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ANALYSIS OF THE ANISOTROPIC PLASTIC DEFORMATION OF MANGANESE SULFIDE INCLUSIONS

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Abstract:

The negative "plastic Poisson's ratio" suggested in earlier studies of MnS deformation is analyzed, and the results are verified experimentally.

In the early stages of deformation where the primary $\{110\}\langle 1\bar{1}0\rangle$ slip predominates, compression along the $[011]$ direction produces contraction along the lateral $[0\bar{1}1]$ direction plus a compensating expansion along the $[100]$ direction. With additional strain, the primary slip is supplemented by secondary $\{111\}\langle 1\bar{1}0\rangle$ slip so that the above contraction diminishes and a more typical deformation appears.

INTRODUCTION

This report is a direct outgrowth of an earlier investigation of the deformation of manganese sulfide inclusions in steel,[1, 2] in which an unusual contraction occurred normal to the $[011]$ compression axis.

In that investigation, cylindrical specimens of steel containing small oriented crystals of MnS were prepared by powder metallurgy and plastically compressed. The flow of the inclusions was determined relative to that of the steel by sectioning the composite and measuring the length-to-width ratio L/W of the inclusion on the plane of section. These L/W ratios, normalized by dividing by the initial L_o/W_o ratio of the inclusion, were compared with the normalized diameter-to-height ratio of the steel cylinder.

Of particular interest here are the deformation characteristics of an inclusion oriented with [011] parallel to the compression axis. When measurements were made on sections parallel to the (100) plane of the inclusion, the L/W ratios were only slightly in excess of unity, indicating a high apparent hardness of the [011] oriented inclusion. However, measurements on a $(0\bar{1}1)$ plane of section resulted in L/W ratios which were much larger (Fig. 1). These observations prompted an analysis of the strains resulting from slip in an MnS crystal compressed along [011].

CRYSTALLOGRAPHIC ANALYSIS

Manganese sulfide has the NaCl crystal structure, and the predominate slip system is $\{110\}\langle 1\bar{1}0\rangle$. A secondary system, $\{111\}\langle 1\bar{1}0\rangle$, has also been reported.[3]

$\{110\}\langle 1\bar{1}0\rangle$ Slip

When a crystal is subject to a uniaxial compressive stress σ_{011} along [011], four slip systems, $(101)[\bar{1}01]$, $(\bar{1}01)[101]$, $(110)[\bar{1}10]$ and $(\bar{1}10)[110]$, will be equally stressed with a shear stress $\tau = 0.25\sigma_{011}$, while the other two $\{110\}\langle 1\bar{1}0\rangle$ systems will be unstressed. The normal strain ε_n along any reference axis $\underline{n}$ resulting from slip is given by Schmid's law:

$$\varepsilon_n = \sum_i \cos\lambda_{in}\cos\phi_{in}\gamma_i, \tag{1}$$

where λ_{in} and ϕ_{in} are the angles between $\underline{n}$ and the $\underline{i}$ slip direction and $\underline{n}$ and the $\underline{i}$ slip plane normal, respectively, and where γ_i is the shear strain on the $\underline{i}$ slip system. For the four active slip systems above, the $\cos\lambda_{in}\cos\phi_{in}$ terms are all equal when the reference axes are the [011] compressive axis, $[0\bar{1}1]$ and [100]. Equation (1) therefore reduces to

$$\varepsilon_n = \cos\lambda_n\cos\phi_n\gamma, \tag{2}$$

where $\gamma = \sum\gamma_i$ for all slip systems. Substituting the appropriate angles,

$$\begin{aligned} \varepsilon_{011} &= 0.25\gamma \\ \varepsilon_{0\bar{1}1} &= 0.25\gamma \\ \varepsilon_{100} &= -0.50\gamma. \end{aligned} \tag{3}$$

The interesting feature of equations (3) is that they predict that the strains in the [011] and [0$\bar{1}$1] directions, resulting from {110}<1$\bar{1}$0> slip, are equal and of the same sign. This implies that when a crystal is compressed along [011], it will contract along the lateral [0$\bar{1}$1] direction and expand rapidly along the [100] direction, maintaining constant volume. This unusual shape change would explain, at least qualitatively, the strong dependence of L/W values on the plane of section. If only {110}<1$\bar{1}$0> slip were occurring, the L/W ratio measured on a (100) section would remain unchanged, while that measured on a (0$\bar{1}$1) section would change rapidly. The unusual fracture behavior of [011] oriented MnS inclusions in a steel matrix observed in (100) and (01$\bar{1}$) sections have been explained on these bases.[2]

