

5D attractors with higher derivatives

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ABSTRACT: We analyze higher derivative corrections to attractor geometries in five dimensions. We find corrected $\text{AdS}_3 \times S^2$ geometries by solving the equations of motion coming from a recently constructed four-derivative supergravity action in five dimensions. The result allows us to explicitly verify a previous anomaly based derivation of the AdS_3 central charges of this theory. Also, by dimensional reduction we compare our results with those of the 4D higher derivative attractor, and find complete agreement.

KEYWORDS: Black Holes in String Theory, Black Holes.

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1. Introduction

The last few years have seen progress in our understanding of corrections to the entropy of black holes in string theory, both at the microscopic and macroscopic levels. On the supergravity side, this has meant studying the effect of higher derivative terms in the action [1–4]. 4D extremal black holes have a near horizon $\text{AdS}_2 \times S^2$ geometry, with moduli fixed by the attractor mechanism [5]. By using the corrected attractor solution and the general Wald entropy formula [6], it is possible to successfully match an infinite series of corrections to the Bekenstein-Hawking area law with the corresponding microscopic degeneracy of states [1–3, 7–12]. For reviews see [13–15].

However, on closer inspection this success actually seems quite mysterious, since only a selected subset of terms in the supergravity action are being used. Namely, one incorporates the supersymmetric completion of certain R^2 terms (as can be captured by corrections to the generalized prepotential), but neglects various R^4 and higher order terms, even though these *a priori* contribute at the order one is working. There is at present no 4D understanding of why these terms can be neglected.

Greater control is achieved by realizing that these black holes admit near horizon $\text{AdS}_3 \times S^2$ geometries [3]. To relate an $\text{AdS}_2 \times S^2$ geometry to $\text{AdS}_3 \times S^2$, one interprets one of the 4D gauge fields as coming from a Kaluza-Klein circle [16]. An $\text{AdS}_3 \times S^2$ region then appears provided that there is vanishing Kaluza-Klein monopole charge ($p^0 = 0$). Alternatively, one can study these black holes in the context of 5D supergravity.

By using the extra symmetries inherent in the 5D near horizon description, one finds that the corrected entropy formula is governed by the coefficients of the Chern-Simons terms in the supergravity action. In this way it is possible to bypass the need to find the full set of higher derivative terms, or to find the explicit values of the near horizon moduli. The key observation is that the entropy formula is controlled by the values of the left and right moving central charges of the associated 1 + 1 dimensional CFT, and due to supersymmetry these are completely determined by gauge and gravitational anomalies.

To verify this picture explicitly, and to find the corrected black hole geometry, one needs to work with the full 5D susy invariant four derivative action, corresponding to the supersymmetric completion of the four derivative Chern-Simons terms. This action appeared recently in [17]. In this paper we find the near horizon $\text{AdS}_3 \times S^2$ geometry by analyzing the BPS conditions and the equations of motion coming from this action. To do this, it is most efficient to employ the “ c -extremization” procedure developed in [3] (or the closely related “entropy-function” developed in [4]). The strategy is to write down a c -function whose critical points correspond to the solutions of the equations of motion. Furthermore, the value of the c -function at a critical point is equal to the average of the left and right moving central charges of the associated CFT. We will show that the result is in precise agreement with the values inferred from the supersymmetry/anomaly based argument, and thereby verify that the entropy is indeed controlled by the Chern-Simons terms. More generally, this same logic leads to the conclusion that there are no further corrections to the central charges from additional higher derivative terms (i.e. more than four derivatives), since we have already taken into account the full set of terms related by supersymmetry to the Chern-Simons terms.

Since this procedure also yields the values of the fixed 5D moduli, we can compare with the known results for the 4D moduli. Writing out the details of the reduction from 5D to 4D we find full agreement.

This article is organized as follows. In section 2 we review c -extremization. In section 3 we illustrate the procedure for the leading order action. In section 4 we analyze the supersymmetry conditions in an off-shell form that applies also when higher derivatives are taken into account. In section 5 we carry out the c -extremization procedure on the full action including all terms that are related to the Chern-Simons term by supersymmetry. We find results that are consistent with the supersymmetry conditions from section 4 and moreover find a central charge that agrees with the one previously found using supersymmetry and anomalies. In section 6 we compare our results with those found for black holes in four dimensions and find complete agreement.

2. Review of c -extremization

The problem of finding an $\text{AdS}_3 \times S^2$ solution¹ to a general higher derivative action can be reduced to the problem of extremizing a single function of the scale sizes and moduli [3]. Furthermore, the value of this function at its critical point is (after suitable normalization)

¹Or more generally an $\text{AdS}_p \times S^q$ solution

equal to the average of the left and right moving central charges of the asymptotic conformal symmetry group of the theory. In this section we review how this works.

