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by: Sandra Lach Arlinghaus, Ph.D.

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SANDRA LACH ARLINGHAUS, Ph.D.

Institute of Mathematical Geography

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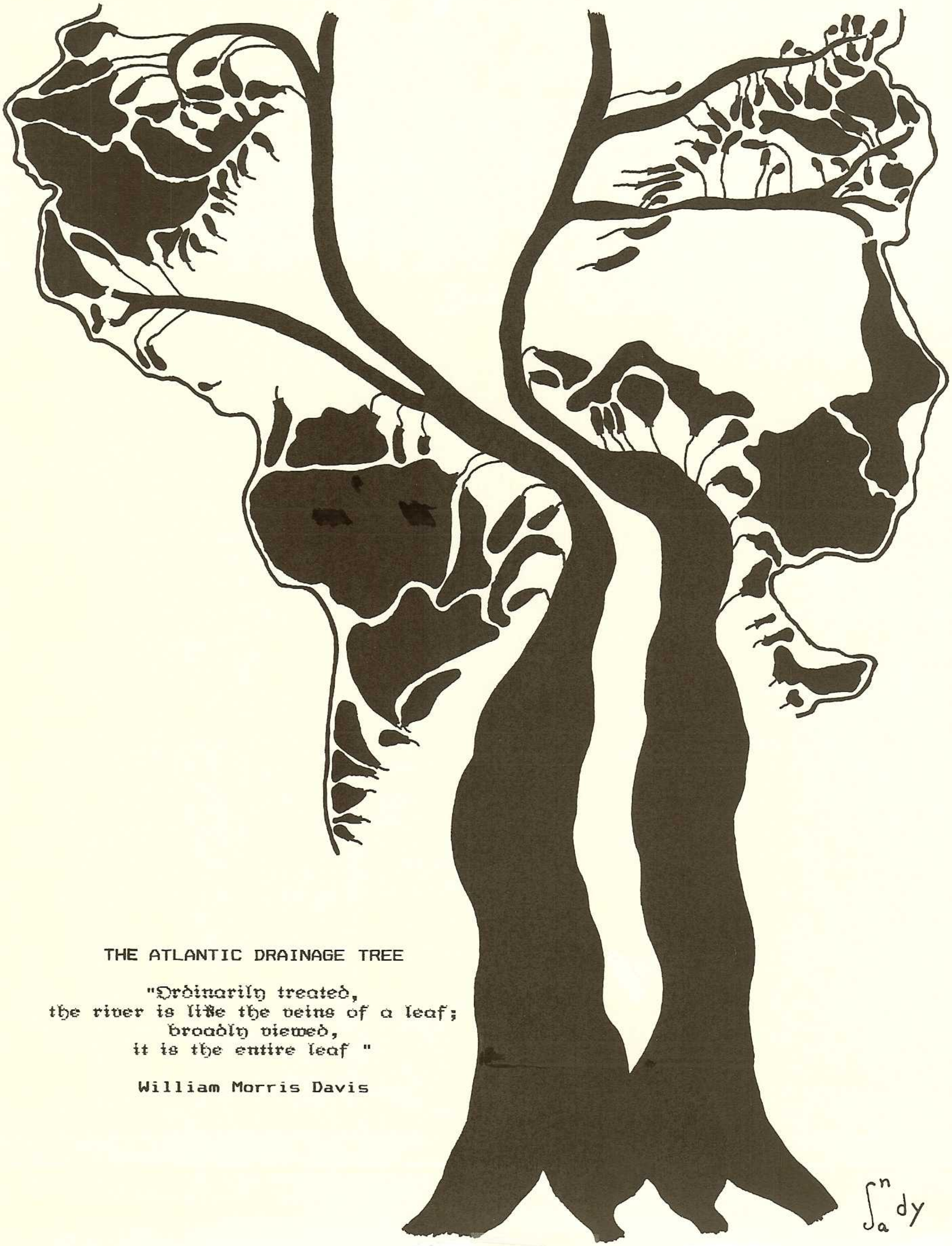
To my husband,

William C. Arlinghaus

with deep appreciation for his continuing encouragement
on these and on many other projects.



THE ATLANTIC DRAINAGE TREE

> "Ordinarily treated,
> the river is like the veins of a leaf;
> broadly viewed,
> it is the entire leaf "

William Morris Davis

ACKNOWLEDGMENT

The author wishes to thank William E. Arlinghaus, Robert F. Austin, H. S. M. Coxeter, Harvey Fleet, Frank Harary, Donald F. Lach, John D. Nystuen, and Sylvia L. Thrupp, for their generosity with time and commentary on, or related to, various essays in this book. Specific acknowledgment follows each essay. Errors that remain are, of course, those of the author alone.

