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Final Report

EVALUATION OF MACHINE ELEMENTS

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Project 2152

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OBJECTIVE

The purpose of this report is to summarize the experience gained on Project 2152. Two prime goals of the project were to study the possibility of a photoelastic stress analysis of an automotive wheel and to study means of determining the "soundness" of spot welds on the same wheels.

ABSTRACT

This report presents the basic approach to two problems:

1. the possibility of a photoelastic stress analysis of an automotive wheel and

2. the means for determining the "soundness" of spot welds on the same wheels.

The methods proposed and used were, respectively,

1. three-dimensional photoelasticity and

2. sonic measurements of decay of pure vibrations.

PART I

PHOTOELASTIC STRESS ANALYSIS, GENERAL

Photoelasticity, as a tool for determining the stress distribution in a piece, is a well-accepted science. The fundamentals are reviewed here in order to make clear the approach taken to the particular problem.

A beam of ordinary light may be thought of as having a "wave-like" character. That is, each ray of the beam may be considered to consist of a sequence of particles oscillating in a sinusoidal fashion such that the oscillation of one particle is caused by a preceding one and is transmitted to a subsequent one. Thus the particles do not travel in the direction of the ray but produce a traveling "wave-like" motion. Taking many rays at once creates the idea of a "beam" of light. The planes of oscillation of the various rays in the beam are randomly oriented with respect to one another. Light beams of this type are not useful in photoelastic studies.

A Polaroid film distributed by the Polaroid Corporation has physical characteristics (dichroic properties) by virtue of which it transmits only those rays of the ordinary beam of light which oscillate in planes parallel to the axis of the film. This film thus has the ability to provide a transmitted beam of orderly oscillating rays of light (plane-polarized) which is useful in photoelastic work. The particle vibrations, if the light is produced from a point source, within the various rays are in phase and perform a simple harmonic type of motion.

A sheet of mica has the optical characteristic of transmitting light in two perpendicular planes. An oncoming ray of plane-polarized light may be treated as having a vector characteristic so that upon entrance into the mica sheet it is resolved into two components whose magnitudes depend only on the orientation of the plane of oscillation of the ray with respect to the planes of propagation of the mica. Thus, if this angle of orientation be 45°, the two components will be of equal magnitude at the start of their transmission through the mica. The velocities of transmissions of the two components within mica are different, so that, if the thickness is properly adjusted, upon emergence the two components are exactly one-fourth a wavelength out of phase. Two simple harmonic motions in perpendicular planes and one-fourth wavelength out of phase combine into a single vector representing a rotating simple harmonic motion. Light of this type is termed "circularly polarized light." Such light is used to bathe the specimens in photoelastic work. It has the ideal property that if it enters a doubly refracting medium, it will always become resolved into two equal plane-polarized components in mutually perpendicular planes, regardless of the orientation of the transmission planes of that medium. (Double refraction is explained below.)

General two-dimensional photoelasticity makes use of models of plastic materials. The plastics which are used have a property called double refraction. Essentially, this means that when the plastic is subjected to a two-dimensional state of stress it has the property of transmitting light in two planes, as mica, and of providing different velocities of transmission in those planes. The difference of velocities in the two planes are shown experimentally to be proportional to the value of the maximum shear stress existent in the material. The two planes of transmission are shown experimentally to lie at 45° angles with the plane of the maximum shear-stress vector.

Thus a beam of circularly polarized light upon being transmitted through a stressed plastic model will emerge as two components, in perpendicular planes and out of phase with one another. The measurements of a photoelastic bench are purely the determination of this phase difference and the orientation of the planes. From the phase difference, knowing the material constants, one may calculate the maximum shearing stress at any point within the model. From the orientation information, one may use the phase-difference information to determine the components of tension and compression at any point within the model.

The preceding paragraphs described the background of photoelastic stress analysis of two-dimensional problems. It was assumed that all model stress vectors lay in planes perpendicular to the beam of light and were uniform along the beam of light (through the thickness of the model). The optical effect was proportional both to the level of shear stress and to the thickness of the model—it was an integrated effect. If in that hypothetical model the stress had varied linearly through the thickness from a positive value to a negative value, being zero at the center, the integrated effect would have been zero. This, generally, is what occurs within a model of an automotive wheel. The spider of the wheel behaves basically as a plate, with a reversal of sign of stress through its thickness. Thus ordinary two-dimensional techniques are not adaptable to this problem.

THE REFLECTION METHOD

One powerful means of studying the stress distribution in the bending of plates makes use of a reflection-type technique. In this method a metal model is used and coated on one side with a thin adherent layer of plastic material. When the metal plate is bent the plastic is stressed with the plate. Generally, the stress level varies linearly from the center of the plate and linearly through the plastic layer. A simple mathematical relation exists among the thickness of the plate, the thickness of the plastic, the stress level at the surface of the plate, and the average stress in the plastic. Thus, if one could find this average plastic stress level photoelastically, he could calculate the stress existent in the model.