{111}<1$\bar{1}$0> Slip

It is possible that [011] compression also activates the {111}<1$\bar{1}$0> slip systems. Four of these, (111)[$\bar{1}$10], (111)[$\bar{1}$01], ($\bar{1}$11)[101] and ($\bar{1}$11)[110], would be equally stressed ($\tau = 0.408\sigma_{011}$), while the other eight would be unstressed. Slip on the stressed systems alone would produce strains

$$\begin{aligned} \varepsilon_{011} &= 0.408\gamma \\ \varepsilon_{0\bar{1}1} &= 0 \\ \varepsilon_{100} &= -0.408\gamma. \end{aligned} \tag{4}$$

Thus, {111} slip, activated by [011] compression, would cause no lateral expansion or contraction along [0$\bar{1}$1].

With combined {111} and {110} slip, the ratio of the contraction strain along [0$\bar{1}$1] to the compressive strain along [011], found by combining equations (3) and (4),

$$\frac{\varepsilon_{0\bar{1}1}}{\varepsilon_{011}} = \frac{\gamma_{110}}{\gamma_{110} + 1.63\gamma_{111}}, \qquad (5)$$

decreases with increasing amounts of $\{111\}$ slip.

EXPERIMENTAL VERIFICATION

Since the prediction that [011] compression should be accompanied by a large plastic contraction normal to the compression axis (i.e., a negative plastic Poisson's ratio) seems intuitively surprising, an experiment was undertaken to test this prediction by compressing a large crystal of rock salt.

A large cleaved crystal of NaCl (Harshaw Chemical Co.) was carefully polished to a nearly cubical rectangular prism with (011), ($0\bar{1}1$) and (100) faces. Each of the dimensions indicated in Fig. 2 varied by 0.0005 in. or less. Resistance strain gages were mounted on the faces so the three principal strains could be measured during the test. To minimize the effect of any non-axiality of loading, parallel gages on opposite faces were wired in series. The crystal was compressed in an Instron machine, with two 0.002 in. teflon films placed between each platen and the (011) faces of the crystal.

The crystal was tested by loading to a predetermined level, unloading, and then measuring the three principal strains. By repeating this procedure with steadily increasing loads, the three plastic strains were determined as a function of stress, as shown in Fig. 3. Plastic strains were also determined by micrometer measurements after loading to 10,000 psi. Although the strain gages had ceased to operate at this level, extrapolation of the strain gage measurements is within the error of the micrometer measurements. It may be noted that the sum of the principal strains in Fig. 3 is not exactly zero. This undoubtedly reflects slight inaccuracy in gage alignment and differing sensitivities of the three gage circuits, rather than a volume change.

At one point during the test, readings of the [$0\bar{1}1$]

strain were made while the crystal was being reloaded. The stress-strain record (Fig. 4) shows increasing positive strains (elastic Poisson expansion) during the elastic reloading. When yielding occurred there was a sudden reversal of the sign of the strain corresponding to the lateral plastic contraction.

It is of interest to compare the lateral contraction along $[0\bar{1}1]$ with the compressive strain along $[011]$. In Fig. 5 the ratio of the strain increments,

$$\frac{\Delta\varepsilon_{0\bar{1}1}}{\Delta\varepsilon_{011}},$$

determined from successive strain measurements, is plotted as a function of applied stress. It may be seen that this ratio falls from nearly unity at yielding to a much lower value at high stresses. This may be interpreted as an indication that initially $\{110\}$ slip predominates but that the fraction of $\{111\}$ slip increases with continued deformation. The fraction of $\{111\}$ slip may be estimated by referring to the right-hand scale on Fig. 5, which was constructed from equation 5. Apparently, nearly equal amounts of $\{111\}$ and $\{110\}$ slip are occurring by the time the stress reaches 4000 psi. At higher stress levels $\{111\}$ slip tends to dominate.