CONTENTS

GETTING A HANDEL ON WATER-GRAPHS*

INTRODUCTION

The problem of how to secure the infrastructure of the Detroit metropolitan region, including the water distribution network, from deliberate abuse or from extensive, accidental, service disruption led to an analysis of that system based, in part, in graph theory. The Detroit water distribution network is comprised of water pipes of varying diameter that link five water plants, which draw water from the Detroit River and from Lake Huron, to households and industries scattered across a five county area of southeastern Michigan [3]. Within the primary distribution network (formed from tubing exclusive of that which leads directly into individual buildings), this network includes many miles of water pipes hooked together at 727 distinct pipe junctions below the streets. When these junctions are viewed as a vertex set, and the piping incident with them as an edge set, this network may be represented conveniently as a graph [5]. The example below illustrates, using an hypothetical network, how viewing the complicated water supply network as a graph revealed the simplicity of its structural components, and it suggests in addition other networks to which such an analysis might be applied.

WATER-GRAPHS

Water pipes of width sufficient (54" in diameter (or more) in the hypothetical example) to carry a volume of water large enough that the system apparently flushes itself of dangerous levels of contaminants will be viewed to comprise a trunk-line of tubing for the distribution network. The diversion of the flow through a prescribed routing pattern is difficult within the trunk-line piping because the valves that guide the course of the flow through such wide pipes require heavy machinery to turn the screw-mechanisms [3]. Typically, such trunk-line tubing leads from water plants into smaller pipe branches more remote, in the network topology, from the water plant. Population close to these large lines are more certain to have access to a continuing source of fresh water, in time of disaster, than are their topologically remote neighbors.

The graph in Figure 1 shows a graph of an hypothetical water distribution network; unlabelled vertices represent pipe junctions, edges of regular width represent water pipes linking these junctions, heavy edges represent the more secure trunk-line, and vertices $v_1,...,v_8$ represent junctions of trunk-line pipes. When access to fresh water is denied at all 8 labelled vertices, 10 fragments are formed in which water deprivation would arise downstream from the trunk-line. (The "fixed" subgraph of heavy edges splits the network into 10 distinct network fragments labelled 1,...,10 in Figure 1.a.) Because this subgraph has a circuit in it [5], one suspects that a subgraph of fewer edges and vertices might force the same degree of fragmentation in the network (Figures 1.b and 1.c).