This procedure was not intensely investigated on the project. A few samples of metal plates coated with unknown plastics were provided by the sponsor on request and were found ineffective. The procedure used was to transmit a beam of polarized light into the plastic and reflect it back from the metal, causing it to be transmitted twice through the plastic layer, thus doubling the optical effect.

Since the project work ceased, an article, "An Analysis of Plastic Behavior of Metals with Bonded Birefringent Plastic," by D. C. Drucker _et al_. of Brown University, appeared in the literature.* This article reported the use of an adhesive layer of Armstrong C-6 on aluminum plates as a photoelastic strain gage. The author has privately applied this technique to a simulated wheel structure. The results are extremely encouraging in that the same model may be loaded and unloaded any number of times without destroying it as is done in the case of stress-coat layers used as strain gages. During gradually applied loads on calibration strips it was observed that metal stress could be measured to an accuracy of ± 400 psi up to about 40,000 psi, using a plastic layer 1/8 inch thick. It may be pointed out that the work on the simulated wheel was qualitative only and covered an area corresponding to the hat or crown of a real wheel.

THREE-DIMENSIONAL PHOTOELASTIC METHOD

The so-called "frozen-stress" techniques of three-dimensional photoelasticity received the majority of the effort of the project. The technique depends on the existence of special "di-phase" characteristics of certain plastic materials. At room temperatures the optical effects in a plastic model disappear as the load is removed. The same is true at high temperature. If, however, the model is slowly cooled and the loads are kept on it and removed only at room temperature, the model will be found to exhibit a permanent photoelastic pattern. The model may be cut into slices or pieces without disturbing the pattern in any way. This technique represents a means of avoiding the cancellation by integration that is observed when viewing through a plate in the usual manner. Cross-sectional views may be made by appropriate slicing techniques. In this manner it is possible to gain a photoelastic pattern of the variation of the stress through the model.

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*_Proc. of Soc. of Exp. Stress Anal._, Vol XII, No. 2.

EXPERIMENTAL WORK

Models of one-half section of the automotive wheel spider were formed as follows. The metal spider was prepared by a chrome plating so that it might be used as a pattern in producing a mold. The holes in the spider were closed by gluing patches of aluminum foil in place. The spider was coated with a thin layer of mold-release grease and wiped with a clean, dry cloth to remove all but a film of the grease. Next the spider was mounted in a metal box so that approximately two-thirds of it was within the box. The box itself was constructed so as to divide in the plane of the spider and when assembled there was clamped into position a thin metal template shaped to fit within 1/16 inch of the outer edge of the spider.

Approximately one-inch clearance existed between the inner walls of the box and the nearest surfaces of the spider. The box was filled to a level slightly higher than the one-half height of the spider with Castolite molding compound. This compound is available in a fluid form. The box was heated in an air oven with good ventilation at 200°F for approximately two hours and then allowed to cool for 16 hours. When the compound had been treated in this way it formed an extremely tough rubber-like substance. The box was divided by slightly lifting off one half and cutting the compound in the 1/16-inch clearance between the template and spider with a long thin blade. When this cut was made the one half of the completed mold was completely lifted off and the spider removed from the other half. The slight drag of the skirt of the spider did not provide any problem in removal due to the flexibility of the cured mold compound. The surface of the mold adjacent to the spider had a high gloss and contained no imperfections. The two halves of the mold adhered well to the inner surfaces of the metal box. The template was left in place and the box reassembled, producing a mold cavity within which to cast the plastic model. It was determined by experience that the mold would deteriorate if heated. This took the form of shrinkage and surface cracking. The mold appears to evolve gas continually while heated and has barely any hot strength.

A mixture of Castolite liquid and the proper amounts of catalysts was prepared for casting in the mold. Considerable difficulty was found in treating this material in preparation for casting and in the casting itself. First, the catalyst is used in a small amount but must be blended well into the honey-like liquid. Blending invariably introduced air bubbles into the mix. Reducing the viscosity by heating in order to clear the bubbles hastened the hardening of the liquid. No solution was found to this problem and all models contained some small bubbles. (Since that time this problem was removed by allowing the uncatalyzed liquid to clear itself of bubbles in a rubber sac. The sac was tied off below the liquid level and the catalysts injected by a hypodermic needle just below the tie-off. This puncture was tied off again and mixing was afforded by violent hand kneading of the sac. The sac could be cut and bubble-free castings made.)

Second, the gas evolved at even room temperature from the mold caused some discoloration of the plastic. This was only objectionable from a visual viewpoint as the slight amber color did not influence the optics of the photoelasticity. (Since that time a new Castolite molding material has been marketed which cures at about 350°F and does not cause any discoloration of the casting.)

