THE UNIVERSITY OF MICHIGAN

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# ATMOSPHERIC DIFFUSION STUDY

# AT THE

# ENRICO FERMI NUCLEAR REACTOR SITE

First Technical Report

A Quantitative Analysis of Diffusion

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# PREFACE

This report presents a quantitative analysis of the diffusion experiments that have been held at the Enrico Fermi plant site between August 1959 and July 1960. The methods utilized, both successful and unsuccessful, are discussed. The net result is a computed value of $C_y$ and $C_z$ for each experimental day using an assumed value of the parameter n.

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TABLE OF CONTENTS

## LIST OF TABLES

## ABSTRACT

This report contains assumed values of Sutton's n together with values of $C_z$ and $C_y$ computed from the data collected during six experimental runs at the Enrico Fermi nuclear power plant. Computations of the transport of mass through a vertical surface are also made to find the efficiency of the aerosol generator and to check to see how valid the assumptions being used were. The results of these computations are given in section VI. The report concludes with a brief quantitative discussion of each of the experimental runs.

# I. INTRODUCTION

The actual quantitative evaluation of the observed data from the experimental runs has presented several problems. It is known that the formulae of Sutton do characterize diffusion under many conditions, especially those conditions when the atmosphere is in a near neutral condition. At the extremes, that is during strong adiabatic lapse rates and strong inversions, Sutton's equations have not been as applicable. In fact it has been found that concentrations at long distances downwind show lower concentrations than Sutton's formula would indicate. Hilst [1] has developed a model for use in atmospheric diffusion under stable conditions at Richland, Washington. Gifford has proposed another model [2] which has a great deal of merit. One of the main advantages of Gifford's model is that it accounts for the influence of large scale turbulence as well as that of smaller scale turbulence on the diffusion and meandering of the plume. For a fixed sampling network this meandering can be a major problem when evaluating the collected tracer data. In the experiments at the Enrico Fermi site where airplane sampling has been done, the problem of meander is not as important when evaluating the data since each traverse of the plume was accomplished in a matter of four minutes or less; the influence of meandering is allowed for in the technique of analyzing the data. Since Hilst's and Gifford's models have not yet come into general usage, it is considered preferable to use Sutton's equations in an engineering analysis such as the present study.

Although there are other theoretical models available, it might be well to quote Dr. Gartrell of the Division of Health and Safety, Tennessee Valley Authority concerning Sutton's model. "Our experience is that the form of the equation appears to be very sound, and so far our data do not point to any better theoretical expression for diffusion than Sutton's equation" [3]. Dr. Katz of the Occupational Health Division, Department of National Health and Welfare of the Canadian government says, "...most of the data indicate more and more that the diffusion theories of Sutton are quite practical and valid and are becoming extremely useful in design" [3]. Adopting the attitude that the Sutton equation is valid, the parameters n, $C_z$, and $C_y$ were computed from the field data obtained.

The present technical report describes the results of the engineering study of diffusion near the Enrico Fermi plant site. Some of the more theoretical aspects of diffusion in transitional states, such as that near Lagoona Beach, will be explored by the writers with the support of a research grant from the National Science Foundation.

## II. THE VALUE OF SUTTON'S n

In the model to be used in evaluating the diffusion of matter from an elevated continuous point source, the concentration anywhere above the ground is given by the following formula [4].

$$X = \frac{Q \exp\left(-y^2/C_y^2\, x^{2-n}\right)}{\pi\, C_y\, C_z\, \bar{u}\, x^{2-n}} \left\{ \exp\left[-\,(z-h)^2/C_z^2\, x^{2-n}\right] + \exp\left[-\,(z+h)^2/C_z^2\, x^{2-n}\right] \right\} \tag{1}$$

where

$X$ = concentration in mass per unit volume, g/cc

$Q$ = source strength in mass per unit time, g/sec

$x$ = distance downwind, meters

$y$ = distance crosswind, meters

$z$ = vertical distance, meters

$\bar{u}$ = mean wind speed, mps

$h$ = effective stack height, meters

$C_y$ = crosswind virtual diffusion coefficient, $m^{n/2}$

$C_z$ = vertical virtual diffusion coefficient, $m^{n/2}$

$n$ = a nondimensional parameter related to the diffusing power of the turbulence

The value of n varies depending upon meteorological conditions with larger values indicating greater stability in the atmosphere.