We look for a solution respecting all $\text{AdS}_3 \times S^2$ isometries. Besides constant scalar fields, we can also have two-form fields proportional to the volume form on S^2 . More generally, we could also have three-form fields proportional to the AdS_3 volume form; these will not make any appearance in this paper, but we note that they would necessitate a modification of some of the following formulas.

Let the action for the theory be of the form

$$S = \frac{1}{4\pi^2} \int d^5x \sqrt{g} \mathcal{L} + S_{\text{CS}} + S_{\text{bdy}}, \tag{2.1}$$

with $G_5 = \frac{\pi}{4}$. Our trial solution takes the form

$$\begin{aligned} ds^2 &= \ell_A^2 ds_{\text{AdS}}^2 + \ell_S^2 d\Omega_2^2 \\ F^I &= \frac{p^I}{2} \epsilon_2 \\ v &= V \epsilon_2 \\ \phi^a &= \text{constant} . \end{aligned} \tag{2.2}$$

Here F^I denote two-form fields strengths with magnetic charges p^I ; v denotes additional two-form field(s); and ϕ^a denote physical and auxiliary scalar fields. ϵ_2 is the volume form on the unit S^2 . We normalize the F^I such that the charges p^I are integer quantized.

Since all covariant derivatives are assumed to vanish, the equations of motion following from (2.1), evaluated on the trial solution (2.2), reduce to extremizing the function $\sqrt{g} \mathcal{L}$ as a function of ℓ_A, ℓ_S, V and ϕ^a . Equivalently, we can extremise the c-function, defined as

$$c = -6\ell_A^3 \ell_S^2 \mathcal{L} . \tag{2.3}$$

When extremizing, we hold fixed the quantized charges p^I , so that all free parameters are determined in terms of the p^I (or else are undetermined). This is the attractor mechanism, fixing the geometry and moduli in terms of the charges.

The choice of normalization in (2.3) is motivated as follows. The theory on AdS_3 has a boundary stress tensor [18] whose trace anomaly is [19]

$$T_i^i = -\frac{c}{12} R^{(2)}, \tag{2.4}$$

where $R^{(2)}$ is the scalar curvature of the conformal boundary metric. The prefactor in (2.3) was chosen such that the c-function evaluated at its critical point is equal to the c appearing in (\bar{a}). In a theory with equal left and right moving central charges, c is the central charge. More generally, the trace anomaly is related to the average:

$$c = \frac{1}{2}(c_L + c_R) . \tag{2.5}$$

Given the central charges, evaluation of the Euclidean black hole action leads to the general formula for the black hole entropy s :

$$s = 2\pi \sqrt{\frac{c_L}{6} \left(L_0 - \frac{c_L}{24} \right)} + 2\pi \sqrt{\frac{c_R}{6} \left(\tilde{L}_0 - \frac{c_R}{24} \right)} . \tag{2.6}$$

Since this expression takes the same form as Cardy’s formula it is convenient for comparison with microscopic results. Here, however, it is just a statement about the on-shell supergravity action. In fact, for $c_L = c_R$ (2.6) agrees with Wald’s entropy formula independently of the microscopic theory [20, 3]².

We will be working with five dimensional supergravity coupled to vector multiplets, which can be thought of as arising from M-theory compactified on CY_3 . The charges p^I then correspond to M5-branes wrapping 4-cycles in CY_3 . The central charges are known to be [22, 23]

$$c_L = 6p^3 + \frac{1}{2}c_2 \cdot p, \quad c_R = 6p^3 + c_2 \cdot p, \tag{2.7}$$

with

$$p^3 = \frac{1}{6}c_{IJK}p^I p^J p^K, \tag{2.8}$$

where c_{IJK} are the triple intersection numbers of the CY_3 , and c_{2I} are the expansion coefficients of the second Chern class. This then gives the following prediction for the extremal value of the c-function

$$c = 6p^3 + \frac{3}{4}c_2 \cdot p. \tag{2.9}$$

This is the result we wish to verify from the explicit higher derivative action.