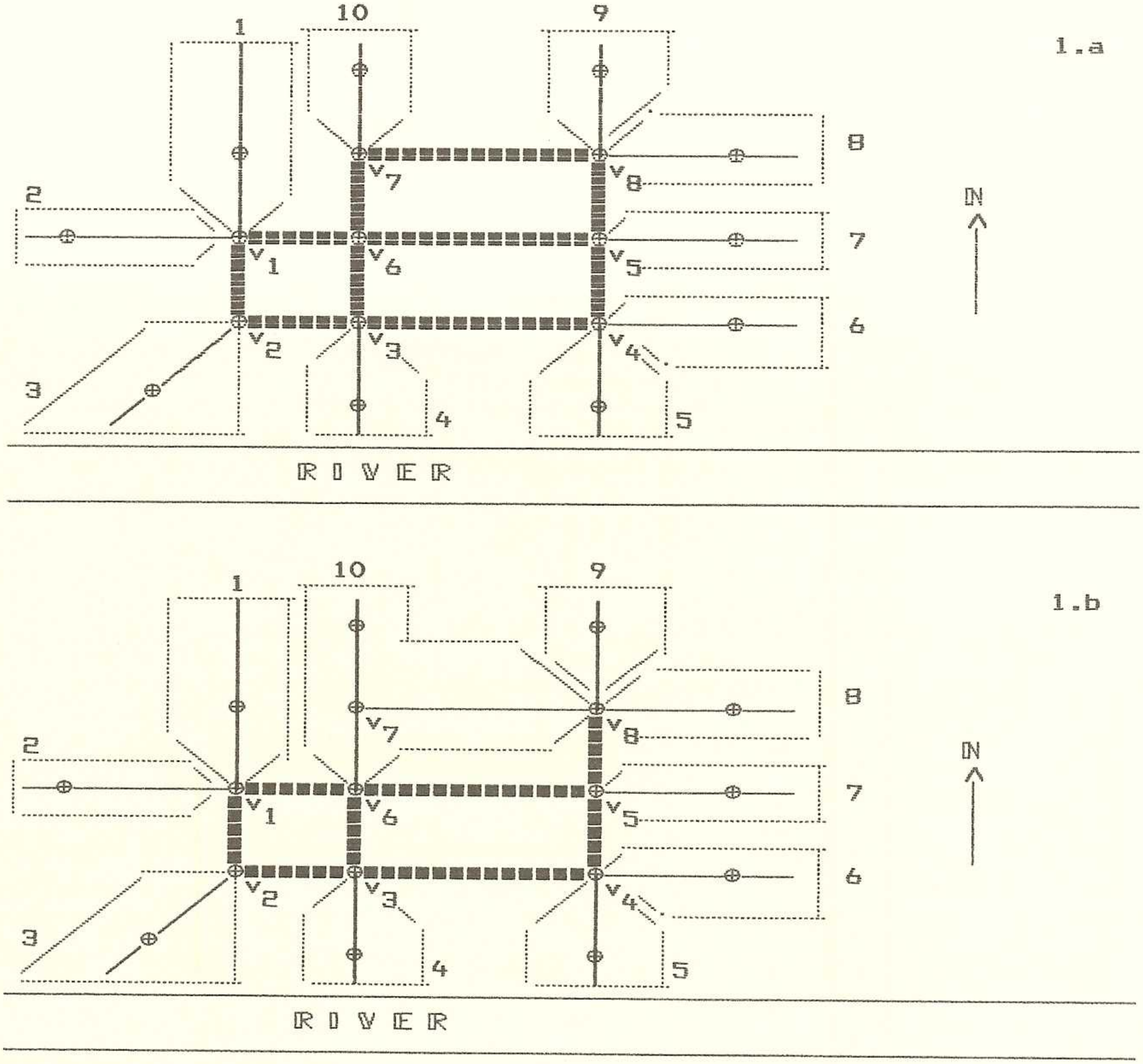
Definition 1

A heart of a network is a fixed subgraph J, with a minimal number of vertices, that splits the entire system into the largest possible number of fragments.

As suggested in the remarks above, the fixed subgraph in Figure 1.a obviously forces maximal fragmentation for this case. The subgraph J based on the vertices v_1, v_2, v_4 v_5, v_6, v_8, which has no circuits, also forces the network to split into ten fragments

FIGURE 1

WATER--GRAPHS

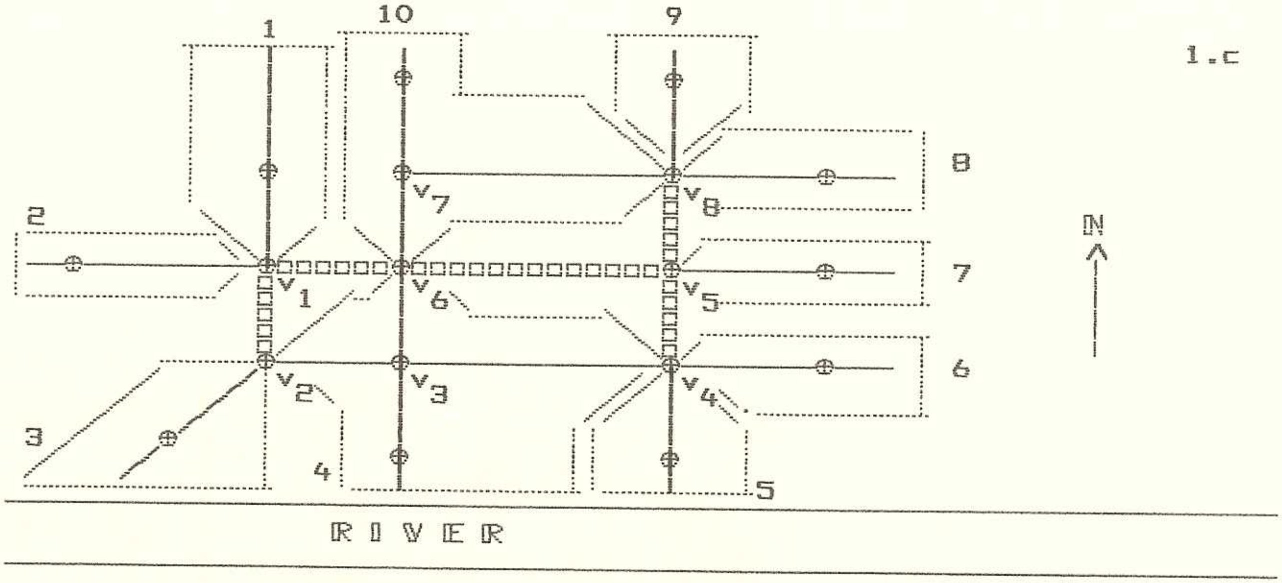


⊕	Vertex: water-pipe junction
⊕■■■■■■■⊕	Fixed subgraph.
⊕□□□□□□□⊕	Network heart.
⊕*******⊕	Heart by-pass.
⊕ 1	Numbered fragment. (Space between fragments is for graphic clarity.)
⊕v_1	Junction between water pipes of diameter 54" or more.

