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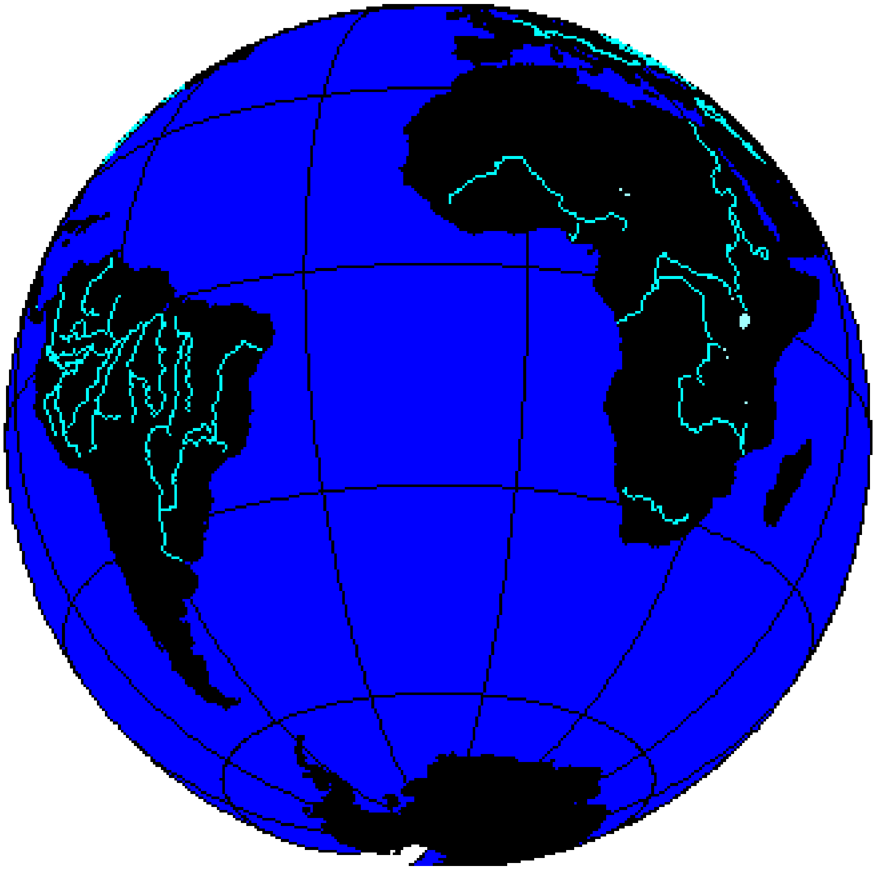

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The purpose of Solstice is to promote interaction between geography and mathematics. Articles in which elements of one discipline are used to shed light on the other are particularly sought. Also welcome are original contributions that are purely geographical or purely mathematical. These may be prefaced (by editor or author) with commentary suggesting directions that might lead toward the desired interactions. Individuals wishing to submit articles or other material should contact an editor, or send e-mail directly to sarhaus@umich.edu.

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Visualizing Accessibility with GIS

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Introduction

As the environmental, social, and health costs of sprawling, automobile dependent development patterns become well understood, accessibility, or walkability, becomes a significant goal of planners, policy makers, and citizens. Our current planning environment is one of auto-mobility, which has the goal of reducing the cost per mile of travel within a metropolitan area. An auto-mobility approach may find success for a 15 minute commute that travels fifteen miles at speeds of sixty plus miles per hour – the cost per mile is relatively low in terms of time and delay. Similarly, an auto-mobility approach to regional travel would be considered a failure when congestion inhibits automobile travel from traveling at maximum speed limits; the cost per mile becomes quite high on account of time delays in traffic.

In contrast, an accessibility focus of development seeks to help people gain access to their destinations at a low cost per trip. In an accessibility-centered approach, popular places to visit cause increased numbers of people on sidewalks and in street intersections. These increases in turn tend to slow down the speeds of automobiles in the area. There is a tradeoff of mobility that favors the pedestrian rather than the automobile.

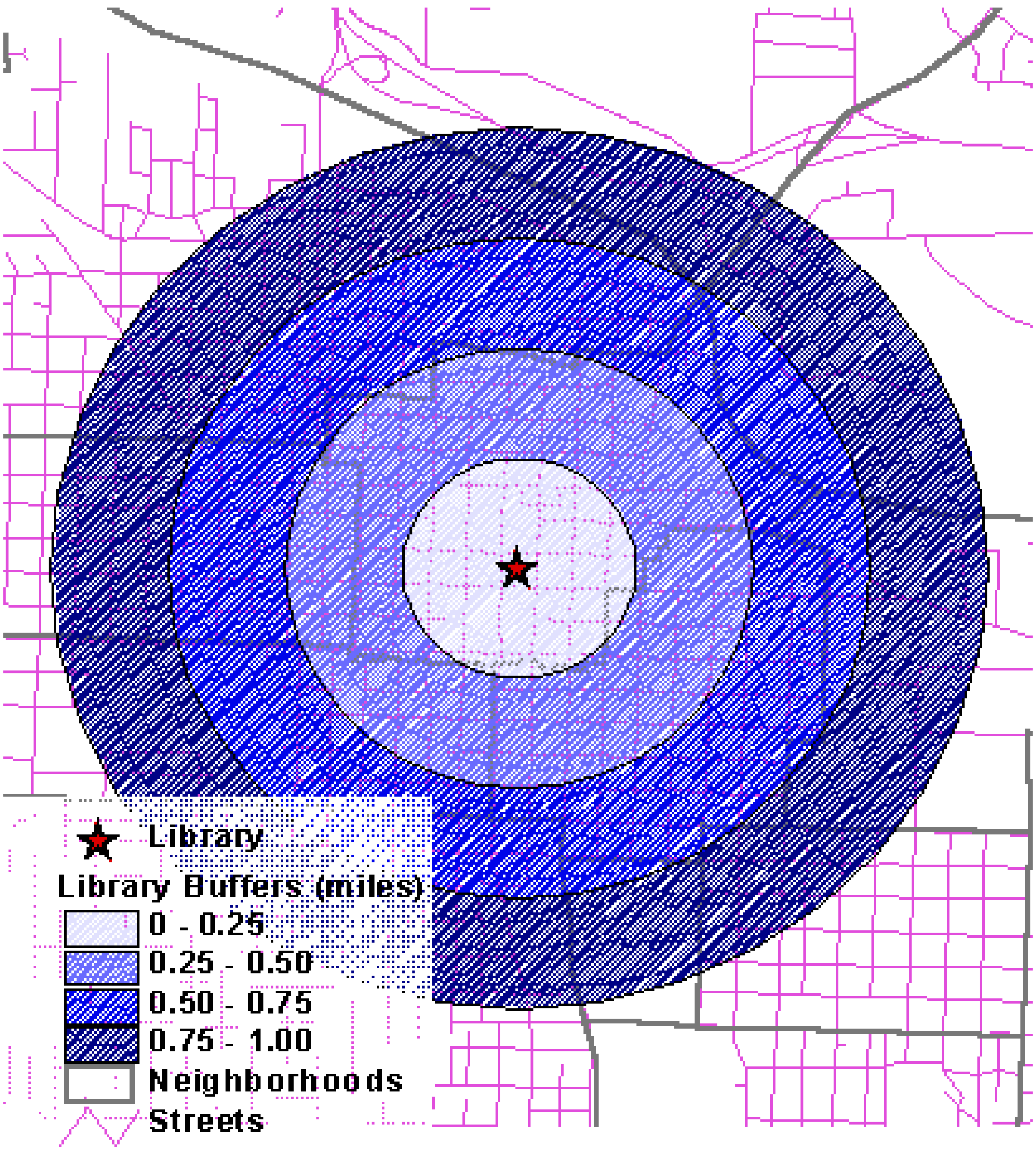
Developers and planners are increasingly incorporating such tradeoffs involving pedestrian accessibility into their visions and plans. They tend to base their decisions on a variety of principles, including increased quality of life, more active community interaction, environmental benefits of reduced automobile dependence, and congestion reduction. These principles are often characterized, in part at least, under a variety of terms: "New Urbanism," "Neotraditional Planning," "Pedestrian Pockets," "Transit Oriented Development," or "Nodal Development." The claimed or potential benefits of these schemes is beyond the scope of the current discussion. The focus here is on visualizing accessibility principles: to visualize is to clarify.

What are the various ways that one can visualize accessibility using Geographic Information

Systems (GIS)? This presentation uses the centralized area of Eugene, Oregon (USA) as the case study. Eugene has a centralized downtown with a gridded street network, has several old, established neighborhoods, and has some newer developments as well. Most of Eugene's topography is flat, except for portions of South Eugene, which ascends up some foothills. Eugene has clearly identified neighborhoods that are recognized by the City and are represented by elected neighborhood association presidents. Measuring accessibility at a neighborhood scale can be facilitated by these pre-existing boundaries of the neighborhoods.

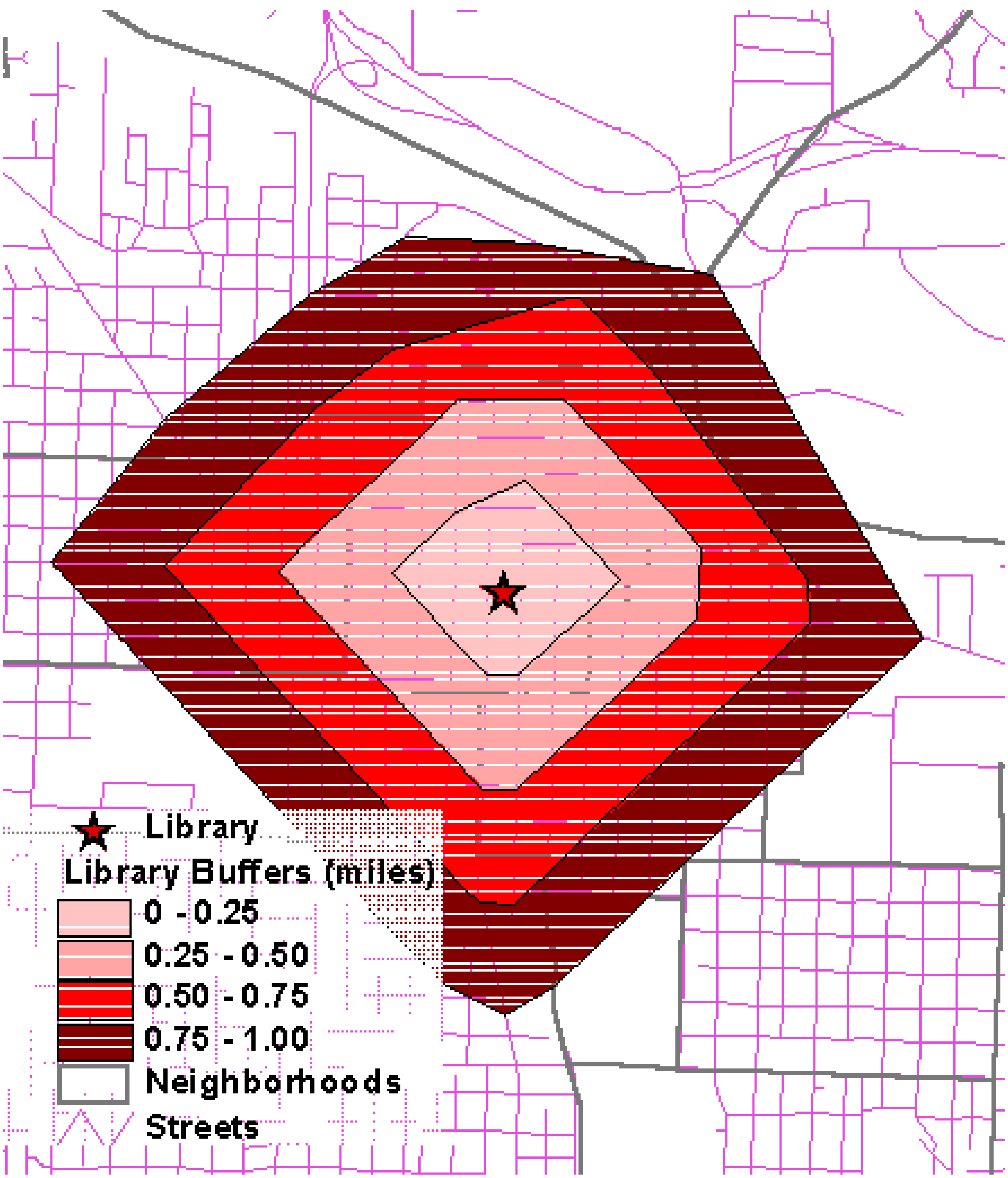
Accessibility through Buffering

Figure 1: 1/4 Mile Buffers as the Crow Flies Around Key Destination



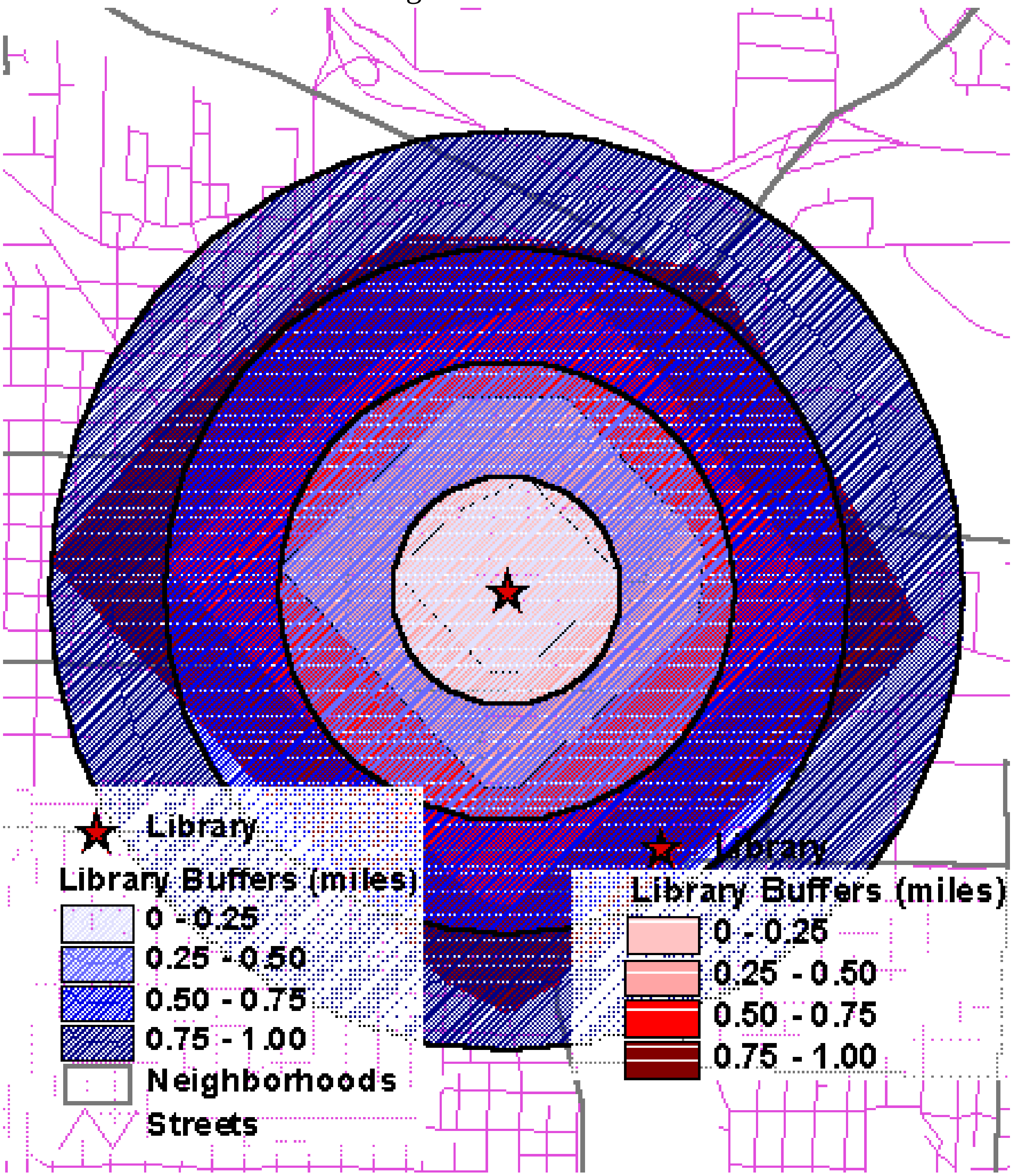
An easy way to visualize accessibility to a specific place is to use "buffer." Buffers target areas all of which are within a given distance of a point, line, or area. Thus, Figure 1 shows four buffers around the Library location. Buffering is a common GIS technique and can be used to quickly identify a geographic area that is considered accessible or walkable to a given location. Planners often consider a ¼ mile distance from a location as being the maximum distance that people are willing to walk to get to the destination they desire. Thus, Figure 1 shows ¼ mile rings of accessibility to a new downtown library that is being constructed in Eugene. The buffer rings (in Figure 1) are "as the crow flies", and do not take into account the actual paths that people may need to take to access the library. Thus, Figure 2 shows ¼ mile rings around the library based on the actual walking path of the street network (assuming that all streets have sidewalks and that there are no other walking-only paths). The diamond shaped buffer rings reflect the gridded street pattern of this part of Eugene.

