

SOLSTICE: An Electronic Journal of Geography and Mathematics.

(Major articles are refereed; full electronic archives available)

- *Solstice* was a Pirelli INTERNETional Award Semi-Finalist, 2001 (top 80 out of over 1000 entries worldwide)
- One article in *Solstice* was a Pirelli INTERNETional Award Semi-Finalist, 2003 (Spatial Synthesis Sampler).
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Congratulations to all *Solstice* contributors.

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Mail

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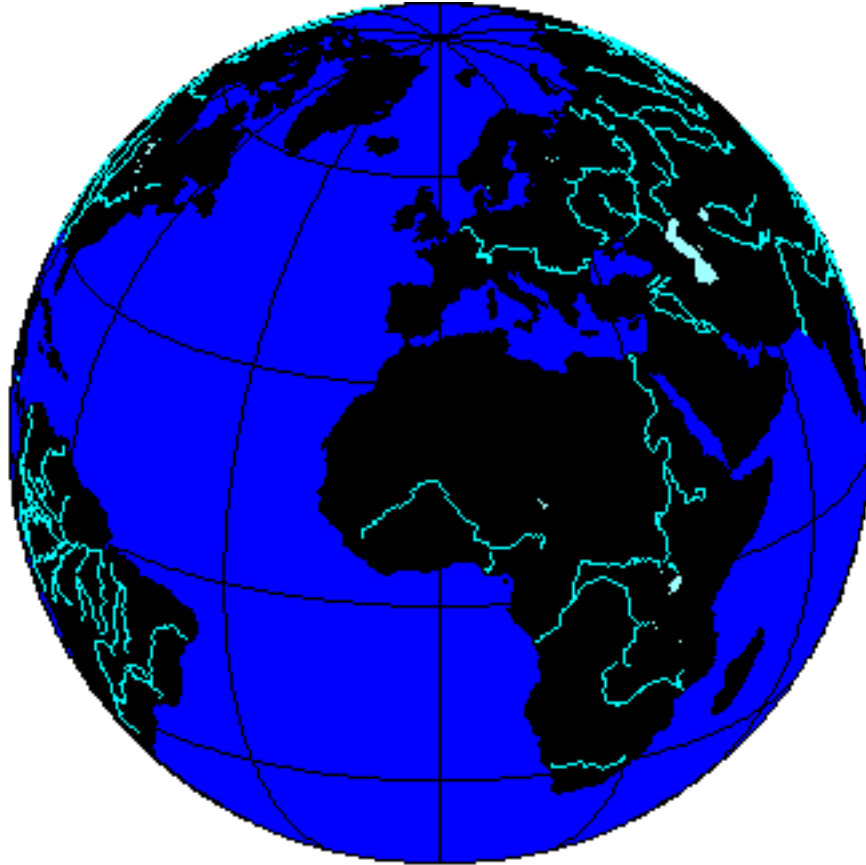
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<http://www.InstituteOfMathematicalGeography.org/>

Solstice:

An electronic journal of geography and mathematics



Earth: with 23.5 degrees north latitude as the central parallel.

Volume XVII
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SOLSTICE: AN ELECTRONIC JOURNAL OF GEOGRAPHY AND MATHEMATICS

<http://www.imagenet.org>

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VOLUME XVII, NUMBER 1

ANN ARBOR, MICHIGAN

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MISSION STATEMENT

The purpose of Solstice is to promote interaction between geography and mathematics. Articles in which elements of one discipline are used to shed light on the other are particularly sought. Also welcome are original contributions that are purely geographical or purely mathematical. These may be prefaced (by editor or author) with commentary suggesting directions that might lead toward the desired interactions. Individuals wishing to submit articles or other material should contact an editor, or send e-mail directly to sarhaus@umich.edu.

SOLSTICE ARCHIVES

Back issues of Solstice are available on the WebSite of the Institute of Mathematical Geography, <http://www.imagenet.org> and at various sites that can be found by searching under "Solstice" on the World Wide Web. Thanks to Bruce Long (Arizona State University, Department of Mathematics) for taking an early initiative in archiving Solstice using GOPHER.

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To cite the electronic copy, note the exact time of transmission from Ann Arbor, and cite all the transmission matter as facts of publication. Any copy that does not superimpose precisely upon the original as transmitted from Ann Arbor should be presumed to be an altered, bogus copy of *Solstice*. The original rug, with errors, serves as the model for creating this weaving of words, blank space, and graphics.

Awards and Recognition

(See [Press Clippings](#) page for other.)

- Sandra L. Arlinghaus and William C. Arlinghaus, Spatial Synthesis Sampler, *Solstice*, Summer 2004. Semi-Finalist, [Pirelli](#) 2003 INTERNETional Award Competition.
 - Sandra Lach Arlinghaus, recipient, The President's Volunteer Service Award, March 11, 2004.
 - [Jeffrey A. Nystuen](#), won the 2003 Medwin Prize in Acoustical Oceanography given by the [Acoustical Society of America](#). The citation was "for the innovative use of sound to measure rainfall rate and type at sea". It is awarded to a young/mid-career scientist whose work demonstrates the effective use of sound in the discovery and understanding of physical and biological parameters and processes in the sea.
 - [Sandra L. Arlinghaus](#), William C. Arlinghaus, and Frank Harary. *Graph Theory and Geography: an Interactive View (eBook)*, published by John [Wiley](#) and Sons, New York, April 2002. Finished as a Finalist in the 2002 Pirelli INTERNETional Award Competition (in the top 20 of over 1200 entries worldwide). [Link](#) to Pirelli website and to downloaded pages concerning this particular competition: [1](#), [2](#), [3](#).
 - *Solstice*, Semi-Finalist, Pirelli 2001 INTERNETional Award Competition in the Environmental Publishing category.
 - *Solstice*, article about it by Ivars Peterson in *Science News*, 25 January, 1992..
 - *Solstice*, article about it by Joe Palca, *Science* (AAAS), 29 November, 1991.
-

Looking At Some Data from Isard's "Methods of Regional Analysis"

For presentation at the Western Regional Science Association meeting,

Santa Fe, NM. February 2006

Waldo Tobler, Geographer

<http://www.geog.ucsb.edu/~tobler>

In his path-breaking book “Methods of Regional Analysis” (MIT Press, 1960) Walter Isard presented several sets of data. Some of these are fictitious, designed to illustrate concepts. Others contain real empirical values. Among the latter are several tables. I have used two of these, not in a research mode, but rather as exploratory visualizations. Of the two matrices one is 36 by 36 in size, the other is 47 by 47. I have used these tables in their original form without attempting any transformations such as biproportional adjustment to equalize marginals, or eigenvector analysis, or partitioning into symmetric and skew symmetric parts to estimate potentials¹. Nor have I attempted to reduce the resolution of the data by combining, or by omitting, rows or columns from the matrices.

Relating to the first of my examples, Isard has four maps of the movement of funds between Federal Reserve cities. These are illustrated on pages 149, 154-156. of the book. He also presented a table² of similar monetary transfers between the thirty-six Federal Reserve cities in the United States. Curiously none of the four maps shown is made using the information from this table. But he argued, rightly in my opinion, that such tables and maps should be used more widely. To date I have not seen much evidence that this suggestion has been followed. An examination of recent issues of the *Annals of Regional Science* convinces one of this fact. Isard thus recognized that insufficient use is often made of the human power of inference from visualizations in research. As an exercise to demonstrate the usefulness of this idea I have used a simple and freely available computer program³ to create several maps from the 36 by 36 table. In order to do this it was only necessary to look up the latitudes and longitudes of the cities on the internet and to find a computer compatible outline map of the United States. The preparation of such maps is then very simple and alternate versions of the maps can be created quickly. The computer program allows one to interactively hover (with a mouse) over a location to read its name, and to hover over the flow symbol to be presented with the numerical value. Thus exploratory research use can be made of the maps. This interaction of course is not possible with the printed versions of the maps⁵.

The thirty six by thirty six matrix yields a possible 1,260 movements so that some parsing is desirable. Such tables are not random, but contain considerable structure. The majority of the volume is contained in a minority of the flows so that deleting flows below the mean tends to leave 75% of the movement volume but requires only 20% of the movement activity. This is a far better strategy then, for example, reducing the resolution by lumping all of the

thirty six cities into the twelve Federal Reserve Districts and thereby reducing the 1260 values to 132 with the attendant loss of geographic detail.

Examples of maps from one set of data hardly constitutes a research project – but inclusion of such fiscal-movement maps can enhance most studies, and a sequence from several time periods is certain to be enlightening, especially if rendered within an animation.

Table and five maps go here

Map 1: Original map from Isard, p.149.

Table I (part)

Map 2: All fiscal transfers during a week (of 1,260 possible)

Map 3: Net transfers greater than $2 \cdot 10^7$ dollars

Map 4: Above average transfers to New York City

Map 5: Above average transfers from New York City

Another important data array is available in an input-output table. As pointed out by Isard⁴ “...the *interregional input-output* approach is most prominent, both in terms of accomplishment and recognition. It represents a fruitful approach for depicting and investigating the underlying processes which bond together the regions of a system and all the separate facets of their economies.” A graphical exercise, similar to that given above, has been undertaken with respect to the New England input/output table from the “Methods” book⁶. In this case there are forty-seven industry groupings and the flows are represented by a 47 by 47 table. The industries are quickly given coordinate locations by a metric multidimensional scaling program⁷. This works somewhat as follows. Consider two industries to be “close” if the quantity of exchanges between them is large. Then construct a diagram in which “close” industries are placed near each other. Do this by positioning a first industry at random. Take the industry “closest” to this and place it on a circle of chosen radius near the first one, thereby establishing a scale for the diagram. The next industry, the third industry, then lies at the intersection of two circles drawn about the first two, with appropriately scaled radii. There is a slight ambiguity and arbitrariness here because two circles can intersect in two places. Chose one of the options and insert the third industry. and then proceed to locate the next (fourth) industry using more circles. The degrees of freedom rapidly decrease and the choices become more difficult. Mathematically the fitting via

circles can be replaced by an iterative procedure that places industries into the diagram so that “close” industries are also close in the diagram⁸. The contacts between industries are then drawn as flows between these locations using the same mapping computer program. Curiously the scaling algorithm has placed most of the industries on the perimeter of a circle with only a few industries in the interior. My suspicion is that the starting configuration - the Gower algorithm was used - began with all items on a circle. The non-metric version - the Torsca algorithm - gave very similar results.

The few interior-lying industries seem to dominate the table. The “Household” category is especially prominent. The important industries exchanging goods and services are easily identified when using the interactive computer program, as are the magnitudes involved. Close visual examination of the original input-output matrix would permit recognition of these major flows, especially when studied by the original compiler of the table. But without such close scrutiny a casual user could find the graphical version(s) more useful and especially rapid. This technique has, to the best of my knowledge, not been used very often, if at all, with input/output tables.

The 47 by 47 matrix could contain as many as 2,162 entries, if the diagonal is ignored, so that a parsing by size is again appropriate, as is seen clearly in the diagrams. Only a little over a dozen entries are significant. As is the case with the movement maps, one example does not constitute a research project but a series of such diagrams, even an animation, can enhance the study of changing regional structure. An opportunity to study regional dynamics thus becomes available. An interesting experiment might be provided by applying the technique to the Leontiev inverse, or to the input/output coefficient matrix.

Input / Output table & maps go here

Table 3 (part).

Map 6: The complete I/O table in graphic form

Map 7: All exchanges above the average

Map 8: Above $5 * 10^8$ dollars, with labels

Map 9: Net exchanges above $2 * 10^7$ dollars

Citations

¹ W. Tobler, 2005, “Estimation of Potentials from Asymmetries”, in Solstice, A refereed electronic journal, XVI, 2 (Winter) at: <http://www.imagenet.org>

² “Table 1. Net Commercial And Financial Flows Between Federal Reserve Zones, Week Ended October 13, 1954”, pages 152-153 of W. Isard, 1960, Methods of Regional Analysis, Cambridge, MIT Press. Careful examination of Isard’s table and the maps reveals that they came from different sources and relate to different time periods. Thus the flows mapped here differ from those illustrated in the book.

³ The program, tutorial, and sample data can be downloaded without charge from: [www.csiss.org\spatial tools\flow mapper](http://www.csiss.org/spatial%20tools/flow%20mapper)

⁴ An alternate graphical method of representing dollar movement is presented in W. Tobler, 1981, “A Model of Geographic Movement”, Geographical Analysis, 13, 1 (Jan):1-20; Table I, Figures 1-6..

⁵ Isard, op. cit., 310; emphasis as in the original.

⁶ “Table 3. New England: Input Requirements And Commodity Balances, 1947, Estimated from National Coefficients”, pages 604-607 of Isard, 1960, op. cit. Also see the discussion of the data in the table on pages 603-617.

⁷ Borgatti, 1996, ANTHROPAC 4.0, Natick, MA, Analytic Technologies. The two-dimensional stress after eight iterations was 0.40. A three dimensional solution was not attempted.

⁸ A simple introduction to one of the iterative methods is given in: W. Tobler, 1996, “A Graphical Introduction to Survey Adjustment“, Cartographica, 33-42.

Total Transfers



Federal Reserve Dollar Flows, above 20 Million
Week ended October 13, 1954



To New York



From New York

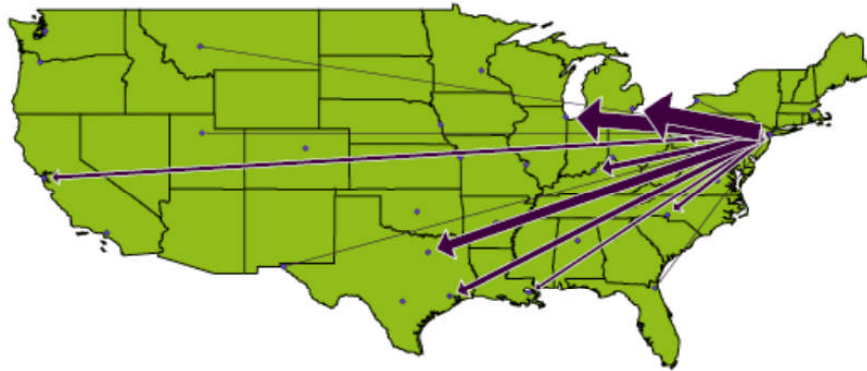
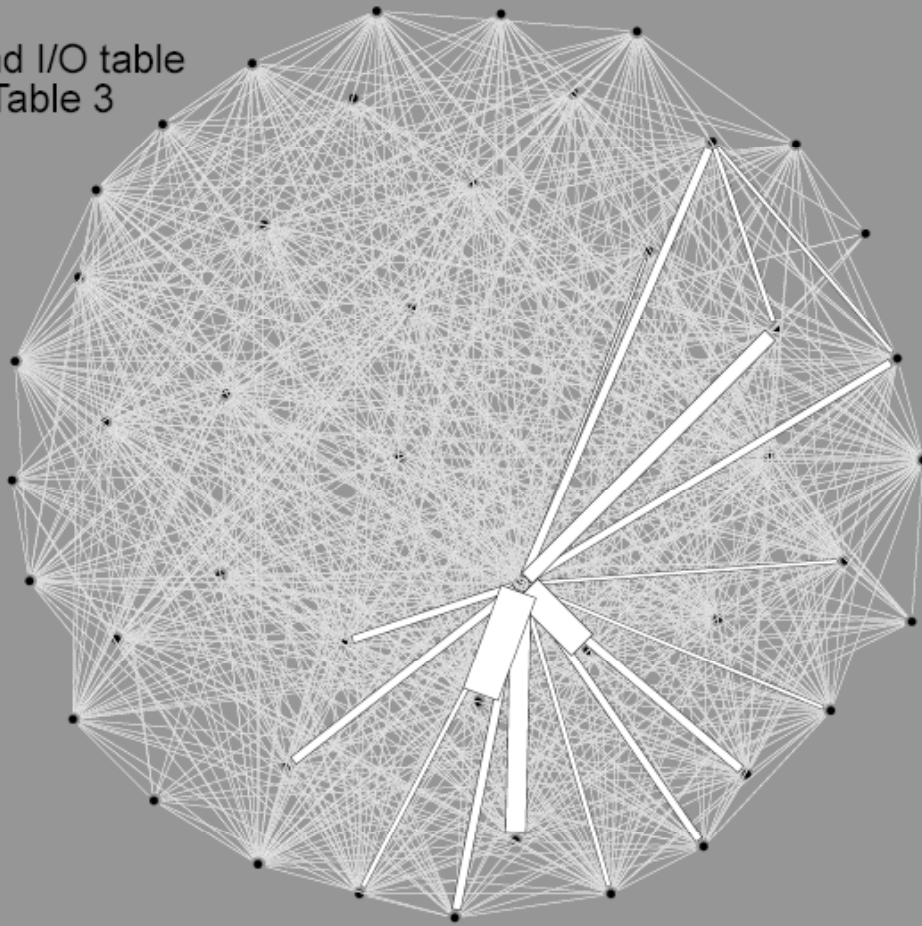


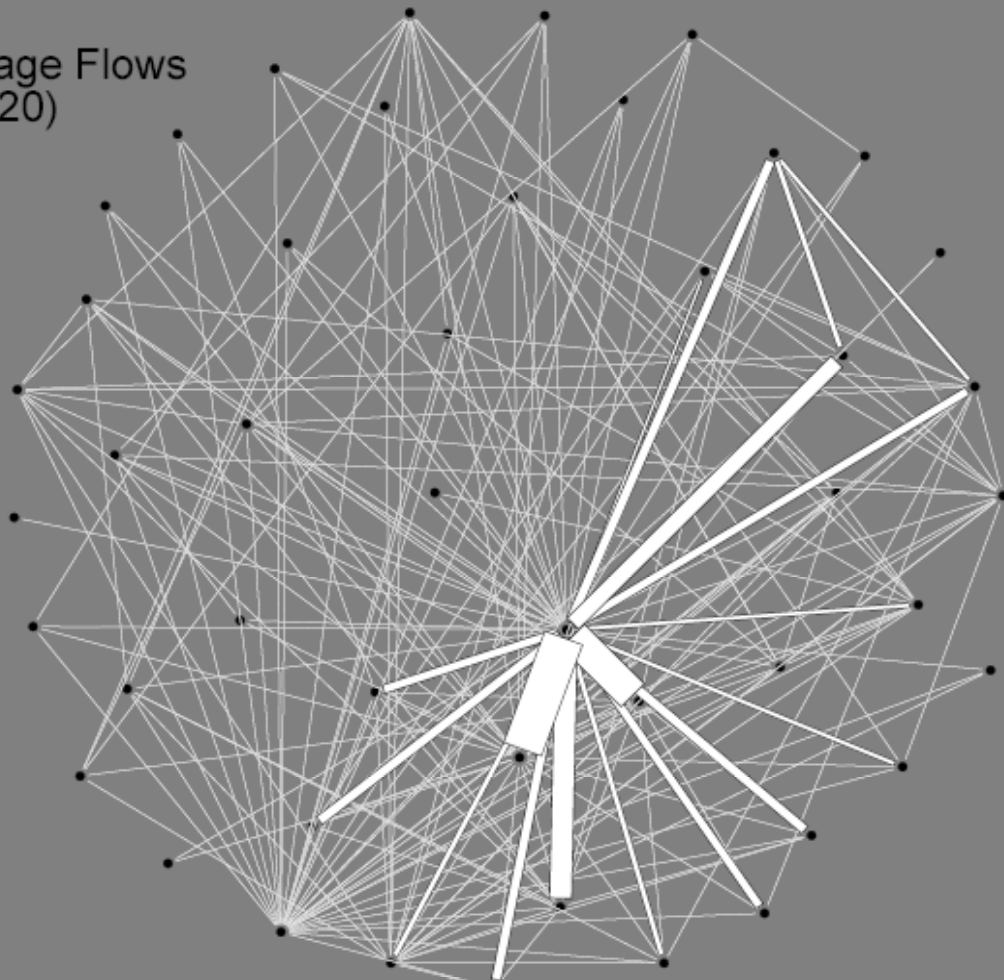
TABLE 3. NEW ENGLAND : INPUT REQUIREMENTS AND COMMODITY

Industry (Sector) Producing	Industry (Sector) Purchasing									
	1. Agriculture and Fisheries	2. Food and Kindred Products	3. Tobacco Manufactures	4. Textile Mill Products	5. Apparel	6. Lumber and Wood Products	7. Furniture and Fixtures	8. Paper and Allied Products	9. Printing and Publishing	10. Chemicals
1. Agriculture and Fisheries	244.5	534.2	7.6	571.2	1.2	8.3	-	1.2	-	57.9
2. Food and Kindred Products	53.6	174.3	0.1	16.5	0.6	*	*	4.1	*	32.8
3. Tobacco Manufactures	-	-	8.1	-	-	-	-	-	-	†
4. Textile Mill Products	1.4	0.1	-	358.0	246.0	0.1	15.4	5.9	1.6	0.6
5. Apparel	1.0	7.2	-	-	124.2	-	0.3	2.8	-	1.4
6. Lumber and Wood Products	3.3	2.9	0.2	4.9	0.1	47.2	20.8	36.8	0.1	2.2
7. Furniture and Fixtures	-	-	-	3.3	-	-	0.4	0.7	-	-
8. Paper and Allied Products	†	16.1	0.6	21.4	1.6	0.2	0.8	358.0	70.3	15.8
9. Printing and Publishing	-	1.4	-	0.6	-	-	-	-	49.8	0.8
10. Chemicals	18.7	51.5	0.2	219.8	9.0	1.1	3.4	25.2	6.3	127.0
11. Products of Petroleum and Coal	10.3	2.1	*	8.2	0.3	3.2	0.1	8.7	0.2	15.5
12. Rubber Products	2.7	0.3	-	3.6	1.1	0.4	0.3	1.2	0.2	†
13. Leather and Leather Products	-	-	-	0.6	3.4	0.2	0.4	-	0.3	-
14. Stone, Clay and Glass Products	1.5	9.0	†	0.3	-	0.6	1.8	3.9	-	12.3
15. Iron and Steel	0.1	0.1	-	-	0.1	0.4	5.2	-	-	0.2
16. Nonferrous Metals	-	-	-	-	-	0.1	0.9	-	0.9	9.0
17. Plumbing and Heating Supplies	-	-	-	-	-	-	-	-	-	-
18. Fabricated Structural Metal Products	-	-	-	-	-	-	0.3	-	-	-
19. Other Fabricated Metal Products	1.9	19.3	0.1	*	0.4	1.5	7.1	2.3	0.1	6.2
20. Agricultural, Mining and Construction Machinery	1.3	-	-	-	-	-	-	-	-	-
21. Metalworking Machinery	-	-	-	-	-	-	-	-	-	-
22. Other Machinery (except electric)	-	0.5	-	9.6	1.3	0.6	0.6	1.9	2.3	†
23. Motors and Generators	-	-	-	-	-	-	-	-	-	-
24. Radios	-	-	-	-	-	-	-	-	-	-
25. Other Electrical Machinery	-	-	-	-	-	-	-	-	-	†
26. Motor Vehicles	2.5	0.1	-	-	-	†	-	-	-	†
27. Other Transportation Equipment	0.2	-	-	-	-	-	-	*	-	†
28. Professional and Scientific Equipment	-	-	-	-	-	-	0.1	0.8	2.1	0.6
29. Miscellaneous Manufacturing	0.1	0.4	-	1.1	16.2	†	0.9	2.1	-	1.4
30. Coal, Gas, and Electric Power	1.4	6.9	†	28.8	2.3	1.0	1.0	17.0	1.9	9.0
31. Railroad Transportation	9.9	19.5	0.2	25.8	3.8	6.2	2.9	30.9	4.4	13.7
32. Ocean Transportation	1.6	4.5	†	3.6	0.7	0.4	*	2.2	*	2.1
33. Other Transportation	12.5	13.0	0.2	21.7	1.6	6.0	2.2	16.1	1.6	4.5
34. Trade	30.6	14.8	0.4	62.6	23.4	2.6	3.2	24.3	2.0	8.3
35. Communications	†	1.5	†	2.5	1.2	0.4	0.3	1.1	2.5	1.1
36. Finance and Insurance	5.4	5.1	†	5.5	1.5	3.3	1.0	2.5	1.5	0.9
37. Rental	53.9	3.2	-	6.9	6.1	0.8	0.9	3.6	4.0	1.6
38. Business Services	0.2	18.9	1.0	19.5	6.1	0.8	3.1	3.0	3.8	20.3
39. Personal and Repair Services	8.3	4.2	*	0.8	0.2	1.8	0.2	0.6	1.3	0.5
40. Medical, Educational and Nonprofit Organizations	-	-	-	-	-	-	-	-	-	-
41. Amusements	-	-	-	-	-	-	-	-	-	-
42. Scrap and Miscellaneous Industries	-	-	-	6.6	-	-	-	34.5	-	5.3
43. Undistributed	-	73.1	1.3	120.3	83.0	38.0	17.7	27.7	39.6	83.2
44. Eating and Drinking Places	-	-	-	-	-	-	-	-	0.1	-
45. New Construction and Maintenance	4.5	4.2	†	10.7	1.0	0.5	0.4	5.8	1.0	1.7
48. Government	18.3	40.3	1.0	175.6	23.8	14.6	6.0	68.5	21.8	36.5
50. Households ^d	431.7	222.3	3.8	902.8	254.3	110.6	57.3	297.9	197.2	164.1
Total Gross Outlays	921.5	1250.7	25.0	2612.9	814.6	251.1	154.9	991.4	416.8	636.9