3. Two derivative analysis

3.1 Five dimensional off-shell supergravity

Following [17] (see [24, 25] for earlier work) we consider superconformal gravity in five dimensions. The local form of the theory is off-shell, meaning that the auxiliary fields in the multiplets are not integrated out. At two-derivative order the bosonic terms in the Lagrangian are³

$$\begin{aligned} \frac{1}{2}\mathcal{L}_0 = & \partial^a \mathcal{A}_i^\alpha \partial_a \mathcal{A}_\alpha^i + \mathcal{A}^2 \left(\frac{1}{8}D - \frac{3}{16}R - \frac{1}{4}v^2 \right) + \mathcal{N} \left(\frac{1}{4}D + \frac{1}{8}R + \frac{3}{2}v^2 \right) + \mathcal{N}_I v^{ab} F_{ab}^I \\ & + \mathcal{N}_{IJ} \left(\frac{1}{8}F_{ab}^I F^{Jab} + \frac{1}{4} \partial_a M^I \partial^a M^J \right) + \frac{1}{48} e^{-1} c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde}. \end{aligned} \tag{3.1}$$

We are taking into account the bosonic fields of two distinct super multiplets: the Weyl multiplet, contains the vielbein e_μ^a , the two-form auxiliary field v_{ab} , and a scalar auxiliary field D ; the vector multiplets enumerated by index $I = 1 \dots n_V$, each containing a one-form gauge field A^I and scalar M^I , with $F^I = dA^I$. Although we will not discuss gauge fixing in detail, it is useful to include a term for the hyper multiplet which contains the Weyl scalar \mathcal{A}_i^α . The index $i = 1, 2$ refers to $SU(2)$ doublets and $\alpha = 1, \dots, 2r$ refers to

²Theories with $c_L \neq c_R$ have gravitational Chern-Simons terms that violate diffeomorphism invariance so Wald’s formula does not apply. [21] generalizes Wald’s formula to this case and shows that agreement with (2.6) is maintained.

³We have omitted fields in the multiplets associated with gauged supergravity or that can be turned off by gauge fixing conformal symmetries. With respect to [17] we have switched the sign in the kinetic term for the scalars M^I and the sign of the Ricci scalar ($R_{\text{here}} = -R_{\text{there}}$).

the $USp(2r)$. The hyper is used to gauge fix the dilatational symmetry and we choose a gauge that satisfies

$$\mathcal{A}^2 = -2, \quad \partial_a \mathcal{A}^i = 0. \quad (3.2)$$

The functions on the scalar manifold are defined by

$$\mathcal{N} = \frac{1}{6} c_{IJK} M^I M^J M^K, \quad \mathcal{N}_I = \partial_I \mathcal{N} = \frac{1}{2} c_{IJK} M^J M^K, \quad \mathcal{N}_{IJ} = c_{IJK} M^K. \quad (3.3)$$

The auxiliary field D appears linearly in (3.1), which means that it acts as a Lagrange multiplier. The resulting constraint determines \mathcal{N} , which can be thought of as the volume of the compactification manifold. Given that we chose $\mathcal{A}^2 = -2$, solving the equation of motion for D implies $\mathcal{N} = 1$. So, at the level of the two derivative action the scalars are described using real special geometry. For a pedagogical introduction see [26].

We can eliminate the auxiliary fields v_{ab} and D by solving their equations of motion. This gives

$$\mathcal{L}_0 = -\mathcal{N} \left[-R + G_{IJ} \partial_a M^I \partial^a M^J + \frac{1}{2} G_{IJ} F_{ab}^I F^{Jab} - \frac{e^{-1}}{24\mathcal{N}} c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde} \right], \quad (3.4)$$

with

$$G_{IJ} = -\frac{1}{2} \partial_I \partial_J (\ln \mathcal{N}) = \frac{1}{2} \left(\frac{\mathcal{N}_I \mathcal{N}_J}{\mathcal{N}^2} - \frac{\mathcal{N}_{IJ}}{\mathcal{N}} \right). \quad (3.5)$$

This is the familiar two derivative Lagrangian in five dimensional supergravity. For our purposes, we will not use (3.4) and instead work with (3.1).

3.2 c-extremization

We now determine, at the two-derivative order, the near horizon $AdS_3 \times S^2$ geometry corresponding to a black string in five dimensions, which we will refer to as the “black string attractor”. The near horizon configuration is given by (2.2) and the central charge as defined in (2.3) is

$$c = -6\ell_A^3 \ell_S^2 \mathcal{L}_0, \quad (3.6)$$

with \mathcal{L}_0 given by (3.1) evaluated on the trial solution (2.2). For this configuration, we will have some simplifications. The Chern-Simons term in (3.1) vanishes and derivatives of the scalars M^I are zero. The Ricci scalar is

$$R = -\frac{6}{\ell_A^2} + \frac{2}{\ell_S^2}. \quad (3.7)$$

By symmetry, we know that the scalars M^I are proportional to the charges p^I , so we write $M^I = m p^I$, with m a constant to be determined. The c -function then becomes,

