T H E U N I V E R S I T Y O F M I C H I G A N

COLLEGE OF ENGINEERING
Department of Mechanical Engineering

Student Project Reports

INVESTIGATION OF DESIGN MEANS FOR HOME LAUNDRY APPLICANCES

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DEVELOPMENT OF A THERMODYNAMIC ACTUATOR

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ABSTRACT

This work describes the development of a linear thermodynamic actuator to replace solenoids as actuation devices in certain applications in washing machines. The investigation includes the characteristics and also considers possibility of development of a multiple-position thermodynamic actuator.

PROBLEM STATEMENT

Whirlpool Corporation products use millions of linear actuators every year. Presently, the electromechanical device known as the solenoid is used exclusively. A solenoid offers relatively instantaneous actuation with two-position capability, fully extended, and fully retracted.

The major goal of this research project was to develop an actuation device that could replace solenoids in certain washing machine applications. One requirement was that the device should possess multiple-position capability, an option which solenoids do not offer.

Since many solenoids are used each year, the development of a cheap reliable substitute might reduce costs. Even if the actuation device did not result in lower unit cost; with its multiple-position capability, it might replace several solenoids and therefore be cost competitive.

CONCEPT AND ANALYSIS

The basic concept of the actuating device used a thermodynamic working fluid to displace a piston upon the addition of heat energy as shown in Figure 1. The working fluid was chosen for its thermodynamic properties. For two reasons, it was desirable that the working fluid operate in its thermodynamic two-phase region. By operating in the two-phase region, a large pressure change could be produced which would act against the load with a small volume change. Secondly, by choosing an appropriate working fluid whose vapor pressure-temperature curve was compatible with ambient conditions, the amount of energy input required would be minimized.

After investigating the thermodynamic properties of various fluids, "Freon 11" (trichlorofluoromethane) was chosen as the working fluid. A temperature-volume diagram of "Freon 11" is presented in Figure 2. At typical ambient conditions, 75°F, the vapor pressure of "Freon 11" is 14.7 psia.

The thermodynamic actuator can by analyzed by considering a control volume, as shown in Figure 1.

Applying the First Law of Thermodynamics to the system gives

$$Q = W + m(u_2 - u_1)$$

where Q = heat which crosses system

m = mass of fluid

u_1 = specific internal energy of fluid before heat transfer

u_2 = specific internal energy of fluid after heat transfer

W = work done by the system on surroundings

The work relation for the system is

$$W_{total} = \int PdV$$

Also the total work is sum of the work done on the spring and the work done on the atmosphere. Thus

$$W_{total} = W_{spring} + W_{atm}$$

$$= \int k \; x \; dx + \int P_{atm} dV$$

where k = spring constant

x = the displacement of the spring

Interestingly enough, by pre-loading the spring, the pressure at which actuation initiates can be controlled. Here the mechanics of the device tend to govern the thermodynamics of the system.

A computer program, shown in Appendix A, was written using the above equations and the properties of the fluid to size the thermoactuator for a given set of conditions.

EXPERIMENTAL PROCEDURE

A prototype actuator was constructed to provide information and to demonstrate the feasibility of the device. For purposes of instrumentation, the prototype was constructed larger than normal size. The cylinder is nearly 3 in. in diameter as compared with a typical installation size of about 1/2 in. diameter.

Figure 3 shows the cylinder apparatus. All parts except the clamping rods, piston rod, spring sleeve, and spring are aluminum. A lip seal was used on the piston (Huva cup, Crane Co.). An O-ring seal was used in the cylinder base. The spring constant of the spring used was 147 lbf/in.

Figures 4 and 5 show the test set-up. The standpipe configuration was finally used to insure that the liquid Freon was always in contact with the heating surface. A flexible heating element was wrapped around the lower portion of the standpipe as shown in Figure 6. The filling port was located at the lowest point in the system.

A variable transformer (Variac) was used to control electrical power imput to the heater. This power was measured with an AC voltmeter and ammeter. Piston deflection was measured with a dial indicator gage.

Each experimental run was conducted as follows: first, the system pressure was nulled along with the dial indicator reading. A vacuum of 21 in. Hg was used to reduce the amount of air in the system. Then the liquid Freon 11 was allowed to flow into the system and the system was filled with Freon to the same level for each run.

Five runs were made at different power levels. The powers were 16, 37, 54, and 94 watts. For each run, the pressure and extension were recorded as a function of time. Data was also taken for the retraction condition (zero input power) with cooling natural convection.

Data are presented in Figures 7-11. Figure 7 is a temperature-volume plot for the heating process and compares two data sets with the predicted results. A data reduction program, listed in Appendix B, was used to obtain thermodynamic properties from the pressure-extension data. Figure 8 is a plot of extension vs. time and Figure 9 shows pressure vs. time for various powers. Figure 10 is a plot of extension vs. time, and Figure 11 shows pressure vs. time—both plots for the cooling or retraction portion of the cycle.

DISCUSSION OF RESULTS

Looking at the temperature-volume diagram of Figure 7, the predicted and measured values (54 and 73 watts) are in good agreement.

The extension (heating) graphs of displacement and pressure vs. time proved interesting. At first glance, it might be expected that the system response would be exponential in nature; with the initial rapid rise in output variable slowly reducing to become nearly constant with time. However, this expected trend is found only at the lowest power input, 16 watts. Higher power response appears to follow a different relationship. This apparent discrepancy may be explained by the inherent non-linear behavior of the system. Further, the experiment was run insufficiently long for the displacements to become constant with time for the higher powers.

The retraction (cooling) performance of the system as shown in Figures 10 and 11 were more conventional. The exponential decay in both deflection and pressure follow classical free convective colling curves.

Caution must be used in extrapolating results from these experiments to the performance of different sized (smaller) actuators. Since the performance of the actuators is primarily heat-transfer limited, no simple set of scaling parameters can be readily defined. However some general trends can be inferred.

Since the dynamic response of the actuator is heat-transfer limited, then the net heat input rate to the fluid is the difference between the heat transferred to the fluid from the electrical heater and the heat lost from the fluid (and the system) via convective heat transfer effects. Suppose that the electrical heater is virtually immersed in the liquid Freon so that the amount of heat to the system is a function of the electrical power only, and not limited by heat transfer surface. The heat loss from the system, however, is a function of the surface area available for heat transfer. Consequently, a smaller actuator, for the same power input, will have a smaller heat transfer surface available for losses. Thus the net heat input to the fluid will be greater with a correspondingly increase in actuator response. Roughly speaking, scaling the actuator down by a factor of 6 in a linear dimension (3-in. diameter piston scaled to 1/2 in.) results in a reduction in area by a factor of 36. Thus response time would be reduced by a factor of 36.

Additional work would include study of the scaling effects just mentioned. Analytical modelling together with the construction and testing of a smaller actuator would be useful. Various methods of forced cooling might be investigated in order to improve the retraction (cooling) response time. Alternate heat sources, other than electrical, could be considered.

CONCLUSIONS

While this research project has not fully investigated all aspects of thermodynamic actuators, some important conclusions can be drawn. A linear thermodynamic actuator has been developed which appears to possess potential for replacing solenoids in certain application in washing machines. The prototype has demonstrated that the concept of a linear thermodynamic actuator is a feasible approach to problems in actuation. The research also showed that the theory correlates well with the experimental results, and that the computer program satisfactorily predicts the performance of these actuators. The dynamic response indicates that this type of actuator should be used for an extension application rather than retraction or rapid cycle. These conclusions suggest that additional work is warranted in the further development of linear thermodynamic actuators.

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3. Lipson, Charles, Design of Experiments.

4. Sonntag, R. E. and G. J. Van Wylen, Fundamentals of Classical Thermodynamics, John Wiley & Sons, Inc., New York, 1965.

5. Streeter, V. L., Fluid Mechanics, McGraw-Hill Book Company, New York, 1971.

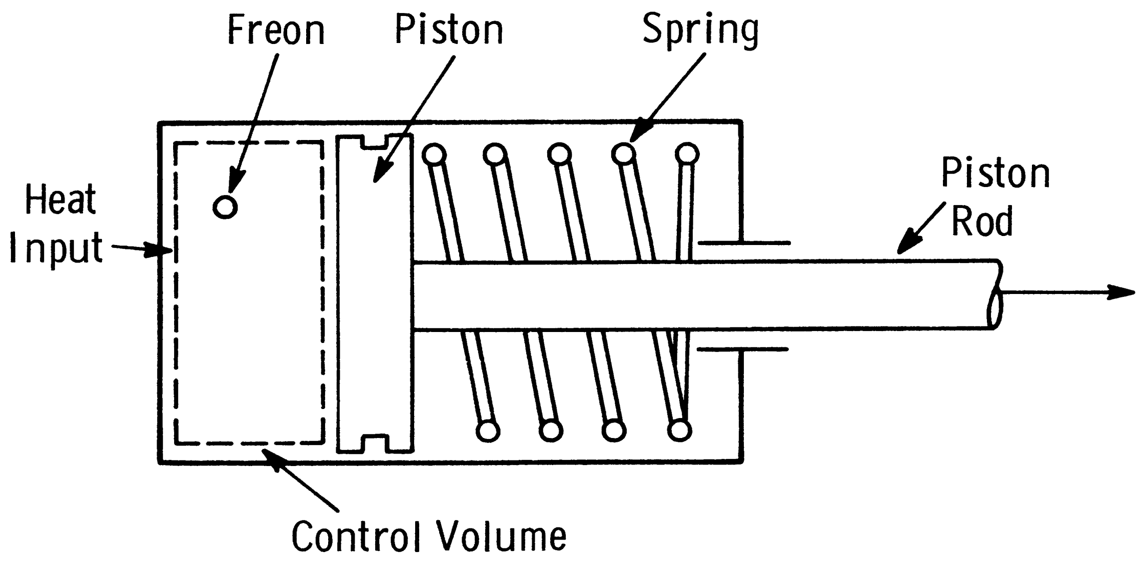

Figure 1. Schematic of actuator.

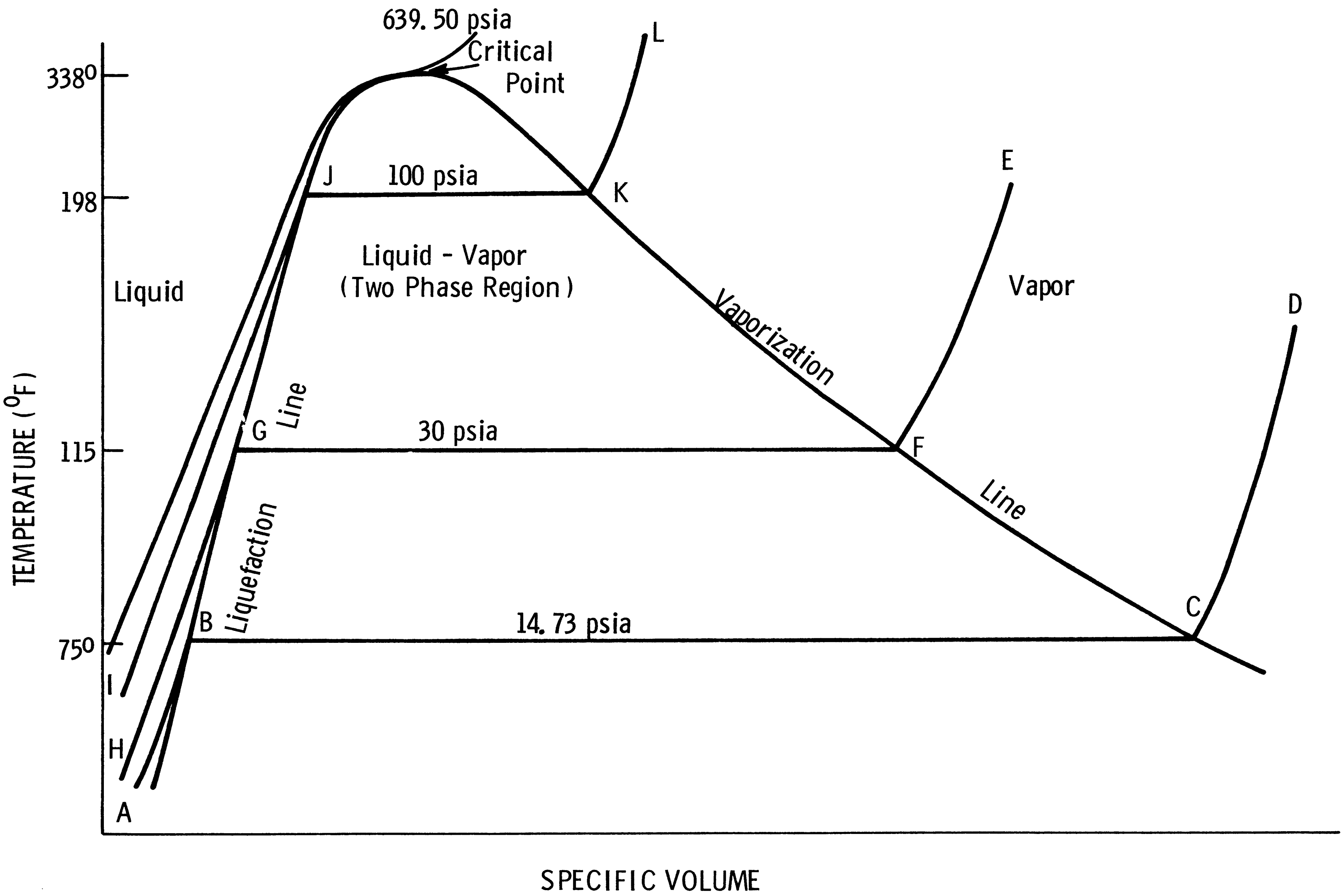

Figure 2. Temperature-volume plot of Freon 11.