Sutton suggests that n may be computed from wind profiles using the following equation [4].

$$\overline{u}/\overline{u}_1 = (z/z_1)^{n/(2-n)}$$

where $\overline{u}$ is the mean wind speed at height z and $\overline{u}_1$ is the mean wind speed at height $z_1$.

DeMarrais has successfully used this method utilizing Bendix-Friez Wind Gradient Recorders at Brookhaven National Laboratories [5]. Such methods have also been used for practical evaluations with exhaust gases from nuclear reactors [6]. The technique was tried on the data from the Enrico Fermi plant site, but unsuccessfully. The value of n computed under the three stability conditions of inversion, weak lapse, and strong lapse varied so markedly that they could not be used. Very recently Barad and Haugen have shown that there is in reality not a single n but two n's, one for lateral diffusion, $n_y$, and one for vertical diffusion, $n_z$ [7]. The method of computation they used was to select various ratios and eliminate several of the unknowns by mathematical manipulations. A ratio of the peak concentrations at two radial distances from the source was tried on the Enrico Fermi data but because of the sampling technique used, the data from the runs plot out as a histogram. It is very difficult from such histograms to find the value of the peak concentration because the histograms do not indicate true normal distributions.

Because of failure in obtaining values of n, it was then decided to assume values of n depending upon the vertical temperature distribution at the time of the sampling. Although such a procedure seems inconsistent at first, it is in reality based upon sound reasoning. By assuming acceptable values for n, the computed values of $C_z$ and $C_y$ may then be compared with values obtained by other experimenters on a common basis. Such a technique has been used at

Harwell, England even though values of n have been computed from wind profiles [6].

Table I shows the lapse rates and the value of n that has been assumed.

TABLE I. Assumed Values of Sutton's n

| Experiment Date | Lapse Rate | n |
|---|---|---|
| 6 August 1959 (run No. 1) | Weak at tower Inversion inland Isothermal over water | 0.25 |
| 27 November 1959 | Strong at tower Lapse over water and land | 0.20 |
| 4 February 1960 | Inversion at tower and land | 0.30 |
| 3 April 1960 | Inversion at tower, land and water | 0.30 |
| 8 May 1960 | Strong in lower levels, inversion above | 0.20 |
| 25 June 1960 | Weak | 0.23 |

The assumed values of n are taken from a table of values of the Sutton parameters as measured by Sutton and cited by Haltiner and Martin [8]. These values are based on 3-minute sampling values, while the sampling period of the airplane at the Enrico Fermi site was 7 1/2, 15, 30, or 60 sec. It is true that the assumed values of n might not be exactly correct but they are the best available, and represent a good first approximation.

One assumption that is implicit in the use of the assumed n values is that n is invariant in time and space. This is not true, of course, since if n is truly a parameter that is related to the diffusing power of the turbulence, then n must change since this diffusing power changes as the transition from land to water and vice versa takes place. The value of n may also vary with distance from the source, even with horizontally homogeneous atmospheric conditions, but precise information on this point is not required for the present engineering analysis.

# III. COMPUTATION OF $C_z$

Once a value of n had been assumed, the evaluation of $C_z$ became simple. Using equation (1) as a starting point, several assumptions were made. The first was that no reflection takes place from the earth's surface. This means that once FP particles hit the ground, they remained there and never became airborne again. Actually, this assumption seems quite valid, especially since particulates are being used as a tracer material. The advantage of this assumption is that the second exponential term in the braces of equation (1) can then be neglected so the equation now may be rewritten as follows:

$$X = \frac{Q \exp\left(-y^2/C_y^2\, x^{2-n}\right)}{\pi\, C_y\, C_z\, \bar{u}\, x^{2-n}} \left\{ \exp\left[\frac{-(z-h)^2}{C_z^2\, x^{2-n}}\right] \right\} \qquad (2)$$

A second assumption to be made is that n, $C_z$, and $C_y$ are constant in time. A third assumption is that $C_y$ is not a function of y; this may be open to question on theoretical grounds, but need not be considered for the present study.