Figure 2: 1/4 Mile Walking Buffers Around Key Destination



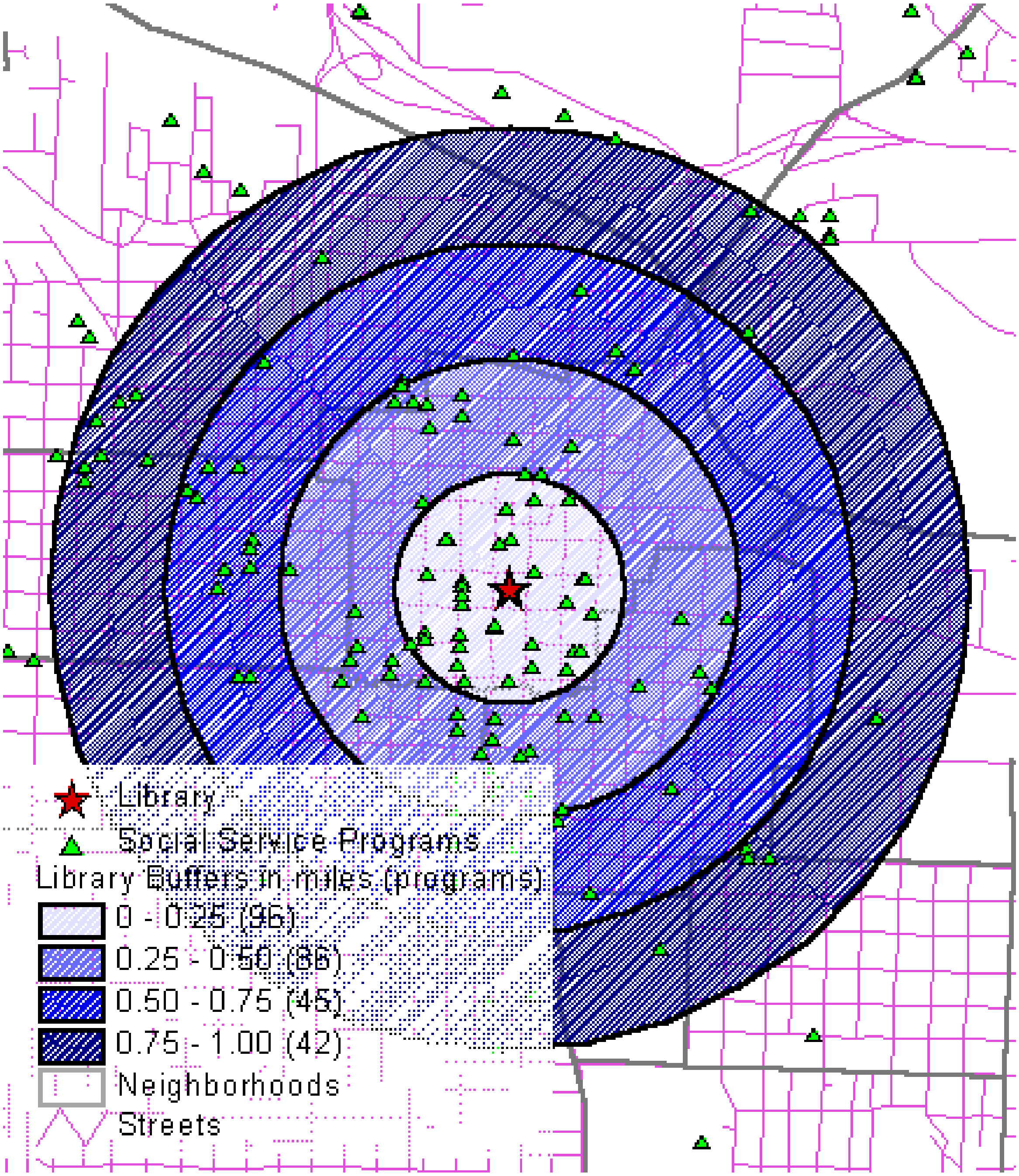
When Figures 1 and 2 are combined as Figure 3, the new Figure shows the overlap between the two different accessibility measurements. In this so-called "Ped Shed" of Figure 3, the ¼ mile buffer area of each technique can be compared by dividing the area of one by the area of the other to calculate a Ped Shed ratio [Rood, n.d.]. Different ratios imply areas that are more or less walkable.

Figure 3: Ped-Shed



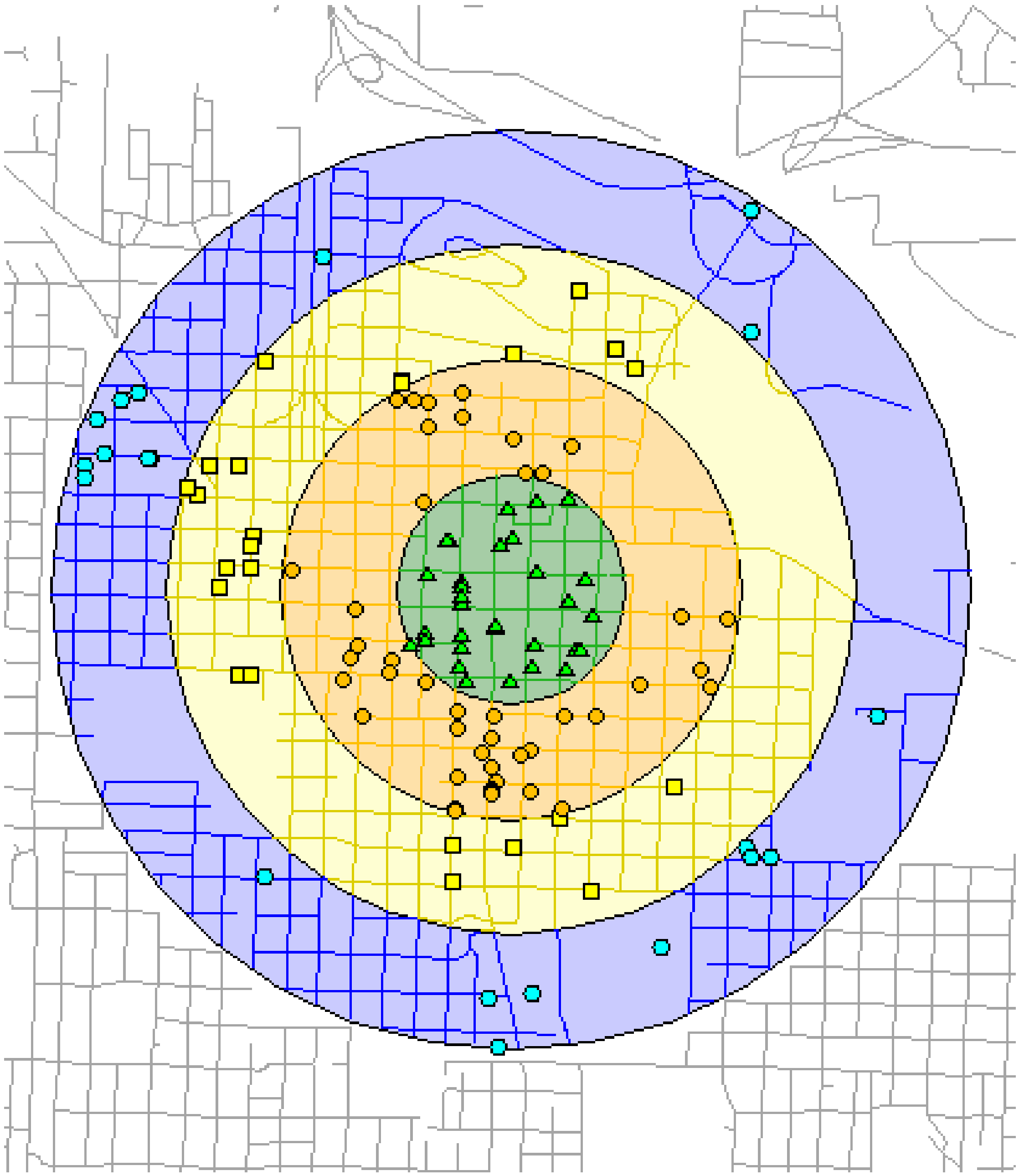
Additional aspects of urban life may also be identified within the walkable buffers. For example, planners at the library may wish to provide sensitive services to people with special needs for social services located near the library. Figure 4, plots the location of social services with the buffer rings to give the library a sense of the type of potential demand it may receive from any of a number of specialized populations.

Figure 4: Potential Social Service Patrons



While the image of Figure 4 is fairly intuitive and easy to read, additional visualization manipulations are possible to increase the clarity of the information being presented. Since the data underlying the image is spatial, data within each buffer can be individually selected and color coded based on its location. Figure 5 illustrates this approach by altering the color of variables (buffer, streets, and social services) based on geographic location. Thus, the visual representation of accessibility is enhanced and the capacity to distinguish or visually segregate the data based on geographical location is improved.

Figure 5: Color Coding Data by Distance



Visualization may be further enhanced by viewing the rings in three dimensions. In Figure 5, there is no discernable change in distance between each ¼ mile buffer and in many cases the line dividing each buffer distance is arbitrary in relation to the movement of people. Instead, tier the distances in a way that conveys the visual message that the geographical dividing lines are not arbitrary, but have real implications for the movement of people through space. Figures 6 and 7 represent the library and its buffers using three dimensional tiers. Color coding of the data within each tier enhances the visual effect.

Figure 6: 3D Tiers of Accessibility

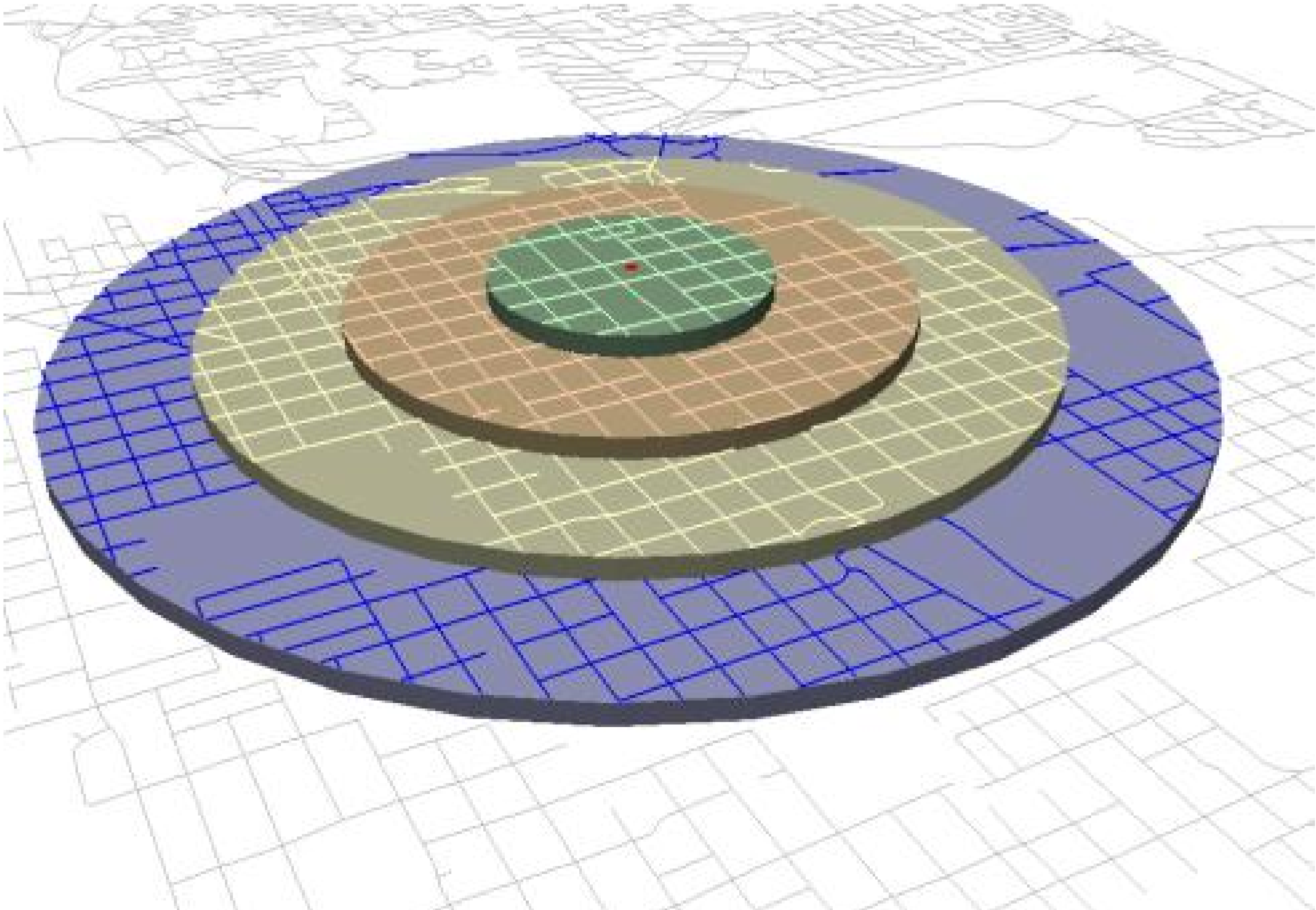
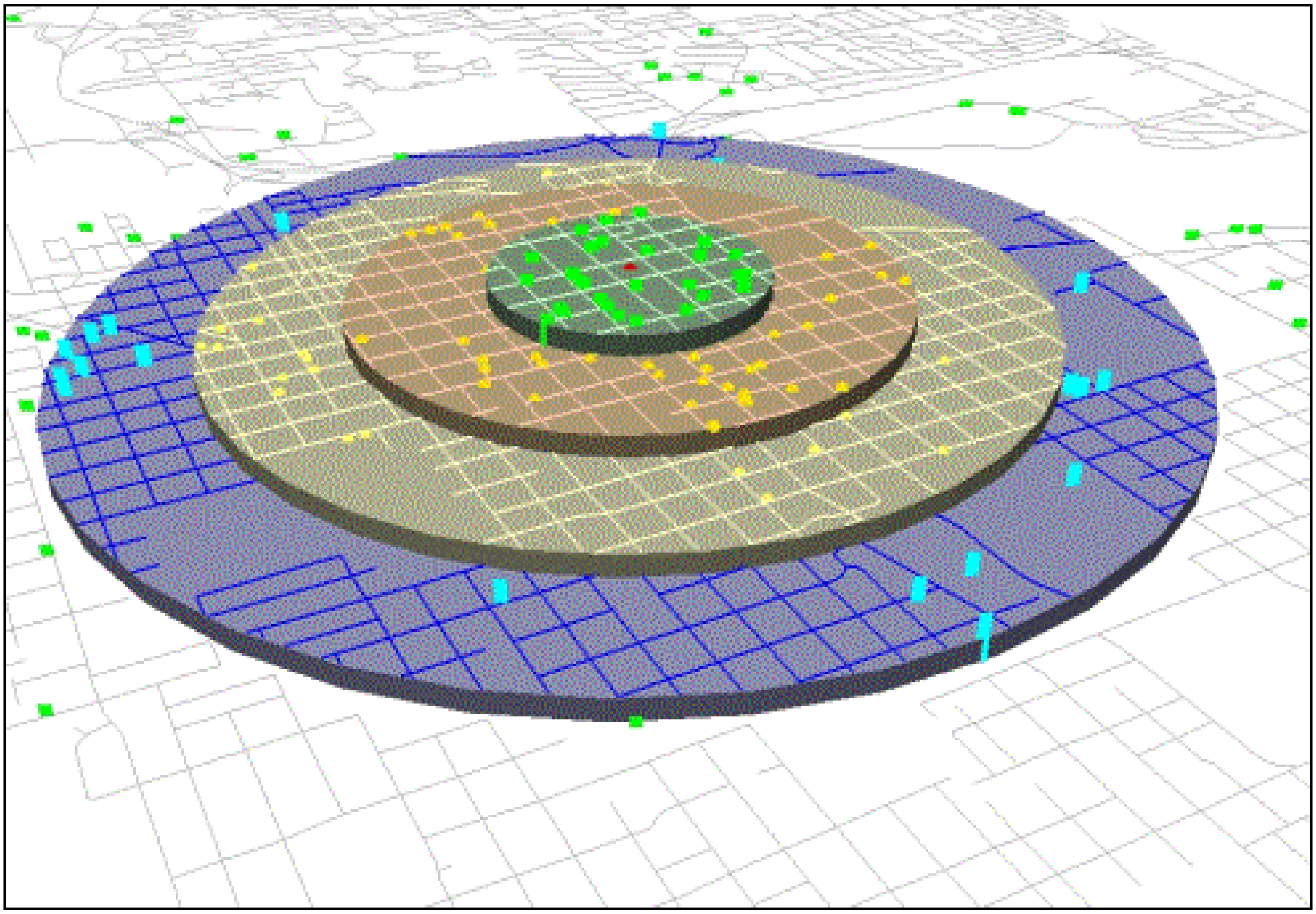