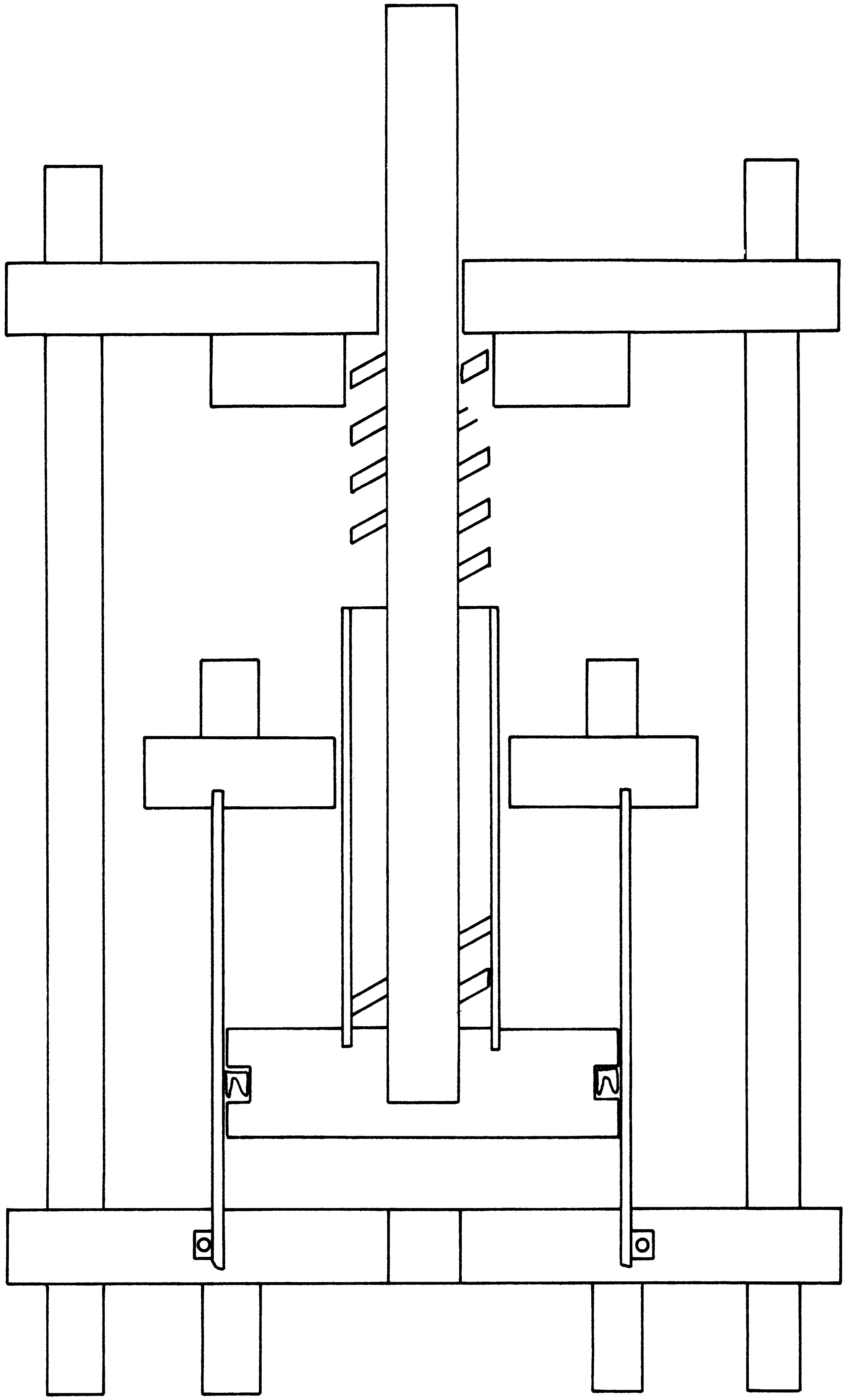

Figure 3. Apparatus.

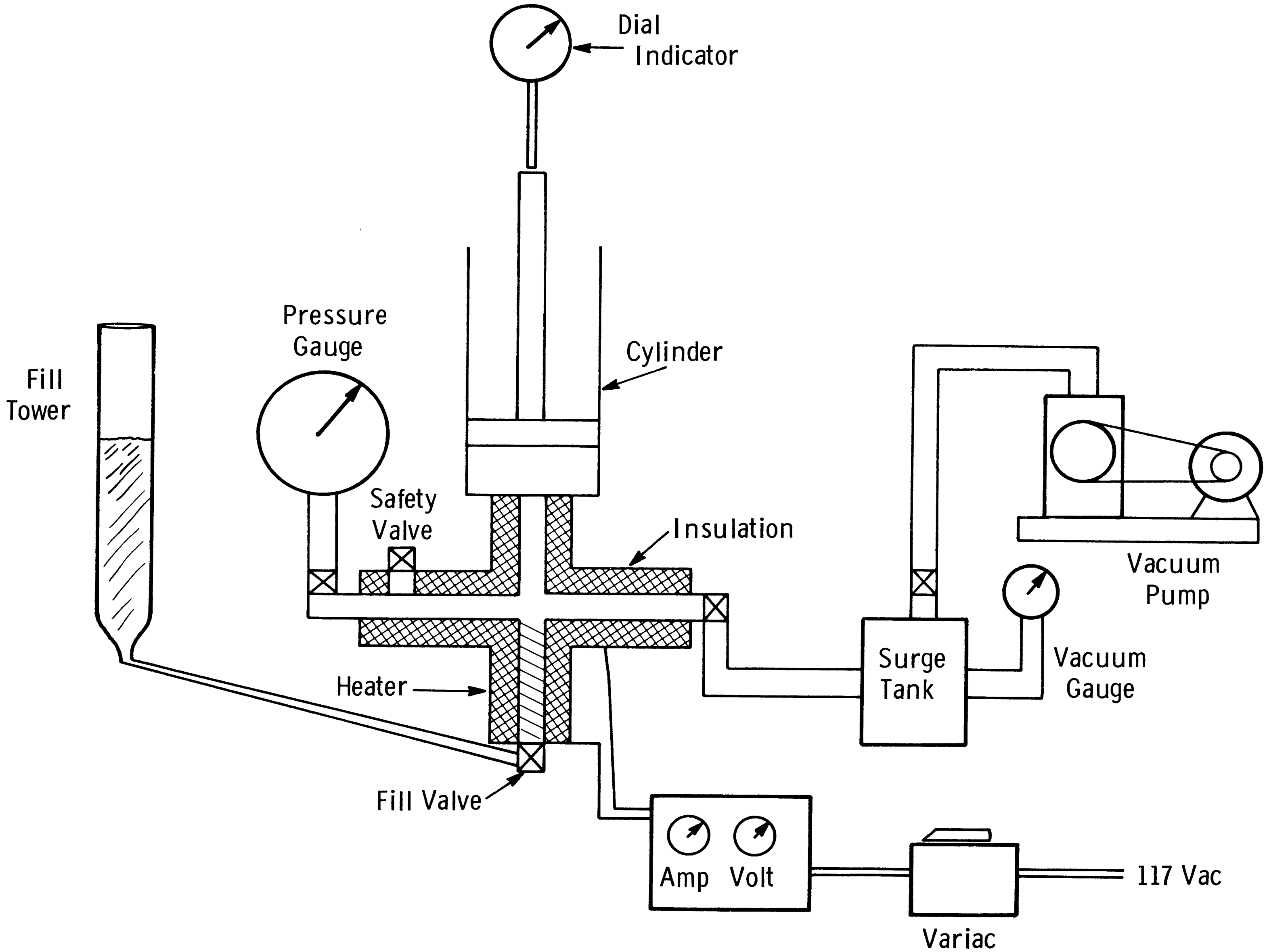

Figure 4. Test set-up.

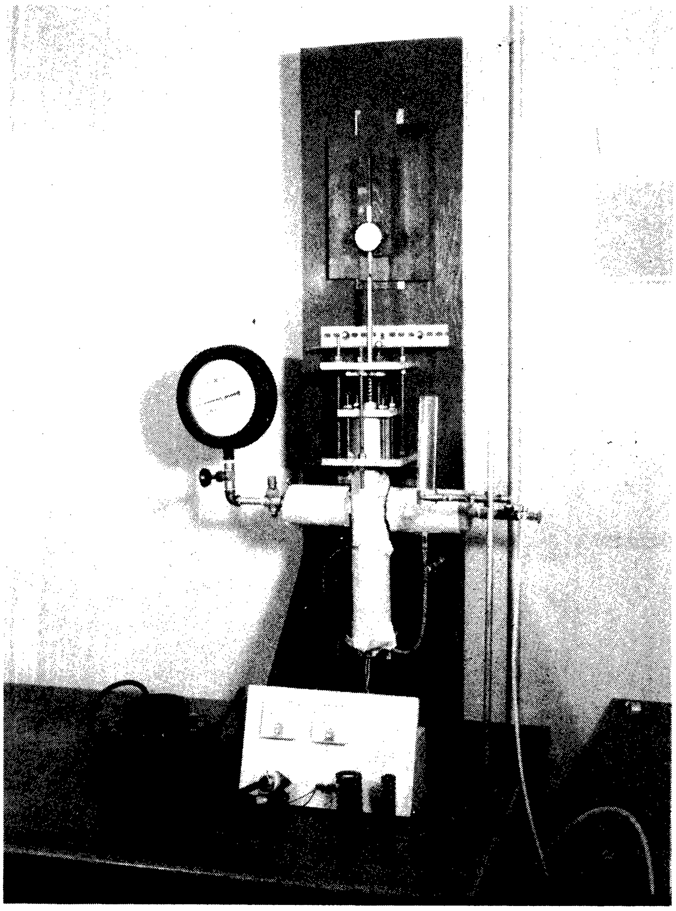

Figure 5. Photograph of experimental apparatus.

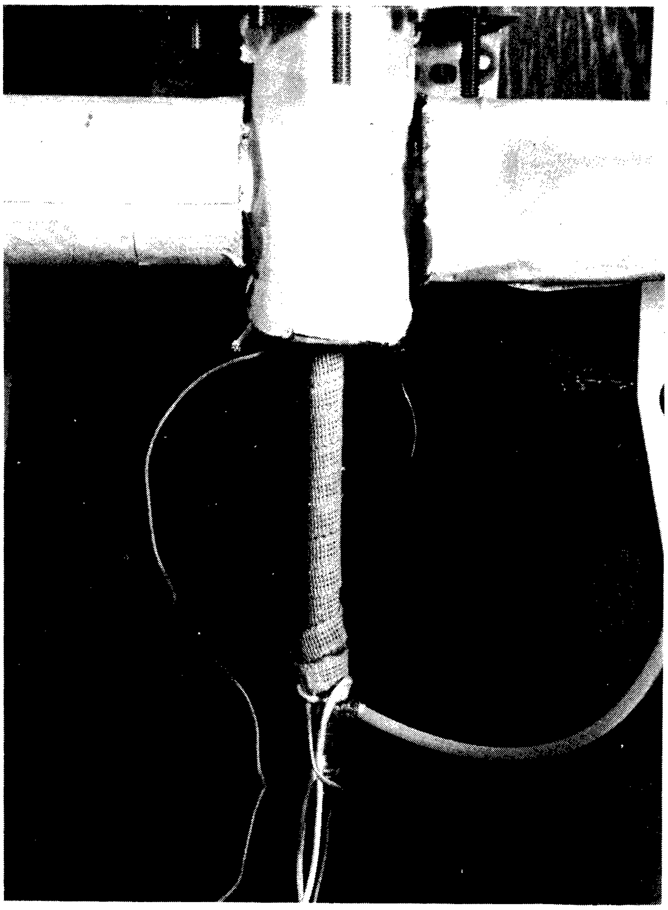

Figure 6. Photograph of standpipe.

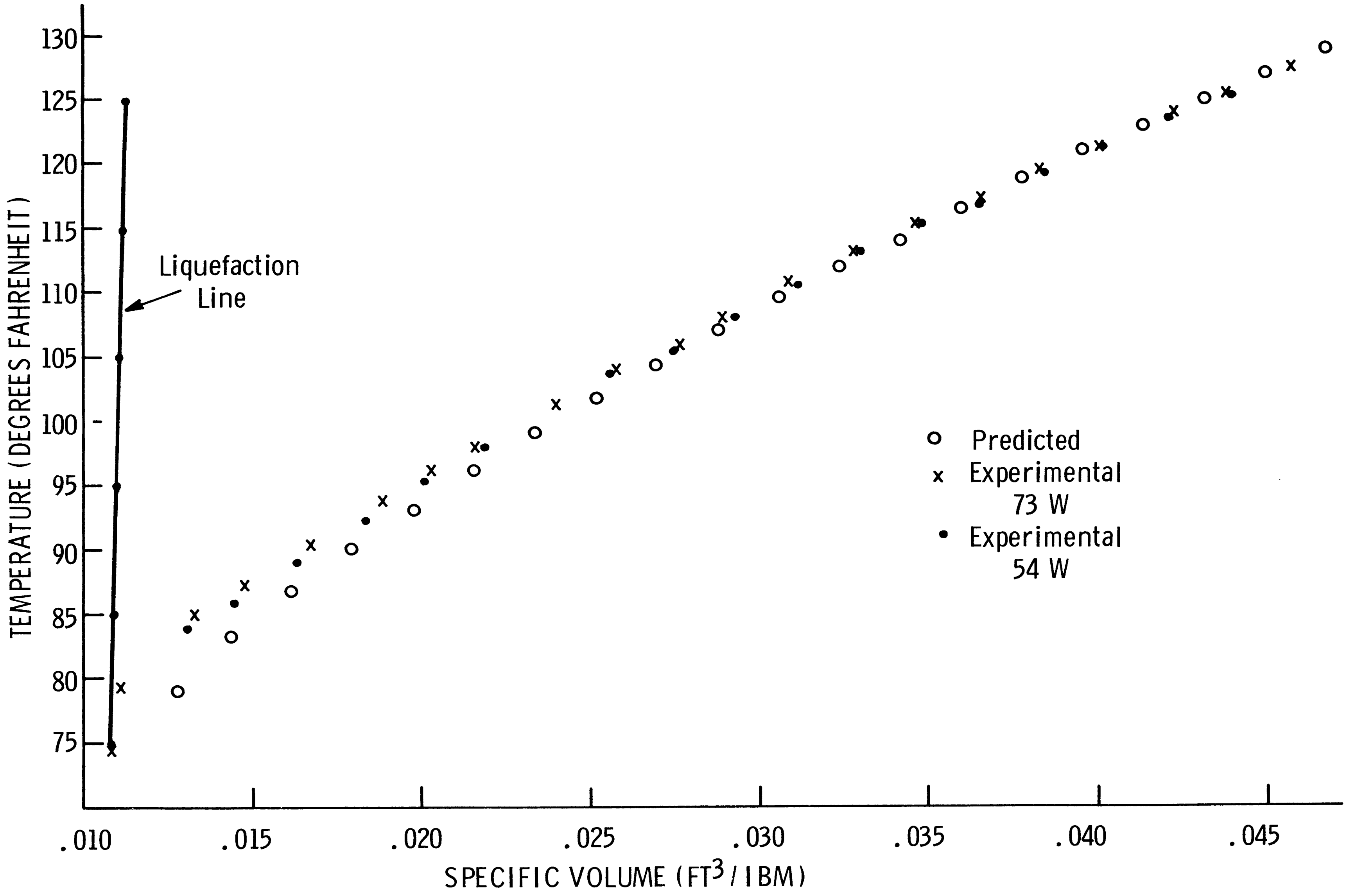

Figure 7. Temperature-volume plot.

