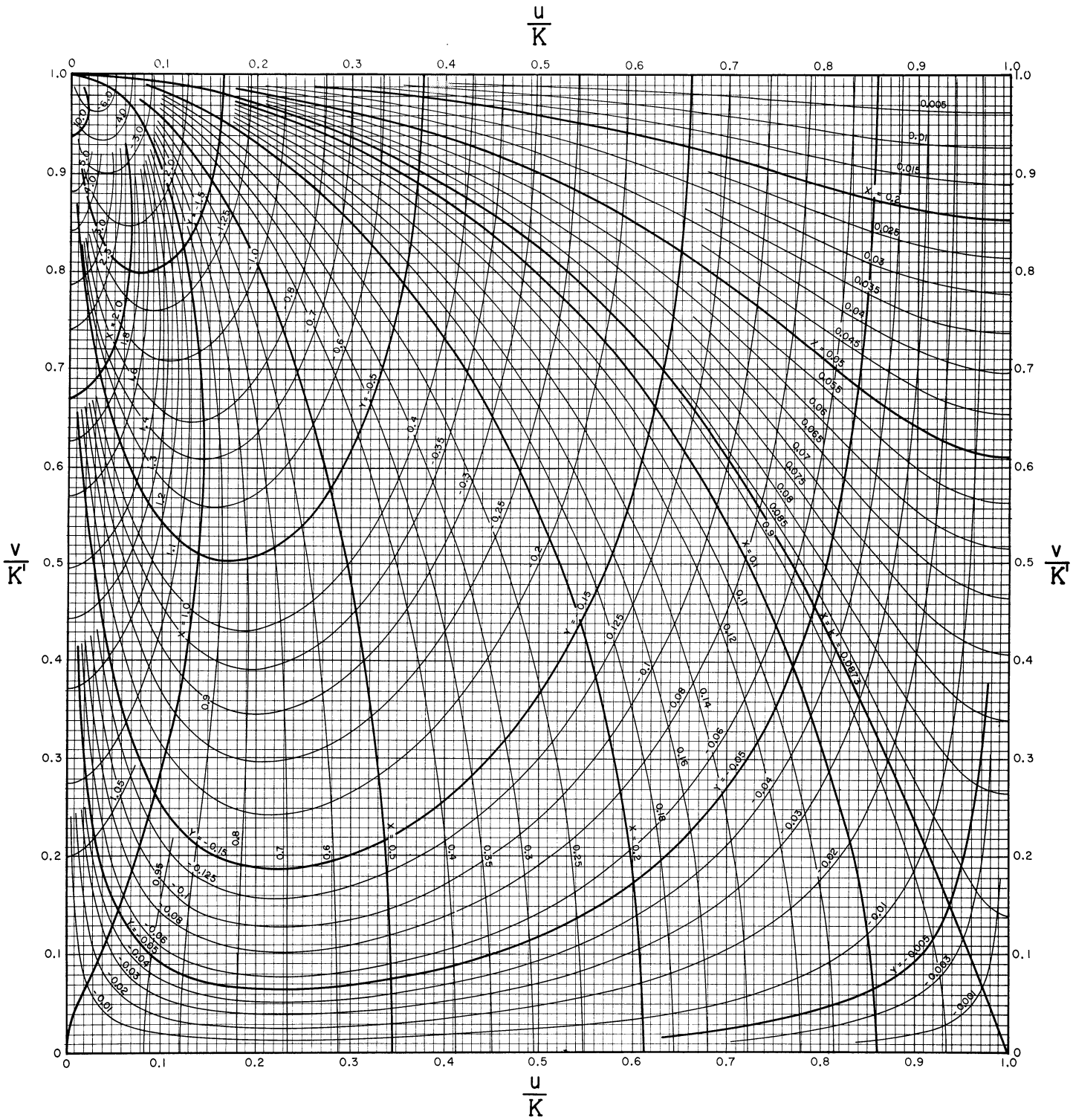
The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 80° = 0.9848$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 85° = 0.996195; K = 3.831742, K' = 1.573792

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.9313	0.7661	0.5790	0.4176	0.2952	0.2087	0.1505	0.1138	0.09358	0.08749
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.9398	0.7676	0.5766	0.4141	0.2922	0.2063	0.1487	0.1124	0.09243	0.08609
	y	0	0.05407	0.07779	0.07375	0.05841	0.04233	0.02910	0.01902	0.01139	0.005318	0
0.2	x	1.051	0.9655	0.7716	0.5687	0.4037	0.2830	0.1992	0.1434	0.1083	0.08903	0.08290
	y	0	0.1140	0.1605	0.1493	0.1168	0.08412	0.05765	0.03764	0.02251	0.01051	0
0.3	x	1.122	1.010	0.7759	0.5537	0.3855	0.2676	0.1874	0.1346	0.1015	0.08343	0.07769
	y	0	0.1871	0.2531	0.2280	0.1750	0.1248	0.08510	0.05542	0.03312	0.01545	0
0.4	x	1.235	1.075	0.7764	0.5289	0.3588	0.2458	0.1711	0.1225	0.09227	0.07579	0.07036
	y	0	0.2840	0.3614	0.3108	0.2324	0.1635	0.1108	0.07197	0.04294	0.02003	0
0.5	x	1.413	1.161	0.7651	0.4907	0.3226	0.2177	0.1505	0.1073	0.08074	0.06627	0.06169
	y	0	0.4226	0.4904	0.3970	0.2878	0.1994	0.1342	0.08686	0.05175	0.02412	0
0.6	x	1.699	1.264	0.7290	0.4346	0.2761	0.1834	0.1258	0.08947	0.06719	0.05512	0.05129
	y	0	0.6334	0.6436	0.4842	0.3392	0.2314	0.1546	0.09970	0.05931	0.02762	0
0.7	x	2.199	1.354	0.6487	0.3567	0.2191	0.1433	0.09771	0.06927	0.05195	0.04259	0.03963
	y	0	0.9683	0.8170	0.5671	0.3839	0.2581	0.1713	0.1101	0.06542	0.03045	0
0.8	x	3.231	1.331	0.5022	0.2555	0.1524	0.09853	0.06680	0.04724	0.03539	0.02900	0.02697
	y	0	1.493	0.9923	0.6373	0.4191	0.2783	0.1837	0.1178	0.06992	0.03253	0
0.9	x	6.381	0.9424	0.2788	0.1339	0.07827	0.05019	0.03391	0.02395	0.01792	0.01468	0.01336
	y	0	2.169	1.131	0.6851	0.4416	0.2909	0.1913	0.1225	0.07267	0.03380	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	2.546	1.184	0.7021	0.4494	0.2952	0.1939	0.1241	0.07360	0.03423	0



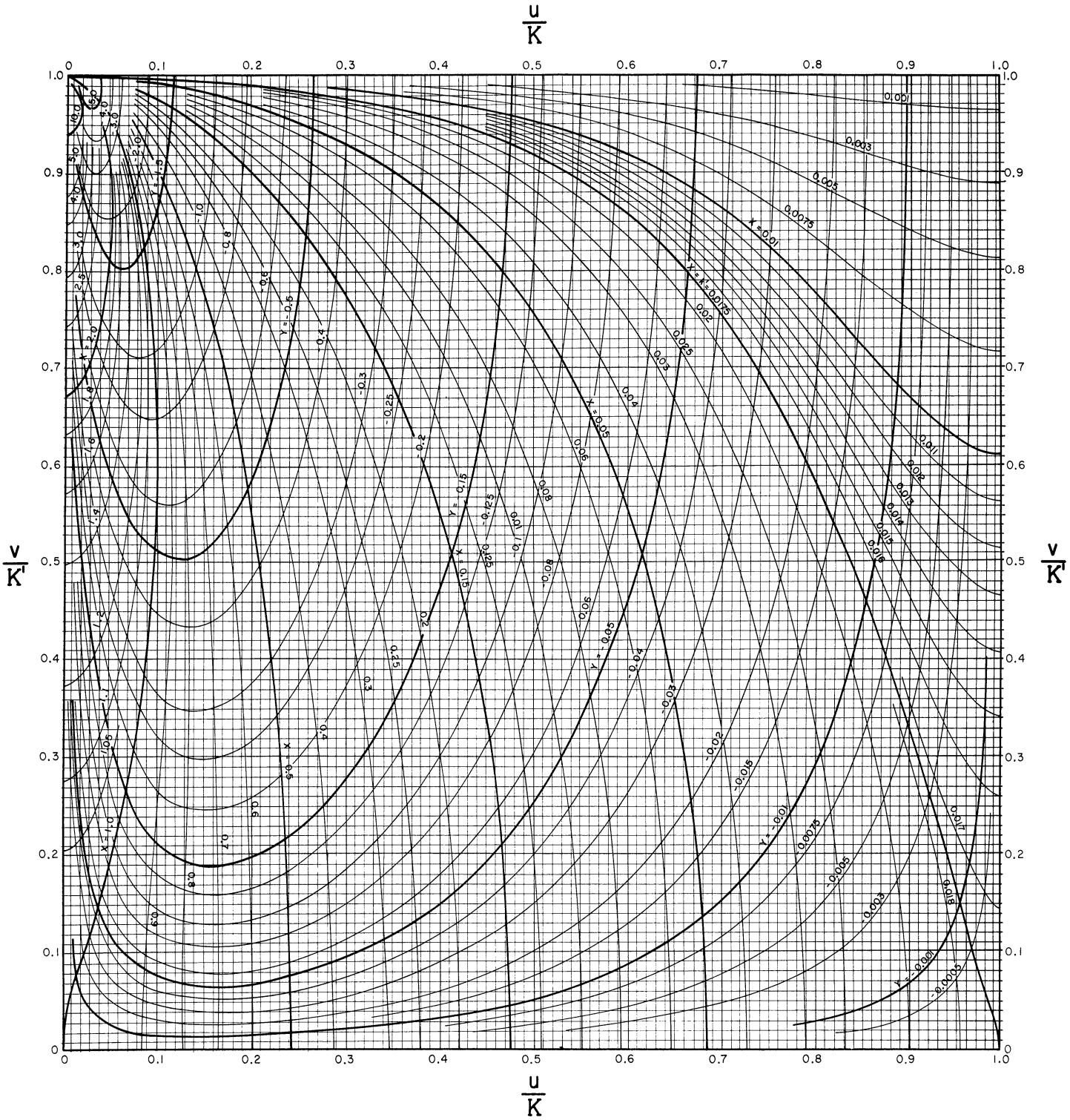
The dn - function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 85° = 0.9962$

THE dn-FUNCTION, x + iy = dn(u + iv)

k = sin 89° = 0.999848; K = 5.434910, K' = 1.570916

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	x	1.000	0.8686	0.6057	0.3774	0.2249	0.1321	0.07760	0.04624	0.02881	0.02009	0.01745
	y	0	0	0	0	0	0	0	0	0	0	0
0.1	x	1.012	0.8740	0.6037	0.3741	0.2224	0.1305	0.07665	0.04567	0.02846	0.01985	0.01724
	y	0	0.06859	0.07605	0.05481	0.03422	0.02032	0.01179	0.006692	0.003585	0.001557	0
0.2	x	1.051	0.8902	0.5970	0.3639	0.2149	0.1258	0.07384	0.04399	0.02740	0.01911	0.01660
	y	0	0.1433	0.1543	0.1094	0.06785	0.04018	0.02331	0.01322	0.007082	0.003076	0
0.3	x	1.122	0.9164	0.5838	0.3464	0.2025	0.1181	0.06922	0.04122	0.02568	0.01790	0.01555
	y	0	0.2314	0.2366	0.1633	0.1002	0.05915	0.03427	0.01943	0.01041	0.004520	0
0.4	x	1.236	0.9504	0.5611	0.3211	0.1852	0.1075	0.06290	0.03743	0.02332	0.01626	0.01412
	y	0	0.3421	0.3243	0.2159	0.1307	0.07676	0.04440	0.02516	0.01347	0.005852	0
0.5	x	1.414	0.9861	0.5245	0.2873	0.1631	0.09423	0.05503	0.03243	0.02038	0.01421	0.01234
	y	0	0.4886	0.4172	0.2658	0.1585	0.09260	0.05347	0.03028	0.01621	0.007040	0
0.6	x	1.701	1.008	0.4685	0.2446	0.1367	0.07854	0.04579	0.02721	0.01694	0.01181	0.01026
	y	0	0.6878	0.5129	0.3115	0.1828	0.1062	0.06123	0.03465	0.01855	0.008056	0
0.7	x	2.203	0.9831	0.3879	0.1932	0.1064	0.06081	0.03539	0.02102	0.01309	0.009123	0.007923
	y	0	0.9561	0.6057	0.3508	0.2028	0.1173	0.06749	0.03818	0.02043	0.008872	0
0.8	x	3.236	0.8449	0.2801	0.1339	0.07282	0.04147	0.02411	0.01431	0.008908	0.006210	0.005393
	y	0	1.289	0.6858	0.3812	0.2178	0.1254	0.07209	0.04076	0.02181	0.009471	0
0.9	x	6.392	0.5145	0.1475	0.06856	0.03700	0.02102	0.01221	0.007247	0.004509	0.003143	0.002730
	y	0	1.610	0.7411	0.4005	0.2270	0.1304	0.07489	0.04234	0.02265	0.009836	0
1.0	x	∞	0	0	0	0	0	0	0	0	0	0
	y	∞	1.752	0.7609	0.4071	0.2301	0.1321	0.07584	0.04287	0.02293	0.009959	0



The dn-function, $z = x + iy = dn(u+iv) = \sqrt{1 - k^2 sn^2(u+iv)}$

$k = \sin 89° = 0.9999$

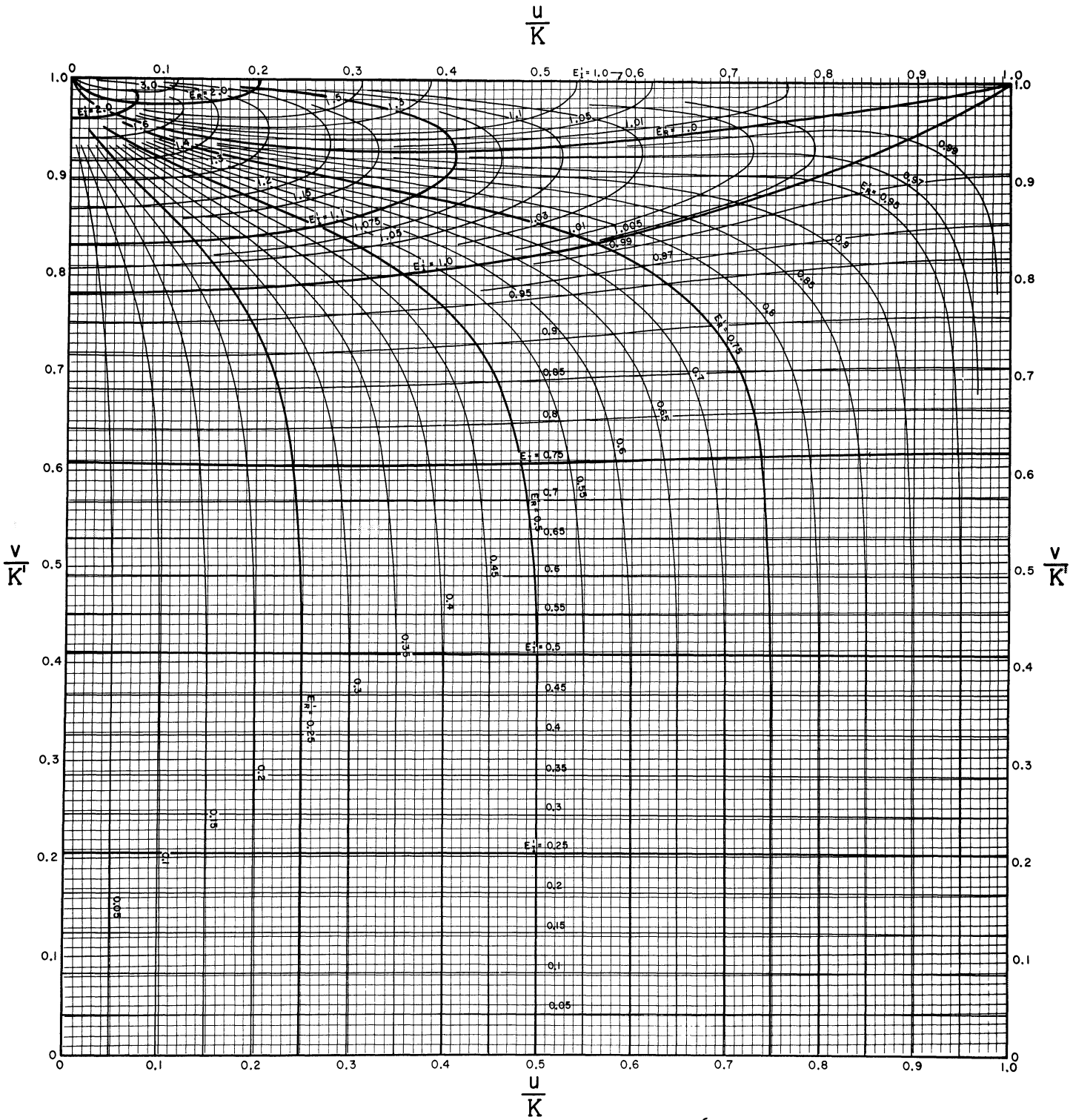
APPENDIX IV

THE E-FUNCTION

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 1° = 0.0174524$; $K = 1.570916$, $K' = 5.434910$, $K' - E' = 4.434158$; $E = 1.570677$, $E' = 1.000752$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1000	0.2000	0.3001	0.4001	0.5001	0.6001	0.7001	0.8000	0.9000	1.000
	E'_I	0.1226	0.1226	0.1226	0.1226	0.1226	0.1226	0.1225	0.1225	0.1225	0.1225	0.1225
0.2	E'_R	0	0.1001	0.2001	0.3002	0.4002	0.5002	0.6002	0.7002	0.8001	0.9001	1.000
	E'_I	0.2452	0.2452	0.2452	0.2451	0.2451	0.2451	0.2451	0.2451	0.2450	0.2450	0.2450
0.3	E'_R	0	0.1002	0.2004	0.3005	0.4006	0.5006	0.6006	0.7005	0.8004	0.9002	1.000
	E'_I	0.3679	0.3679	0.3678	0.3678	0.3677	0.3677	0.3676	0.3675	0.3675	0.3674	0.3674
0.4	E'_R	0	0.1006	0.2011	0.3015	0.4018	0.5019	0.6018	0.7015	0.8011	0.9006	1.000
	E'_I	0.4909	0.4908	0.4907	0.4906	0.4904	0.4902	0.4900	0.4898	0.4897	0.4896	0.4895
0.5	E'_R	0	0.1017	0.2033	0.3045	0.4052	0.5056	0.6053	0.7045	0.9032	0.9017	1.000
	E'_I	0.6147	0.6146	0.6144	0.6139	0.6134	0.6128	0.6121	0.6116	0.6112	0.6109	0.6108
0.6	E'_R	0	0.1052	0.2099	0.3135	0.4156	0.5165	0.6155	0.7131	0.8095	0.9050	1.000
	E'_I	0.7412	0.7410	0.7401	0.7388	0.7371	0.7353	0.7335	0.7319	0.7307	0.7298	0.7295
0.7	E'_R	0	0.1162	0.2306	0.3413	0.4469	0.5488	0.6453	0.7378	0.8270	0.9140	1.000
	E'_I	0.8758	0.8750	0.8721	0.8680	0.8627	0.8573	0.8521	0.8476	0.8441	0.8419	0.8412
0.8	E'_R	0	0.1563	0.3027	0.4333	0.5443	0.6430	0.7271	0.8022	0.8711	0.9364	1.000
	E'_I	1.038	1.035	1.023	1.008	0.9910	0.9746	0.9604	0.9490	0.9409	0.9362	0.9343
0.9	E'_R	0	0.3810	0.6443	0.7845	0.8457	0.8856	0.9089	0.9300	0.9521	0.8756	1.000
	E'_I	1.332	1.301	1.229	1.156	1.099	1.057	1.029	1.010	0.9979	0.9913	0.9892
1.0	E'_R	∞	4.119	2.159	1.549	1.276	1.137	1.063	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