DISCUSSION

A negative "plastic Poisson's ratio" is generally possible along some reference axis if deformation occurs by slip on a single system. Single slip activated by a uniaxial stress along x should produce a strain along a reference axis y (90° to x) of $\varepsilon_y = \alpha\varepsilon_x$. Since

$$\alpha = \frac{\cos\lambda_x \cos\phi_x}{\cos\lambda_y \cos\phi_y},$$

α will be positive whenever $\cos\lambda_y\cos\phi_y$ has the same sign as $\cos\lambda_x\cos\phi_x$; i.e., if λ_x and ϕ_x are both between 0 and 90°,

α will be positive if λ_y and ϕ_y are both between 0 and 90°. Unless $\phi_x + \lambda_x = 90°$ (i.e., the slip direction, the slip plane normal and x are coplanar), there will be some position of the y axis for which α is positive (Fig. 6).

Two factors make the effect observed in [011] compression of NaCl unique. One is the magnitude of α, which approaches unity at small strains. The other is the fact that the effect occurs under multiple slip (four $\{110\}\langle 1\bar{1}0\rangle$ slip systems active) so that y is a principal strain axis.

CONCLUSIONS

Compression along the [011] direction of crystals with an NaCl-type structure results in a negative plastic Poisson's ratio, which decreases in magnitude with increasing stress from nearly one at the start of the test. This observation is explained in terms of slip on the primary $\{110\}\langle 1\bar{1}0\rangle$ and secondary $\{111\}\langle 1\bar{1}0\rangle$ slip systems, and can be used to understand the flow patterns observed in [011] oriented MnS inclusions imbedded in steel. It offers support for the explanation of the unusual behavior of MnS inclusions in terms of a lateral contraction.

ACKNOWLEDGMENTS

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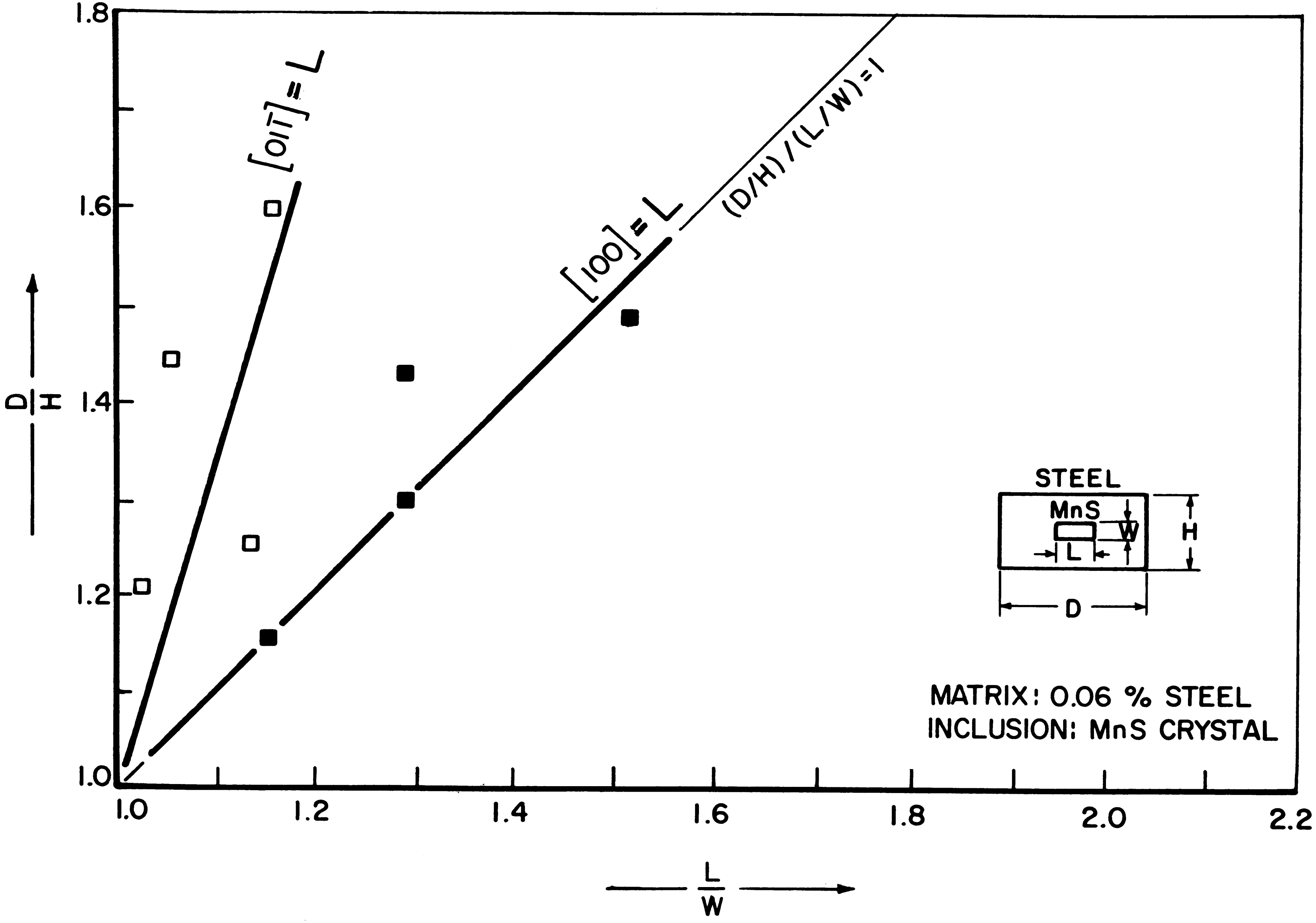