Figure 7: 3D Tiers of Accessibility with Social service Program Locations

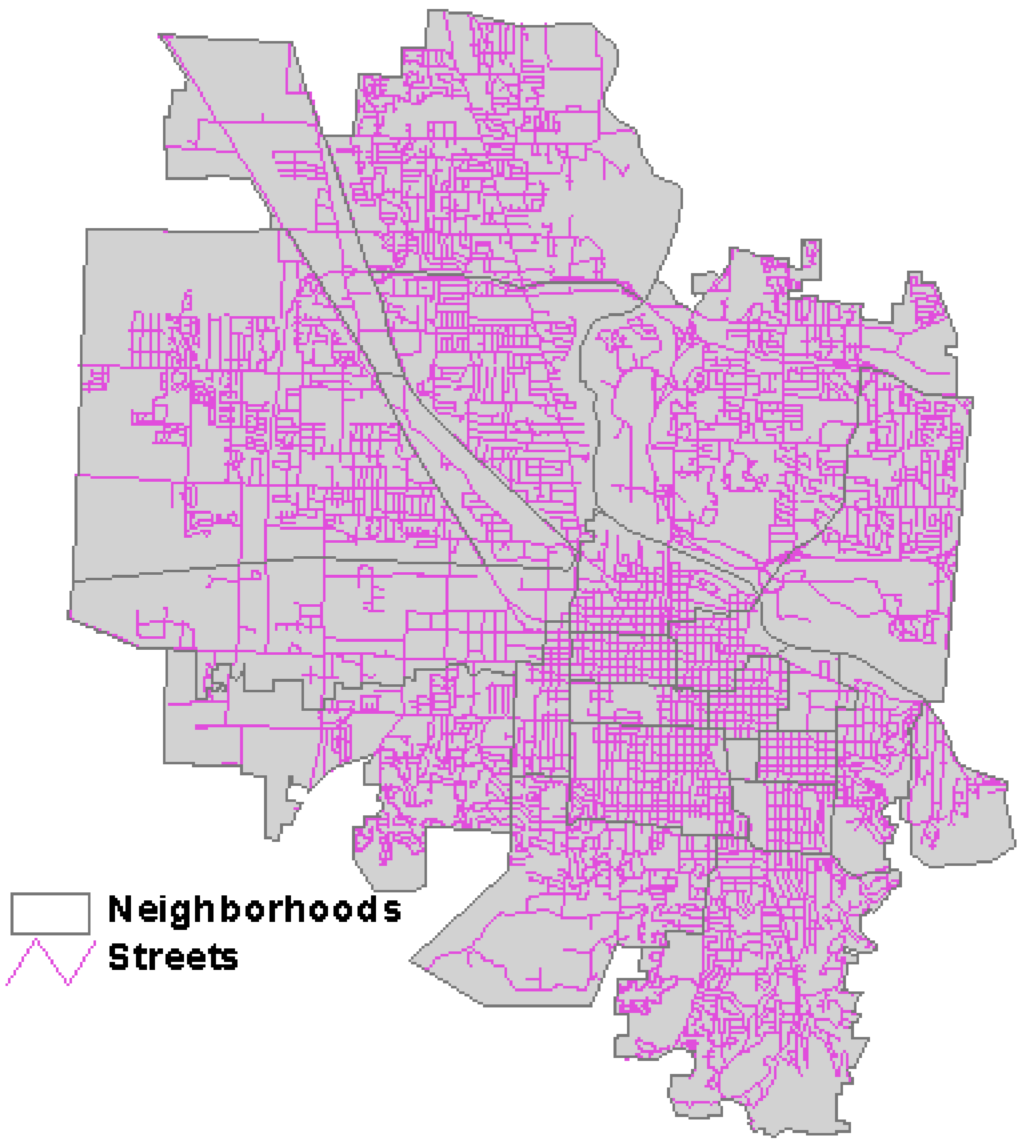


Accessibility through Intersection Density

The images above visualized accessibility in terms of the distance to a specific place. One might, instead, look across a landscape to ascertain which sub-areas are characterized by potentially more accessible movement patterns. Some areas within a region may have street networks (and therefore sidewalk networks) that are more conducive to walkability. Thus, accessibility may be visualized by investigating different patterns of street networks. Within the development schemes mentioned at the outset (New Urbanism, Nodal Development, and so forth), one idea is that street patterns that are based on a grid are more accessible than non-grid patterns. Within a gridded street network, there are redundant paths that walkers can use to access the same destination. This increase in path choice can be represented by areas with numerous street intersections and thus relatively great accessibility. One way to view this idea is to consider the difference in numbers of intersections and accessibility between a downtown street network grid suburban development with many cul-de-sacs. Regions with higher concentrations of intersections are regions with higher potentials for accessibility. The following series of images visualizes this characterization of accessibility.

Figure 8 shows the street pattern within the central Eugene Neighborhoods. The downtown core is located at about the center of the map. From only this simple map of one layer, it is visually possible to get a sense of which areas in Eugene are more walkable.

Figure 8: Eugene Street Network



Although one can get a general sense of accessible places by simply viewing the street layer, it is possible to perform a series of calculations based on the location and density of intersections (or cul-de-sacs). By viewing the concentration of intersections, one can get a better grasp of the connectivity of the street network across space. Figure 9 and Figure 10 visualize the street network based on the location of intersections and cul-de-sacs (or dead-ends).

Figure 9: Intersection and Dead End Points

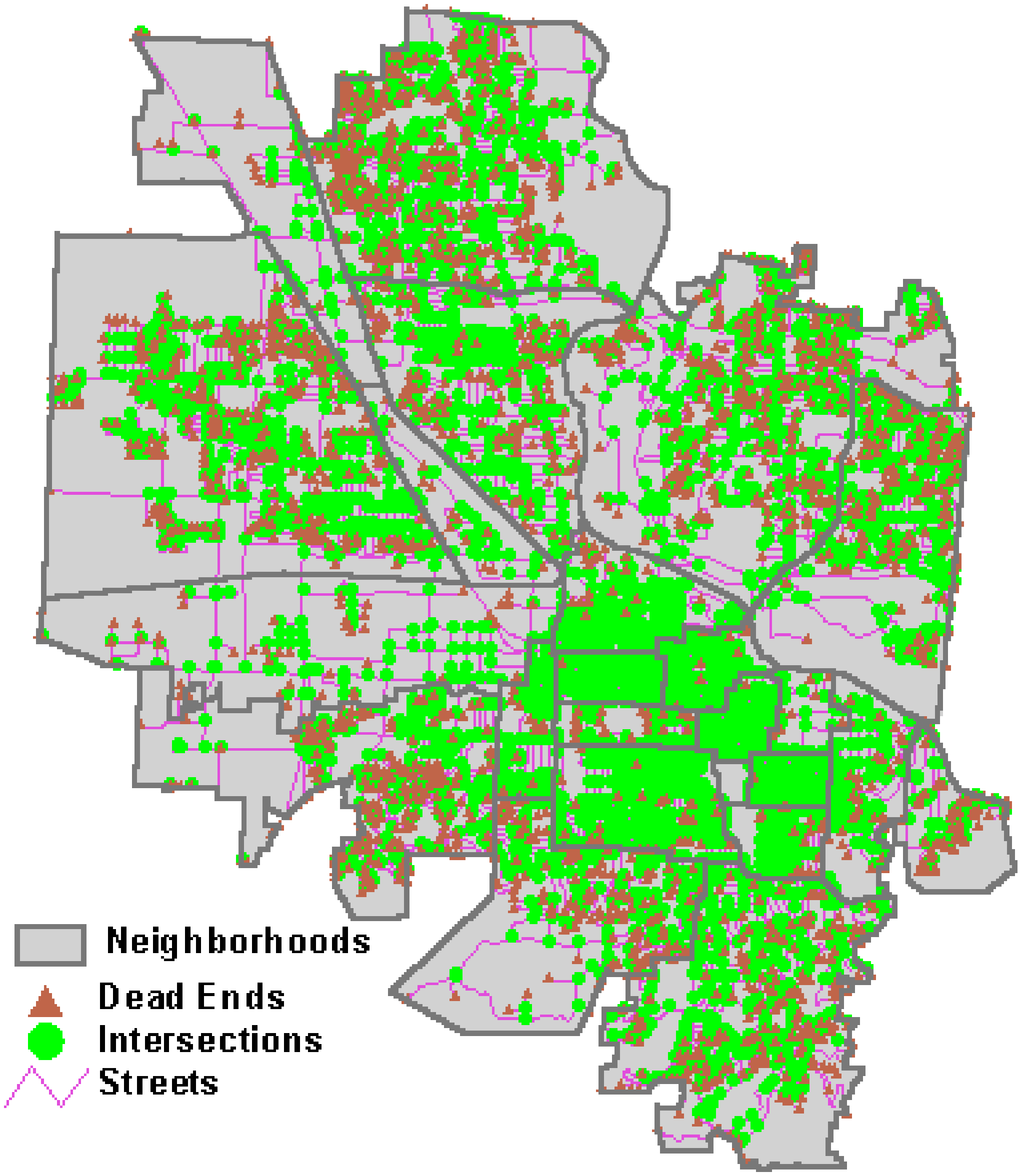
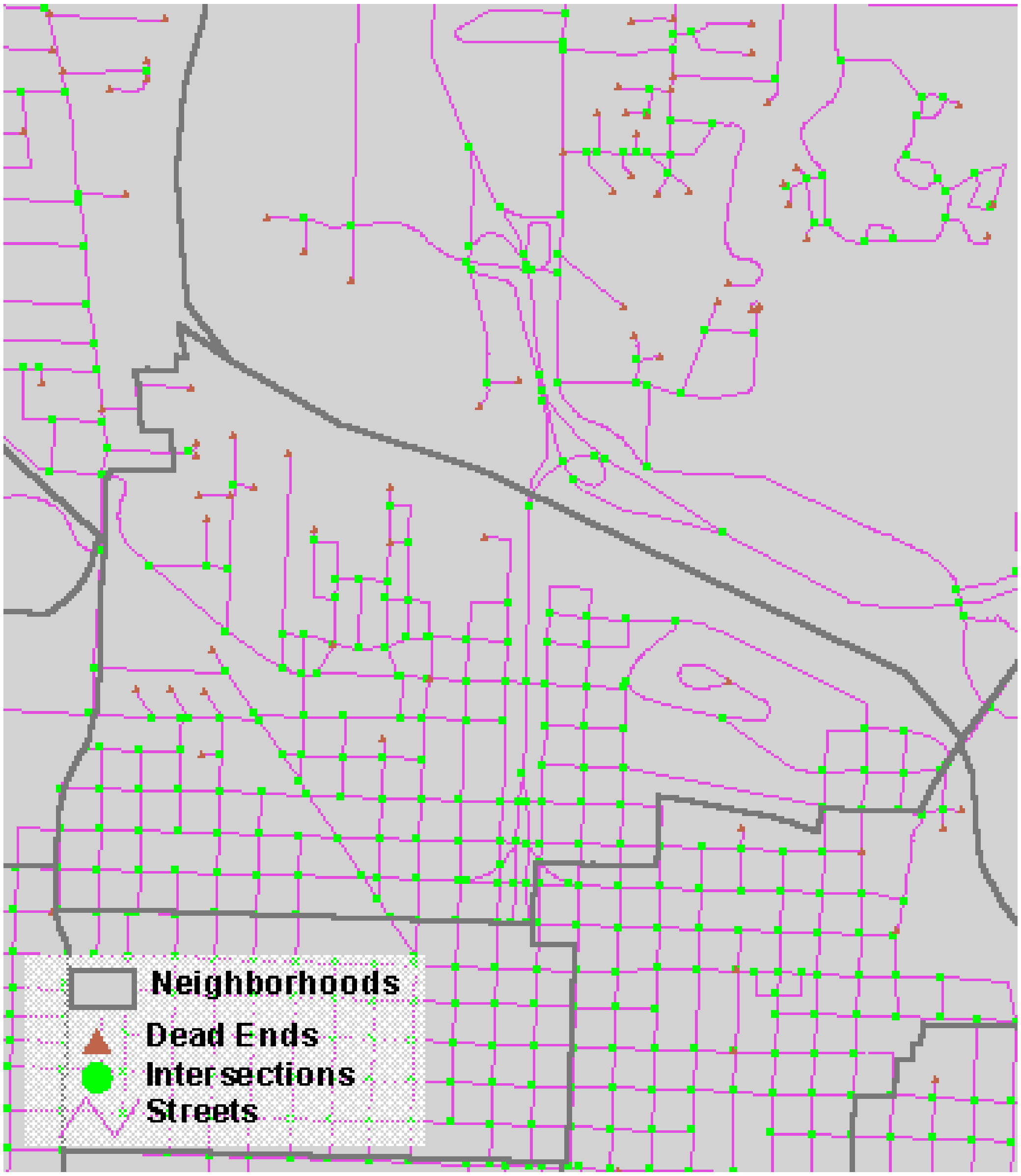


Figure 10: Close-up of Intersection and Dead End Points



Visualizing concentrations of intersections is helpful, but it may be that one would want to characterize the different neighborhoods in Eugene based on the density of intersections within the neighborhoods. Neighborhoods with higher intersection density (intersections per square mile) might be considered as more accessible than those neighborhoods with lower intersection densities. Figure 11 visualizes the aggregation of intersections within each neighborhood divided by the total area of each neighborhood to calculate a relative intersection density figure. Figure 12 visualizes a similar calculation, but is based on the concentration of cul-de-sacs – areas that can be classified as having low accessibility.