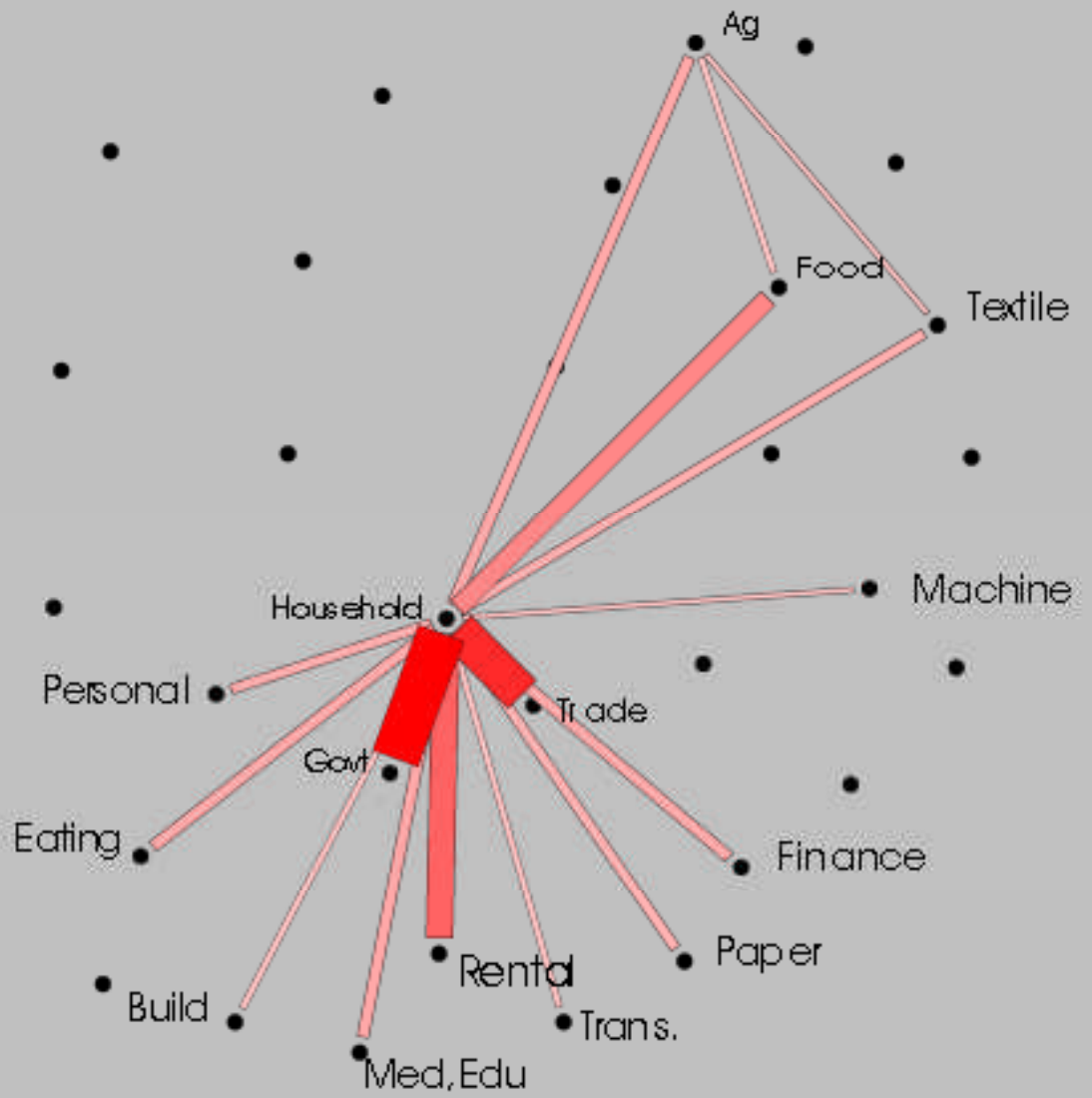
New England I/O table
Isard 1960 Table 3



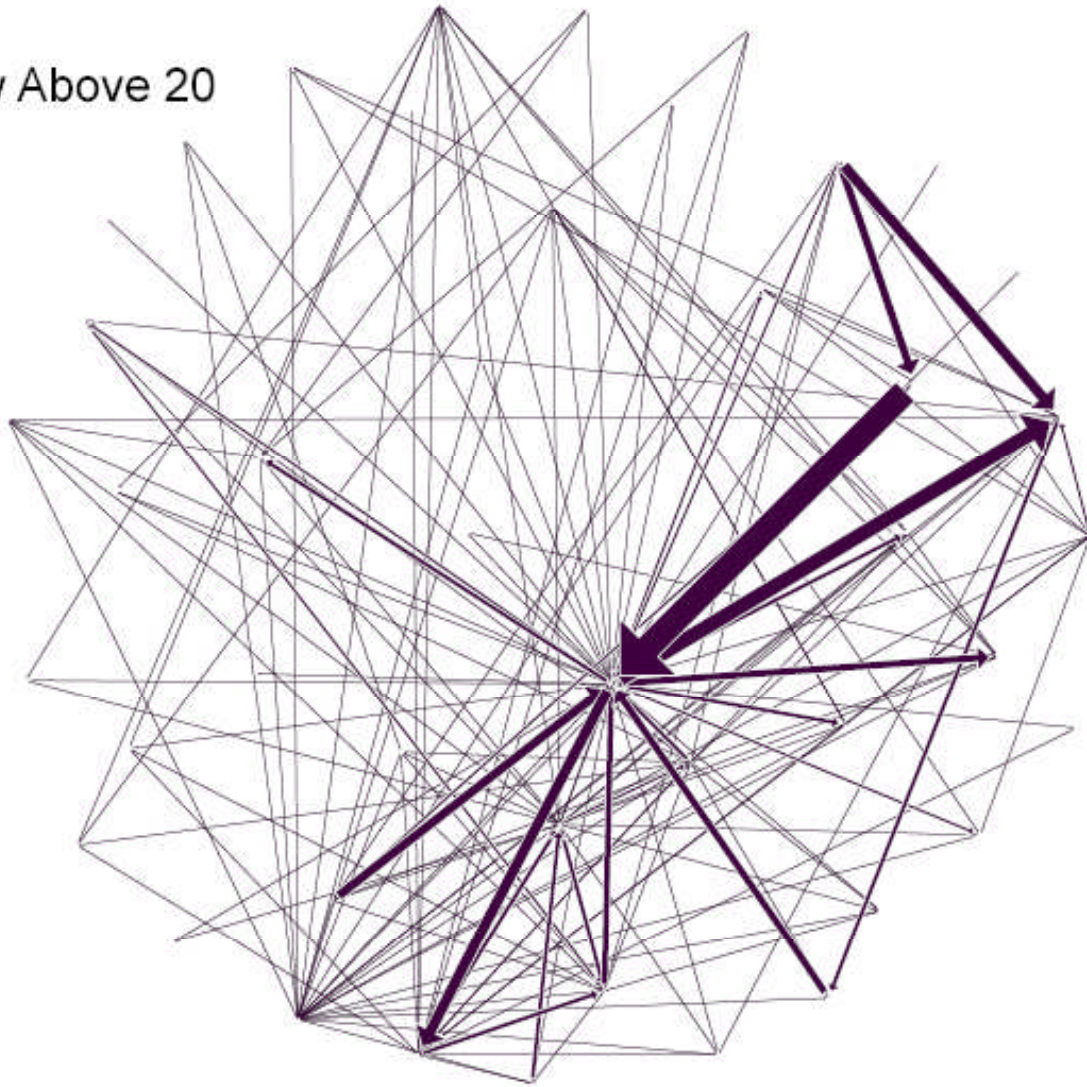
Above Average Flows
(average is 20)



Gross



Net Flow Above 20



MAPS AND DECISIONS, II: Ambiguity.

Sandra L. Arlinghaus and William C. Arlinghaus

General Introduction

Tournament level duplicate bridge is a card game that is a sport. As is the case with sports, generally, there is an overseeing body: in basketball it is the National Basketball Association (NBA); in bridge it is the American Contract Bridge League for North America (ACBL) and the World Bridge Federation (WBF) for all nations in the world. The ACBL is a non-profit organization based in Memphis, Tennessee. The ACBL has about 150,000 members in the U.S.A., Canada, and Mexico. The WBF has more than 10 million members. The ACBL owns two buildings in Memphis where they house a large staff to maintain records, databases, publications, and a host of other operations associated with this business in the entertainment/sports sector of the business world. The second author of this work is currently a member of the Board of Directors of the ACBL. This Board, as do equivalent boards of other corporations, sets policy for the organization, makes decisions that affect the entire population of ACBL members, and oversees the work of the Chief Executive Officer. There are 25 Board members, each representing one geographical "district" of the ACBL. Thus, the members of the Board of Directors are also referred to, even though their charge is to represent the interests of the entire ACBL, "District Directors."

Ambiguity

The second author is the District Director for District 12, including all of the lower peninsula of Michigan, part of the upper peninsula, and northwestern Ohio. The map in Figure 1 shows the geographic extent of each of the districts with respect to the conterminous USA and suggests boundaries in Canada and Mexico (to see a full characterization, visit the ACBL website). In addition, Figure 1 shows the count of members for each district as of March 2005, according to ACBL databases. One reason for an individual district director to care about maps of all districts is individual district norms can be evaluated against national ones. A national view may help to put local issues in perspective. Decisions made at the national level affect individual districts. The idea is a straightforward one employed in many arenas: map the data at one scale in order to make cross-scale evaluations of various sorts. As long as the data and maps work well together, the process works well. When that mesh is not there, however, sometimes extra insight for decision-making comes from the mere process of mapping the data as the case below will show.

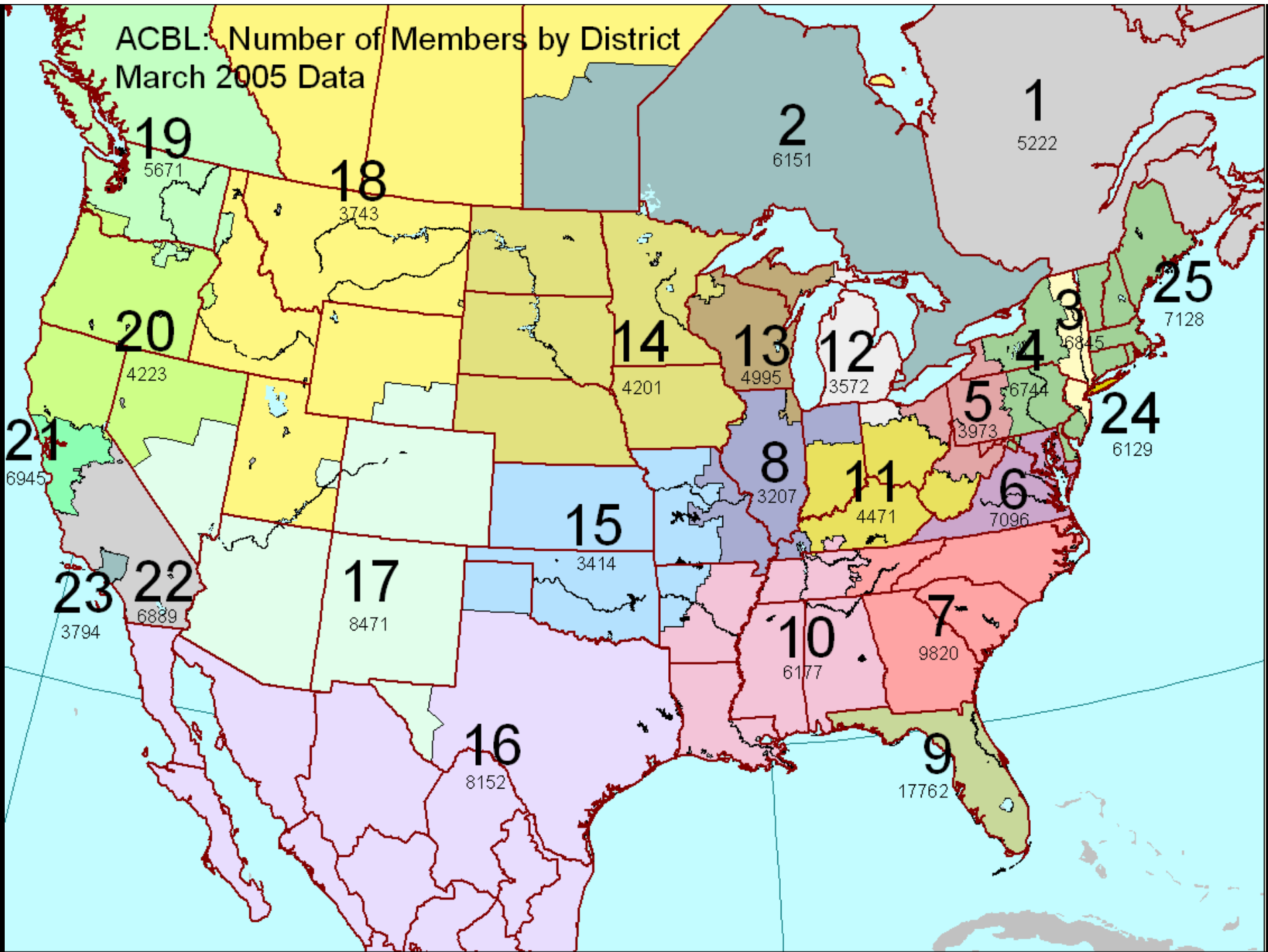


Figure 1. ACBL membership mapped by District, based on March 2005 data.

Each ACBL district is further subdivided into a number of "units." The attached clickable map shows each district and its units for the conterminous USA. Click on any unit and the corresponding part of the underlying database pops up showing district number for that unit, unit number, and number of ACBL members by unit. This interactive map is a useful tool for getting a quick count on where there are concentrations of members and where there are not. The former is important in considering where tournaments might best be located to serve large numbers of current members while the latter is important in looking for regions of potential recruitment (when coupled with other information such as density counts). When the lower 48 of the US is mapped as a choropleth map on the number of members by unit the map in Figure 2 is the result. The map was shaded in the GIS using shades of red; the light yellow areas are regions of no data (and these are identical independent of the method used to partition the data to produce the red regions).

It might be plausible that there are units in rural California or eastern Oregon and Idaho with 0 members. It is not reasonable, however, to believe that all of southeastern Michigan, including the entire Detroit metropolitan area and Ann Arbor contains no members. A look back at the clickable map, at southeastern Michigan reveals the same situation (although it is much more evident in the single, static map below). Is there a blunder in the record-keeping? Take a closer look; it reveals far more. What appears to be a single unit in Southeastern Michigan is in fact two units (from other ACBL data):

- the Michigan Bridge Association, MBA (roughly containing all of Detroit and Ann Arbor areas and suburbs between the two)
- the Southern Michigan Bridge Association, SOMBA (roughly containing suburbs to the north of Detroit, but not Detroit)

One might wonder why there is this separation that belies, for example, the idea of standard metropolitan areas of various sorts; it is useful to employ census measures and such in relation to bridge territories, but this split makes that process difficult. The reason for the separation is historical and is apparently based in perception issues of various sorts that are hopefully no longer part of the contemporary scene. Some time ago, therefore, the ACBL allowed anyone living in the southeastern area of Michigan to choose which unit he or she wished to join. Because the ACBL accumulates data by ZIP code and assigns member ZIP code to units in this and other databases, those in which members are allowed to choose units do not show up and are recorded as missing data: there is no assignment algorithm that applies to these members. There is ambiguity in choice: one member in Detroit may choose the MBA and that member's next door neighbor (in the same ZIP code) may choose SOMBA. In most other locales, all members in any given ZIP code are all assigned, with no choice, to a particular unit: they show up in the database. The mapping process made evident a densely populated region of the bridge database that has no count in decisions made using that database.

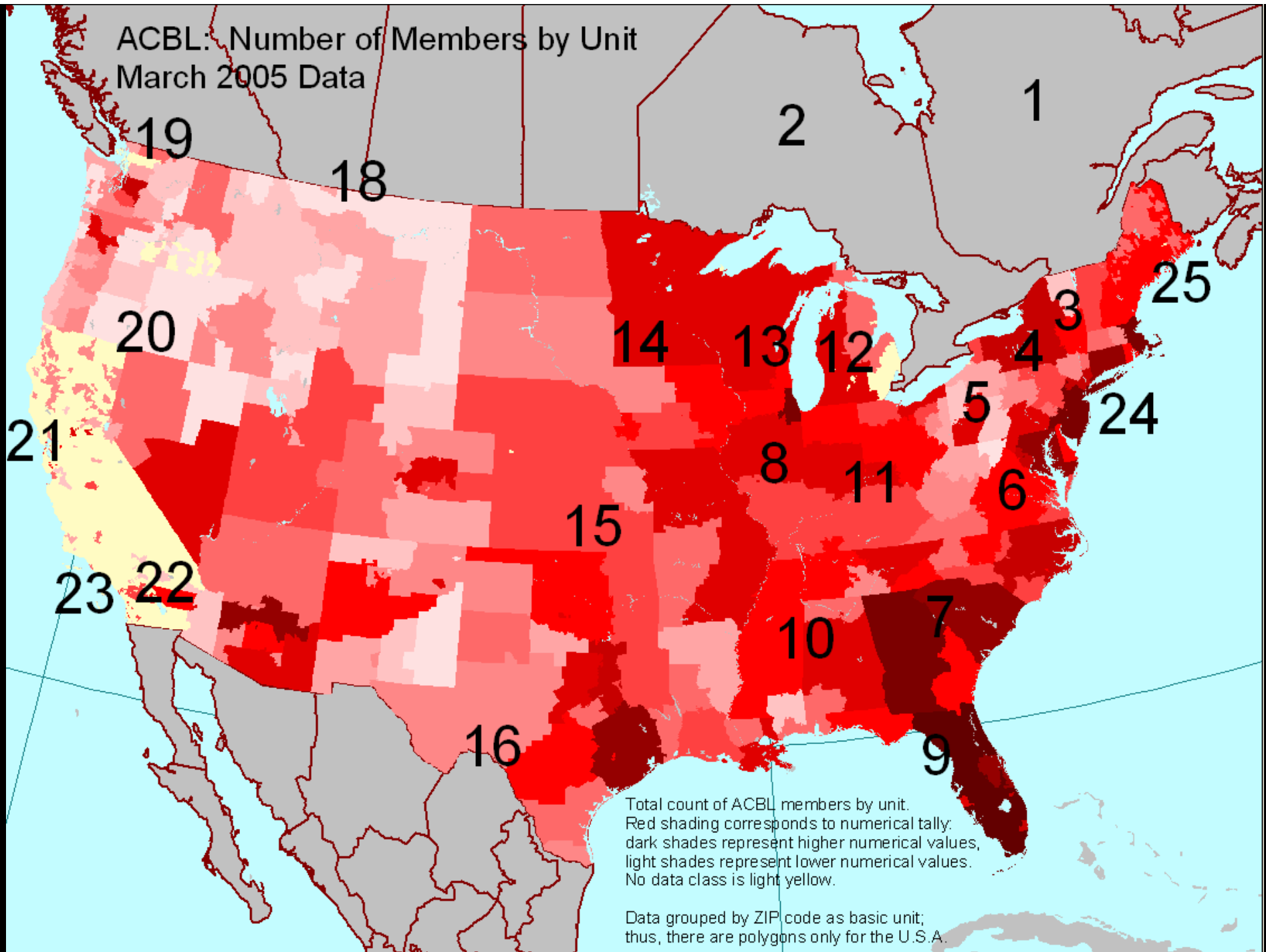


Figure 2. Ambiguity: note the light yellow area in southeastern Michigan.

Thanks to Jay Baum, ACBL CEO, Rick Beye, Carol Robertson, Richard Oshlag, and Ed Evers, ACBL, for providing the materials directly to Sandra Arlinghaus, who then created the map sets using GIS software (ESRI, ArcView 3.2) that forges a dynamic link between underlying database and outline base map. Graphic adjustments of various kinds were made in Adobe PhotoShop or Adobe Illustrator.

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<http://www.InstituteOfMathematicalGeography.org/>

MAPS AND DECISIONS, III: Redistricting.

Sandra L. Arlinghaus and William C. Arlinghaus

General Introduction

Tournament level duplicate bridge is a card game that is a sport. As is the case with sports, generally, there is an overseeing body: in basketball it is the National Basketball Association (NBA); in bridge it is the American Contract Bridge League for North America (ACBL) and the World Bridge Federation (WBF) for all nations in the world. The ACBL is a non-profit organization based in Memphis, Tennessee. The ACBL has about 150,000 members in the U.S.A., Canada, and Mexico. The WBF has more than 10 million members. The ACBL owns two buildings in Memphis where they house a large staff to maintain records, databases, publications, and a host of other operations associated with this business in the entertainment/sports sector of the business world. The second author of this work is currently a member of the Board of Directors of the ACBL. This Board, as do equivalent boards of other corporations, sets policy for the organization, makes decisions that affect the entire population of ACBL members, and oversees the work of the Chief Executive Officer. There are 25 Board members, each representing one geographical "district" of the ACBL. Thus, the members of the Board of Directors are also referred to, even though their charge is to represent the interests of the entire ACBL, "District Directors."

Redistricting

The second author was also a member of the "Redistricting Committee," an ACBL Board committee, during the year 2005. A quick glance at the numbers in Figure 1, however, suggests that perhaps the system might work better if the size, shape, or number of districts were altered. ACBL staff and management has been interested, for a number of years, in having a more streamlined organization. One way to consider such a process is to reduce the number of districts and shrink the size of the Board of Directors. Naturally, Directors from different regions have various viewpoints on this matter. During 2005, a "redistricting" committee was convened to consider the possibility of redistricting the ACBL. Maps offered some useful information for decisions; because the maps were created in a GIS with database and map interaction, it was relatively straightforward to create various scenarios for the Redistricting Committee to consider.