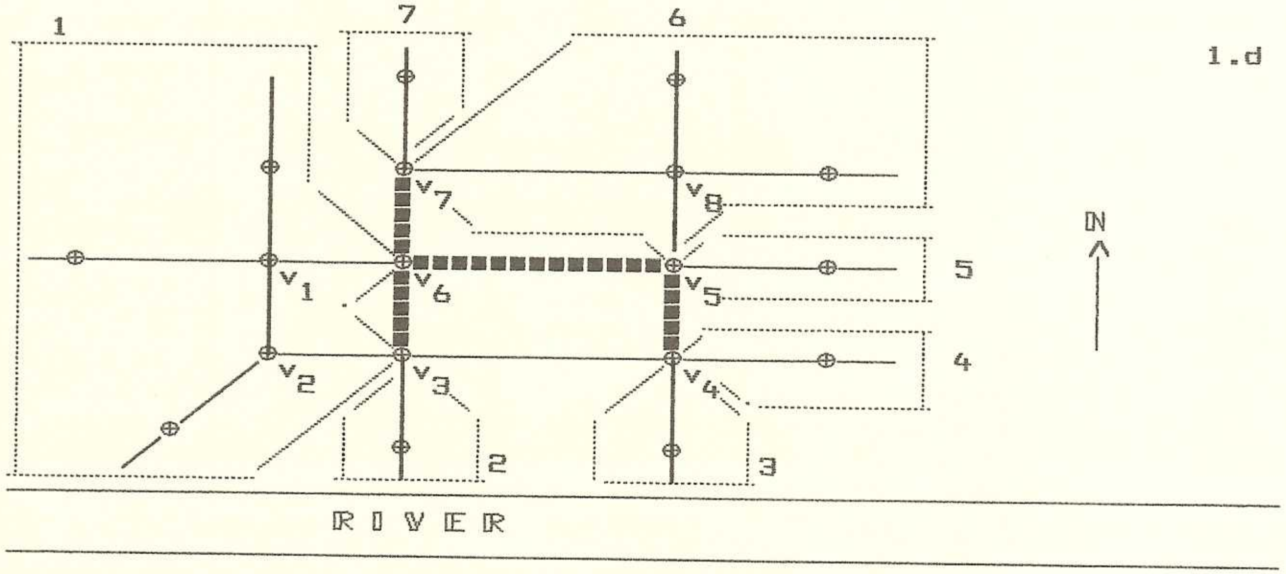
(Figure 1.c) not necessarily identical to the fragments in Figure 1.a. The fixed subgraph of Figure 1.d has a smaller number of vertices than does that in Figure 1.c, but it does not force maximal fragmentation. In this case, it is easy to see by exhaustion that the subgraph in Figure 1.c is the heart of this network; there is no graph of fewer edges, linking some subset of vertices of $\{v_1, \ldots, v_8\}$, which forces maximal fragmentation of the entire graph into ten distinct fragments.

In the much more complicated Detroit Water Supply Network, the use of this idea permitted rapid determination of a fixed subgraph based on about 100, of the 727, critical pipe junctions which forced the entire network to fall apart into about a dozen distinct fragments. A subgraph of this fixed subgraph, based on around 30 vertices served as the network heart. When population dot maps, showing the geographic positions of residential and industrial populations were superimposed on the fragments, a quick estimate of the potential disaster to these populations, coming from water deprivation or contamination, was possible, and has been supplied to, and used by, the Detroit Water and Sewerage Department [2; 7].

Identification of key fragments also provided means to determine geographic positions for by-pass routes to surround the heart. To remove fragmentation downstream from the heart, a by-pass link had to pass through each fragment across a location where there was already a pipe junction (vertex) present. Simple calculations of percentage improvement in transmission reliability, using multiple edges in the by-pass, together with a ranking of percentages for these key fragments, served as a step fundamental to



1.c



1.d

⊕	Vertex: water-pipe junction
⊕■■■■■■⊕	Fixed subgraph.
⊕□□□□□□□⊕	Network heart.
⊕*******⊕	Heart by-pass.
⊕┄┄┄┄ 1	Numbered fragment. (Space between fragments is for graphic clarity.)
$\oplus v_1$	Junction between water pipes of diameter 54" or more.

the development of an open-ended "Richter"-type logarithmic scale to measure the extent of potential disaster [4].

Parallel real-world scenarios (1) in the hauling of wastes and toxic substances across freeway junctions linked by superhighways, (2) in the terrorizing, by political or street-gang criminals, of fragments of the population along elevated railroad lines, or (3) in analyzing biological issues pertaining to human circulatory systems, are not difficult to imagine. Therefore, what is most important is to return to the theory so that its application to diverse settings, representing problems of significance to society at large, is facilitated. (Some of these applications will be examined in the next essay.)

Definition 2 [8, p. 6]

Let H be a subgraph of a graph G. A vertex of attachment of H in G is a vertex of H that is linked to a vertex of G that is not a vertex of H.

In Figure 1.c the subgraph H, forming Fragment 10, has vertices of attachment at v_6 and at v_8, since each of these is linked to a vertex of G not in Fragment 10; v_7 is not a vertex of attachment of H in G as it is adjacent only to other vertices of H.

Definition 3 [8, p. 9]

Given a fixed subgraph J of G. Another subgraph H of G is detached modulo J if every vertex of attachment of H in G is also a vertex of J.