In Figure 11 there is a clear pattern of higher accessibility in the centralized area of Eugene, the location with the tightest grid pattern of development. This is the oldest developed portion of Eugene and was developed before the predominance of automobiles. The lighter colored neighborhoods out to the west are areas where more industrial development has occurred and the street network, and thus the density of intersections, follows a much less dense pattern. In Figure 12, the areas that have a more characteristic suburban style of development are clearly visualized.

The southern portion is hilly and the street network tends to transect the mountains in long straight swaths with few intersecting streets. The dark area to the north in Figure 12 is an area more recently developed and follows a street pattern much more characteristic of the post-war suburban approach.

Figure 11: Street Intersection Density by Neighborhood

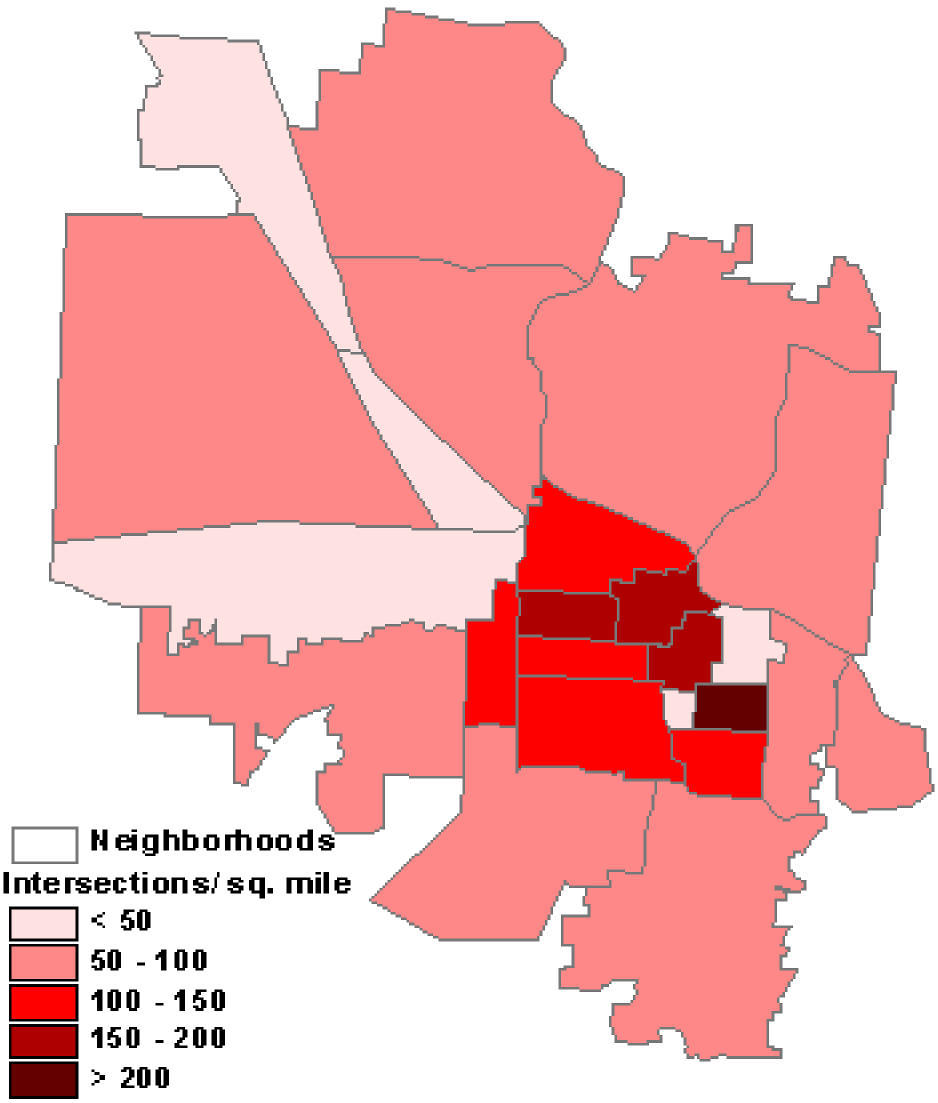
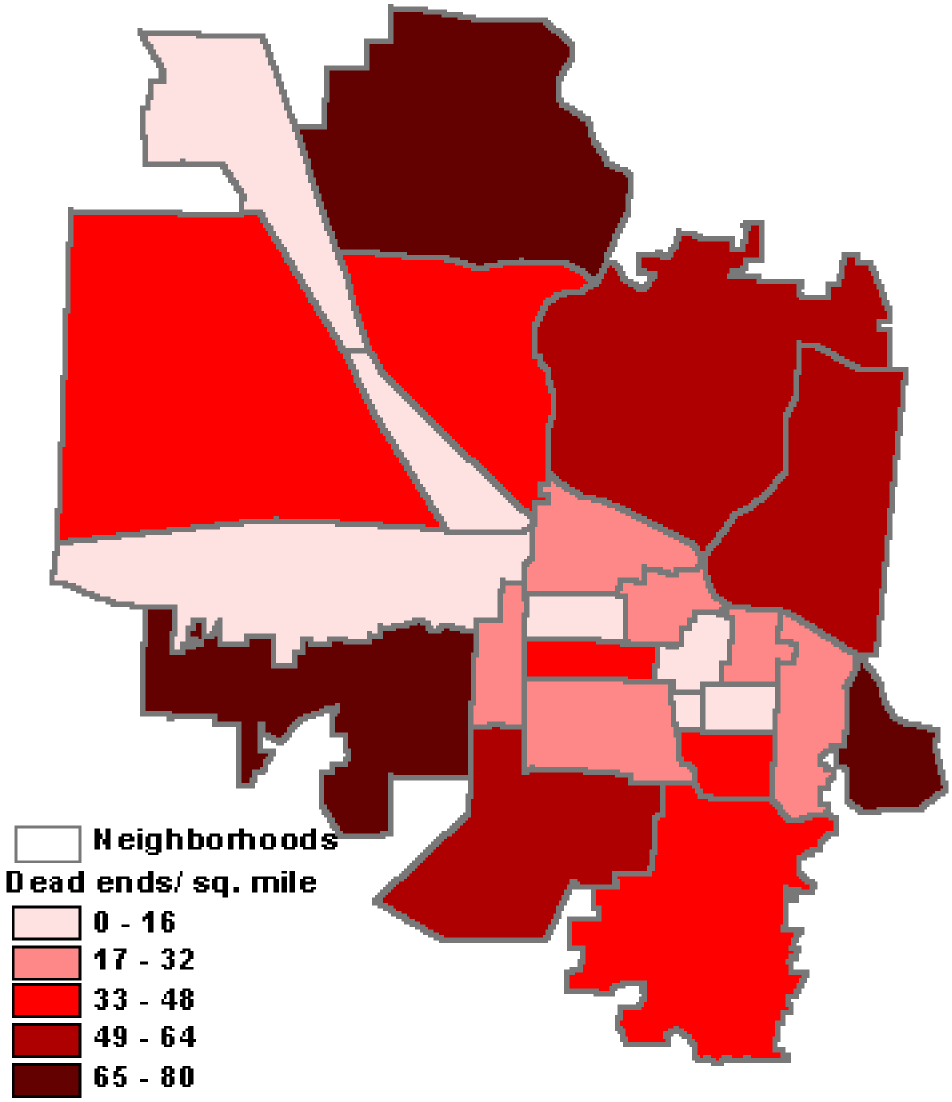
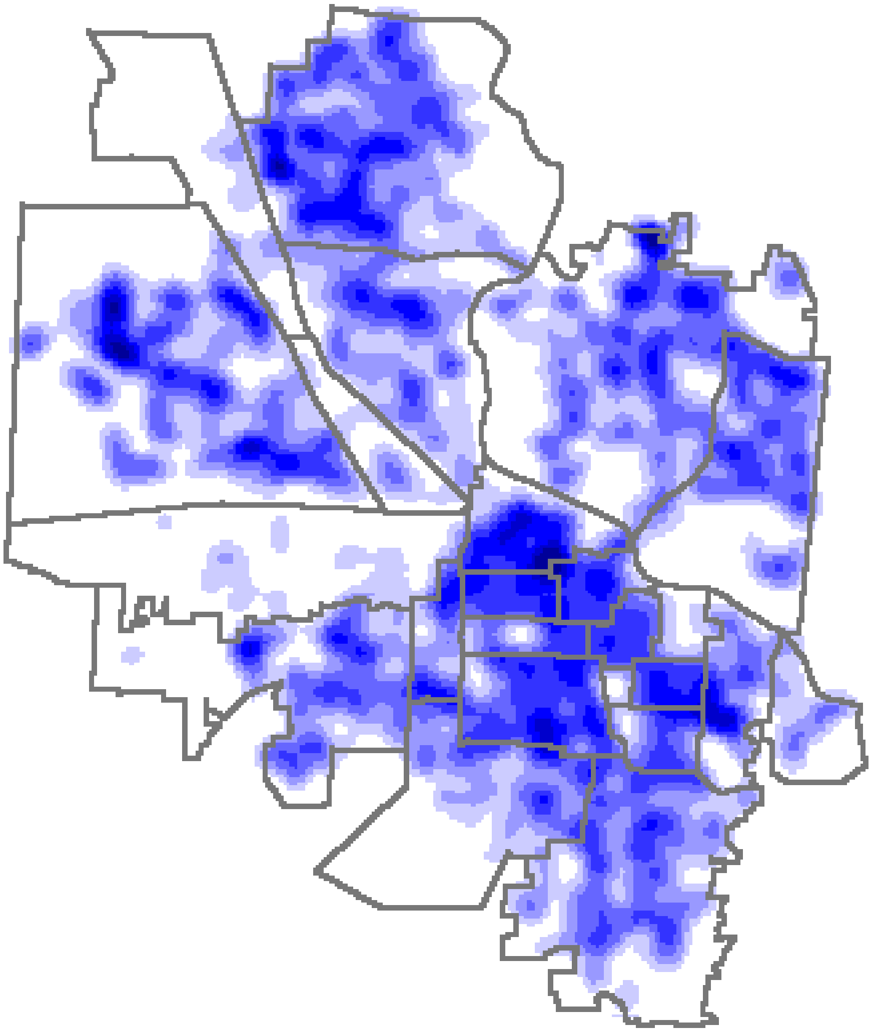


Figure 12: Dead End Density by Neighborhood



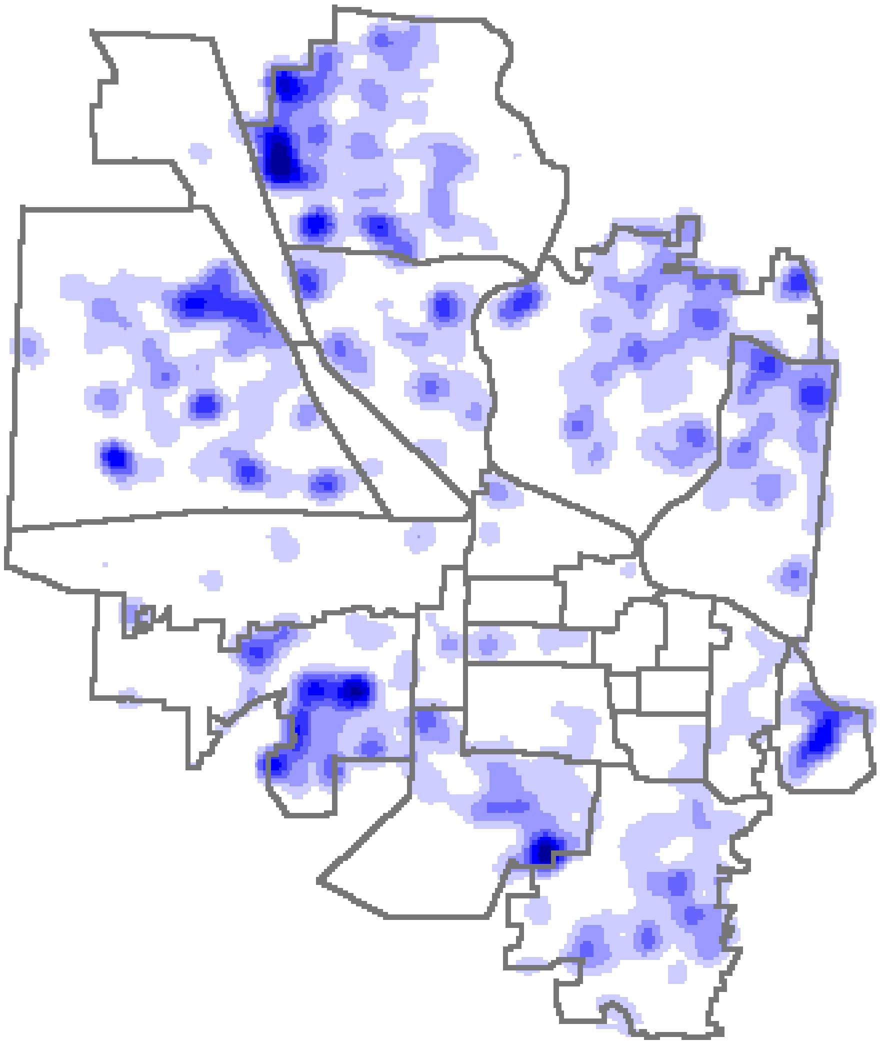
The figures above aggregate intersections to specific Eugene neighborhoods, which allows one to visualize accessibility on a neighborhood by neighborhood basis. Aggregating intersections to these pre-defined boundaries, however, is a bit artificial in nature. Alternatively, as shown in Figure 13, intersection density can be calculated by exact location in space. The intersection density of each spatial location can be calculated and then visualized based on the number of intersections that surround it. By transforming the vector data above to raster data (cells), a computation of the intersections within a ¼ mile of each cell can be calculated and displayed. Individual cells that are centrally located in relation to many intersections will appear in darker colors. Thus, regions of high intersection density can be visualized independent of the arbitrary borders of neighborhoods (or city boundaries, census tracts, and so forth). The neighborhood boundaries in Figure 13 are displayed, however, to give reference to the intersection density visualization.

Figure 13: Intersection Density by Point Location



The same type of calculation and visualization can be conducted on the density of dead-end streets or cul-de-sacs as shown in Figure 14.