$$c = -12\ell_A^3 \ell_S^2 \left(\frac{1}{4} (p^3 m^3 - 1) D - \frac{1}{4} (p^3 m^3 + 3) \left(\frac{3}{\ell_A^2} - \frac{1}{\ell_S^2} \right) + \frac{1}{\ell_S^4} \left((3p^3 m^3 + 1) V^2 + 3p^3 m^2 V \right) + \frac{3p^3 m}{\ell_S^4 8} \right), \quad (3.8)$$

with p^3 given by equation (2.8). Extremizing (3.8) with respect to D imposes $m^3 = p^{-3}$ and the equation for V gives

$$V = -\frac{3}{8}p . \tag{3.9}$$

The extremization of (3.8) with respect to the radii ℓ_A and ℓ_S results in

$$\ell_A = 2\ell_S, \quad \ell_A = p . \tag{3.10}$$

Finally, by extremizing the c -function with respect to m we find $D = 12p^{-2}$. Summarizing, our result for the parameters of the solution is

$$M^I = \frac{p^I}{p}, \quad \ell_A = 2\ell_S = p, \quad D = \frac{12}{p^2}, \quad V = -\frac{3p}{8} . \tag{3.11}$$

Inserting (3.11) in (3.8), the central charge for the black string in the two derivative theory is

$$c = 6p^3 = c_{IJK}p^I p^J p^K . \tag{3.12}$$

The value of c agrees with the expectation (2.9) to the leading order in charges. The new feature is verifying that c -extremization off-shell (i.e. keeping auxiliary fields) is consistent.

4. Susy variations

We would like to determine corrections to the attractor solution from the higher derivative terms in the action. A strong constraint comes from the fact that the attractor solution exhibits maximal supersymmetry. Furthermore, in the off-shell formulation the supersymmetry transformations are independent of the detailed form of the action (i.e. they are the same for the two and four derivative actions). With this in mind, we now analyze the constraints from supersymmetry.

The supersymmetry variations are

$$\begin{aligned} \delta\psi_\mu^i &= \mathcal{D}_\mu \varepsilon^i + \frac{1}{2}v^{ab}\gamma_{\mu ab}\varepsilon^i - \gamma_\mu \eta^i, \\ \delta\chi^i &= D\varepsilon^i - 2\gamma^c\gamma^{ab}\varepsilon^i \mathcal{D}_a v_{bc} - 2\gamma^a \varepsilon^i \epsilon_{abcde}v^{bc}v^{de} + 4\gamma \cdot v \eta^i, \\ \delta\Omega^I i &= -\frac{1}{4}\gamma \cdot F^I \varepsilon^i - \frac{1}{2}\gamma^a \partial_a M^I \varepsilon^i - M^I \eta^i, \\ \delta\zeta^\alpha &= \gamma^a \partial_a \mathcal{A}_j^\alpha \varepsilon^j - \gamma \cdot v \varepsilon^j \mathcal{A}_j^\alpha + 3\mathcal{A}_j^\alpha \eta^j . \end{aligned} \tag{4.1}$$

The first two transformations come from the fermions in the Weyl multiplet, the gravitino ψ_μ^i and an auxiliary Majorana spinor χ^i . From the vector multiplets we have the gaugino Ω_i^I and the hyper multiplet contributes with ζ^α . We are using the notation $\gamma \cdot v = \gamma_{ab}v^{ab}$.

4.1 Supersymmetry constraints for the black string attractor

The supersymmetry transformations (4.1) simplify dramatically when evaluated on our trial background (2.2). The attractor has maximal supersymmetry, meaning all variations must vanish. In our background this reduces to solving

$$\begin{aligned}
 \delta\psi_{\mu}^i &= \mathcal{D}_{\mu}\varepsilon^i + \frac{1}{2}v^{ab}\gamma_{\mu ab}\varepsilon^i - \gamma_{\mu}\eta^i = 0, \\
 \delta\chi &= D\varepsilon^i + 4\gamma \cdot v\eta^i = 0, \\
 \delta\Omega^{Ii} &= -\frac{1}{4}\gamma \cdot F^I\varepsilon^i - M^I\eta^i = 0, \\
 \delta\zeta^{\alpha} &= (-\gamma \cdot v\varepsilon^j + 3\eta^j)\mathcal{A}_j^{\alpha} = 0.
 \end{aligned}
 \tag{4.2}$$

