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Progress Report

CONTROL OF THE PROPERTIES OF RESEARCH SOIL SYSTEMS

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## ABSTRACT

This report covers a series of experiments made with a precision bevameter on sand and mixtures of sand with other materials. The object of the tests was to ascertain the range over which the cohesive and friction moduli of deformation of a soil can be controlled, thus exerting a controlling influence upon the dimensionless parameters employed during similitude testing of wheels.

The results indicate that such control is possible to a moderate degree when using the types of materials studied.





## INTRODUCTION

A fundamental problem in land locomotion is the interaction of the wheel-soil system. Since the mechanics of this system involves the properties of all its constituents, the problem is related to the physical properties of the soil in contact with the wheel as well as that of the wheel itself.

The soil can vary from a more or less plastic clay to sands and hard surfaces. Little is known about the science of soils, particularly about the complete range of possible soil values applicable to the soil-wheel system. The result is that an empirical approach is necessary to solve the immediate problems, at least for the present.

One such approach is the Bekker<sup>1</sup> soil-value system, represented by Eq. (1):

$$P = (k_c/b + k_\phi)Z^n \quad (1)$$

This relates the soil pressure  $P$  psi to sinkage  $Z$  in. by a physical dimension "b" in. of the measuring device employed: the narrow width of the imprint in the soil, or the radius of a circular foot if used, and three parameters  $k_c$ ,  $k_\phi$ , and  $n$ . The latter three parameters are a function of the soil itself.

These coefficients are defined as follows:

$k_c$  = cohesive modulus of soil deformation

$k_\phi$  = frictional modulus of soil deformation

$n$  = exponent of deformation

It is well recognized by those working in the field that the pressure  $P$  and sinkage  $Z$  of the device is not accurately represented by the relation shown at all values of  $P$  and  $Z$ . However, the main error involved is at small values of  $Z$ , outside the range of vehicle sinkages at which trouble develops. Thus the relation permits considerable theoretical analyses and prediction, and the development of trends, saving much money and time.



## SOIL VALUES

It has been indicated above that Eq. (1) can be used for the solution of land-locomotion problems provided that the soil values  $k_c$ ,  $k_\phi$ , and  $n$  are known for the soil in question. There is one other approach to the subject in which these parameters assume considerable importance: in dimensionless analysis where the object is to use the principles of similitude to employ small-scale tests in the laboratory to predict full-scale vehicle performance. In other words, the approach is to employ in land locomotion the equivalent of wind-tunnel tests of airplanes, a technique which has been employed for many years to check drag, stability, etc., of a model of the plane before its actual construction.

Such similitude studies of land-locomotion problems have recently been made.<sup>2-4</sup> To take complete advantage of these studies, with respect to their application to full-scale wheeled vehicles, examination of the relationships given below is necessary.

$$k_c = c_1 d k_\phi \quad (2)$$

$$W = c_2 k_\phi d^{n+2} \quad (3)$$

$$D = c_3 d \quad (4)$$

These relationships should be met if the model rules are to be satisfied and complete similitude obtained under the conditions assumed in Ref. 2. Soil-value quantities must change as the scale of the model changes. Fortunately for the studies reported in Ref. (2), the material employed was sand for which  $k_c = 0$  and useful work was accomplished without control of  $k_c$  and  $k_\phi$ .

To extend the studies to soils possessing values for both  $k_c$  and  $k_\phi$  it follows that means must be found to vary, independently,  $k_c$ ,  $k_\phi$ , and  $n$  if at all possible. The studies reported here, conducted with materials, other than natural soils, were aimed at obtaining sufficient variation in the laboratory soil-system parameters to permit the construction of dimensionless graphs from model tests, over a range of values which allow application to the range of wheel sizes currently used. To achieve the complete natural range of values of  $k_c$ ,  $k_\phi$ , and  $n$ , it should be possible to prepare soils or their equivalent that would permit:

$$k_c \text{ to vary from } 0 \text{ to } \geq 20.0$$

$$k_\phi \text{ to vary from } 0 \text{ to } \geq 30.0$$

$$n \text{ to vary from } 0.5 \text{ to } \geq 3.0$$

At the same time these soils must maintain their properties constant while in contact with the varying average laboratory atmosphere for extended periods of time.

Research conducted at the Detroit Tank Arsenal Land Locomotion Laboratory on artificial soils<sup>5</sup> provides a limited approach to the problem, but independent variation of  $k_c$ ,  $k_\phi$ , and  $n$  would be difficult to achieve by this method alone.

The experiments to be described represent a further attempt to produce stable soils accompanied with at least a moderate independent variation in both  $k_c$  and  $k_\phi$  and their ratio of the materials, using the Bekker Sinkage Parameters.

## THE LABORATORY SOIL

Various possible laboratory soils composed of clay, sand, and glycol are described in Ref. 5. Various other possible soil ingredients were also proposed but not used, some of which are employed in the research now under discussion.

The ideal soil mass, as defined in Ref. 5, should have the following characteristics:

- (a) Stability of the solid state regardless of time, oxidation, or other chemical interaction.
- (b) Controllability of the soil values and parameters.
- (c) Reproducibility of the mass in any quantity using standard or easily obtained materials.

The materials employed at the Arsenal satisfy these requirements to a high degree, but the glycol is also hygroscopic, and thus will pick up moisture from the atmosphere. The properties of the soil can change somewhat with time, particularly those mixtures representing viscous muds where high glycol contents are employed. Figure 1 shows the three-phase diagram of clay, glycol and water, plainly indicating the wide range of soil properties that could be expected from any one blended combination of clay and glycol, given sufficient time and atmospheric humidity. Fortunately, with proper organization the test data can be secured before such time elapse occurs.

The importance of moisture content in a given soil (an average sandy farm soil of Wayne County, Michigan) can be seen from Table I.<sup>6</sup>

TABLE I  
EFFECT OF MOISTURE CONTENT ON SOIL PROPERTIES

Moisture, %	Soil Properties				
	$\phi$ , deg	c	$k\phi$	$k_c$	n
19	36	0.60	9.00	20.00	0.16
20	38	0.53	7.00	16.00	0.17
22	36	0.25	2.20	2.50	0.18

It follows that stable soils under all conditions of time, temperature, and humidity are necessary, or else the magnitudes of the soil parameters must be determined at each testing.

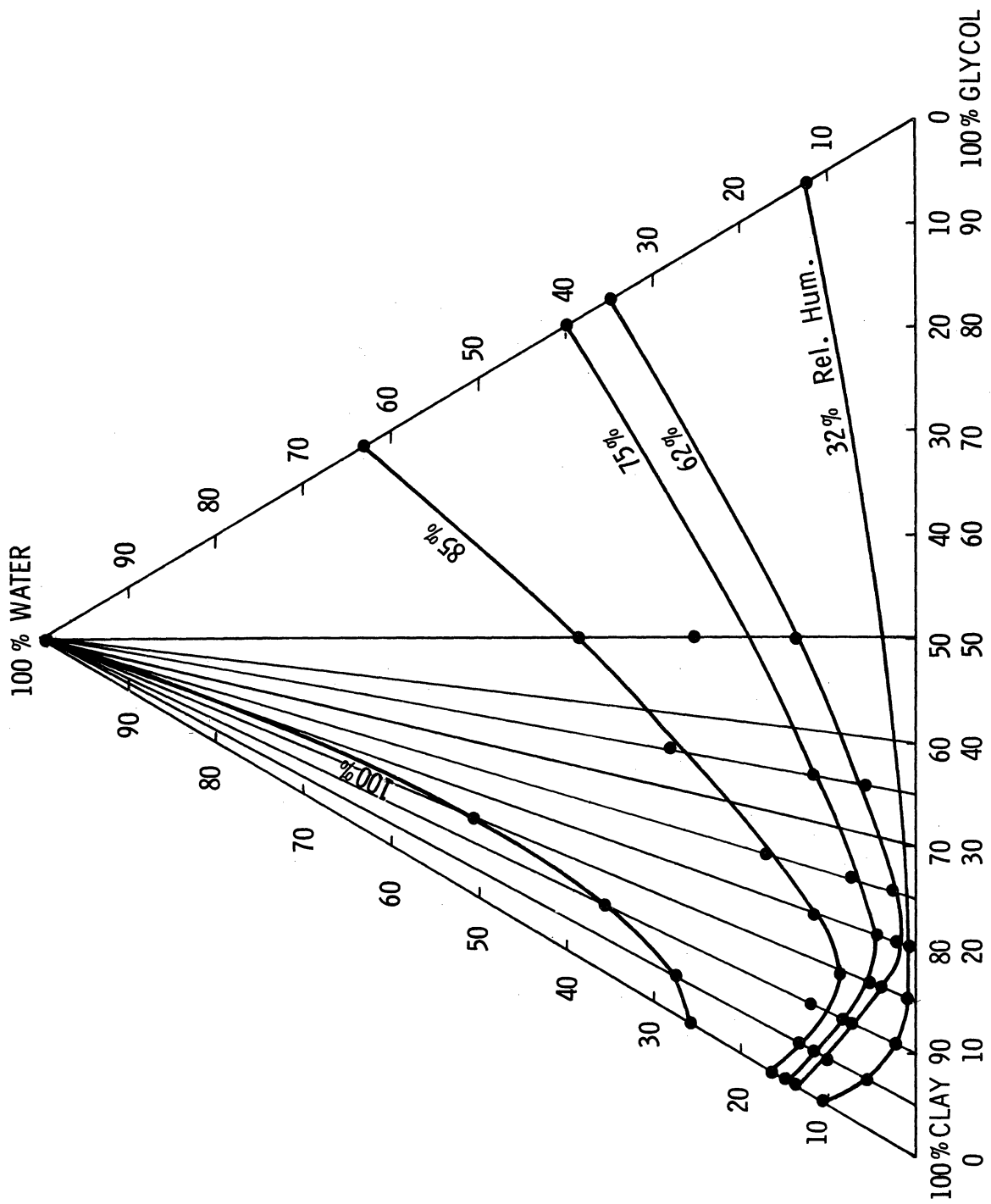


Fig. 1. Composition of the 3-component substance (water, clay, glycol) in equilibrium with humid air at 75°F, 1 atmos. pressure.