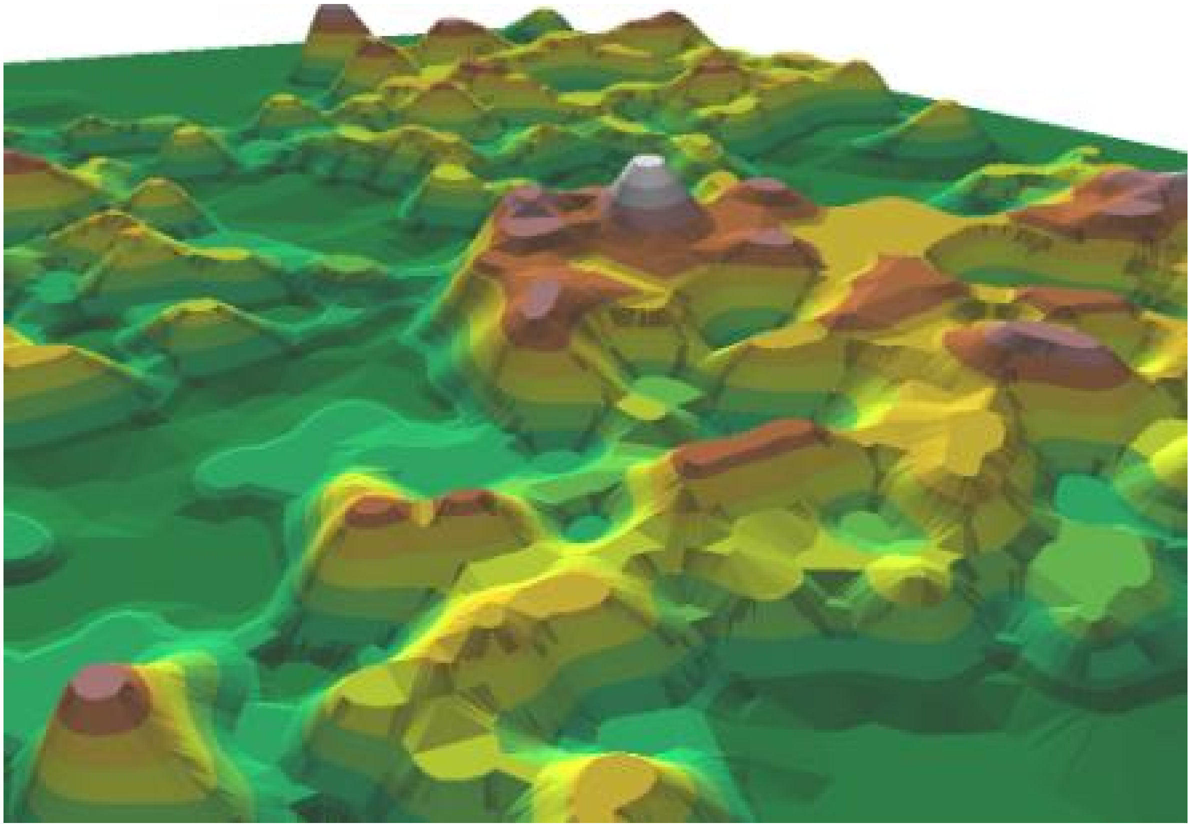
Figure 14: Dead End Density by Point Location



Figures 13 and 14 suggest locations where development has occurred in a way that is highly walkable and highly unwalkable.

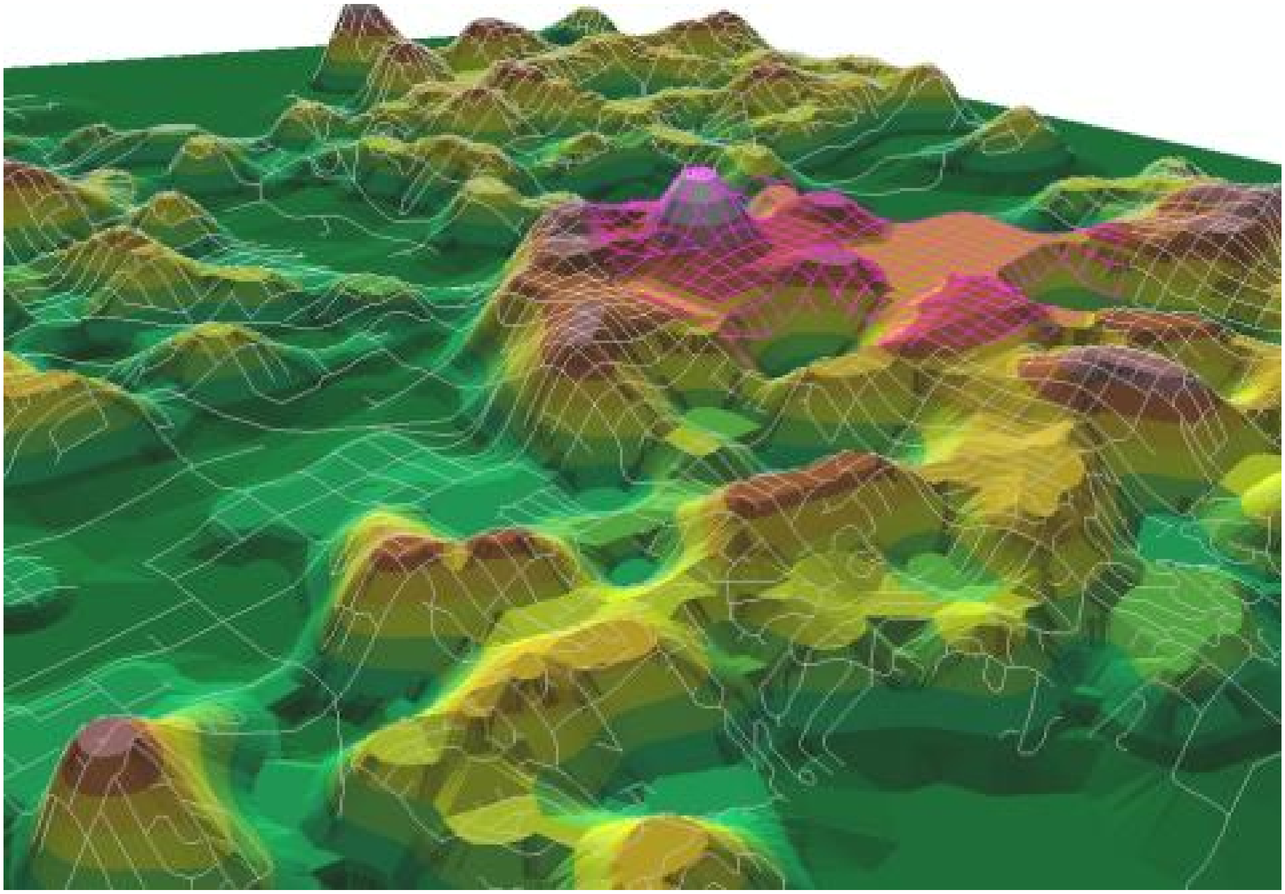
Accessibility using these raster-based calculations can also be viewed in three dimensions, using density of intersections in space, rather than actual elevation of land features, to create the topographic effect. Figure 15 visualizes the central Eugene area using this strategy, with mountain peaks representing areas of highest accessibility (concentration of intersections) and low areas representing places of low accessibility.

Figure 15: Elevation by Intersection



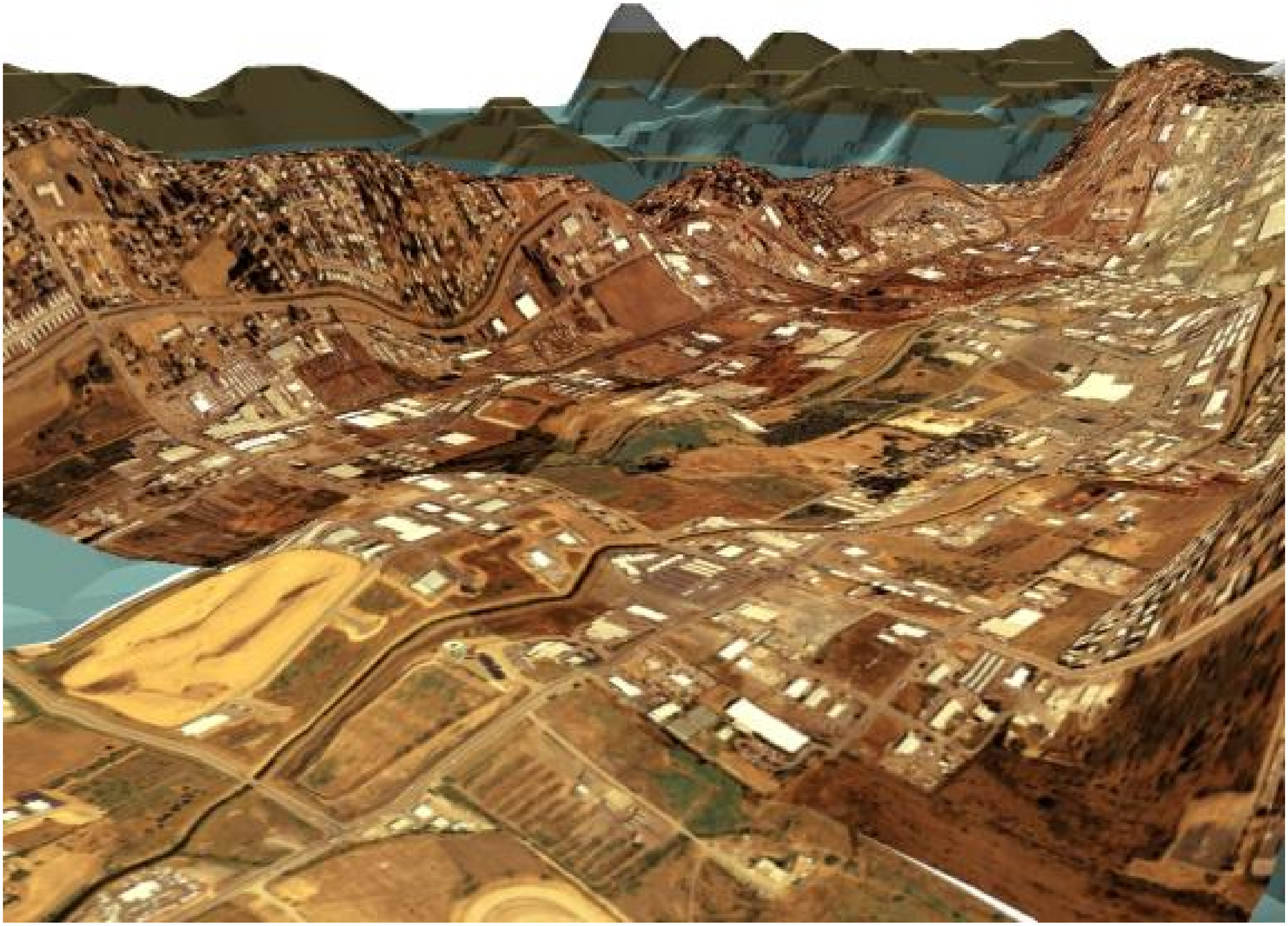
The three dimensional approach can be further augmented by overlaying the street network on top of the intersection topography to help visualize the concept of accessibility. Figure 16 illustrates this combination with streets within ¼ mile of the library highlighted in pink. That Figure also shows that the location of the new library is on the most accessible land of downtown Eugene. While not shown explicitly, the center of the pink streets (the location of the library) is just to the right of the tallest mountain peak (the location of the highest intersection density).

Figure 16: Intersection Elevation and Streets



Finally, aerial photographs can be draped on top of this new intersection topology to allow one to visualize the actual development of an area in relation to intersection density. Figure 17 visualizes accessibility using color aerial photos and the intersection-based topography. Some areas on the image below do not have aerial photos displayed in order to reveal the underlying connectivity as illustrated in Figures 15 and 16.

Figure 17: Intersection Elevation with Aerial Photos



In Figure 17, then, one can visualize the landscape of a city in a new way based on accessibility. Areas of high accessibility can be represented as mountain peaks (or alternatively as flat spaces) and the photographs of actual development can be viewed with this new underlying elevation. A policy connection, as well as a visual connection, might then be made between development patterns and accessibility.

Rood, T. (n.d.). Ped Sheds. Congress for a New Urbanism, Internet: http://www.cnu.org/cnu_reports/CNU_Ped_Sheds.pdf

COST PROXY MODELS IN RURAL TELEPHONE COMPANIES

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Introduction

The Federal-State Joint Board on Universal Service, CC Docket 96-45 ("Joint Board") published its *Recommended Decision* on November 8, 1996. Among other topics considered in that document, the Joint Board discussed the use of cost proxy models to determine the cost of network construction and by extension the cost of unbundled network elements. The Joint Board specified that the "technology assumed in [a cost proxy] model should be the least-cost, most efficient and reasonable technology for providing the supported services that is currently available for purchase."[1] Furthermore, the Joint Board specified that: "All underlying data should be verifiable, engineering assumptions reasonable, and outputs plausible."

Subsequent reports by the FCC and filings by interested parties have documented widespread and deep-rooted philosophical concerns within the telecommunications industry regarding cost proxy models *per se*. The cost proxy models created to date may be appropriate for the larger, urban area-based, incumbent local exchange carriers (ILECs) such as the former Bell operating companies and GTE; no opinion on that issue is offered here. However, it is clear that the cost proxy model procedures and unit prices proposed by the FCC are wholly unsuitable for use in rural areas. This report summarizes several areas in which this fact is evident, with particular emphasis on unit price input choices.

Geographic Considerations

Rural telephone companies face numerous geographic problems not experienced, for the most part, by large, urban-based ILECs. Among the distinctions that have a significant impact on the cost of network construction are the following factors.

Terrain

Many rural companies are located in areas with significant physical relief. Steep slopes pose particular obstacles to construction. For example, aerial plant placed in service in areas with steep slope often requires supplementary guying and support structures. As a second example, buried plant placed in service in areas with steep slope often must be placed at greater depth or with greater attention to cover and compaction to minimize the risk of cable exposure through erosion.

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[1] Federal-State Joint Board on Universal Service, CC Docket 96-45, *Recommended Decision*, November 8, 1996, ("Joint Board Decision"), paragraph 277.

These and numerous other issues related to terrain must be addressed by rural telephone companies during network construction. In each instance, the unit cost of construction is higher than would be the case in level terrain. The same issues also affect network maintenance costs and network upgrade costs. Rural telephone companies should be permitted to <u>adopt higher unit prices to accommodate the factor of terrain</u> - both slope and degree of terrain irregularity (roughness).

Although the cost proxy model includes a variable for slope, the model is unsatisfactory because it provides only partial consideration of terrain through its use of an "average slope" factor. Average slope may be a meaningful variable in urban areas where minimal variation is the general rule. Moreover, in urban areas, large volume contracts permit construction contractors to average costs and minimize the perceived effect of price differences due to terrain. However, the tremendous variations in slope that companies often face in rural areas, and the generally much smaller contracts for construction, render this simple measure inadequate. Rural telephone companies should be permitted to <u>adopt higher slope adjustment factors</u>.

Soil/Rock Conditions

Many rural companies are located in areas with significant adverse lithologic conditions. Construction in areas with rocky soil conditions is significantly more expensive than construction in new suburban sub-divisions. Indeed, many rural telephone companies must dedicate a significant proportion of their construction budget to rock sawing, rock drilling and similar placement activities. The comparatively high cost of such methods and the small size of the rural telephone companies mean that the relative cost impact of placing cable in rocky conditions is higher than it would be for urban companies. Rural telephone companies should be permitted to <u>adopt higher rock and rocky soil adjustment factors</u>.

Similarly, many rural companies are located in areas with significant adverse pedologic conditions. Coastal areas such as those in the Carolinas contain significant amounts of sand, which abrades plow shows and related equipment much faster than does suburban topsoil. Rural telephone companies should be permitted to <u>adopt higher sandy soil adjustment factors</u>.