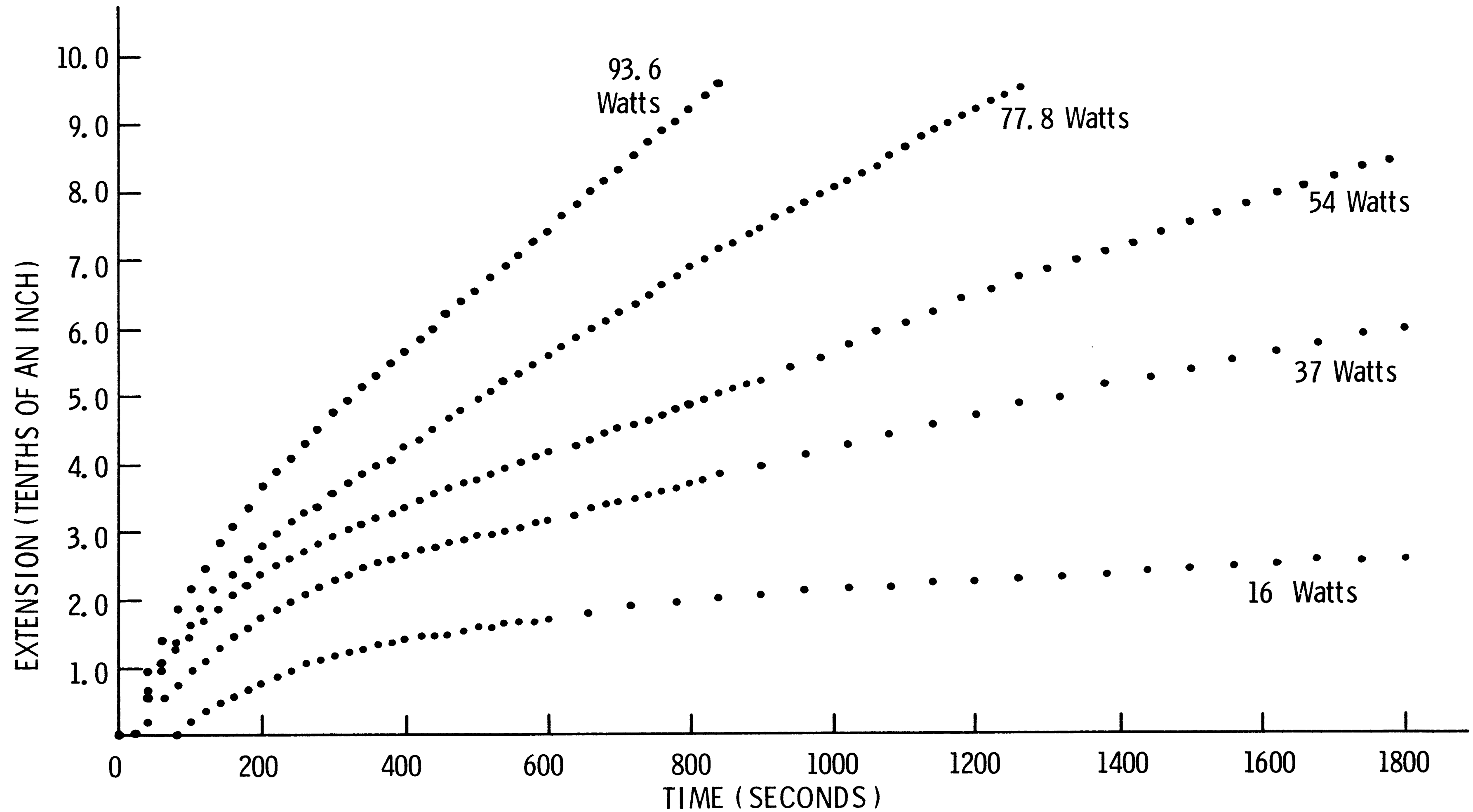

Figure 8. Extension vs. time for heating.

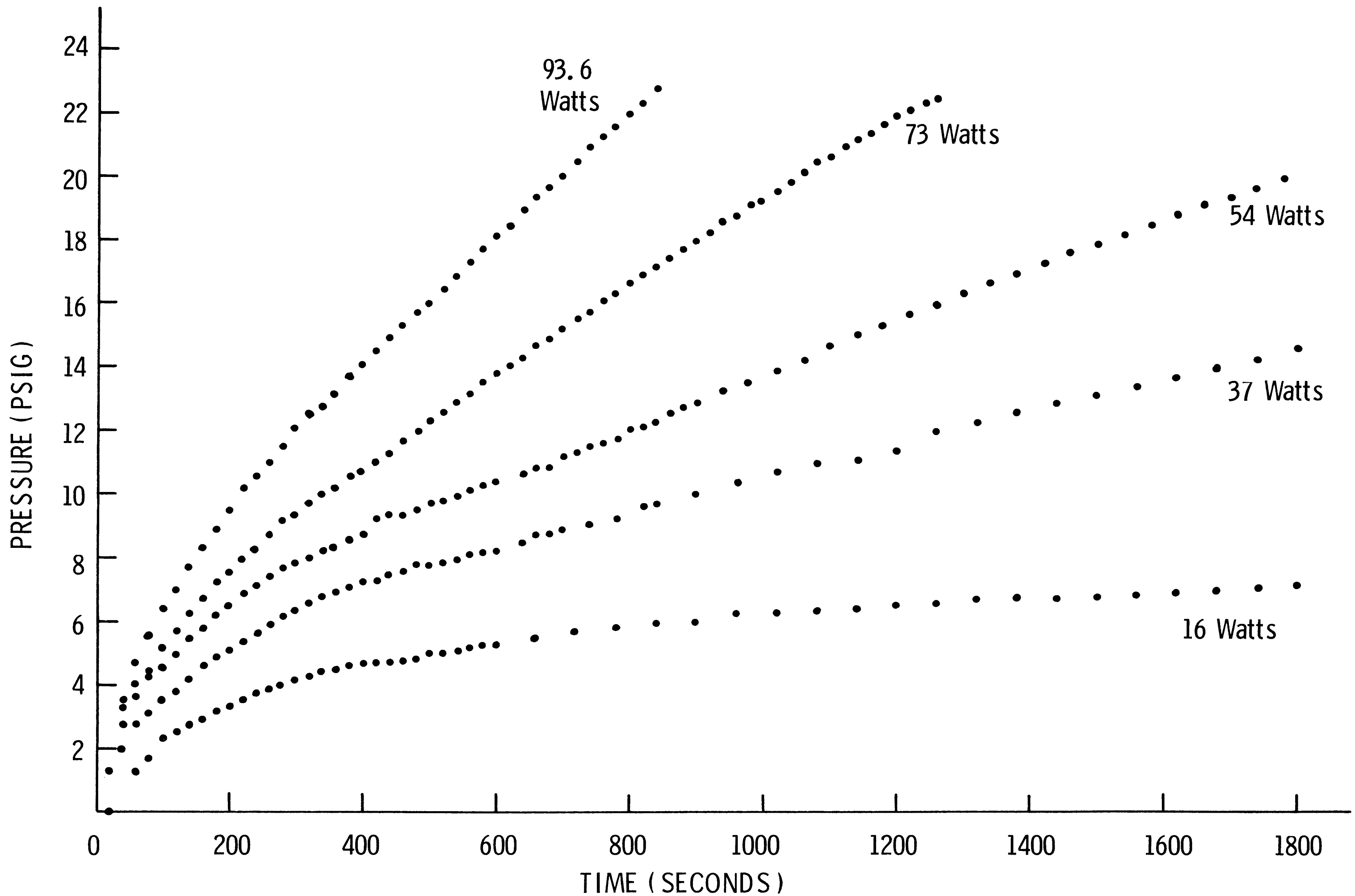

Figure 9. Pressure vs. time for heating.

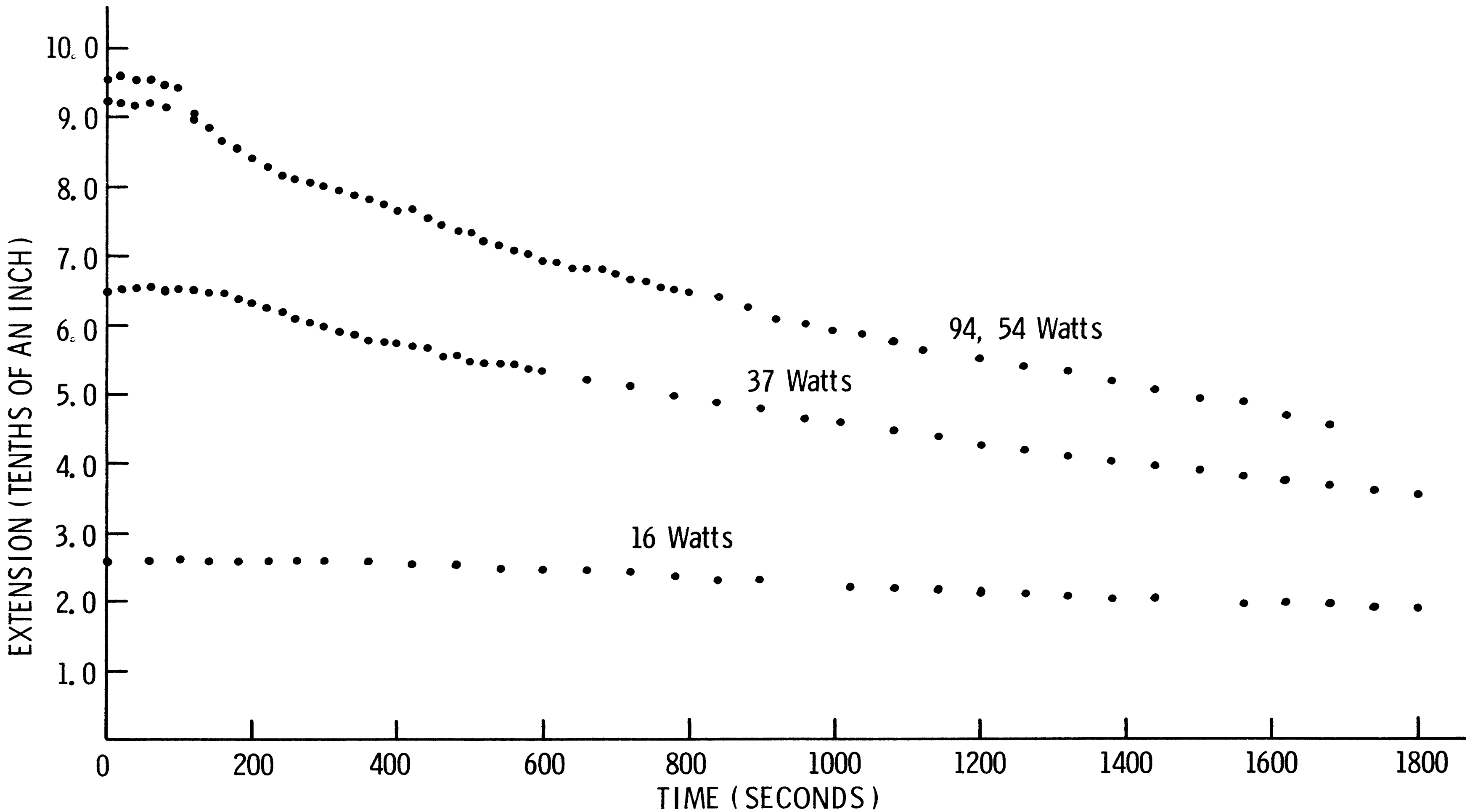

Figure 10. Extension vs. time for cooling.

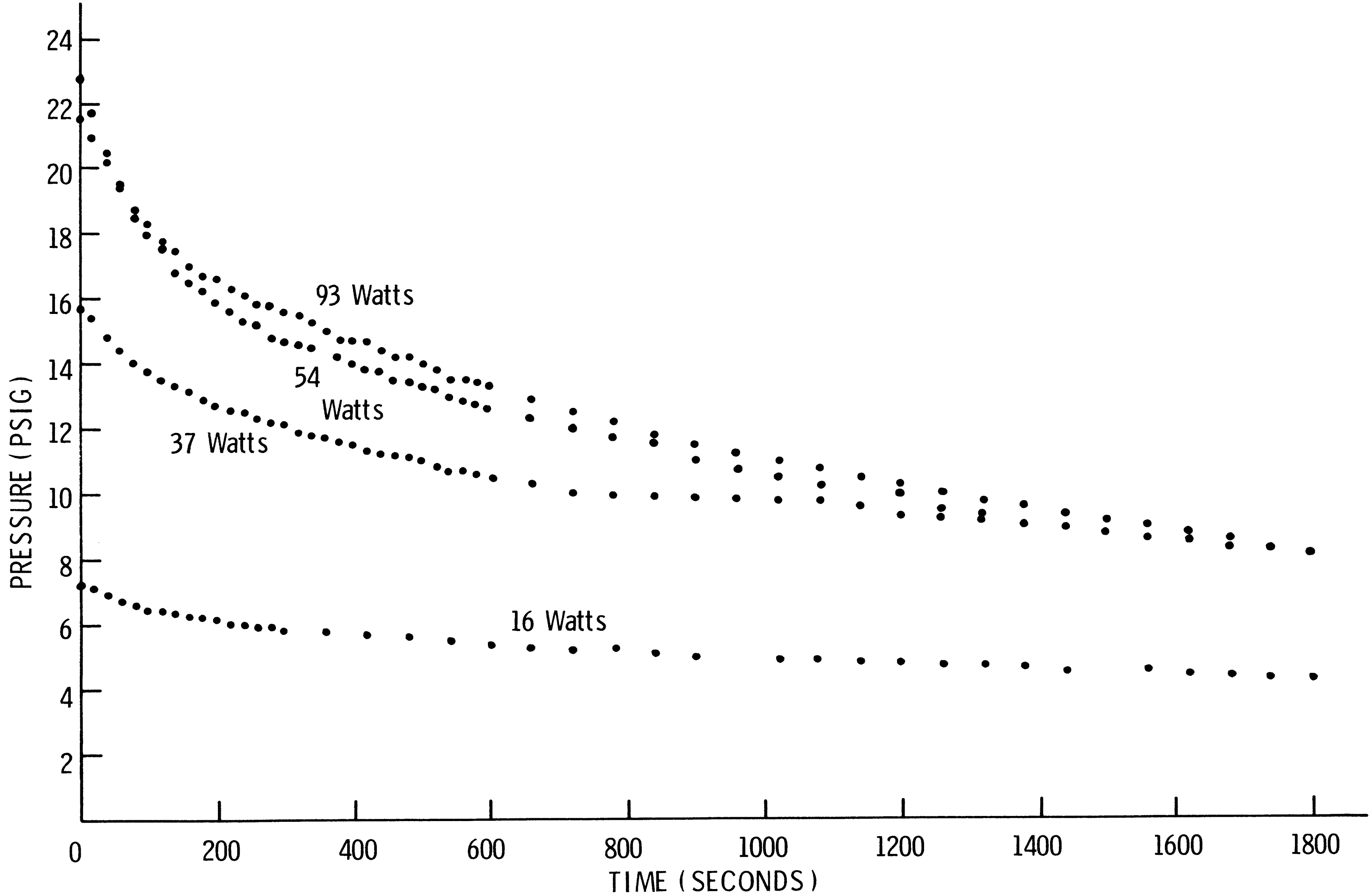

Figure 11. Pressure vs. time for cooling.

ACKNOWLEDGMENTS

I would like to take this opportunity to thank my advisor Professor Robert Keller for his suggestions and guidance. Also, I would like to thank Mr. Ray Allen and Mr. Sheldon Roll for their help in constructing the experimental apparatus.