Third, the time involved in making a casting was extremely long. Original attempts to remove the casting within the same day as pouring resulted in a limp form with sticky, wrinkled surfaces. When the casting was left in the mold for a period of seven days it could be removed with an excellent surface (high polish) but not fully hardened. In this form the material had excellent machining characteristics, better than Plexiglas. The model was mounted on a form and mill cut across a true diameter to form a half spider model. A second half spider model was made, using the metal pattern to form a new mold since the two halves of the real spider are different. The cut edges of the two pieces were wetted by their own monomer properly catalyzed, fitted together, and allowed to stand on a form for several days. During this time the cemented joint cured to essentially the same degree as the original pieces. No permanent stress was introduced during any of these operations as the material was incapable of maintaining it. The complete spider model was thus made and was essentially a perfect reproduction of the original, although between 2 and 3% smaller due to shrinkage. The model was not completely cured at this point. Only one really good model was made. Many one-half models with imperfections were made and served a certain degree of usefulness.

The fourth difficulty in casting had to do with the seal between the two halves of the mold. The cut edges never fitted exactly, with the result that a row of tiny air pockets formed into the casting, causing serious imperfections around the flange of the spider. This problem was partially solved by deliberately cutting this region back, or beveling it, on the two open mold halves. In this case the casting had a flash of material which had to be removed by scraping and sanding. This was not a good solution but was practical at the time.

At approximately this stage of the project the sponsor requested a transfer of attention to the welding problem and no models of a rim section were prepared.

Curved sections of thin plastic rings of Castolite about 1/8 inch thick, one inch wide, and eight inches long were cast between glass plates of the same radius as the inner-rim radius. When these had reached a semi-cured state (hard enough to handle without causing surface marks) they were coated with a thin layer of mold-release compound, with the exception of a 1/2-inch circular area on the outside of one ring and inside the other. These points were coated with a drop of catalyzed plastic and nestled together so that the two areas matched, and lightly clamped.

No residual stress developed from the semi-curing of this simulated spot-welded area. The welded pieces were then cured by heating in an oil bath from room temperature to 200°F at a rate of 2°F per hour, holding that temperature for two hours and cooling at the same rate. No residual stress could be observed after this curing cycle. The pieces were again placed in the oil bath and raised at 2°F per hour to the critical temperature of 245°F. The pieces were wedged apart, causing them to bend away from each other against the restraint of the spot weld. The temperature was held for 1/2 hour and lowered at 2°F intervals to room temperature. A stress pattern could be observed about the weld point in an ordinary transmission through the dual thickness.

A slice 1/8 inch thick was taken through the two pieces containing the center of the weld on a glass-cutter's disc. A cross-sectional view of the specimen was made on the photoelastic bench by immersing the cut slice in a fluid of the same index of refraction so as to clear the cut surfaces. A good stress pattern not unlike that about a simple crack in a two-dimensional stress field was observed.

No further analysis of this weld was made, as the sponsor at this time requested all work to cease.

CONCLUSIONS

Two basically different means of photoelastic analysis of automotive wheels were studied. Neither means accomplished its goal in the time taken.

RECOMMENDATIONS

Any further photoelastic stress analysis of wheel structures should focus its attention upon the reflection method for the general wheel area. This method is proven and would allow an easy examination of shear stress throughout the wheel. To gain the true tensile or compressive stresses would require considerably more basic research. The analysis of stress in the immediate vicinity of the spot welds should use partial models and the frozen-stress technique.

PART II

VIBRATIONAL FORM OF AN AUTOMOTIVE WHEEL

It is possible to consider one form of vibrational modes of an automotive wheel without introducing a mathematical analysis. A common experience of striking a freely supported wheel a sharp blow with a metal hammer is known to be the exciting of a musical noise. Careful attention to this noise will allow the observer to deduce that there is a tonal change with time after striking. The general sound level deteriorates and the tone becomes purer, resulting finally in a pure sounding hum of decreasing magnitude. The air-pressure fluctuations by which this tone is created are caused by the movement of air adjacent to the oscillating wheel.

Consider, temporarily, a circular metal ring supported in a horizontal plane by threads. One form of oscillation that this ring may have consists of a pure radial-type motion. As an example, assume that four quarter points on the ring are stationary but that all other points have a radial motion. If at one instant the radial motion to the right of a stationary point was outward, then the radial motion to the left of that point would be inward. The ring would be seen to be divided into four regions, of which an alternate two would move outward while the other alternate two were moving inward. As one motion reached its limiting amplitude and stopped, so would the other. A reversal of the motions would then occur. Thus the shape of the ring would continually change with time, repeating at regular periods of time. At any one instant the shape would correspond to the form generated if one should plot, from the undeformed ring as a circular axis, radial variations corresponding to two complete cycles of a sine wave. This shape is termed a "normal mode of vibration." The unmoving points are termed "nodes" of the vibrating ring.