The research on the value of the soil parameters, covered by this report, can be considered an investigation of the manner in which the individual parameters  $k_c$ ,  $k_\phi$ , etc., can be controlled and varied by other means than the variation of moisture content or glycol constituent. Variations are introduced by controlling the shape of soil particle, the size of particle, or its cohesive force as measured by its degree of magnetization, i.e., the effect of artificial materials on the control of soil parameters is examined as set out below.

## MAGNETIC SOILS

As already pointed out, it is very desirable to be able to vary  $k_c$  without material change of  $k_\phi$ , if the relationship given by Eq. (2) is to be satisfied for complete fulfillment of the model rules. At least the ratio of  $k_c$  and  $k_\phi$  must be variable over a wide range, even if individual variation proves impossible.

In considering this problem, it occurred to us that a material capable of being magnetized might present a variable  $k_c$  with little change in  $k_\phi$ ; in other words varying magnetic attraction between the particles might represent a soil with variable cohesion. To this end, a supply of barium ferrite was obtained from the D. M. Steward Manufacturing Company of Chatanooga, Tennessee, which gave the properties, listed in Table II, of the material when magnetized.

TABLE II

### TYPICAL PROPERTIES OF BARIUM FERRITE

Residual Induction, $B_r$ (gausses)	2190
Coercive Force, $H_c$ (oersteds)	1850
Intrinsic Coercive Force, $H_c$ (oersteds)	3450
Maximum Energy Product, $B_d H_d$	$1.03 \times 10^6$
Permeance Coefficient, $B_m/H_m$ at $B_d H_d$ max	1.160
Temperature Coefficient of Residual Induction	0.18%/°C
Coefficient of Linear Thermal Expansion	$10 \times 10^{-6}/°C$
Curie Temperature (°C)	450
Apparent Density	4.70 gr/cm <sup>3</sup>
Electrical Resistivity (ohm/cm at 25°C)	$10 \times 10^8$

Samples of this material in the nonmagnetized state show very little cohesion. At the same time the angle of repose ( $\phi$ ) is extremely small; a sample spreads out over a wide area and has little depth even in the center. When magnetized, it has considerable magnetic attraction which appears to have the same effect on a mass as cohesion. At the same time such a mass will form into a distinct cone-like mound having a definite angle of repose compared to the

nonmagnetic variety, which might indicate a change in  $k_\phi$  also. It follows that variation of the degree of magnetization may produce a reasonably wide variation of  $k_c$ ,  $k_\phi$ , and  $n$ . An additional advantage is relative economy, as compared with glycol.

#### GLASS BEADS

One other material considered for the control of soil properties was spherical glass beads, with an average diameter of 0.010 to 0.015 in. Thus in particle size the beads are approximately in the same range as the sand particles in use; but by themselves they have no value of  $\phi$ , the angle of repose, since, being almost perfect spheres, they will roll out to a single layer, provided they are dry and not mixed with oil or other viscous or cohesive binder. But if they are mixed with the sand employed in the routine tests, the frictional characteristics of the soil may change sufficiently to be of practical importance. The inclusion in plastic soils with or without the sand of Ref. 5 could also contribute to a possible increased range of  $k_c$ ,  $k_\phi$ , etc.

To study these effects, bevameter tests were run with a series of both circular and rectangular plates, with the object of obtaining the range of soil properties that could be produced by the use of the materials being considered.



## TEST APPARATUS

To obtain soil properties with a high degree of precision, and thus obtain closely reproducible test results, a special penetrometer had been developed at this laboratory. This equipment removed much of the scatter and many of the discontinuities of previous bevameters employed. The equipment consists of an hydraulically operated plunger to which the penetrating foot is attached, held in place by a sensitive ring gage to which strain gages are attached to record the load on the foot. Displacement of the foot is recorded by a linear potentiometer driven by the plunger movement. This equipment is similar to many others employed for the purpose in question. The differences are (1) the size, and thus the sensitivity of the ring; and (2) the supply of fluid to the plunger is via a constant-flow control valve, permitting accurate control of the plunger velocity regardless of the loads applied. This appears to be responsible for the absence of steps in the curves. The pressure-sinkage data are plotted directly on an Autograf X-Y recorder.

Figure 2 is a diagram of the system. The electrical circuit is shown in Fig. 3. The feet employed for the plunger are shown in Fig. 4. The size of the box containing the soil was considered large enough to render the boundary effects of both bottom and sides negligible as judged by the tests of Refs. 2 and 7. The soil can thus be considered homogeneous with semi-infinite boundary conditions.

Figures 5 and 6 show the type of data recorded for a sand and barium ferrite, respectively. The material was well stirred between runs. Several runs are superimposed on one another, indicating the degree of reproducibility, particularly when allowance is made for the difficulty of accurately determining the position of zero sinkage for the start of the graft.

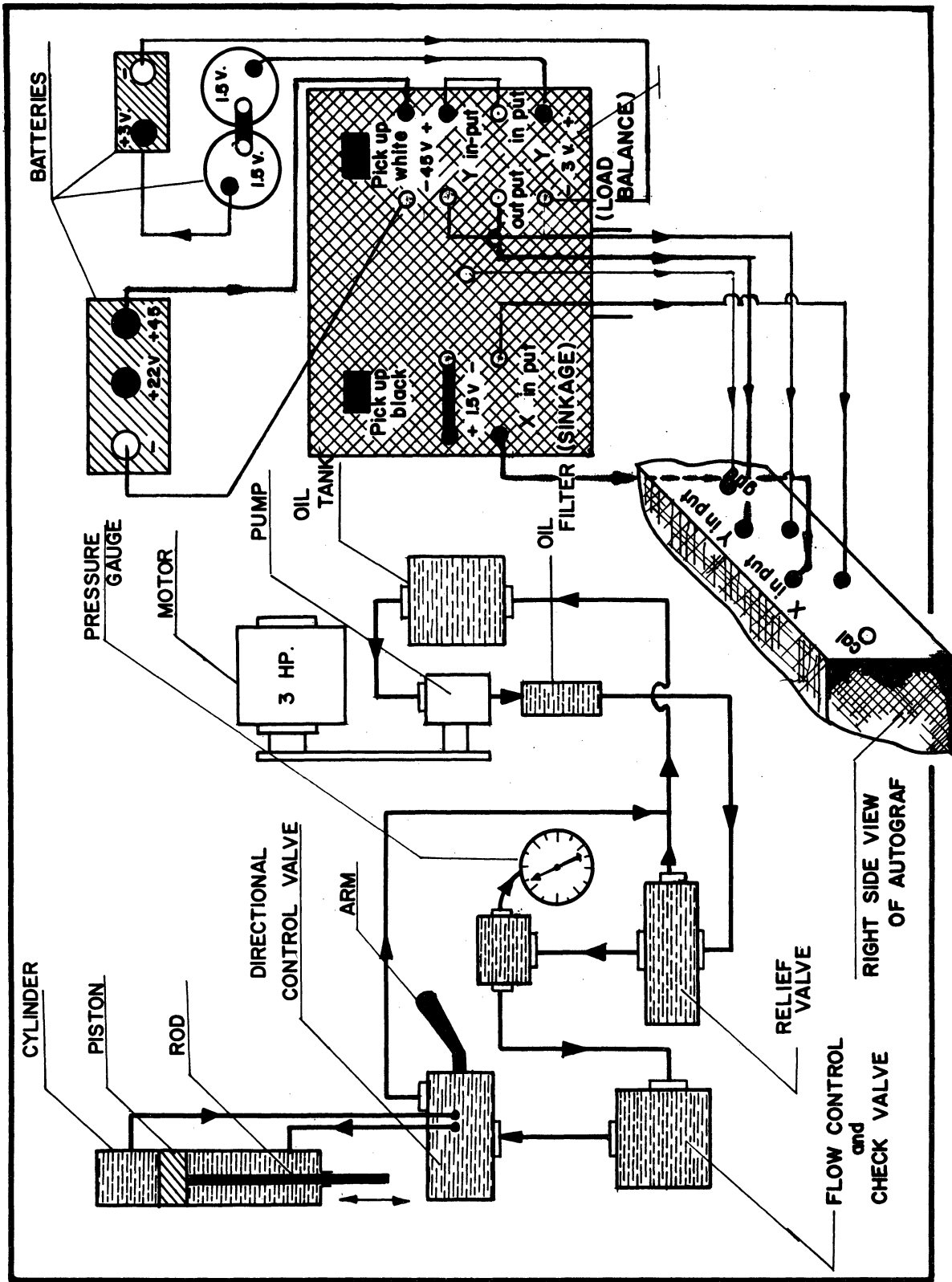


Fig. 2. Diagram of test apparatus.