Fig. 1. Deformation characteristics of [011] oriented crystals of MnS imbedded in steel specimens. The ratios L/W and D/H are measures of the apparent deformation of the inclusion and specimen, respectively. When the specimens were sectioned so that L = [100], the L/W ratios were greater than when L = $[0\bar{1}1]$.

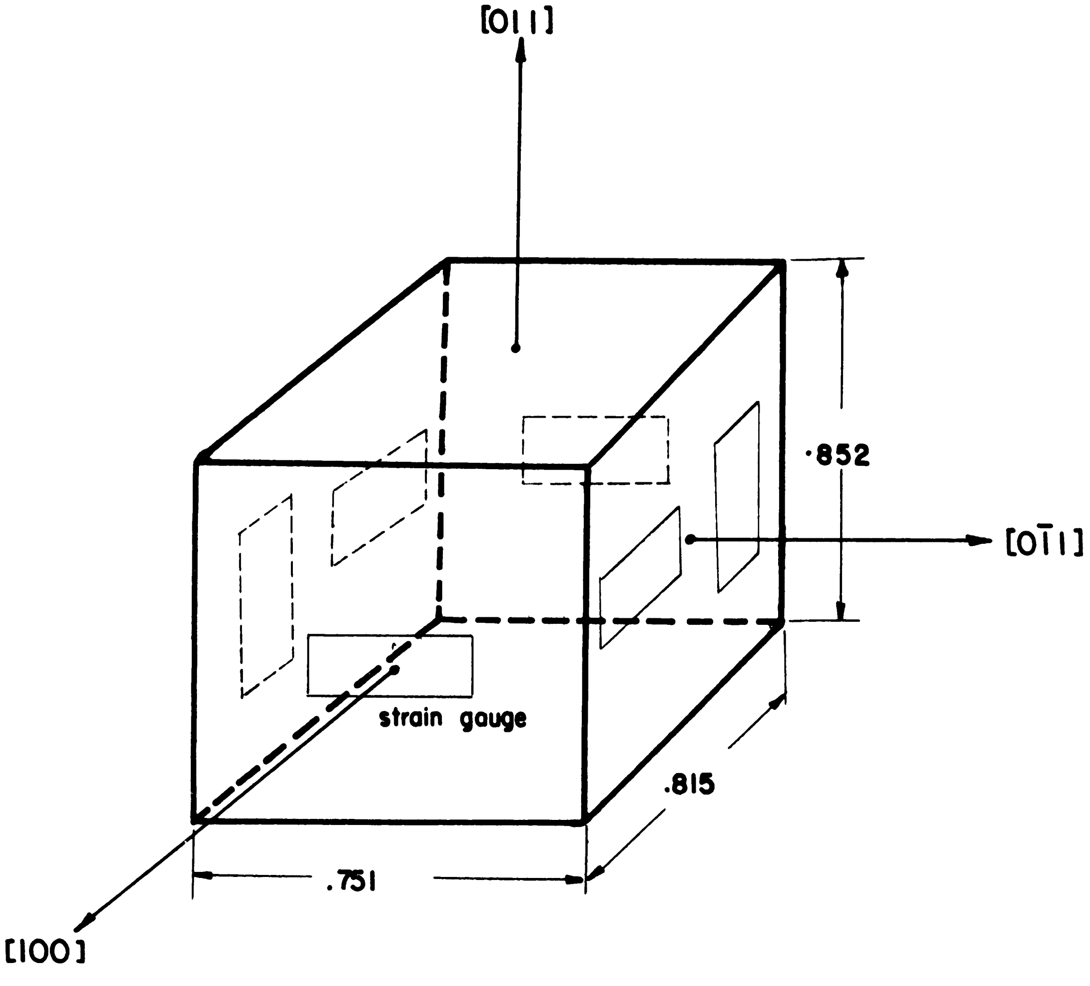

Fig. 2. Initial dimensions and strain gage locations for the NaCl crystal compressed along [011].