APPENDIX A

COMPUTER PROGRAM

PROGRAM VARIABLES

1 PROP (I, J) Freon-property matrix

Temperature -	column 1
Pressure -	colume 2
v_F	column 3
v_g	column 4
H_F	column 5
H_g	column 6

2. DIAM = cylinder diameter (inches)
3. VINT = initial volume $(\text{inches})^3$
4. XINT = initial quality
5. PRESSI = initial pressure (absolute pressure)
6. SPRK = spring constant (lbf/in)
7. PRPRS = gage pressure (psig)
8. FPRE = preload force (lbf)
9. SPRY = preload distance (in)

TSAT = saturation temperature (°F)

SPVDL = specific volume (ft^3/ lbm)

ITINT = initial enthalpy (Btu/ lbm)

VINT = initial internal energy (Btu/lbm)

FREMAS = freon mass (lbm)

EXTEN(I) = extension (inches)

DELTAV = delta volume $(\text{inches})^3$

VOL = new volume $(\text{inches})^3$

X = quality
H = Enthalpy (But/lbm)
v = internal energy (But/lbm)

CAPU(I) = internal energy (Btu)

CAPVINT = initial internal energy (btu)

W = work
VF = specific volume (saturated liquid)
VG = specific volume (saturated vapor)
HF = enthalpy (saturated liquid)
HG = enthalpy (saturated vapor)
UF = internal energy (saturated liquid)
UG = internal energy (saturated vapor)

```
$LIST WHIRL
      C       THIS PROGRAM WILL DETERMINETHE REACTION OF THE ACTU-
      C       ATOR UPON THE ADDITION OF HEAT. THIS ANALYSIS ASSUMES
      C       THAT NO HEAT IS TRANSFERED TO OR FROM THE SURROUNDINGS
      C       READ INITIAL CONDITIONS, FREON 11 PROPERTIES
              DIMENSION PROP(22,6),CAPU(22),EXTEN(22)
              REAL INTERP
              DO 5 I=1,22
            5 READ (5,100) (PROP(I,J),J=1,6)
          100 FORMAT(6F10.6)
              WRITE(6,106) ((PROP(I,J),J=1,6),I=1,22)
          106 FORMAT(' ',6(2X,F10.6))
              DO 81 K=1,40
              READ(5,101) DIAM,FLENG,XINT,PRESSI,SPRK
              WRITE(6,88)
           88 FORMAT('1',2X,'THE DATA SET FOR THIS RUN IS:')
              WRITE(6,44)
           44 FORMAT('0',5X,'DIAMETER',3X,'FREELENGTH',2X,'INT.QUAL'
             1,2X,'INT.PRESS',2X,'SPRING K')
              WRITE(6,107) DIAM,FLENG,XINT,PRESSI ,SPRK
          107 FORMAT('0',2X,5(F10.4,1X))
          101 FORMAT(5F10.4)
              AREA=3.1459*DIAM**2/4
              VINT=AREA*FLENG/1728
              PRPRS=PRESSI-14.70
              FPRE=PRPRS*AREA
              SPRX=FPRE/SPRK
              CALL GIV(PRESSI,PROP,XINT,TSAT,SPVOL,HINT,UINT)
              FREMAS=VINT/SPVOL
              WRITE (6,50)
           50 FORMAT('4',1X,'EXTEN',2X,'VOL',2X,'PRESS',3X,'TSAT',
             13X,'X',3X,'ENTHALP',1X,'INTENER',2X,'SPECVOL',3X,
             2'WORK',3X,'HTTRAN')
              WRITE(6,52) VINT,PRESSI,TSAT,XINT,HINT,UINT,SPVOL
              WRITE(6,55) FREMAS,FPRE,SPRX
           55 FORMAT(' ',3X,'FREMAS=',F7.4,3X,'PRELOAD FORCE='
             1,F7.4,3X,'PRELOAD X =',F5.3)
           52 FORMAT('0',6X,F5.4,F7.3,1X,F6.2,1X,F4.3,1X,F7.3,
             11X,F7.3,1X,F9.6,2X)
      C       UINT IS THE INITIAL INTERNAL ENERGY
      C        NOW DETERMINE CHANGE AS VOLUME IS INCREASED
      C        INCREMENTALLY
              W=0
              Q=0
              DO 6 I=1,20
               EXTEN(I)=I*0.05
               DELTAV=AREA*EXTEN(I)/1728
              VOL=VINT+DELTAV
              SPVOL=VOL/FREMAS
              PRESS=PRESSI+EXTEN(I)*SPRK/AREA
      C       DETERMINE PROPERTIES
              CALL DETERM(PRESS,PROP,SPVOL,TSAT,X,H,U)
              CAPU(I)=FREMAS*U
              IF(I .GT. 1) GO TO 12
              CAPUNT=FREMAS*UINT
              U2U1=(CAPU(I)-CAPUNT)
              GO TO 13
           12 U2U1=CAPU(I)-CAPU(I-1)+U2U1
           13 CONTINUE
```

```
C     WORK DETERMINATION
C     IF YOU DESIRE THE INCREMENTAL WORK BETWEEN ANY TWO
C     EXTENSIONS,SIMPLY OMIT THE +W ON THE END OF THE
C     THE STATEMENTS FOR BB AND CC.
      X1=SPRX
      PATM=14.70
      BB=PATM*AREA*EXTEN(I)+W
      IF (I .GT. 1) GO TO 14
      W=.5*SPRK*((X1+EXTEN(I))**2-(X1)**2)+BB
      GO TO 15
   14 ZZ=.5*SPRK*(X1+EXTEN(I-1))**2-.5*SPRK*X1**2
      CC=PATM*AREA*(EXTEN(I)-EXTEN(I-1))+W
      W=.5*SPRK*((X1+EXTEN(I))**2-X1**2)-ZZ+CC
   15 CONTINUE
      Q=U2U1+W/(12*778)
      WRITE(6,51) EXTEN(I),VOL,PRESS,TSAT,X,H,U,SPVOL,W,Q
   51 FORMAT(' ',1X,F4.2,1X,F5.4,F7.3,1X,F6.2,1X,F4.3,
     11X,F7.3,1X,F7.3,1X,F9.6,1X,F7.3,1X,F7.4)
    6 CONTINUE
   81 CONTINUE
      END
      SUBROUTINE DETERM(PRESS,PROP,SPVOL,TSAT,X,H,U)
      DIMENSION PROP(22,6)
      REAL INTERP
      TSAT=INTERP(PRESS,PROP,2,1)
      VF=INTERP(PRESS,PROP,2,3)
      VG=INTERP(PRESS,PROP,2,4)
      HF=INTERP(PRESS,PROP,2,5)
      HG=INTERP(PRESS,PROP,2,6)
      UF=HF-144*PRESS*VF/778
      UG=HG-144*PRESS*VG/778
      X=(SPVOL-VF)/(VG-VF)
      H=HF+X*(HG-HF)
      U=UF+X*(UG-UF)
      RETURN
      END
      REAL FUNCTION INTERP(VALUE,PROP,IX,IY)
      DIMENSION PROP(22,6)
      DO 10 I=1,22
      IF(PROP(I,IX) .LT. VALUE) GO TO 9
      GO TO 11
    9 CONTINUE
   10 CONTINUE
   11 YL=PROP(I-1,IY)
      YH=PROP(I,IY)
      XL=PROP(I-1,IX)
      XH=PROP(I,IX)
      INTERP=YL+(YH-YL)*(VALUE-XL)/(XH-XL)
      RETURN
      END
      SUBROUTINE GIV(PRESS,PROP,XINT,TSAT,SPVOL,H,U)
      DIMENSION PROP(22,6)
      REAL INTERP
      TSAT=INTERP(PRESS,PROP,2,1)
      VF=INTERP(PRESS,PROP,2,3)
      VG=INTERP(PRESS,PROP,2,4)
      HF=INTERP(PRESS,PROP,2,5)
      HG=INTERP(PRESS,PROP,2,6)
      UF=HF-144*PRESS*VF/778
      UG=HG-144*PRESS*VG/778
```

```
      SPVOL=VF+XINT*(VG-VF)
      H=HF+XINT*(HG-HF)
      U=UF+XINT*(UG-UF)
      RETURN
      END
END OF FILE
```

APPENDIX B

DATA REDUCTION PROGRAM

```
$LIST PROPCALC
C THIS PROGRAM IS FOR THE REDUCTION OF THE TEST DATA OBTAINED
C ED. THE PRESSURE IS FEED IN AS GAUGE AND EXTENSION IN IN.
      DIMENSION PROP(22,6)
      REAL INTERP
      DO 5 I=1,21
    5 READ (5,100) (PROP(I,J),J=1,6)
  100 FORMAT(6F10.6)
      WRITE(6,106) ((PROP(I,J),J=1,6),I=1,21)
  106 FORMAT(' ',6(2X,F10.6))
      WRITE(6,39)
   39 FORMAT('4',2X,'EXTEN',2X,'VOLUME',3X,'PGAUGE',3X,
     1'PABSOL',4X,'TSAT',3X,'SPECVOL',1X,'QUALITY',2X,
     2'ENTROPY')
C NOTE THAT ON THE 'DO 81' STATEMENT THAT 100 IS THE
C LIMIT ON THE NUMBER OF DATA SETS.
      DO 81 K=1,100
      READ(5,333) PRESSI,EXTEN
  333 FORMAT(2F10.5)
      DIAM=2.875
      AREA=3.1459*DIAM**2/4
      VINT=1.96/1728
      FREMAS=0.103
C  FOR THIS SPECIAL CASE DIAM=2.875;VINT=1.96 IN3;AND
C  FREON MASS EQUALS .103LBM.
      VOL=VINT+AREA*EXTEN/1728
      SPVOL=VOL/FREMAS
      PRESS=PRESSI+14.696
      CALL DETERM(PRESS,PROP,SPVOL,TSAT,X,S)
      WRITE(6,92) EXTEN,VOL,PRESSI,PRESS,TSAT,SPVOL,X,S
   92 FORMAT(' ',2X,F5.4,2X,F6.5,2X,F7.4,2X,F7.4,2X,F7.3
     1,2X,F8.7,2X,F5.4,2X,F8.7)
   81 CONTINUE
      END
      SUBROUTINE DETERM(PRESS,PROP,SPVOL,TSAT,X,S)
      DIMENSION PROP(22,6)
      REAL INTERP
      TSAT=INTERP(PRESS,PROP,2,1)
      VF=INTERP(PRESS,PROP,2,3)
      VG=INTERP(PRESS,PROP,2,4)
      SF=INTERP(PRESS,PROP,2,5)
      SG=INTERP(PRESS,PROP,2,6)
      X=(SPVOL-VF)/(VG-VF)
      S=SF+X*(SG-SF)
      RETURN
      END
      REAL FUNCTION INTERP(VALUE,PROP,IX,IY)
      DIMENSION PROP(22,6)
      DO 10 I=1,22
      IF(PROP(I,IX) .LT. VALUE) GO TO 9
      GO TO 11
    9 CONTINUE
   10 CONTINUE
   11 YL=PROP(I-1,IY)
      YH=PROP(I,IY)
      XL=PROP(I-1,IX)
      XH=PROP(I,IX)
      INTERP=YL+(YH-YL)*(VALUE-XL)/(XH-XL)
      RETURN
      END
END OF FILE
```

AN EXPERIMENTAL TESTING PROCEDURE FOR DETERMINING HEATER CHARACTERISTICS

Robert Handler

INTRODUCTION

Recent advances in the development of heating elements have allowed new design approaches to drying devices for clothes dryers. An important problem is the determination of heater performance characteristics. Once these characteristics are known, then the effect on performance caused by changes in heater configuration must be found.

The purpose of this research is to develop a testing procedure which can be used to determine heater performance characteristics. Three major characteristics described the effectiveness of a heater. These are:

(1) thermodynamic heater efficiency, ηth
(2) velocity profile at heater exit, $V_2(x,y)$
(3) temperature profile at heat exit, $T_2(x,y)$

It should be noted that for some applications, only one of these characteristics may be important while in others all three may be important. It will be shown, for instance, that all heaters have high thermal efficiency (greater than 90%) and hence efficiency may be relatively unimportant factor in the selection of any particular heater.

Once a method has been developed to determine these characteristics, changes can be made to improve one or more of these characteristics. For instance, altering the heater element configuration will change $T_2(x,y)$ and $V_2(x,y)$ while leaving ηth unchanged. Insulation or holes placed in the side of the heater box will change ηth but leave $T_2(x,y)$ and $V_2(x,y)$ unchanged.

THERMODYNAMIC ANALYSIS

The system analyzed is shown in Figure 1. Air flows past the heating element from the inlet, station 1, to the outlet, station 2. The electrical input power, $\dot{E}$, acts to heat the air while the heat loss rate, $\dot{L}$, represents the losses across the walls of the duct.