Using these assumptions, we can now integrate equation (2) across the wind or in the y direction. The result is called the integrated crosswind concentration, $X_{ICC}$, which is given by

$$X_{ICC} = \frac{2\, Q \exp\left[\frac{-(z-h)^2}{C_z^2\, x^{2-n}}\right]}{\pi\, C_y\, C_z\, \bar{u}\, x^{2-n}} \int_0^{\infty} \exp\left[-\, y^2/C_y^2\, x^{2-n}\right] dy$$

Performing the indicated operation, the result is

$$X_{ICC} = \frac{Q \exp \left[ \frac{-(z-h)^2}{C_y^2 \; x^{2-n}} \right]}{\pi^{1/2} \, \bar{u} \, C_z \, x^{1-n/2}} \tag{3}$$

By taking ratios of equation (3) at two adjacent heights at the same distance from the source, we can solve for $C_z$.

$$\frac{X_{iICC}}{X_{iiICC}} = \frac{\dfrac{Q \exp \left[ \frac{-(z_i-h)^2}{C_z^2 \; x^{2-n}} \right]}{\pi^{1/2} \, \bar{u} \, C_z \, x^{1-n/2}}}{\dfrac{Q \exp \left[ \frac{-(z_{ii}-h)^2}{C_z^2 \; x^{2-n}} \right]}{\pi^{1/2} \, \bar{u} \, C_z \, x^{1-n/2}}} \tag{4}$$

where $X_{iICC}$ is the integrated crosswind concentration at level "i" which is $z_i$ meters above the surface and $X_{iiICC}$ is the integrated crosswind concentration at level "ii." $C_z$ can be found from the following formula which follows from equation (4).

$$C_z = \left[ \frac{(z_{ii}-h)^2 - (z_i-h)^2}{R \; x^{2-n}} \right]^{1/2} \tag{5}$$

where R is the natural logarithm of the ratio of the integrated crosswind concentrations.

The values of $C_z$ computed for the various experiments are presented in section VI where the results are summarized.

## IV. COMPUTATION OF $C_y$

The value of $C_y$ was computed directly from a modified version of equation (1). Most of the experimental runs had one definite peak at each level of sampling. It was assumed that this highest value was the peak concentration. A good check on the validity of this assumption is obtained from the sample taken while the plane climbs to the new altitudes and gets ready to sample. If the plume were missed or the peak were missed, this extra sample may have a large particle count. Because such large counts were obtained on two occasions, the experimental data obtained on those days, 5 August 1959 and 28 November 1959, had to be discarded.

The maximum concentration occurs when y = 0, or in the centerline of the plume. Thus equation (1) can be written for the maximum concentration

$$X_{max} = \frac{Q}{\pi C_y C_z \bar{u} x^{2-n}} \left\{ \exp \left[ - \frac{(z-h)^2}{C_z^2 x^{2-n}} \right] \right\} \tag{6}$$

$$C_y = \frac{Q \exp \left[ - \frac{(z-h)^2}{C_z^2 x^{2-n}} \right]}{X_{max} \pi C_z \bar{u} x^{2-n}} \tag{7}$$

It was found after computing several values of $C_y$ that the values appeared to be too high. Two major reasons for such high values suggested themselves. The first was that the mean wind speed used in the computation, $\bar{u}$, was actually the mean wind speed from the 100 ft aerovane at the meteorological tower. It was decided to use the 500 ft wind which would be at a height sufficient to be above the main frictional effects and would give a better average wind speed in the layer being sampled. Since there was no way to measure the 500 ft wind it was estimated using the well known 1/7th power law. Thus the wind speed at 500 ft may be found as follows.

$$\bar{u}_{500} = \bar{u}_{100} \left[ 500/100 \right]^{1/7}$$

$$\bar{u}_{500} = 1.26 \quad (\bar{u}_{100}) \qquad (8)$$

The second reason for high values of $C_y$ comes from the fact that the source strength, Q, was obtained initially by considering the input to the aerosol generator rather than the output of the generator into the atmosphere. The output might not actually be the same as the input due to agglomeration of particles and therefore fallout near the source, thinning of the plume due to hitting the tower structure, aerovane, or bivane at the level of emission, and exaggerated shear conditions caused by not pointing the mouth of the generator directly into the wind. The remedy for this situation will be discussed in the next section.

Values of $C_y$ are given in section VI where the results are summarized.

# V. MASS TRANSPORT THROUGH A VERTICAL SURFACE

As indicated in the last section, the computed values of $C_y$ seemed to be quite high. It was felt that at least one of the reasons for this was the fact that the weighed inputs of the aerosol generator were being used for the value of Q, the source strength. In order to arrive at the output efficiency and as a check to see if the sampling techniques were adequate, it was decided to compute the transport of mass through a section of a vertical cylindrical wall at the arcs where sampling took place. By making such a computation, the number of particles passing through the cylinder were compared with the number put into the aerosol generator. In this manner, the output efficiency of the generator was estimated. It has been assumed that the efficiency of the plane sampling unit is 90% for particles in the size range 1-10 $\mu$. This figure has been used by the Stanford University Aerosol Laboratory who developed the equipment and techniques used in the sampling procedure [9].