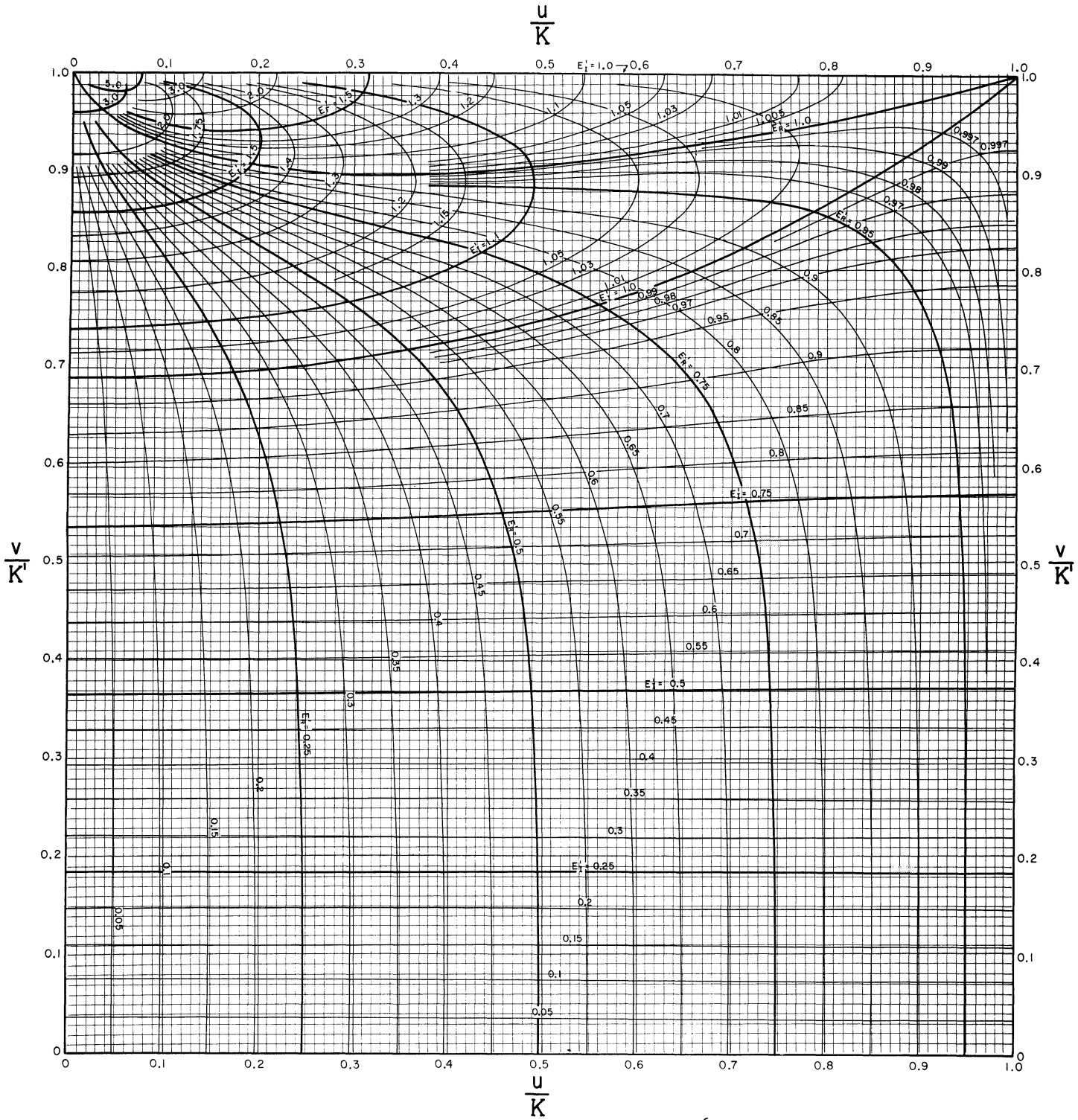
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 1° = 0.0175$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 5° = 0.0871557$; $K = 1.573792$, $K' = 3.831742$, $K' - E' = 2.819078$; $E = 1.567809$, $E' = 1.012664$

$\frac{v}{K'}$	$\frac{u}{K}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1004	0.2007	0.3010	0.4012	0.5012	0.6012	0.7010	0.8007	0.9004	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1005	0.2009	0.3013	0.4015	0.5016	0.6015	0.7013	0.8009	0.9005	1.000
	E'_I	0.1360	0.1359	0.1359	0.1357	0.1356	0.1354	0.1352	0.1351	0.1349	0.1349	0.1348
0.2	E'_R	0	0.1009	0.2017	0.3024	0.4028	0.5029	0.6028	0.7024	0.8017	0.9009	1.000
	E'_I	0.2723	0.2722	0.2720	0.2717	0.2713	0.2708	0.2703	0.2699	0.2696	0.2694	0.2693
0.3	E'_R	0	0.1019	0.2036	0.3049	0.4058	0.5061	0.6058	0.7049	0.8035	0.9019	1.000
	E'_I	0.4095	0.4094	0.4089	0.4082	0.4072	0.4062	0.4052	0.4043	0.4035	0.4031	0.4029
0.4	E'_R	0	0.1041	0.2077	0.3106	0.4124	0.5130	0.6122	0.7104	0.8075	0.9039	1.000
	E'_I	0.5489	0.5485	0.5475	0.5458	0.5438	0.5415	0.5393	0.5374	0.5358	0.5349	0.5345
0.5	E'_R	0	0.1089	0.2169	0.3231	0.4268	0.5278	0.6261	0.7219	0.8158	0.9082	1.000
	E'_I	0.6928	0.6920	0.6896	0.6860	0.6815	0.6767	0.6720	0.6679	0.6646	0.6626	0.6619
0.6	E'_R	0	0.1202	0.2379	0.3510	0.4584	0.5596	0.6551	0.7457	0.8326	0.9169	1.000
	E'_I	0.8473	0.8453	0.8397	0.8314	0.8214	0.8109	0.8010	0.7925	0.7861	0.7821	0.7807
0.7	E'_R	0	0.1485	0.2891	0.4164	0.5288	0.6270	0.7138	0.7920	0.8643	0.9330	1.000
	E'_I	1.027	1.022	1.008	0.9868	0.9635	0.9407	0.9206	0.9043	0.8925	0.8854	0.8830
0.8	E'_R	0	0.2342	0.4327	0.5816	0.6872	0.7634	0.8221	0.8714	0.9160	0.9584	1.000
	E'_I	1.279	1.261	1.214	1.155	1.099	1.052	1.015	0.9885	0.9707	0.9605	0.9572
0.9	E'_R	0	0.6525	0.9511	1.016	1.006	0.9871	0.9747	0.9719	0.9769	0.9871	1.000
	E'_I	1.835	1.702	1.463	1.279	1.163	1.093	1.049	1.022	1.005	0.9966	0.9938
1.0	E'_R	∞	4.119	2.159	1.549	1.276	1.136	1.063	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E - function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

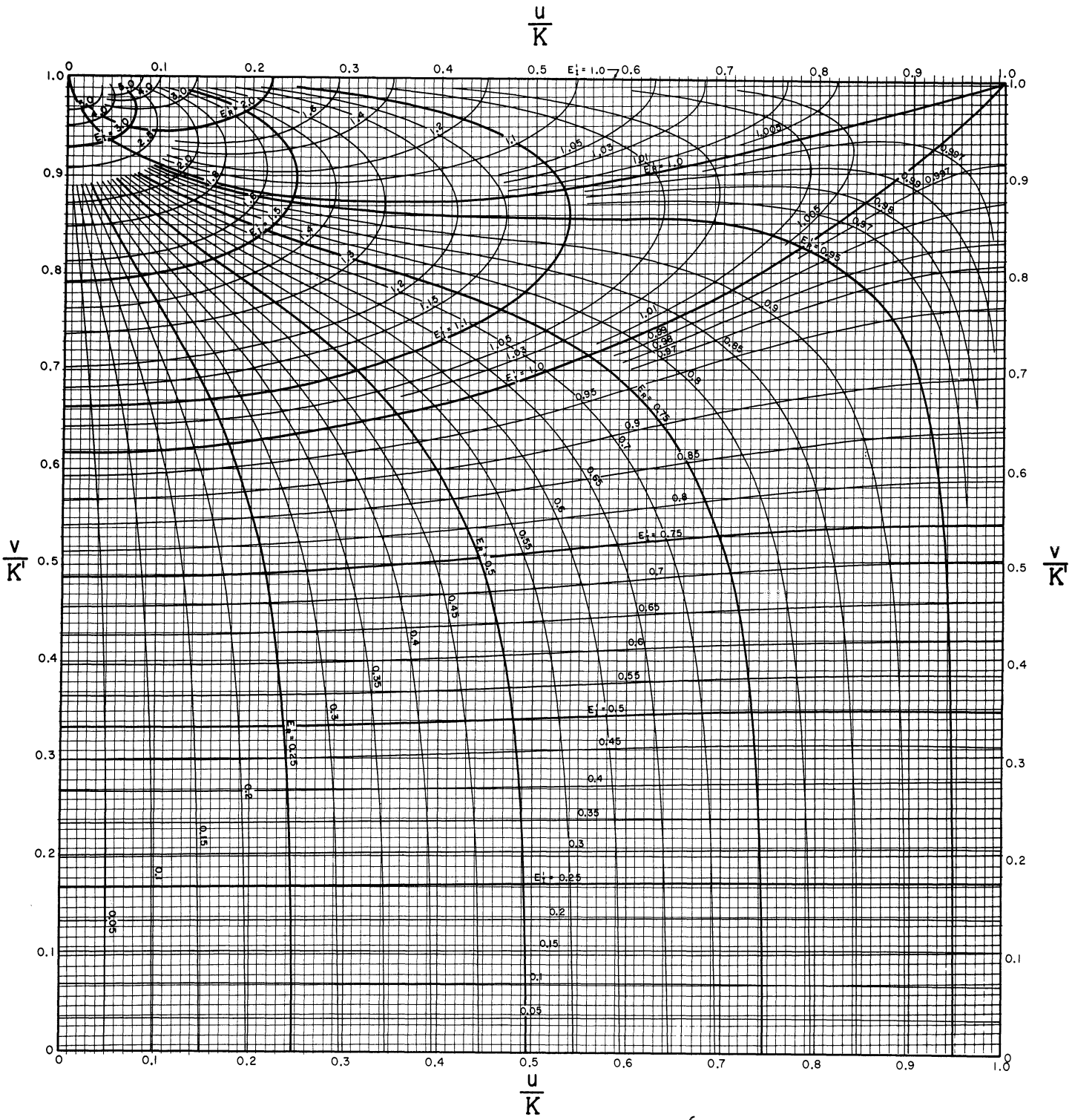
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 5^\circ = 0.0872$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 10° = 0.173648$; $K = 1.582843$, $K' = 3.153385$, $K' - E' = 2.113271$; $E = 1.558887$, $E' = 1.040114$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1015	0.2029	0.3040	0.4046	0.5049	0.6046	0.7039	0.8029	0.9015	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1018	0.2035	0.3048	0.4056	0.5059	0.6056	0.7047	0.8034	0.9018	1.000
	E'_I	0.1494	0.1493	0.1489	0.1484	0.1477	0.1469	0.1462	0.1455	0.1450	0.1447	0.1446
0.2	E'_R	0	0.1029	0.2055	0.3075	0.4088	0.5092	0.6087	0.7074	0.8054	0.9028	1.000
	E'_I	0.2997	0.2994	0.2986	0.2973	0.2957	0.2939	0.2921	0.2905	0.2893	0.2885	0.2882
0.3	E'_R	0	0.1052	0.2098	0.3134	0.4156	0.5163	0.6154	0.7130	0.8094	0.9049	1.000
	E'_I	0.4525	0.4519	0.4502	0.4476	0.4443	0.4407	0.4372	0.4341	0.4316	0.4301	0.4295
0.4	E'_R	0	0.1097	0.2183	0.3249	0.4289	0.5300	0.6281	0.7236	0.8170	0.9089	1.000
	E'_I	0.6102	0.6090	0.6057	0.6005	0.5942	0.5873	0.5807	0.5749	0.5703	0.5675	0.5665
0.5	E'_R	0	0.1188	0.2352	0.3475	0.4544	0.5557	0.6516	0.7429	0.8306	0.9159	1.000
	E'_I	0.7777	0.7753	0.7685	0.7583	0.7460	0.7330	0.7207	0.7102	0.7022	0.6972	0.6955
0.6	E'_R	0	0.1378	0.2700	0.3926	0.5036	0.6035	0.6937	0.7764	0.8538	0.9277	1.000
	E'_I	0.9654	0.9602	0.9456	0.9244	0.9000	0.8755	0.8534	0.8352	0.8217	0.8135	0.8108
0.7	E'_R	0	0.1822	0.3475	0.4868	0.5994	0.6903	0.7657	0.8310	0.8901	0.9458	1.000
	E'_I	1.198	1.185	1.150	1.103	1.053	1.007	0.9694	0.9403	0.9200	0.9081	0.9042
0.8	E'_R	0	0.3092	0.5457	0.6951	0.7826	0.8366	0.8752	0.9078	0.9386	0.9692	1.000
	E'_I	1.552	1.508	1.403	1.284	1.183	1.105	1.049	1.010	0.9853	0.9713	0.9668
0.9	E'_R	0	0.8829	1.151	1.138	1.078	1.030	1.001	0.9877	0.9860	0.9913	1.000
	E'_I	2.403	2.101	1.650	1.363	1.204	1.114	1.060	1.028	1.009	0.9986	0.9954
1.0	E'_R	∞	4.119	2.159	1.549	1.276	1.137	1.062	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