The energy equation, for steady state conditions, simply states that the difference between the heat input rate and the heat loss rate is equal to the difference between the leaving and entering enthalpy flux. Then

$$\dot{E} - \dot{L} = \dot{H}_2 - \dot{H}_1 \tag{1}$$

where $\dot{H}_1$ = entering enthalpy flux

$\dot{H}_2$ = leaving enthalpy flux

Since the fluid passing through the heater is a mixture of dry air and water vapor, the expression for the leaving enthalpy flux is the sum of the enthalpy flux of the dry air, $\dot{H}_{A_2}$, and the water vapor, $\dot{H}_{V_2}$

$$\dot{H}_2 = \dot{H}_{A_2} + \dot{H}_{V_2} \tag{2}$$

Now the leaving enthalpy flux of the dry air is written as

$$\dot{H}_{2_A} = \int h_{2_A} \, d\dot{m}_2$$

where h_{2_A} = enthalpy of dry air

$d\dot{m}_2$ = differential mass flux

But enthalpy is the product of the specific heat at constant pressure, CP_{A_2} and the temperature, $T_2(x,y)$. Note that the exit temperature is a function of position at the duct exit station. Thus

$$h_{2_A} = CP_{A_2} \, T_2(x,y)$$

The mass flow rate, in differential form, is the product of the density of the air, ρA_2 its velocity, $V_2(x,y)$, (velocity is also a function of positions at the duct exit station) and the differential area, dA_2. Combining these several expressions results in the expression for the enthalpy flux of the air at the exit station as

$$\dot{H}_{2_A} = \int_{A_2} CP_2 \, T_2(x,y) \, \rho A_2 \, V_2(x,y) \, dA_2 \tag{3}$$

In a manner similiar to that used for air, the enthalpy flux of the water vapor is written as the

$$\dot{H}_{2_V} = \int h_{2_V} \, dm_{V2}$$

But now the definition of absolute humidity, ω, is introduced as the ratio of the mass flow rate of water vapor and the mass flow rate of dry air as

$$\omega \equiv \frac{\dot{m}_V}{\dot{m}_A}$$

then the differential mass flow rate of water vapor is related to that for air as

$$d\dot{m}_{V_2} = \omega \, d\dot{m}_{A_2}$$

And the enthalpy of water vapor is written in terms of the specific heat of water vapor, CP_{V_2}, and the temperature, $T_2(x,y)$ as

$$h_{V_2} = CP_{V_2} \, T_2(x,y)$$

Combining these several equations then results in the expression for water vapor ethalpy flux at the exit station as

$$\dot{H}_{V_2} = \int_{A_2} CP_{V_2} \, T_2(x,y) \, \omega_2 \, \rho A_2 \, V_2(x,y) \, dA_2 \tag{4}$$

Since as described, in equation (2), the total exit enthalpy flux is the sum of the vapor and air enthalpy fluxes, equations (3) and (4) are combined to give

$$\dot{H}_2 = \int_{A_2} [CP_{A_2} T_2(x,y)\ \rho A_2\ V_2(x,y) + \omega\ CP_{V_2}\ T_2(x,y)\ \rho A_2\ V_2(x,y)]\ dA_2$$

which is simplified to

$$\dot{H}_2 = \int CP_{A_2} \left(1 + \omega_2 \frac{CP_{V_2}}{CP_{A_2}}\right) \rho_{A_2}\ T_2(x,y)\ V_2(x,y)\ dA_2 \qquad (5)$$

A similar approach is used to develop an expression for the entering enthalpy flux at station 1. The problem is considerably simplied by the assumption of uniformity of both the velocity and temperature profiles at the inlet station. Thus the enthalpy flux is the sum of the enthalpy fluxes of the air and water vapor as

$$\dot{H}_1 = \dot{H}_{A_1} + \dot{H}_{V_1}$$

And the air enthalpy flux, $\dot{H}_{A_1}$ is written as the integral of the enthalpy and the mass flow rate

$$\dot{H}_{A_1} = \int_{A_1} h_{A_1}\ d\dot{m}A_1$$

where the differential for the mass flow rate is the product of the density, velocity, and differential area as

$$d\dot{m}_{A_1} = \rho A_1\ V_1\ dA_1$$

Thus the expression for the enthalpy flux of the entering air is

$$\dot{H}_{A_1} = \int CP_{A_1}\ T_1\ \rho A_1\ V_1\ dA_1$$

but since none of the variables are a function of position station 1 the integral can be written directly as

$$\dot{H}_{A_1} = CP_{A_1} T_1 \rho A_1 V_1 A_1$$

Note that the mass flow rate, $\dot{m}_{A_1}$, is the product of density, velocity, and area,

$$\dot{m}_{A_1} = \rho A_1 V_1 A_1$$

Then

$$\dot{H}_{A_1} = CP_{A_1} T_1 \dot{m}_{A_1} \tag{6}$$

Now the expression for the enthalpy flux of the entering water vapor is written as

$$\dot{H}_{V_1} = CP_{V_1} T_1 \omega_1 \dot{m}_{A_1} \tag{7}$$

Combining equations (6) and (7) for entering enthalpy flux gives

$$\dot{H}_1 = CP_{A_1} T_1 \dot{m}_{A_1} + CP_{V_1} T_1 \omega \dot{m}_{A_1}$$

or

$$\dot{H}_1 = m_{A_1} T_1 CP_{A_1}\left(1 + \omega_1 \frac{CP_{V_1}}{CP_{A_1}}\right) \tag{8}$$

Finally equations (5) and (8) for the enthalpy fluxes at both inlet and exit stations can be combined with the first law of thermodynamics, equation (1), to yield

$$\dot{E} - \dot{L} = CP_{A_1}\left(1 + \omega_2 \frac{CP_{V_2}}{CP_{A_2}}\right) \int \rho A_2 \, T_2(x,y) \, V_2(x,y) \, dA_2 - \dot{m}_{A_1} T_1 \, CP_{A_1}\left(1 + \omega_1 \frac{CP_{V_1}}{CP_{A_1}}\right)$$

Some further simplifying assumptions are now made concerning the constant specific heats of both air and water vapor along with the realization that the water vapor content of the air, as it passes through the heater remains unchanged and hence the absolute humidity is also constant. Thus

$$CP_A = CP_{A_1} = CP_{A_2}$$

$$CP_V = CP_{V_1} = CP_{V_2}$$

$$\omega = \omega_1 = \omega_2$$

So that a final expression can be written for the first law which is:

$$\dot{E} - \dot{L} = CP_A \left(1 + \omega \frac{CP_V}{CP_A}\right) \int \rho A_2 [T_2(x,y) - T_1]\, V_2(x,y)\, dA_2 \tag{9}$$

The humidity term can be interpreted as a correction term since its value is generally quite small as compared with unity.

Thermal efficiency ηth can be defined as

$$\eta th = \frac{\dot{H}_2 - \dot{H}_1}{\dot{E}} \tag{10}$$

Thermal efficiency can them be written in terms of enthalpy fluxes using equation (9) as

$$\eta th = \frac{CP_A \left(1 + \omega \frac{CP_V}{CP_A}\right) \rho A_2 [T_2(x,y) - T_1]\, V_2(x,y)\, dA_2}{\dot{E}}$$

EXPERIMENTAL PROCEDURE

INSTRUMENTATION AND EXPERIMENTAL DESIGN

Following the directions indicated by the thermodynamics analysis, a system was designed and constructed to evaluate the various terms of the thermal efficiency definition. A system schematic is shown in Figure 2, a photograph of the installation in Figure 3. Air is drawn into the inlet, through the heater box, and finally through the blower. The inlet ducting is used to develop uniform flow uniformity. A "Flexatherm" resistance heating element was used in the heater. A measuring station is located downstream of the heater to obtain air velocity and temperature information as a function of position.

The inlet temperature, T_1, is measured with a mercury thermometer, the exit temperature, $T_2(x,y)$, and velocity, $V_2(x,y)$ are measured at various x and y positions by means of probes mounted on a traverse mechanism as shown in Figure 4. The mechanism consists of a moveable upper plate, used to vary the x-position, and a gear system used to vary the y position of the probes. Figure 5 shows the inside of the traverse. A total head tube and an iron-constantan thermocouple are located so that velocity and temperature can be measured at the same location.

Read-out devices are shown in Figure 6. The three principal measurements Y, T_2 and ΔP_2 are taken to a three-position switch. The thermocouple signal, in millivolts, is amplified so that all three signals are in the 0- to 1-volt range. Voltage and current delivered to the heating element are measured by voltmeters and ammeters as shown in Figure 6. Regulation of current and voltage for the low power heater (approximately 1 kw) was accomplished by use of a variable transformer. Figure 7 shows the actual instrumentation used. A H-P 3440A digital voltameter was used to measure the output signals. An instrumentation amplifier was used for the thermocouple signal. A sensitive manometer (Baraocell electronic manometer, CGS Corp.) was used to measure the difference in total and static pressure heads of the velocity signal.

Figures 8 and 9 show methods used in the measurement of temperature and position. Temperature can be measured as a differential quantity as indicated in Figure 8. The digital voltmeter then indicates the temperature differential, $T_2 - T_1$. The vertical or y position of the velocity and temperature traverse was measured as shown in Figure 9. The shaft on the traverse mechanism is attached to a potentiometer and a change in shaft position then changes the output voltage. This voltage is then converted into a y-position value.

The velocity, V_x, of the air at station x if found from the difference between the total and static pressures according to Bernoulli's equation as

$$V_x = \sqrt{3440\ T_x\ \Delta P_x / P_x}$$

where T_x = the absolute temperature measured at station x, deg. R.

ΔP_x = the pressure difference between the total and static pressures, psi

P_x = the static pressure at station x, psia

TEST PROCEDURE

The procedure for obtaining data for a typical run is outlined:

(1) Turn on blower
(2) Turn on flexatherm heater and wait until temperature has stabilized
(3) Record heater current, and voltage
(4) Record static pressures at inlet and exit
(5) Record barometric pressure, wet and dry bulb temperatures
(6) Proceed to take temperature and velocity reading at the outlet of the heater by setting the x-position and varying y.

This procedure, assuming 20 to 30 measurements of outlet velocity and temperature are taken, takes approximately 15 to 20 min to complete. It should also be noted that due to the turbulence of the flow, the velocity and temperature at the outlet are both periodic functions of time. Thus,

$$T_2(x,y,t) = T_{2_{AV}}(x,y) \sin(\omega t + \psi)$$

$$V_2(x,y,t) = V_{2_{AV}}(x,y) \sin(\omega t + \psi)$$

Since only the average or steady value of these measurements is desired, they can be obtained by removing the oscillatory position of the signal using a highly damped meter.

RESULTS AND DISCUSSION

SMALL FLEXATHERM HEATER

Data were taken for the small heater in 0.1-volt increments in the vertical or y-direction at the heater exit for values of x-locations of 0.125, 0.25, 0.50, 0.75, 1.0, 1.25, 1.75, 2.0, 2.25, and 2.375 in. Since the computer program is written for data at 16 arbitrary locations at the heater exit, and y-readings used for the computer input were those at 0.4, 0.7, 1.0, and 1.3 volts, and x-values of 0.5, 1.0, 1.5, and 2.0 in. Using these values of temperature and velocity, then enthalpy fluxes/ft^2 for each x-station are found. A linear integration of these values then gives an approximation to the three-dimensional volume as indicated in Figure 10. The linearization, particularly near the wall, of the temperature and velocity profiles, resulted in a considerable error in the final determination of thermal efficiency as indicated in Figure 11. The actual enthalpy flux profile is probably as shown; while the linear interpolation between two adjacent points gives a lower value than is acutally the case. Thus the thermal efficiency of 52% for the small heater as found from the computer calculations is clearly erroneous. A better approximation to the actual efficiency can be obtained by realizing that:

$$\text{net enthalpy flux} = \rho A V_{AV}\ CP(TAV_2 - TAMBIENT)$$

or

$$\text{net enthalpy flux} = (0.073\ \frac{LBM}{FT3})\ (\frac{2.75 \times 4.50}{144}\ FT^2)\ (19.5\ \frac{FT}{SEC})$$

$$(.24\ \frac{B}{LBM\text{-}°F})\ (110.7 - 80) = 0.91\ B/SEC$$

$$\text{Then;}\ \eta th = \frac{\text{NET ENTHALPY FLUX}}{\dot{E}} = \frac{.91}{.92} = 0.99$$

More accurate values of thermal efficiency can be obtained by developing a computer program which will take into account all data taken (as well as that in the thermal boundary layer near the walls of the test section). Typical plots of velocity and temperature versus vertical position for X - 1.0 in. are shown in Figures 12 and 13.

LARGE FLEXATHERM HEATER

Various runs were made with the large flexatherm heater. One run was taken with no insulation, another run was also made with holes punched in the sides of the heater box. Computer output is shown in the Appendix for these runs. Again, Compute calculated efficiencies are seen to be about 50%. These erroneous values of efficiency are easily corrected when the assumption about the thermal boundary layer is recognized. Corrected calculations reveal efficiencies well above 90%. Methods used in such corrected calcualtions are shown in the Appendix.

CONCLUSIONS

It is evident that from an examination of the plots of Figure 12 and 13 that there is considerable variation in velocities and temperatures at the outlet of the heater. Velocities are seen to vary as much as 15 ft/sec over the full range of y-positions. Similarly, temperatures are seen to vary as much at 30°F over the range of y-positions. The implications are clear:

(1) That experimental velocity and temperature profiles, because of the complex nature of the system under consideration, must be obtained in order to develop accurate values for the thermal efficiency;

(2) The irregularities in temperature and velocity fields indicate that flow blockage is occurring and that hot spots are present at the exit of the heater.

PRACTICAL IMPLICATIONS

The practical implications of this study are these:

(1) That such a well-defined, accurate, and quick procedure as developed and described in this report should be of aid in the design and improvement of heating elements.

(2) That the criterion of thermal efficiency as a basis for the judgment of heater performance may be erroneous as it appears that heaters are by nature extremely efficient heat transfer devices.

(3) That the use of velocity and temperature profiles will clearly be of more use to the practicing engineer than efficiency determinations as they will indicate where, for example, the heater is likely to fail during prolonged use.

(4) Such a procedure will enable the engineer to use techniques of trial and error to determine optimum heater configurations (i.e., that which gives uniform temperature and velocity profiles).

In conclusion, it is hoped that this procedure will be of use to those engineers interested in the improvement of heating elements.

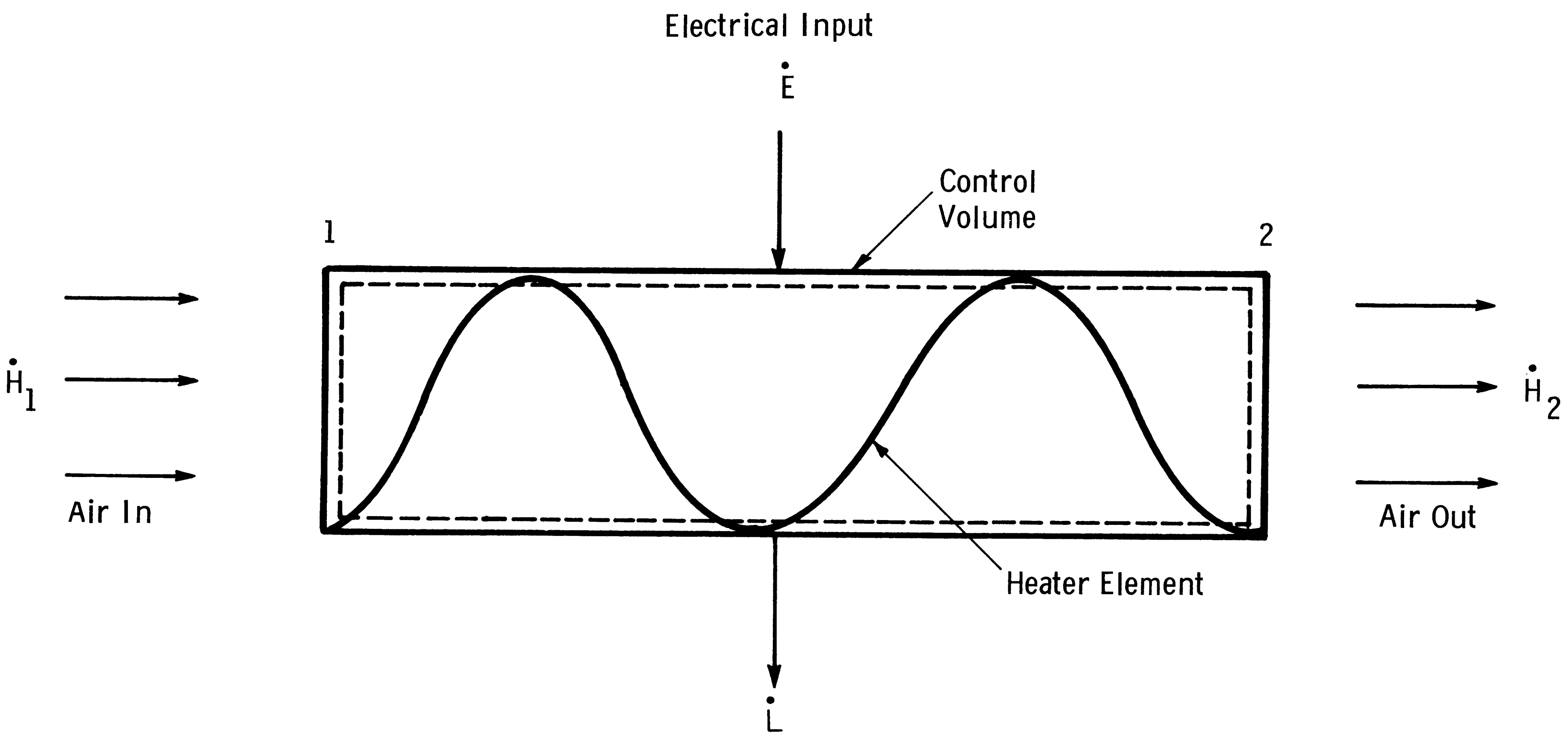

Figure 1. System schematic.