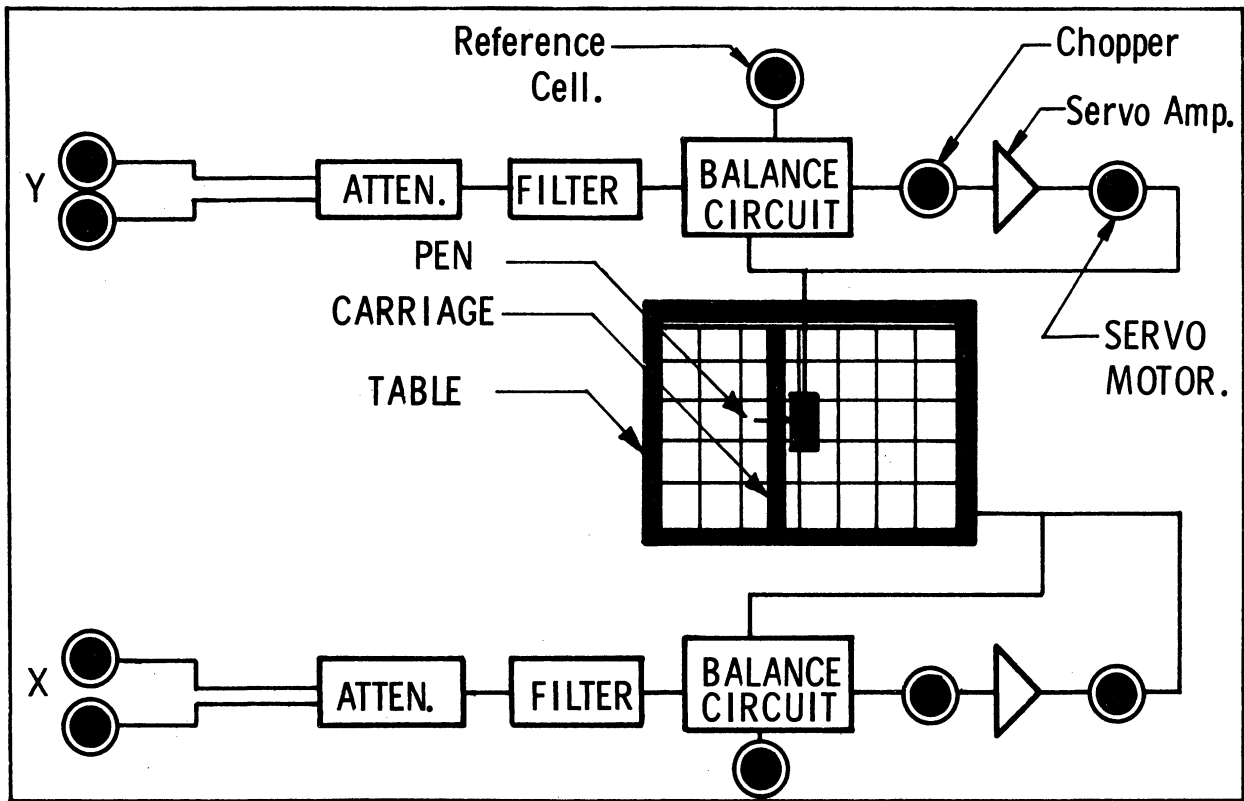


Fig. 3. Electrical circuit.

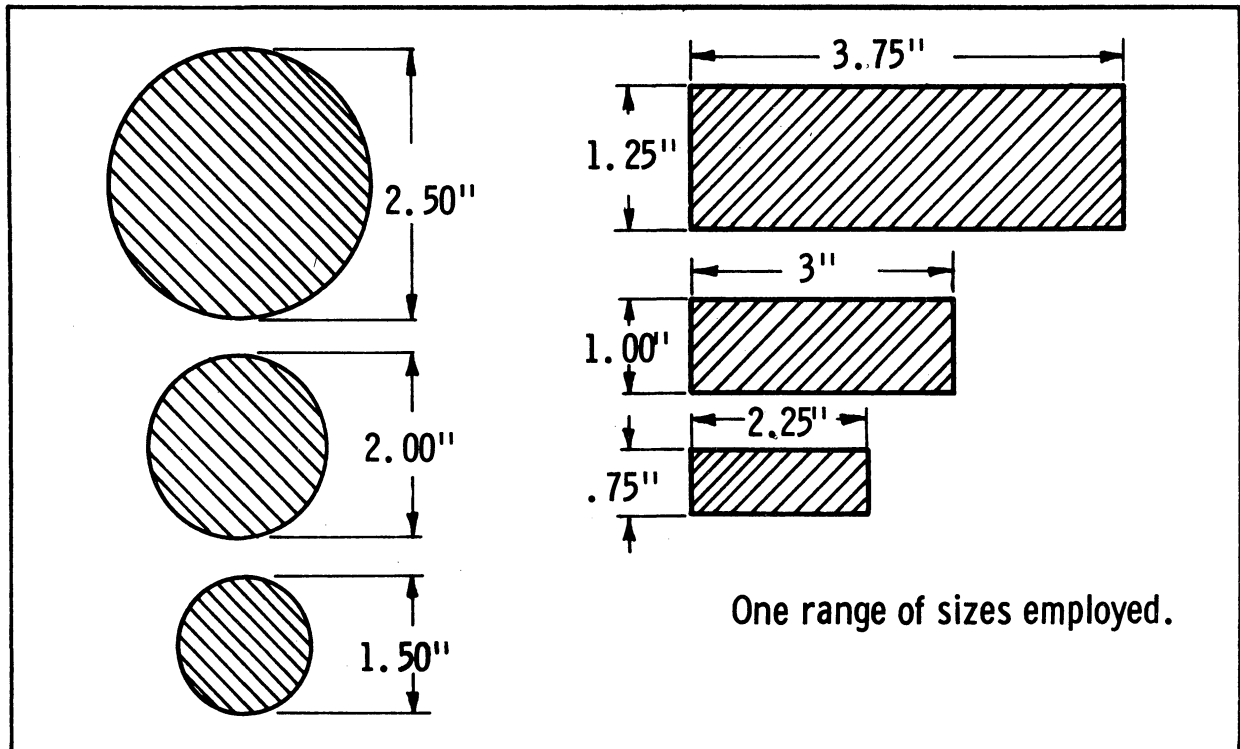


Fig. 4. One range of sizes of feet employed for plunger.

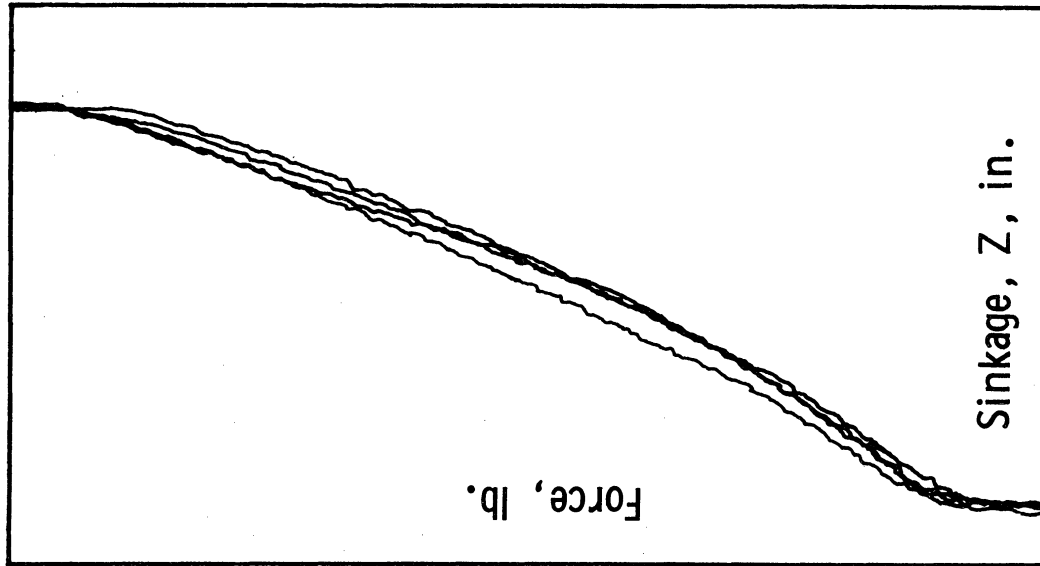


Fig. 5. Pressure and sinkage relationship of sand with stirring.

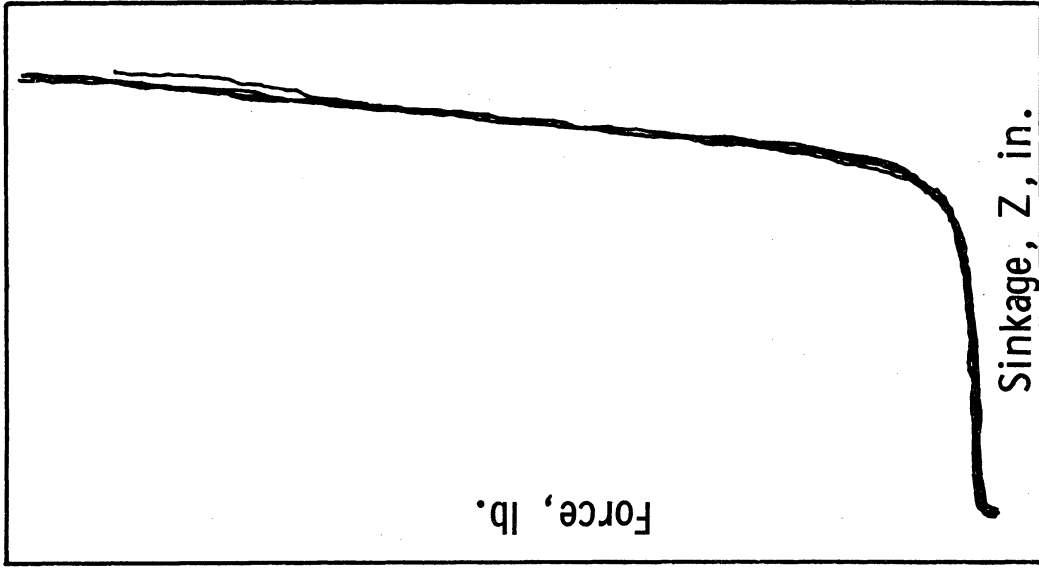


Fig. 6. Pressure and sinkage relationship of nonmagnetic barium ferrite with stirring.