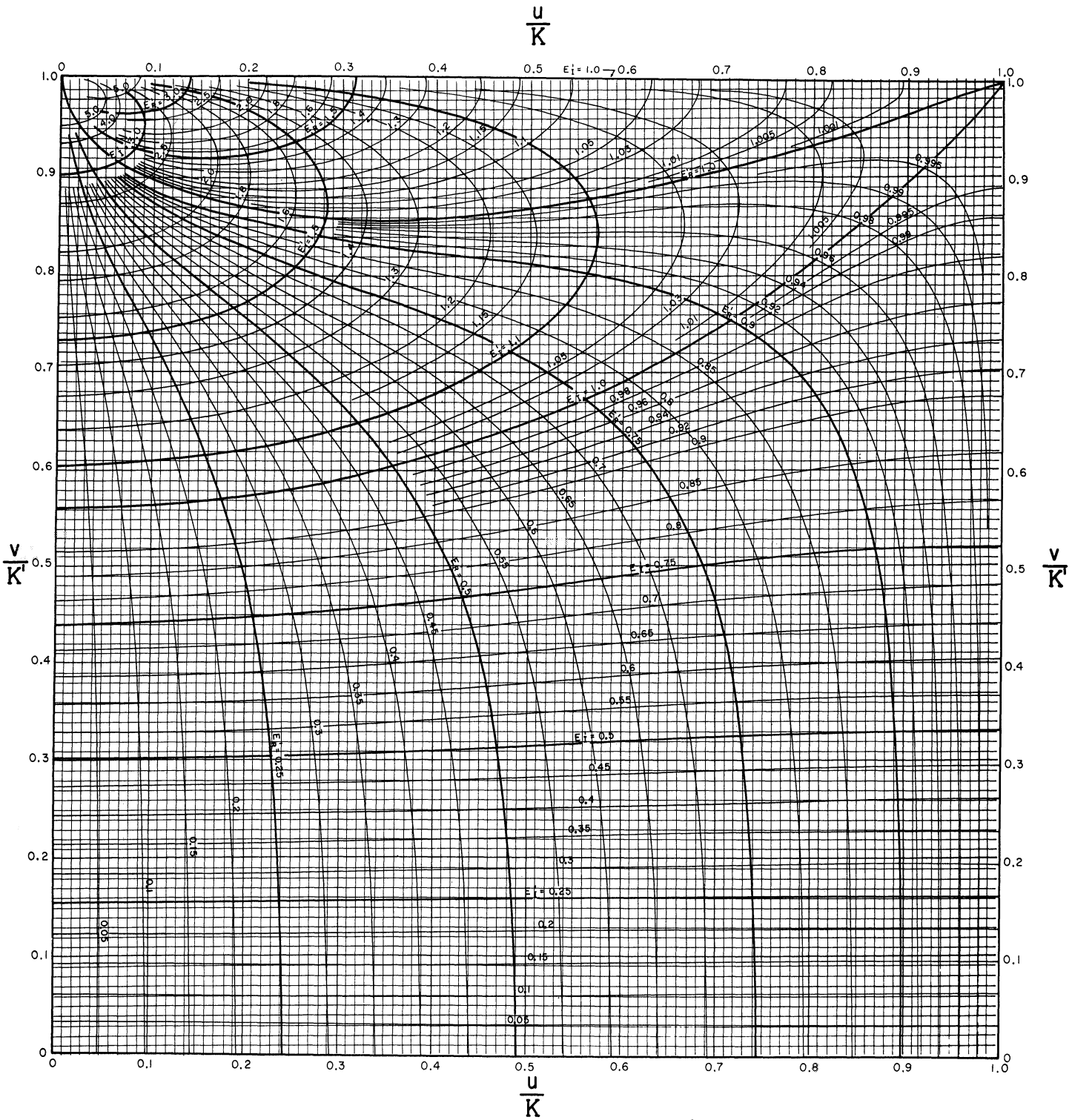
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 10° = 0.1737$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 15° = 0.258819$; $K = 1.598142$, $K' = 2.768063$, $K' - E' = 1.691658$; $E = 1.544150$, $E' = 1.076405$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1034	0.2065	0.3090	0.4105	0.5110	0.6105	0.7089	0.8064	0.9034	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1040	0.2075	0.3104	0.4121	0.5127	0.6120	0.7102	0.8074	0.9039	1.000
	E'_I	0.1639	0.1636	0.1628	0.1615	0.1598	0.1580	0.1563	0.1547	0.1535	0.1527	0.1524
0.2	E'_R	0	0.1058	0.2109	0.3150	0.4175	0.5182	0.6172	0.7146	0.8105	0.9055	1.000
	E'_I	0.3297	0.3290	0.3270	0.3239	0.3201	0.3160	0.3120	0.3084	0.3055	0.3037	0.3031
0.3	E'_R	0	0.1094	0.2178	0.3243	0.4282	0.5293	0.6275	0.7231	0.8166	0.9087	1.000
	E'_I	0.4997	0.4983	0.4945	0.4887	0.4815	0.4737	0.4662	0.4596	0.4544	0.4512	0.4501
0.4	E'_R	0	0.1164	0.2308	0.3416	0.4479	0.5492	0.6457	0.7381	0.8272	0.9142	1.000
	E'_I	0.6780	0.6755	0.6684	0.6576	0.6445	0.6307	0.6176	0.6062	0.5975	0.5921	0.5902
0.5	E'_R	0	0.1296	0.2551	0.3734	0.4830	0.5838	0.6766	0.7630	0.8445	0.9230	1.000
	E'_I	0.8721	0.8672	0.8536	0.8334	0.8097	0.7855	0.7632	0.7445	0.7306	0.7220	0.7191
0.6	E'_R	0	0.1560	0.3024	0.4330	0.5458	0.6427	0.7270	0.8021	0.8711	0.9364	1.000
	E'_I	1.097	1.087	1.059	1.020	0.9759	0.9339	0.8973	0.8681	0.8472	0.8346	0.8305
0.7	E'_R	0	0.2154	0.4026	0.5490	0.6580	0.7397	0.8043	0.8588	0.9080	0.9546	1.000
	E'_I	1.389	1.364	1.300	1.217	1.135	1.064	1.008	0.9665	0.9384	0.9222	0.9169
0.8	E'_R	0	0.3802	0.6433	0.7834	0.8504	0.8851	0.9086	0.9299	0.9520	0.9756	1.000
	E'_I	1.855	1.773	1.590	1.402	1.253	1.147	1.074	1.025	0.9945	0.9776	0.9722
0.9	E'_R	0	1.080	1.292	1.213	1.118	1.053	1.014	0.9961	0.9908	0.9936	1.000
	E'_I	3.030	2.497	1.811	1.431	1.236	1.130	1.069	1.033	1.011	0.9998	0.9963
1.0	E'_R	∞	4.121	2.159	1.549	1.276	1.137	1.063	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

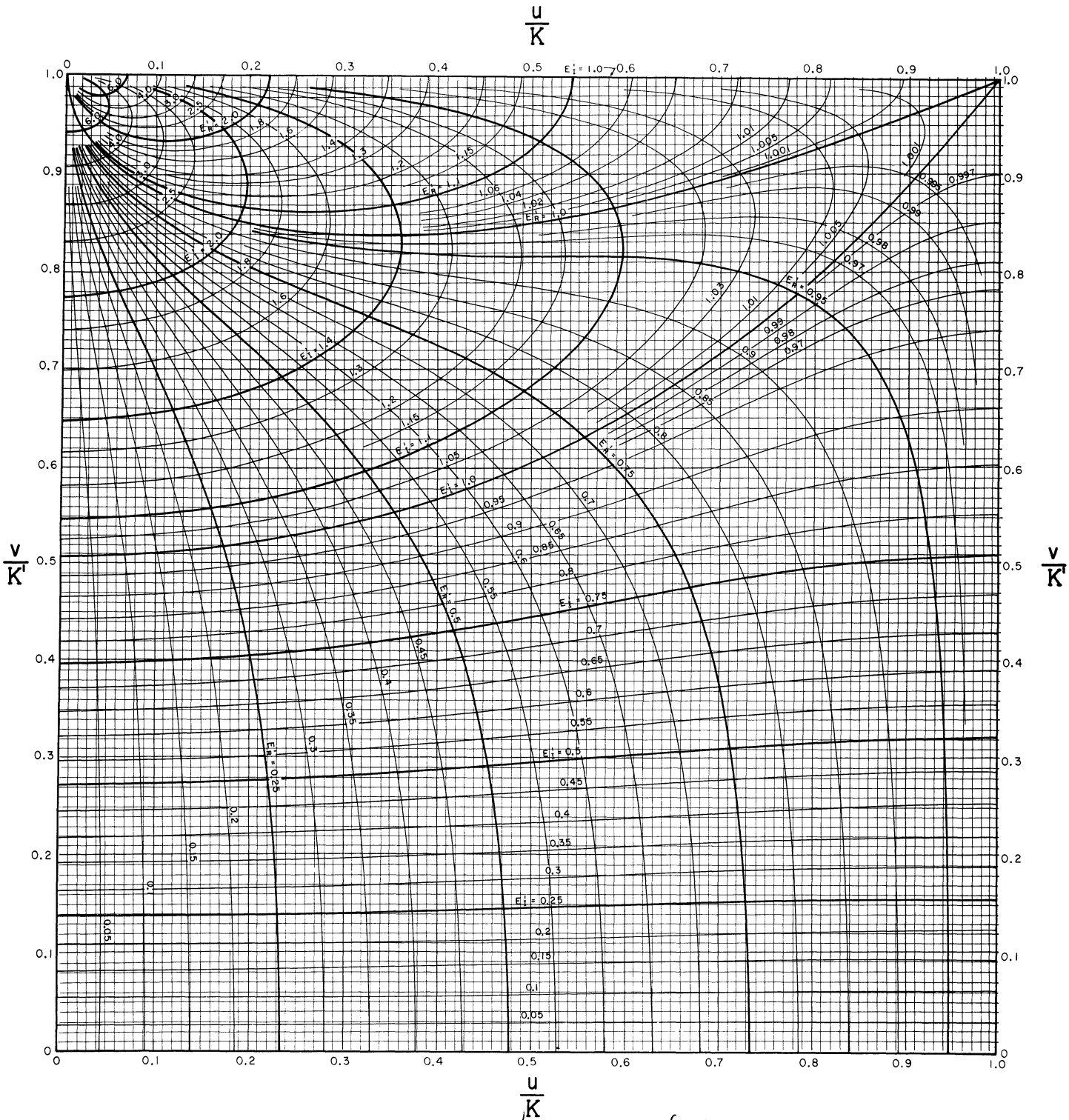
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 15° = 0.2588$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 20° = 0.342020$; $K = 1.620026$, $K' = 2.504550$, $K' - E' = 1.386172$; $E = 1.523799$, $E' = 1.118378$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1062	0.2118	0.3162	0.4189	0.5198	0.6187	0.7159	0.8115	0.9060	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1070	0.2133	0.3182	0.4212	0.5222	0.6209	0.7177	0.8128	0.9067	1.000
	E'_I	0.1811	0.1806	0.1789	0.1763	0.1732	0.1698	0.1664	0.1634	0.1611	0.1597	0.1592
0.2	E'_R	0	0.1096	0.2181	0.3247	0.4287	0.5299	0.6281	0.7237	0.8171	0.9089	1.000
	E'_I	0.3651	0.3638	0.3600	0.3542	0.3470	0.3393	0.3318	0.3253	0.3202	0.3169	0.3158
0.3	E'_R	0	0.1147	0.2277	0.3376	0.4434	0.5447	0.6417	0.7349	0.8250	0.9130	1.000
	E'_I	0.5556	0.5531	0.5460	0.5353	0.5222	0.5083	0.4950	0.4835	0.4746	0.4691	0.4672
0.4	E'_R	0	0.1242	0.2452	0.3606	0.4691	0.5703	0.6647	0.7535	0.8380	0.9197	1.000
	E'_I	0.7585	0.7540	0.7411	0.7220	0.6993	0.6757	0.6537	0.6350	0.6210	0.6123	0.6094
0.5	E'_R	0	0.1415	0.2768	0.4011	0.5128	0.6122	0.7013	0.7824	0.8578	0.9298	1.000
	E'_I	0.9841	0.9755	0.9518	0.9173	0.8780	0.8390	0.8039	0.7753	0.7543	0.7416	0.7373
0.6	E'_R	0	0.1755	0.3362	0.4736	0.5866	0.6793	0.7569	0.8245	0.8859	0.9438	1.000
	E'_I	1.253	1.236	1.188	1.124	1.055	0.9915	0.9381	0.8968	0.8677	0.8506	0.8449
0.7	E'_R	0	0.2500	0.4574	0.6076	0.7100	0.7815	0.8356	0.8808	0.9219	0.9613	1.000
	E'_I	1.614	1.572	1.466	1.338	1.217	1.118	1.042	0.9885	0.9527	0.9323	0.9257
0.8	E'_R	0	0.4518	0.7336	0.8584	0.9041	0.9216	0.9329	0.9456	0.9614	0.9800	1.000
	E'_I	2.212	2.076	1.1789	1.518	1.319	1.184	1.095	1.037	1.002	0.9819	0.9756
0.9	E'_R	0	1.262	1.402	1.267	1.146	1.068	1.024	1.002	0.9940	0.9950	1.000
	E'_I	3.767	2.918	1.965	1.493	1.265	1.145	1.077	1.037	1.013	1.001	0.9967
1.0	E'_R	∞	4.118	2.158	1.549	1.276	1.136	1.062	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

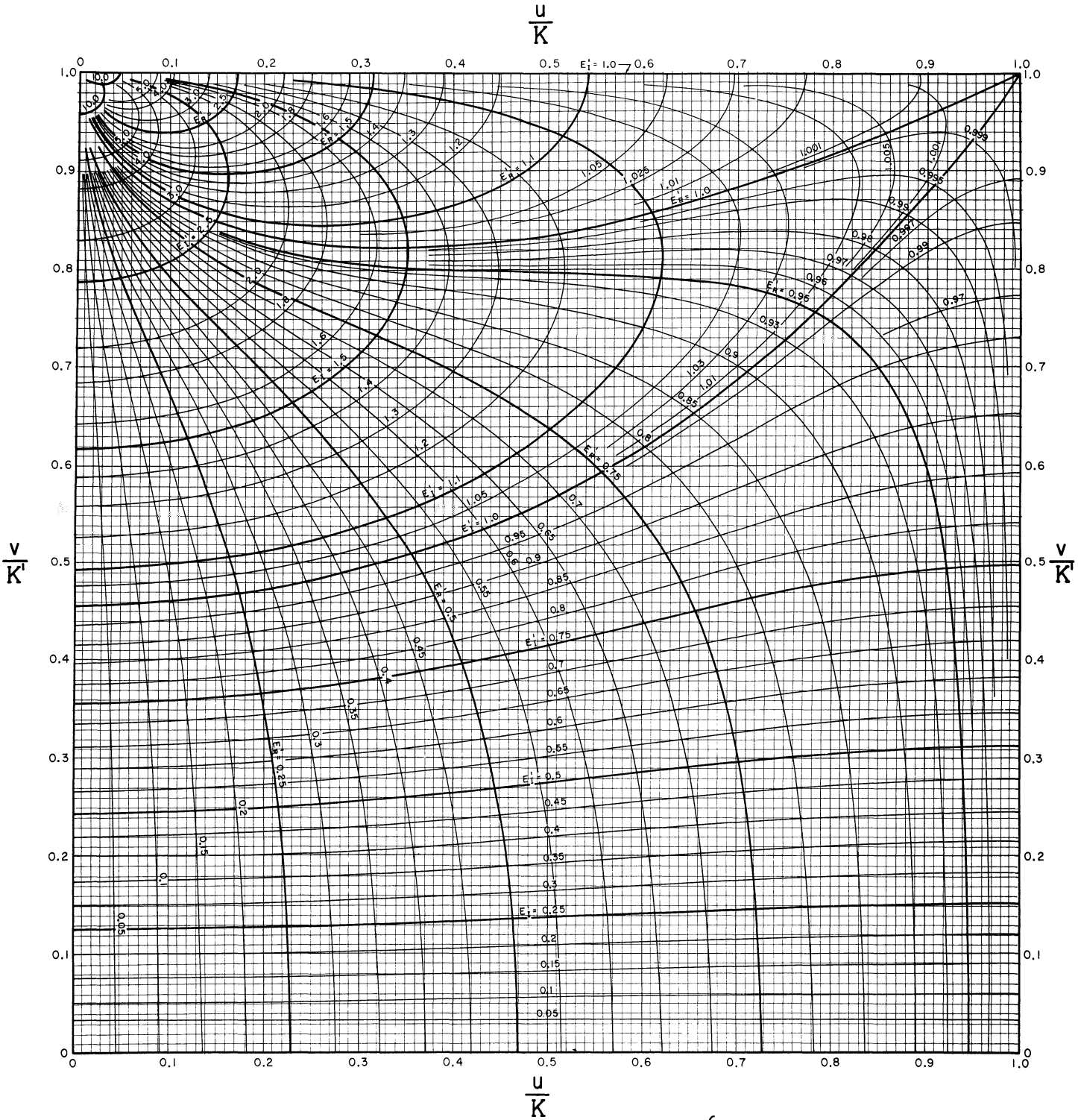
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 20° = 0.3420$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 25° = 0.422618$; $K = 1.648995$, $K' = 2.308787$, $K' - E' = 1.144959$; $E = 1.498115$, $E' = 1.163828$

$\frac{v}{K'}$	$\frac{u}{K}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1099	0.2188	0.3257	0.4300	0.5313	0.6295	0.7249	0.8180	0.9094	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1109	0.2207	0.3283	0.4330	0.5343	0.6323	0.7273	0.8197	0.9103	1.000
	E'_I	0.2023	0.2013	0.1984	0.1940	0.1885	0.1827	0.1771	0.1721	0.1683	0.1659	0.1651
0.2	E'_R	0	0.1144	0.2271	0.3368	0.4427	0.5441	0.6413	0.7347	0.8249	0.9130	1.000
	E'_I	0.4087	0.4064	0.3998	0.3899	0.3778	0.3650	0.3527	0.3420	0.3338	0.3287	0.3269
0.3	E'_R	0	0.1211	0.2395	0.3533	0.4611	0.5626	0.6580	0.7482	0.8344	0.9179	1.000
	E'_I	0.6244	0.6201	0.6081	0.5901	0.5686	0.5461	0.5249	0.5069	0.4932	0.4847	0.4818
0.4	E'_R	0	0.1331	0.2616	0.3820	0.4925	0.5931	0.6849	0.7697	0.8492	0.9254	1.000
	E'_I	0.8573	0.8497	0.8285	0.7974	0.7611	0.7244	0.6908	0.6629	0.6423	0.6296	0.6254
0.5	E'_R	0	0.1549	0.3005	0.4308	0.5438	0.6410	0.7257	0.8013	0.8706	0.9362	1.000
	E'_I	1.121	1.107	1.069	1.014	0.9540	0.8958	0.8450	0.8045	0.7754	0.7579	0.7521
0.6	E'_R	0	0.1967	0.3720	0.5150	0.6266	0.7138	0.7844	0.8447	0.8990	0.9502	1.000
	E'_I	1.444	1.416	1.341	1.242	1.141	1.051	0.9783	0.9233	0.8855	0.8635	0.8562
0.7	E'_R	0	0.2867	0.5131	0.6637	0.7574	0.8180	0.8621	0.8991	0.9333	0.9667	1.000
	E'_I	1.888	1.822	1.659	1.470	1.303	1.171	1.075	1.008	0.9648	0.9403	0.9323
0.8	E'_R	0	0.5259	0.8196	0.9241	0.9485	0.9507	0.9518	0.9576	0.9685	0.9833	1.000
	E'_I	2.645	2.432	2.008	1.640	1.386	1.221	1.116	1.049	1.008	0.9853	0.9782
0.9	E'_R	0	1.435	1.492	1.308	1.167	1.079	1.030	1.005	0.9962	0.9960	1.000
	E'_I	4.660	3.383	2.123	1.557	1.295	1.161	1.085	1.041	1.015	1.001	0.9971
1.0	E'_R	∞	4.115	2.157	1.548	1.276	1.136	1.062	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