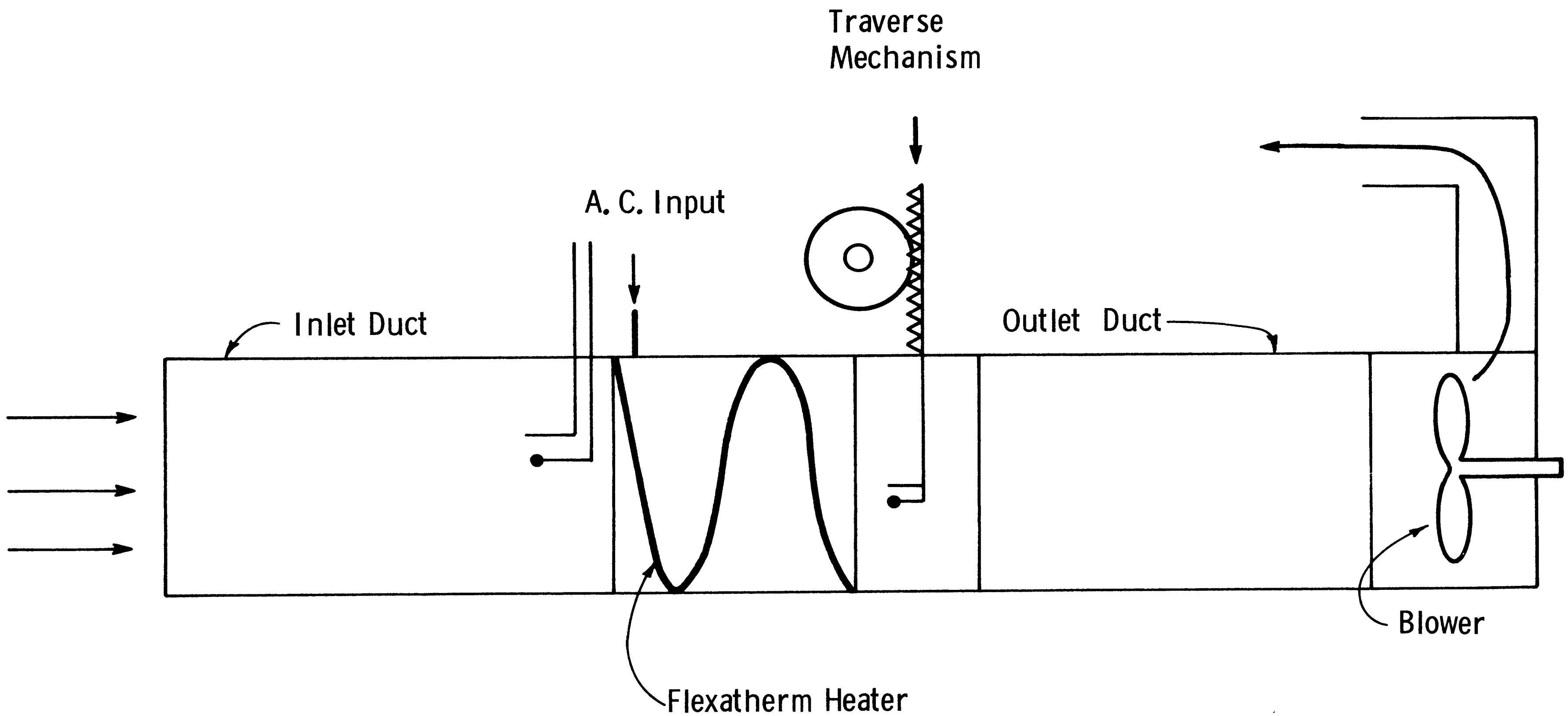

Figure 2. Experimental system.

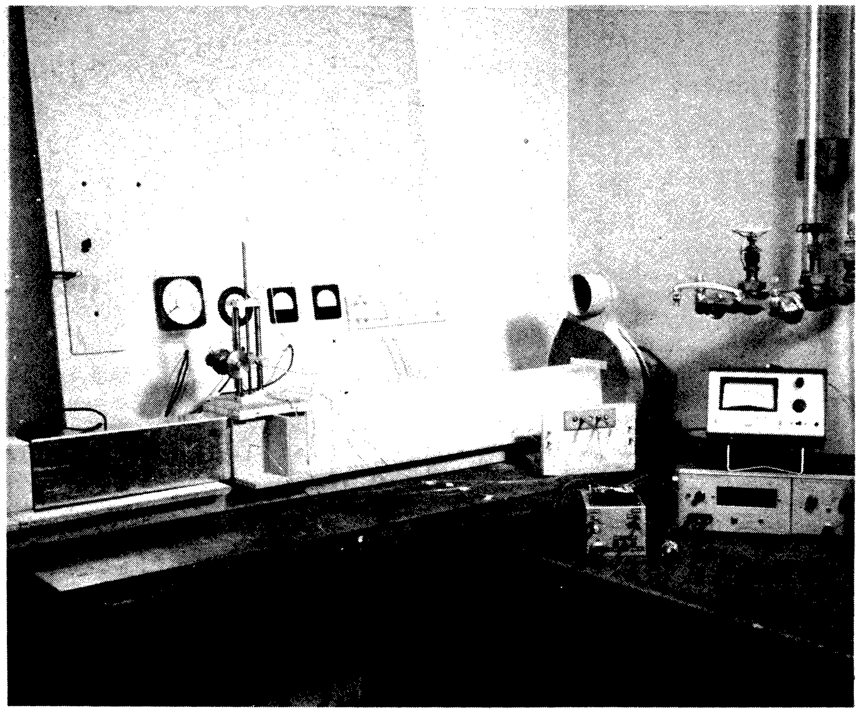

Figure 3. Overall view of experimental installation.

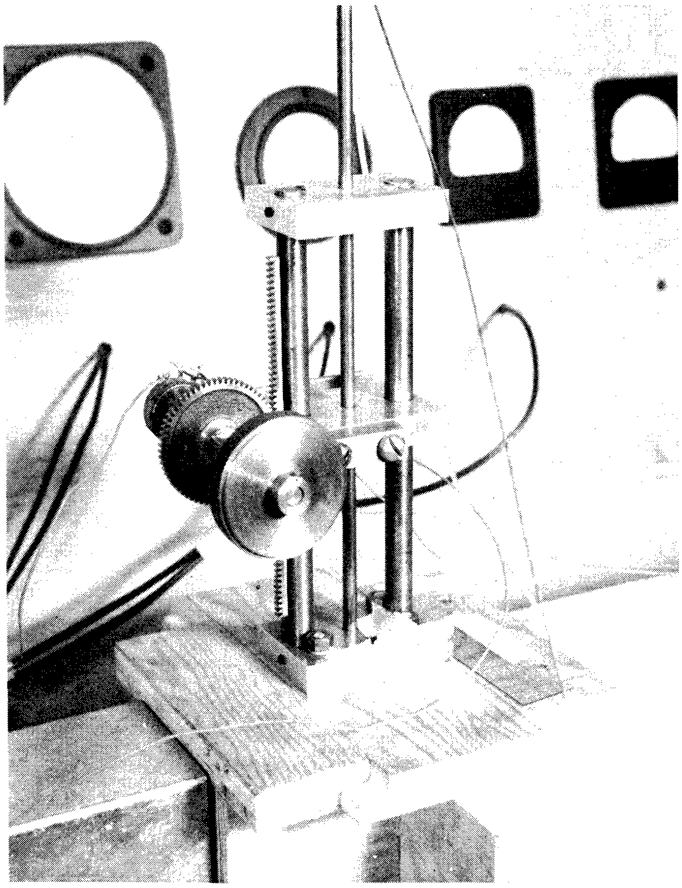

Figure 4. Traverse mechanism.

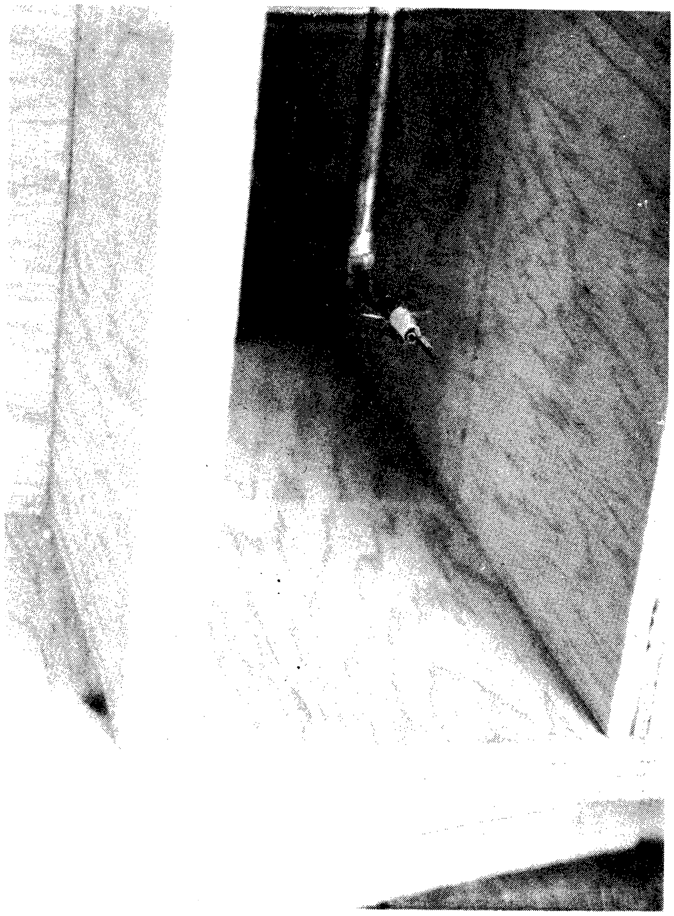

Figure 5. Close-up view of probes attached to traverse mechanisms.

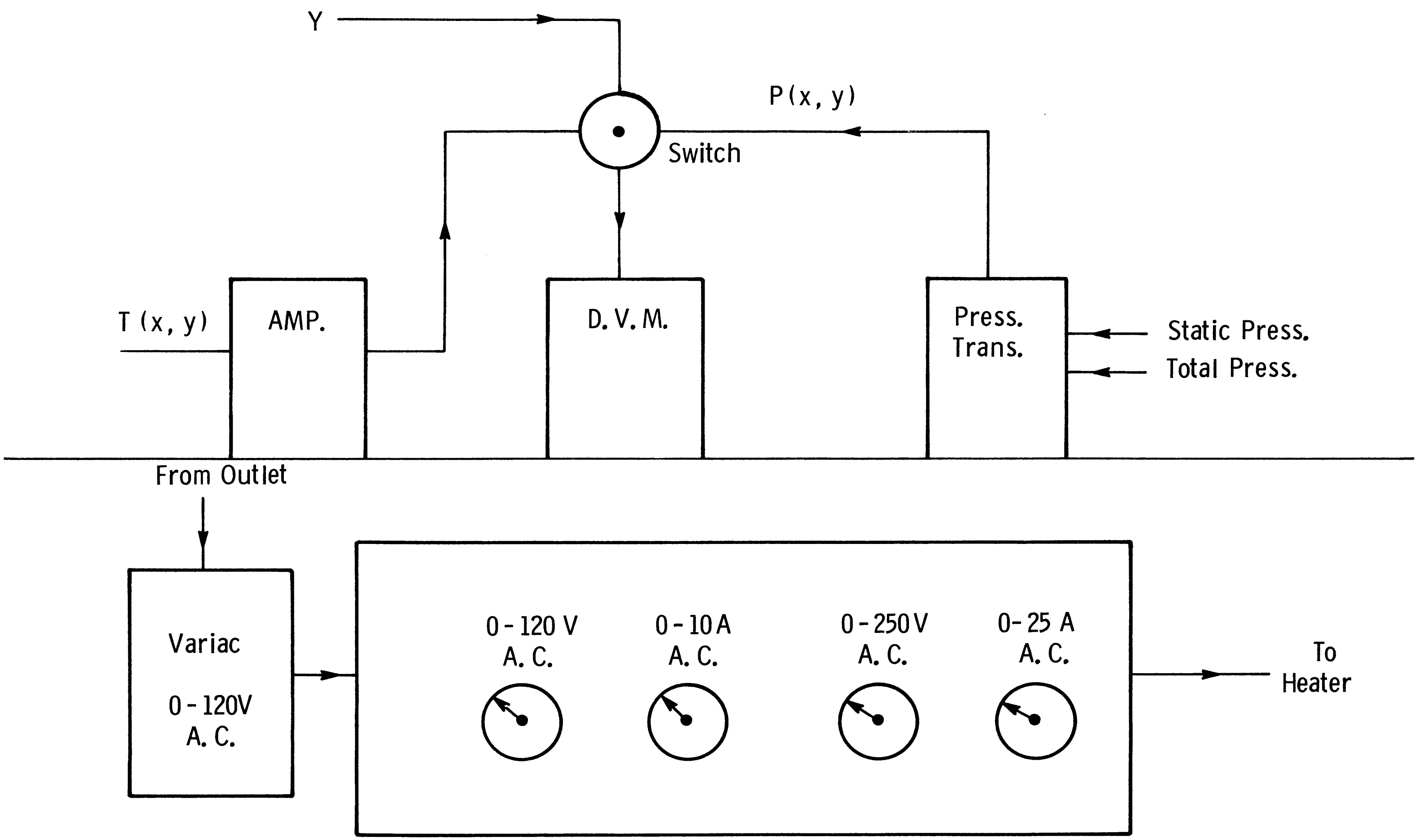

Figure 6. Read-out devices.

Figure 7. Instrumentation.