## PROCEDURE

In all soil testing with a bevameter, there is one great difficulty in duplicating results, particularly when a relatively small sample of the soil, contained in a box of modest size, is concerned: assuring that the material is in exactly the same state as regards humidity, compaction, etc., at the start of each test.

The barium ferrite employed was a rather fine powder and thus did not lend itself well to the air-lift method of reconditioning previously employed successfully; the air flow through this finely powdered soil deposited most of it outside the container. After considerable testing the method of "pouring" was chosen as quite acceptable for the small volumes involved in the tests. This consists of pouring the used sample into a second container and then pouring it back into the test container. This flow process removed stress and compaction satisfactorily.

The container was then placed on the bevameter table and carefully leveled. The zero setting of the sinkage scale was determined. The test was then carried out with a plunger speed of 6 fpm, which was held constant for all the tests with the ferrite, sand, and glass beads. In each case a series of runs was made for each foot size. These results were then averaged for the points shown in the various graphs of the P-Z relationship.



## RESULTS

The results obtained from the apparatus are typified by the curves of Figs. 5 and 6, which show that the reproducibility of the results is good.

Such force-sinkage curves were then averaged and the results were plotted on log-log paper as shown in Fig. 7. For Eq. (1) to represent the recorded results exactly, the points of Fig. 7 should lie on a straight line. A typical line has been drawn in; although it is not strictly through all the points, a close approximation to the straight line is seen over a portion of the graph, particularly if:

- (1) the first one or two points plotted from such figures as Figs. 5 and 6, are neglected where the sinkage is small, say  $< 0.5$  in.;
- (2) The last few points at high load are also disregarded, where the sinkage is  $> 4-5$  in.

These points can be neglected to a first approximation for the following reasons. (a) It is very difficult to determine the exact point at which the bevameter foot first touches the soil and the point at which pressure is exerted by it on the sand, since the top layer appears to be compressed to some extent and, in some cases, voids are taken up with practically no sinkage. Thus the zero setting of pressure and sinkage has some error; since the first point plotted on curves of the type shown in Fig. 7 has been for  $Z = 0.25$  in., an error of 0.05, which is easily possible in the zero setting, would change the picture a great deal. (b) Secondly, it seemed that the upturn at the upper end of the diagram could be due to boundary effects from the bottom of the containing vessel. This was checked by running bevameter tests with soils of varying thickness relative to the size of the penetrometer. The results are shown in Fig. 8; four depths of a given sand were employed and the P-Z relationships were obtained. The graphs show a high degree of agreement, except at the readings for  $Z = 0.25$  and for  $Z > 4-1/2$  in. approx. The upper part of the curves show that, for a 6-in. depth of soil and a 2-1/2-in. diameter foot or  $r = 1-1/4$  in., departure from the approximate straight line occurs at about  $Z = 3-1/2$  in. or  $a(r/Z-D) = 0.5$  approx., where  $D =$  depth of soil above the hard surface to the penetrating plate at its point of departure from the straight line. For the 9-in. depth, departure occurs at  $Z = 4-1/2$  in. or  $r/Z-D = 0.28$ ; for both the 12- and 14-1/2-in. depths of soils, departure from the straight line occurs at about 5 in. or  $r/Z-D = 0.18$  and 0.3. It is concluded that for a 2-1/2-in.-diam ft, the soil should be at least 12 in. deep. Basing the relationship on the radius of the foot, the required depth of soil below the foot at the maximum penetration should be about five times the dimension "b" of Eq. (1) if depth effect is to be avoided. Since only one size

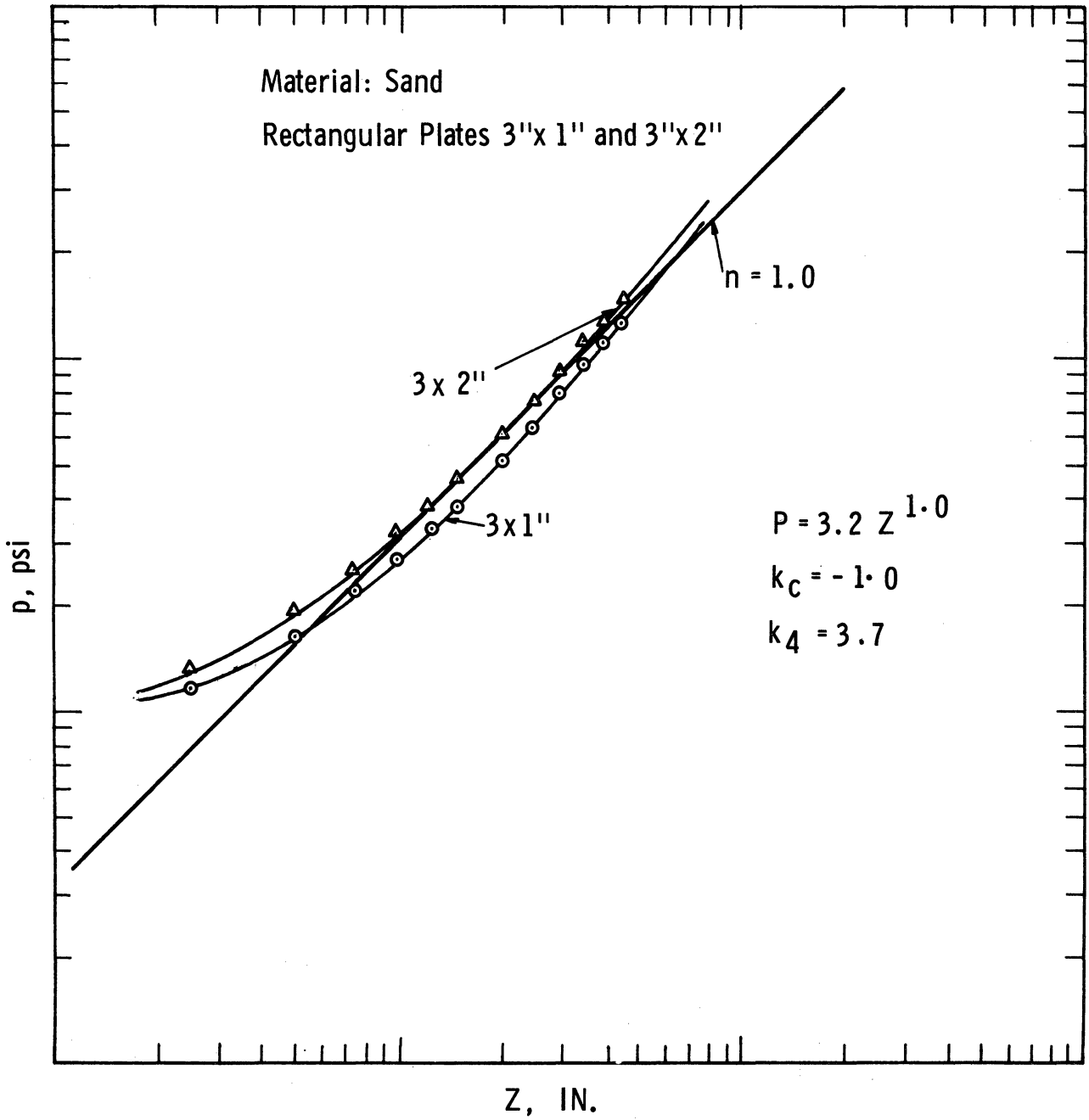


Fig. 7. Pressure vs. sinkage of a flat plate in sand.