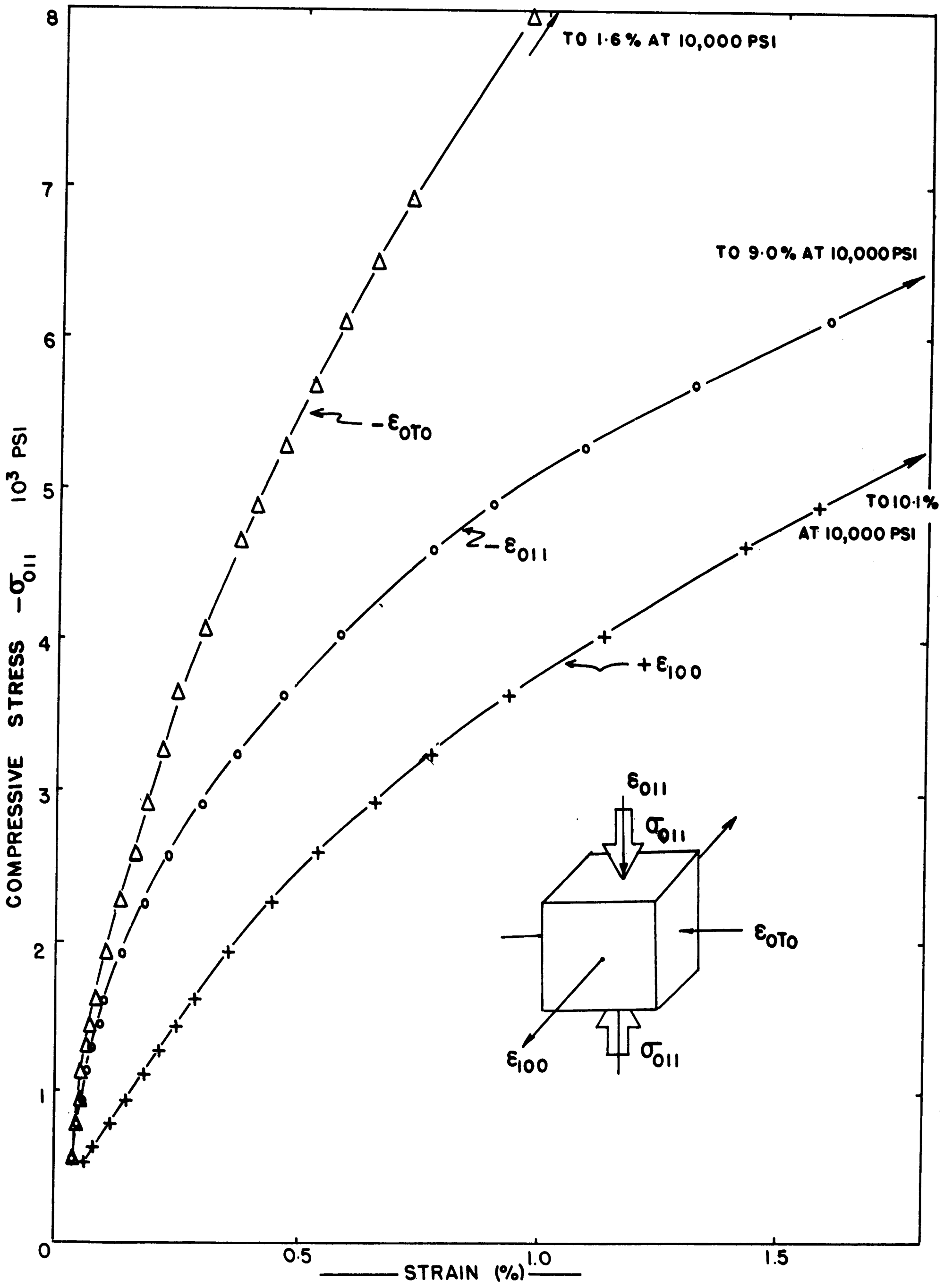

Fig. 3. Variation of the three principal strains ε_{011}, $\varepsilon_{0\bar{1}1}$ and ε_{100} with the compressive stress σ_{011}. Both ε_{011} and $\varepsilon_{0\bar{1}1}$ are negative (compressive), indicating a lateral contraction along [0$\bar{1}$1]. Micrometer readings of the three strains after $\sigma_{0\bar{1}1}$ reached 10,000 psi are indicated.