From the gaugino variation it is clear that the scalars M^I are proportional to the charges p^I , so we can write $M^I = mp^I$, where the constant of proportionality m will be determined by the remaining equations. The last equation in (4.2) gives

$$\eta^i = \frac{1}{3}\gamma \cdot v\varepsilon^i.
 \tag{4.3}$$

Inserting (4.3) in (4.2) we get for the gravitino variation

$$\left(\mathcal{D}_{\mu} + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}v^{ab}\gamma_{\mu}\gamma_{ab} \right) \varepsilon^i = 0,
 \tag{4.4}$$

and for the auxiliary field and gaugino

$$\begin{aligned}
 \left(D + \frac{4}{3}(\gamma \cdot v)^2 \right) \varepsilon^i &= 0, \\
 \left(-\frac{1}{4}\gamma \cdot F^I - \frac{m}{3}p^I\gamma \cdot v \right) \varepsilon^i &= 0.
 \end{aligned}
 \tag{4.5}$$

Solving (4.5) on the ansatz (2.2), we find

$$mV = -\frac{3}{8}, \quad D = \frac{16}{3} \frac{V^2}{\ell_S^4}.
 \tag{4.6}$$

Finally, from the gravitino variation (4.4)⁴ we get a relation between the radii, ℓ_A and ℓ_S , and the auxiliary field V

$$\ell_A = 2\ell_S, \quad V = -\frac{3}{8}\ell_A.
 \tag{4.7}$$

The relations between the moduli shown in equations (4.6) and (4.7) hold independently of the action. Since the supersymmetry variations are exact off-shell, these results will not change for higher derivatives theories.

It is important to note that supersymmetry does not fully determine the values of the moduli and as presented here, one of the fields is unconstrained. This should be expected, since there are gauge symmetries unrelated to supersymmetry transformations

⁴See [27, 28] and references therein for details on manipulation of the gravitino variation.

that we have not imposed. For example, in the leading order theory described by (3.1), the scalars are described using real special geometry, where the volume \mathcal{N} is fixed. This comes about from fixing the superconformal theory to Poincare supergravity using the equation of motion for D , and is not related to the fact that the theory is supersymmetric. When higher derivatives terms are included in the theory, one should similarly expect to use at least one equation of motion from the off-shell theory to specify the solution completely.

Without loss of generality, we will take the AdS_3 radius as the undetermined modulus, and so we can summarize our results as

$$\ell_S = \frac{1}{2}\ell_A, \quad m = \frac{1}{\ell_A}, \quad V = -\frac{3}{8}\ell_A, \quad D = \frac{12}{\ell_A^2}. \quad (4.8)$$

Comparing with the two derivative c -extremization, we can see that (4.8) agrees with (3.11). The piece of information that is missing from the supersymmetry constraints is the relation between ℓ_A and the charges p^I . At the level of the two derivative theory, this is simply $\ell_A = p$. As we will show in the next section, when higher derivatives are taken into account, the AdS_3 radius will be modified and the value of the corrected moduli will be determined by our procedure.

5. c -extremization including higher derivatives

We are now ready to discuss higher derivative corrections to the central charge. As mentioned in the introduction, we want to verify that the Chern-Simons term controls the corrections to the central charge. From anomaly arguments, this term is given by

$$\sqrt{g}\mathcal{L}_{\text{CS}} = -\frac{c_2 I}{48 \cdot 2} A^I \wedge \text{Tr}(R \wedge R) = \frac{c_2 I}{24 \cdot 16} \epsilon_{abcde} A^{Ia} R^{bcfg} R^{de}_{fg}. \quad (5.1)$$

The Chern-Simons term by itself is not supersymmetric and therefore extra terms should be included. The four derivative supersymmetric completion of (5.1) was computed in [17], and the relevant terms for our discussion are

$$\begin{aligned} \mathcal{L}_1 = & \frac{c_2 I}{24} \left(\frac{1}{16} e^{-1} \epsilon_{abcde} A^{Ia} C^{bcfg} C^{de}_{fg} + \frac{1}{8} M^I C^{abcd} C_{abcd} + \frac{1}{12} M^I D^2 + \frac{1}{6} F^{Iab} v_{ab} D \right. \\ & - \frac{1}{3} M^I C_{abcd} v^{ab} v^{cd} - \frac{1}{2} F^{Iab} C_{abcd} v^{cd} + \frac{8}{3} M^I v_{ab} \hat{D}^b \hat{D}_c v^{ac} \\ & + \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_a v_{bc} + \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_b v_{ca} - \frac{2}{3} e^{-1} M^I \epsilon_{abcde} v^{ab} v^{cd} \hat{D}_f v^{ef} \\ & + \frac{2}{3} e^{-1} F^{Iab} \epsilon_{abcde} v^{cf} \hat{D}_f v^{de} + e^{-1} F^{Iab} \epsilon_{abcde} v^c \hat{D}^d v^{ef} \\ & \left. - \frac{4}{3} F^{Iab} v_{ac} v^{cd} v_{db} - \frac{1}{3} F^{Iab} v_{ab} v^2 + 4M^I v_{ab} v^{bc} v_{cd} v^{da} - M^I (v_{ab} v^{ab})^2 \right), \quad (5.2) \end{aligned}$$

with C_{abcd} the Weyl tensor defined as

$$C^a{}_b{}^c{}_d = R^a{}_b{}^c{}_d + \frac{1}{6} R \delta^a_{[c} \delta^b]_d - \frac{4}{3} \delta^a_{[c} R^b]_d. \quad (5.3)$$