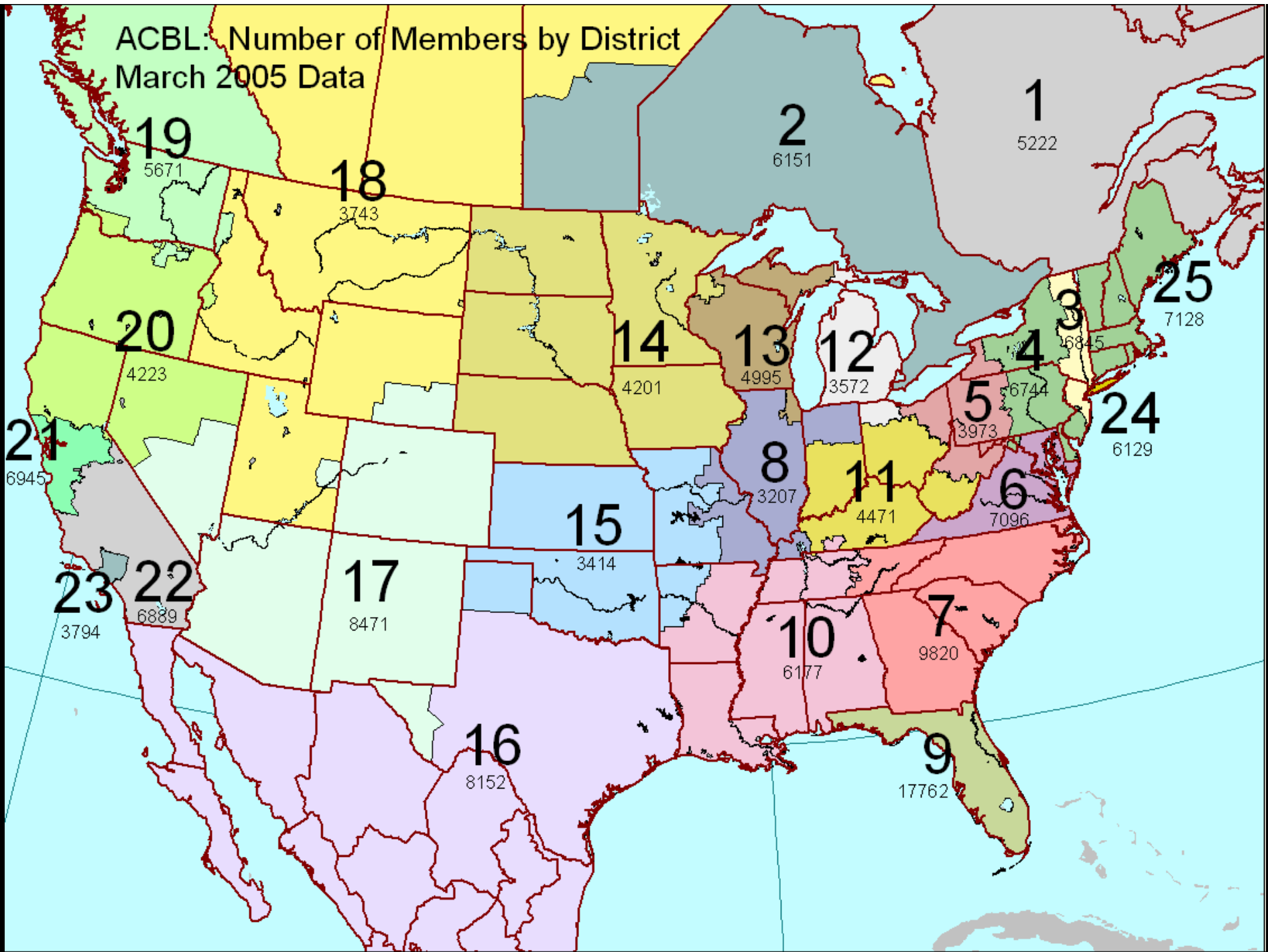
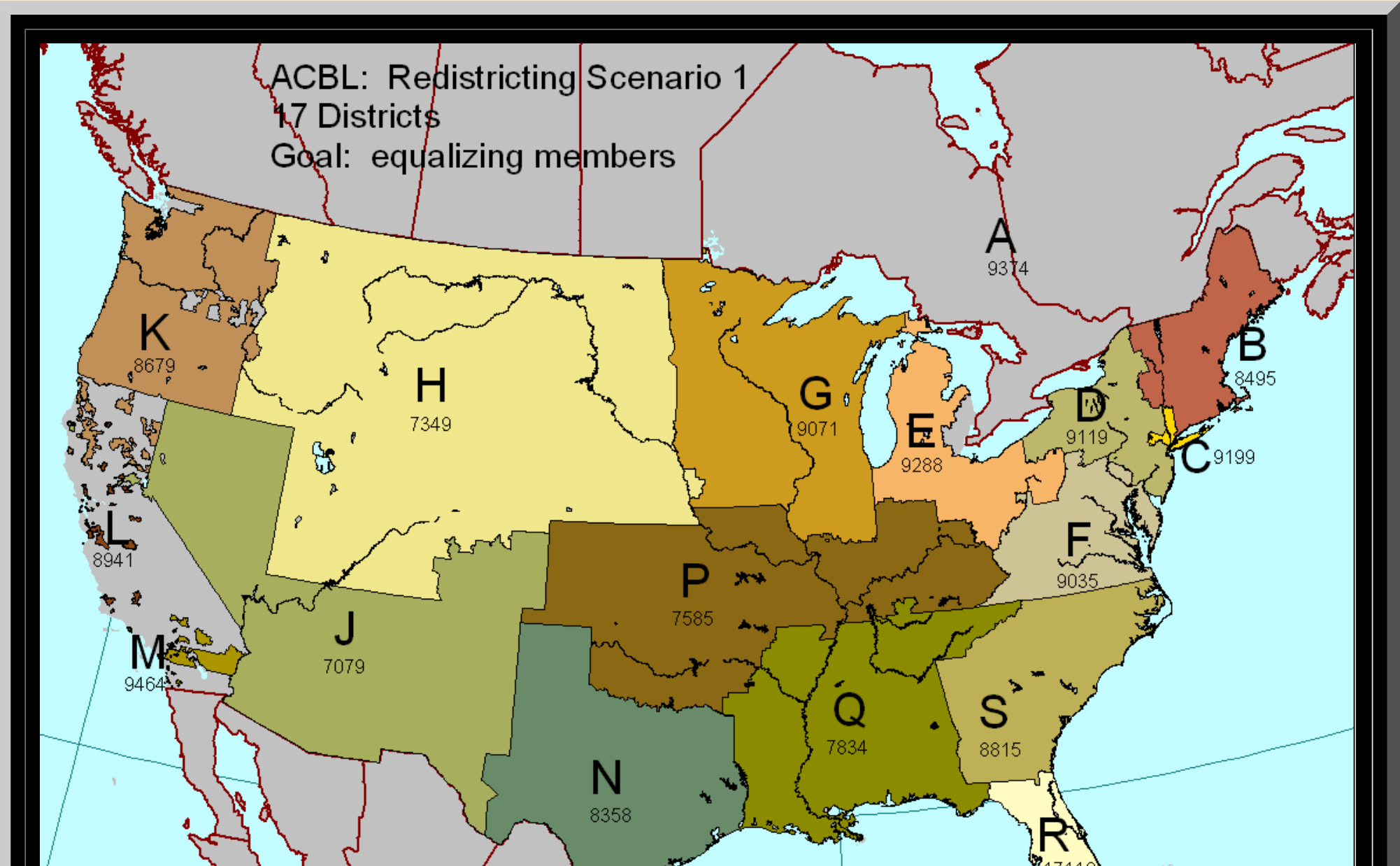
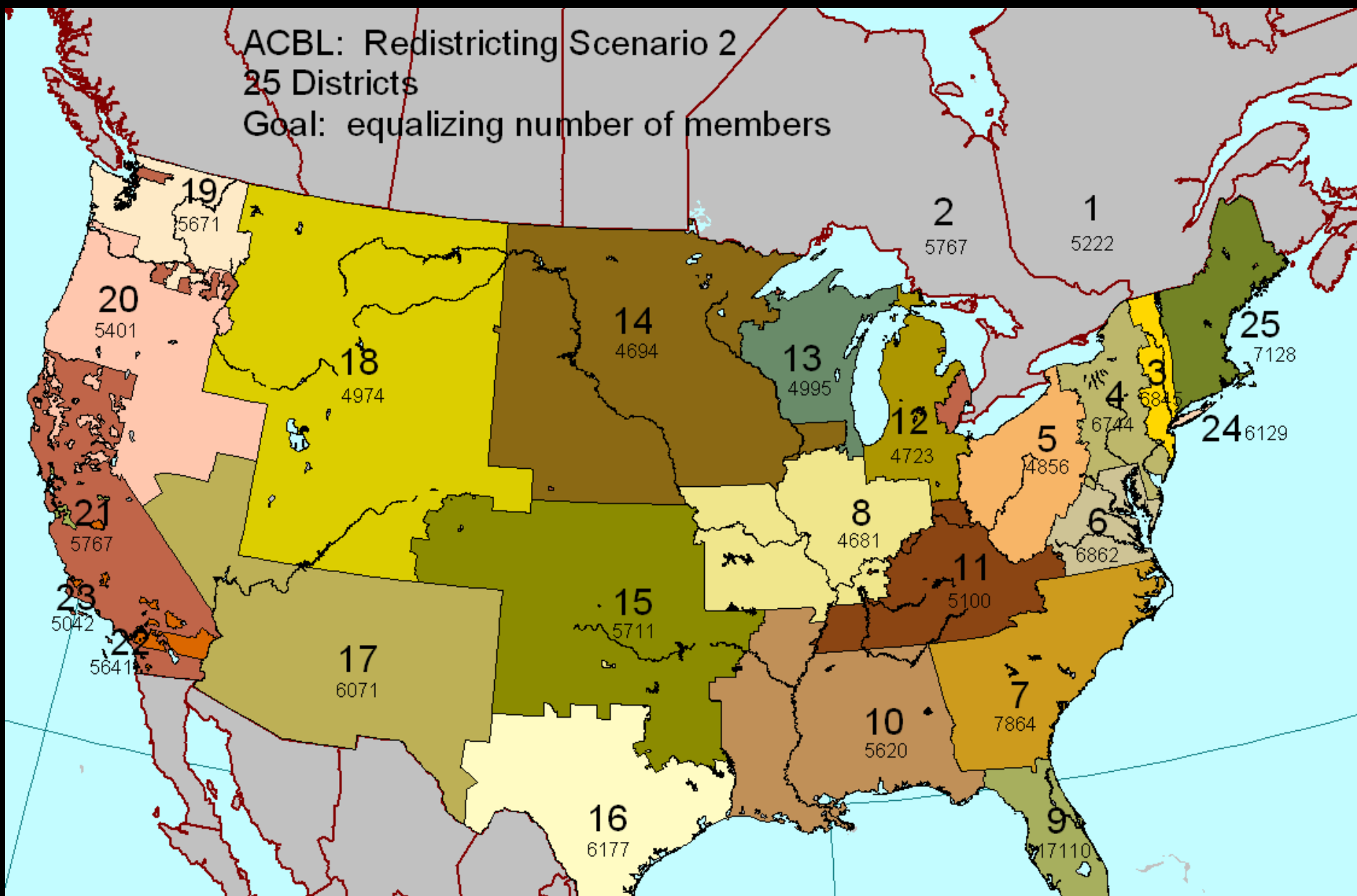
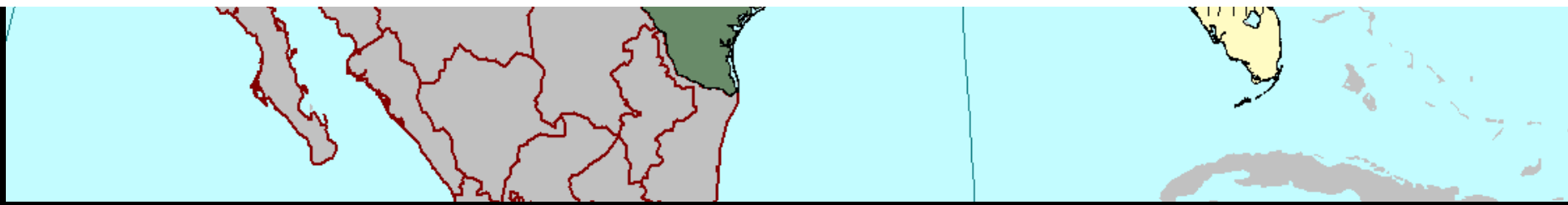


Figure 1. ACBL membership mapped by District, based on March 2005 data.

Two of the more plausible possibilities are presented below in Figure 2a, with 17 districts, and in Figure 2b, with 25 districts. In each the goal was to balance, insofar as possible, the number of members in each district (viewing Florida as an outlier). The map in Figure 2a has 17 districts responding to management's desire to have a smaller Board of Directors. The map in Figure 2b has 25 districts; no district director loses a position. In this case, the maps used in conjunction with other information, led the Redistricting Committee to keep the status quo--changing district boundaries creates a great deal of work in the conversion process so that a realignment of the sort proposed in Figure 2b seemed not worth the effort. After careful analysis of the evidence, including the evidence of maps, the decision was reached to retain the current pattern shown here in the map in Figure 1.







Many thanks to Al Levy, Chair, ACBL Redistricting Committee and to members of that committee. Thanks to Jay Baum, ACBL CEO, Rick Beye, Carol Robertson, Richard Oshlag, and Ed Evers, ACBL, for providing the materials directly to Sandra Arlinghaus, who then created the map sets using GIS software (ESRI, ArcView 3.2) that forges a dynamic link between underlying database and outline base map. Graphic adjustments of various kinds were made in Adobe PhotoShop or Adobe Illustrator.

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MAPS AND DECISIONS, IV: Club Data.

Sandra L. Arlinghaus and William C. Arlinghaus

General Introduction

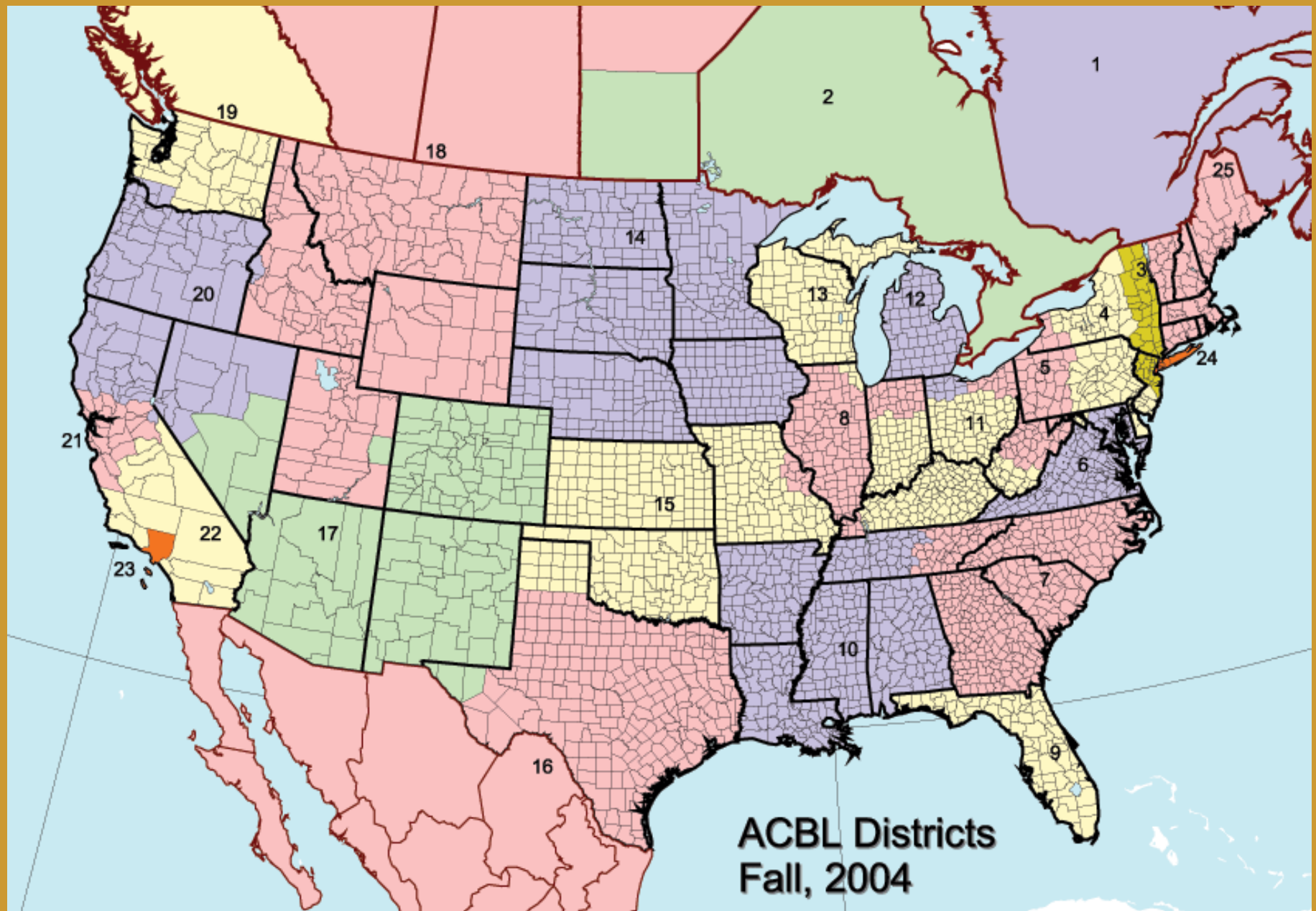
Tournament level duplicate bridge is a card game that is a sport. As is the case with sports, generally, there is an overseeing body: in basketball it is the National Basketball Association (NBA); in bridge it is the American Contract Bridge League for North America (ACBL) and the World Bridge Federation (WBF) for all nations in the world. The ACBL is a non-profit organization based in Memphis, Tennessee. The ACBL has about 150,000 members in the U.S.A., Canada, and Mexico. The WBF has more than 10 million members. The ACBL owns two buildings in Memphis where they house a large staff to maintain records, databases, publications, and a host of other operations associated with this business in the entertainment/sports sector of the business world. The second author of this work is currently a member of the Board of Directors of the ACBL. This Board, as do equivalent boards of other corporations, sets policy for the organization, makes decisions that affect the entire population of ACBL members, and oversees the work of the Chief Executive Officer. There are 25 Board members, each representing one geographical "district" of the ACBL. Thus, the members of the Board of Directors are also referred to, even though their charge is to represent the interests of the entire ACBL, "District Directors."

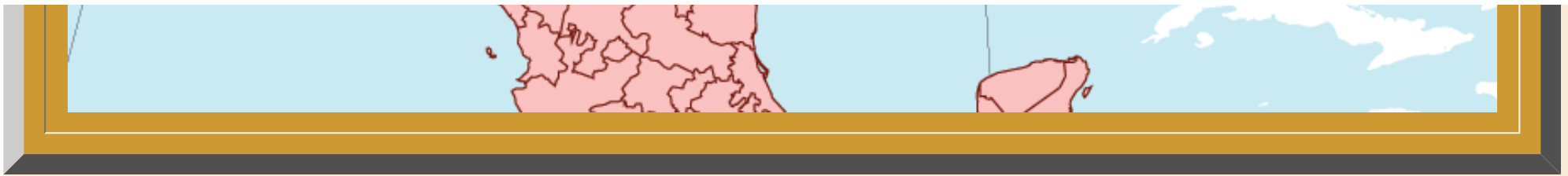
Club Data

Duplicate bridge is played in tournaments that vary in level from "national championships" to "regional" or "sectional" championships. There are tournament opportunities throughout the nation on a fairly regular basis, including three North American Bridge Championships per year. Regionals and Sectionals are more frequent and cater to geographic regions more localized than that of the continent. If, however, one wishes to play on a daily basis, then playing at local bridge clubs may be an attractive alternative. Clubs are also often a pleasant place for beginners to learn, away from the intense competition of the tournament scene. Aileen Osofsky, National Goodwill Chair of the ACBL, expressed to the first author a desire to have a map/data system that would enable her to pinpoint groupings clubs in an effort to more easily extend goodwill to newcomers and promote bridge as a sport to younger players. In this situation the goodwill and recruitment policy is informed and guided by maps.

- Click on a district in the map below and the list of clubs, arranged by unit, from the ACBL database will pop up. **Use the tabs at the bottom to guide your path through the data, or come back to the map and click on a different district.**

- Click [here](#) and a database with a filter applied will come up in Excel so the user can sort the database by any column; click on the down arrow in the desired column header.





Many thanks to Aileen Osofsky, National Goodwill Chair, ACBL. Thanks to Jay Baum, ACBL CEO, Rick Beye, Carol Robertson, Richard Oshlag, and Ed Evers, ACBL, for providing the materials directly to Sandra Arlinghaus, who then created the map sets using GIS software (ESRI, ArcView 3.2) that forges a dynamic link between underlying database and outline base map. Graphic adjustments of various kinds were made in Adobe PhotoShop or Adobe Illustrator.

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3D Atlas of Ann Arbor: The Google Earth® Approach

Part I

Sandra Lach Arlinghaus

Note: Google Earth® and Google SketchUp® are both names trademarked by Google®.

The virtual reality files in the first edition of the *3D Atlas of Ann Arbor* were all made using a strategy involving Geographic Information System (GIS) software coupled with 3D graphic design software. The process was effective but often time-consuming and the files created were quite large. Indeed, in the latter vein, alternative strategies, such as linking one file to another in a hierarchical pattern were employed so that the reader never had too large a file on screen at a single time. Another issue involving the creation of these files was cost of the software. The process involved many thousands of dollars worth of software (and of time of experts using the software). Small cities had faint hope of being able to maintain a state-of-the-art 3D system without substantial donations of time, software, or both.

Google Earth® offers a different approach. The interactive Earth viewer is, by now, probably familiar to most readers of *Solstice*. The spectacular shiny Earth-ball causes most to exclaim at its beauty on first sight. Further probing of the interactive capability brings additional admiration for its capabilities. Figure 1, below, shows an animated set of images from Google Earth®. It begins with the Earth and then zooms in to scenes of Chicago,.

- The first frame of Chicago shows only flat color aerials of the city superimposed on the surface of the Earth. The second through sixth frames show scenes of the city with buildings, in 3D, erected on the surface.
- The second frame shows an overhead scene along the Chicago River and is the locale to which Google Earth® takes the viewer, automatically, when "Chicago" is typed into the Google Earth® search box.
- The third frame shows a view of downtown Chicago, looking west from the eastern end of Navy Pier. The tall curvilinear apartment building at the foot of the pier is Lakepointe Tower.
- The fourth frame shows the curved outline of Lakepointe Tower up close with a view of the downtown looking to the southwest.
- The fifth frame moves to the south of Lakepointe Tower, looking west along the Chicago River through the locks. These images recall with fondness the field trips led by Harold Mayer (then Professor of Geography at the University of Chicago) that the author participated in as a pre-collegiate student at the University of Chicago Laboratory Schools.
- The sixth frame shows the Wrigley Building, Chicago's "Wedding Cake" (a la Carl Sandburg).

Notice that only the flat aerial has texture; the building surfaces have no textures on them, although many of the buildings have been modeled to look like reality--that is, they are not mere rectangles extruded to boxes. The reader wishing to get a good look at Chicago should download Google Earth® (a free download from google.com) and drive around interactively within the model. Then, try looking at a city that is familiar and see what memories emerge.

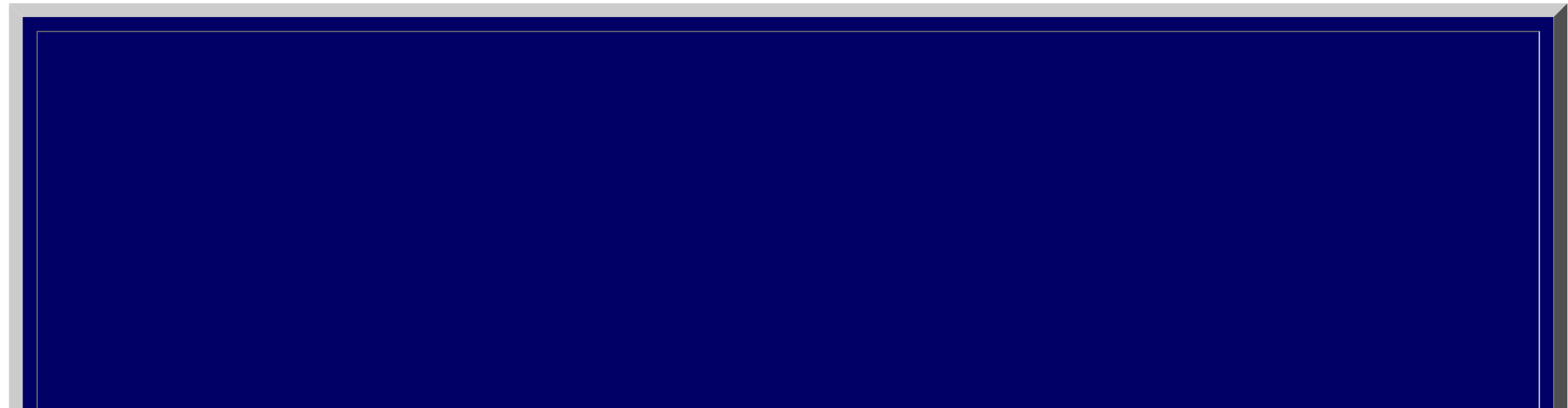




Image © 2006 TerraMetrics
Image © 2006 NASA
© 2006 Europa Technologies

© 2005 Google™

Pointer 55°26'58.36" N 103°50'21.86" W

Streaming ||||| 100%

Eye alt 3953.94 mi

Figure 1: Chicago scenes.