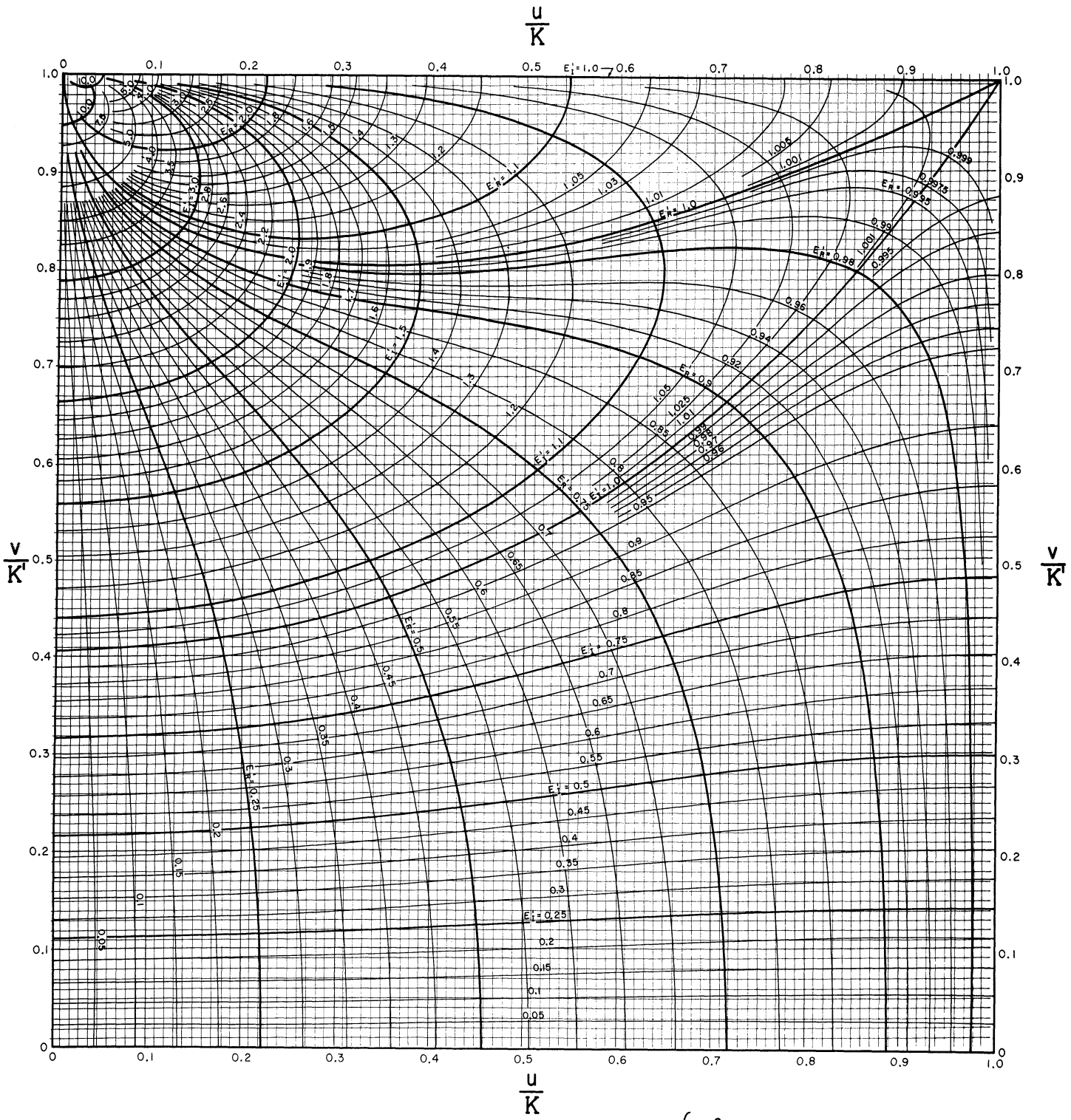
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 25° = 0.4226$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 30° = 0.500000$; $K = 1.685750$, $K' = 2.156516$, $K' - E' = 0.945460$; $E = 1.467462$, $E' = 1.211056$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1146	0.2276	0.3377	0.4439	0.5456	0.6429	0.7362	0.8261	0.9136	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1159	0.2301	0.3410	0.4476	0.5494	0.6463	0.7389	0.8280	0.9146	1.000
	E'_I	0.2290	0.2273	0.2226	0.2154	0.2066	0.1974	0.1886	0.1810	0.1752	0.1716	0.1704
0.2	E'_R	0	0.1202	0.2381	0.3516	0.4594	0.5611	0.6569	0.7475	0.8340	0.9177	1.000
	E'_I	0.4636	0.4599	0.4492	0.4333	0.4141	0.3941	0.3752	0.3591	0.3469	0.3393	0.3367
0.3	E'_R	0	0.1286	0.2535	0.3717	0.4815	0.5828	0.6762	0.7629	0.8447	0.9232	1.000
	E'_I	0.7109	0.7041	0.6848	0.6564	0.6229	0.5887	0.5570	0.5306	0.5108	0.4987	0.4946
0.4	E'_R	0	0.1435	0.2803	0.4058	0.5181	0.6176	0.7062	0.7865	0.8608	0.9313	1.000
	E'_I	0.9814	0.9694	0.9359	0.8877	0.8328	0.7785	0.7301	0.6908	0.6621	0.6448	0.6390
0.5	E'_R	0	0.1699	0.3268	0.4626	0.5761	0.6704	0.7500	0.8197	0.8829	0.9423	1.000
	E'_I	1.293	1.271	1.212	1.129	1.041	0.9580	0.8879	0.8333	0.7947	0.7720	0.7644
0.6	E'_R	0	0.2200	0.4102	0.5576	0.6661	0.7468	0.8099	0.8631	0.9109	0.9560	1.000
	E'_I	1.682	1.638	1.525	1.380	1.237	1.115	1.019	0.9491	0.9016	0.8743	0.8653
0.7	E'_R	0	0.3265	0.5703	0.7184	0.8013	0.8505	0.8852	0.9146	0.9429	0.9713	1.000
	E'_I	2.231	2.128	1.885	1.619	1.396	1.228	1.109	1.028	0.9758	0.9468	0.9375
0.8	E'_R	0	0.6038	0.9025	0.9831	0.9863	0.9747	0.9671	0.9673	0.9743	0.8960	1.000
	E'_I	3.185	2.860	2.257	1.771	1.456	1.259	1.137	1.060	1.013	0.9881	0.9801
0.9	E'_R	0	1.601	1.569	1.340	1.182	1.088	1.035	1.008	0.9979	0.9968	1.000
	E'_I	5.772	3.912	2.292	1.625	1.328	1.178	1.094	1.045	1.017	1.002	0.9974
1.0	E'_R	∞	4.110	2.155	1.547	1.275	1.136	1.062	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

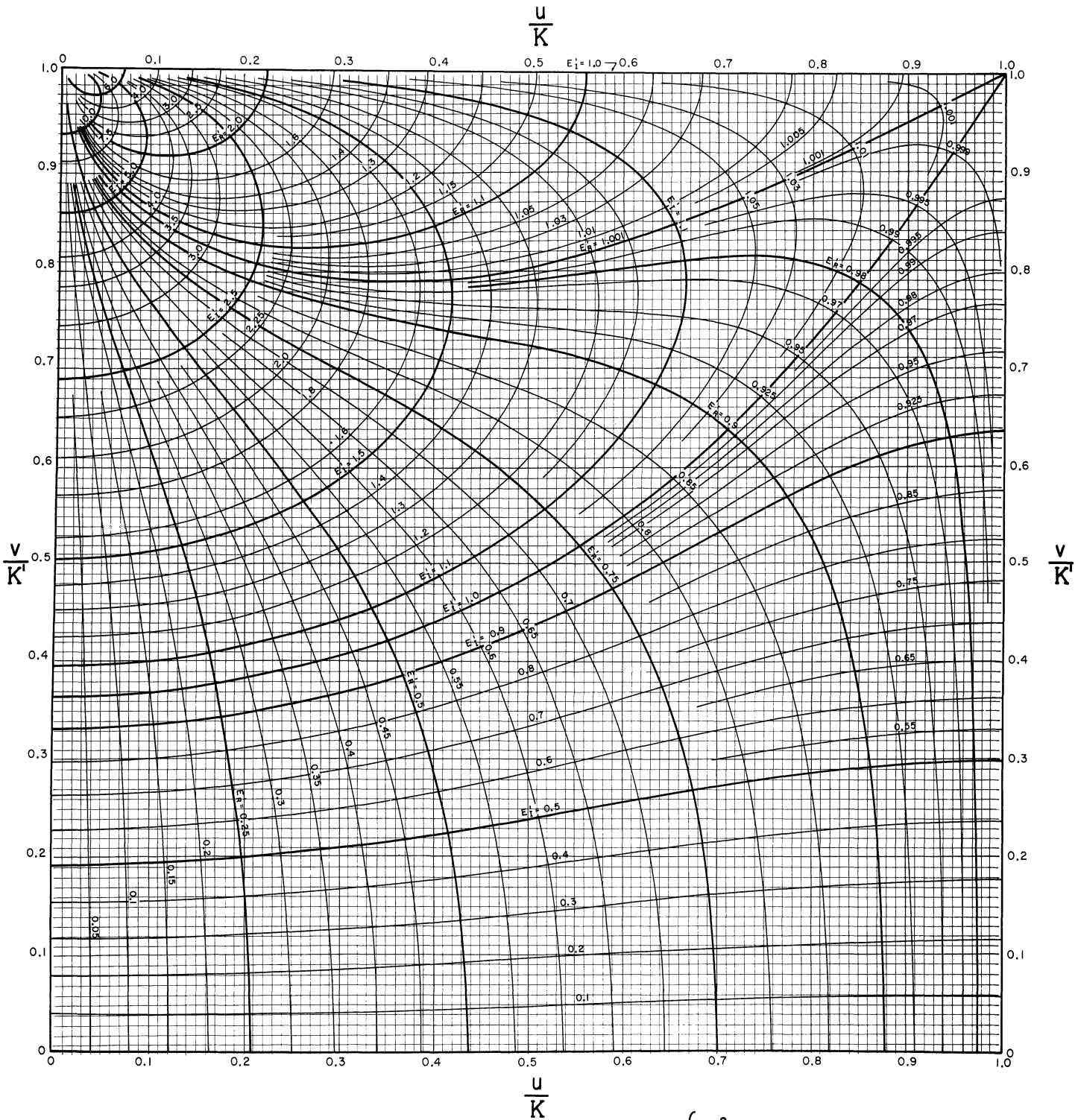
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 30° = 0.5000$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 35° = 0.573576$; $K = 1.731245$, $K' = 2.034715$, $K' - E' = 0.776035$; $E = 1.432291$, $E' = 1.258680$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1205	0.2387	0.3526	0.4610	0.5631	0.6591	0.7496	0.8357	0.9186	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1221	0.2417	0.3566	0.4654	0.5675	0.6630	0.7527	0.8378	0.9197	1.000
	E'_I	0.2634	0.2607	0.2532	0.2420	0.2286	0.2146	0.2015	0.1904	0.1820	0.1769	0.1751
0.2	E'_R	0	0.1274	0.2513	0.3692	0.4792	0.5810	0.6750	0.7623	0.8444	0.9231	1.000
	E'_I	0.5343	0.5284	0.5117	0.4870	0.4578	0.4278	0.4001	0.3769	0.3596	0.3490	0.3454
0.3	E'_R	0	0.1375	0.2698	0.3929	0.5048	0.6055	0.6963	0.7791	0.8559	0.9289	1.000
	E'_I	0.8222	0.8115	0.7816	0.7381	0.6879	0.6377	0.5924	0.5552	0.5279	0.5113	0.5057
0.4	E'_R	0	0.1554	0.3016	0.4325	0.5462	0.6439	0.7287	0.8040	0.8726	0.9373	1.000
	E'_I	1.141	1.122	1.071	0.9982	0.9177	0.8402	0.7730	0.7195	0.6812	0.6583	0.6507
0.5	E'_R	0	0.1869	0.3558	0.4969	0.6100	0.7002	0.7743	0.8378	0.8948	0.9482	1.000
	E'_I	1.514	1.480	1.390	1.268	1.142	1.028	0.9339	0.8625	0.8131	0.7842	0.7747
0.6	E'_R	0	0.2458	0.4512	0.6016	0.7055	0.7786	0.8340	0.8802	0.9217	0.9613	1.000
	E'_I	1.987	1.920	1.751	1.543	1.347	1.186	1.063	0.9751	0.9168	0.8836	0.8728
0.7	E'_R	0	0.3698	0.6296	0.7719	0.8423	0.8800	0.9056	0.9282	0.9512	0.9752	1.000
	E'_I	2.668	2.512	2.159	1.792	1.499	1.288	1.144	1.047	0.9862	0.9525	0.9417
0.8	E'_R	0	0.6863	0.9830	1.036	1.019	0.9949	0.9798	0.9752	0.9790	0.9882	1.000
	E'_I	3.873	3.389	2.546	1.918	1.532	1.301	1.159	1.072	1.019	0.9905	0.9816
0.9	E'_R	0	1.761	1.634	1.366	1.194	1.094	1.039	1.011	0.9993	0.9974	1.000
	E'_I	7.187	4.528	2.481	1.700	1.364	1.196	1.104	1.050	1.019	1.003	0.9976
1.0	E'_R	∞	4.100	2.151	1.545	1.273	1.135	1.062	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E - function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