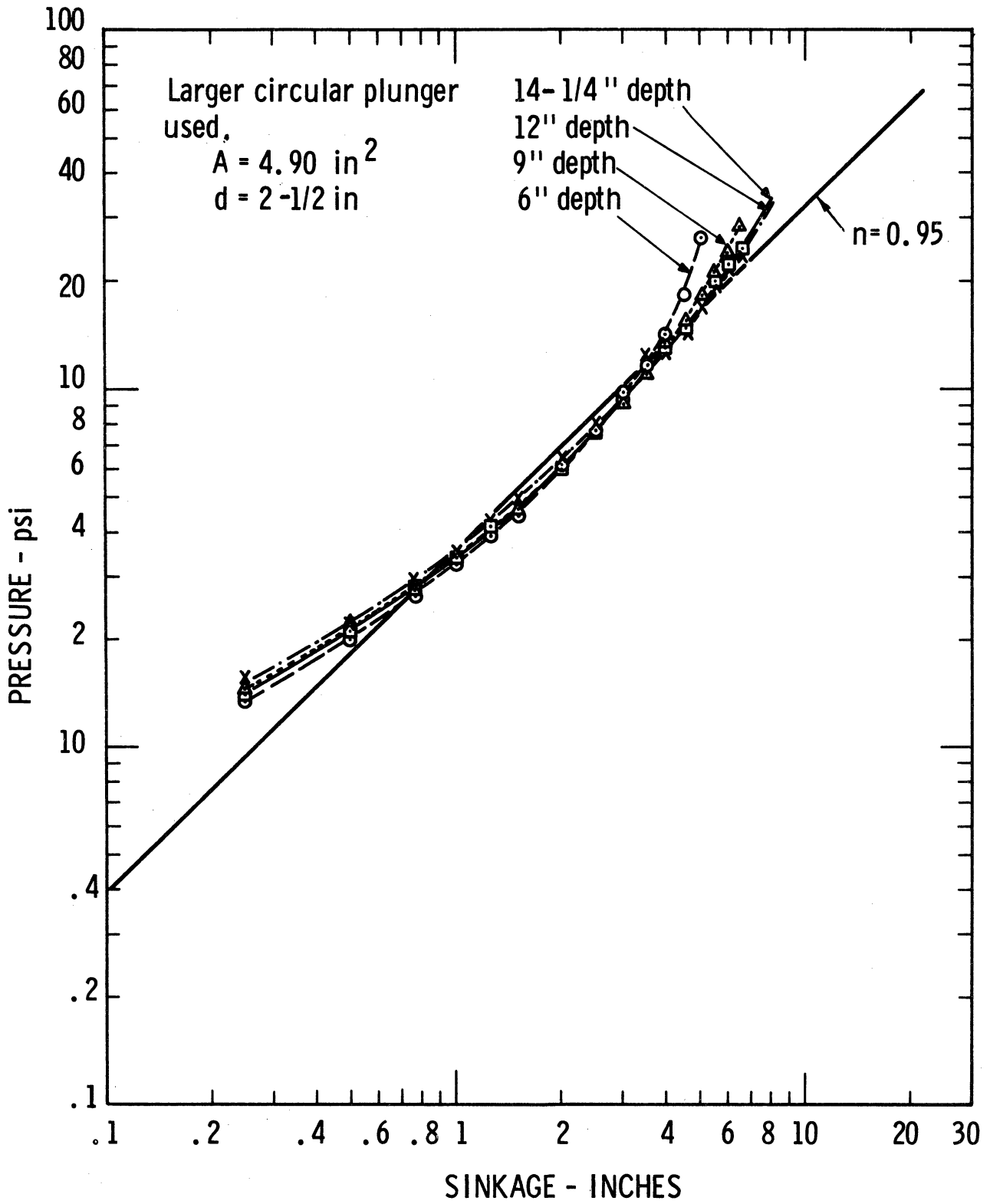


Fig. 8. 100% sand.

of foot was employed for Fig. 8, the values of  $k_c$  and  $k_\phi$  cannot be determined; however, for the sand in question we can write

$$P = 3.5Z^{0.95}$$

which agrees well with other experiments with similar materials.

As a result of the above tests, the analytical relationships between P, Z, and n which follow were all determined by neglecting the value of Z for sinkages  $< 0.5$  and  $> 4$  in. and using a total soil depth of at least 12 in.

With the above limitations, tests with the following materials were carried out:

- (a) Sand
- (b) Nonmagnetic barium ferrite
- (c) Magnetic barium ferrite
- (d) Glass bead of from 0.010 to 0.015 in. in diam (approx.)
- (e) A mixture of 25% glass beads and 75% sand
- (f) A mixture of 50% glass beads and 50% sand
- (g) A mixture of 75% glass beads and 25% sand

All the above materials were dry. It follows that the cohesive force would be expected to be quite small and  $k_c \approx 0.0$ . Tests were made with three sizes of both circular and rectangular footings; theoretically, since  $k_c$  is zero or at least quite small, all the P-Z curves should almost superimpose on one another and cross the line of  $Z = 1$  in. almost at one point. The results substantiate this. Typical plots for some of the materials are shown in Figs. 9 and 10, the former is for the circular plates and the latter for rectangular ones. The soil parameters obtained from all the graphs are as shown in Table III. Substantial agreement exists between the two types of footings. When more than one such set of curves are averaged (as was generally the case), the results tend to be closer than those given here.

The effect of the speed with which the footing penetrated the soil was examined for any change of magnitude of n,  $k_c$ , and  $k_\phi$  that might have resulted. This effect is recorded in Fig. 11. Table IV gives the maximum and minimum relations as calculated.

Since the speed-effect curves for rectangular and circular feet are all recorded with but one size of plate, the individual values of  $k_c$  and  $k_\phi$  cannot be determined in this case. But it is safe to assume  $k_c \approx 0.0$  since this has been determined for this material by other tests.

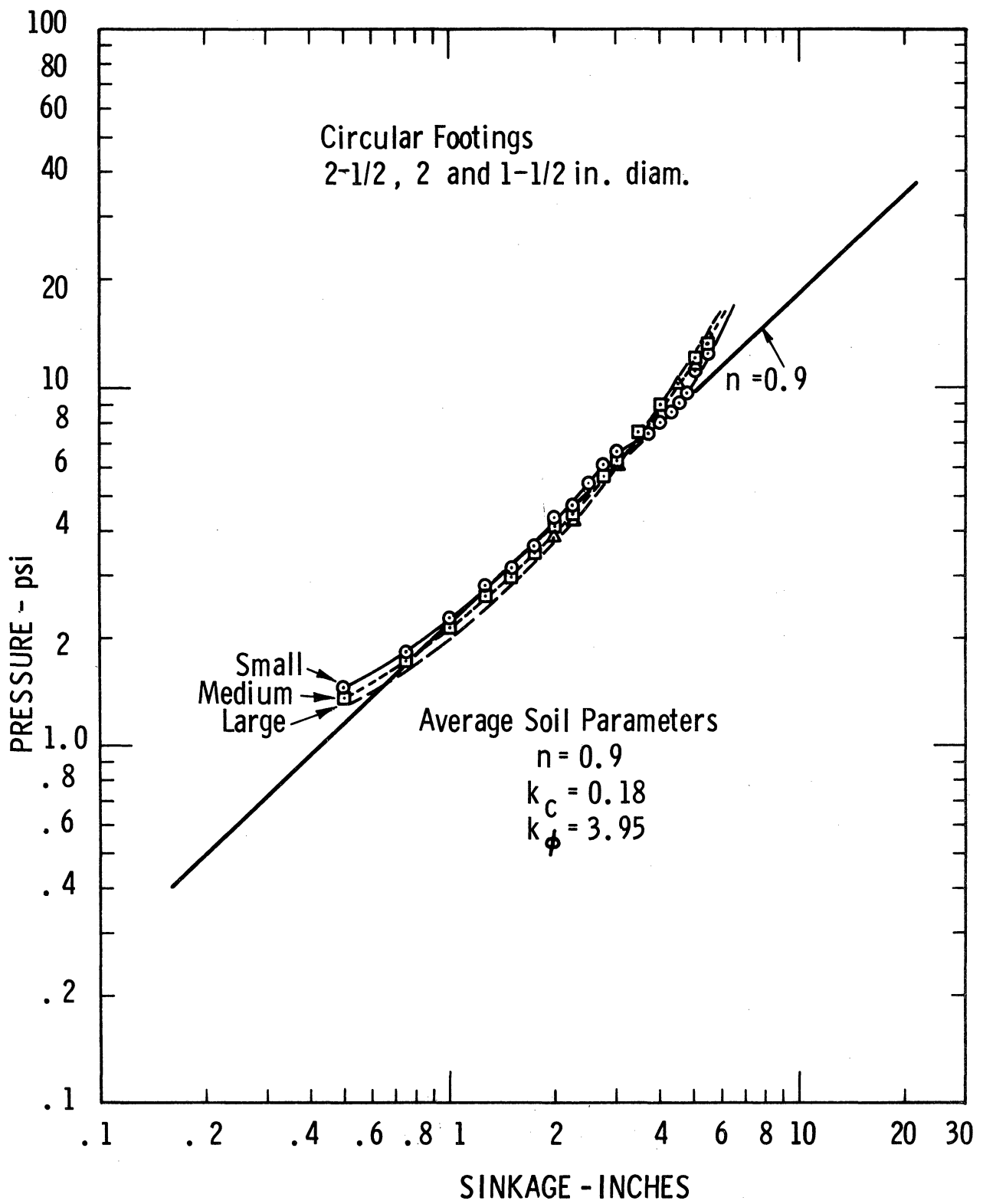


Fig. 9. 100% glass beads, 0.010 to 0.015 in. diam.