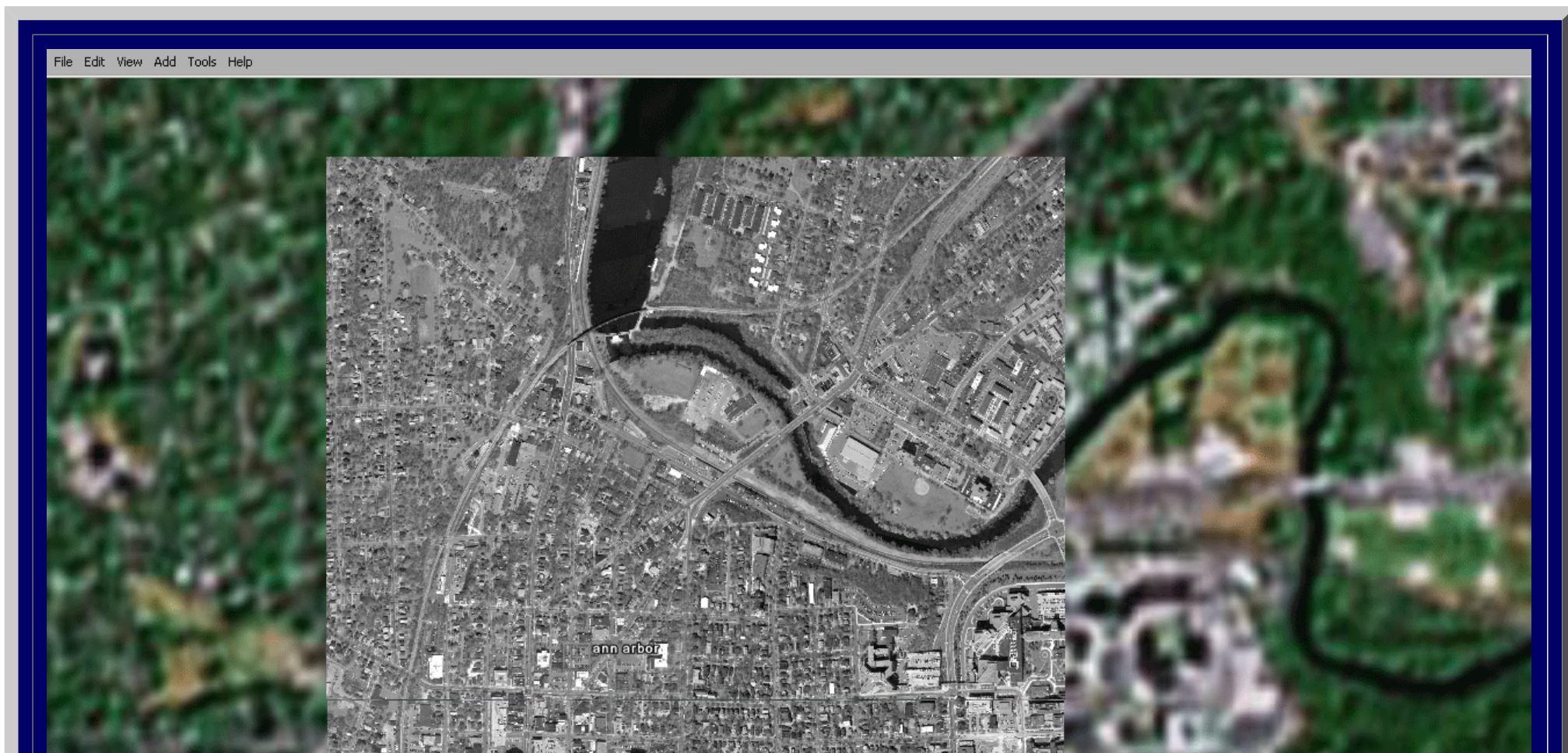
Beyond using Google Earth®, one might also be motivated to wish to create files to upload to Google Earth®. Look, for example, at Ann Arbor in Google Earth®. There is only a low resolution aerial (Figure 2) giving the viewer almost no information.

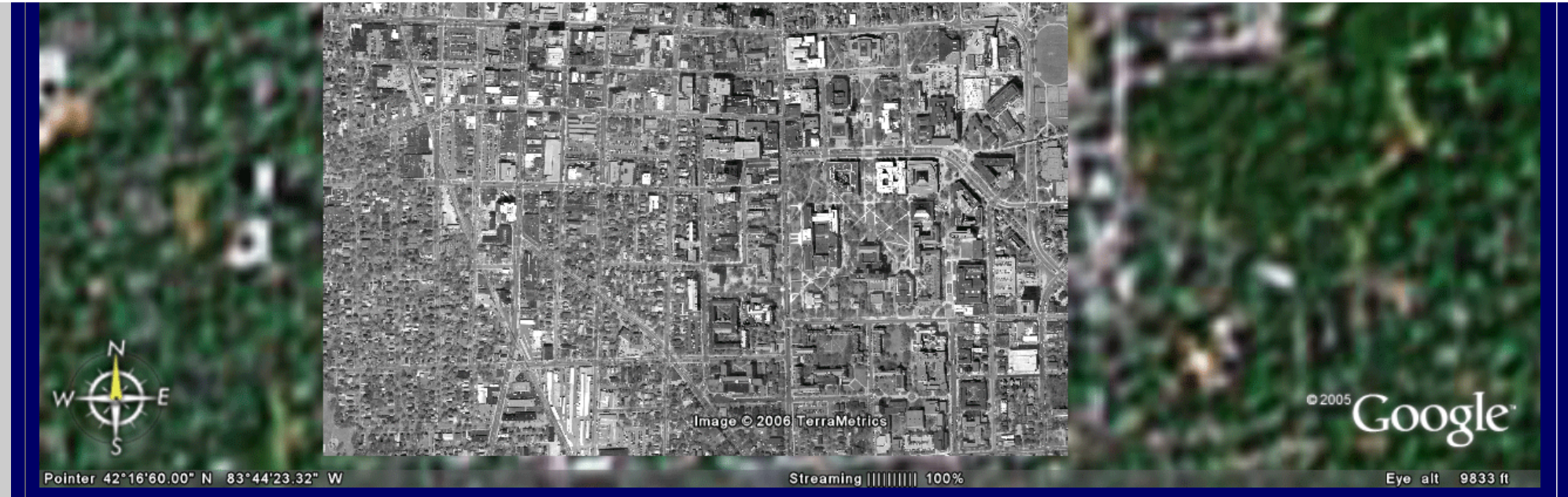




Figure 2. Default view of Ann Arbor in Google Earth®.

One nice feature of Google Earth® is that one can upload aeriels directly to the viewer and reposition them in relation to the coordinates already present. Figure 3 shows more detailed aeriels of Ann Arbor uploaded to Google Earth® (after being aligned with the coordinate system in relation to the location of the Huron River in downtown Ann Arbor). Simply click on the "Add" pulldown and navigate to the files to be uploaded and then reposition them or resize them as needed. The uploaded aeriels are available in the library of The University of Michigan; the City of Ann Arbor supplied them to the author. The first image in Figure 3 illustrates how the alignment was made with respect to the river. The second image shows the image from Figure 2 with aeriels superimposed.





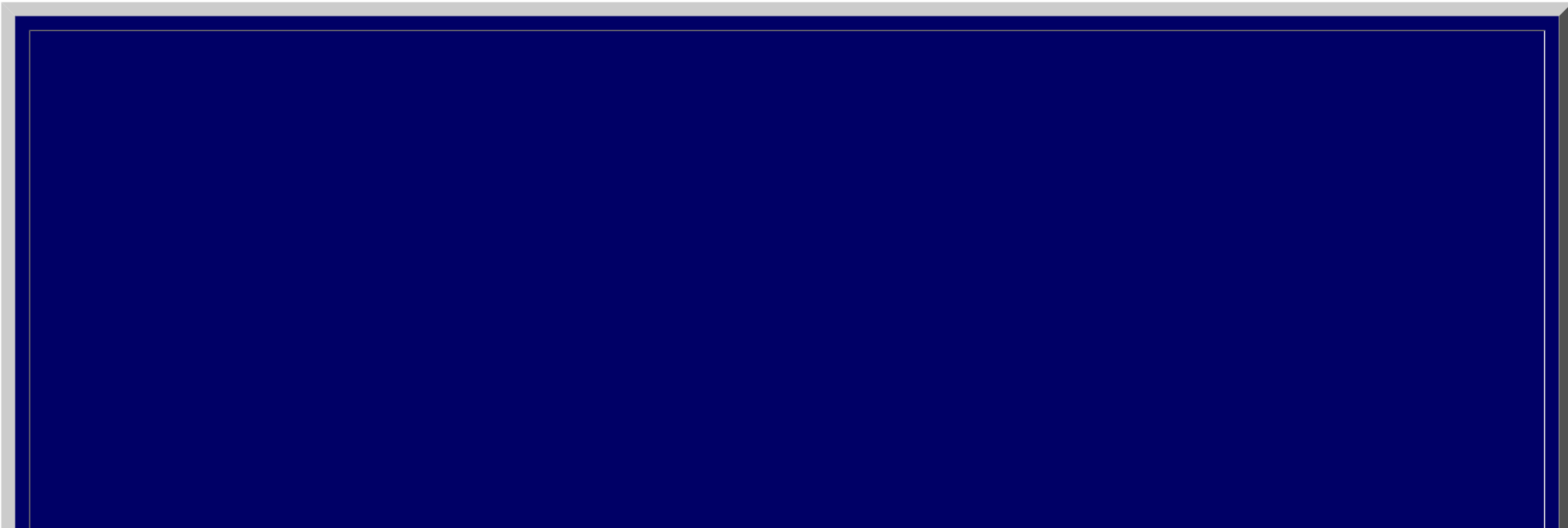
File Edit View Add Tools Help





Figure 3. A better view of part of Ann Arbor (top) and a closeup of central Ann Arbor.

As was the case with Chicago, the flat aerials give a good view of the buildings; however, if one tips the image on its side, no buildings are displayed (Figure 4). Unlike the case with Chicago, the Ann Arbor aerials are shot only in black and white.



File Edit View Add Tools Help

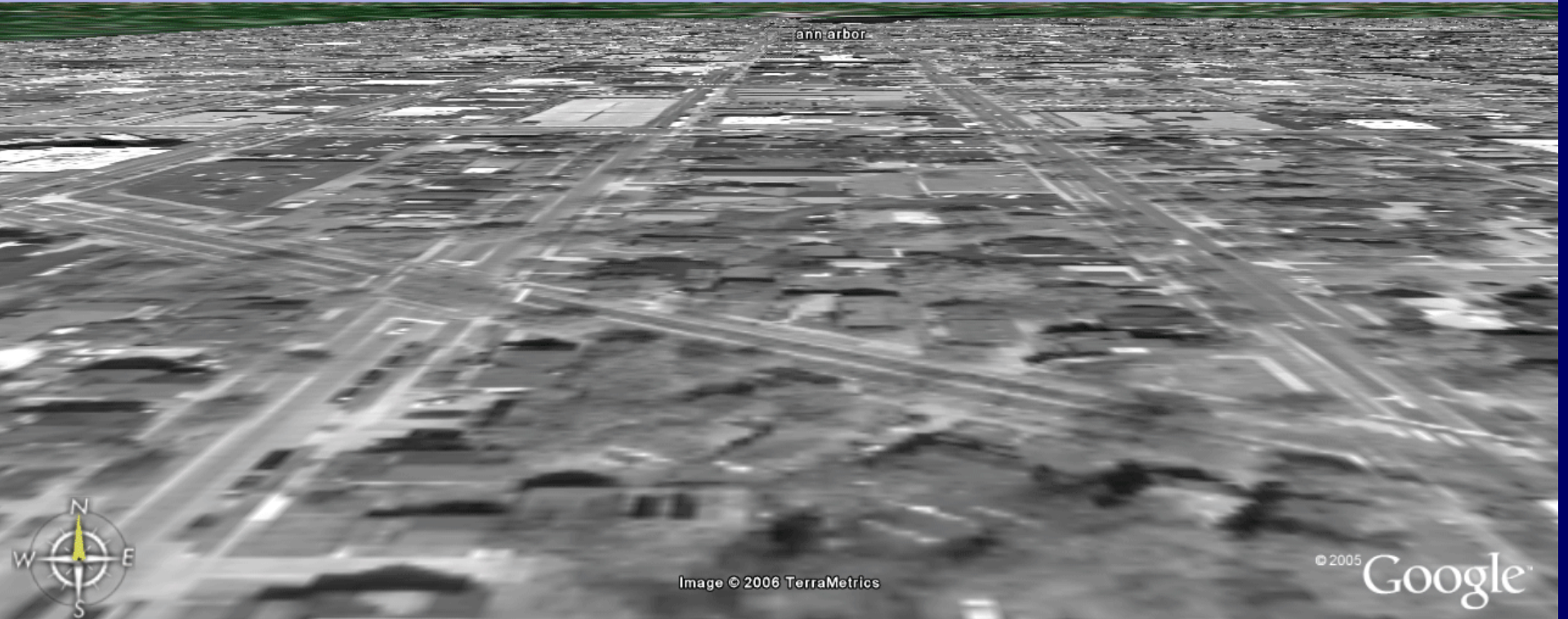


Image © 2006 TerraMetrics

© 2005 Google

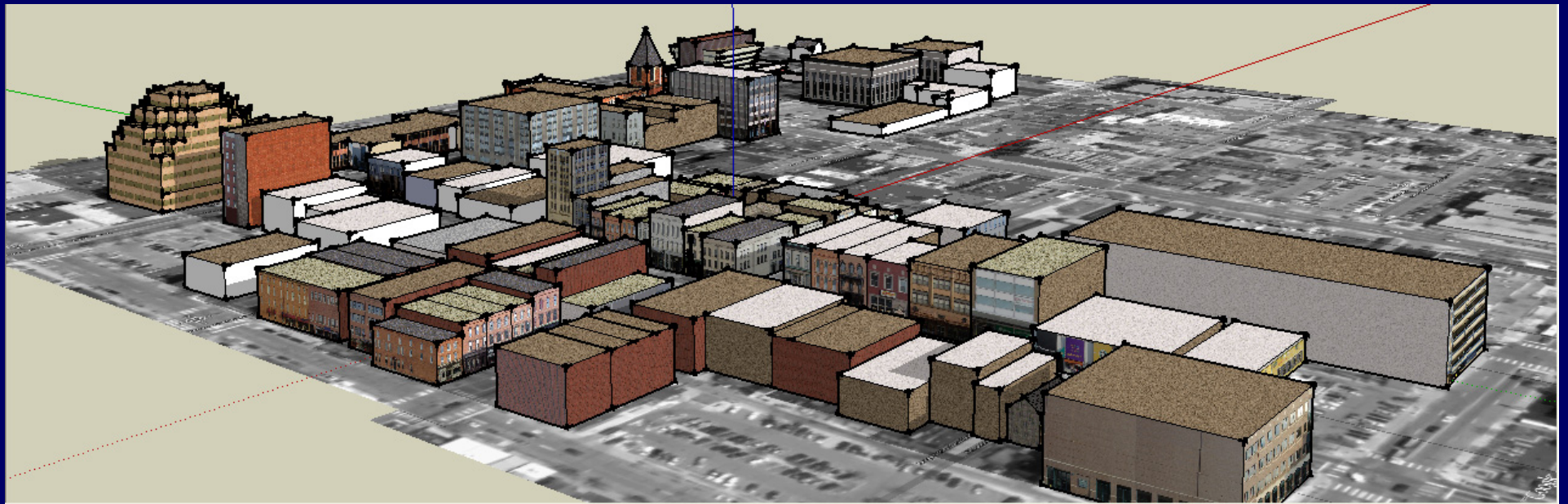
Pointer 42°16'30.41" N 83°44'44.09" W

Streaming ||||| 100%

Eye alt 112 ft

Figure 4. Ann Arbor, aerial only, lateral view shows no extruded buildings.

With the memory of the beautiful scenes of Chicago in mind, it is easy to ask if one might upload 3D images of Ann Arbor (or elsewhere) into Google Earth®. Another free download from Google®, called "SketchUp®" does indeed permit such uploads. The premise is elegant: the aerials uploaded into Google Earth® are coordinatized by virtue of their proper placement in the Google Earth® coordinate system. Google SketchUp® permits the direct download of these aerials while retaining the coordinate information. In SketchUp®, the user can add 3D buildings (digitizing them from the aerial and subsequently extruding them), model the buildings, apply photographic textures to them, set shadows, and a host of other operations. When done, simply press the upload button and the buildings will be uploaded back to Google Earth® in the correct position on the globe using the embedded coordinated system. Information lost in the upload is building material textures; only the flat surface textures of the aerial transfer through the interface. Perhaps future versions of the Google Earth/SketchUp® pair will permit such upload. The sequence of images below, in Figure 5, shows some of the features of SketchUp®.



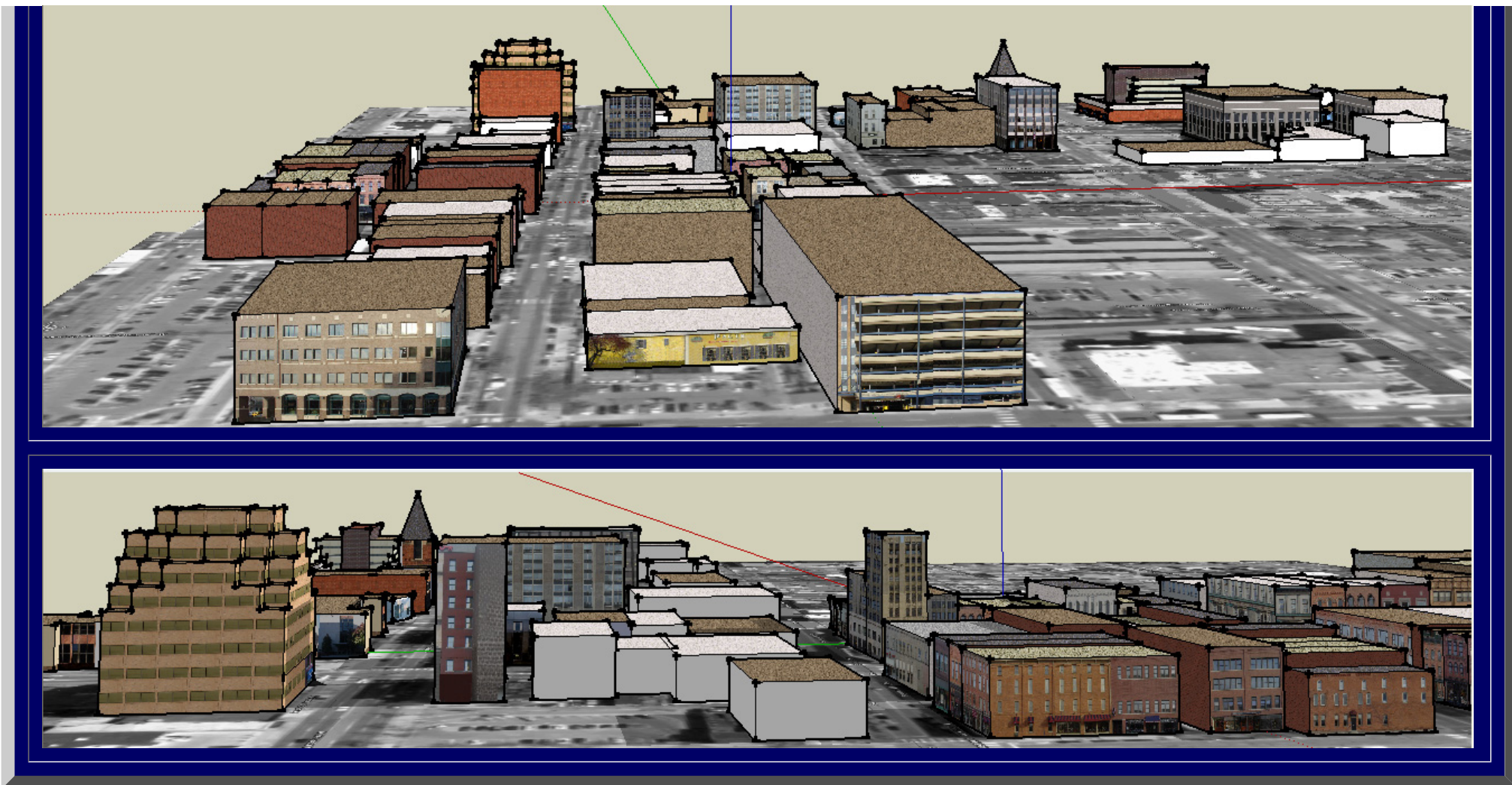
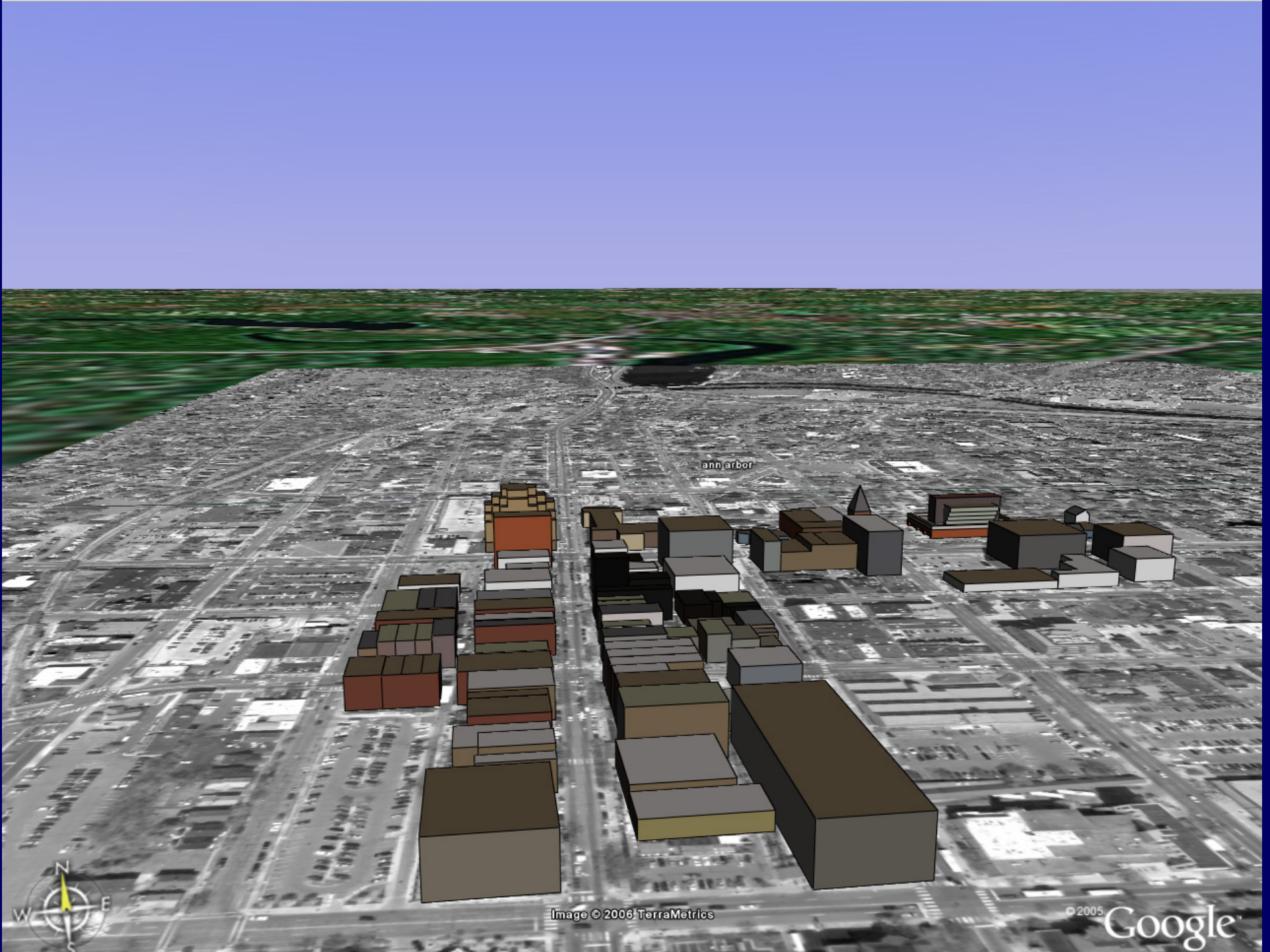


Figure 5. Downtown Ann Arbor (part) displayed in Google SketchUp®.