The double covariant derivative of v_{ab} has curvature contributions⁵ given by

$$v_{ab} \hat{\mathcal{D}}^b \hat{\mathcal{D}}_c v^{ac} = v_{ab} \mathcal{D}^b \mathcal{D}_c v^{ac} + \frac{2}{3} v^{ac} v_{cb} R_a{}^b + \frac{1}{12} v_{ab} v^{ab} R. \quad (5.4)$$

5.1 Central charge and moduli corrections

Using the c -extremization procedure explained in section 2, we will find the corrected central charge and moduli in the higher derivative theory for the 5D black string. Including the four derivative Lagrangian, the central charge becomes

$$c = -6\ell_A^3 \ell_S^2 (\mathcal{L}_0 + \mathcal{L}_1), \quad (5.5)$$

where \mathcal{L}_0 and \mathcal{L}_1 are given by (3.1) and (5.2) evaluated in the $AdS_3 \times S^2$ background. Using (2.2) and the attractor value for the moduli $M^I = mp^I$, the supersymmetric four derivative contributions to the central charge are

$$\begin{aligned} \mathcal{L}_1 = \frac{c_2 \cdot p}{24} \left[\frac{m}{4} \left(\frac{1}{\ell_A^2} - \frac{1}{\ell_S^2} \right)^2 + \frac{2V^3}{3\ell_S^8} + 4m \frac{V^4}{\ell_S^8} + m \frac{D^2}{12} + \frac{DV}{6\ell_S^4} \right. \\ \left. - \frac{2}{3} m \frac{V^2}{\ell_S^4} \left(\frac{3}{\ell_A^2} + \frac{5}{\ell_S^2} \right) + \frac{1}{2} \frac{V}{\ell_S^4} \left(\frac{1}{\ell_A^2} - \frac{1}{\ell_S^2} \right) \right]. \quad (5.6) \end{aligned}$$

On our trial background $\hat{\mathcal{D}}_a v_{bc} = 0$. The two derivative contribution to (5.5) is still given by (3.8).

According to c -extremization we should now extremize with respect to all parameters. It would be extremely difficult to do this were it not for the guidance provided by the supersymmetry analysis in the previous section. For example, the variation of (5.5) with respect to m gives

$$\begin{aligned} \frac{3p^3 m^2}{4} \left(D - \frac{3}{\ell_A^2} + \frac{1}{\ell_S^2} \right) + \frac{3p^3}{\ell_S^4} \left(3m^2 V^2 + 2mV + \frac{1}{8} \right) + \\ + \frac{c_2 \cdot p}{48} \left[\frac{1}{4} \left(\frac{1}{\ell_A^2} - \frac{1}{\ell_S^2} \right)^2 + 4 \frac{V^4}{\ell_S^8} + \frac{D^2}{12} - \frac{2V^2}{3\ell_S^4} \left(\frac{3}{\ell_A^2} + \frac{5}{\ell_S^2} \right) \right] = 0. \quad (5.7) \end{aligned}$$

It is easy to verify that the moduli given in (4.8) do indeed satisfy this equation.

Before proceeding with the remaining extremization conditions, recall that the BPS conditions (4.2) leave one modulus undetermined. The missing information about the attractor can be simply obtained from the c -extremization procedure. The simplest is to consider the equation for the auxiliary field D

$$\frac{\partial c}{\partial D} = 0 \quad \Rightarrow \quad p^3 m^3 = 1 - \frac{1}{72} c_2 \cdot p \left(mD + \frac{V}{\ell_S^4} \right). \quad (5.8)$$

Using (4.8) and solving for the AdS_3 radius (ℓ_A) we find

$$\ell_A^3 = p^3 + \frac{1}{12} c_2 \cdot p. \quad (5.9)$$

⁵The sign difference with respect to [17] is coming from the difference in curvature convention.