Suppose the network heart represented in Figure 1.c is J. Then the subgraph H of fragment 10 is detached modulo J, since each of its vertices of attachment, v_6 and v_8, is also a vertex of the subgraph

J. However, the subgraph composed of v_7, the unlabelled vertex adjacent to it on the north, and the corresponding edge is not detached modulo J since neither vertex, attaching H to G, is a vertex of J.

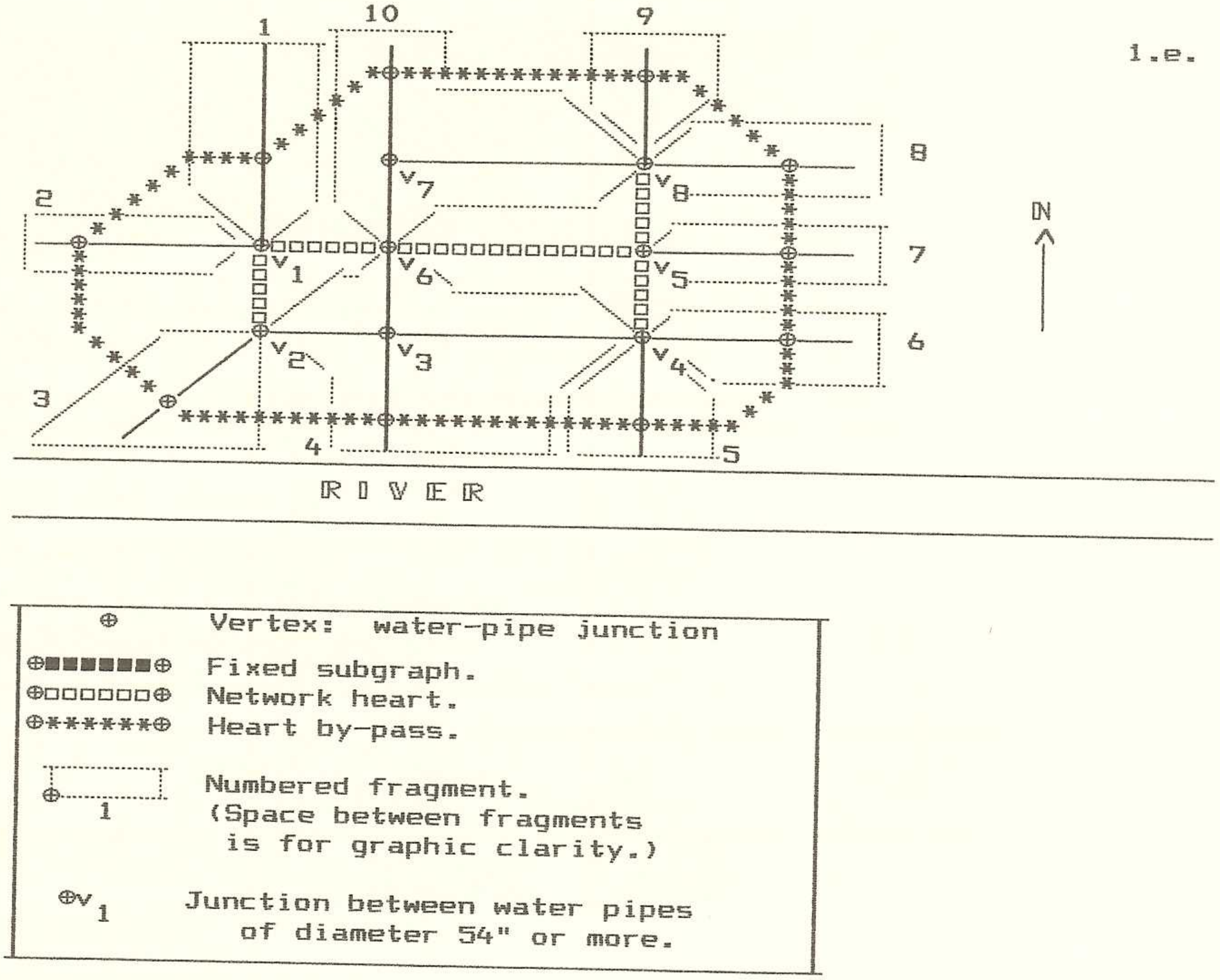
When these definitions are applied to water-graphs it is evident that the set of network fragments that arises downstream from the heart, J, of a water-graph (as a result of disruption in service at the vertices of J), is in one-to-one correspondence with the set of subgraphs, in the graphical network, that are detached modulo J. This observation provided a systematic means for determining the set of network fragments relative to any heart and a handle on analyzing, rapidly, the complicated Detroit water-graph.

<u>Definition 4</u> [8, p. 10]

A graph G is J-connected if it has no (non-trivial) subgraphs detached modulo J.