The files created from SketchUp® for upload into Google Earth® are quite small in size so that one can imagine creating a great many buildings and still having the files run smoothly. Figure 6 shows the upload of the material in Figure 5 to Google Earth® (**important:** turn off the terrain switch in Google Earth®). While the photographic textures do not upload, their influence is present as the buildings are more than the uniform shade of light gray (as in the Chicago file). Scale of buildings and other objects is made easy from photographs coupled with the onboard "tape measure" in Google SketchUp®. One might model any level of detail for the buildings, for street furniture, and so forth.

File Edit View Add Tools Help



ann arbor



Image © 2006 TerraMetrics

© 2005 Google

Pointer 0°00'00.00" N 0°00'00.00" E

Streaming ||||| 100%

Eye alt 526 ft

File Edit View Add Tools Help

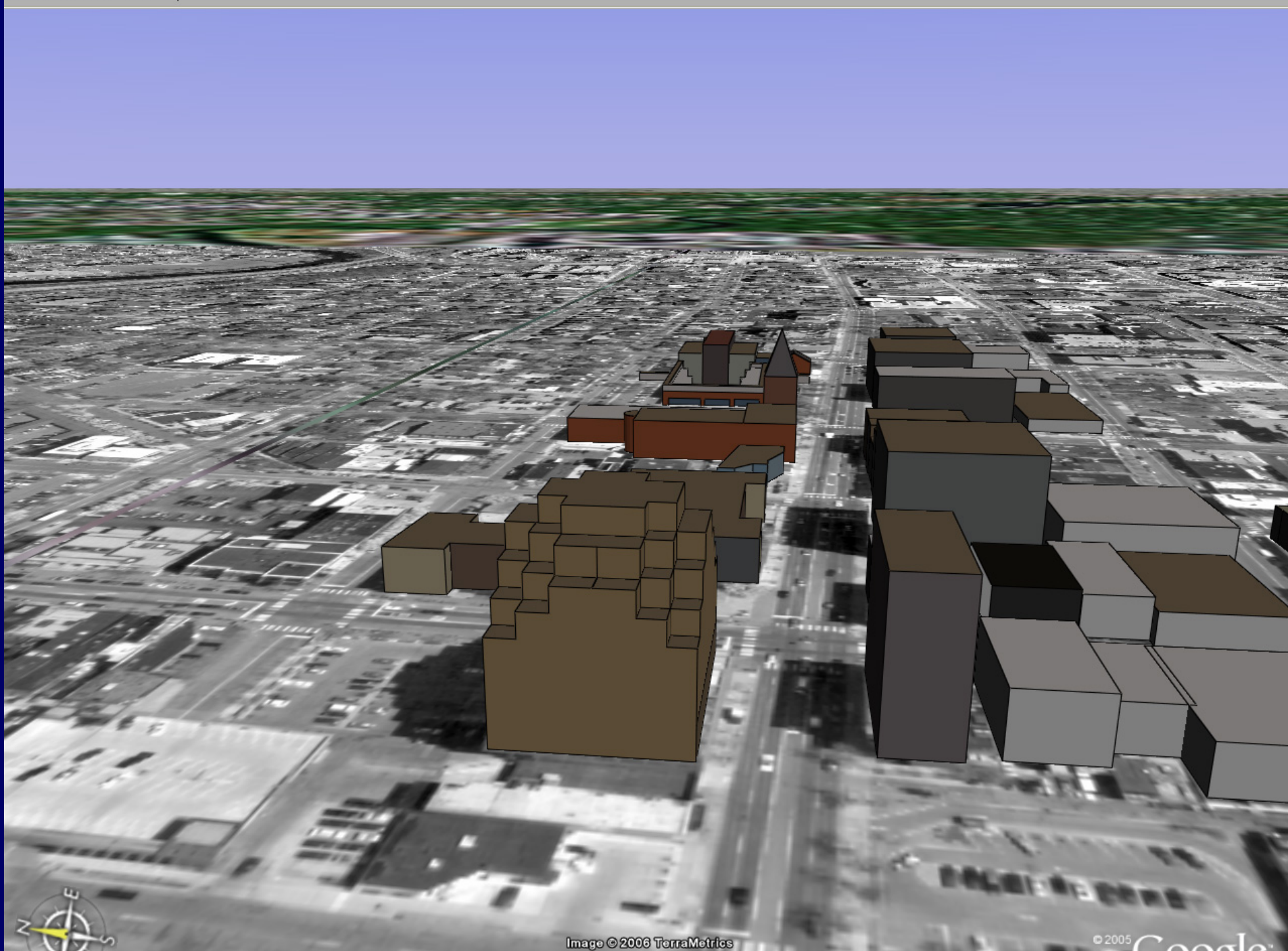


Image © 2006 TerraMetrics

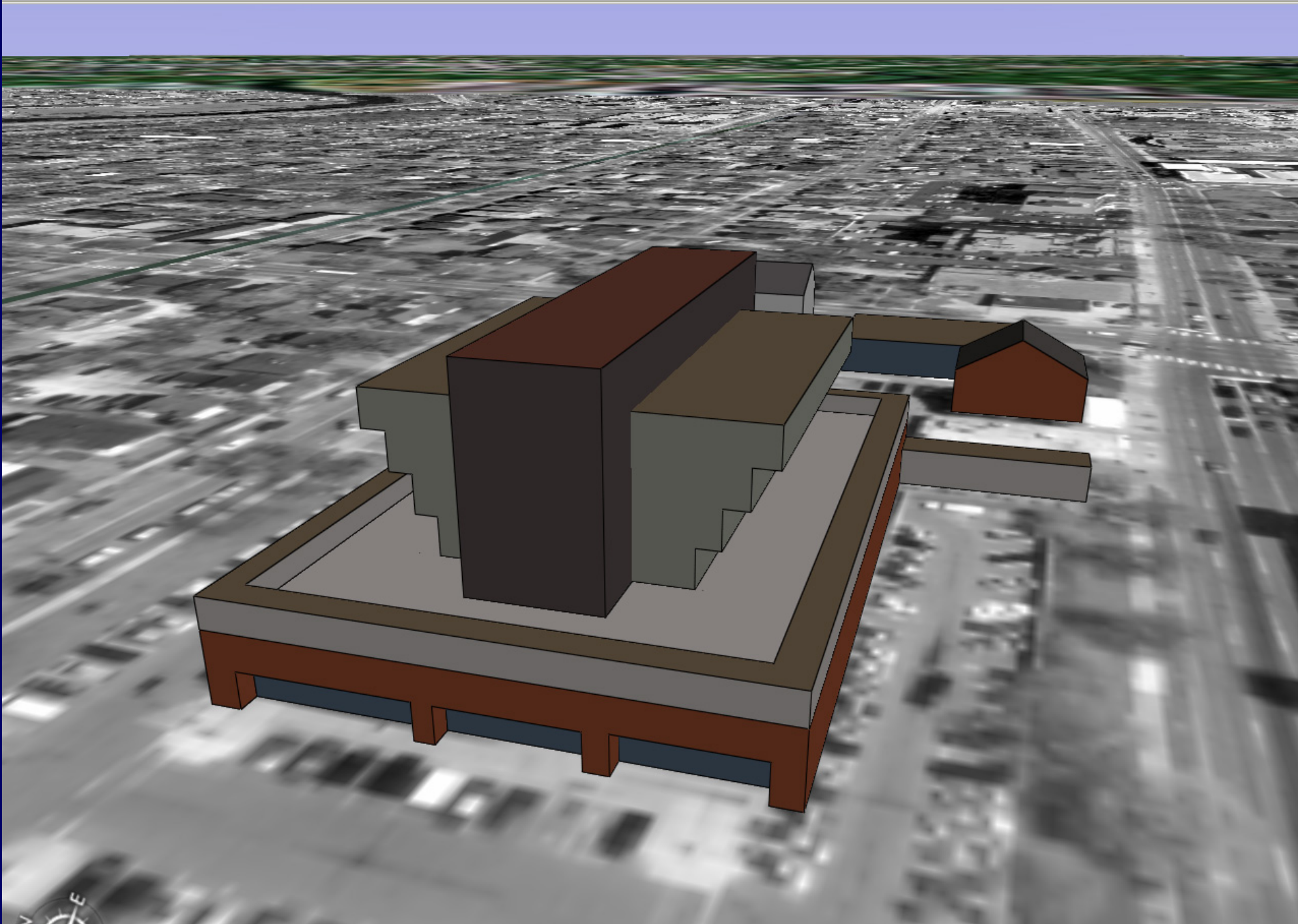
© 2005 Google

Pointer 42°17'01.02" N 83°44'05.18" W

Streaming ||||| 100%

Eye alt 277 ft

File Edit View Add Tools Help



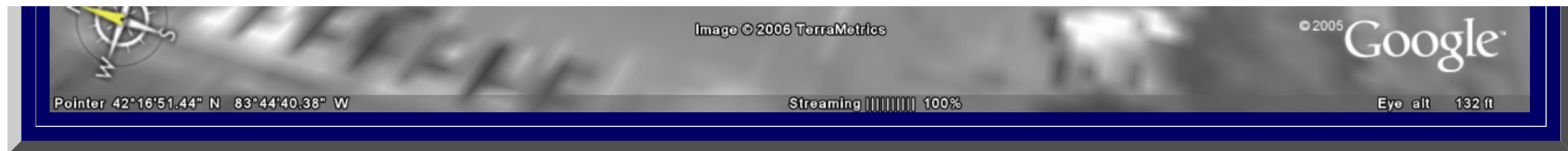
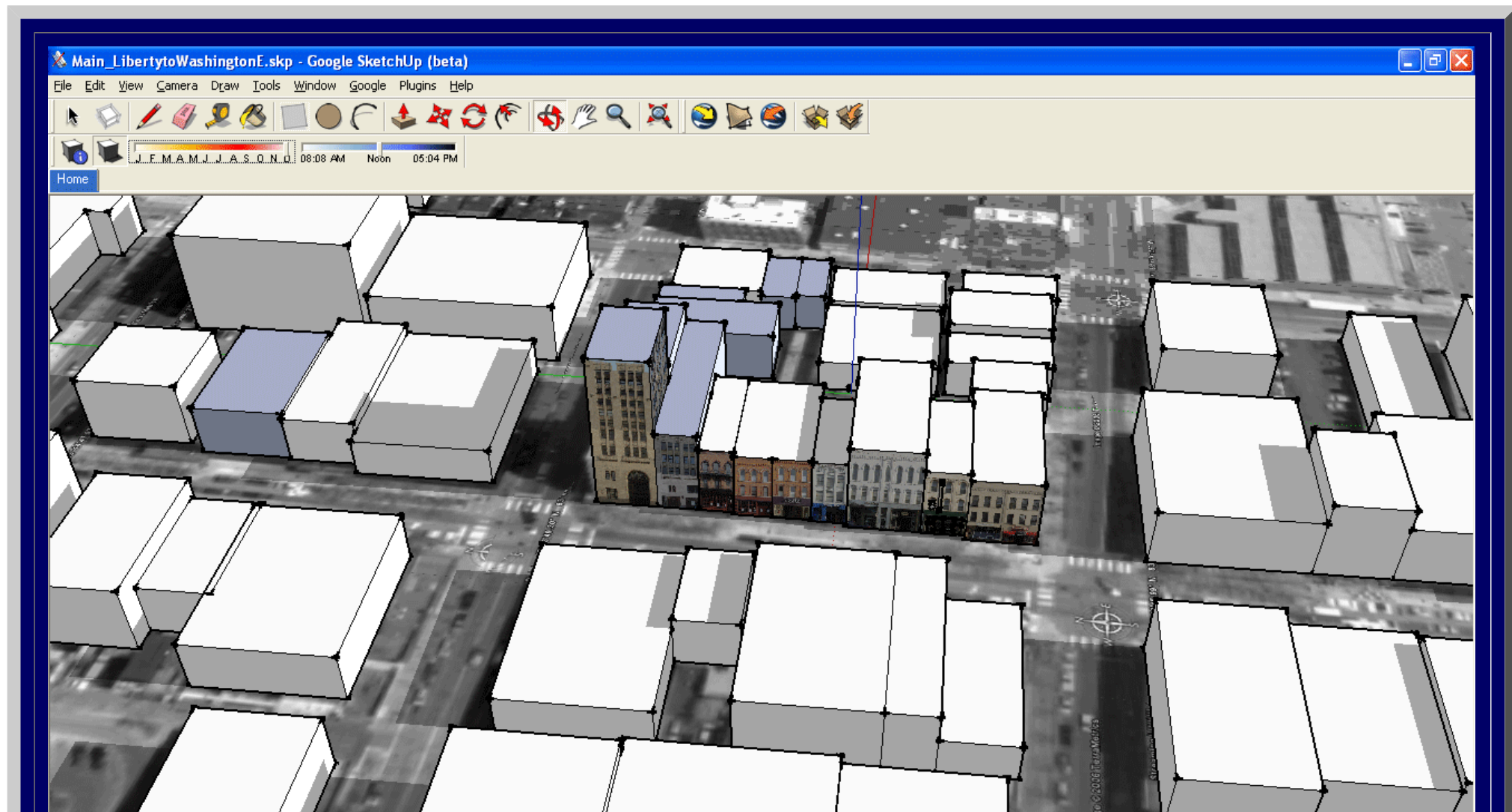


Figure 6. Google SketchUp® materials uploaded to Google Earth®.

The modeling of buildings, as opposed to the mere extrusion of buildings from digitized footprints, is quite straightforward once one works through the interactive tutorials. Another useful feature is the ability to set shadow patterns according to month of the year and time of day. Figure 7 shows an animated sequence of two views of the downtown, emphasizing shadow position at noon in each of the 12 months. The darkest shadow is that inherited from the aerial; watch the lighter gray shadow dance across it. When the SketchUp® files are uploaded to Google Earth® (Figure 6), the shadows do not upload. The shadow present in the Google Earth® image is that photographed in the uploaded aerial.



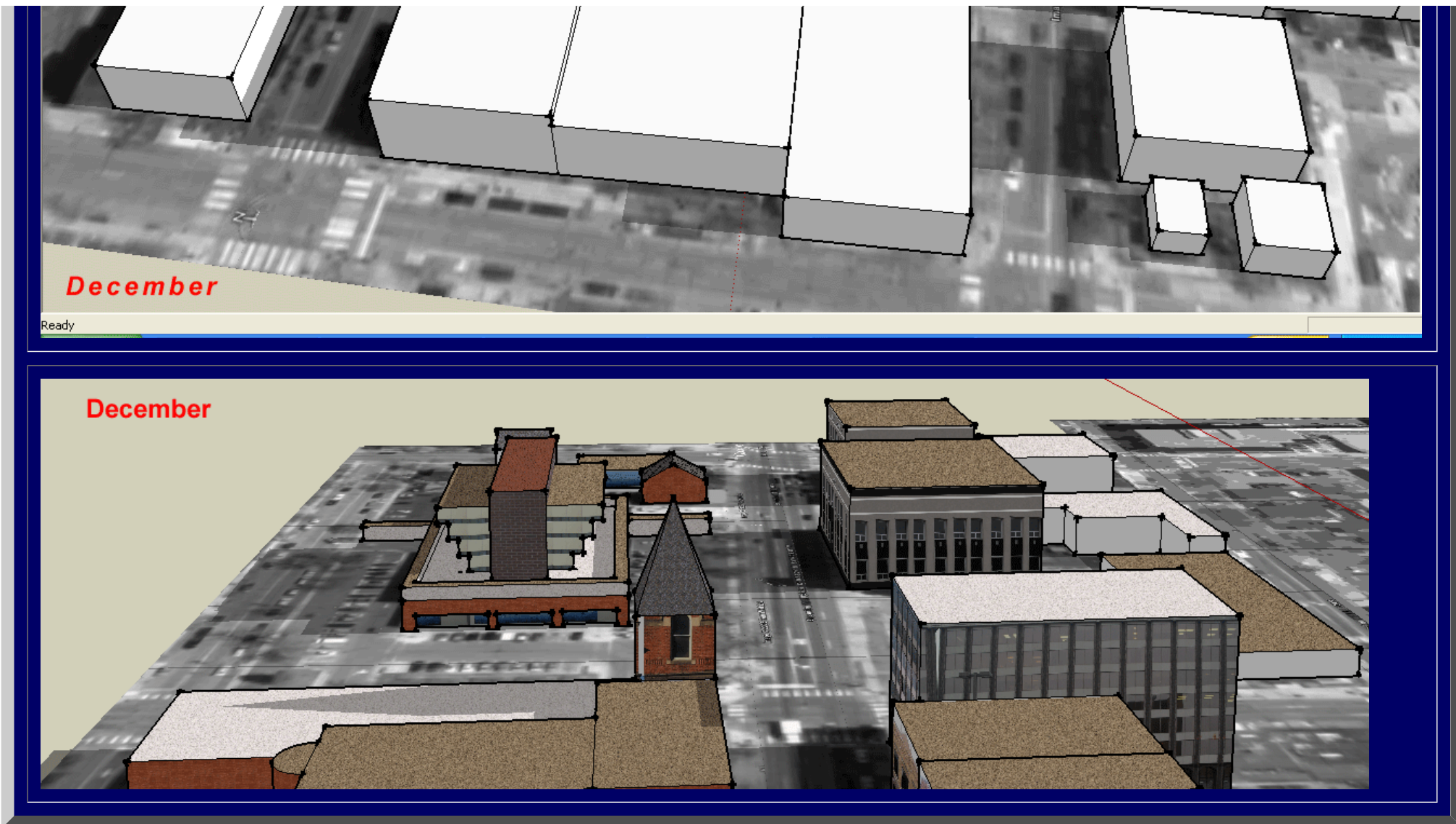


Figure 7. Animations emphasizing shadow position.

The images in Figures 1 to 8 are all based on screen captures from interactive products. As such, they present only a small part of the story. To see the Ann Arbor files, in Google Earth®, download Google Earth® to your hard drive. Save the following files to your desktop (scanned by Norton and virus-free at upload):

- Image files: [Link 1](#), [Link 2](#).
- Buildings file: [Link 3](#).

Then, in Google Earth®, go to Open and navigate to the files on your desktop and open each of the three. Then, type in "Ann Arbor" and you should be able to navigate the buildings for yourself! Notice the compass.

If you wish to view the scene with the photographic textures and shadows at varying positions, or to add new buildings of your own as an experiment in planning, then download Google SketchUp® (link below with the library of textures used) and install it and also download the SketchUp® file and Open it in SketchUp®.

- SketchUp® executable file with suitable library: [Link 4](#).
- SketchUp® file of Ann Arbor: [Link 5](#).

During the past 6 months presentations of the *3D Atlas of Ann Arbor*, 1st Volume (without Google Earth/SketchUp®), were given by the author to various groups of community leaders. Particular wishes, beyond what was presented in that material were:

- a desire to model buildings
- a desire to introduce detail at the level of a single building (awnings, window displays, design elements of the building, and so forth)
- a desire to measure the effect of shadows
- a desire to be able to add buildings oneself, in considering possible sites for future buildings
- a desire to build an Ann Arbor game
- a desire to be able to own easy-to-use software and do all of the modeling on a home computer, or a city council computer, with no additional purchase of software.

The Google Earth/SketchUp® package enabled the entire wish list. It is an important tool to add to the glittering array of software already employed by many urban and environmental planners.

What remains, beyond the obvious (but time-consuming) completion of all buildings and field-checking of heights and facades, is:

- to integrate the effect of terrain
- to learn to transform the files into a format that will play out in an immersion CAVE or other interesting 3D visualizations.

On June 9, 2006, the author presented the material in this article to an invited group at the 3D Laboratory of the Duderstadt Center at The University of Michigan: John Nystuen, Gwen Nystuen, Fred Goodman, Ann Larimore, and Bart Burkhalter. Click here for photos from the event: [photo 1](#); [photo 2](#).

The author thanks Michael Batty, University College London for his encouragement in with respect to the Google Earth® upload. She thanks Lars Schumann of the 3D Laboratory of The University of Michigan for pointing her to Google SketchUp®. She thanks Professor Klaus-Peter Beier (The University of Michigan College of Engineering) and the staff of the 3D Laboratory, as well as Matthew Naud (Environmental Coordinator, City of Ann Arbor), for their continuing interest and encouragement in all aspects of this project over the past 4 years. Merle Johnson of the City of Ann Arbor kindly supplied images of various sorts for this and for related projects. For a full list of all individuals associated with this project, please see the online *3D Atlas of Ann Arbor* also on the IMAge website.

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3D Atlas of Ann Arbor: The Google Earth® Approach

Part II

Sandra Lach Arlinghaus

Note: Google Earth® and Google SketchUp® are both names trademarked by Google®.

Part I of this work concluded with the comment below (cyan background in table below). During the meeting on June 9 (yellow background in table below), in which Part I was displayed to a set of viewers, the authored noted (as does Part I) that textures placed on buildings in Google SketchUp® did not transfer to Google Earth®.

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Lars Schumann informed the author on June 12 that a new version of Google Earth® and Google SketchUp® had just been released and that the new version supported textures in Google Earth® (among other things). The links below are links to files to load directly into the most recent version of Google Earth®. Download the files, and then open them in Google Earth® using the File|Open command. Also upload the aerial. Then, you should see a section of downtown Ann Arbor, Michigan, complete with building textures applied to modeled buildings (completely redone from the effort in Part I). Given that the photographic textures were in good shape and needed little extra work, it took about 15 hours to build the model of 6 blocks of the downtown-- a small effort when compared with previous work. The image that follows the links is a screen capture from the smaller of the two files, showing textured buildings in Google Earth®.

- Aerial link: [image](#). Load directly into Google Earth®. It will be positioned correctly.
- Main Street, historic district (first image below)
 - Buildings, [small file](#) (3.6 Mb): Load directly into Google Earth® and drive around the streets--3 blocks.
 - Buildings, [medium file](#) (4.4 Mb): Load directly into Google Earth® and drive around the streets--4 blocks. Source of the image below.
- Huron Street, buildings (9.5 Mb) [link](#) to file for Google Earth®; interesting modeling of buildings in Google SketchUp® (second and third images below).
- All Buildings, [large file](#) (10.8Mb): Load directly into Google Earth® using a Pentium 4+ machine with at least 1GB of RAM.