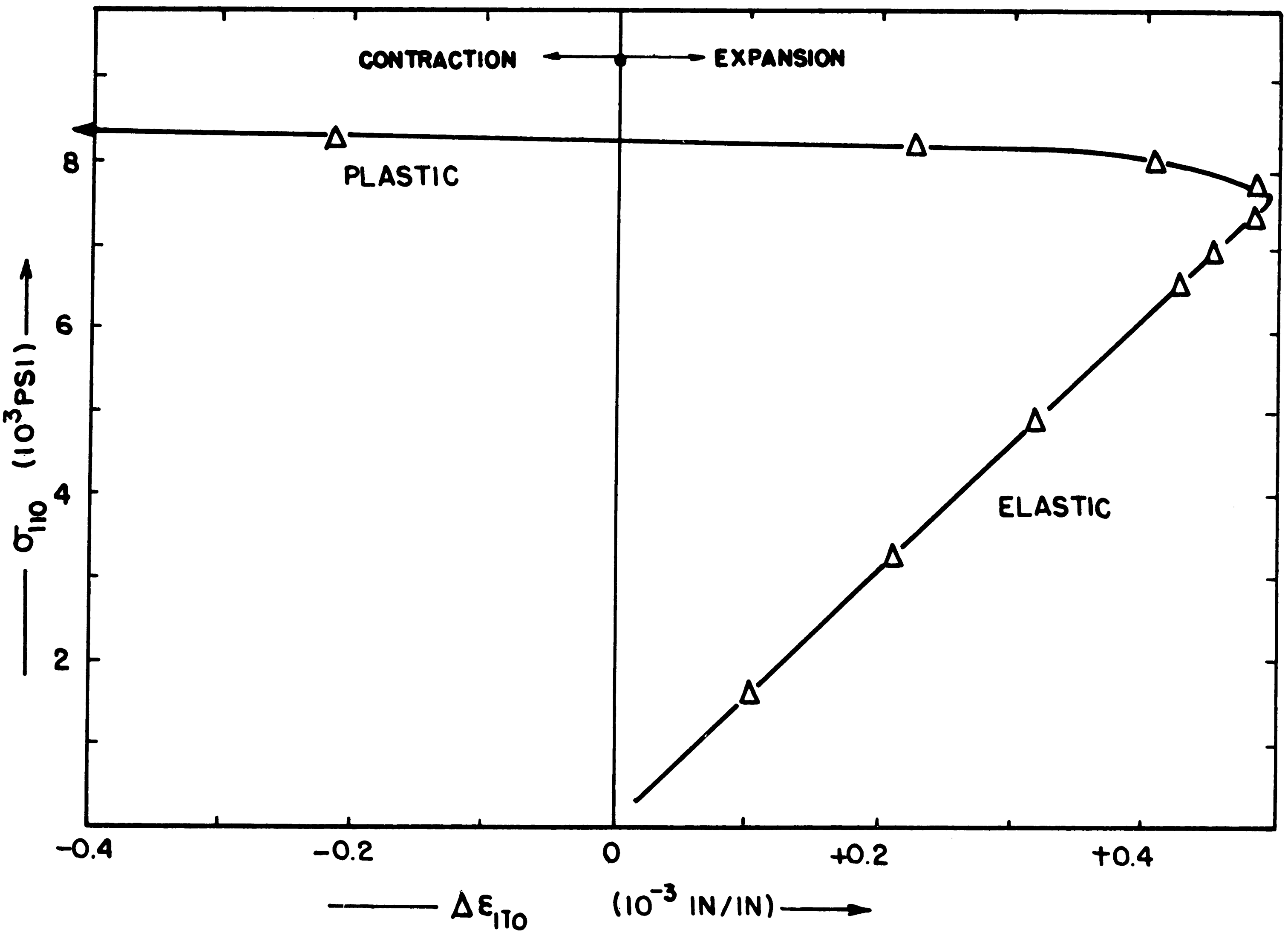

Fig. 4. The change of the lateral strain along $[0\bar{1}1]$, $\Delta\varepsilon_{0\bar{1}1}$ with the compressive stress, σ_{011} during reloading. During the elastic reloading, normal Poisson expansion resulted in positive values of $\Delta\varepsilon_{0\bar{1}1}$. However, plastic flow at yielding caused a large contraction along $[0\bar{1}1]$ (negative $\Delta\varepsilon_{0\bar{1}1}$).