In Figure 1.c there are ten non-trivial subgraphs detached modulo J. When the heart is supplemented by a peripheral by-pass, C,pumping in fresh water from remote positions (Figure 1.e), the graph $(G \cup C)$ becomes $(J \cup C)$-connected. That is, there are no fragments of $(G \cup C)$ which can be detached from $(J \cup C)$. The only sort of subgraph for which this might not be evident is for one like the subgraph, H, comprised of v_7, the unlabelled vertex adjacent to it on the north, and the corresponding edge. By Definition 3, H is detached modulo (J u C) if every vertex of attachment of H in $(G \cup C)$ is also a vertex of $(J \cup C)$ By Definition 2, v_7 is a vertex of attachment of H in $(G \cup C)$ because v_7 is linked to a vertex of $(G \cup C)$ that is not a vertex of H (namely, v_8). Thus, v_7 is a vertex of attachment of H in $(G \cup C)$ that is not a vertex of $(J \cup C)$. Therefore, by Definition 3, H is not detached modulo $(J \cup C)$. Finally, Definition 4 ensures that $(G \cup C)$ is $(J \cup C)$-connected. From a practical standpoint, this suggests that more of the underlying population has increased security of access to fresh water when the water supply network is connected modulo a fixed subgraph.

1.e.



Definition 5

A circuit, C, introduced into a water-graph G with heart J, is said to by-pass the heart effectively if there are no fragments in $(G \cup C)$ which can be detached modulo the subgraph $(J \cup C)$.

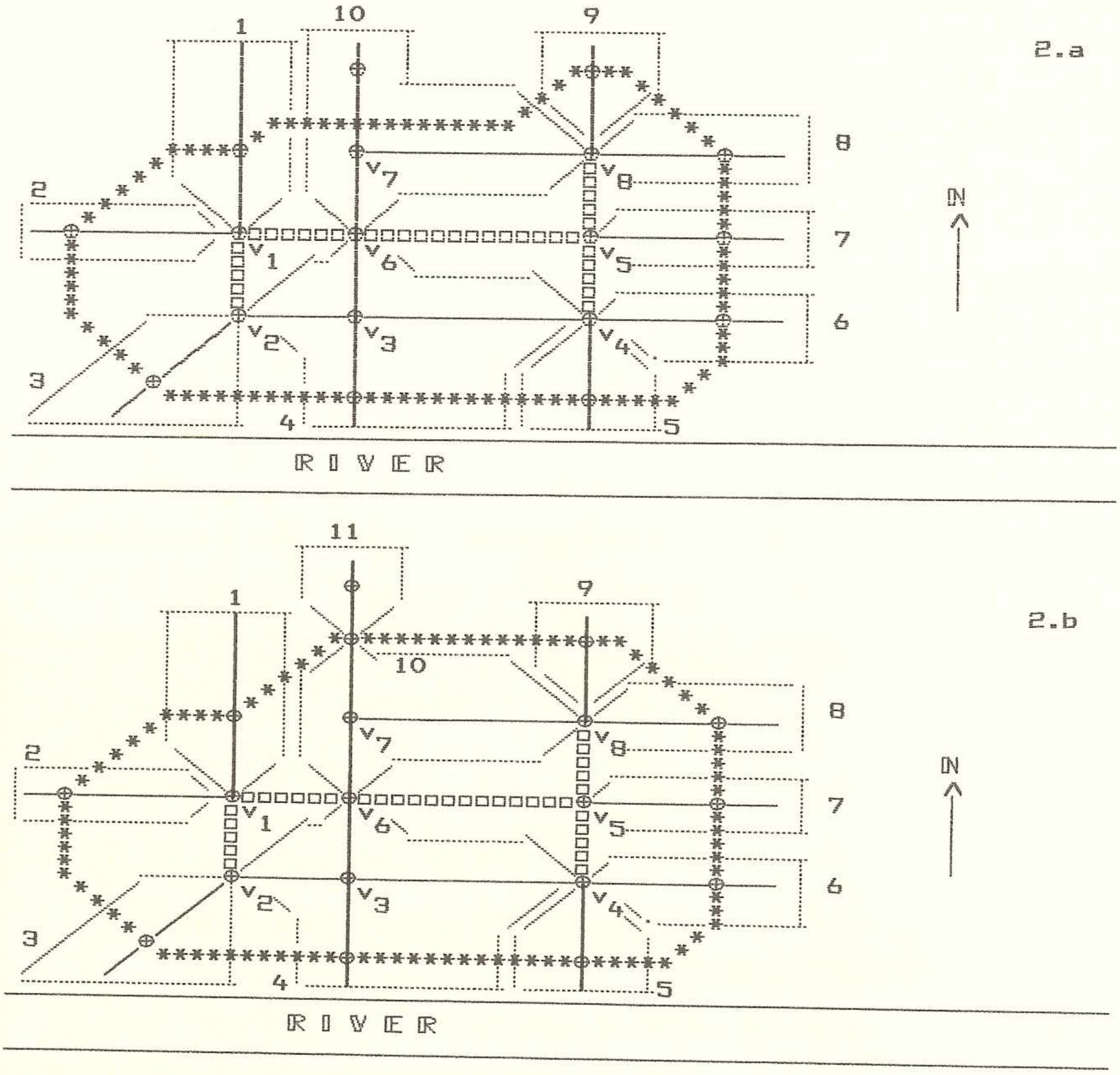
This Definition holds as an "if and only if" statement in cases where the following statement holds: if the edges added in C do not force $(G \cup C)$ to become $(J \cup C)$-connected, then C is not an effective heart by-pass. In terms of the example in Figure 1,