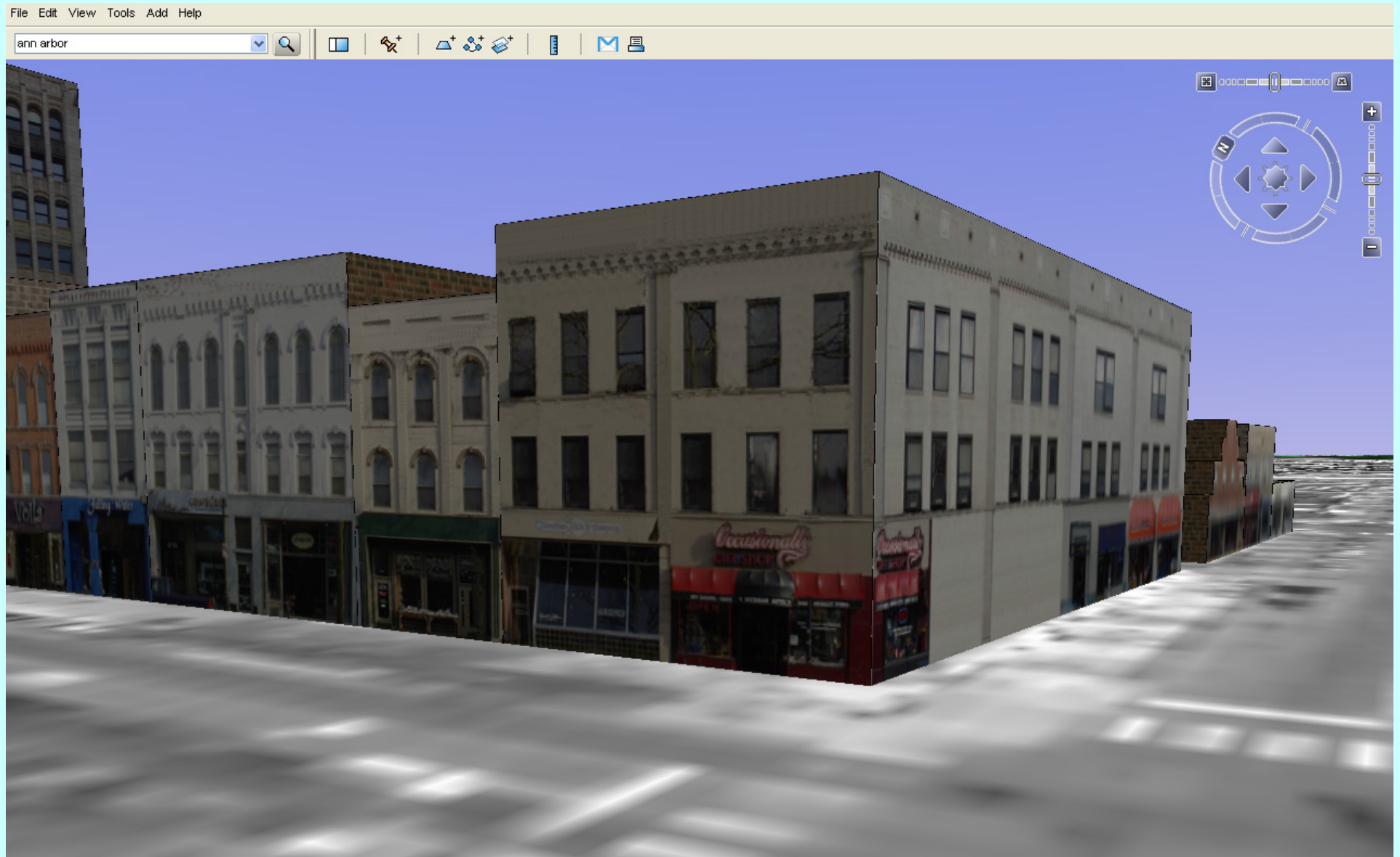


Image © 2006 DigitalGlobe
© 2006 Europa Technologies

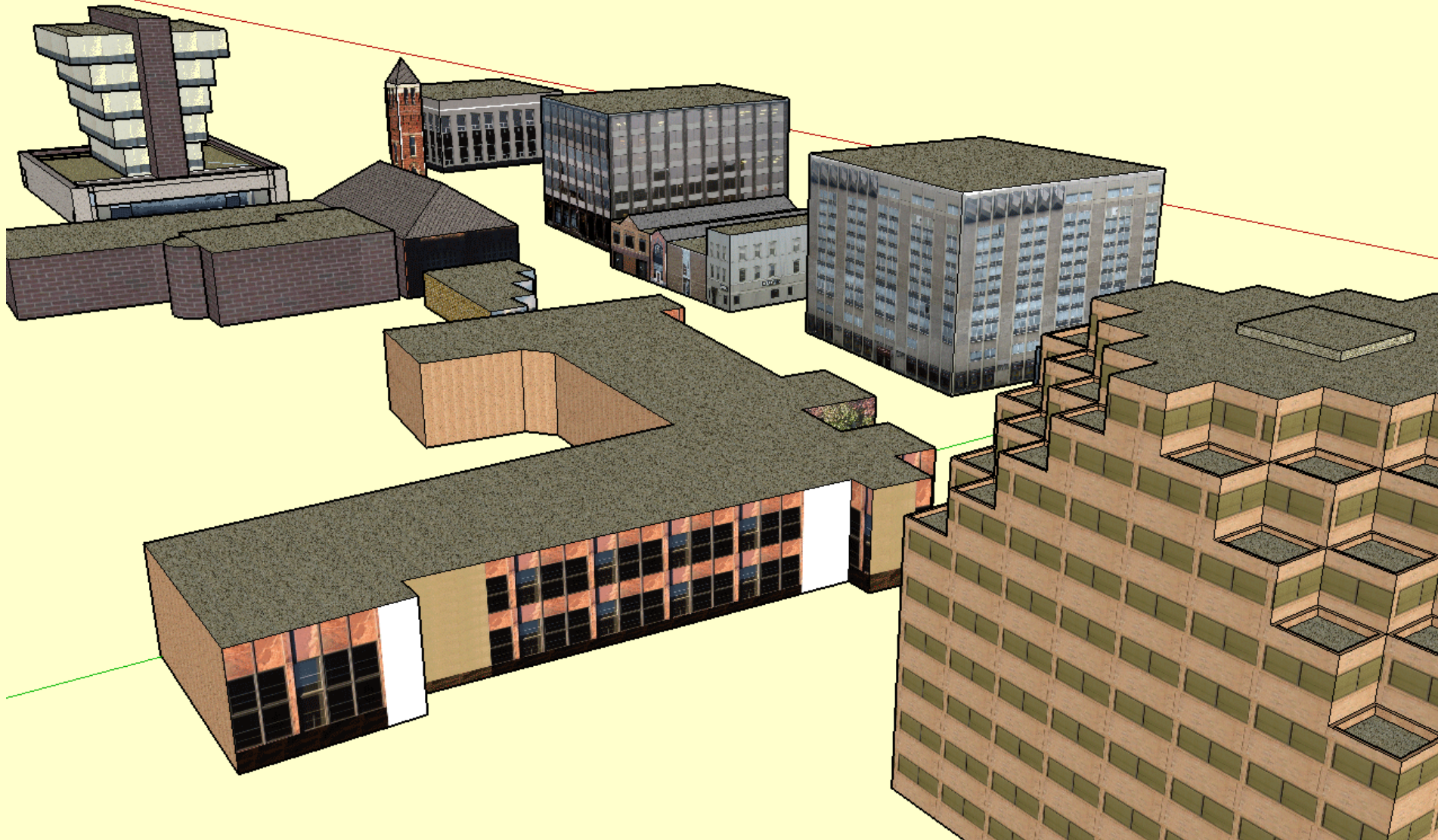
© 2005 Google™

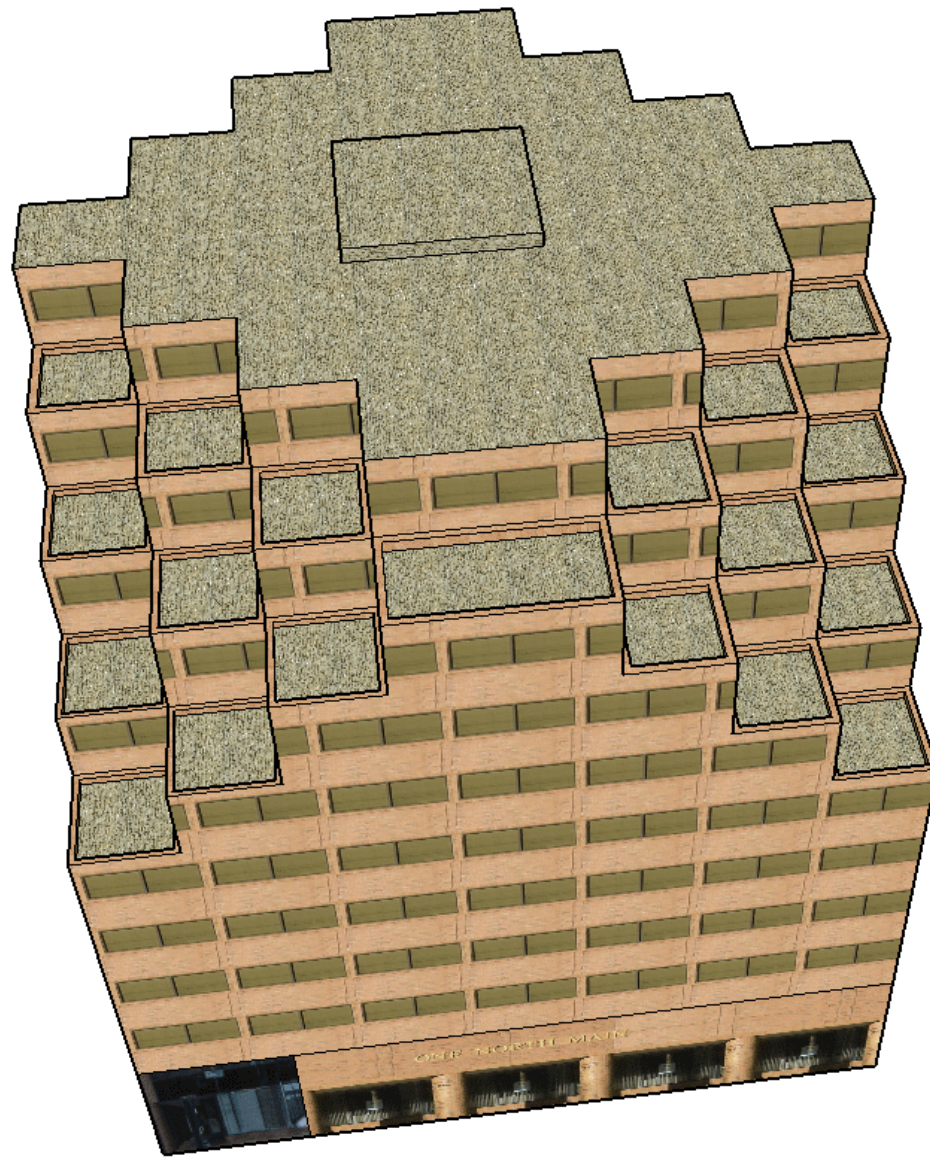
Image © 2006 TerraMetrics

Pointer 42°16'45.39" N 83°44'49.15" W

Streaming ||||| 100%

Eye alt 21 ft





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RESEARCH ANNOUNCEMENT

Zipf's Hyperboloid?

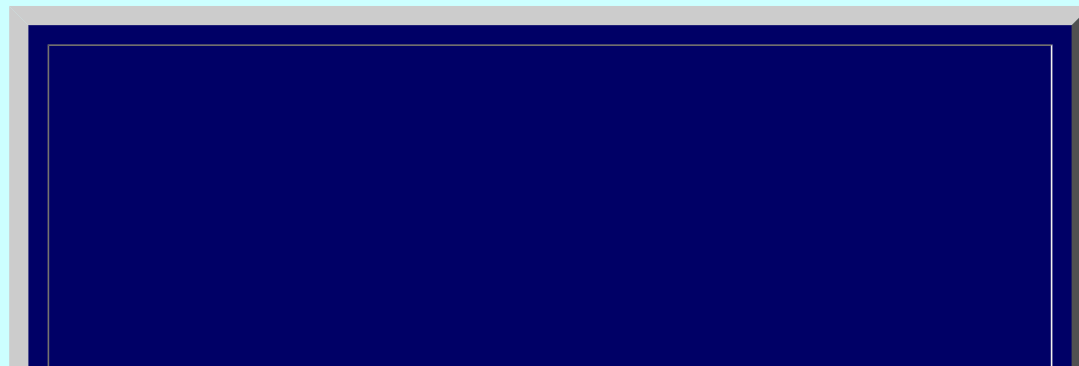
[Sandra L. Arlinghaus](#), The University of Michigan

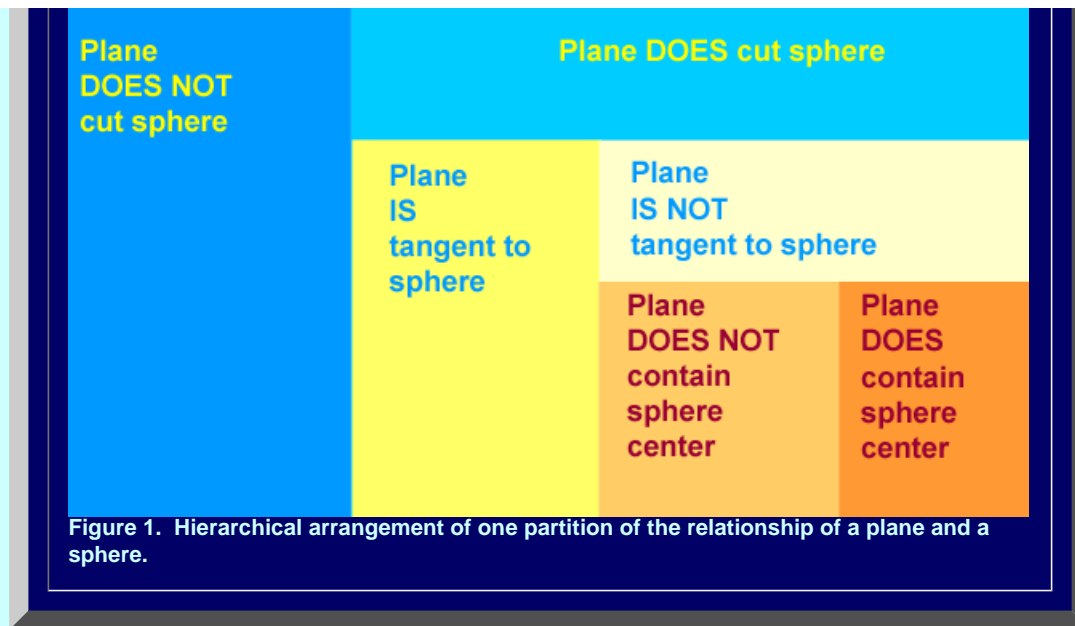
[Michael Batty](#), University College London

For most of us, the model of the Earth as a sphere is sufficient for our needs. Geodetic scientists and others do important research to develop models that are more accurate than the simple, but clearly imperfect, spherical model of the Earth's surface. When the spherical model is used as the domain in a transformation carrying the Earth's surface to the plane, to create flat maps of the Earth's surface, further distortion is necessarily the result. These derived maps reflect both the inaccuracy of the base model and of its transformation to a different surface. It is these imperfect maps, nonetheless, that often serve as foundation material for planning and other decision making processes that circumscribe our everyday lives. The nature of the base models which subtly envelop our lives is critical to understand: specific applications often fall from broad conceptual models. To illustrate the thought processes developed here, we begin with a detailed analysis of a familiar surface: the sphere. Thus, the reader has that model in mind as a benchmark against which to visualize less familiar surfaces and processes.

THE SPHERE

To navigate a surface, be that navigation in the real or virtual world, one needs some sort of chart and method for finding paths from here to there. On the spherical model of the Earth, one conventional system involves the use of parallels and meridians. To understand the nature of this system consider a plane and a sphere in three dimensional space. There are two broad logical relationships between them: either the plane does not intersect the sphere or the plane does intersect the sphere. This relationship serves as a mathematical partition of the space: it separates the space into mutually exclusive, yet exhaustive, classes. Within the second class, one might further specify the partition: either the plane is tangent to the sphere or it is not tangent to the sphere. In the latter instance the plane cuts the sphere, as a cutting plane, in a circular cross-section, much as a knife might cut into an orange. With the "orange" idea in mind, the class of circular cross sections can then be further partitioned into circles containing the center of the sphere and circles not containing the center of the sphere. A diagram illustrating the hierarchical arrangement of this partition appears in Figure 1.





When the sphere in the above paragraph is specified to be a representation of the Earth's surface, then two other points, in addition to the center of the sphere, become involved in the system: the north and south poles. These points lie at opposite ends of a diameter of the sphere: they are antipodal (opposite feet). Circles containing the center of the Earth are conventionally called great circles: they are the largest circles possible and are geodesics on this surface. A geodesic is a line along which shortest paths are measured and these differ, both in shape and in number, according to the surface traversed. In the Euclidean plane, geodesics are unique; on the sphere they are not (antipodal points, for example, have two distinct shortest paths joining them along any single great circle). All other circles, those not containing the center of the Earth, are called small circles. There is an infinite number of great circles and an infinite number of small circles. There is, however, exactly one great circle that bisects the distance between the poles: that great circle is called the equator. Its role is like that of the horizontal axis in the Cartesian coordinate system. A set of small circles, whose cutting planes are parallel to the cutting plane of the equator, become grid lines on either side of the horizontal axis. This set is called a set of parallels. Notice, however, that it is the cutting planes that generate the small circles, and not the small circles themselves, that are parallel. The parallels specify location to the north and south of the equator but not east to west. One way to do so is to introduce a set of lines that cut across the horizontal set in an orthogonal fashion: a set of great circles that intersect each other at the poles functions in this manner. The halves of these great circles, from pole to pole, are called meridians (half circles reflecting the earth-sun relationship that on the equinoxes, and only on the equinoxes, any full meridian spends half the day in light and half in night). One meridian is typically singled out for unique characterization as the "vertical axis" meridian; historical convention followed recently in many parts of the world is to use the meridian passing through the Royal Observatory in Greenwich, England. This unique meridian is the Prime Meridian. This arrangement of a grid on a sphere, often called a graticule, is shown in Figure 2. The location of a point *P* on the sphere may be specified in terms of parallels and meridians. However, different observers may create different graticules. Thus, location is relative to graticule choice as is shown Figures

2a and 2b: in Figure 2a, *P* is located at about one and a half parallels to the north of the Equator and almost three meridians to the west of the Prime Meridian while in Figure 2b, *P* is about eight parallels to the north of the Equator and about 14 meridians to the west of the Prime Meridian. Both are correct; the navigation to *P* depends on how the system was set up. This relative location strategy does not permit replication of results.



Figure 2a*.

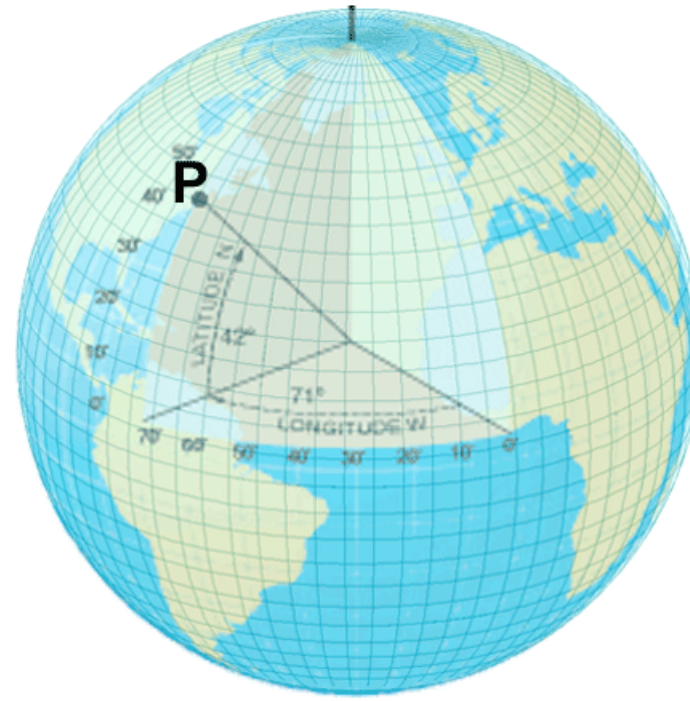


Figure 2b*.

What is needed to produce a natural replication of results is the introduction of some standard system of measure: to convert relative to absolute location. Most people agree to a measure of circles that splits them into some number of equally sized wedge-shaped pieces, measured from the center to the perimeter. Some use degrees, some use radians, but basically the idea is the same: transformation from one system to another is easy. In either of the graticules in Figure 2, the point *P* lies 42 degrees north of the equator (measure of latitude) and 71 degrees west of the prime meridian (measure of longitude). Conventional circular measure standardizes location on the sphere.

On flat maps, however, circular measurements can seem odd. When one understands clearly, however, the base model from which the flat map was derived and the need for circular measure as a standardizing device to ensure replication of results, then the presence of circular measure notation on a flat, rectangular map does make sense. Specific use of circular measure notation for locational precision falls from the broad spherical model of the

Earth; the structure of the geometry is clear independent of sphere radius.

ZIPF'S RANK-SIZE RULE

The view of the Earth as a sphere, and its geometry that has served as a conventional abstract model, has endured for centuries. Beyond the natural world derived from the Earth's planetary relation to the sun, one might wonder what other surfaces could arise as interesting models and what their associated geometries might be. One set of systems of interest to both authors (albeit in different ways) is the set of systems of cities. Batty has investigated this set from the standpoint of "power laws" and particularly in relation to the basic power law of Zipf. A set of lectures given by Batty at The University of Michigan, in 2003, stimulated a response from Arlinghaus to investigate the geometry of the Zipf configuration from an unconventional approach.

A set of cities may be rank-ordered according to population (or any number of other variables). When it is, and the largest city is assigned the rank of 1, the next largest the rank of 2, and so forth, it follows that there is an inverse relationship between rank and population size. Zipf characterized this relationship as the "rank-size rule." The pattern inferred from plotting actual data corresponds to the equation $xy=K$ which represents an hyperbola in the plane. West, Brown, Enquist, and Savage characterize this rule as:

The empirically observed regularity is that settlements of rank r in the descending (size) array of settlements have a size equal to $1/r$ of the size of the largest settlement in the system. In other words, when the population size of towns is plotted against their frequency on a logarithmic scale, we see an approximately straight line with an exponent of -1 . This relationship is known as the 'rank-size rule'.

Figure 3 illustrates how the hyperbolic forms emerge. Figure 4 shows a copy of the original transformed Zipf curves using incorporated places in the U.S.A. from 1790 to 1930. Figure 5 shows representation of a set of rectangular hyperbolae in the plane, typically in the first quadrant with x and y axes as asymptotes. Figures 6a and 6b show an update of the original Zipf plot by Batty and his colleagues.