Forested Areas/ Parks/Protected Areas

Many rural companies are located in areas with significant amounts of land reserved for state and national forests, state and national parks, nature preserves, military bases and other public uses. Cumulatively, the presence of these large reserve areas often forces inefficient construction methodologies to be adopted. For example, the shortest route to a remote serving unit cannot necessarily be used if it crosses a military base or contravenes other regulations. Similarly, the rights of Native American property holders (of reservations and other holdings) must be observed and appropriate permit fees must be paid for crossing such property even if permission is obtained. These factors contribute to increases in the cost of construction. Rural telephone companies should be permitted to <u>define and adopt a factor to control for increased construction costs related to the presence of public lands</u>.

Demographic Considerations
By definition, many rural companies are located in areas with relatively small populations and relatively low population densities. Both demographic factors force rural telephone companies to incur significantly higher construction costs.

Population Size
The five largest ILECs serve approximately 80% of the population of the United States. Cumulatively, the top ten ILECs serve almost 95% of the population. This factor of the size of the subscriber base is significant for several aspects of cost proxy model use.

Perhaps most significant, equipment manufacturers design, develop and, at least in the first instance, market equipment primarily for their larger customers. Manufacturers offer substantial discounts for large volume equipment purchases. Indeed, manufacturers have been known to provide equipment to large customers below cost at certain times (for example, early in the product cycle to encourage adoption and late in the year to supplement annual unit sales records).

No such volume discounts for central office and other equipment are available to rural telephone companies. The central office equipment (switch) pricing information contained in the cost proxy model is extremely poor, as argued in several FCC filings and as acknowledged by several model designers, and inappropriate for rural areas. Rural telephone companies should be permitted to define and adopt appropriate unit prices for switches and related equipment

Population Density
Although rural telephone companies may serve only approximately 5% of the US population, they do so over approximately 70% of the land area of the nation. The corresponding low population density for the typical rural telephone company forces such a company to incur disproportionately higher costs to provide service.

Customer Drops
The costs of terminals and drops vary greatly between zones of different population density. Within more densely populated areas, where subscribers are concentrated closer together, a design engineer can spread installation costs over a larger number of subscribers, particularly when pre-cabling subdivisions. Rural telephone companies should be permitted to adopt appropriate unit prices for drops.

This factor also affects the cross-connect or comparable flexibility-point technologies available to rural carriers. With greater drop spacing, the size of access cabinets is proportionately smaller. Rural telephone companies should be permitted to adopt appropriate unit prices for network interface devices.

Distances to subscribers - 1
Rural telephone companies must provide service from a single central office over a substantially larger area than would a large, urban ILEC. Even if one considers the use of remote serving unit technology, the physical network construction cost incurred by the rural telephone companies are substantially higher on a per-customer basis. To maintain network quality for the provision of contemporary services to schools, hospitals, and libraries, and of course, typical subscribers, as

well as enhanced services such as 911, rural telephone companies must engineer their networks with very different assumptions from those guiding the cost proxy model developers. Rural telephone companies should be permitted to define and adopt appropriate loop length calculation methodologies appropriate to the greater physical areas served. In passing, we note that these relatively long loops also will cause the rural telephone companies to incur greater maintenance and operating costs, further justification for modification of the unit costs.

Distances to subscribers - 2

In general, the length of drops to subscribers is greater in rural areas than in urban areas. This is a function of the greater average distance of the customers from the main roads, which itself is a function of the comparatively larger average land holdings typical of rural areas. This spatial characteristic affects the cost proxy model in another significant way. The FCC has determined that actual customer locations should be used with the cost proxy model, accepting the suggestion to use actual geocoded data if available and road network information where actual data are not available. However, According to the FCC's *Fifth Report & Order*, "the majority of commenters indicate that their geocode success rates decrease in rural areas."[2] Complicating the problem is the fact that the larger land holdings render the alternative (that is, use of the road network as a surrogate) non-viable without significant modification. Rural telephone companies should be permitted to define and adopt appropriate mechanisms for calculating rural subscriber locations.

Commercial Considerations

Transportation Costs

The relatively remote nature of rural telephone companies also contributes to higher network construction costs. Rural telephone companies incur higher transportation costs for equipment and material than do urban companies located closer to production facilities. Even in cases where urban carriers are located at some distance, the larger volume of purchases ensures discounts for transportation that are not available to smaller rural telephone companies. Rural telephone companies should be permitted to define and adopt a factor to incorporate equipment and material transportation costs into the unit price scheme. Alternatively, this problem offers further evidence for the need for flexibility in defining unit prices.

Other Service Costs

As with transportation costs, large urban ILECs can demand and expect to receive substantial discounts for construction service prices based on volume. No such volume discounts for construction services are available to rural telephone companies. Similarly, rural telephone companies can expect to pay proportionately higher costs for splicing services (and equipment such as fusion splicers), inspection services, locating services, maintenance and repair services, equipment installation and test services and other similar professional/technical services. Rural telephone companies should be permitted to define and adopt a factor to incorporate professional and technical costs into the unit price scheme. Alternatively, as with transportation, this problem offers further evidence for the need for flexibility in defining unit prices.

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[2] Federal-State Joint Board on Universal Service, CC Docket 96-45 and CC Docket 97-160, *Fifth Report & Order*, October 28, 1998, paragraph 34, footnote 71.

Structure Sharing

All versions of the cost proxy models (whether submitted and/or adopted) endorse sharing network construction costs among several companies where feasible. In brief, the concept assumes that several companies could use some or all support structures in a telephone network simultaneously. For example, in theory several companies could bury cables in a common trench with shared conduits and innerducts.

There are several tangible practical issues associated with structure sharing in rural areas that cost proxy models ignore. Most significant for rural telephone companies is the assumption that shared trench and conduit construction is even an economically feasible option. The predominant placement techniques in rural areas are direct cable plowing and aerial cable placement. For obvious reasons, the opportunities for structure sharing when directly plowing cable are limited. However, numerous problems also limit the opportunity for structure sharing with aerial placement.

The number of companies that may attach facilities to a pole depends primarily on the height of the pole, the class of the pole, and the number of pre-existing attachments. The height of the pole is a factor because federal, state, and local laws and ordinances, as well as safety considerations, mandate certain minimum clearances over roadways and railroad tracks below the cable span. Similarly, the class of the pole, which corresponds to the diameter of the pole, determines the total load that the pole may bear and the support guying required. Other parameters, such as the weight of the cable, also influence the minimum height at which users may attach cables to poles. In combination, these constraints determine the maximum theoretical number of cables that users may attach.

Rural aerial plant generally must cover significant distances at minimum cost through areas not reached by high volume roadways. This dictates that aerial plant will be constructed with poles that are placed at greater intervals than in urban areas. To reduce costs further, shorter poles are used. In combination, this means that mid-span sag will bring the cable much lower to the ground that the cost proxy model designers anticipated. Because the poles are smaller, there are fewer opportunities for structure sharing due to the reduced load-bearing capability of the poles. Consequently, rural telephone companies must be permitted to make significant changes to the assumed percentage of structure sharing in any cost proxy model.

Conclusion

The cost proxy models currently proposed by the FCC were built using input values (unit prices, engineering practices, structure sharing assumptions and similar variables) that were defined by the experience of large, predominantly urban-area ILECs. Such values are completely unsuitable for small, rural telephone companies for the reasons outlined here.

The large urban ILECs recognize the financial and commercial disincentives to providing services in rural areas that have been outlined here. That is why the large urban ILECs frequently have traded properties in rural areas, either to eliminate the problem by getting rid of the franchise area or to aggregate territories to achieve volume discounts in purchasing, transport and construction.

The question of the applicability of cost proxy models in the context of universal service remains open to public debate. To ameliorate the specific issues noted here and to accommodate the concerns of universal service, rural carriers must be allowed significant latitude in redefining, and in some cases supplementing, input values.

Predicting Pre-monsoon Thunderstorms – A Statistical View through Propositional Logic

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Abstract

Thunderstorms very close to a monsoon and not so close to a monsoon are considered in this analysis. Some important predictors are considered. Pearson Correlation Coefficient and lag-1 autocorrelation coefficients are calculated to create necessary universes of discourse for propositional logic. The purpose is to make regression analysis more convenient for prediction of the pre-monsoon thunderstorm weather phenomenon.

Key words: Predictor, Predictand, Pre-monsoon, Pearson Correlation Coefficient, Autocorrelation, Propositional Logic

1. Introduction

The severe thunderstorm is a very important weather phenomenon in Gangetic West Bengal (GWB) during March to May. Thunderstorms of this period are called pre-monsoon thunderstorms. The present study encompasses some important parameters associated with pre-monsoon thunderstorms and tries to identify one or more important predictors and predictands so that in the future, regression analysis (simple or multiple) can be done conveniently to study pre-monsoon thunderstorms. Moreover, the study further tries to understand whether a particular predictor can be used with the same predictand to analyze the thunderstorms very close to a monsoon as well as thunderstorms not very close to a monsoon. Because the dataset is vastly complex, the study has relied on the kind of robust summary offered by a linguistic proposition applied to statistical measures. Two consecutive months of pre-monsoon season are considered in this study.

2. Data

The dataset consists of the values of some parameters associated with severe thunderstorms of April and May occurring over GWB between 1987 and 1998. The total number of thunderstorms considered in this study is 130. Parameters considered for this study are:

- Duration (d) of the thunderstorm
- Change in air pressure (ΔP) during the thunderstorm
- Change in surface temperature (Δ T) during the thunderstorm
- Maximum wind speed (v) associated with the thunderstorm
- Change in relative humidity (Δ R/H) during the thunderstorm.

3. Methodology
The methodology adopted in the present study consists of

- Calculation of Pearson Correlation Coefficient (PCC)
- Testing for persistence
- Propositional logic

3.1 Pearson Correlation Coefficient (PCC)
Pearson Correlation Coefficient (PCC) measures the degree of association between two variables ‘x’ and ‘y’. Mathematically PCC is defined as

$$\rho_{xy} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}\ \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{1}$$

where, $n \rightarrow$ Total number of observations

$\bar{x} \rightarrow$ mean of the variable ‘x’

$\bar{y} \rightarrow$ mean of the variable ‘y’

In the present paper the following PCC’s are calculated;

$\rho_{d\Delta P}$, $\rho_{d\Delta T}$, ρ_{dv}, $\rho_{d\Delta R/H}$, $\rho_{v\Delta P}$, $\rho_{v\Delta T}$, $\rho_{v\Delta R/H}$, $\rho_{\Delta R/H\Delta P}$, $\rho_{\Delta R/H\Delta T}$, $\rho_{v\Delta R/H}$

The aforesaid quantities are calculated separately for April and May for each year.

3.2 Testing for persistence
Persistence means existence of statistical dependence among successive values of the same variable (Wilks, 1995). Persistence is measured by lag-1 autocorrelation defined as;

$$r_1 = \frac{\sum_{i=1}^{n}\left[(x_i - \bar{x}_-)(x_{i+1} - \bar{x}_+)\right]}{\left[\sum_{i=1}^{n}(x_i - \bar{x}_-)^2 \sum_{i=2}^{n}(x_i - \bar{x}_+)^2\right]^{1/2}} \tag{2}$$

where $\bar{x}_- =$ Mean of first (n-1) data values
$\bar{y}_+ =$ Mean of last (n-1) data values

In the present paper, PCC’s mentioned in section 3.1 are considered as a dichotomous random variable X defined as

$$X = 1 \quad \text{if } |\rho| < 0.5$$
$$= 0 \quad \text{if } |\rho| \geq 0.5 \quad (3)$$

Then, sequences of entries 0 and 1 are constructed separately for April and May. In the next step, the lag-1 autocorrelation coefficient is calculated for each sequence.

3.3 Propositional logic

Propositional logic is a generalized logic that includes all possible values between 0 and 1. In this logic, a relationship is required to express the distribution of the truth of a variable (Klir and Folger, 2000). A function called a "membership function" is needed to indicate the extent to which a variable 'x' has the attribute 'F'. Membership functions are defined on a universe of discourse indicated by the research variable.

In the present paper, lag-1 autocorrelations create the required universes of discourse. X1 is the universe of discourse for April and X2 is the universe of discourse for May. The proposition 'P' tested for the present study is:

> "The degree of association between any pair of parameters is consistently very high is very true".

Thus, the membership function is framed as

$$\mu(x) = 0 \quad \text{for } x < 0.5$$
$$x/0.6 \quad \text{for } 0.5 \leq x \leq 0.6 \quad (4)$$
$$1 \quad \text{for } x > 0.6$$

3. Result and discussion

From the above study the following are found (Fig.1) to be highly true:

(a) In the month of April, maximum wind speed associated with a severe thunderstorm is mostly dependent upon the change in air pressure during the thunderstorm. But, the dependence is less in the month of May.
(b) In the month of May, maximum wind speed associated with a thunderstorm depends mostly upon the change in relative humidity during the thunderstorm. Whereas, in April, maximum wind speed has no relationship with the change in relative humidity during the thunderstorm.
(c) Change in the surface temperature during a thunderstorm depends highly upon duration of the thunderstorm in April. But, in May, they have almost no association.
(d) In the month of May, change in relative humidity during thunderstorms depends upon the duration of the thunderstorm. Whereas, in April, these two parameters have no association.
(e) Degree of association between change in air-pressure and change in relative humidity remains almost the same in the months of April and May.

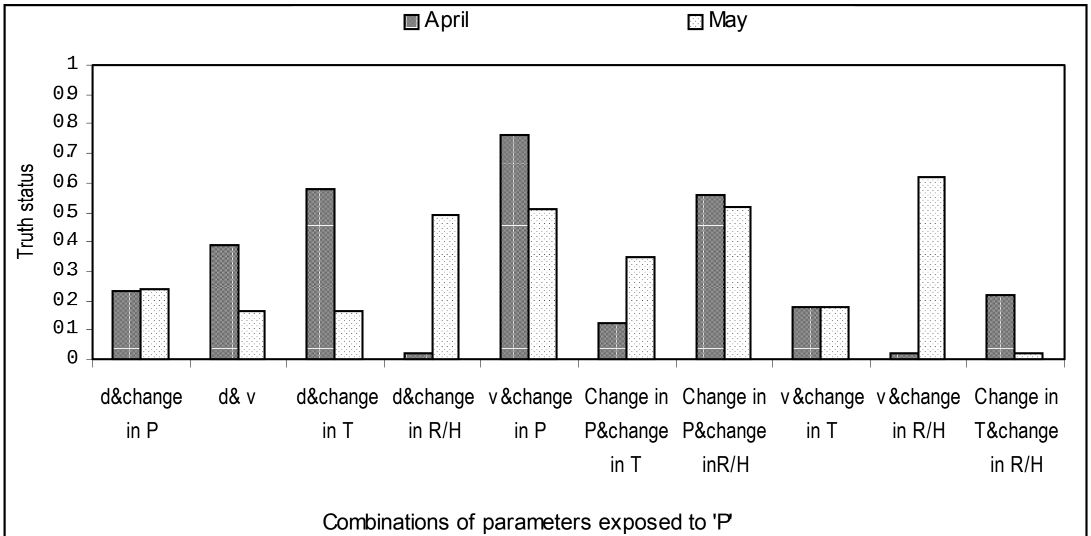

Fig.-1Schematic presentation of the truth status of the proposition 'P'applied to different pairs of parameters associated w ith sev ere pre-monsoon thunderstorms ov er GWB.

4. Conclusion

From the above study it can be concluded, in the study area, that if maximum wind speed associated with a severe thunderstorm is considered as a predictand, then change in air pressure is a good predictor in the month of April and change in relative humidity is a good predictor in the month of May. Duration of a thunderstorm can be used as a good predictor with change in surface temperature as predictand in the month of April and with change in relative humidity as predictand in the month of May. It can further be concluded that relation between change in air pressure and change in relative humidity does not change in spite of advancement of monsoon.