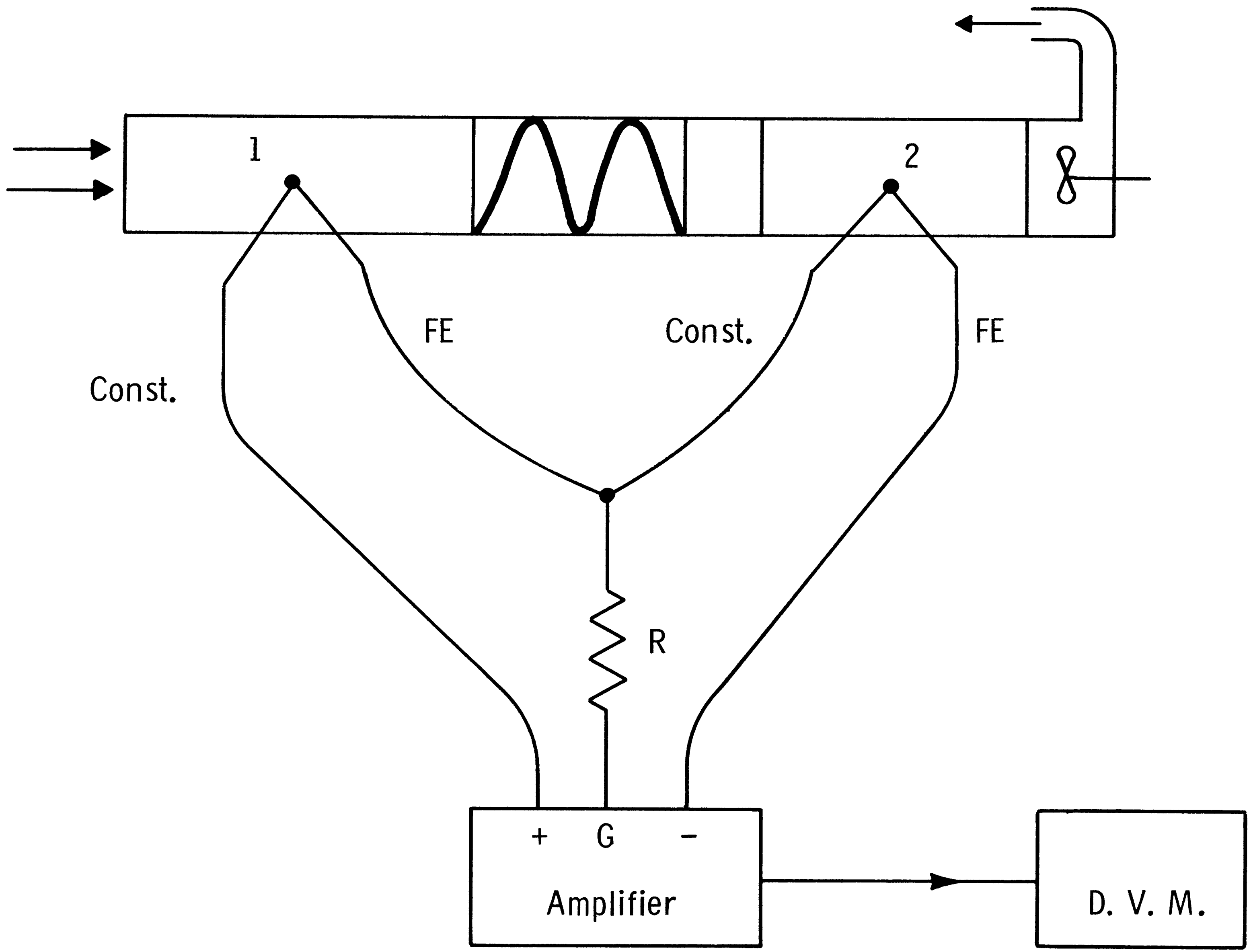

Figure 8. Temperature measurement.

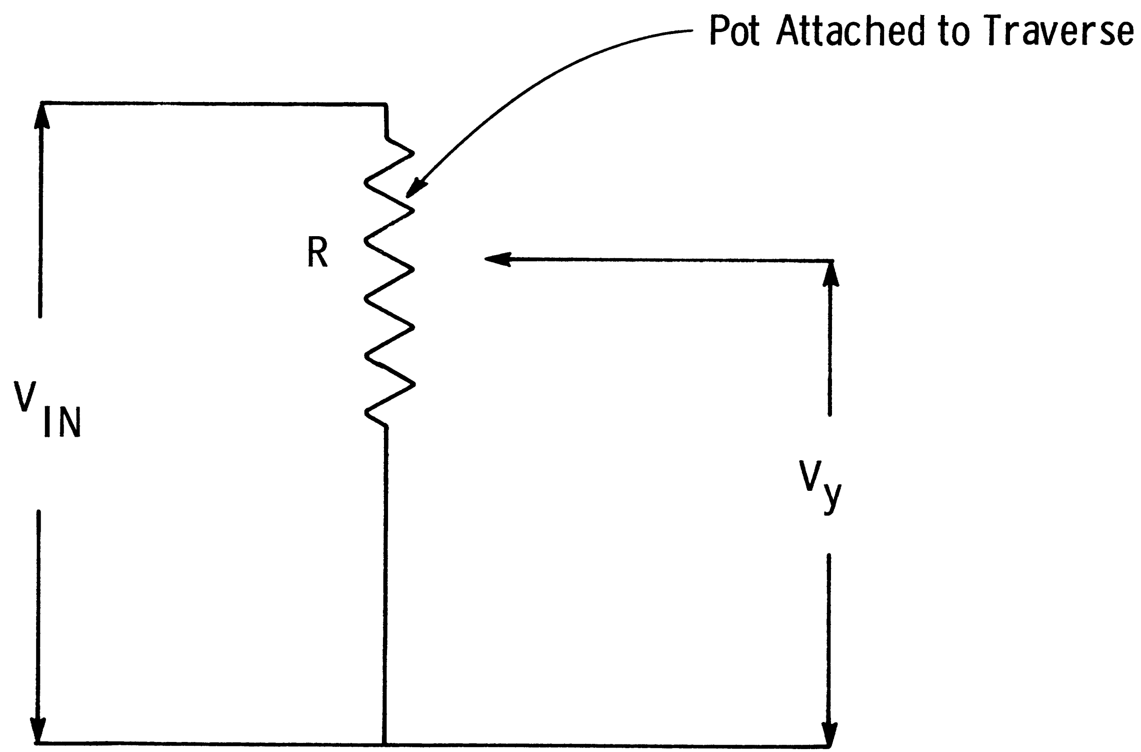

Figure 9. Vertical position transducer.

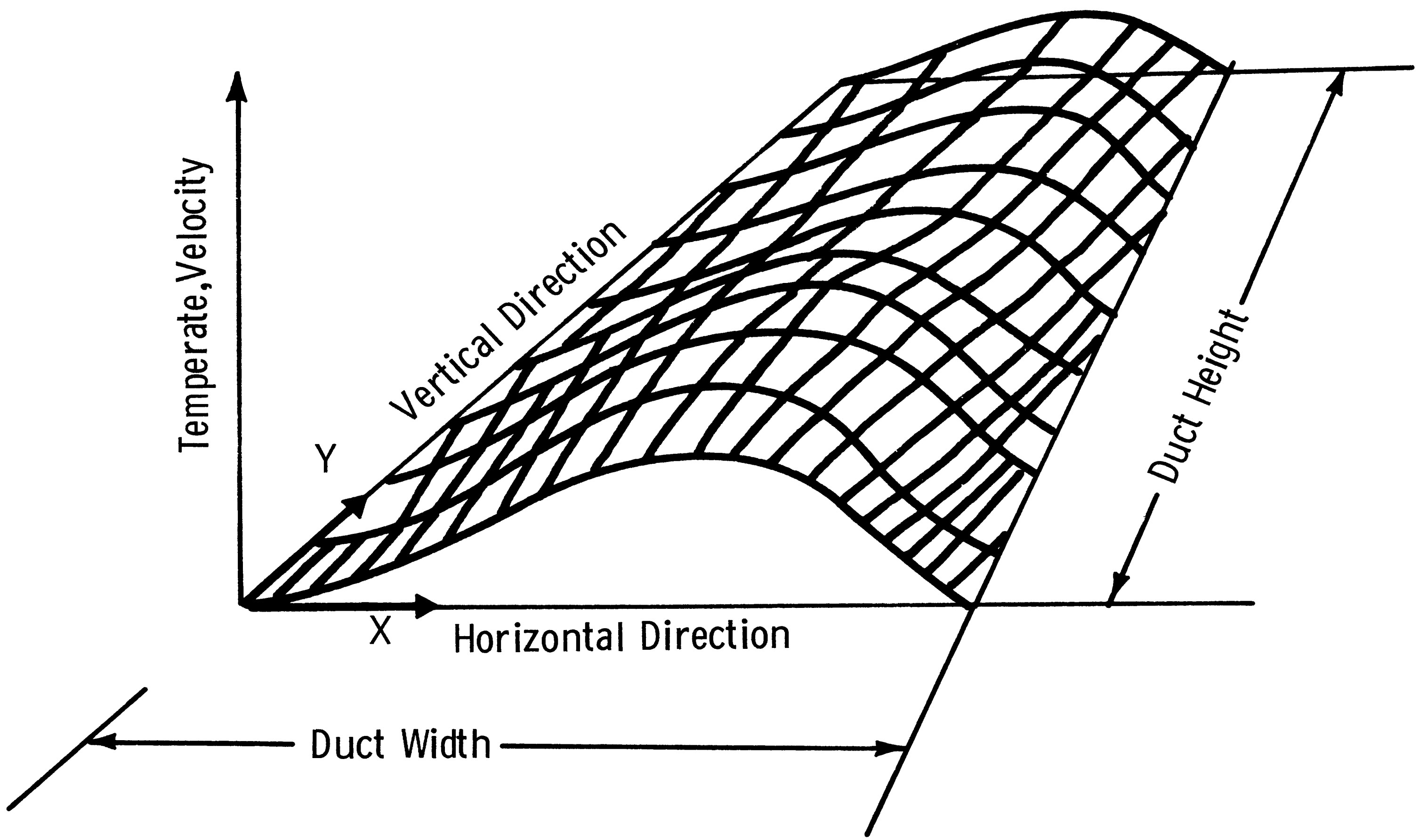

Figure 10. Sketch illustrating three-dimensionality of temperature and velocity field at heater exit.

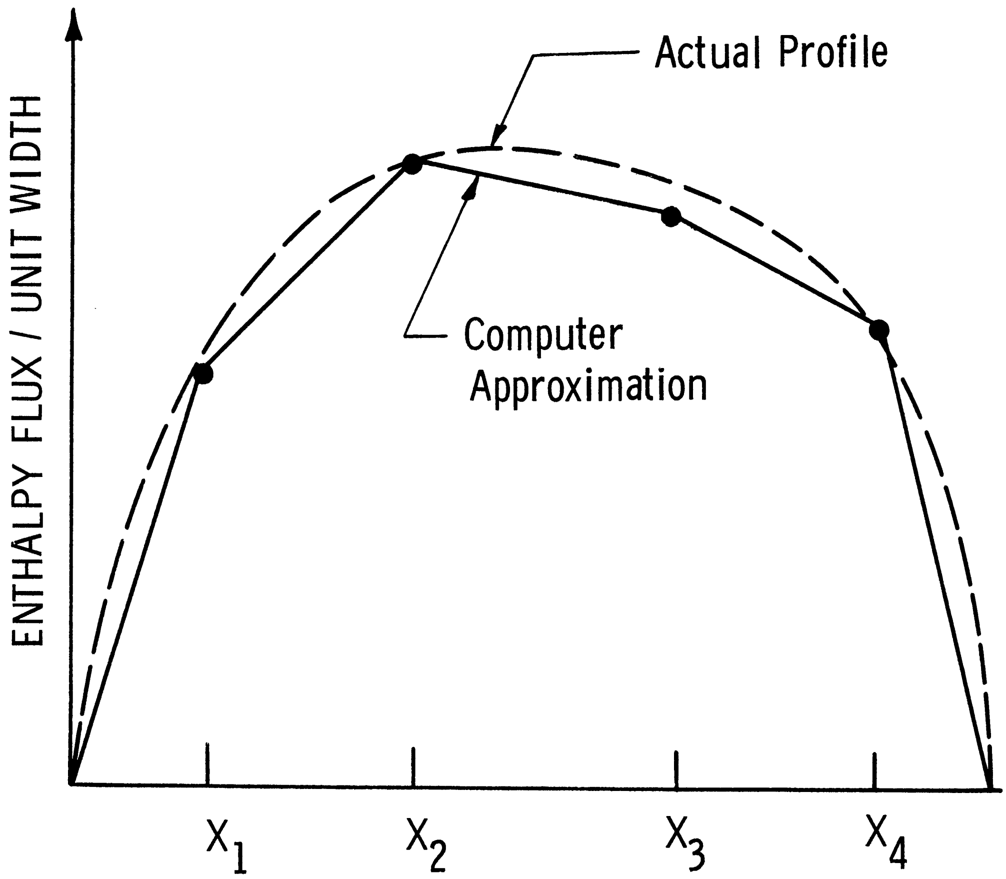

Figure 11. Example of the difference between actual profile and the linearized profile used in the computer program.

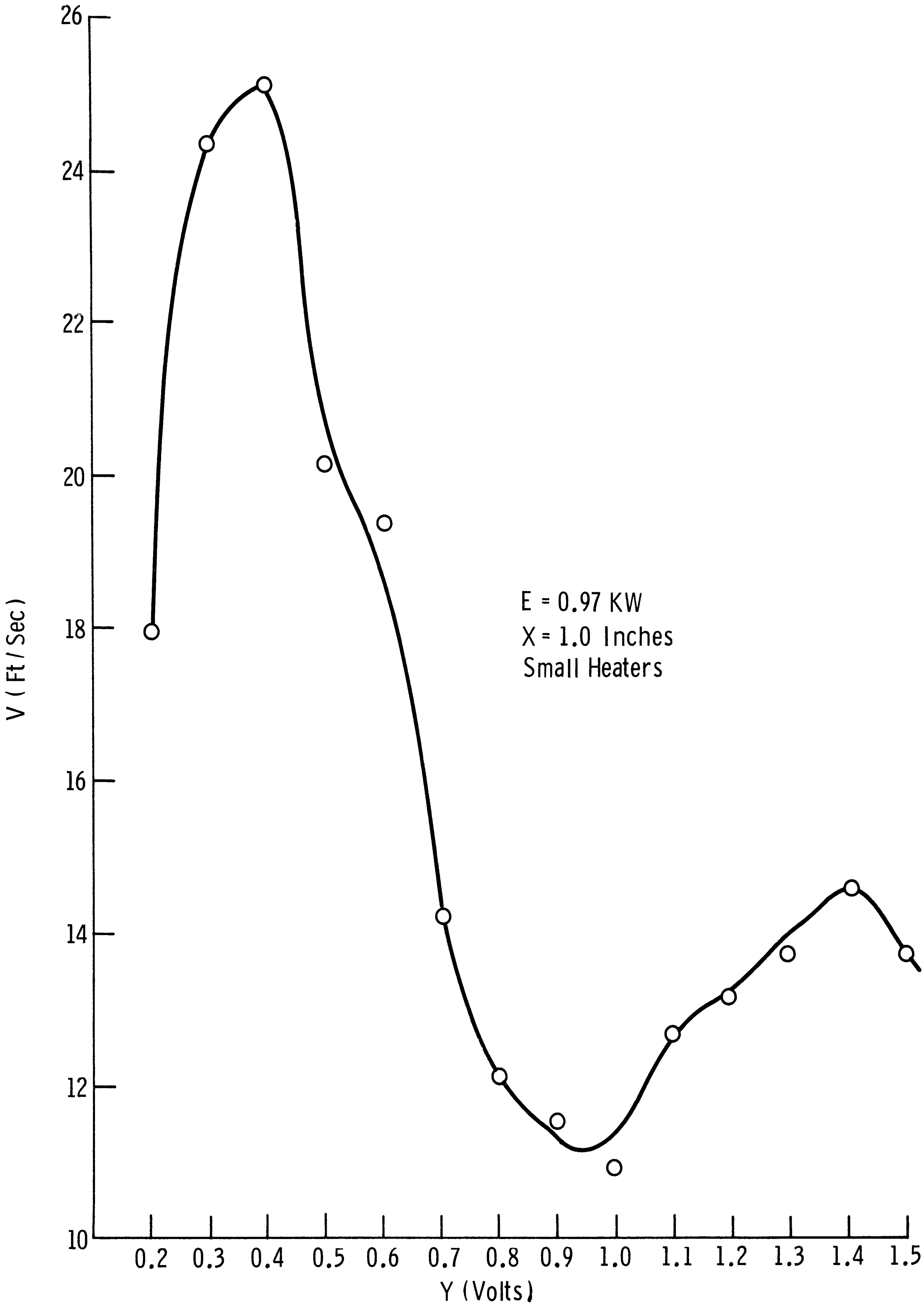

Figure 12. Typical velocity profile at heater exit.