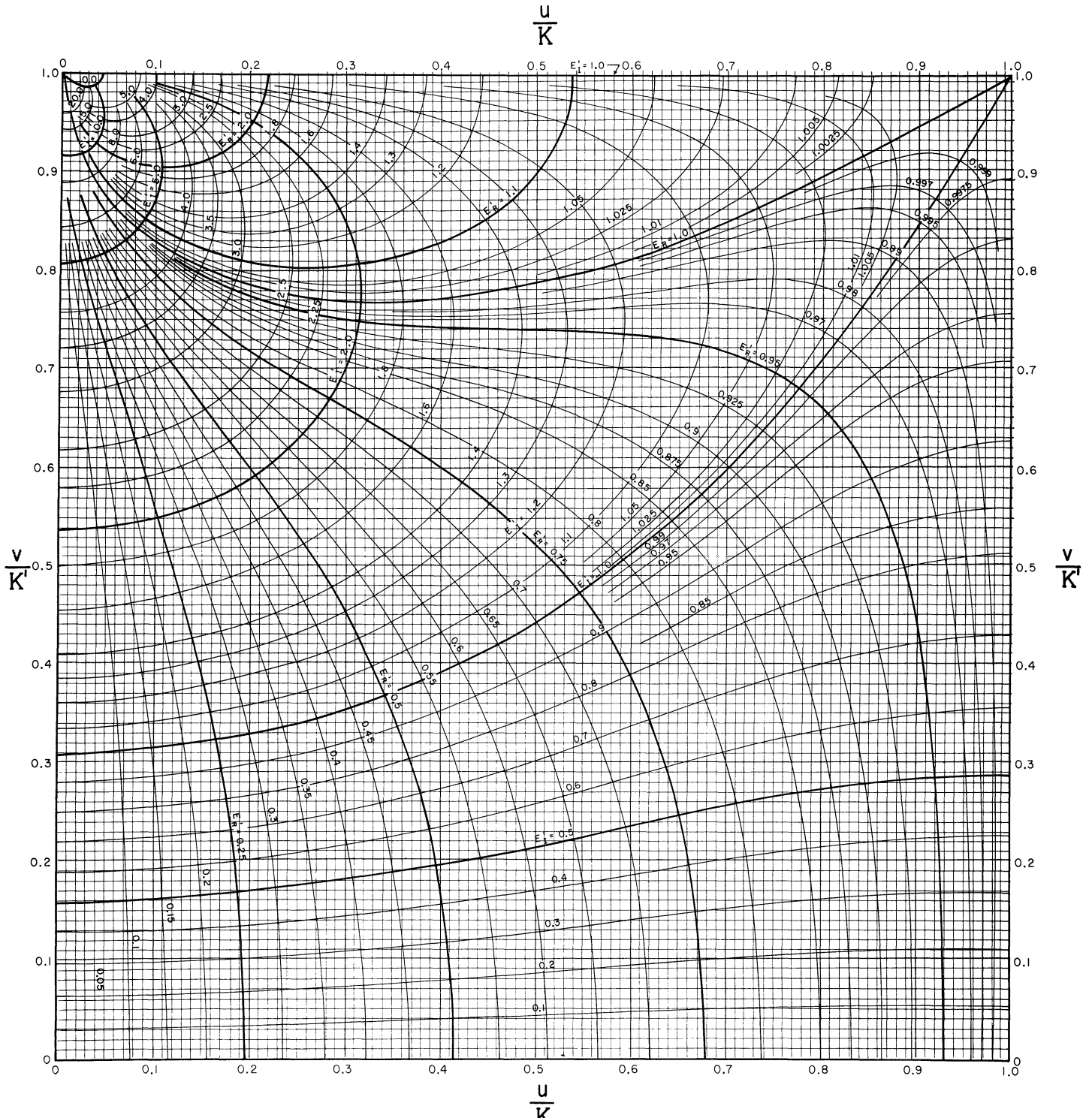
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 35° = 0.5736$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 40° = 0.642788$; $K = 1.786769$, $K' = 1.935581$, $K' - E' = 0.630042$; $E = 1.393140$, $E' = 1.305539$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1277	0.2522	0.3707	0.4815	0.5840	0.6783	0.7654	0.8468	0.9244	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1297	0.2557	0.3753	0.4866	0.5888	0.6825	0.7687	0.8491	0.9256	1.000
	E'_I	0.3088	0.3047	0.2930	0.2758	0.2556	0.2350	0.2161	0.2005	0.1888	0.1817	0.1793
0.2	E'_R	0	0.1360	0.2672	0.3900	0.5024	0.6039	0.6956	0.7790	0.8561	0.9291	1.000
	E'_I	0.6277	0.6185	0.5927	0.5550	0.5115	0.4679	0.4284	0.3960	0.3723	0.3579	0.3531
0.3	E'_R	0	0.1481	0.2889	0.4173	0.5311	0.6308	0.7184	0.7966	0.8680	0.9350	1.000
	E'_I	0.9689	0.9524	0.9066	0.8411	0.7674	0.6954	0.6321	0.5813	0.5447	0.5228	0.5155
0.4	E'_R	0	0.1692	0.3258	0.4623	0.5768	0.6720	0.7524	0.8221	0.8848	0.9434	1.000
	E'_I	1.351	1.322	1.244	1.136	1.020	0.9121	0.8207	0.7497	0.6999	0.6705	0.6608
0.5	E'_R	0	0.2061	0.3880	0.5339	0.6454	0.7307	0.7985	0.8556	0.9065	0.9540	1.000
	E'_I	1.804	1.752	1.616	1.440	1.263	1.108	0.9846	0.8931	0.8310	0.7952	0.7835
0.6	E'_R	0	0.2747	0.4955	0.6471	0.7447	0.8094	0.8569	0.8961	0.9318	0.9661	1.000
	E'_I	2.388	2.286	2.035	1.741	1.475	1.265	1.110	1.002	0.9315	0.8918	0.8790
0.7	E'_R	0	0.4174	0.6912	0.8243	0.8808	0.9068	0.9239	0.9403	0.9586	0.9788	1.000
	E'_I	3.241	3.006	2.496	1.996	1.618	1.356	1.182	1.068	0.9964	0.9575	0.9451
0.8	E'_R	0	0.7743	1.061	1.085	1.047	1.012	0.9906	0.9820	0.9830	0.9900	1.000
	E'_I	4.773	4.054	2.891	2.088	1.619	1.347	1.184	1.084	1.025	0.9928	0.9828
0.9	E'_R	0	1.916	1.689	1.387	1.204	1.099	1.042	1.012	1.000	0.9979	1.000
	E'_I	9.038	5.265	2.699	1.788	1.406	1.218	1.115	1.056	1.021	1.003	0.9978
1.0	E'_R	∞	4.085	2.144	1.541	1.271	1.134	1.061	1.024	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

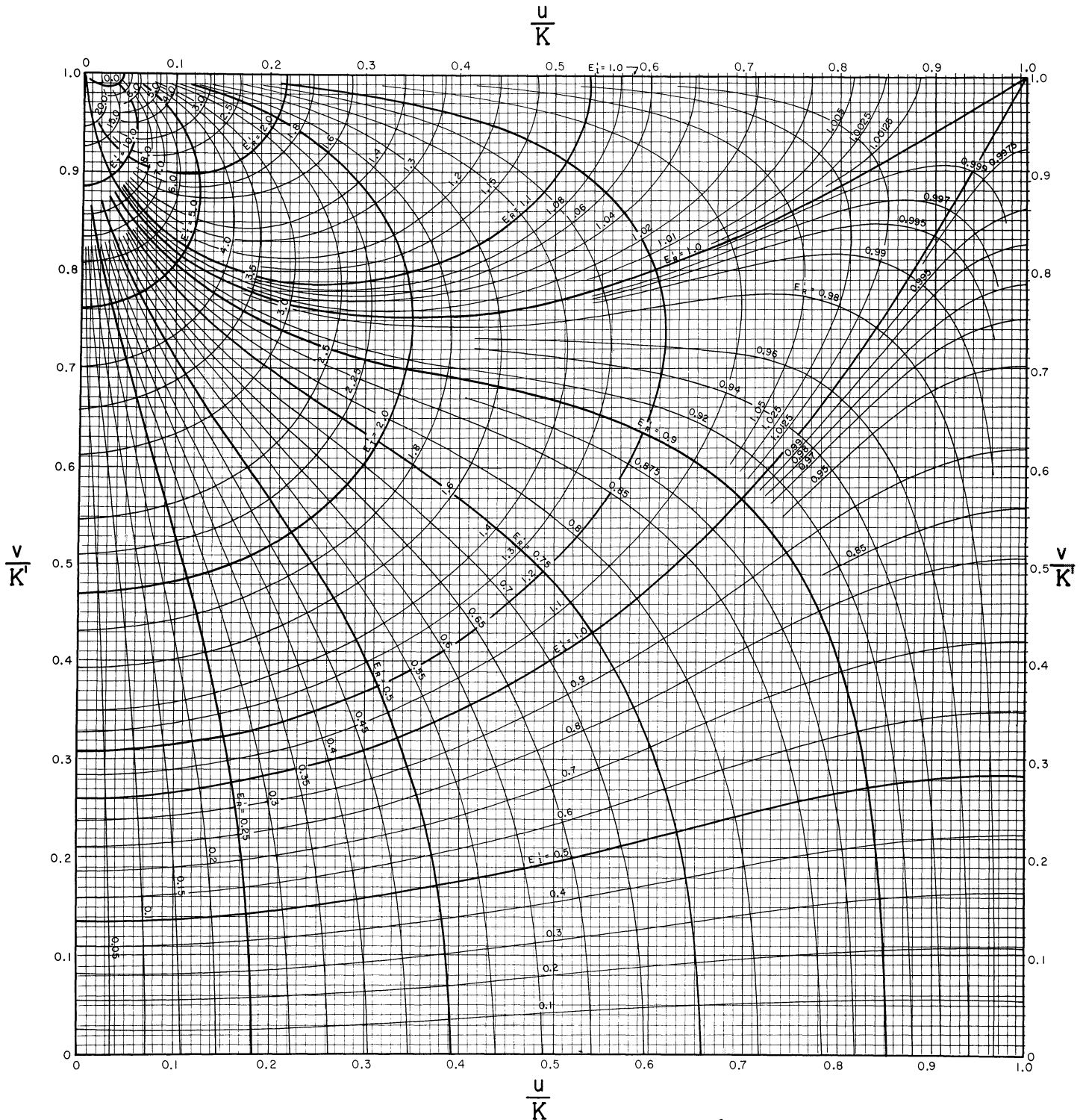
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 40° = 0.6428$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 45° = 0.707107$; $K = K' = 1.854075$, $K' - E' = 0.503431$; $E = E' = 1.350644$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1365	0.2685	0.3924	0.5059	0.6084	0.7004	0.7835	0.8596	0.9310	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1388	0.2727	0.3977	0.5116	0.6137	0.7050	0.7870	0.8619	0.9321	1.000
	E'_I	0.3704	0.3640	0.3460	0.3198	0.2898	0.2599	0.2332	0.2115	0.1957	0.1862	0.1831
0.2	E'_R	0	0.1463	0.2860	0.4144	0.5292	0.6301	0.7188	0.7976	0.8690	0.9357	1.000
	E'_I	0.7542	0.7399	0.7003	0.6434	0.5792	0.5164	0.4612	0.4170	0.3852	0.3662	0.3599
0.3	E'_R	0	0.1605	0.3111	0.4453	0.5608	0.6589	0.7426	0.8155	0.8809	0.9416	1.000
	E'_I	1.168	1.142	1.072	0.9740	0.8667	0.7649	0.6779	0.6099	0.5618	0.5334	0.5240
0.4	E'_R	0	0.1852	0.3535	0.4954	0.6101	0.7020	0.7771	0.8409	0.8974	0.9496	1.000
	E'_I	1.635	1.590	1.472	1.313	1.147	0.9978	0.8752	0.7825	0.7187	0.6816	0.6695
0.5	E'_R	0	0.2281	0.4239	0.5737	0.6824	0.7618	0.8228	0.8733	0.9179	0.9596	1.000
	E'_I	2.195	2.116	1.913	1.657	1.410	1.202	1.042	0.9262	0.8491	0.8051	0.7909
0.6	E'_R	0	0.3072	0.5435	0.6943	0.7837	0.8392	0.8786	0.9112	0.9413	0.9707	1.000
	E'_I	2.928	2.773	2.403	1.987	1.629	1.358	1.164	1.031	0.9465	0.8993	0.8842
0.7	E'_R	0	0.4700	0.7552	0.8757	0.9169	0.9313	0.9404	0.9512	0.9653	0.8919	1.000
	E'_I	4.013	3.658	2.922	2.245	1.758	1.434	1.224	1.090	1.007	0.9621	0.9479
0.8	E'_R	0	0.8682	1.137	1.128	1.072	1.027	0.9997	0.9878	0.9865	0.9917	1.000
	E'_I	5.987	4.917	3.313	2.290	1.722	1.401	1.212	1.098	1.031	0.9950	0.9838
0.9	E'_R	0	2.066	1.734	1.402	1.210	1.102	1.044	1.014	1.001	0.9984	1.000
	E'_I	11.53	6.173	2.961	1.893	1.456	1.244	1.128	1.062	1.024	1.004	0.9979
1.0	E'_R	∞	4.062	2.133	1.535	1.268	1.132	1.060	1.023	1.007	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

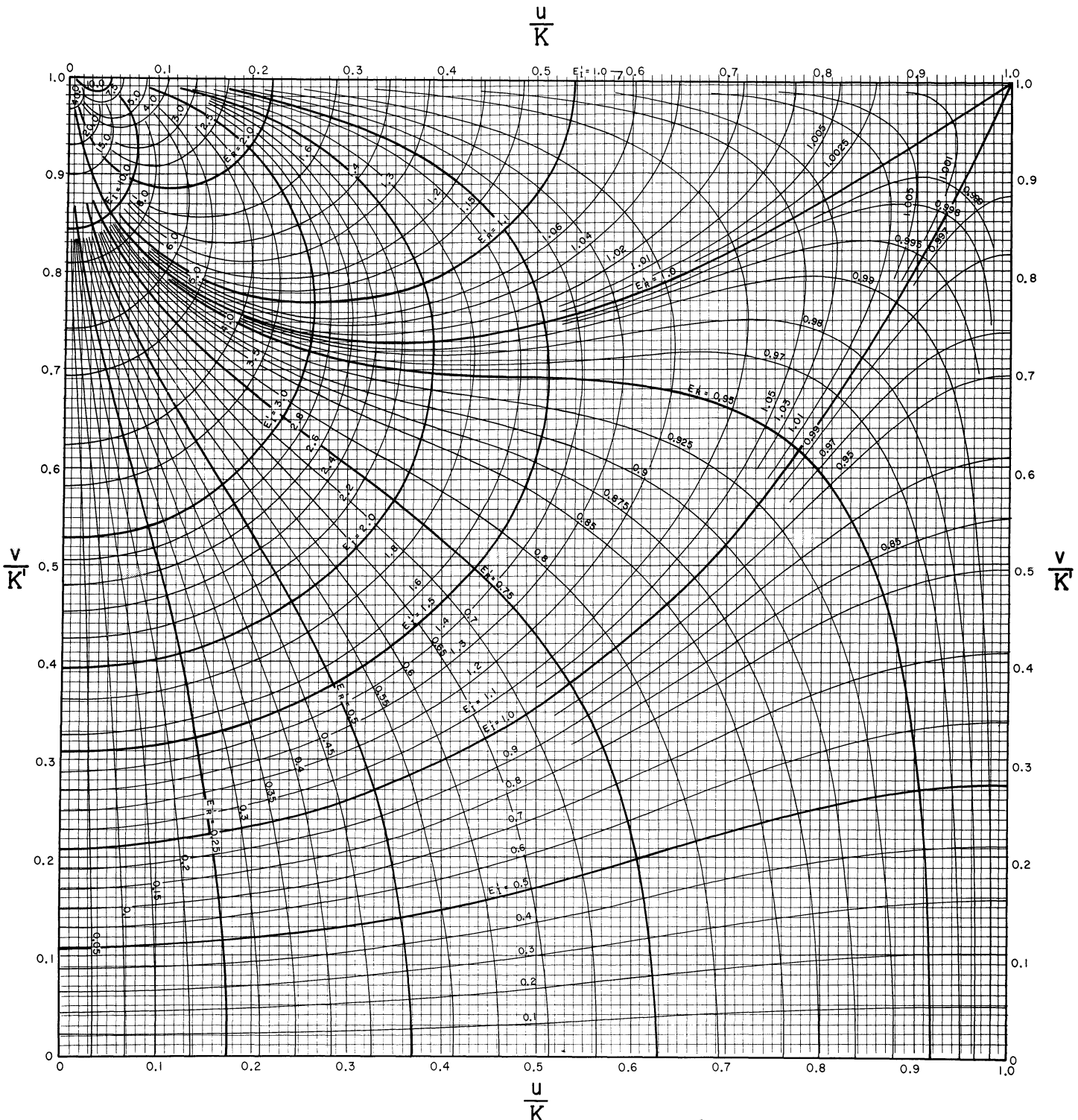
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 45° = 0.7071$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 50° = 0.766044$; $K = 1.935581$, $K' = 1.786769$, $K' - E' = 0.393629$; $E = 1.305539$, $E' = 1.393140$

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1472	0.2882	0.4183	0.5347	0.6368	0.7259	0.8040	0.8739	0.9383	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1499	0.2930	0.4243	0.5408	0.6425	0.7305	0.8075	0.8762	0.9394	1.000
	E'_I	0.4568	0.4467	0.4187	0.3787	0.3341	0.2910	0.2535	0.2240	0.2029	0.1905	0.1864
0.2	E'_R	0	0.1586	0.3084	0.4430	0.5600	0.6597	0.7447	0.8181	0.8832	0.9429	1.000
	E'_I	0.9315	0.9091	0.8476	0.7613	0.6666	0.5768	0.5002	0.4405	0.3986	0.3739	0.3658
0.3	E'_R	0	0.1752	0.3371	0.4774	0.5941	0.6897	0.7688	0.8358	0.8946	0.9485	1.000
	E'_I	1.446	1.406	1.298	1.151	0.9942	0.8508	0.7321	0.6418	0.5795	0.5433	0.5314
0.4	E'_R	0	0.2038	0.3851	0.5324	0.6463	0.7338	0.8030	0.8603	0.9102	0.9560	1.000
	E'_I	2.032	1.963	1.782	1.546	1.309	1.103	0.9392	0.8191	0.7383	0.6920	0.6769
0.5	E'_R	0	0.2534	0.4639	0.6167	0.7210	0.7934	0.8471	0.8908	0.9293	0.9652	1.000
	E'_I	2.743	2.620	2.312	1.940	1.595	1.316	1.109	0.9630	0.8678	0.8144	0.7972
0.6	E'_R	0	0.3441	0.5955	0.7429	0.8224	0.8681	0.8994	0.9255	0.9503	0.9751	1.000
	E'_I	3.683	3.444	2.891	2.302	1.820	1.468	1.226	1.064	0.9623	0.9064	0.8885
0.7	E'_R	0	0.5284	0.8215	0.9256	0.9505	0.9537	0.9554	0.9611	0.9713	0.9848	1.000
	E'_I	5.091	4.547	3.477	2.556	1.929	1.528	1.274	1.115	1.019	0.9666	0.9502
0.8	E'_R	0	0.9683	1.209	1.167	1.093	1.039	1.007	0.9927	0.9895	0.9931	1.000
	E'_I	7.680	6.069	3.848	2.539	1.846	1.465	1.245	1.115	1.038	0.9973	0.9846
0.9	E'_R	0	2.207	1.771	1.413	1.214	1.104	1.045	1.015	1.002	0.9988	1.000
	E'_I	15.00	7.329	3.286	2.023	1.158	1.276	1.145	1.070	1.027	1.005	0.9980
1.0	E'_R	∞	4.027	2.118	1.526	1.263	1.129	1.059	1.023	1.006	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