5. Acknowledgement

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Disaggregation and Targeting of Universal Service Support

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Introduction

The actual cost of providing telecommunications services in rural America is generally higher, per customer, than is the cost of providing these services in urban areas. This difference is due in part to the lower density of population of rural areas. Rural carriers, in contrast to urban carriers, have fewer customers to share basic fixed costs (for example, switches) and these customers are separated by greater distances, increasing outside plant costs, than are their urban counterparts. The disparity in costs is also related to the economies of scale and economies of skill enjoyed by large urban carriers that are not available to rural carriers. For example, the Federal Communications Commission’s forward-looking economic cost model shows a cost of $866.27, without adjustment for overhead costs, to provide a local loop in a Wyoming wire center, compared to a cost of $9.97 to provide a local loop in a New York City wire center.

During the era of monopoly service, the disparity in costs between rural and non-rural service was addressed through implicit subsidies between geographic areas and classes of service. Between 1984 and 1996, a series of opinions, rulings and regulations began to coalesce and focus attention on the need to restructure these subsidies. The Telecommunications Act of 1996 had three primary goals: to promote competition, to reduce regulation, and to ensure all Americans receive the benefits of telecommunications (that is, to ensure universal service). Under the provisions of the Act, universal service would continue to be subsidized, satisfying the third of these goals, but the previously implicit subsidies would be transformed into explicit subsidies. This transformation would also further the first goal by making federal universal service support portable to all certified eligible telecommunications carriers.

Federal universal service support consists of three categories of support, with a fourth category scheduled for implementation in the near future. The first category is High

Cost Loop Support, or HCLS. This category focuses on the costs associated with high-cost local loop outside plant costs. The second category is Long Term Support, or LTS. This category provides support for the interstate loop cost of rate-of-return carriers that participate in the National Exchange Carrier Association (NECA) common line pool. The FCC tentatively has concluded that LTS should be merged with Interstate Common Line Support (ICLS) as of July 1, 2003, after which participation in the NECA common line pool would not be required for receipt of support. The third category is Local Switching Support, or LSS. As its name suggests, this category focuses on the relatively higher costs for carriers with fewer than 50,000 access lines of providing basic switching services. All three categories of support are affected by a process known as the "averaging" of support.

Averaging of Support

Under existing embedded cost mechanisms, federal universal service high-cost support for rural carriers is averaged across all lines served by a carrier within its study area. The FCC's definition of a "study area" confirms a specific service territory and states that "[study area boundaries shall be frozen as they are on November 15, 1984" to reflect "[a] telephone holding company's operations within a single state. Figure 1 represents an hypothetical study area with a centrally located town (the shaded oval). The difficulty with averaging support across all lines served by a carrier within a study area is that the support in low-cost areas of a study area may exceed the cost of serving those areas while support in high-cost areas may be insufficient to offset the higher cost of serving those areas.

The Rural Task Force (RTF), an independent advisory panel appointed by the Federal–State Joint Board on Universal Service to provide guidance on universal service issues affecting rural telephone companies, produced a series of six white papers detailing the results of its inquiry and its recommendations to the Federal Communications Commissions (FCC), which culminated in an FCC Order.[1] Two of these white papers were dedicated to the question of averaging support. The RTF recommended that rural carriers should be permitted to depart from study area averaging and to disaggregate and target per-line high-cost universal service support (that is, HCLS, LTS and LSS) to geographic areas below (that is, smaller than) the study area level. Disaggregation to this finer level of granularity would define per-line support that would reflect the actual cost of providing service in particular geographic sub-areas within the study area.

Disaggregation Paths
The RTF stated that rural carriers needed flexibility in the manner in which federal high-cost universal service support is disaggregated and targeted due to variations in the characteristics and operating environments of rural carriers. To provide this flexibility, the RTF recommended a disaggregation system that consisted of three options or "paths." These paths would allow rural carriers to identify zones of relative cost variation (if any) and to develop appropriate methods of specifying which zones should receive more support.

The FCC adopted the RTF's recommendation of three paths for disaggregation and targeting of high-cost universal service support. The FCC agreed that there should be flexibility in the manner in which support was disaggregated and targeted for rural carriers. The FCC confirmed that support should be disaggregated and targeted below the study area level to ensure that per-line level of support would be more closely associated with the cost of providing services.

- *Path One*

  Path One allows a carrier to certify to the state commission or other appropriate regulatory authority that it does not want to disaggregate support (Figure 2). However, a state could require disaggregation and targeting of support, either on its own motion or on the motion of an interested party, in which case the carrier would be required to disaggregate its support zones. After selection, the plan will remain in effect until a state commission or appropriate regulatory authority requires, on its own motion or upon petition by an interested party (including the affected carrier), a change to a different disaggregation and targeting methodology. The rationale for these restrictions was the desire to eliminate "gaming" of the system.
- *Path Two*

  Path Two is available to carriers that want state commission review and approval of a relatively complex disaggregation plan. Path Two allows a carrier to disaggregate and target support to multiple levels below a wire center. A disaggregation and targeting method can be tailored with precision, subject to state approval, to the cost and geographic characteristics of the carrier and the competitive and regulatory environment (Figure 3). The plan must show a per-line amount of support for each element in each disaggregation zone. Path Two provides the most flexibility in the development of a disaggregation plan, but also provides for regulatory approval to ensure that the methodology implemented is competitively neutral.
- *Path Three*

  Path Three would permit carriers to self-certify a method of disaggregation with the state commission or other appropriate regulatory authority. Path Three Permits carriers to choose 1) a disaggregation plan of up to two cost zones per wire center (Figure 4) or 2) a disaggregation plan that complies with a prior regulatory determination.

Under the terms of Path Three, self-certifying carriers must provide state regulators (or other appropriate regulatory authority) and the Universal Service Administrative Company (USAC) with a description of the rationale used to disaggregate support, including the methods and data, and a discussion of how the plan complies with the self-certification guidelines. If the plan uses a benchmark, it must be generally consistent with how the total study area level of support for each category of costs (HCLS, LSS and LTS) is derived, to enable a competitor to compare the disaggregated costs used to determine support for each zone. The plan must show a per-line amount of support for each element in each disaggregation zone.

Levels of Support

The FCC order requires that an incumbent carrier's total support for a given study area using the chosen disaggregated method must equal the total support available in that study area on a non-disaggregated (that is, averaged) basis. In this way, the FCC sought to limit the impact of disaggregation on the universal service funding requirements.

The FCC also requires that the relative per-line support relationships between disaggregation zones for each disaggregated category of support must remain fixed over time and that such relationships must be made publicly available. That is, the FCC requires that the per-line support for each category of support (HCLS, LTS & LSS) in each disaggregation zone must be determined so the relative support relationships between zones will be maintained.

The FCC recognized that there is some variation in costs with different categories of support. Specifically, the HCLS and LTS mechanisms support loop costs and therefore share similar cost characteristics. Carriers would be required to allocate the same ratio of HCLS and LTS to each disaggregation zone. However, a carrier's local switching cost characteristics might differ from its loop cost characteristics in different disaggregation zones. Therefore, it would be allowed to allocate a different ratio for LSS to the extent that the cost characteristics of providing loop and switching service in disaggregation zones differ.

The FCC requires that the product of all of the ILEC's lines for each cost zone multiplied by the per-line support for those zones when added together must equal the sum of the ILEC's total level of support. FCC requires that per-line support amounts for each zone must be recalculated whenever an ILEC's total annual support changes. The recalculated support amount must be based on the changed support amounts and lines at that point in time.

After a CLEC is designated as a competitive eligible telecommunications carrier (CETC) in a rural study area, determination of per-line amounts of support for the CLEC will be based on the ILEC's total support levels, lines and disaggregated support relationships.

Timeframes
In its order, the FCC directed carriers to choose a disaggregation path within 270 days of the effective date of the rules adopted in the order. The order stated that carriers that failed to do so would not be permitted to disaggregate and target support unless ordered to by an appropriate regulatory authority. This requirement meant that carriers were required to submit their disaggregation plans to USAC no later than March 18, 2002. The FCC's Multi-Association Group (MAG) Order extended this date to May 15, 2002. (We note in passing that this extension resolved an ambiguity in the original order that in at one case suggested a date of March 15, rather than March 18.)

A carrier electing Path Two or Path Three must, by May 15, 2002, file with the relevant state regulatory authority its proposed disaggregation plan or its self-certified disaggregation plan. State approval of a carrier's proposed disaggregation plan pursuant to Path Two would not be required by that date, but the disaggregation plan could not go into effect until approval was received.

After selection, the Path will remain in effect until a state commission or appropriate regulatory authority requires a change to a different disaggregation and targeting methodology. Such a requirement could be based on its own motion or on petition by an interested party (including the affected carrier).

Restrictions
The FCC adopted several general restrictions for all paths.

- *Competitive Carrier Designated*

  For study areas in which a CLEC was designated as a CETC prior to the effective date of these rules, an ILEC could elect Path Three only to the extent that it was self-certifying a disaggregation and targeting plan that had already been approved by the state.
  In all other instances in which an eligible CLEC had been designated as a CETC prior to the effective date of these rules, the ILEC must seek prior state approval of its

disaggregation and targeting plan under Path Two.

- *Certifying Boundaries*
  Rural ILECs must submit to USAC maps in which the boundaries of the designated disaggregation zones of support are clearly specified, which USAC will make available for public inspection by competitors and other interested parties.
- *Algorithm Used*

  FCC required that, when submitting information in support of self-certification, a carrier must provide USAC with publicly available information that allowed competitors to verify and reproduce the algorithm used to determine zone support levels. The carrier also must demonstrate that the underlying rationale is reasonably related to the cost of providing service for each cost zone within each disaggregated category.
- *Certification*
  FCC requires carriers electing Path One to submit to USAC a copy of the certification to the state commission or appropriate regulatory authority certifying that it will not disaggregate and target support.

  Carriers selecting Path Two must submit a copy to USAC of the order approving the disaggregation plan submitted by the carrier to the state commission or appropriate regulatory authority. Carriers selecting Path Two also must submit a copy of the disaggregation plan approved by the state commission or appropriate regulatory authority.

  Carriers selecting Path Three must provide the state and USAC with a description of the rationale used to disaggregate support, including the methods and data, and a discussion of how the plan complies with the self-certification guidelines. The plan must show a per-line amount of support for each element in each disaggregation zone.
- *MAG Plan*
  The initial purpose of the disaggregation and targeting Paths was to allocate appropriate levels of support to geographic sub-areas within a study area. This purpose has been extended, at least by implication, as a result of the MAG Plan's application of the same zones.[2]

  In the MAG Plan Order, the FCC ordered that the RTF system for geographic disaggregation and targeting below the study area level would also apply to the newly-defined Interstate Common Line Support (ICLS) category of portable high-cost universal service support. The FCC noted that disaggregation by allowing ILECs to target explicit universal service support to regions within a study area that costs relatively more to serve would ensure that a competitive entrant would receive targeted support only if it also serves the high-cost region. Disaggregation would prevent the competitive entrant from receiving greater support than was needed to serve relatively low-cost regions, a circumstance that would give the competitive carrier a potential price advantage over the incumbent.

The FCC noted that the same three paths would be available for the disaggregation of ICLS as for other types of support defined in the RTF Order. The MAG Plan Order extended the deadline for selecting a path to May 15, 2002 and reaffirmed that after that date a carrier would not be permitted to disaggregate and target support unless ordered to do so by a state commission or other appropriate regulatory authority.

The MAG Plan order confirmed that a carrier's choice of disaggregation paths would remain in place for four years, unless a state commission or other appropriate regulatory authority ordered disaggregation and targeting of support in a different manner. Rate-of-return carriers would be required to select identical disaggregation zones for all forms of high-cost universal service support, with the exception of forward-looking intrastate high-cost support received by non-rural carriers that are also rate-of-return carriers. For example, if a rural rate-of-return carrier self-certified two cost zones per wire center under Path Three, it would be required to disaggregate all forms of high-cost universal service support -- HCLS, LTS, LSS and ICLS -- to the same two cost zones per wire center. The FCC noted that there was no reason why support should be allocated differently in different disaggregation zones.

The FCC reaffirmed that there is some variation in costs with different categories of support. The HCLS, LTS and ICLS mechanisms support loop costs and share similar cost characteristics; carriers are required to allocate the same ratio of HCLS, LTS and ICLS to each disaggregation zone. However, a carrier's local switching cost characteristics might differ from its loop cost characteristics in different disaggregation zones. Therefore, it would be allowed to allocate a different ratio for LSS to the extent that the cost characteristics of providing loop and switching service in disaggregation zones differ.

The FCC rules for the disaggregation and targeting of portable ICLS and LTS apply to both rural and non-rural rate-of-return carriers. Non-rural rate-of-return carriers are required to adopt a disaggregation and targeting path only for their receipt of ICLS and LTS. Non-rural intrastate high-cost support, including forward-looking high-cost support and interim hold-harmless support, will continue to be targeted to high-cost wire centers, consistent with FCC rules for targeting such support to high-cost wire centers.

Conclusion

Although the deadline for filing a disaggregation plan may have passed by the time this article appears, that deadline may have been extended as it was with the release of the MAG Plan. In any event, any carrier may file a request with the appropriate regulatory authority to investigate a change in disaggregation at any time. Therefore, it remains appropriate to consider whether, and how, a rural rate-of-return carrier should disaggregate and target support.

The range of specific circumstances that rural carriers face prevents us from offering a general recommendation regarding an approach to disaggregation. The significant amounts of support fund that are affected by this issue justify careful analysis of each individual case to ensure the availability of all appropriate support.

[1] Rural Task Force, Competition And Universal Service, White Paper No. 5, September 2000; Rural Task Force, Disaggregation And TargetingOf Universal Service Support, White Paper No. 6, September 2000; *Federal-State Joint Board on Universal Service*, CC Docket No. 96-45, Recommended Decision, FCC 00J-4 (Joint Board released December 22, 2000) and FCC Fourteenth Report And Order, Twenty-Second Order On Reconsideration, And Further Notice Of Proposed Rulemaking In CC Docket No. 96-45, and Report And Order In CC Docket No. 00-256, released May 23, 2001.

[2] FCC Second Report And Order And Further Notice Of Proposed Rulemaking In CC Docket No. 00-256, Fifteenth Report And Order In Cc Docket No. 96-45, And Report And Order In CC Docket Nos. 98-77 And 98-166, released November 8, 2001.

L^1-CONVERGENCE OF COSINE SERIES WITH HYPER SEMI-CONVEX COEFFICIENTS

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and
S.S.BHATIA

Abstract. In this paper we obtain a necessary and sufficient condition for L^1-convergence of the Fourier cosine series with hyper semi-convex coefficients. Results of Bala R. and Ram B. [1] have been obtained as a special case.

2000 Mathematics Subject Classification: 42A20. 42A32

KEY WORDS AND PHRASES. Cesàro means, L^1-convergence, hyper semi-convexity.

1. Introduction. Consider

$$g(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx \tag{1.1}$$

to be the cosine series with partial sums defined by

$$S_n(x) = \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos kx$$

and let $g(x) = \lim_{n\to\infty} S_n(x)$

Concerning the L^1-convergence of cosine series (1. 1) Kolmogorov [3] proved his well known theorem:

Theorem A. If $\{a_n\}$ is a quasi-convex null sequence, then for the L^1-convergence of the cosine series (1. 1) it is necessary and sufficient that $\lim_{n\to\infty} a_n \log n = 0$.