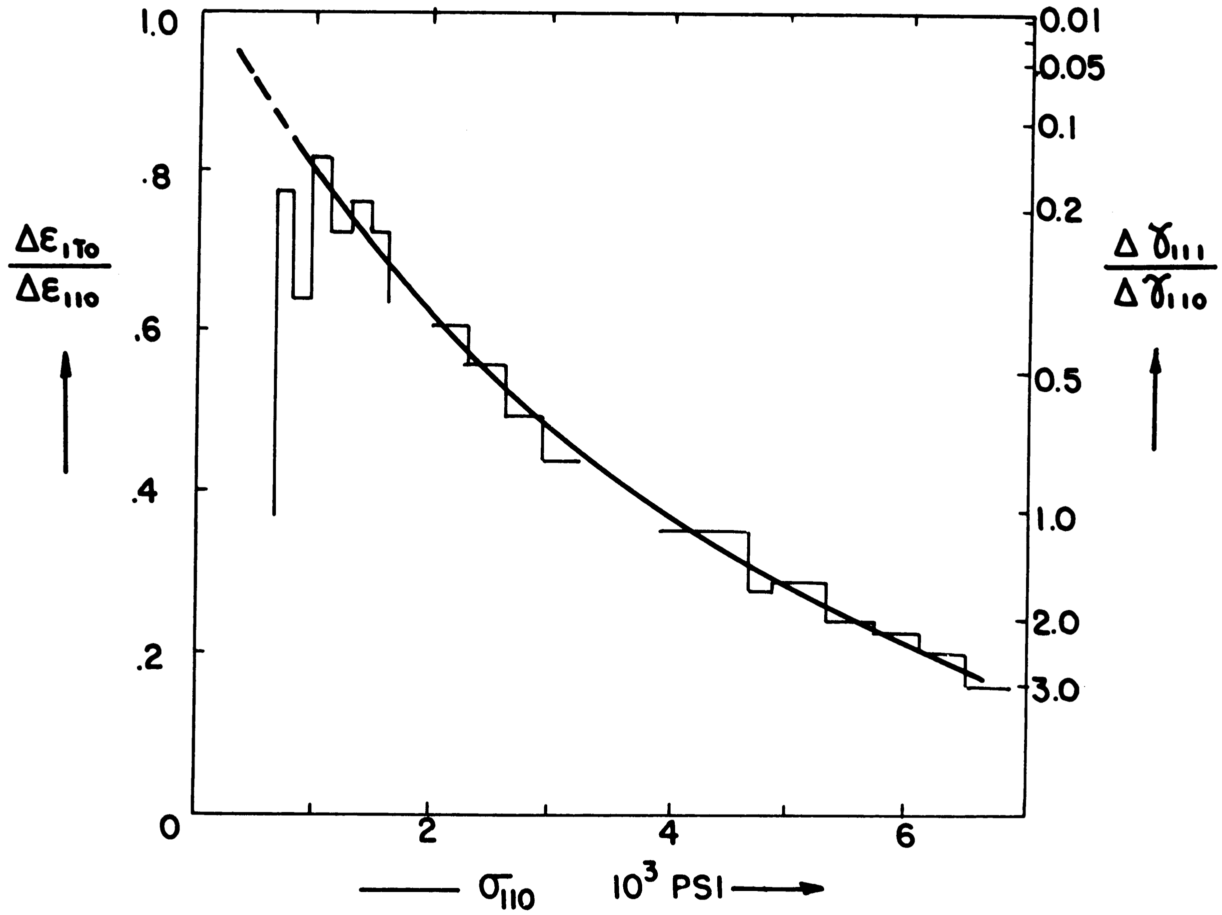

Fig. 5. Ratio of incremental contraction along $[0\bar{1}1]$ and $[011]$ as a function of compressive stress σ_{011}. The high initial values of $\Delta\varepsilon_{0\bar{1}1}/\Delta\varepsilon_{011}$ indicate that slip is primarily on $\{110\}$ planes. At higher loads, an increasing fraction of the slip occurs on $\{111\}$ planes.