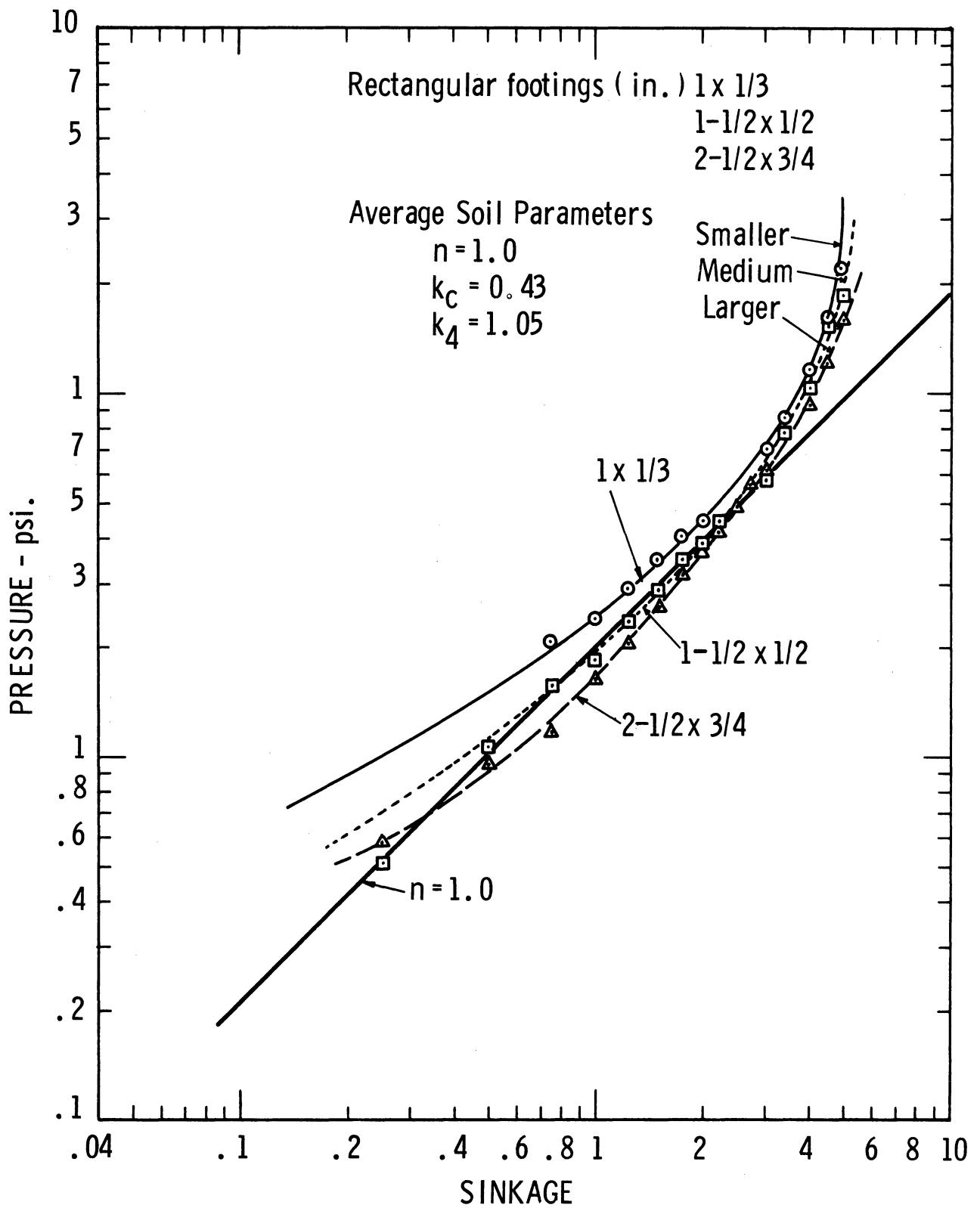


Fig. 10. Poured magnetic barium ferrite.

TABLE III

SOIL PARAMETERS FOR INDICATED MATERIALS,  
OBTAINED WITH CIRCULAR AND RECTANGULAR FOOTINGS

Material	n		$k_c$		$k_\phi$	
	Circular	Rect.	Circular	Rect.	Circular	Rect.
Dry Sand	0.95	0.95-1.0	-0.60	-0.64	3.1	3.3-3.7
100% Glass Beads	0.9-0.91	0.91-1.06	+0.175	+ .182	1.96	1.96
25% Glass Beads and 75% Sand	0.9	0.95	-0.7	-0.53	3.95	3.68
50% Glass Beads and 50% Sand	1.0	1.0	-0.36	-0.48	2.97	3.01
75% Glass Beads and 25% Sand	1.06	1.06	-0.703	-0.461	2.79	2.43
Nonmagnetic Barium Ferrite	0.68	0.65	0.93	1.21	0.37	0.69
Magnetic Barium Ferrite	1.0	1.00	0.34	0.43	1.22	1.05

TABLE IV

CALCULATED MAXIMUM AND MINIMUM RELATIONS  
OF INDICATED PARAMETERS

Speed, fpm	Circular Footing, 2 in. Diam	
	n	$k_\phi$
5.0	0.95	2.9
6.5	0.95	3.0

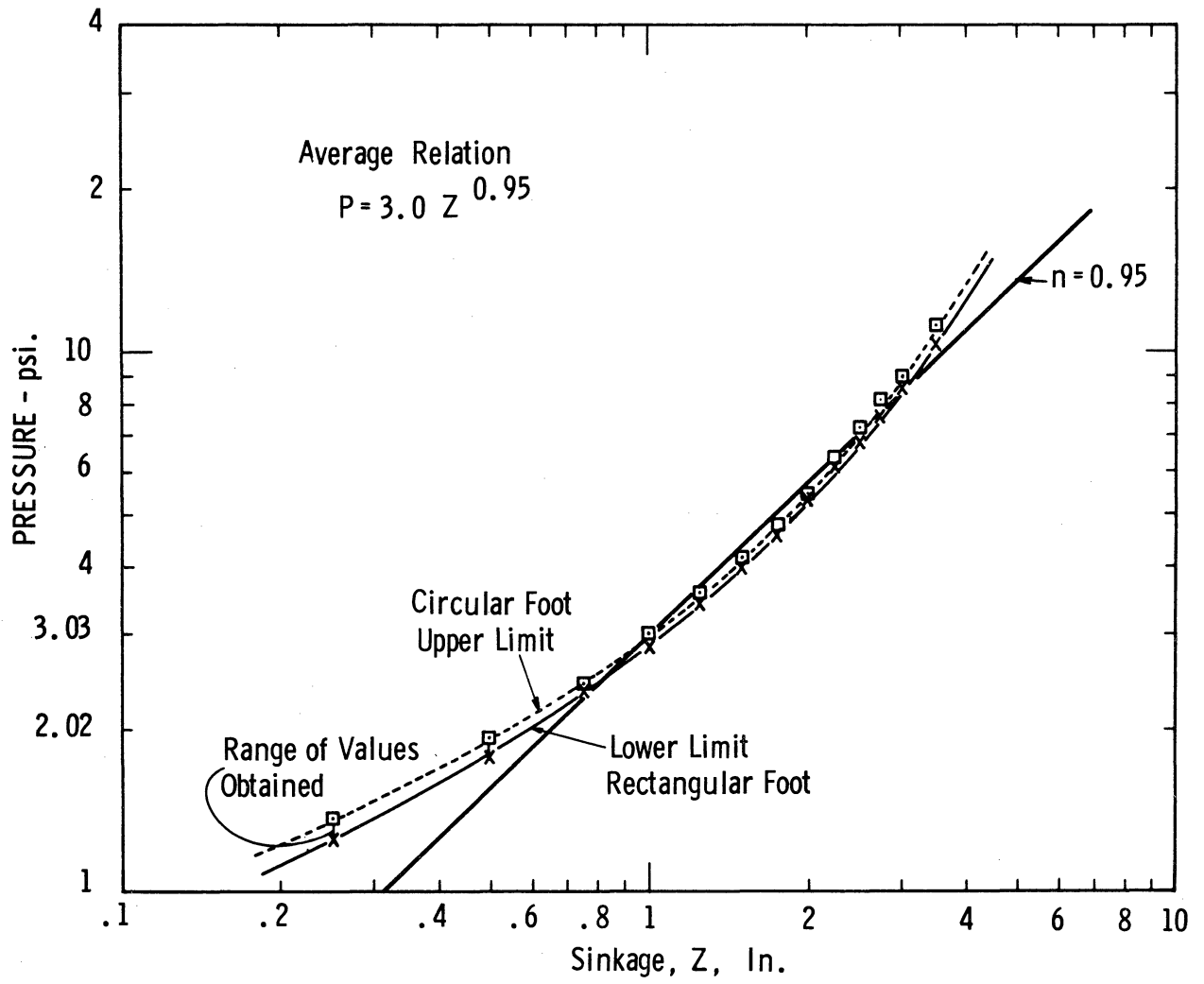


Fig. 11. Effect of speed of penetration in sand.

## DISCUSSION

The values of  $k_c$  and  $k_\phi$  obtained from such diagrams as Figs. 9 and 10 result in  $k_c$  varying from positive to negative for the same material in different tests. This is considered to be because, for the materials in question,  $k_c \approx 0.0$  and small errors in the observed data or even in drawing the curves through the points produce positive or negative values for  $k_c$  in such cases. But it must be admitted that some doubt exists about the correctness of this explanation, due to the accuracy with which the observed data were repeated in subsequent check tests, and the consistently small negative values obtained. In Fig. 12 the actual test points are plotted to show the rather tight grouping of most of the points at their averaged values, plus a few scattered results which in almost all cases still do not change the sign of the coefficients. The values given in Table III result from the averaging of several runs using the straight-line log-log relationship  $P = (k_c/b + k_\phi)Z^n$  in the conventional manner. Employing the appropriate values from Table III in this formula and calculating the pressure and sinkage values, the calculated points do fit in very well with the mean plotted values. It is concluded that no major error in  $k_c$  and  $k_\phi$  exists by employing the average of a number of runs with small, medium, and large plates, at least for  $Z$  between 0.7 and 4.5 in.

Table III indicates that for the range of materials examined change in the value of "n" for all these dry, large-particle materials is negligible. But for the fine powdery barium ferrite this parameter has been reduced to  $n = 0.68$ , which, when magnetized, is re-established at 1.0 as for the other materials.

### VALUES OF $k_c$

The object of the tests was to produce an independent variation in the magnitude of  $k_c$ . The average data indicate a possible variation of its magnitude from -0.7 to +0.93 for circular plates and -0.53 to +1.21 for rectangular ones.

The variation produced in  $k_c$  is small as is to be expected since all the materials, with the exception of the barium ferrite, were granular, with little if any cohesion. A cohesive coefficient of 0.9 to 1.2 was recorded with the magnetic barium ferrite.

In general, it was observed of all the test results that there was less variation between tests for rectangular plates than for circular ones.