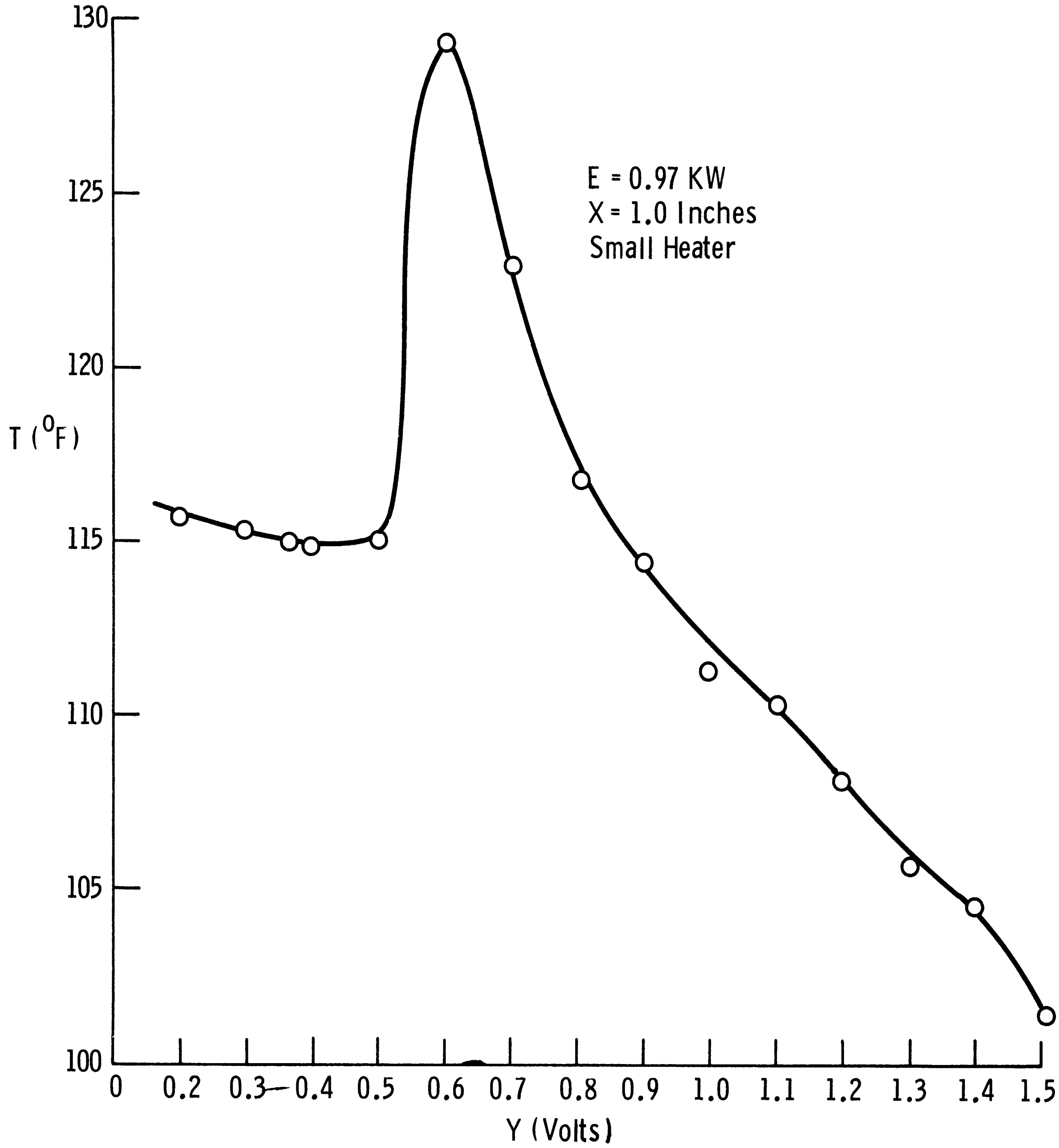

Figure 13. Typical temperature profile at heater exit.

APPENDIX

COMPUTER PROGRAM FOR DATA REDUCTION

In order to facilitate and to formalize the data reduction procedures, a computer program was written. Since the program was written after hand calculations had been made; the sequence and methods used in the program closely follow the conventional data reduction methods and procedures.

Certain tables were put into the program: steam tables (for determining humidity calculations), thermocouple voltage-temperature calibrations, and the calibration information for the y-position of the transducer as a function of the potentiometer voltage reading.

Then all of the data was read into the computer—the atmospheric conditions, electrical quantities, inlet conditions, along with the temperature and pressure quantities for each of 16 locations, in terms of voltages.

Data reduction then followed; first thermocouple voltages were converted into temperatures, y-position voltages converted into displacements in inches, the pressure differences between total and static pressures, in voltages, were converted into a velocity in ft/sec. Humidity calculations were next, followed by determinations of the enthalpy fluxes at the 16 stations at the heater exit. Next integration of these local enthalpy fluxes into a single, average or overall value was found.

Finally electrical quantities were used to find the heat supplied to the heater, and then thermal efficiency was found. After printing the majority of the information found, reduced, and calculated, the next set of data was read in and the calculation procedure repeated, until all input data had been processed.

A copy of the computer program, together with a set of data for the small heater is included. A list of the nomenclature follows.

NOMENCLATURE

Variable	Computer	Units
ΔP in	PR	psi
Y	YV	Volts
X	X	in.
	TV	Volts
Amps	Amp	Amps
Volts	Volts	Volts
Wet bulb	WB	°F
Dry bult	DB	°F
Baro	Baro	in. Hg
PStatic exit	PSE	psi
PS in	PSI	psi
TIN	TIN	°F
PVAP	PVAP	psi
TVAP	TVAP	°F
HVAP	HVAP	Btu/lbm
TMV	TMV	mv } Thermocouple
T acutal	TA	°F } calibration
YA	YA	in.
TB	YB	in./volt
	TCAL	mv/v
Duct width	XW	in.
Duct Depth	YD	in.
Exit vapor enthalpy	HV OUT	Btu/lbm

```
      REAL LOSS
       DIMENSION PR(16),YV(16),TV(16),T(16),Y(16),V(16),PVAP(10),
     1 HVAP(10),TMV(10),TACT(10),HVOUT(16), X(4),TVAP(10),
     2HFLUX(16),AREA(4)
       NAMELIST/EXPDAT/YV,TV,PR,WB,DB,BARO,AMPS,VOLTS,PSE, PSI,TCAL,YA,YB
       DATA PVAP,TVAP,HVAP,TMV,TACT,YD,XW,X/.36292,.94924,2.2230,4.7414,
     C9.340,17.186,29.840,49.200,77.693,117.992,70.,100.,130.,160.,190.,
     C220.,250.,280.,310.,340., 1092.1,1105.1,1117.8,1130.2,1142.1,1153.
     C4,1164.0,1173.8,1182.5,1190.1,1.07,1.94,2.82,3.71,4.61,5.51,6.42,7
     C.33,8.25,9.17,70.,100.,130.,160.,190.,220.,250.,280.,310.,340., 4.
     C4,2.5,.5,1.0,1.5,2.0/
      WRITE(6,33)
   33 FORMAT( '0STEAM TABLES. TEMPS,PRESSURE,ENTHALPY')
      WRITE(6,30)(TVAP(I),I=1,10)
      WRITE(6,30)(PVAP(I),I=1,10)
      WRITE(6,30)(HVAP(I),I=1,10)
      WRITE(6,34)
   34 FORMAT('0THERMOCOUPLE DATA, DEG   MILLIVOLTS')
      WRITE(6,30)(TACT(I),I=1,10)
      WRITE(6,30)( TMV(I),I=1,10)
      WRITE(6,30)( X(I),I=1,4)
   30 FORMAT(10F10.3/)
    5  READ (5,EXPDAT)
      WRITE(6,35)
   35 FORMAT('0RAW DATA, YPOSITION,THERMOCOUPLE, PRESSURE')
        WRITE (6,20) ((I,YV(I),TV(I),PR(I)),I=1,16 )
   20      FORMAT (I4,2F10.3,F10.4/)
      WRITE(6,36)
   36 FORMAT('0WET BULB,DRYBULB, BAROMETER,CURRENT,VOLTAGE,IN AND EXIT S
     1TATIC PRESSURE, TCAL, POSIT CAL,YA,YB')
        WRITE (6,21) WB,DB,BARO,AMPS,VOLTS,PSI,PSE,TCAL,YA,YB
   21     FORMAT(10F10.3/)
C  TEMP CON  MV TO DEG F
       DO 50 I=1,16
      TEMP=TV(I)*TCAL
      DO 49 J=1,9
      IF(TEMP.LT.TMV(J).OR.TEMP.GT.TMV(J+1)) GO TO 49
       T(I)=TACT(J)+(TACT(J+1)-TACT(J))/(TMV(J+1)-TMV(J))*(TEMP-TMV(J))
   49      CONTINUE
   50      CONTINUE
C  Y POSITION AND VELOCITY CONV
       DO 60 I=1,16
           Y(I)=YA+YB*YV(I)
   60     V(I)=SQRT(3440.*PR(I)*(460.+T(I))/(BARO*0.491+PSE))
      TAV=0.
      VAV=0.
      DO 22 I=1,16
      TAV=TAV+T(I)
   22 VAV=VAV+V(I)
      TAV=TAV/16.
      VAV=VAV/16.
       WRITE(6,23) TAV,VAV
   23 FORMAT(2F12.3/)
      WRITE(6,37)
   37 FORMAT('1CORRECTED DATA, POSITION,VELOCITY,TEMP')
      WRITE(6,31)((I,Y(I),V(I),T(I)),I=1,16)
   31 FORMAT(I4,2F10.3,F10.3/)
C  SATURATION PRESSURE VIA WB
      DO 70 I=1,9
      IF(WB.LT.TVAP(I).OR.WB.GT.TVAP(I+1)) GO TO 70
```

```
      PV=PVAP(I)+(PVAP(I+1)-PVAP(I))/(TVAP(I+1)-TVAP(I))*(WB-TVAP(I))
 70     CONTINUE
C  SPECIFIC HUMIDITY
          PAIR=BARO*14.691/29.92+PSE
      W=0.622*PV/(PAIR-PV)
C  INLET VAPOR ENTHALPY
     DO 71 I=1,9
     IF(DB.LT.TVAP(I).OR.DB.GT.TVAP(I+1)) GO TO 71
      HVAPIN=HVAP(I)+(HVAP(I+1)-HVAP(I))   /(TVAP(I+1)-TVAP(I))
    1/(DP-TVAP(I))
 71     CONTINUE
     WRITE(6,38)
 38  FORMAT('OVAPOR PRESSURE,AIR PRESS,SPEC. HUMID,INLET VAPOR ENTH.')
     WRITE(6,26)  PV,PAIR,W,HVAPIN
 26  FORMAT(4F12.4/)
C  OUTLET VAPOR ENTHALPY
        DO 72 J=1,16
      DO 73 I=1,9
     IF(T(J).LT.TVAP(I).OR.T(J).GT.TVAP(I+1)) GO TO 73
      HVOUT(J)=HVAP(I)+(HVAP(I+1)-HVAP(I))/(TVAP(I+1)-TVAP(I))*(T(J)-
    1TVAP(I))
 73     CONTINUE
 72        CONTINUE
C  OUTLET ENTHALPY FLUX--LOCAL
        DO 80 I=1,16
 80  HFLUX(I)=144./53.3*(PAIR-PV)/(460.+T(I))*V(I)*(0.24*(T(I)-DB)
    1+W*(HVOUT(I)-HVAPIN))
     WRITE(6,39)
 39  FORMAT('OEXIT VAPOR ENTH.   ENTHALPY FLUX')
     WRITE(6,32)((I,HVOUT(I),HFLUX(I)),I=1,16)
 32  FORMAT(I4,2F10.3/)
C  OUTLET ENTHALPY FLUX AT EACH X STATION
        DO 85 J=1,4
        I=1+4*(J-1)
 85  AREA(J)=(YD-Y(I))*HFLUX(I)/2.
    1+(Y(I+0)-Y(I+1))*(HFLUX(I+1)+HFLUX(I+0))/2.
    2+(Y(I+1)-Y(I+2))*(HFLUX(I+2)+HFLUX(I+1))/2.
    3+(Y(I+2)-Y(I+3))*(HFLUX(I+3)+HFLUX(I+2))/2.
    4+Y(I+3)*HFLUX(I+3)/2.
     WRITE(6,40)
 40  FORMAT('OENTHALPY FLUX/WIDTH AT FOUR X STAS.')
     WRITE(6,28) (AREA(I),I=1,4)
 28  FORMAT(4F10.4/)
C  OUTLET ENTHALPY FLUX
     ZETA=AREA(1)*X(1)/2.
    1+(X(2)-X(1))*(AREA(2)+AREA(1))/2.
    2+(X(3)-X(2))*(AREA(3)+AREA(2))/2.
    3+(X(4)-X(3))*(AREA(4)+AREA(3))/2.
    4+(XW-X(4))*AREA(4)/2.
     ZETA=ZETA/144.
       EDOT=AMPS*VOLTS*3.413/3600.
       LOSS=EDOT-ZETA
         EFF=ZETA/EDOT
     WRITE(6,41)
 41  FORMAT('OWATTS IN,LOSS,EFF.,NET HEAT FLUX')
     WRITE(6,28) EDOT,LOSS,EFF,ZETA
     GO TO 5
     END
```