Definition. A sequence $\{a_n\}$ is said to be semi-convex if $\{a_n\} \to 0$ as $n \to \infty$, and

$$\sum_{n=1}^{\infty} n|\Delta^2 a_{n-1} + \Delta^2 a_n| < \infty, \qquad (a_0 = 0)$$

where

$$\Delta^2 a_n = \Delta a_n - \Delta a_{n+1}$$

It may be remarked here that every quasi-convex null sequence is semi- convex.

Bala R. and Ram B. [1] have proved that Theorem A holds true for cosine series with semi-convex null coefficients in the following form:

Theorem B. If $\{a_k\}$ is a semi–convex null sequence, then for the convergence of the cosine series in the metric space L, it is necessary and sufficient that $a_{k-1} \log k = o(1)$.

We define $\{a_n\}$ to be hyper semi-convex of order α, in the following way:

Definition. A sequence $\{a_n\}$ is said to be hyper semi-convex, if

$$\{a_n\} \to 0 \text{ as } n \to \infty,$$

$$\sum_{n=1}^{\infty} n^{\alpha+1}|(\Delta^{\alpha+2}a_{n-1} + \Delta^{\alpha+2}a_n)| < \infty, \qquad \text{for } \alpha = 0,1,2\ldots\ldots\ldots,$$

$$(a_0 = 0).$$

By definition, hyper semi-convexity of order zero is same as semi-convexity.

The purpose of this paper is to generalize the Theorem B for the cosine series with hyper semi-convex null coefficients.

2. Notation and Formulae. In what follows, we use the following notation [4]:

Given a sequence S_0, S_1, $S_2,\ldots\ldots\ldots\ldots$, we define for every $\alpha = 0,1,2\ldots\ldots\ldots$,

the sequence S_0^α, S_1^α, $S_2^\alpha,\ldots\ldots\ldots$, by the conditions

$$S_n^0 = S_n,$$

$$S_n^\alpha = S_0^{\alpha-1} + S_1^{\alpha-1} + S_2^{\alpha-1} + \ldots\ldots + S_n^{\alpha-1} \qquad (\alpha = 1,2\ldots\ldots, n = 0,1,2\ldots\ldots).$$

Similarly for $\alpha = 0,1,2\ldots$, we define the sequence of numbers

$A_0^\alpha, A_1^\alpha, A_2^\alpha,\ldots\ldots$ by the conditions

$$A_n^0 = 1,$$

$$A_n^\alpha = A_0^{\alpha-1} + A_1^{\alpha-1} + A_2^{\alpha-1} + \ldots\ldots + A_n^{\alpha-1} \qquad (\alpha = 1,2\ldots\ldots,\ n = 0,1,2\ldots\ldots).$$

where A_p^α denotes the binomial coefficients and are given by the following relations.

$$\sum_{p=0}^{\infty} A_p^\alpha x^p = (1-x)^{-\alpha-1}$$

and $\tilde{S}_n^{\cdot}\, s$ are given by

$$\sum_{p=0}^{\infty} S_p^\alpha x^p = (1-x)^{-\alpha} \sum_{p=0}^{\infty} S_p x^p$$

Also

$$A_n^\alpha = \sum_{p=0}^{n} A_p^{\alpha-1}, \qquad A_n^\alpha - A_{n-1}^\alpha = A_n^{\alpha-1}$$

$$A_n^\alpha = \binom{n+\alpha}{n} \simeq \frac{n^\alpha}{\Gamma\alpha+1} \qquad (\alpha \neq -1,-2,-3\ldots\ldots)$$

Also for $0 < x \leq \pi$, let

$$\tilde{S}_n^0 = \tilde{S}_n = \sin x + \sin 2x + \ldots\ldots\ldots\ldots + \sin nx$$

$$\tilde{S}_n^1 = \tilde{S}_1 + \tilde{S}_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \tilde{S}_n$$

$$\tilde{S}_n^2 = \tilde{S}_1^1 + \tilde{S}_2^1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \tilde{S}_n^1$$

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$$\tilde{S}_n^k = \tilde{S}_1^{k-1} + \tilde{S}_2^{k-1} \ldots\ldots\ldots\ldots\ldots\ldots + \tilde{S}_n^{k-1}$$

The conjugate Cesàro means $\tilde{T}_n^\alpha$ of order α of $\sum a_n$ will be defined by

$$\tilde{T}_n^\alpha = \frac{\tilde{S}_n^\alpha}{A_n^\alpha}.$$

2. Lemma. The following Lemma will be used for the proof of our result.

Lemma [2]. If $\alpha \geqslant 0,\ p \geqslant 0,$

(*i*) $\epsilon_n = o(n^{-p})$,

(*ii*) $\sum_{n=0}^{\infty} A_n^{\alpha+p} |\Delta^{\alpha+1} \epsilon_n| < \infty$, then

(*iii*) $\sum_{n=0}^{\infty} A_n^{\lambda+p} |\Delta^{\lambda+1} \epsilon_n| < \infty$, for $-1 \leq \lambda \leq \alpha$ and

(*iv*) $A_n^{\lambda+p} \Delta^\lambda \epsilon_n$ is of bounded variation for $0 \leq \lambda \leq \alpha$ and tends to zero as $n \to \infty$.

3. Main Result. We prove the following theorem:

Theorem 3.1. Suppose $\{a_n\}$ is a hyper semi–convex null sequence. Then the cosine series (1.1) converges in the metric space L if and only if $|a_{n-1}| \log n \to 0$ as $n \to \infty$.

If we take $\alpha = 0$, then this theorem reduces to the Theorem B of Bala R. and Ram B. [2].

Proof. We have

$$\begin{aligned} S_n(x) &= \sum_{k=1}^{n} a_k \cos kx \\ &= \frac{1}{2\sin x} \sum_{k=1}^{n} a_k \cos kx 2 \sin x \\ &= \frac{1}{2\sin x} \sum_{k=1}^{n-1} (a_{k-1} - a_{k+1}) \sin kx + a_{n-1} \frac{\sin nx}{2 \sin x} \\ &\qquad + a_n \frac{\sin(n+1)x}{2\sin x} \end{aligned}$$

$$= \frac{1}{2\sin x} \sum_{k=1}^{n-1} (\Delta a_k + \Delta a_{k-1}) \sin kx + a_{n-1} \frac{\sin nx}{2\sin x} + a_n \frac{\sin(n+1)x}{2\sin x}$$

Applying Abel's transformation, we have

$$g_n(x) = \frac{1}{2\sin x} \sum_{k=1}^{n-2} (\Delta^2 a_k + \Delta^2 a_{k-1}) \sum_{\nu=1}^{k} \sin \nu x + (\Delta a_{n-1} + \Delta a_{n-2}) \sum_{\nu=1}^{n-1} \sin \nu x$$
$$+ a_{n-1} \frac{\sin nx}{2\sin x} + a_n \frac{\sin(n+1)x}{2\sin x}$$

$$= \frac{1}{2\sin x}[\sum_{k=1}^{n-2}(\triangle^2 a_{k-1} + \triangle^2 a_k)\tilde{S}_k^0(x) + (\triangle a_{n-1} + \triangle a_{n-2})\tilde{S}_{n-1}^0(x)$$

$$+a_{n-1}\frac{\sin nx}{2\sin x} + a_n\frac{\sin(n+1)x}{2\sin x}$$

If we use Abel's transformation $\alpha + 1$ times, we have

$$S_n(x) = \frac{1}{2\sin x}\left[\sum_{k=1}^{n-(\alpha+2)}(\triangle^{\alpha+2}a_{k-1} + \triangle^{\alpha+2}a_k)\tilde{S}_k^\alpha(x) + \sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-1}\tilde{S}_{n-k-1}^k(x)\right]$$

$$+\frac{1}{2\sin x}\left[\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-2}\tilde{S}_{n-k-1}^k(x)\right]$$

$$+a_{n-1}\frac{\sin nx}{2\sin x} + a_n\frac{\sin(n+1)x}{2\sin x}.$$

$$= \frac{1}{2\sin x}\left[\sum_{k=1}^{n-(\alpha+2)}(\triangle^{\alpha+2}a_{k-1} + \triangle^{\alpha+2}a_k)\tilde{S}_k^\alpha(x) + \sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-1}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right]$$

$$+\frac{1}{2\sin x}\left[\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-2}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right] + a_{n-1}\frac{\sin nx}{2\sin x} + a_n\frac{\sin(n+1)x}{2\sin x}$$

Since $\tilde{S}_n$ and $\tilde{T}_n$ are uniformly bounded on every segment $[\varepsilon, \pi - \epsilon]$, $\varepsilon > 0$.

$$f(x) = \lim_{n\to\infty} S_n(x)$$

$$= \frac{1}{2\sin x}\left[\sum_{k=1}^{\infty}(\triangle^{\alpha+2}a_{k-1} + \triangle^{\alpha+2}a_k)\tilde{S}_k^\alpha(x)\right]$$

Thus

$$f(x) - S_n(x) = \frac{1}{2\sin x}\left[\sum_{k=n-(\alpha+1)}^{\infty}(\triangle^{\alpha+2}a_{k-1} + \triangle^{\alpha+2}a_k)\tilde{S}_k^\alpha(x)\right]$$

$$-\frac{1}{2\sin x}\left[\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-1}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right]$$

$$-\frac{1}{2\sin x}\left[\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-2}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right]$$

$$-\left(a_{n-1}\frac{\sin nx}{2\sin x} + a_n\frac{\sin(n+1)x}{2\sin x}\right)$$

$$\|f(x) - S_n(x)\| \le \int_0^\pi \left|\frac{1}{2\sin x}\left[\sum_{k=n-(\alpha+1)}^{\infty}(\triangle^{\alpha+2}a_{k-1} + \triangle^{\alpha+2}a_k)\tilde{S}_k^\alpha(x)\right]\right|dx$$

$$+\int_0^\pi\left|\frac{1}{2\sin x}\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-1}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right|dx$$

$$+\int_0^\pi\left|\frac{1}{2\sin x}\left[\sum_{k=0}^{\alpha}(\triangle^{k+1}a_{n-k-2}A_{n-k-1}^k\tilde{T}_{n-k-1}^k(x)\right]\right|dx$$

$$+\int_0^\pi\left|\left(a_{n-1}\frac{\sin nx}{2\sin x} + a_n\frac{\sin(n+1)x}{2\sin x}\right)\right|dx$$

$$\begin{aligned}
\|f(x) - S_n(x)\| &\le C\left[\sum_{k=n-(\alpha+1)}^{\infty} (\triangle^{\alpha+2} a_{k-1} + \triangle^{\alpha+2} a_k) \int_0^{\pi} \left|\tilde{S}_k^{\alpha}(x)\right| dx\right] \\
&+ C\left[\sum_{k=0}^{\alpha} A_{n-k-1}^{k} |\triangle^{k+1} a_{n-k-1}| \int_0^{\pi} \left|\tilde{T}_{n-k-1}^{k}(x)\right| dx\right] \\
&+ C\left[\sum_{k=0}^{\alpha} A_{n-k-1}^{k} |\triangle^{k+1} a_{n-k-2}| \int_0^{\pi} \left|\tilde{T}_{n-k-1}^{k}(x)\right| dx\right] \\
&+ \int_0^{\pi} \left|\left(a_{n-1}\frac{\sin nx}{2\sin x} + a_n \frac{\sin(n+1)x}{2\sin x}\right)\right| dx \\
&\le C\left[\sum_{k=n-(\alpha+1)}^{\infty} A_k^{\alpha} |(\triangle^{\alpha+2} a_{k-1} + \triangle^{\alpha+2} a_k)| \int_0^{\pi} \left|\tilde{T}_k^{\alpha}(x)\right| dx\right] \\
&+ C\left[\sum_{k=0}^{\alpha} A_{n-k}^{k} |\triangle^{k+1} a_{n-k-1}| \int_0^{\pi} \left|\tilde{T}_{n-k-1}^{k}(x)\right| dx\right] \\
&+ C\left[\sum_{k=0}^{\alpha} A_{n-k}^{\alpha} |\triangle^{k} a_{n-k-2}| \int_0^{\pi} \left|\tilde{T}_{n-k-1}^{k}(x)\right| dx\right] \\
&+ \int_0^{\pi} \left|\left(a_{n-1}\frac{\sin nx}{2\sin x} + a_n \frac{\sin(n+1)x}{2\sin x}\right)\right| dx \\
&\le C_1 \sum_{k=n-(\alpha+1)}^{\infty} A_k^{\alpha+1} |(\triangle^{\alpha+2} a_{k-1} + \triangle^{\alpha+2} a_k)| \\
&+ C_1 \sum_{k=0}^{\alpha} A_{n-k+1}^{k} |\triangle^{k+1} a_{n-k-1}| \\
&+ C_1 \sum_{k=0}^{\alpha} A_{n-k+1}^{\alpha} |\triangle^{k} a_{n-k-2}| \\
&+ O(a_{n-1}\log n) \qquad (C_1 \text{ is an absolute constant})
\end{aligned} \tag{3.1}$$

The first three terms of the above inequality are of $o(1)$ by the Lemma and the hypothesis of theorem.

Because,

$$\begin{aligned}
&\int_0^{\pi} \left|\left(a_{n-1}\frac{\sin nx}{2\sin x} + a_n \frac{\sin(n+1)x}{2\sin x}\right)\right| dx \\
&\le |a_{n-1}| \int_0^{\pi} |D_n(x)| dx \\
&= O(a_{n-1}\log n) \qquad \text{as } \int_0^{\pi} |D_n(x)| dx \sim \log n.
\end{aligned}$$

$\int_0^{\pi} |f(x) - S_n(x)| dx \to 0$ if and only if $|a_{n-1}| \log n \to 0$ as $n \to \infty$.

This completes the proof of the theorem.

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Spatial Synthesis: A Research Program

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"Spatial Analysis" is a term in current use in a variety of disciplines: from geography to regional analysis, to economics, to anthropology, to (no doubt) a host of others [1]. Merriam-Webster's online Collegiate Dictionary defines "analysis" as a "separation of a whole into its component parts" [2]. Often, however, one wishes to consider not only separation but also composition: the composition of the whole from a set of parts. Thus, the same source defines "synthesis" as "the composition or combination of parts or elements so as to form a whole."

We take the occasion of this Winter Solstice issue to invite the world at large to come together and offer a synthesis of ideas involving spatial concepts and theories, as a part of a broadly-based research program. Volume I of this work will concern the concept of spatial hierarchy. Book 1 (by the authors of this article) of Volume I, to appear, is entitled Centrality and Hierarchy: Regular Lattices, Geometry, and Number Theory. Other topics, that we might foresee, involve more books on Centrality and Hierarchy within Volume I as well as Volumes on topics such as (but not limited to): Distance and Geodesic; Adjacency and Connection; Minimax, Absolute/relative, and Density; Scale, Orientation, and Dimension; Partition, Separation, and Diffusion; and, Transformation and Symmetry.

The authors of this article would assemble, edit, obtain reviews, and work to obtain a publisher for a series of eBooks entitled "Spatial Synthesis." In doing so, they would draw on their recent experience in publishing an eBook, and in developing websites, to make the final product one that employs a variety of interactive tools for communicating information on the internet [3, 4]. Issues involving agreements concerning publication would be dealt with at the outset according to the format of the publisher. If you would like to submit an idea for preliminary review, for suitability for inclusion, or if you would like to suggest yet other directions for this synthesis of spatial concepts and theories, please feel free to e-mail us or send e-mail attachments to: sarhaus@umich.edu. We wish to have this work be synthetic: from its method of creation through its content formulation. Please consider joining this venture in spatial synthesis.

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