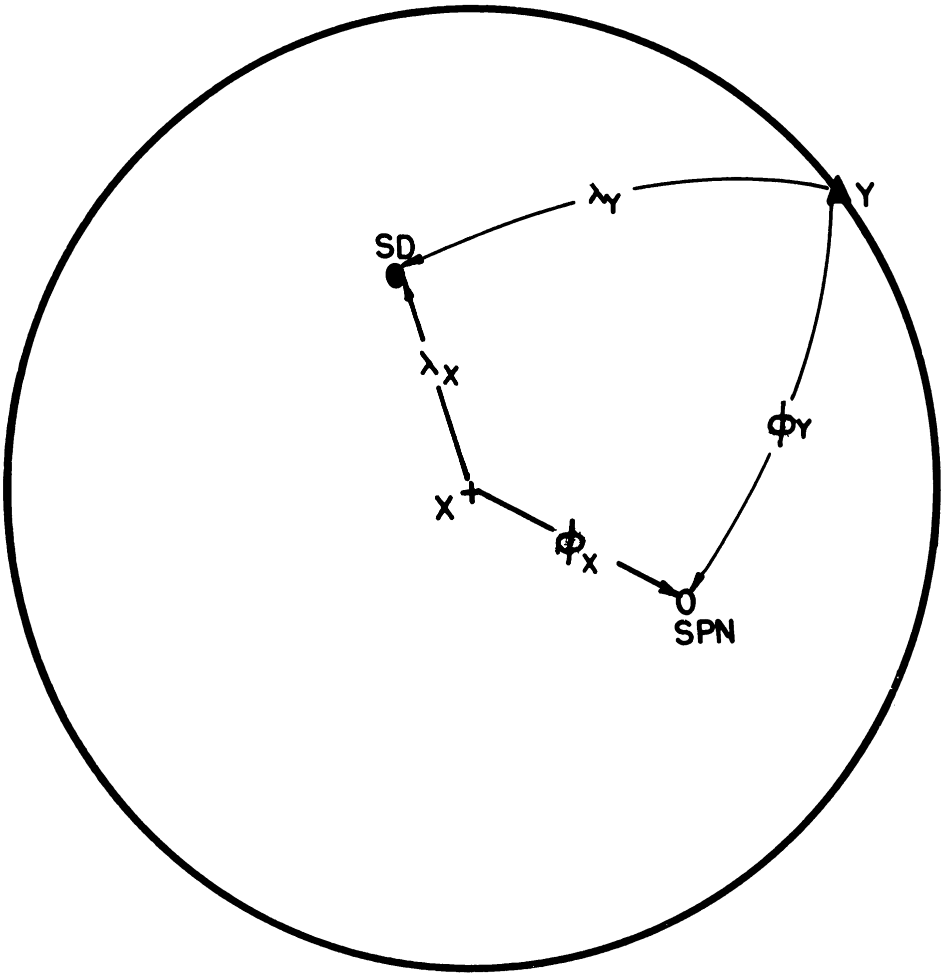

Fig. 6. Stereogram showing the orientation of a single slip plane normal (SPN) and slip direction (SD) activated by uniaxial tension or compression along x. Unless SPN, SD and x are coplanar, there will be a location of a lateral reference axis y for which $\cos\lambda_y \cos\phi_y$ is positive, and consequently ε_y will be of the same sign as ε_x.