i) suppose the circuit, C, is not incident with a vertex in every fragment (in Figure 2.a it does not pass through any pipe-junction in Fragment 10). Any fragment through which C does not pass is detached modulo J. Because such a fragment is also detached modulo $(J \cup C)$ in $(G \cup C)$, C is therefore not an effective by-pass for J, by Definition 5. (In Figure 2.a, Fragment 10 is "left out.")

ii) Suppose the circuit C is incident with a vertex in every fragment (Figure 2.b). To force $(G \cup C)$ to be not connected modulo $(J \cup C)$, there must remain a fragment in $(G \cup C)$ which is detached from $(J \cup C)$ (Fragment 11 in Figure 2.b). Thus, by Definition 5, the circuit C would not serve as an effective by-pass.

This Definition, derived from Tutte's notion of a vertex of attachment, served as the basis in the Detroit study for determining specific percentage improvements in water transmission to particular fragments resulting from the construction of various by-pass links. These percentages were used constructively by the Detroit Department of Water and Sewerage to demonstrate to the Detroit City Council the need for by-pass tubing to increase redundancy in the water supply network.

FIGURE 2



⊕	Vertex: water-pipe junction
⊕■■■■■■■⊕	Fixed subgraph.
⊕□□□□□□□⊕	Network heart.
⊕*******⊕	Heart by-pass.
⊕……… 1	Numbered fragment. (Space between fragments is for graphic clarity.)
$\oplus v_1$	Junction between water pipes of diameter 54" or more.

QUESTIONS USEFUL IN GUIDING FURTHER APPLICATIONS

Related graph-theoretical questions, whose solutions would appear useful in guiding further applications in this or in a similar context, include:

1) finding conditions under which network hearts are, or are not, unique.

2) finding existence and uniqueness conditions under which a circuit serves as an effective by-pass to an arbitrary network-heart.

3) proving minimality conditions involving the number of vertices in a network heart.

4) determining whether or not the network heart is necessarily a tree, and under what conditions it is or is not a tree [1].

Tactical ideas involved in the real-world settings described above have endured throughout history: from the Graeco-Roman phalanx to the admonition of the British geographer and historian, Halford J. Mackinder [6], that

> "Who rules East Europe commands the Heartland;
> Who rules the Heartland commands the World Island;
> Who rules the World Island commands the World."

What is different here is the use of vertices of attachment and J-connection to investigate these tactical locations.

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Entries in references 2, 4, and 7 also appear in:

Snyder, James C. and Sandra Arlinghaus, Michael Brabec, Rolf Deininger, Allan Feldt, Ronald Inglehart, Peter Meier, John Nystuen, Mitchell Rycus. "Detroit Water And Sewerage Department Pollution and Detection Abatement: Preliminary Report." With appendices. (Ann Arbor, Michigan: Studies In Urban Security Group, College of Architecture and Urban Planning, University of Michigan.

and in,

Snyder, James C. and Sandra Arlinghaus, Michael Brabec, Rolf Deininger, Lynda Duke, Allan Feldt, Ronald Inglehart, Thomas Lyons, Peter Meier, John Nystuen, James Segedy, Mitchell Rycus. "Security Of The Water System, Final Report Tasks 1, 2, 3." (Ann Arbor, Michigan: Studies In Urban Planning, University of Michigan, 1986).

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TERROR IN TRANSIT:*

A GRAPH THEORETIC APPROACH TO THE PASSIVE DEFENSE OF URBAN NETWORKS.

INTRODUCTION

Terrorist activities frequently involve actions regarded as sensational within the culture of the reporting news media. All too often, however, one thinks of terrorist activity as an outrageous atrocity committed far beyond North American borders--borders which might be perceived as never having been penetrated by an invading army using tanks and conventional weapons let alone by a "sneaky" tactic designed to obtain press coverage to use as leverage in gaining some political or cultural advantage. Yet terrorist activities are present in our society, although frequently such actions are dismissed, complacently, as "different" from those which occur in nations not on our continent. The lessons of history suggest that such continental isolationism, in viewpoint, is at least as dangerous in today's "shrinking" world as it was in the "larger" world prior to World War I.

Indeed, the random poisoning of headache remedies in supermarkets is no less a terrorist activity than is the hijacking of an airplane; both affect, directly, only a few individuals while striking fear into the hearts of large numbers of potential users of the system (marketing or transportation). Further, both offer extortion as an opportunity to the terrorists to extract support for their causes by issuing threats, be these threats economic or corporal.