The computation for the mass transport follows this line of reasoning. First, a thin slice of the atmosphere at the radius under examination is considered. Everytime the plane passes horizontally through the plume, the sampler actually collects the number of particles in a long thin strip of $\pi D^2/4$ in cross section, where D is the diameter of the circular orifice of the sampler. For ease in computation, let us consider that this cross sectional area can be represented by a rectangle of height D and breadth $\pi D/4$.

Also consider that if sampling takes place at a height of 100 ft, then the average concentration in the band 50-150 ft may be taken effectively as the same as that at 100 ft. Similarily, the average concentration in the band 150-250 ft is taken to be the same as that collected at 200 ft. Let us designate these bands as $W_z$ where the subscript z indicates the level of the sampling.

In any one strip, $W_z$, the number of particles in this band is given by the number of particles in the sampled strip multiplied by the number of strips that make up the band. Symbolically this is:

$$\text{No. of particles/band} = \sum_{-\ell}^{+\ell} (\text{Count on sampler})(W_z/D) = K_z,$$

where $\sum_{-\ell}^{+\ell}$ (Count on sampler) = total particle count on sampler for an arc at level z.

Assuming that there is an average wind speed $\bar{u}_z$ in the layer $W_z$, we can determine the time, $t_z$, needed for particles to pass through this thin strip $\pi/4D$ in breadth. The value of $\bar{u}_z$ is also obtained from the 1/7th power law profile, equation 8; it is assumed to be constant for the band, $W_z$.

Mathematically then,

$$t_z = \pi D/4\ \bar{u}_z$$

The number of particles that will pass through the band of height $W_z$ may then be determined by dividing the number of particles/band by the time needed to fill this band.

$$K = \frac{K_z}{t_z} = \frac{\sum_{-\ell}^{+\ell} (\text{Count on sampler})(W_z/D)}{\pi D/4\ \bar{u}_z} = \text{particles/sec}$$

The total rate of particle passage through this cylindrical wall is computed by adding up all the bands from the lowest to the highest.

$$K_T = \sum_{z=a}^{z=d} K$$

where a = height of lowest band and d = height of highest band.

This value of $K_T$ is the number of particles that is transported through a cylindrical wall from 50 ft above the surface to the top of the sampling array. Recall that the emission height is 56 ft above the ground. For all practical purposes then, the value $K_T$ represents only one half of the particles emitted since one half will be below 56 ft while the other half is above 56 ft. Thus the total number of particles emitted is 2 $K_T$.

The value of 2 $K_T$ was compared with the value of Q to compute the efficiency of output.

$$(2\ K_T/Q)(100) = \%\ \text{efficiency}$$

Table II shows the computed efficiencies for the experimental runs that were made.

Table II in effect shows that when the aerosol generator was working properly, that is at a constant output, the computed efficiencies were quite reasonable and logical. However, on 3 April, 4 May, and 25 June the feed motor was not operating at a constant output, and thus there are such inconsistencies as 43.2% at 2 km and 68.0% at 4 km. Also at 8 km on 25 June there is an efficiency of 102%. Such a figure arises from the fact that the computed number of particles/second are compared with the weighed number of particles. Therefore, if the aerosol generator stopped emitting FP material for a short while, as it did on 25 June 1960 prior to the 4 km run, then the average number of particles emitted during the entire time of the experiment would be lower than if the generator operated at a constant output for the entire period. This means Q is smaller than it would normally have been. In such a case 2 $K_T$ might be larger thus allowing an efficiency of over 100%.

TABLE II. Computed Efficiencies at Several Radii on the Experimental Days

| Day | Radius (km) | Efficiency % | Mean Efficiency % |
|---|---|---|---|
| 6 August 1959 (run No. 1) | 2 | 55.7 | 50.9 |
| | 4 | 46.0 | |
| 27 November 1959 | 2 | 59.0 | 40.5 |
| | 4 | 22.0 | |
| 4 February 1960 | 2 | 50.3 | 48.1 |
| | 4 | 45.8 | |
| 3 April 1960 | 2 | 43.2 | 55.6 |
| | 4 | 68.0 | |
| 4 May 1960 | 2 | 14.6 | 14.6 |
| 25 June 1960 | 2 | 89.1 | |
| | 4 | 24.0 | 71.7 |
| | 8 | 102.0 | |

Once the efficiency was determined, it was then possible to substitute the computed number of particles transported through the cylindrical wall as the real source strength. In cases where several values of efficiency were determined for one experiment, the average efficiency was used as the correction factor. Values of $C_y$ were then recomputed using the new Q value. Also the values of $C_z$ were recomputed.