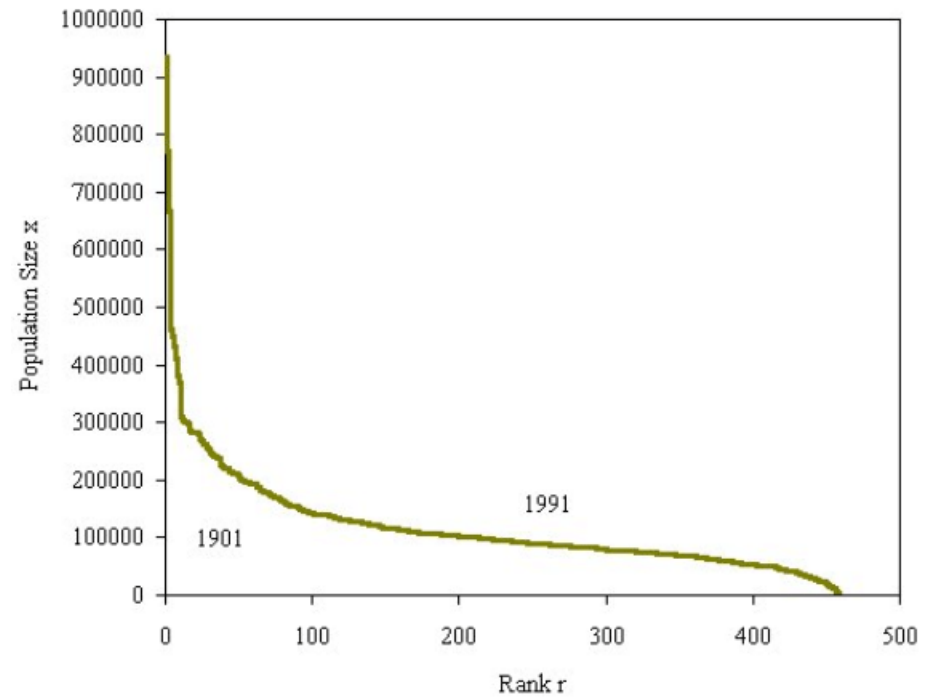
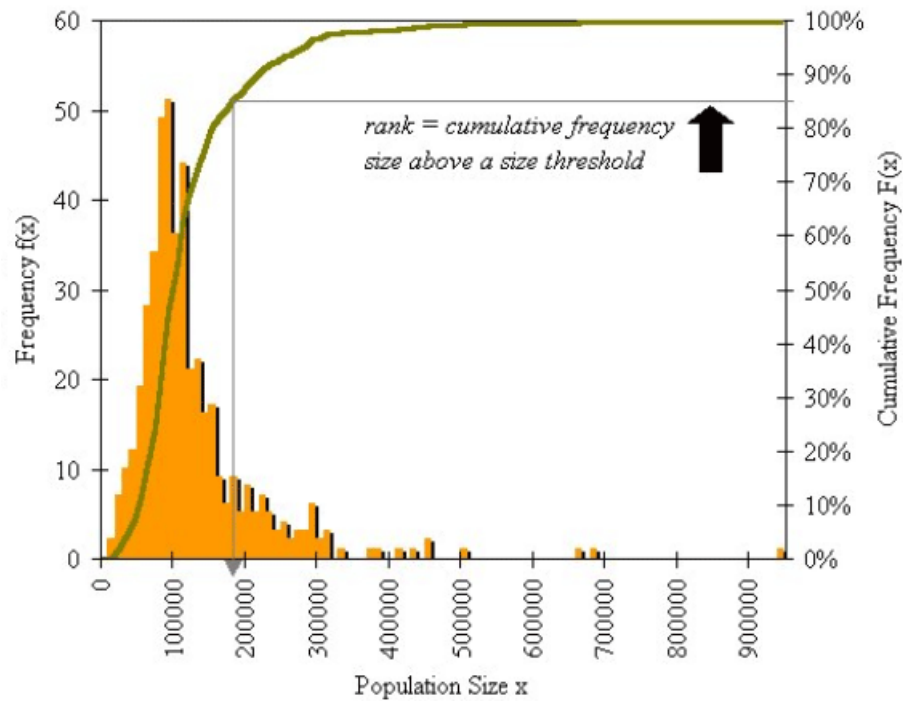


Figure 3a* and Figure 3b*. City Size Distributions: The Rank-Size Rule. If we examine the size distribution of cities, we find they are not normally distributed by lognormally distributed, and can be approximated by a power law.

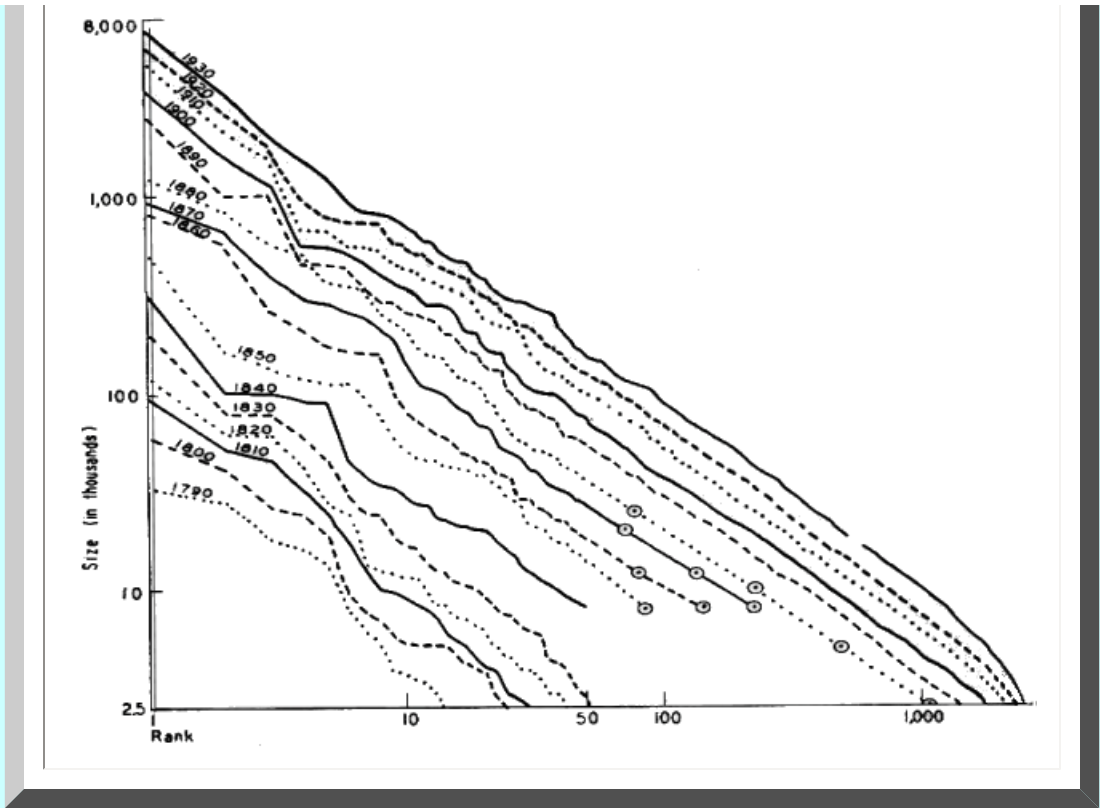
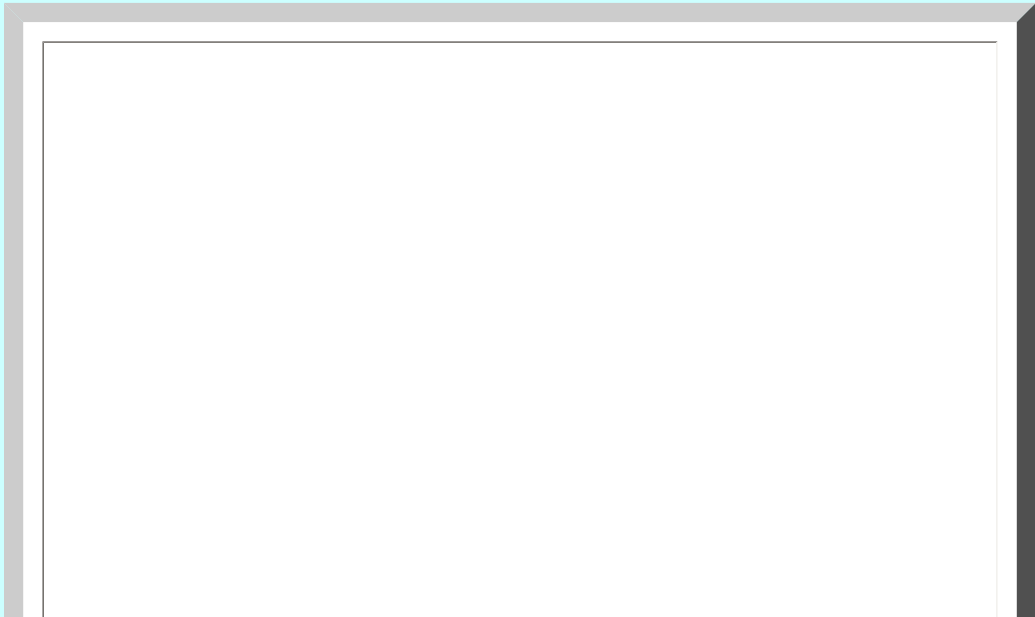


Figure 4*. Original Zipf plot using incorporated places in the U.S.A. from 1790 to 1930.



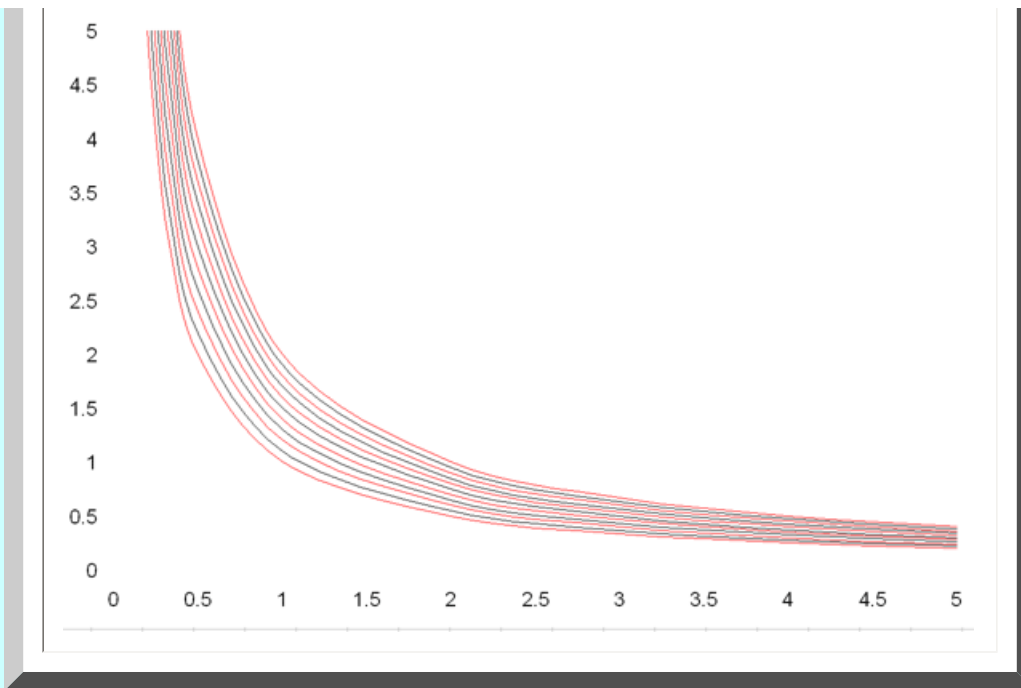


Figure 5. A set of rectangular hyperbolae, $xy = K$ ($K=1.0, 1.1, 1.2, \dots, 2.0$) graphed in the first quadrant of the plane.



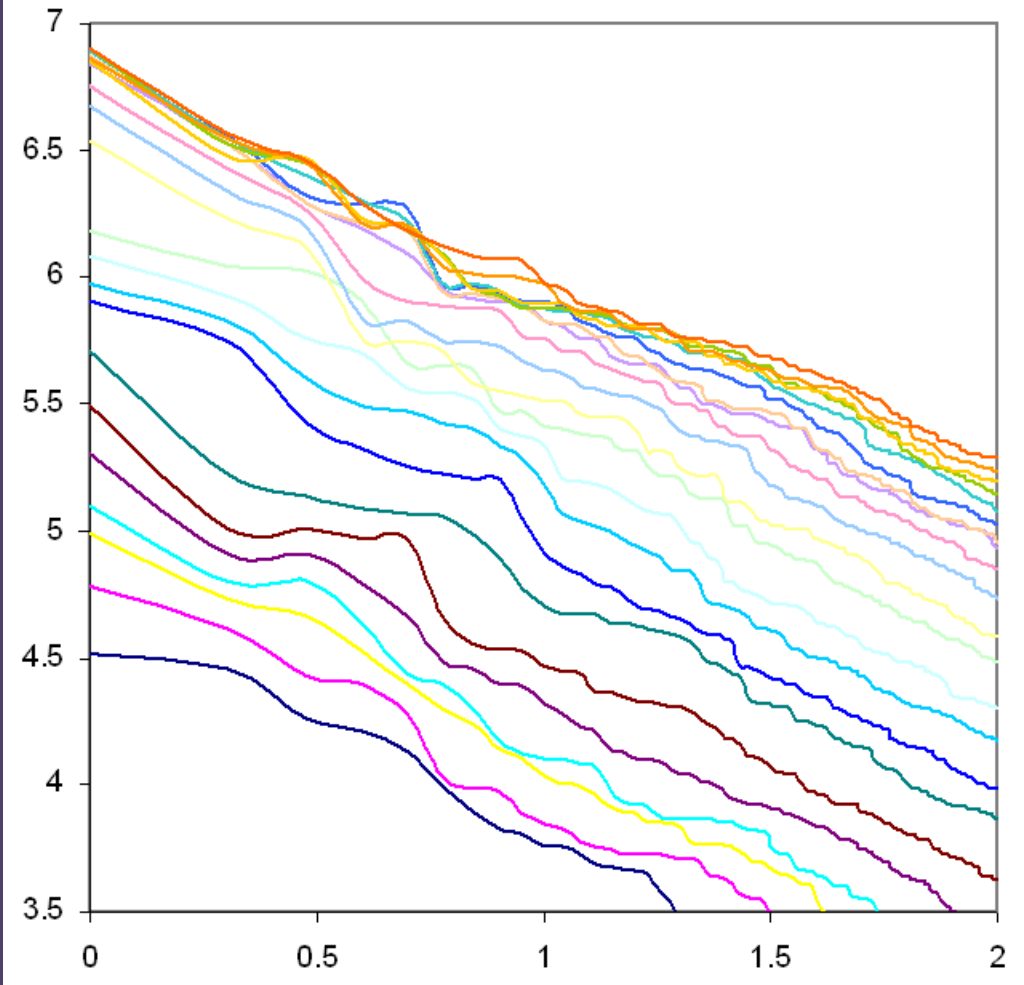


Figure 6a*.

Year	r-squared	exponent
1790	0.975	0.876
1800	0.968	0.869
1810	0.989	0.909
1820	0.983	0.904
1830	0.990	0.899
1840	0.991	0.894
1850	0.989	0.917
1860	0.994	0.990
1870	0.992	0.978
1880	0.992	0.983
1890	0.992	0.951
1900	0.994	0.946
1910	0.991	0.912
1920	0.995	0.908
1930	0.995	0.903
1940	0.994	0.907
1950	0.990	0.900
1960	0.985	0.838
1970	0.980	0.808
1980	0.986	0.769
1990	0.987	0.744
2000	0.988	0.737

Figure 6b*.

Figures 6a and 6b. Figure 6a shows a reworking of Zipf's Law by Batty and colleagues to bring it up to date. Figure 6b shows the data used to create the updated Zipf plot.

THE HYPERBOLOID OF TWO SHEETS

Part of the rationale for plotting on log paper and converting to straight lines is that lines are easier than curves to handle. Such conversion, however, implies a functional relationship that may or may not be there. Why convert curves as models, whose equations are known, to what they in fact are not? Log paper masks the true shape of the curves. Instead, Arlinghaus wished to look for a surface from which to derive these curves, as a set, in order to understand, rather than to mask, fundamental geometric structure.

The branches of the hyperbola in the 1st and 3rd quadrants may be rotated about the line $y=x$ to generate a surface of revolution composed of two dish or bowl-shaped objects facing outward from the origin; follow this [link](#) and scroll down to find an image containing an hyperboloid of two sheets that the user can manipulate ("sheets" is the 3-dimensional equivalent of the 2-dimensional "branches").

To ease visualization of the pattern of intersecting planes with the hyperboloid sheets, consider the shapes in an upright position. In analogy with the sphere, one sees, looking at the animations in Figure 7, that the parallels are circular sections sliced by the cutting plane in Figure 7a and that the meridians are hyperbola of varying curvature (Figure 7b) reminiscent of the pattern in the Zipf plot and its hyperbolic representation (Figure 5).

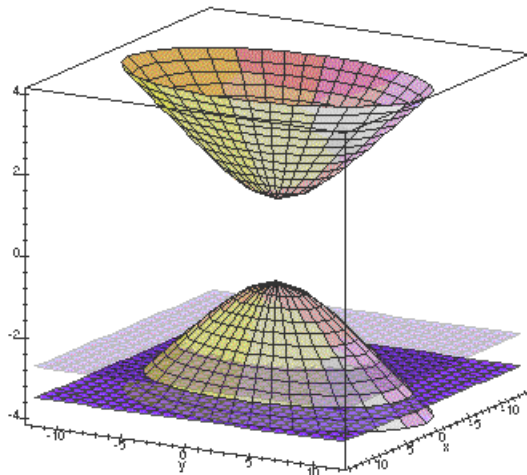


Figure 7a*. Click [here](#) to link to movie version in which the reader can control frames and stop the animation at any point desired.

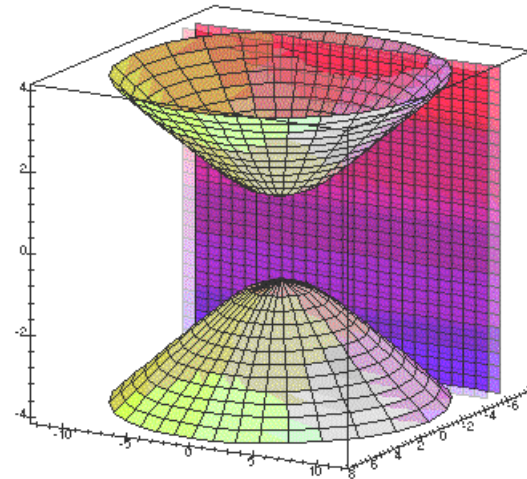
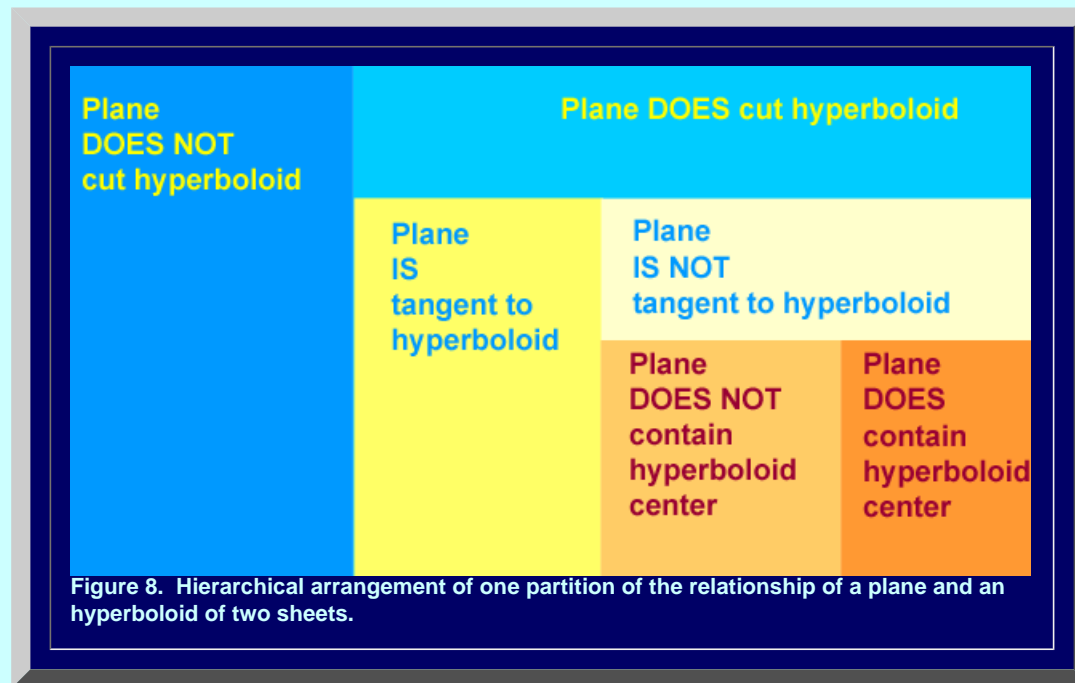


Figure 7b*. Click [here](#) to link to movie version in which the reader can control frames and stop the animation at any point desired.

The basic geometric configuration, in terms of underlying structure is similar to that of the sphere although of course the shapes of the hyperboloid of two sheets and the sphere appear very different from each other. Once one understands the geometry of the unit sphere, one understands the geometry of any sphere; the same is true for the hyperboloid of two sheets. The center of the solid, in each case, is the origin of the three dimensional

coordinate system. Each solid has two poles: on the sphere, these are opposite ends of a diameter of the surface. On the hyperboloid, these are opposite ends of the axis of bilateral symmetry of the surface, piercing each bowl at its base. The "great hyperbolae" on the hyperboloid pass through a pole; they serve as geodesics on the surface. The "small hyperbolae" do not pass through a pole. Indeed, a hierarchy for the hyperboloid of two sheets matches the one for the sphere shown in Figure 1. It is shown below in Figure 8.



In terms of the Zipf plots, the farther a small hyperbola is from a great hyperbola, as suggested in the animation in Figure 7b, the larger is the biggest city and all subsequent cities by rank in the smaller hyperbola. When the planes are parallel, visualization is easy in either two or three dimensions. When they are not, and curves cross each other in the plane, visualization on the three dimensional surface permits visual disentanglement of form.

MAPPING THE HYPERBOLOID OF TWO SHEETS

Cartographers have a permanent job because it is not possible to map precisely a sphere on a plane (stereographic projection is the best one can do by the one-point compactification theorem of topology). Nonetheless, flat maps are easily portable and have other nice characteristics. Thus, if the hyperboloid of two sheets is the one that drops out the ideal models of hyperbolae for the Zipf plot, then how might one map this surface back to the plane? Harrison, on a cover of *Scientific American* in November of 1975 suggested one possibility (Figure 9) in his rendering of Raisz's earlier orthoapsidal projection of the orthographic projection to the plane of a globe on an hyperboloid of two sheets. Harrison's "globe" is depicted on part of an hyperboloid of two sheets.

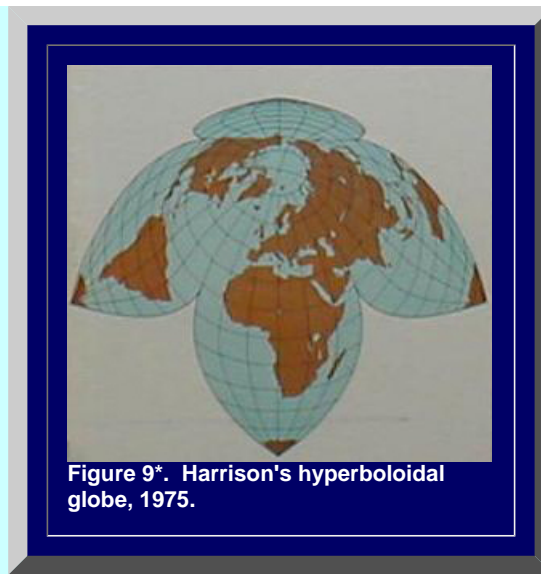


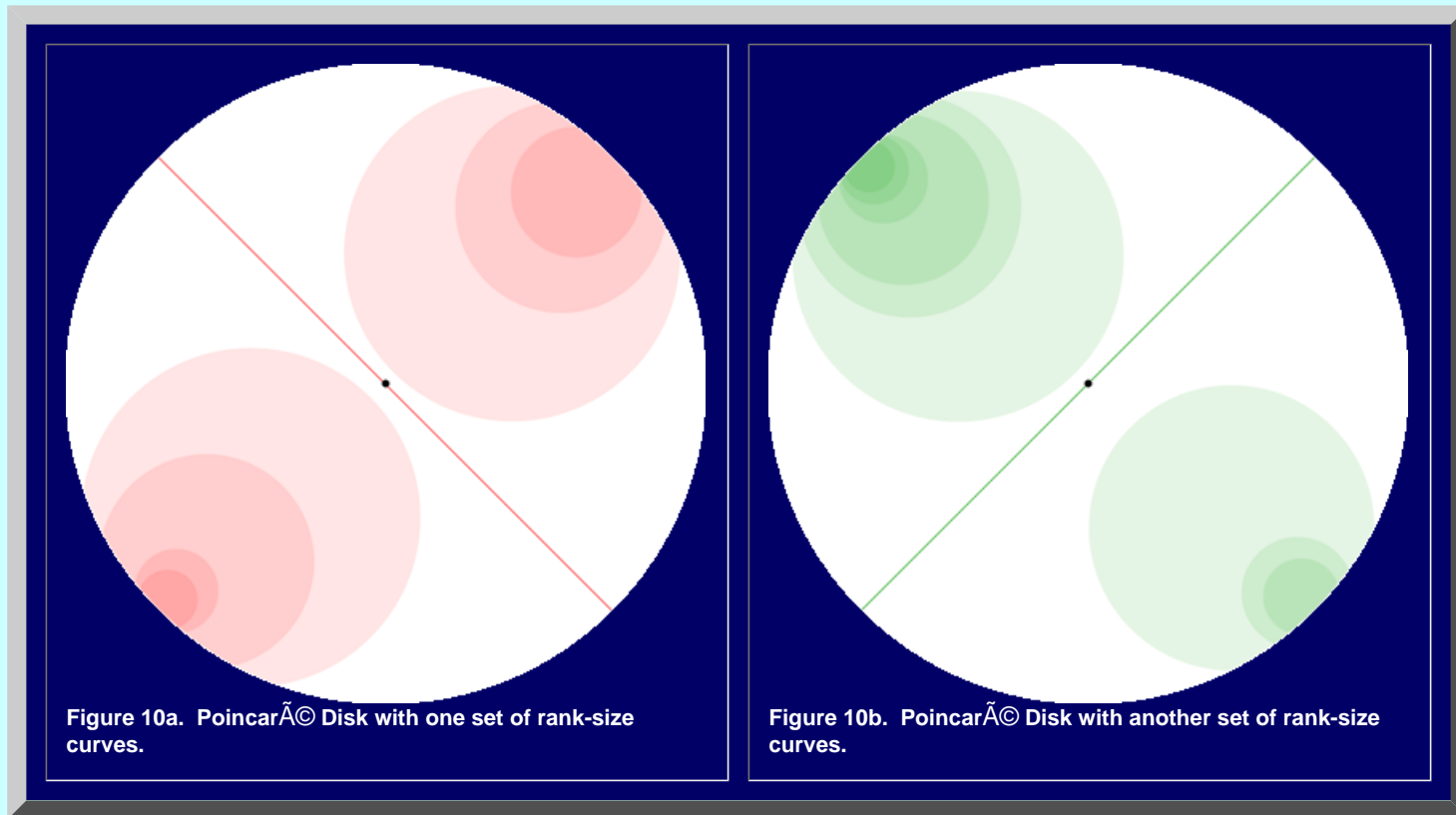
Figure 9*. Harrison's hyperboloidal globe, 1975.