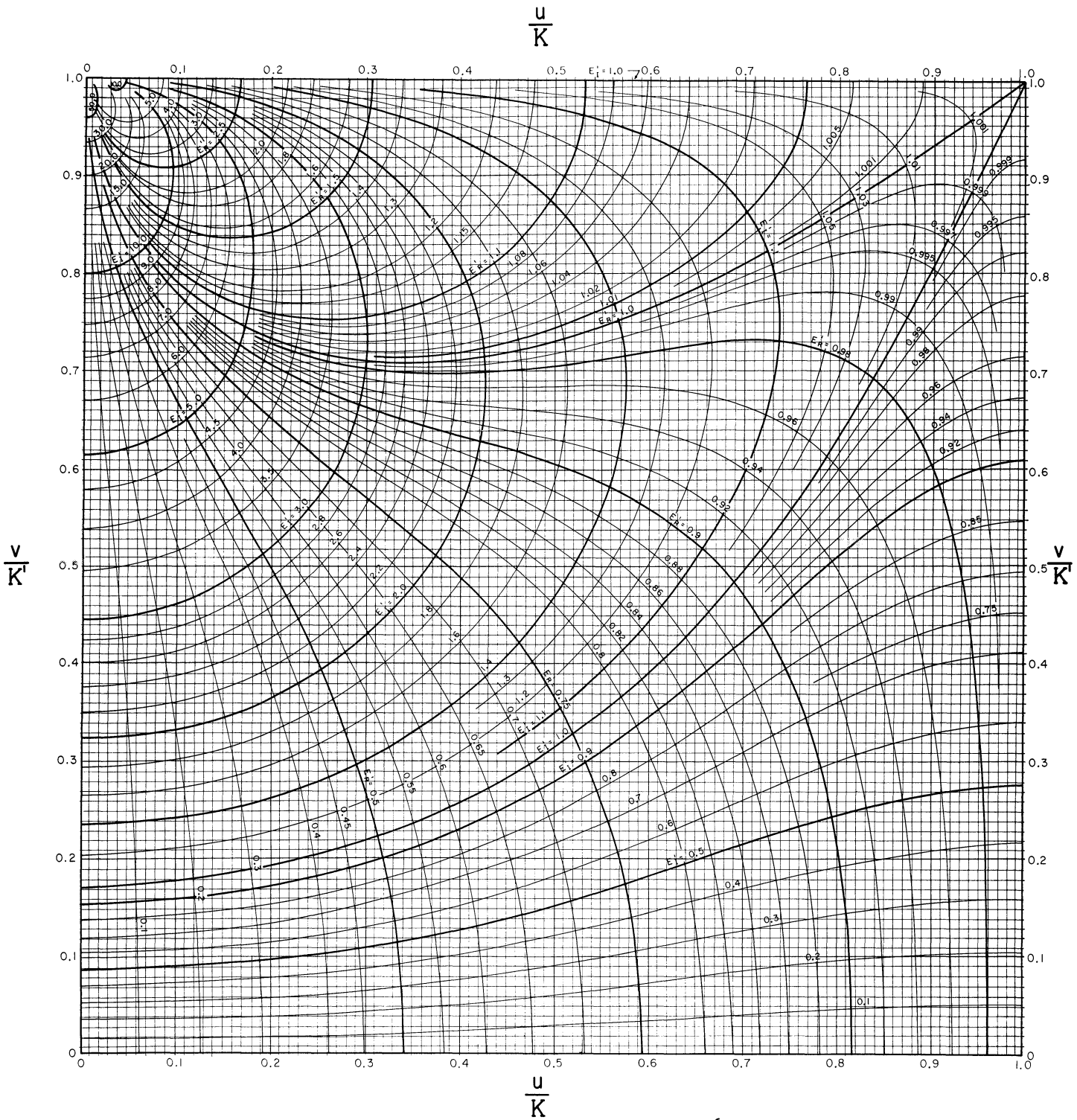
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 50° = 0.7660$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 55° = 0.819152$; $K = 2.034715$, $K' = 1.731245$, $K' - E' = 0.298954$; $E = 1.258680$, $E' = 1.432291$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1602	0.3120	0.4490	0.5682	0.6694	0.7546	0.8268	0.8897	0.9463	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1633	0.3175	0.4557	0.5748	0.6752	0.7593	0.8303	0.8919	0.9474	1.000
	E'_I	0.5831	0.5668	0.5224	0.4606	0.3935	0.3310	0.2785	0.2384	0.2107	0.1945	0.1893
0.2	E'_R	0	0.1734	0.3349	0.4764	0.5953	0.6930	0.7734	0.8405	0.8984	0.9506	1.000
	E'_I	1.191	1.155	1.057	0.9245	0.7832	0.6542	0.5479	0.4678	0.4129	0.3812	0.3709
0.3	E'_R	0	0.1926	0.3674	0.5140	0.6312	0.7235	0.7970	0.8573	0.9091	0.9557	1.000
	E'_I	1.852	1.789	1.619	1.393	1.163	0.9601	0.7980	0.6787	0.5985	0.5526	0.5377
0.4	E'_R	0	0.2257	0.4214	0.5735	0.6854	0.7675	0.8299	0.8803	0.9234	0.9626	1.000
	E'_I	2.612	2.502	2.220	1.864	1.521	1.235	1.017	0.8613	0.7593	0.7018	0.6833
0.5	E'_R	0	0.2826	0.5087	0.6628	0.7611	0.8254	0.8714	0.9082	0.9405	0.9706	1.000
	E'_I	3.543	3.347	2.871	2.320	1.835	1.459	1.189	1.006	0.8881	0.8232	0.8025
0.6	E'_R	0	0.3860	0.6516	0.7926	0.8605	0.8958	0.9192	0.9392	0.9589	0.9793	1.000
	E'_I	4.784	4.406	3.566	2.719	2.063	1.606	1.300	1.102	0.9796	0.9132	0.8922
0.7	E'_R	0	0.5933	0.8895	0.9733	0.9813	0.9737	0.9688	0.9701	0.9769	0.9875	1.000
	E'_I	6.663	5.810	4.228	2.961	2.145	1.643	1.335	1.145	1.031	0.9710	0.9521
0.8	E'_R	0	1.075	1.277	1.199	1.110	1.049	1.014	0.9970	0.9922	0.9945	1.000
	E'_I	10.15	7.668	4.551	2.858	2.004	1.546	1.287	1.134	1.046	0.9997	0.9852
0.9	E'_R	0	2.337	1.796	1.417	1.214	1.104	1.045	1.015	1.002	0.9991	1.000
	E'_I	20.06	8.858	3.709	2.193	1.599	1.316	1.165	1.080	1.031	1.006	0.9981
1.0	E'_R	∞	3.976	2.095	1.513	1.255	1.125	1.057	1.022	1.006	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

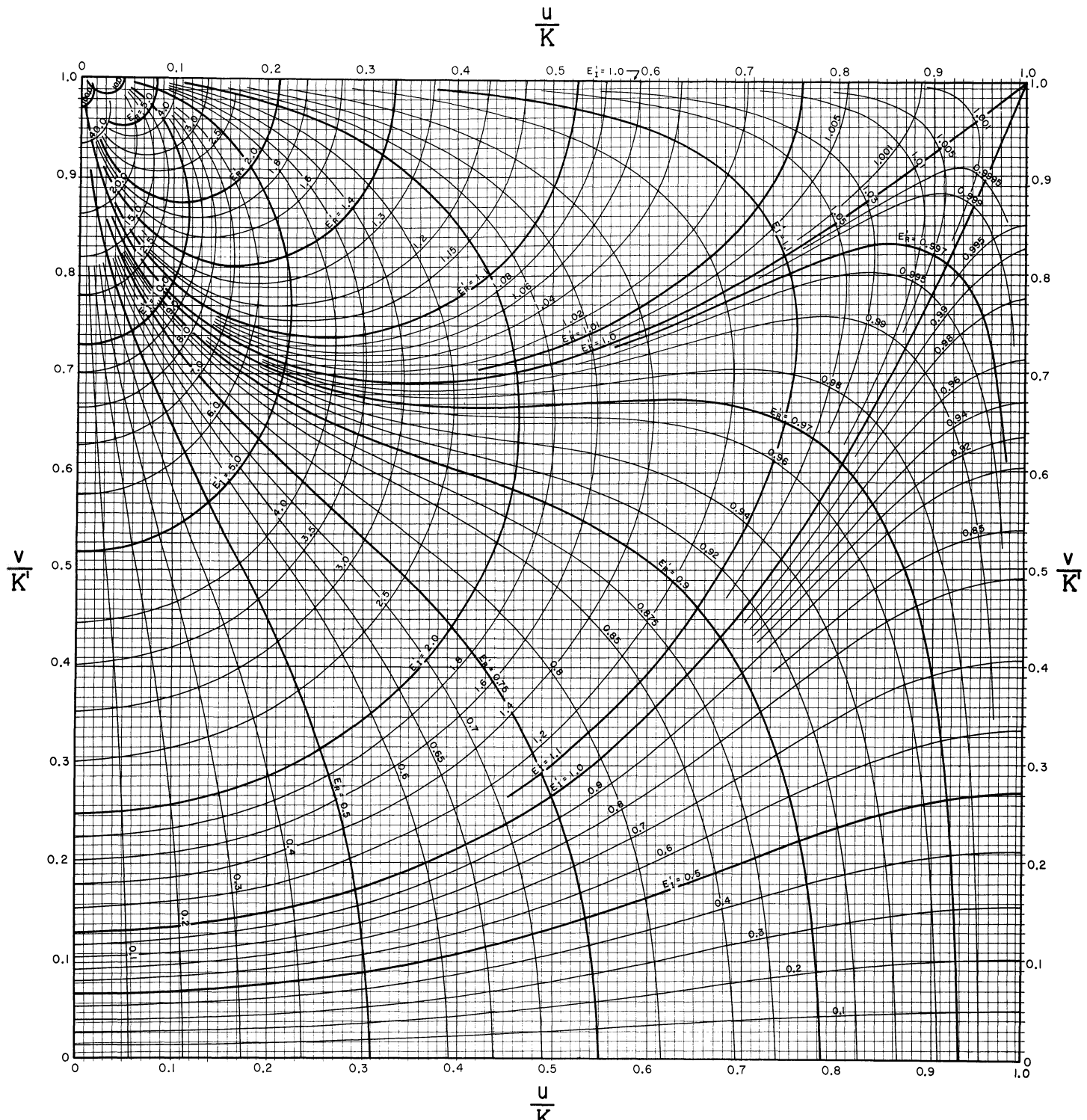
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 55° = 0.8192$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 60° = 0.866025$; $K = 2.156516$, $K' = 1.685750$, $K' - E' = 0.218288$; $E = 1.211056$, $E' = 1.467462$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1760	0.3406	0.4855	0.6072	0.7064	0.7866	0.8519	0.9067	0.9550	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1797	0.3468	0.4927	0.6141	0.7123	0.7911	0.8551	0.9088	0.9559	1.000
	E'_I	0.7778	0.7507	0.6778	0.5792	0.4763	0.3843	0.3103	0.2557	0.2192	0.1984	0.1917
0.2	E'_R	0	0.1914	0.3665	0.5153	0.6355	0.7301	0.8046	0.8646	0.9147	0.9587	1.000
	E'_I	1.590	1.530	1.371	1.161	0.9454	0.7570	0.6084	0.5005	0.4288	0.3883	0.3753
0.3	E'_R	0	0.2135	0.4030	0.5559	0.6726	0.7602	0.8270	0.8800	0.9242	0.9633	1.000
	E'_I	2.479	2.373	2.097	1.743	1.396	1.105	0.8813	0.7230	0.6196	0.5617	0.5431
0.4	E'_R	0	0.2515	0.4629	0.6191	0.7275	0.8028	0.8577	0.9007	0.9368	0.9692	1.000
	E'_I	3.507	3.325	2.868	2.317	1.811	1.409	1.114	0.9119	0.7827	0.7114	0.6886
0.5	E'_R	0	0.3166	0.5587	0.7119	0.8023	0.8575	0.8955	0.9253	0.9515	0.9761	1.000
	E'_I	4.774	4.452	3.691	2.856	2.159	1.645	1.291	1.057	0.9110	0.8319	0.8069
0.6	E'_R	0	0.4339	0.7119	0.8430	0.8975	0.9222	0.9379	0.9521	0.9672	0.9833	1.000
	E'_I	6.481	5.859	4.542	3.298	2.389	1.784	1.394	1.148	0.9995	0.9201	0.8951
0.7	E'_R	0	0.6657	0.9582	1.018	1.009	0.9914	0.9807	0.9782	0.9821	0.9901	1.000
	E'_I	9.082	7.695	5.292	3.512	2.432	1.793	1.412	1.182	1.047	0.9758	0.9536
0.8	E'_R	0	1.187	1.338	1.225	1.122	1.056	1.018	1.000	0.9946	0.9957	1.000
	E'_I	13.94	10.00	5.523	3.290	2.213	1.652	1.340	1.160	1.056	1.002	0.9858
0.9	E'_R	0	2.451	1.809	1.415	1.211	1.102	1.044	1.015	1.003	0.9994	1.000
	E'_I	27.85	10.99	4.287	2.422	1.707	1.370	1.192	1.092	1.036	1.007	0.9982
1.0	E'_R	∞	3.903	2.062	1.495	1.245	1.119	1.054	1.021	1.006	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

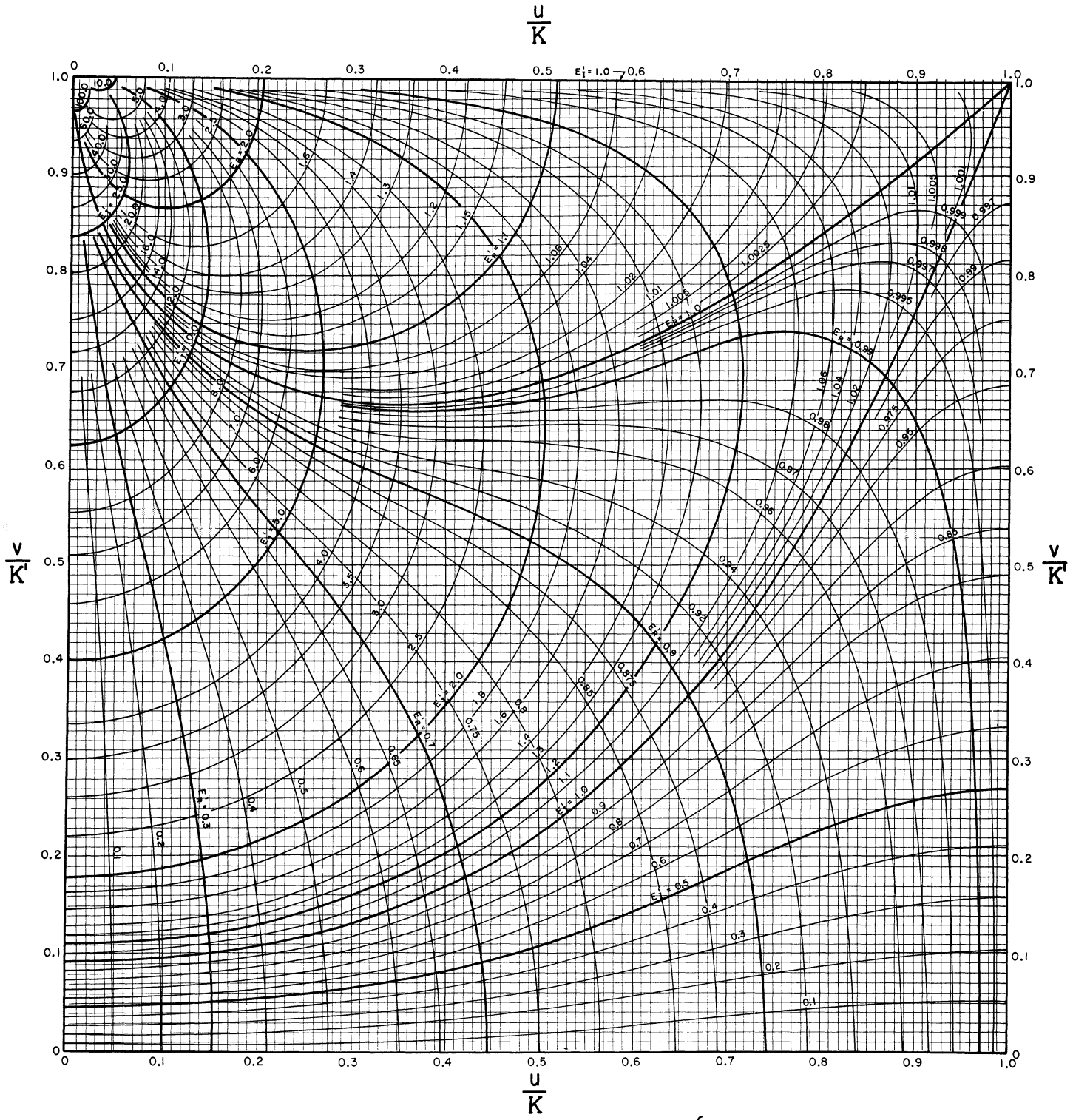
Contours of $E'_R = \dfrac{E_R}{E}$, and $E'_I = \dfrac{E_I}{K' - E'}$

$k = \sin 60° = 0.8660$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 65° = 0.906308$; $K = 2.308787$, $K' = 1.648995$, $K' - E' = 0.150880$; $E = 1.163828$, $E' = 1.498115$