An automotile wheel corresponds to this illustration of a circular ring. The rim of the wheel may be considered as an equivalent ring. The spider may be considered as having a distributed effect or contribution to this equivalent ring. However, due to the fact that the spider is not angularly symmetric, the contribution to the equivalent ring is not angularly symmetric. Thus the equivalent ring, representing the wheel mathematically, has a pattern of variation repeating four times and corresponding in position to the position of the spider in the rim.

Now this equivalent ring will oscillate essentially like a true ring. However, while in a true ring the nodes may be any set of four quarter points, in this ring there are only two sets of natural quarter points. These are either those points corresponding to the center of the legs or to the center of the scallops of the spider. It is possible for the wheel to oscillate, using either of these sets of nodal points. In fact, sometimes there is a transfer from one set to the other during free oscillation. This transfer is not too common as the periods of the two motions are slightly different.

It is seen that the motion described is accomplished by a bending of the ring in its own plane. In the automobile wheel the bending characteristics of the spider are not continuous with those of the rim, with the result that there is some "working" at the joints. The joints consist of two things: one, the eight spot welds, and two, any point of physical contact.

It has been stated that a freely vibrating wheel will gradually decrease its amplitude with time. This is due to a loss of energy in three fashions; first, air pumping (the noise we hear, for instance, represents radiated energy), second, internal friction (hysteresis looping), and third, energy lost in the joint of spider to rim. It is this last form which is important to our analysis.

We may deduce, and prove experimentally, that the least energy loss in the joint would occur when the eight welds were solid nuggets and when no other mechanical contact existed between wheel and rim. Any deviation from this state will produce a higher rate of loss.

In addition we may deduce that the physical spacing of the four nodal points is a function of the equality of the eight welds. That is, if one of the eight should be missing, a definite measurable shift (5° to 15°) of the nodes would occur. If one should be of half the area of the others, a lesser but measurable shift would occur.

EXPERIMENTAL WORK

An experimental setup was constructed consisting of the following:

1. An enclosing box to isolate the wheel from external noises.
2. An anvil upon which to rest the hub of the wheel.
3. A microphone with which to pick up noise generated by the wheel.
4. An amplifier to boost the electrical signal from the microphone.
5. An exponential peak recording voltmeter to produce a plot of the sound level.
6. An oscillograph to observe visually the level and quality of the noise signal being picked up.
7. An electromagnet mounted near the wheel to apply periodically varying radial force.

8. An a-c sine wave signal generator to produce a varying period signal to the electromagnet.
9. An amplifier to boost the power of this signal.

In this setup it was possible electromechanically to excite the wheel to oscillate with large amplitudes in a pure, four-noded, mode shape. By positioning the microphone circumferentially above the rim, it was possible to determine the position of the nodes. The wheel was excited to a fixed amplitude and the magnet suddenly removed so that the wheel oscillated freely. The time dependency of the dying motion was plotted on a moving waxpaper on the recording exponential voltmeter. The number of seconds required for a standard change in level of the free oscillation was recorded and used in correlation with the physical information on the wheel.

The sponsor provided several sets of wheels made under what he considered to be controlled conditions. No comment can be made regarding the control methods used. The results of each set of tests were plotted as distribution diagrams, making four tests per wheel, with the only difference being a 90° rotation of the wheel on its anvil for each test. The distribution diagrams overlapped approximately 30 to 40% areawise between each neighboring classification of "excellent, good, marginally satisfactory, and unsatisfactory." A further set of samples containing systematically missing or poor welds was also tested. In this case the single parameter of time measured was not sufficient to classify the wheel as defective, although in each case the missing or poor weld could be located by studying the shifting of the nodal points.

In each of the test groups one or more nonconforming wheels were found. That is, the group of "excellent" wheels might show one wheel with a "poor" time constant. In each such case it was possible to find the cause of this in a nonstandard contact of the spider flange to the rim. Thus some welds had not been confined completely and had "flashed." Local distortions of the flange were apparently due to nonuniform welding procedures.

No attempt was made to conclude what a satisfactory weld was. However, excellent quality-control-type results were gained in these tests. At no time did it appear proper to propose any more measurements than the time constant. This was due to the complications that arise in interpreting nodal shifts, weld flashing, heat distortions, etc.

CONCLUSIONS

A quality-control test requiring approximately one minute was devised which measured the gradation or degradation of a continuously produced product. The test was influenced by factors associated with

1. Press fit.
2. Heat of weld.
3. Local distortions near welds.

RECOMMENDATIONS

Any further work in this area should be mechanized to include both a display of the nodal positions and time constant, and operators trained to interpret interrelationships of these quantities with physical appearance of the wheel.

No attempt to use this method with "continuous spider wheels" should be made.