Beyond that particular projection, however, one might ask about general mapping strategy. Once again, a look to the past finds answers. When the hyperboloid of two sheets is orthographically mapped to the plane (through the origin) the visualization is as follows. First, imagine that the plane is extended to include points at infinity so that the concept of parallel lines is removed. The Euclidean space is converted to a non-Euclidean one. Now, orthographic projection from the origin sends the upper sheet to the interior of a disk whose boundary is composed of the points at infinity: what had been unbounded become compressed inside a single disk. Points on the boundary of the disk are places where the hyperbolae touch the asymptotes (at infinity). In this model, a great hyperbola, a geodesic passing through a pole (bottom of the bowl), maps to a diameter of the disk; small hyperbolae map to circular arcs with arc endpoints on the disk boundary (images in this [link](#) suggest how some of these mappings occur). This systematic mapping of the hyperboloid of two sheets carries the viewer into the non-Euclidean world of hyperbolic geometry. The disk is the "Poincaré" disk. The Poincaré disk boundary served as the limit circle of M.C. Escher's "Circle Limit" series [[Peterson](#), Science News]; the interiors reflect the characteristics described above of hyperbolic geometry modeled within the limit circle. This hyperbolic geometry model served as the base for Lobachevsky's geometry which has often been realized in relation to space-time problems in a variety of disciplines.

ZIPF'S HYPERBOLOID?

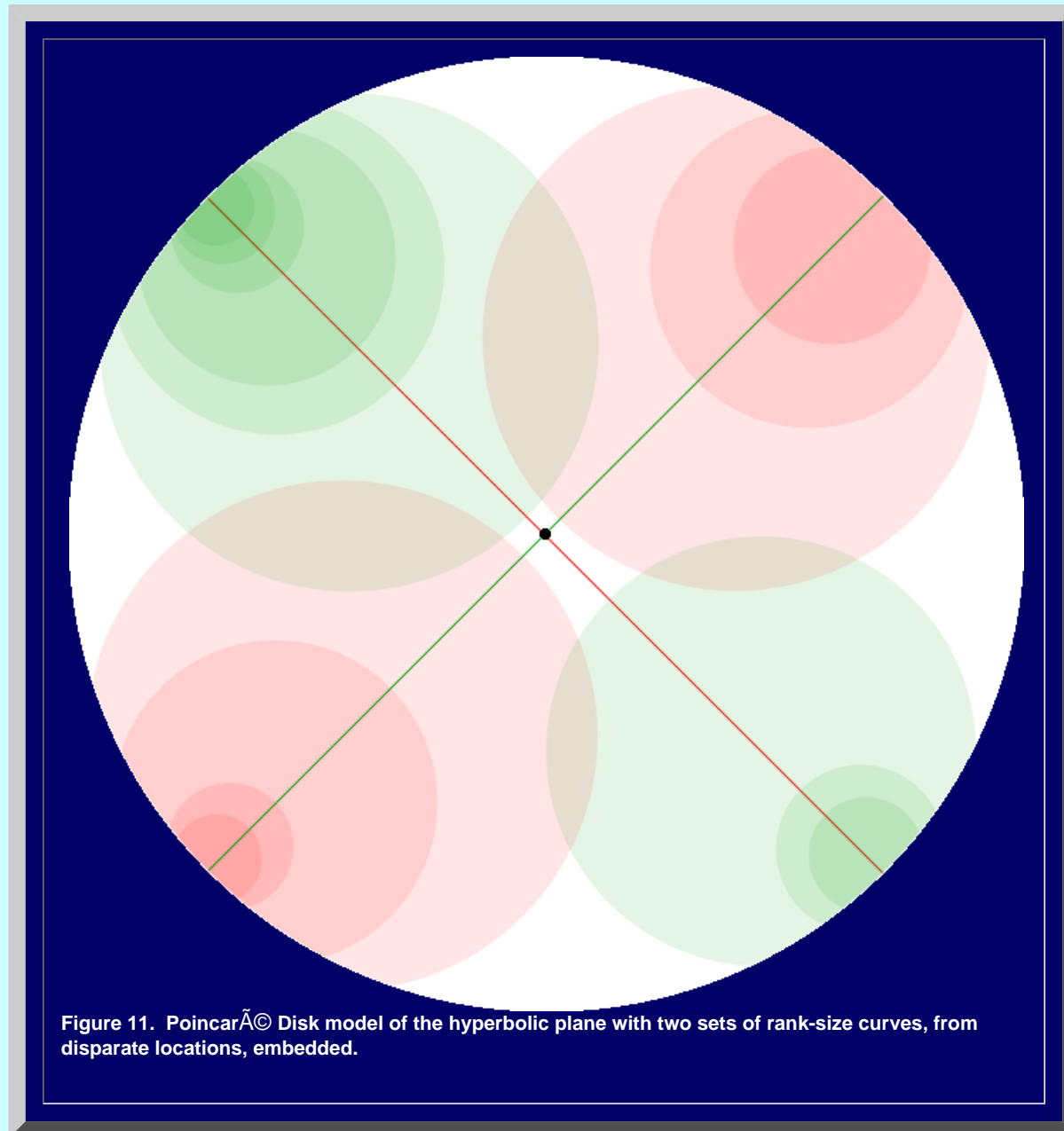
If Lobachevsky's hyperboloidal geometry is a natural base from which to drop Zipf curves, as Zipf's Hyperboloid, then how might that geometry be useful in understanding the emergence, timing, and decay of these systems? Figure 10 shows mappings of Zipf curves in a compact form in the Poincaré disk. Because the hyperbolae that pass a pole on the hyperboloid of revolution of two sheets map to diameters of the Poincaré Disk model of the hyperbolic plane, one has the freedom in the Zipf model to choose any Zipf hyperbola as the one about which to perform the revolution. Thus, one might tie such choice to cultural or other considerations. Suppose one wished, for example, to consider pre- and post-World War II patterns. Then, 1950 might be selected as a date about which to create the surface of revolution. Thus, in Figure 10a, imagine that the red diameter represents a U.S.A. rank-size database from 1950,

extrapolated according to some procedure to infinity in either direction. Then, the red circles, each with 10 percent fill, on either side, represent the corresponding extrapolated U.S.A. rank-size curves on either side of 1950; those farther from the diameter are farther from 1950. Further, in Figure 10b, imagine that the green diameter represents a European rank-size (extrapolated) database from 1950. The green circles on either side of the diameter, each with 10 percent fill, represent the corresponding European rank-size (extrapolated) databases on either side of 1950. These figures have both been drawn with no crossings of rank-size curves. Were there crossings, the corresponding circles would have perimeters that intersect, affecting fill concentration (which might be used to suggest urban or other concentration). They have also been drawn assuming data on both sides of the date selected for revolution; naturally, if no data were available on one side, then no fill would appear on that side.



When the two Disk layers are superimposed, as in Figure 11, one advantage to the compact Poincaré Disk model becomes clear: sets of rank-size curves from different parts of the world can be clearly visualized, simultaneously. It is perhaps an interesting question to consider what overlapping regions might represent: perhaps an opportunity for a Thiessen-style of transformation leading to rank-size art? Indeed, the simple pattern in Figure 11 suggests the style of deeper pattern that can arise in association with complex issues: it is the method of pattern creation so beautifully depicted in the art of Escher's Circle Limit series (inside Poincaré Disks).

Naturally, extrapolated rank-size curves need not be used; if instead, actual data only is used, the circles do not extend to the edge of the limit circle (points at infinity) of the Poincaré Disk. We show the extrapolated curves here to indicate the power of the method. Not only may it be used for the simultaneous display of complex datasets from disparate locations but also it handles finite as well as infinite processes.



Conjecture 1: Raisz's 1943 orthoapsidal projection of the hyperboloid to the plane is intellectually part of the broader 19th century Poincaré-Beltrami model of the hyperbolic plane.

Conjecture 2: The non-Euclidean Lobachevskian geometry of the hyperbolic plane is a natural geometry in which to view Zipf rank-size processes.

Conjecture 3: The same or other non-Euclidean geometries are natural geometries in which to view power-law processes.

Indeed, an Atlas of non-Euclidean maps based on appropriate surfaces would then become useful for Zipf and power-law processes. Beyond that, examination of the uses of the non-Euclidean Lobachevskian geometry in space-time problems from the past should be enlightening. Further, how might the capability to characterize broad systems through a single model enhance our understanding of the multiple connection patterns between and within urban systems? Recent applications of the non-Euclidean Lobachevskian geometry in computer scientific analysis of internet connections patterns would seem important to consider. There are many exciting avenues to probe in this clearly multidisciplinary approach; thus, we offer a glimpse of our thinking about these complex issues in order to entice others, from a variety of disciplines and with a varied set of experiences, to join us in these thought processes.

FUTURE DIRECTIONS

- Create actual Poincaré Disk models from real-world data
- Work on Conjectures.
- Work out detail of hyperboloidal model
 - Is one hyperboloid sufficient to model a system?
 - Are multiple hyperboloids helpful and if so what is their meaning (perhaps in terms of complex curve crossings or other complexity)?
- Recast the model characterization in terms of level curves, vertical profiles, and conventional topographic considerations. Take advantage of the substantial science already built-up in the field of terrestrial mapping.
- Connection to Diophantine equations: the equation $xy=K$ admits only composite solutions for K when all of x , y , and K are to be integers.
- View of the hyperboloidal geometry as one part of broader classifications of
 - Transformations of a cartographic nature [Tobler]: here, only orthographic was mentioned, in the spirit of Raisz's orthoapsidal projection.
 - Transformations of a synthetic geometric nature, particularly in association with fractals and their characterization of central place geometry [Arlinghaus, 1985; Arlinghaus and Arlinghaus, 1989].
- Assessment of properties of Lobachevskian geometry and its implications for thinking about systems of cities.
- Assessment of use of applications of Lobachevskian geometry in disparate academic fields [Rhodes and Semon, for example] and implications for thinking about systems of cities.
 - Evaluation of existing use of hyperboloid of two sheets for analysis of complex internet connections [Munzner], particularly as they mirror complex urban connections.
 - Evaluation of existing space-time use of the geometry and implications for thinking about the rise and fall of systems of cities [Batty].
- Virtual reality of figures to allow readers interesting visual experiences.

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- Figures 2a and 2b. Use of part of a [globe](#) found at Geosystems.com in the 1990s that is no longer available. Permission was granted earlier for use of this globe elsewhere. Here, a modified form is used as a backdrop to figures created in ArcView GIS (ESRI, TM).

- Figures 3a and 3b. Batty, Michael, Sendai 2003 Powerpoint presentation.
- Figure 4. Zipf, G. K. Based on the 1949 reference as found in Batty, above, and modified in Adobe Photoshop (TM).
- Figures 6a and 6b. Batty, Michael, Sendai 2003 Powerpoint presentation.
- Figures 7a and 7b. Animations are modified from base animations of John F. Putz, Professor of Mathematics and Computer Science, Alma College, Alma Michigan 48801, putz@alma.edu.
- Figure 9. Modified form of a scanned image found on [eBay](#).

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Book Review:

Norman J. Wildberger

Divine Proportions: Rational Trigonometry to Universal Geometry

Wild Egg Pty Ltd, Sydney Australia, 2005

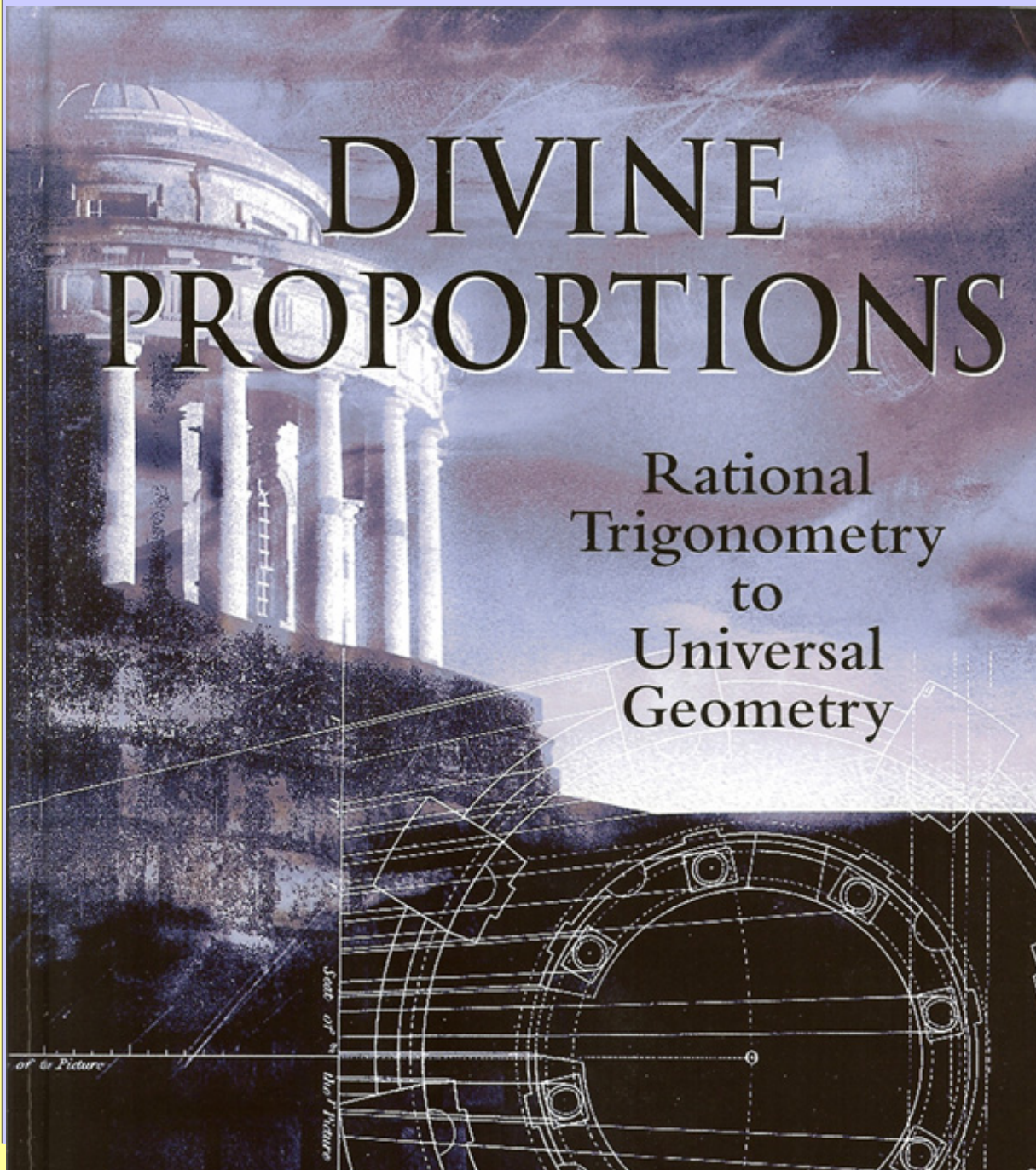
The image below shows the front cover of the book under review; click on the image to see the back cover of the book.

Use the back button on your browser to return to this review.

<http://wildegg.com>

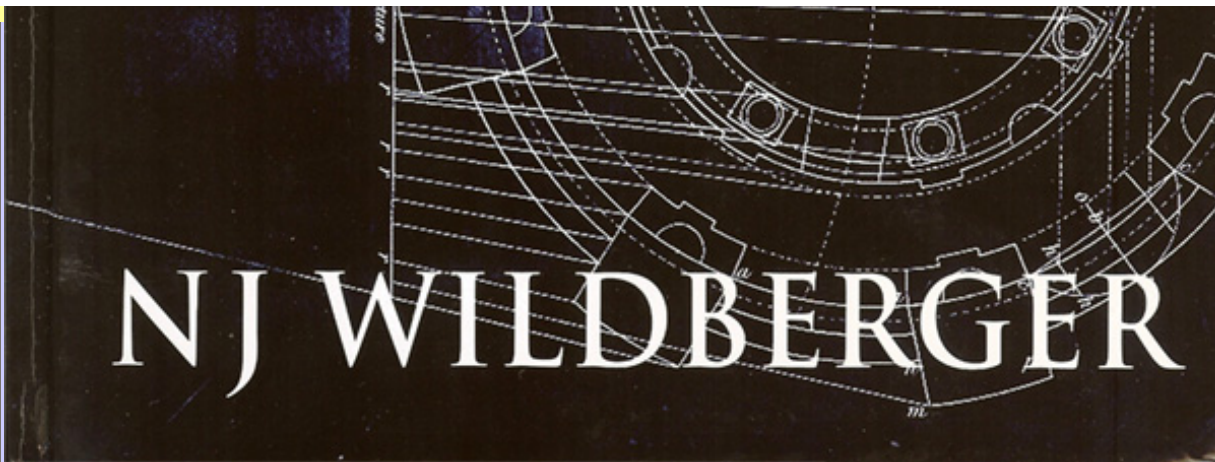
<http://web.maths.unsw.edu.au/~norman/>

email: n.wildberger@unsw.edu.au



Wildberger's work is more than merely novel: it may stand conventional applications of trigonometry and geometry on their heads! Consider his removal of the concepts of "distance" and "angle" from these subjects and his replacement of them by "quadrature" (distance squared) and "spread" (sine of an angle squared). The effect is to move trigonometry and geometry away from an almost-linear approach to a quadratic one. Thus, one no longer sees pages of unwieldy-looking trigonometric functions. These are replaced with intuitively simpler characterizations, based on the concept of separation, that are easy to calculate. To quote from the *Introduction* to Wildberger's elegantly written and produced document:

The new form of trigonometry developed here is called *rational trigonometry*, to distinguish it from *classical*



trigonometry, the latter involving $\cos \theta$, $\sin \theta$ and the many trigonometric relations currently taught to students. An essential point of rational trigonometry is that quadrance and spread, not distance and angle, are the right concepts for metrical geometry (i.e. a geometry in which measurement is involved).

Quadrance and spread are quadratic quantities, while distance and angle are almost, but not quite, linear ones. The quadratic view is more general and powerful. At some level, this is known by many mathematicians. When this insight is put firmly into practice, as it is here, a new foundation for mathematics and mathematics education arises which simplifies Euclidean and non-Euclidean geometries, changes our understanding of algebraic geometry, and often simplifies

difficult practical problems. Quadrance measures the separation of points, and spread measures the separation of lines.

What makes sense for the student and researcher from an intuitive standpoint may not, however, make sense to those who are set within rigid institutional guidelines in academic curricular development or in academic publishing. Imagine a world in which conventional trigonometry and its representation is no longer the norm. All books that use these functions must now be rewritten--perhaps a fine opportunity for the world of electronic publishing in which mathematics can come to life through animation, virtual reality, and other state-of-the-art tools.

While this latter prospect of extensive document conversion is a daunting one, that would seem unlikely to happen based on the presence of a single work adopting the "quadrature" and "spread" approach, those in disciplines such as mathematical geography that have relatively little invested in traditional institutional rigidity have an opportunity to integrate promising

new conceptual approaches into our own theoretical work. If we demonstrate great success, then others more steeped in convention, may see merit in the approach for their own research.

The implications for mapping, of the earth-sphere to the plane, might be probed: what would a "sinusoidal" map look like when expressed in terms of "spread"? One would expect the same map would result using either approach but the route by which one gets there would be different. The history of our discipline is one that is rich in example of the merit of exploration, field study, and different paths: these terms need not be confined to the "real" world but apply equally in the world of the mind.

For example, looking at existing maps within a different light might open up deep and interesting cartographic research areas. Further, one might imagine a total recasting of the graticule in which rotation matrices contain no trigonometric functions so that even a beginning student can flip from one earth-based coordinate system to another in a somewhat intuitive manner. As we, on

planet Earth, continue to look outside toward other planets, theoretical material that makes modeling more intuitive is important--indeed, it is critical.

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Solstice: An Electronic Journal of Geography and Mathematics, Institute of Mathematical Geography, Ann Arbor, Michigan.

Volume XVII, Number 1.

<http://www.InstituteOfMathematicalGeography.org/>

Mail

(in reference to material from *Solstice*, Volume XVI, Number 2, December 2005)

Mail from (other than about submitted manuscripts):

- From Joseph J. Kerski (Ph.D., Geographer, USGS, Denver, Colorado): Sandy: One of my all time favorite sites linking mathematics and geography that you're probably familiar with: <http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/> There is a film of this that we saw when we were kids in school, and then years later I saw it at the Smithsonian.
 - From Marc Schlossberg (Ph.D., Assistant Professor, Planning, Public Policy, and Management, University of Oregon, Eugene, Oregon): Howdy - how are things going these days? Out here, I've been messing around with ArcPad and community-based GIS data gathering with PDAs. It's been fun. Let me know how things are going out there.
 - Peter Beier (Ph.D., Director, 3D Laboratory, University of Michigan) notes that <http://www.zillow.com/> is an interesting site that lists real estate and values.
-

PseudoCoup[®] — a *Psubtle Puzzle*

by William C. Arlinghaus

V	B		R					
	D	C	P		V		S	
		R		S		V		
	S	P		D		B	R	
D								P
	M	B		C		S	G	
		V		M		D		
	C		D		B	G	V	
					C		P	B

Code for Bridge Coup:

B: BATH COUP

C: CROCODILE COUP

D: DESCHAPELLES COUP

G: GRAND COUP

M: MORTON'S FORK COUP

P: PITT COUP

R: ROBERT COUP

S: SCISSORS COUP

V: VIENNA COUP

Challenge:

In the manner of a well-known contemporary puzzle, place the letters into the 9 by 9 table above in such a way that:

- Each row contains every letter exactly once
- Each column contains every letter exactly once
- Each outlined 3 by 3 sub-box contains every letter exactly once.

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V	B	S	R	G	M	P	D	C
M	D	C	P	B	V	R	S	G
P	G	R	C	S	D	V	B	M
C	S	P	M	D	G	B	R	V
D	V	G	B	R	S	C	M	P
R	M	B	V	C	P	S	G	D
B	P	V	G	M	R	D	C	S
S	C	M	D	P	B	G	V	R
G	R	D	S	V	C	M	P	B

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