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.1955	0.3753	0.5286	0.6521	0.7481	0.8217	0.8789	0.9249	0.9640	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.1998	0.3822	0.5364	0.6592	0.7538	0.8259	0.8817	0.9266	0.9649	1.000
	E'_I	1.101	1.053	0.9271	0.7624	0.5985	0.4591	0.3525	0.2775	0.2291	0.2023	0.1937
0.2	E'_R	0	0.2132	0.4043	0.5605	0.6809	0.7709	0.8383	0.8901	0.9316	0.9672	1.000
	E'_I	2.254	2.148	1.874	1.523	1.183	0.9008	0.6887	0.5415	0.4473	0.3954	0.3789
0.3	E'_R	0	0.2388	0.4448	0.6035	0.7181	0.7996	0.8587	0.9035	0.9397	0.9710	1.000
	E'_I	3.520	3.333	2.858	2.276	1.737	1.305	0.9916	0.7785	0.6441	0.5708	0.5476
0.4	E'_R	0	0.2824	0.5108	0.6693	0.7722	0.8394	0.8861	0.9213	0.9502	0.9758	1.000
	E'_I	4.991	4.671	3.893	3.003	2.231	1.649	1.243	0.9756	0.8103	0.7211	0.6931
0.5	E'_R	0	0.3566	0.6144	0.7638	0.8440	0.8894	0.9191	0.9421	0.9623	0.9814	1.000
	E'_I	6.817	6.253	4.975	3.657	2.622	1.901	1.424	1.121	0.9383	0.8410	0.8105
0.6	E'_R	0	0.4891	0.7761	0.8928	0.9324	0.9467	0.9552	0.9642	0.9749	0.9871	1.000
	E'_I	9.293	8.211	6.046	4.149	2.850	2.027	1.518	1.207	1.024	0.9275	0.8975
0.7	E'_R	0	0.7462	1.026	1.058	1.032	1.006	0.9909	0.9854	0.9868	0.9924	1.000
	E'_I	13.09	10.71	6.897	4.311	2.835	1.998	1.514	1.229	1.066	0.9811	0.9549
0.8	E'_R	0	1.303	1.389	1.243	1.129	1.060	1.022	1.003	0.9966	0.9968	1.000
	E'_I	20.23	13.63	6.952	3.909	2.508	1.798	1.412	1.193	1.069	1.006	0.9862
0.9	E'_R	0	2.543	1.805	1.403	1.202	1.097	1.042	1.015	1.003	0.9996	1.000
	E'_I	40.76	14.15	5.129	2.754	1.860	1.445	1.229	1.109	1.043	1.009	0.9982
1.0	E'_R	∞	3.799	2.015	1.469	1.230	1.111	1.050	1.019	1.005	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

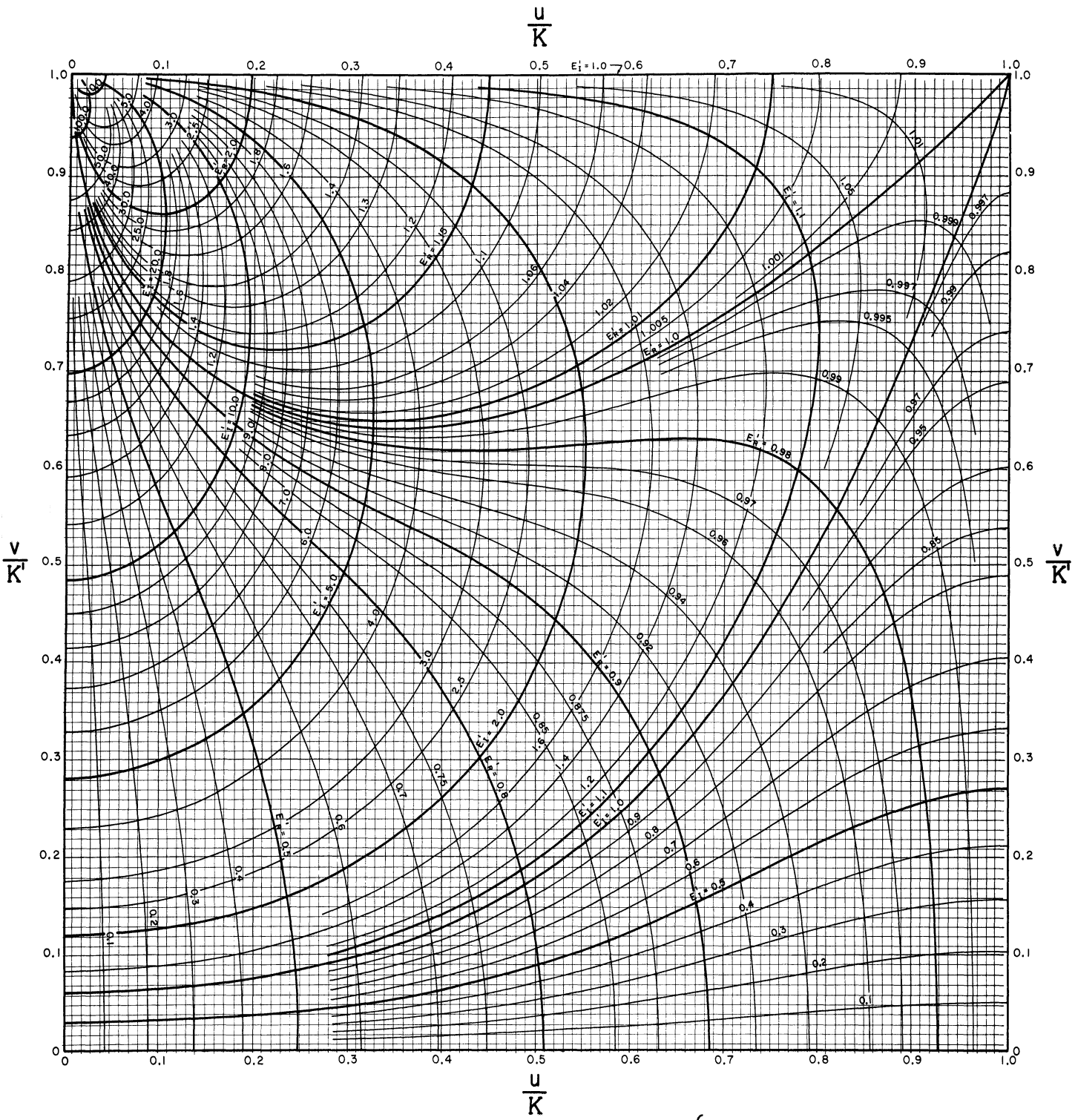
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 65° = 0.9063$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 70° = 0.939693$; $K = 2.504550$, $K' = 1.620026$, $K' - E' = 0.0962267$; $E = 1.118378$, $E' = 1.523799$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.2199	0.4177	0.5797	0.7036	0.7942	0.8595	0.9072	0.9436	0.9733	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.2248	0.4254	0.5879	0.7106	0.7994	0.8631	0.9096	0.9449	0.9739	1.000
	E'_I	1.697	1.604	1.366	1.070	0.7928	0.5714	0.4124	0.3065	0.2413	0.2063	0.1954
0.2	E'_R	0	0.2403	0.4498	0.6132	0.7318	0.8152	0.8739	0.9164	0.9489	0.9758	1.000
	E'_I	3.475	3.271	2.756	2.131	1.561	1.116	0.8024	0.5961	0.4701	0.4029	0.3819
0.3	E'_R	0	0.2697	0.4944	0.6576	0.7678	0.8412	0.8913	0.9274	0.9553	0.9786	1.000
	E'_I	5.436	5.075	4.189	3.164	2.272	1.605	1.147	0.8526	0.6746	0.5805	0.5512
0.4	E'_R	0	0.3197	0.5660	0.7243	0.8191	0.8767	0.9144	0.9417	0.9634	0.9823	1.000
	E'_I	7.723	7.107	5.672	4.131	2.885	2.004	1.425	1.061	0.8449	0.7317	0.6966
0.5	E'_R	0	0.4041	0.6762	0.8177	0.8853	0.9201	0.9417	0.9582	0.9727	0.9865	1.000
	E'_I	10.58	9.497	7.176	4.958	3.339	2.277	1.613	1.208	0.9730	0.8510	0.8133
0.6	E'_R	0	0.5530	0.8433	0.9405	0.9641	0.9685	0.9709	0.9754	0.9822	0.9907	1.000
	E'_I	14.47	12.42	8.582	5.509	3.555	2.386	1.694	1.287	1.055	0.9360	0.8994
0.7	E'_R	0	0.8355	1.090	1.090	1.049	1.017	0.9991	0.9916	0.9911	0.9946	1.000
	E'_I	20.48	16.01	9.542	5.570	3.449	2.301	1.660	1.295	1.091	0.9875	0.9558
0.8	E'_R	0	1.420	1.426	1.250	1.129	1.060	1.023	1.005	0.9983	0.9978	1.000
	E'_I	31.81	19.83	9.250	4.876	2.958	2.015	1.515	1.239	1.087	1.010	0.9865
0.9	E'_R	0	2.601	1.781	1.381	1.188	1.089	1.039	1.014	1.003	0.9998	1.000
	E'_I	64.52	19.30	6.471	3.275	2.096	1.558	1.282	1.133	1.051	1.011	0.9983
1.0	E'_R	∞	3.653	1.950	1.433	1.210	1.100	1.044	1.017	1.005	1.001	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

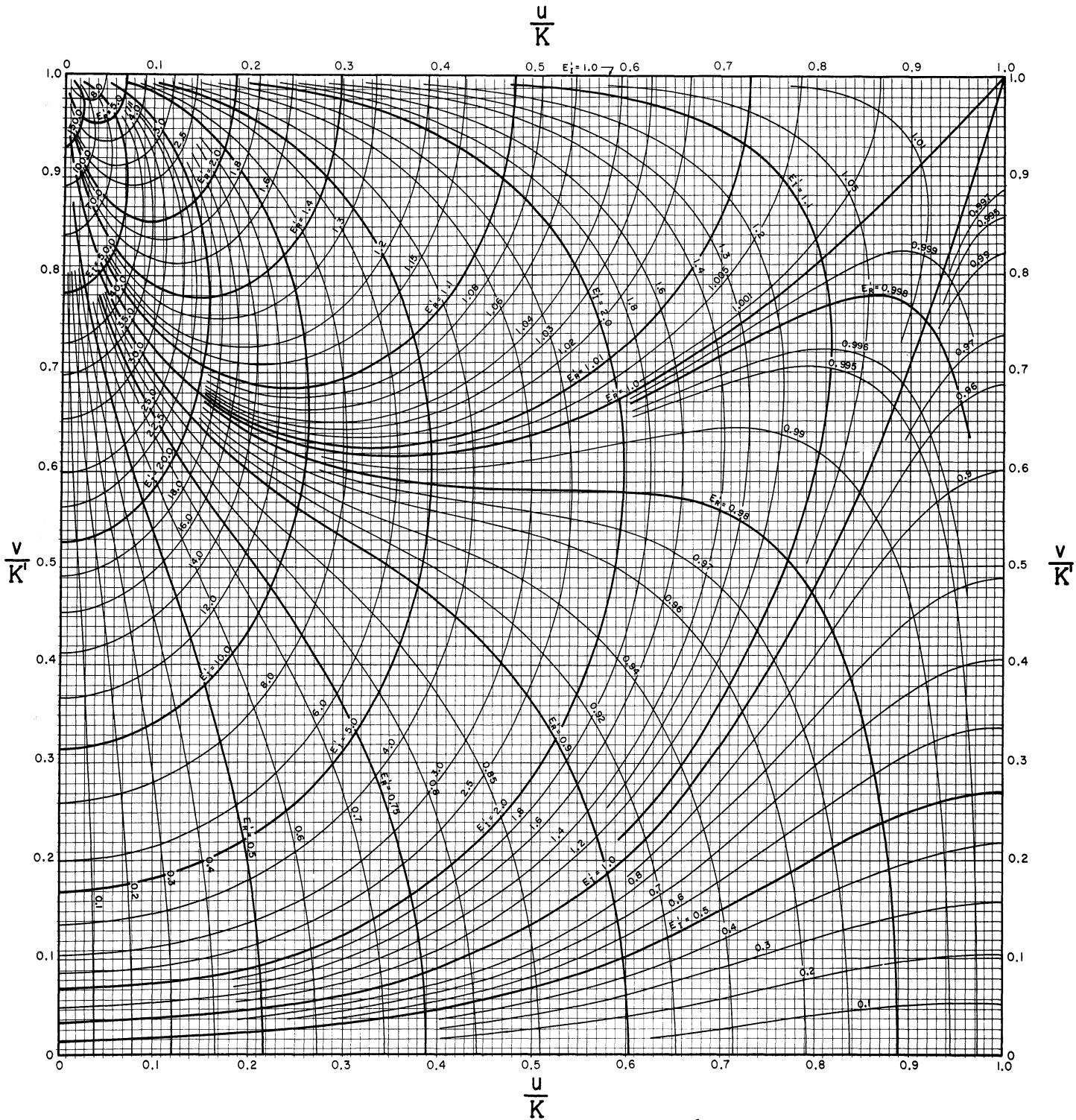
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 70° = 0.9397$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 75° = 0.965926$; $K = 2.768063$, $K' = 1.598142$, $K' - E' = 0.0539915$; $E = 1.076405$, $E' = 1.544150$

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.2512	0.4705	0.6408	0.7621	0.8443	0.8989	0.9359	0.9621	0.9824	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.2568	0.4790	0.6491	0.7687	0.8488	0.9018	0.9376	0.9631	0.9828	1.000
	E'_I	2.984	2.776	2.261	1.661	1.141	0.7589	0.5057	0.3487	0.2576	0.2110	0.1967
0.2	E'_R	0	0.2748	0.5056	0.6746	0.7884	0.8623	0.9104	0.9427	0.9659	0.9840	1.000
	E'_I	6.116	5.658	4.547	3.290	2.233	1.474	0.9794	0.6759	0.5008	0.4115	0.3842
0.3	E'_R	0	0.3087	0.5538	0.7188	0.8213	0.8842	0.9240	0.9508	0.9703	0.9860	1.000
	E'_I	9.577	8.768	6.874	4.843	3.220	2.101	1.390	0.9611	0.7162	0.5918	0.5540
0.4	E'_R	0	0.3659	0.6300	0.7839	0.8672	0.9136	0.9419	0.9612	0.9759	0.9884	1.000
	E'_I	13.63	12.25	9.226	6.243	4.033	2.591	1.707	1.186	0.8924	0.7443	0.6993
0.5	E'_R	0	0.4617	0.7444	0.8720	0.9247	0.9487	0.9626	0.9730	0.9823	0.9912	1.000
	E'_I	18.71	16.31	11.52	7.358	4.584	2.897	1.907	1.338	1.022	0.8634	0.8155
0.6	E'_R	0	0.6278	0.9116	0.9833	0.9908	0.9867	0.9843	0.9852	0.9888	0.9940	1.000
	E'_I	25.66	21.15	13.48	7.981	4.772	2.975	1.970	1.407	1.100	0.9470	0.9009
0.7	E'_R	0	0.9338	1.146	1.112	1.059	1.024	1.005	0.9965	0.9948	0.9966	1.000
	E'_I	36.43	26.82	14.53	7.824	4.504	2.800	1.891	1.395	1.128	0.9961	0.9566
0.8	E'_R	0	1.531	1.441	1.242	1.121	1.056	1.022	1.006	0.9996	0.9987	1.000
	E'_I	56.83	32.02	13.48	6.596	3.731	2.375	1.681	1.310	1.112	1.016	0.9868
0.9	E'_R	0	2.609	1.729	1.344	1.165	1.077	1.033	1.012	1.003	1.000	1.000
	E'_I	115.8	28.93	8.915	4.201	2.503	1.746	1.368	1.170	1.065	1.014	0.9983
1.0	E'_R	∞	3.447	1.858	1.383	1.181	1.085	1.037	1.014	1.004	1.000	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

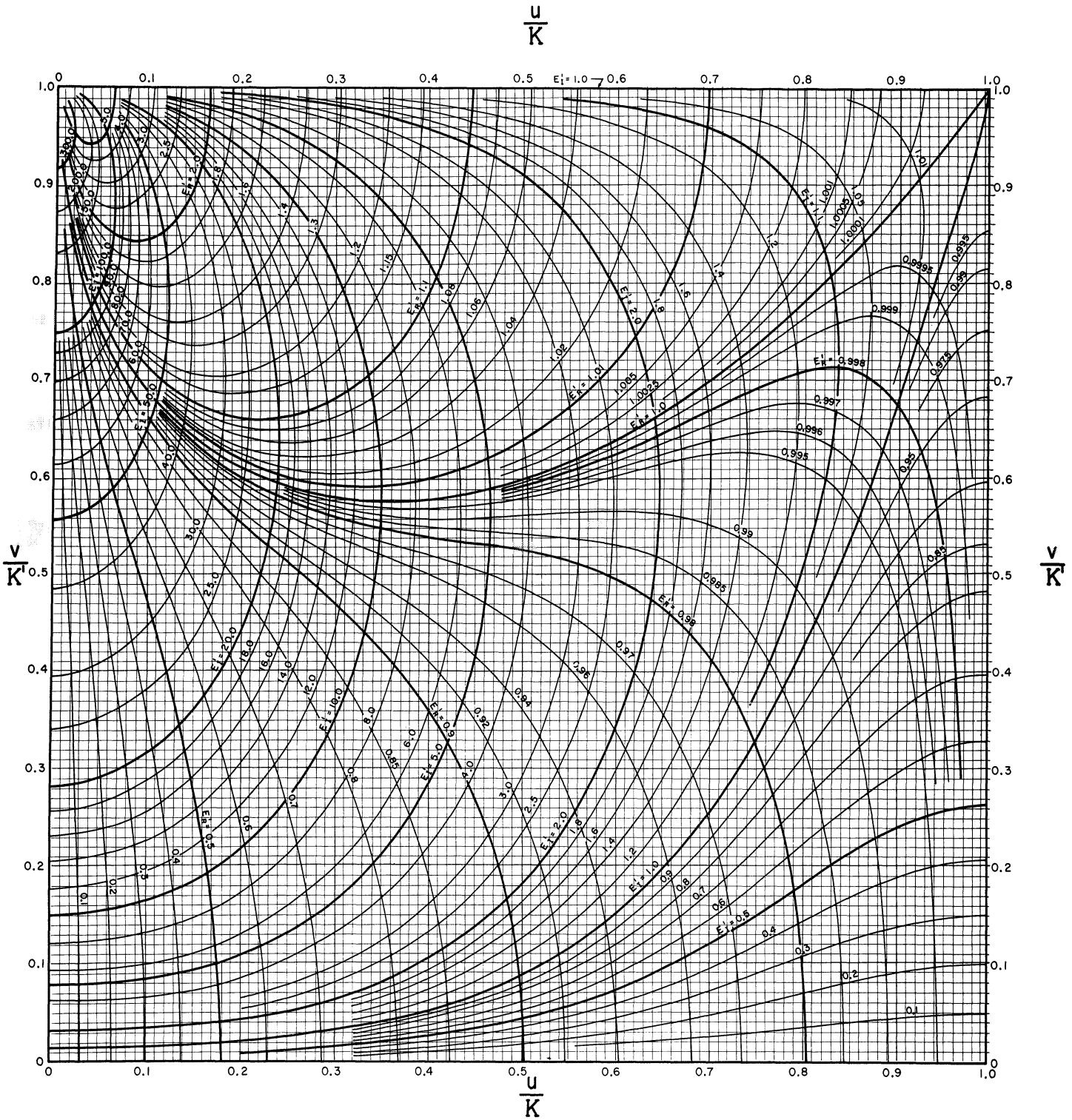
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 75° = 0.9659$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

k = sin 80° = 0.984808; K = 3.153385, K' = 1.582843, K' - E' = 0.0239556; E = 1.040114, E' = 1.558887