The solution is now fully specified. At this point it is straightforward to vary the c -function (5.5) also with respect to V , ℓ_A , ℓ_S and show that the resulting equations are satisfied by (4.8) and (5.9). All in all we have found a solution that extremizes the c -function and shown that this solution is supersymmetric. Since we proceeded somewhat indirectly, we have not excluded the existence of other solutions with the same charge configuration but no supersymmetry.

Evaluating the c -function for our solution we find the corrected central charge

$$c = 6p^3 + \frac{3}{4}c_2 \cdot p . \tag{5.10}$$

This is precisely the result (2.9) that was predicted from supersymmetry and anomalies.

It is worth noting that the simple form of this result comes about in a rather nontrivial way in the present procedure. The radius of curvature ℓ_A (5.9) is a nontrivial function of the charges and it enters in the denominator of the Lagrangian (5.6). It is only due to intricate cancellations that the final result (5.10) becomes a linear function of the charges p^I . That this works out correctly provides a rather stringent consistency check on the entire framework.

5.2 Small black holes

One of the benefits of considering higher derivative corrections is that we can find smooth solutions in cases where the two-derivative action would yield a naked singularity. These are so-called “small black holes” [8, 9]. In particular, if we choose charges p^I such that $p^3 = 0$ but $c_2 \cdot p \neq 0$, then the two-derivatives formulas (3.11) become singular, while the four derivative formulas given in this section are well behaved. We can further check that the full action expanded around the small black hole solution exhibits no obvious pathologies. On the other hand, since some of the moduli M^I now vanish, some of the internal compactification cycles are becoming small, and so one should be alert to potentially large corrections from non-perturbative effects not included here. See [9] for more discussion.

6. Comparison between 5D and 4D attractor formulas

So far we have focussed on the $AdS_3 \times S^2$ attractor geometry near an effective string in five dimensions. Most recent works on higher curvature corrections are in the context of black holes in four dimensions and their $AdS_2 \times S^2$ near horizon attractor geometry. It is instructive to work out the detailed comparison between the four and five dimensional perspective in view of the higher derivative corrections. This is achieved by wrapping the string on a circle and dimensionally reducing. In terms of black hole entropy counting, a recent discussion of the relation between the AdS_3 and AdS_2 viewpoints can be found in [29].

A good starting point is the AdS_3 geometry written in Poincare coordinates as

$$ds_3^2 = \frac{\ell_A^2}{y^2}(dw^+ dw^- + dy^2) . \tag{6.1}$$

Introducing the coordinates [30]

$$\begin{aligned} w^+ &= \frac{1}{2\pi T_L} e^{2\pi T_L(x^5+t)}, \\ w^- &= x^5 - t - \frac{\ell_A^4 \pi T_L}{U^2}, \\ y &= \frac{\ell_A^2}{U} e^{\pi T_L(x^5+t)}, \end{aligned} \tag{6.2}$$

the line element becomes

$$ds_3^2 = \frac{U^2}{\ell_A^2} (dx_5^2 - dt^2) + \ell_A^2 \frac{dU^2}{U^2} + \pi^2 \ell_A^2 T_L^2 (dx_5 + dt)^2. \tag{6.3}$$

Wrapping the string on a circle corresponds to imposing the periodicity condition $x^5 \equiv x^5 + 2\pi R$, which amounts to identifications on the AdS_3 space that change the causal structure to that of a black hole [31]. This is clearest if we introduce the Schwarzschild-type coordinates

$$\begin{aligned} \rho^2 &= (\pi^2 \ell_A^2 T_L^2 + \frac{U^2}{\ell_A^2}) R^2, \\ x_5 &= R\phi, \\ \tau &= \frac{\ell_A}{R} t. \end{aligned} \tag{6.4}$$

Then the line element becomes

$$ds_{\text{BTZ}}^2 = -N^2 d\tau^2 + N^{-2} d\rho^2 + \rho^2 (d\phi + N_\phi d\tau)^2, \tag{6.5}$$

where

$$\begin{aligned} N &= \frac{\rho}{\ell_A} - \frac{\pi^2 \ell_A T_L^2 R^2}{\rho}, \\ N_\phi &= \frac{\pi^2 \ell_A T_L^2 R^2}{\rho^2}. \end{aligned} \tag{6.6}$$

We are interested in the direct product of the three dimensional geometry just introduced and an additional S^2 . Kaluza-Klein reduction on x^5 takes us from 5D to 4D. The compactification yields a 4D dilaton with near horizon value

$$e^{-2\Phi} = \rho_{\text{hor}} = \pi \ell_A T_L R. \tag{6.7}$$

So far we have just reviewed a standard construction. The issue we wish to emphasize is that these considerations are purely geometric and thus hold regardless of the details of the action. The corrections due to higher curvature terms enter only through the relation between parameters in the geometry and the underlying microscopic parameters. For example, we found earlier that the AdS_3 radius is

$$\ell_A = p_R, \tag{6.8}$$

where

$$p_R^3 = \frac{1}{6} \left(c_{IJK} p^I p^J p^K + \frac{1}{2} c_2 \cdot p \right) . \tag{6.9}$$

We also found the attractor values of the 5D scalars as

$$X^I = \frac{p^I}{p_R} . \tag{6.10}$$

Note that the corrections again appear through p_R .