## VI. RESULTS OF COMPUTATIONS

Table III presents the mean computed values for $C_z$ and $C_y$ for the several experiments.

TABLE III. Mean Computed Values of $C_z$ and $C_y$ at the Enrico Fermi Plant Site

| Day | n | $\overline{C}_z$ | $\overline{C}_y$ |
|---|---|---|---|
| 6 August 1959 (run No. 1) | 0.25 | 0.15 | 0.38 |
| 27 November 1959 | 0.20 | 0.14 | 0.64 |
| 4 February 1960 | 0.30 | 0.08 | 0.54 |
| 3 April 1960 | 0.30 | 0.09 | 0.61 |
| 8 May 1960 | 0.20 | 0.13 | 0.44 |
| 25 June 1960 | 0.23 | 0.14 | 0.37 |

The above table indicates that for near neutral conditions, the computed values are reasonable. For the inversion conditions -- higher values of n, -- both the value of $C_z$ and $C_y$ are larger than would be anticipated. As noted earlier, as the atmosphere departs more from the neutral condition, the less accurate is Sutton's equation. Therefore, it would be expected that the computed values of $C_z$ and $C_y$ will be higher than usual. Some of this difference between the computed values and those in Haltiner and Martin [9] may be attributed to the fact that the values in Haltiner and Martin are based on 3-minute sampling times whereas the Enrico Fermi sampling times are the order of 1 minute or less. Some difference is to be expected because of the special atmosphere and terrain conditions near the site.

## VII. QUANTITATIVE DISCUSSION OF THE EXPERIMENTS

Using the computed and assumed values from Table III, some appropriate remarks relative to the diffusion on the days of the various experiments may be made.

a. Experiment of 6 August 1959 -- The computed values of the several parameters do indicate that the atmosphere was in a state of near equilibrium. Diffusion under such conditions is relatively moderate. The computations thus verify what was anticipated from the qualitative discussion in the first progress report.

b. Experiment of 27 November 1959 -- With the strong lapse rate conditions that existed on this day, it was quite definite from the outset that the diffusion potential at the plant site would be high. The computed values of $C_z$ and $C_y$ bear this out although the value of $C_z$ seems a little low, or $C_y$ is a little high. Perhaps this is a result of a nearly isothermal layer above that containing the strong lapse rate. This isothermal layer acted to contain the FP material in the lower layer.

c. Experiment of 4 February 1960 -- Using the assumed value of $n = 0.30$, the value of $C_z$ is in the expected range, but that of $C_y$ is somewhat high. This is attributed to the fact that the lapse rate was changing as the plume moved inland. Thus from an inversion in the lower levels it changed to a lapse condition inland. Aloft, above 800 ft, a strong inversion existed. Under such conditions, the value of $C_y$ would be high relative to $C_z$, as on 27 November

1959. Hence diffusion was moderate although at first glance the situation would not lead to such a conclusion.

d. Experiment of 3 April 1960 -- The value of $C_y$ appears to be quite high for an inversion condition. However, the sampling was carried out over the land where the surface roughness and changing characteristics of the lapse rate would certainly cause the plume to spread out horizontally. This type of day is typical at the site with a lake breeze induced inversion. The computations indicate that diffusion would be moderate and indeed substantially higher than would have been supposed from the first analysis of the data.

e. Experiment of 8 May 1960 -- The computed value of $C_y$ looks satisfactory, considering that this period was categorized as one of strong lapse rate. The value of $C_z$ is low because the layer with strong lapse rate was only 100-200 ft thick. Above the strong lapse rate was an inversion which would certainly dampen vertical diffusion. Diffusion may be characterized by saying that it was moderate.

f. Experiment of 25 June 1960 -- The effect of a late afternoon lake breeze is shown here in the low value of $C_z$. The value of $C_y$ is in the anticipated range so that in the overall picture, diffusion is relatively high.

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