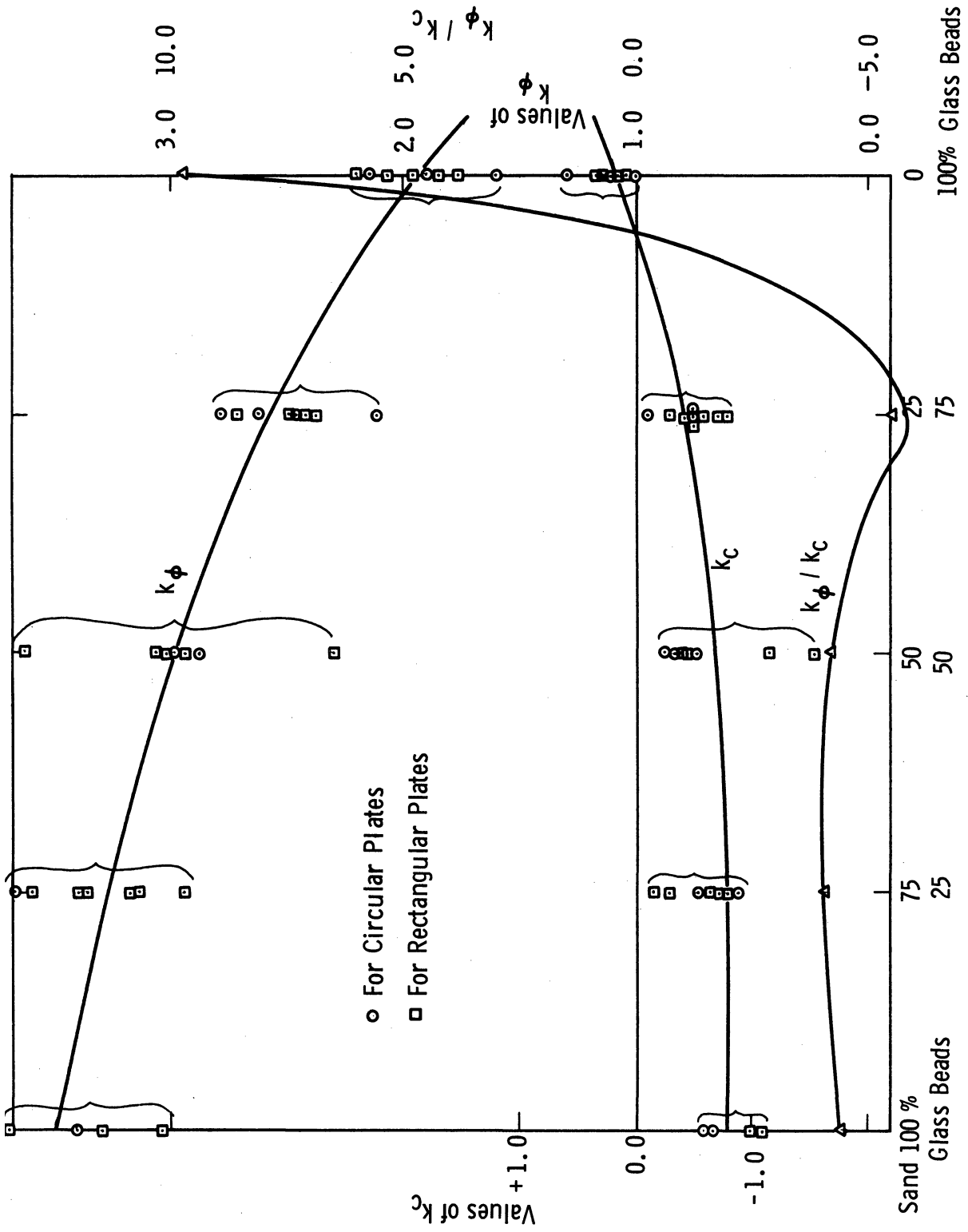


Fig. 12. Averaged values of actual test points.



## VALUES OF $k_\phi$

The corresponding variation of  $k_\phi$  is from 0.37 to 3.95 for circular plates and from 0.69 to 3.7 for rectangular ones. Here a significant change of values has been achieved. Taking the sand and bead combinations only, where for sand  $k_\phi = 3.1$  to 3.7 and for beads  $k_\phi = 1.96$ , a variation of about 1.8:1 is seen, a range of considerable interest as far as similitude study is concerned. If the ferrite is included, a range from 0.5 to 3.5 or 7:1 is possible.

## VALUE OF $n$

Table III shows that the value of " $n$ " in Eq. (1) changes from 0.65 to 1.06. If the nonmagnetic barium ferrite is neglected, the range is from 0.9 to 1.06. In other words, " $n$ " for the materials so far tested is approximately constant at  $n = 1$ .

## GENERAL

The analysis so far has been based on the fact that Eq. (1) represents the relationship between  $P$  and  $Z$  with sufficient accuracy for a soil. This equation is an approximation of the results obtained by soil-penetration tests in the manner described; yielding important results for an engineering approach to the problem of soil-vehicle relationships. The relationship employed departs from the test results mainly at very low sinkage of little importance in practical problems.

The question arises what, if anything, could be added to Eq. (1) to lend a more scientific approach to the whole load-sinkage relationship. In parallel work at The University of Michigan,<sup>8</sup> the equation

$$P = C + (k_c/b + k_\phi)Z^n$$

was suggested as a refinement of the Bekker relationship, in which the magnitude of " $C$ " depends on the width " $b$ " of the plate employed in the tests.

The addition of such a constant " $C$ " does not raise any particular problem for the soil-value-system parameters, but it would seriously complicate the equations for wheel sinkage, etc., deduced from its use in any theoretical considerations. The effect of general soil stress state on the bevameter parameters, also examined in Ref. 8, should also result in some further modification of Eq. (1) if bevameter tests are to be represented completely.

Turning again to the main problem of this report, it may be an advantage to list the similitude relationships in a slightly different form. If  $W$ ,  $D$ ,  $d$ , and  $L$  are considered the independent variables and  $Z$  and  $R$  the dependent ones,

where

- W = Load on wheel, lb
- D = Depth of soil above hardpan, in.
- d = diameter of wheel, in.
- $\alpha$  = aspect ratio of wheel
- Z = Depth of sinkage, in.
- R = Drag of wheel, lb,

then for similitude the following conditions must be met:

$$\frac{k_{\phi}}{k_c} d = \text{Constant} \quad (5)$$

$$\frac{W}{k_{\phi} d^{n+2}} = \text{Constant} \quad (6)$$

$$\frac{D}{d} = \text{Constant} \quad (7)$$

When  $k_c \approx 0$ , Eq. (5) disappears and thus, for sand where  $k_c = 0$ , model work is a possibility by the use of Eqs. (6) and (7). It follows that the main importance of the present test program arises as soon as  $k_c$  has some magnitude. Then to provide a sufficient band of model tests to cover all practicable wheels, as  $d$  is varied the ratio  $k_{\phi}/k_c$  must also vary if true similitude is desired for all cases.

The work to date has shown that there are some possible ways in which the cohesive and deformation moduli of the Bekker soil-value system can be varied independently, at least over a moderate range of values. The important feature is the ratio of the two [see Eq. (5)]. Figure 13 shows the range of  $k_c$  and  $k_{\phi}$ , together with values of their ratio for the magnetic soil (the degree of magnetization is unknown). For the sand and glass bead combination, the value of  $k_c$  is so close to zero, actually varying from positive to negative values, that it is believed that the ratio of the parameters loses some of its significance. But the variation of  $k_{\phi}$  is a reasonable amount and could have a significant effect in extending the range of similitude testing for full scale and models, particularly when the limitation arising from Eq. (5) is taken into account for a given soil bin with a fixed value of  $D$ , the soil depth.

If the curve of Fig. 12 for  $k_c$  is taken as drawn, then the ratio  $k_{\phi}/k_c$  varies from -4.4 for pure sand to +9.7 for glass beads. The major change occurs as the composition approaches 100% glass beads.

Typical load-sinkage curves with various sizes of rectangular plates are shown in Figs. 14a, 14b, and 14c. The initial portion of the record consists of an almost vertical rise in load with a negligible deformation. Examining the initial part of the curves on the basis of the pressure per unit area under the plate, one obtains the data of Table V. An approximately constant stress