We are also interested in the thermodynamics for a black string excited to level $|q_0|$. The corrected formula for the entropy is

$$S = 2\pi \sqrt{\frac{c_L |q_0|}{6}} = 2\pi \sqrt{p_L^3 |q_0|} , \tag{6.11}$$

where the corrections enter through

$$p_L^3 = \frac{1}{6} \left(c_{IJK} p^I p^J p^K + c_2 \cdot p \right) . \tag{6.12}$$

The energy of the excitations is $E_L = |q_0|/R$ and so the first law of thermodynamics gives the temperature

$$T_L = \frac{1}{\pi R} \sqrt{\frac{|q_0|}{p_L^3}} . \tag{6.13}$$

We see again that it is the combination p_L that appears in the thermodynamics.

We next consider the corrections of some less obvious quantities. For example, the precise value for the 4D dilaton (6.7) is

$$e^{-2\Phi} = p_R \sqrt{\frac{|q_0|}{p_L^3}} . \tag{6.14}$$

The string frame radius of the very near horizon AdS_2 close to the 4D black hole is inherited from the AdS_3 [32] and so its value is simply ℓ_A . The AdS_2 radius in 4D Einstein frame is therefore

$$R_0^2 = e^{-2\Phi} \ell_A^2 = p_R^3 \sqrt{\frac{|q_0|}{p_L^3}} . \tag{6.15}$$

This expression agrees with the result previously found directly in four dimensions (for a good review see [13] ⁶).

We can also determine the 4D scalars. We are considering the simple situation with $q_{I \neq 0} = 0$ and $p^0 = 0$, where there are no M2-branes wrapping the 2-cycles of the CY, nor any magnetic charge of the Kaluza-Klein gauge field from reduction along x^5 . In this case the 4D scalars other than the dilaton $g^{(0)}$ are purely imaginary.⁷ Combining the 5D

⁶The AdS_2 radius is the 4D central charge so $R_0^2 = |Z|^2$. Combining (6.16) and (6.17) in [13] gives our (6.15) after notation has been adapted.

⁷M2-brane charges q_I are easily incorporated, as they correspond to Wilson lines for the gauge fields: $w^I \sim \frac{1}{R} A_5^I \sim c^{IJ} q_J$, with c^{IJ} being the inverse of $c_{IJK} p^K$.

scalars (6.10) with the dilaton we find [33]

$$z^I = ie^{-2\Phi} X^I = ip^I \sqrt{\frac{|q_0|}{p_L^3}}. \quad (6.16)$$

This expression also agrees with results previously found directly in four dimensions⁸. Note that in 4D the p_L introduced in (6.12) controls both the scalars and the thermodynamics.

From the 5D point of view the charge q_0 corresponds to AdS₃ angular momentum. In a two-derivative theory the angular momentum can be read off from the metric via

$$j = \frac{\rho_{\text{hor}}^2}{4G_3 \ell_A}. \quad (6.17)$$

Applying this formula to the corrected metric yields (in our units $G_3 = \frac{1}{4\ell_A^2}$)

$$j = \frac{p_R^3}{p_L^3} |q_0|. \quad (6.18)$$

The mismatch between j and q_0 is due to the fact that the expressions for conserved quantities as surface integrals themselves receive corrections from the higher derivative terms. It would be instructive to derive these corrected expressions, as was done in [34] for the gravitational Chern-Simons term.

In this section we have focussed on extremal black holes with $T_L \neq 0$ and $T_R = 0$. However, one of the nice features of the AdS₃ framework is that our 5D results easily extend to the non-extremal case $T_{L,R} \neq 0$. The 5D attractor formulas are unchanged, and the general entropy formula is given in (2.6). So higher derivative corrections to the entropy are under control even for these non-BPS, non-extremal black holes [3].

Another feature of the 5D setup is that we can make contact with higher derivative corrections to black rings [35]. Black rings have a near horizon AdS₃ × S² region of the same type as studied here. Therefore our results for the corrected attractor geometry will also apply to the near horizon region of black rings. In particular, it should be possible to find explicit solutions for “small black rings”.

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⁸Adapting the notation of (6.18) in [13] gives our (6.16)

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