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.2938	0.5389	0.7148	0.8283	0.8972	0.9383	0.9632	0.9792	0.9906	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.3004	0.5481	0.7228	0.8338	0.9007	0.9403	0.9643	0.9797	0.9908	1.000
	E'_I	6.661	6.045	4.604	3.080	1.903	1.134	0.6771	0.4203	0.2831	0.2171	0.1976
0.2	E'_R	0	0.3213	0.5764	0.7470	0.8505	0.9108	0.9461	0.9675	0.9814	0.9915	1.000
	E'_I	13.66	12.31	9.218	6.062	3.699	2.190	1.305	0.8113	0.5489	0.4229	0.3858
0.3	E'_R	0	0.3604	0.6270	0.7882	0.8778	0.9271	0.9553	0.9724	0.9839	0.9925	1.000
	E'_I	21.41	19.02	13.82	8.822	5.274	3.089	1.836	1.146	0.7816	0.6073	0.5560
0.4	E'_R	0	0.4260	0.7053	0.8473	0.9148	0.9485	0.9671	0.9788	0.9871	0.9939	1.000
	E'_I	30.50	26.47	18.31	11.18	6.500	3.756	2.228	1.401	0.9681	0.7619	0.7012
0.5	E'_R	0	0.5343	0.8190	0.9243	0.9598	0.9734	0.9806	0.9859	0.9907	0.9954	1.000
	E'_I	41.93	34.95	22.41	12.88	7.236	4.124	2.450	1.561	1.100	0.8812	0.8171
0.6	E'_R	0	0.7176	0.9772	1.017	1.010	0.9997	0.9945	0.9932	0.9944	0.9969	1.000
	E'_I	57.62	44.70	25.50	13.58	7.344	4.142	2.482	1.616	1.172	0.9632	0.9019
0.7	E'_R	0	1.041	1.184	1.118	1.059	1.025	1.007	0.9998	0.9977	0.9983	1.000
	E'_I	82.00	55.23	26.41	12.85	6.724	3.790	2.322	1.571	1.189	1.009	0.9571
0.8	E'_R	0	1.624	1.424	1.215	1.102	1.046	1.018	1.005	1.000	0.9994	1.000
	E'_I	128.3	62.71	23.30	10.40	5.358	3.092	1.991	1.436	1.156	1.025	0.9870
0.9	E'_R	0	2.539	1.638	1.286	1.132	1.059	1.025	1.009	1.002	1.000	1.000
	E'_I	262.5	51.84	14.52	6.247	3.362	2.121	1.530	1.236	1.087	1.018	0.9983
1.0	E'_R	∞	3.152	1.728	1.312	1.142	1.064	1.027	1.010	1.003	1.000	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

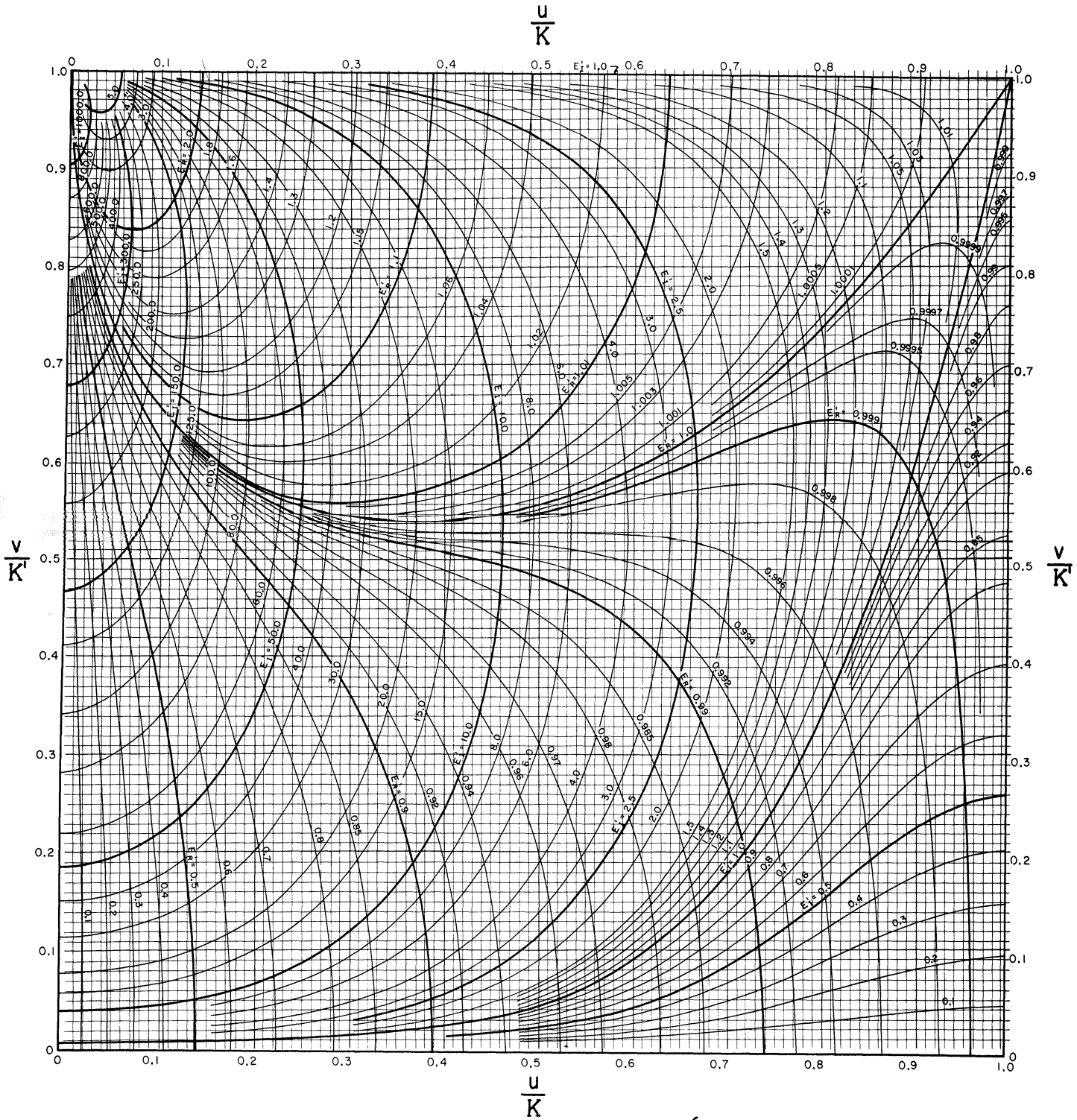
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 80° = 0.9848$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 85° = 0.996195$; $K = 3.831742$, $K' = 1.573792$, $K' - E' = 0.00598306$; $E = 1.012664$, $E' = 1.567809$

$\frac{v}{K'}$		$\frac{u}{K}$ 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.3610	0.6376	0.8094	0.9031	0.9507	0.9745	0.9865	0.9930	0.9970	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.3688	0.6468	0.8160	0.9068	0.9526	0.9754	0.9869	0.9932	0.9971	1.000
	E'_I	26.52	22.93	15.41	8.747	4.532	2.261	1.130	0.5881	0.3365	0.2282	0.1982
0.2	E'_R	0	0.3935	0.6751	0.8359	0.9180	0.9583	0.9781	0.9882	0.9938	0.9973	1.000
	E'_I	54.40	46.54	30.60	17.05	8.734	4.335	2.164	1.130	0.6502	0.4440	0.3868
0.3	E'_R	0	0.4392	0.7246	0.8687	0.9359	0.9673	0.9824	0.9902	0.9947	0.9976	1.000
	E'_I	85.30	71.53	45.22	24.41	12.28	6.045	3.016	1.582	0.9202	0.6359	0.5571
0.4	E'_R	0	0.5148	0.7984	0.9140	0.9595	0.9788	0.9879	0.9928	0.9958	0.9981	1.000
	E'_I	121.6	98.57	58.60	30.22	14.84	7.235	3.610	1.911	1.130	0.7951	0.7024
0.5	E'_R	0	0.6361	0.9002	0.9701	0.9870	0.9919	0.9940	0.9956	0.9971	0.9986	1.000
	E'_I	167.3	128.1	69.52	33.79	16.12	7.784	3.897	2.094	1.268	0.9156	0.8180
0.6	E'_R	0	0.8326	1.032	1.034	1.016	1.005	1.000	0.9985	0.9984	0.9990	1.000
	E'_I	230.2	159.6	75.87	34.34	15.90	7.622	3.851	2.120	1.332	0.9955	0.9025
0.7	E'_R	0	1.155	1.189	1.098	1.044	1.018	1.006	1.001	0.9995	0.9995	1.000
	E'_I	328.1	188.3	74.49	31.21	14.07	6.747	3.481	1.996	1.323	1.037	0.9574
0.8	E'_R	0	1.667	1.352	1.156	1.067	1.028	1.010	1.003	1.000	0.9998	1.000
	E'_I	514.2	197.7	61.79	24.14	10.73	5.238	2.830	1.743	1.253	1.045	0.9871
0.9	E'_R	0	2.332	1.482	1.196	1.083	1.034	1.013	1.004	1.001	1.000	1.000
	E'_I	1054	145.7	36.07	13.60	6.198	3.248	1.970	1.396	1.138	1.029	0.9984
1.0	E'_R	∞	2.703	1.533	1.211	1.088	1.037	1.014	1.005	1.001	1.000	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



The E-function, $E_R + iE_I = E(u+iv) = E(\omega) = \int dn^2\omega\, d\omega$

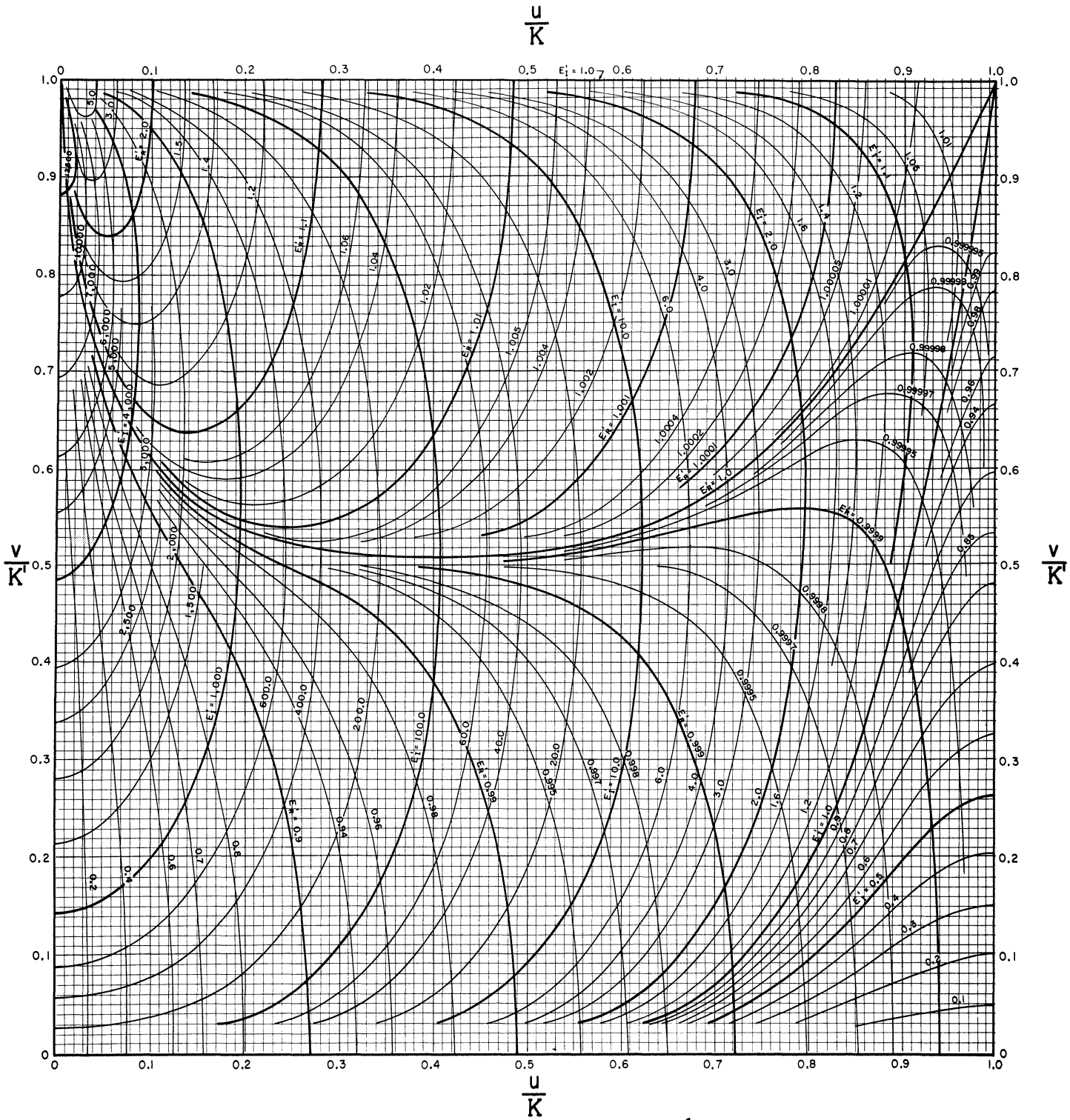
Contours of $E'_R = \frac{E_R}{E}$, and $E'_I = \frac{E_I}{K' - E'}$

$k = \sin 85° = 0.9962$

THE E-FUNCTION, $E_R + iE_I = E(u + iv)$; $E'_R = \frac{E_R}{E}$, $E'_I = \frac{E_I}{K' - E'}$

k = sin 89° = 0.999848; K = 5.434910, K' = 1.570916, K' - E' = 0.000239249; E = 1.000752, E' = 1.570677

$\frac{v}{K'}$		$\frac{u}{K}$ = 0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	E'_R	0	0.4953	0.7953	0.9256	0.9740	0.9909	0.9967	0.9988	0.9995	0.9998	1.000
	E'_I	0	0	0	0	0	0	0	0	0	0	0
0.1	E'_R	0	0.5046	0.8025	0.9288	0.9752	0.9913	0.9969	0.9988	0.9995	0.9998	1.000
	E'_I	662.1	496.4	239.1	92.32	32.72	11.28	3.890	1.383	0.5379	0.2624	0.1984
0.2	E'_R	0	0.5337	0.8241	0.9383	0.9787	0.9925	0.9973	0.9989	0.9996	0.9998	1.000
	E'_I	1358	998.8	467.1	177.4	62.45	21.49	7.416	2.641	1.033	0.5089	0.3871
0.3	E'_R	0	0.5864	0.8603	0.9536	0.9842	0.9944	0.9979	0.9992	0.9996	0.9999	1.000
	E'_I	2130	1510	671.1	248.2	86.46	29.66	10.24	3.660	1.446	0.7252	0.5575
0.4	E'_R	0	0.6698	0.9106	0.9734	0.9912	0.9969	0.9988	0.9994	0.9997	0.9999	1.000
	E'_I	3037	2028	835.2	297.9	102.4	35.00	12.09	4.350	1.748	0.8999	0.7028
0.5	E'_R	0	0.7952	0.9738	0.9965	0.9992	0.9995	0.9997	0.9998	0.0998	0.9999	1.000
	E'_I	4180	2532	939.2	320.7	108.6	36.98	12.80	4.655	1.918	1.026	0.8183
0.6	E'_R	0	0.9782	1.046	1.021	1.007	1.002	1.001	1.000	0.9999	0.9999	1.000
	E'_I	5752	2962	959.9	312.5	104.3	35.39	12.31	4.553	1.948	1.100	0.9028
0.7	E'_R	0	1.235	1.122	1.043	1.015	1.005	1.001	1.000	1.000	1.000	1.000
	E'_I	8205.	3181	875.6	271.9	89.53	30.36	10.67	4.064	1.847	1.125	0.9575
0.8	E'_R	0	1.559	1.190	1.062	1.021	1.007	1.002	1.001	1.000	1.000	1.000
	E'_I	12870	2918	675.0	201.4	65.71	22.39	8.049	3.244	1.633	1.109	0.9871
0.9	E'_R	0	1.875	1.238	1.075	1.024	1.008	1.002	1.001	1.000	1.000	1.000
	E'_I	26390	1846	369.7	107.6	35.15	12.27	4.713	2.185	1.338	1.062	0.9984
1.0	E'_R	∞	2.016	1.256	1.079	1.026	1.008	1.003	1.001	1.000	1.000	1.000
	E'_I	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000



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