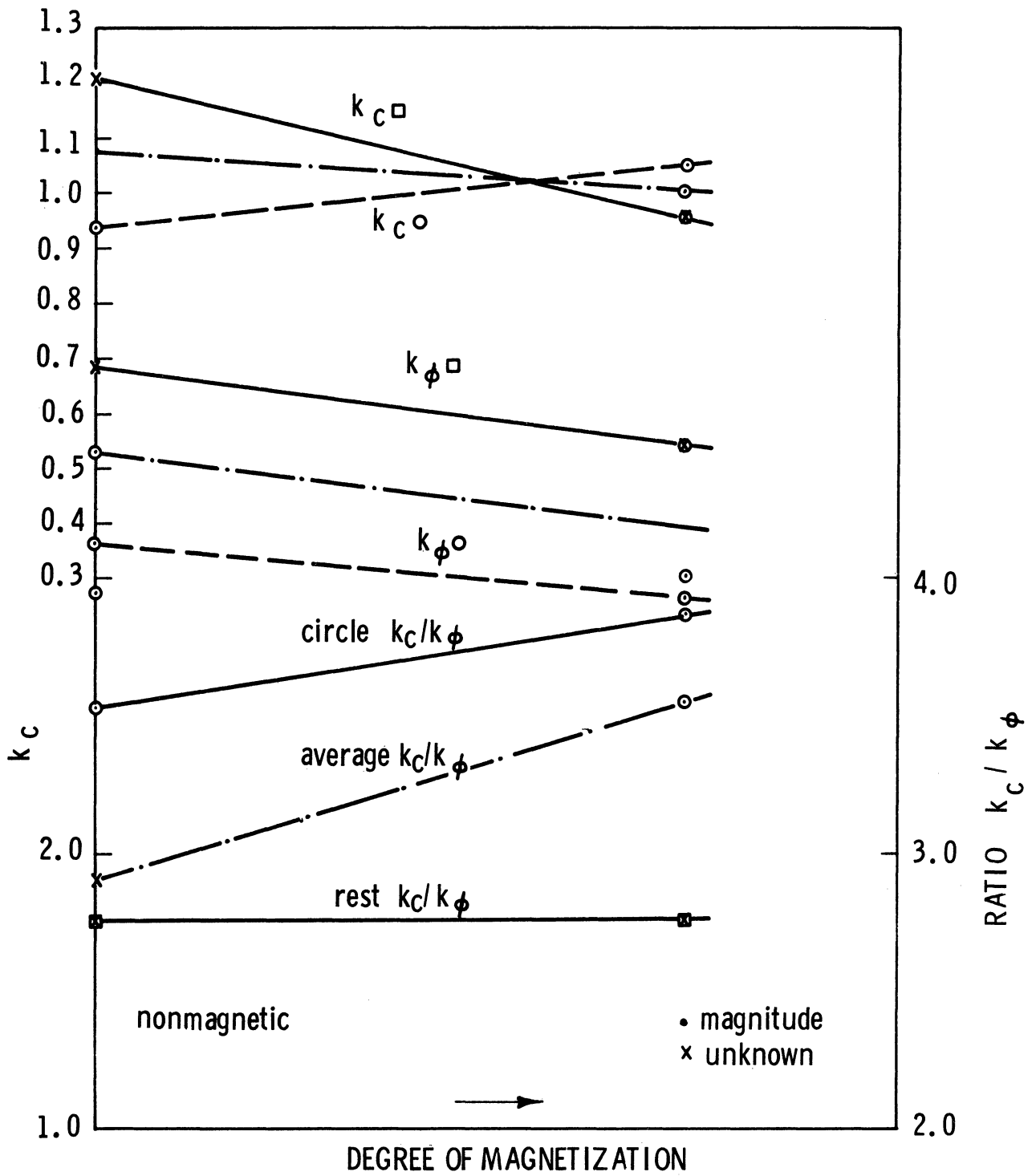


Fig. 13. Magnetic barium ferrite.

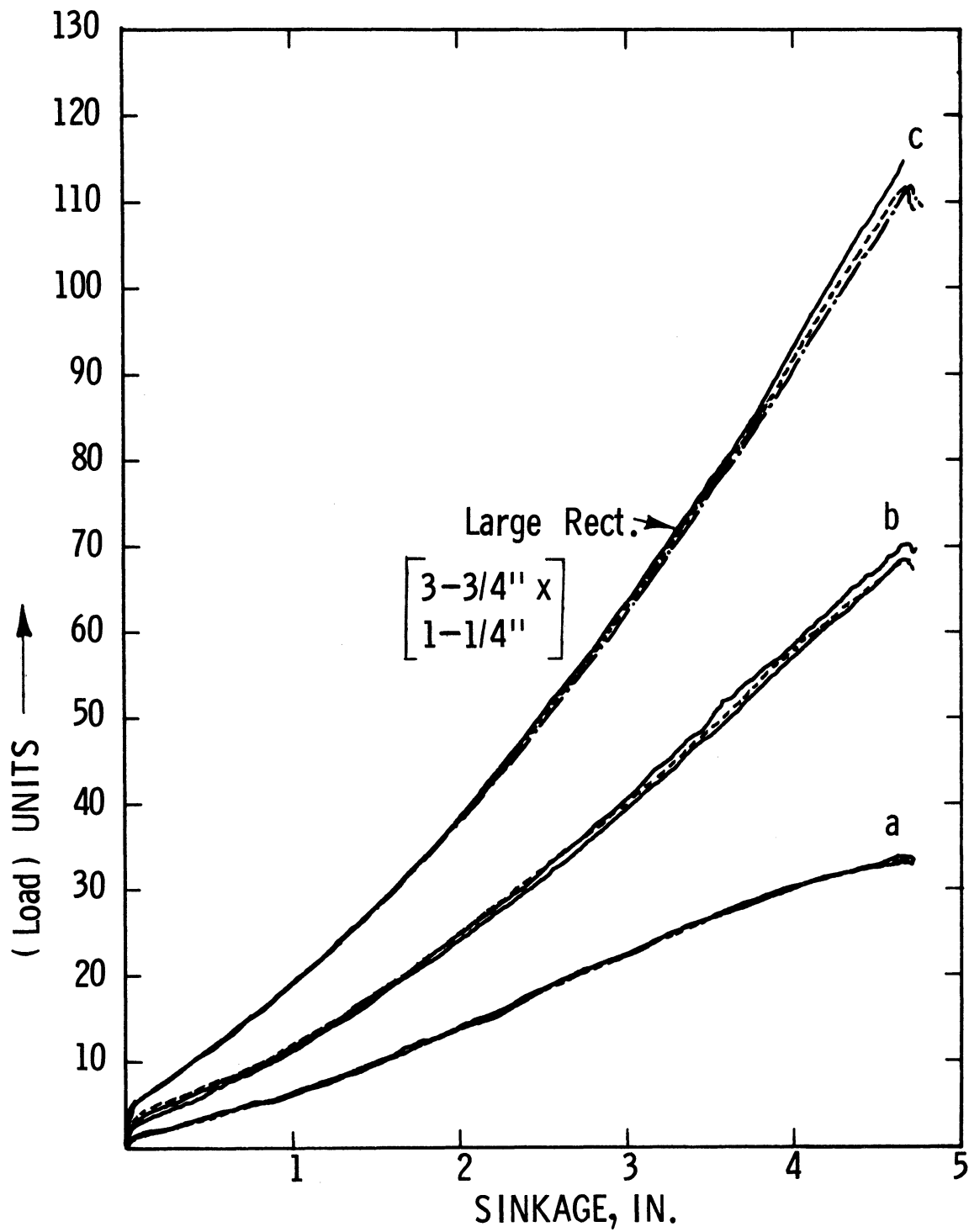


Fig. 14. Typical load-sinkage curves.

TABLE V

MATERIAL: 75% GLASS BEADS, 25% SAND

Plate and Area sq in.	Load, lb	Penetration, Z, in.	Stress, psi
2-1/4" x 3/4" 1.687 sq in.	1.2	Too small to read	0.71
2-1/2" diam 1.767 sq in.	1.5	Too small to read	0.85
3" x 1" 3 sq in.	2.7	.015	0.90
2" diam 3.14 sq in.	2.5	.02"	0.80
3-3/4" x 1-1/4" 4.675 sq in.	4.7	.02"	0.99
2-1/2" diam 4.906 sq in.	5.0	.02"	1.02

under the various plates is seen. The exact process occurring during this phase of the load-sinkage curve is difficult to establish. In Ref. 8 the initial visible effect is claimed to be a compaction accompanied with a gradual increase in the boundary-layer development; however, this definitely involves a displacement which according to Fig. 14 has not yet occurred. It follows that the very first load increase must take place with an almost negligible sinkage, perhaps as follows:

- (a) The sand is left in some state of particle arrangement and density for some depth by the soil-conditioning process employed.
- (b) When load is applied, no penetration occurs until a force is exerted sufficient to cause the uppermost sand particles to reorientate themselves against the friction between particles, and to slide or shear on one another an insignificantly small amount to produce a more or less rigid solid surface in contact with the penetrating plate on one side and the sand at greater depths on the other.
- (c) Further increase in load produces an increased boundary layer attached to the plate accompanied by some small measurable sinkage resulting

from further compaction and slip with the start of flow away from the stressed region to places of no stress, most probably toward the sand surface as shown in Ref. 8.

- (d) When the attached boundary has been completely formed to the penetrating plate, the remaining operation is mainly one of flow from below the boundary layer outward and upward.

The result is the curves as shown in Fig. 14, with little or no sinkage for a given increase in load, followed by a fairly rapid increase in sinkage for a further moderate increase in load, and finally a more or less constant functional change of P and Z approximating Eq. (1).

One other explanation could be the assumption of an initial elastic phase where the modulus of compression would have to be of very considerable magnitude. This does not seem to fit all the observations.

That this initial change of stress without sinkage is in some manner associated with the material of the soil is established by comparison of Figs. 5 and 6 (the latter showing the load-sinkage curve for nonmagnetic barium ferrite). For this material, due to its finely powdered state, there is little or no need to apply load of much magnitude to reorient the particles and produce a flow. Also due to the fine nature of the material and its powdered condition, its density is probably below that of its natural compacted state, that is, unless it has been consolidated. It follows that the first portion of Fig. 6 shows a very small vertical rise followed by sinkage, producing mainly compaction. With little or no applied pressure, some of this sinkage is definitely due to flow of the finely powdered loose surface material; how much, is difficult to say. It is seen that after some consolidation the load-sinkage curve does assume the normal shape. The great difference between Figs. 5 and 6 establishes that the soil itself and its preparation does control the resulting curves obtained in bevameter tests.

## CONCLUSIONS

As a result of analysis of the test data herein reported, it can be concluded that:

1. It is possible to control the values of  $k_c$  and  $k_\phi$  with some degree of independence over a range of values that should prove useful for similitude testing when employing frictional materials.
2. The value of the ratio  $k_\phi/k_c$  can be varied from approximately -4.0 to +10.0 by the use of various combinations of sand and glass beads of approximately the same particle size.
3. By the use of barium ferrite in the nonmagnetized and magnetized states, the values of  $k_c$  and  $k_\phi$  can be moderately varied; however, the ratio  $k_\phi/k_c$  varies only from 0.28 to 0.34 in the process.
4. Additional tests with materials having greater magnitudes of  $k_c$  are desirable to evaluate fully the usefulness of the methods employed.
5. To cover the total possible variation of the parameters and their ratios, further test work should be undertaken, such as:
  - (a) variation of bead size in relation to sand particles;
  - (b) variation (and measurement) of the degree of magnetization;
  - (c) combinations of these materials with clays and loams, etc.





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