In what follows, a number of hypothetical scenarios, which are probably as likely to occur in the mind of a science-fiction writer as in the mind of a potential terrorist, are examined to see how a graph-theoretic technique which is already in use might be applied

1) to yield insight into the scope of the potential for disaster; and,

2) to suggest ways to relieve this potential.

Because multi-layered systems appear to be vulnerable to such attack, particularly where a shift from one level to another occurs, transit systems of both a conventional nature (those transporting people) as well as of an institutional nature (those conveying information) will be focused upon as the source for this set of scenarios.

BRIEF DESCRIPTION OF GRAPH-THEORETIC TECHNIQUE

Any network may be characterized as a graph, G. The set of vertices, V(G), of the graph, G, represents a set of origins and destinations within the real-world system, and the set of edges, E(G), represents channeled flow linking destinations (independent of the nature of that flow). (Convention in the use of the word "graph" follows Harary [8].) Some systems have more redundancy in linkage pattern than do others; consequently, one may view destinations in networks as serving different sorts of functions within the underlying graph-theoretical structure of the network. For example, in the graph representing a network that is comprised of a single circuit (Figure 1.a), every vertex v_i has the following property: the removal of vertex v_i would not prevent access through the system from any one of the remaining vertices to any other (Figure 1.B).

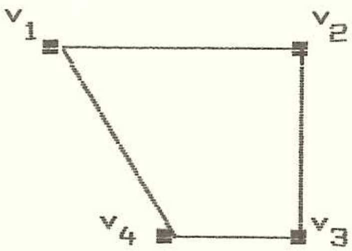


Figure 1.a

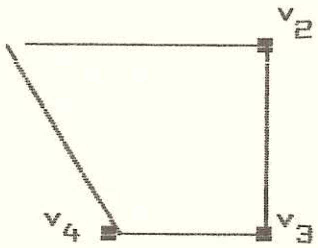


Figure 1.b

In more complicated networks, however, the corresponding graph-theoretic structure may be such that the removal of certain vertices forces the graph to separate into a number of fragments.

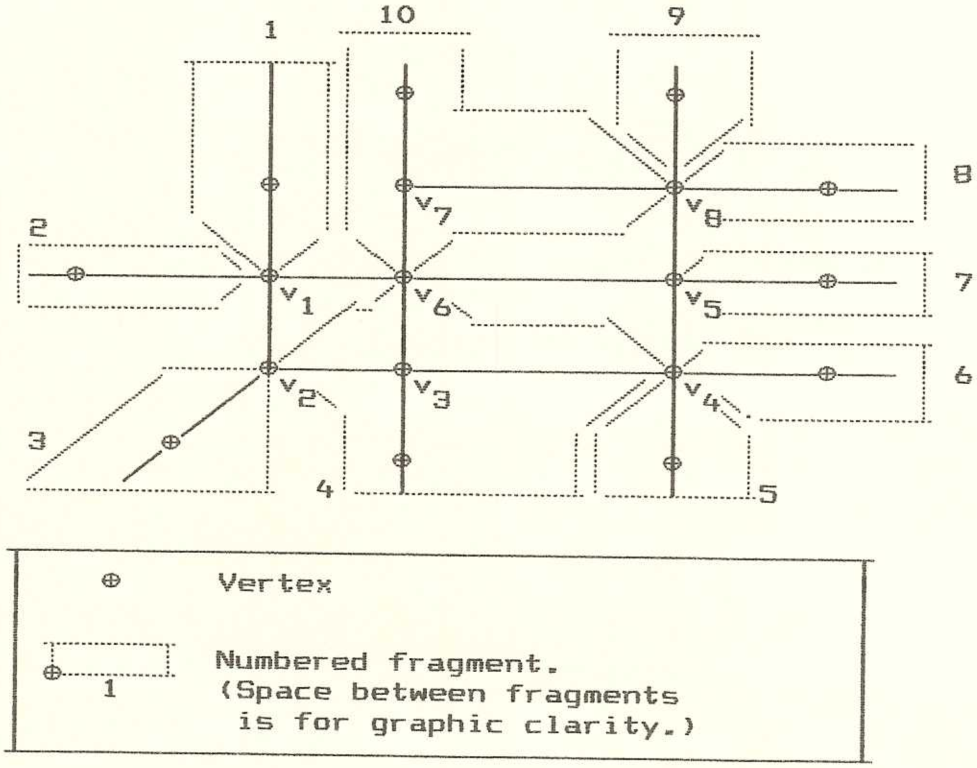


Figure 2

In Figure 2, the removal of vertices v_1, v_2, v_4, v_5, v_6, and v_8, forces the network to split into 10 distinct fragments hanging from a central tree-like structure. The tree of fewest edges which forces maximal fragmentation of the entire graph, is called the network heart (Figure 3). A set of edges which reconnects the fragments is called a by-pass (Figure 3). (For a more detailed description of the procedure, see the previous essay.) The network used as a base in Figures 2 and 3 is derived from the pattern of the expressway system surrounding central Detroit [2].

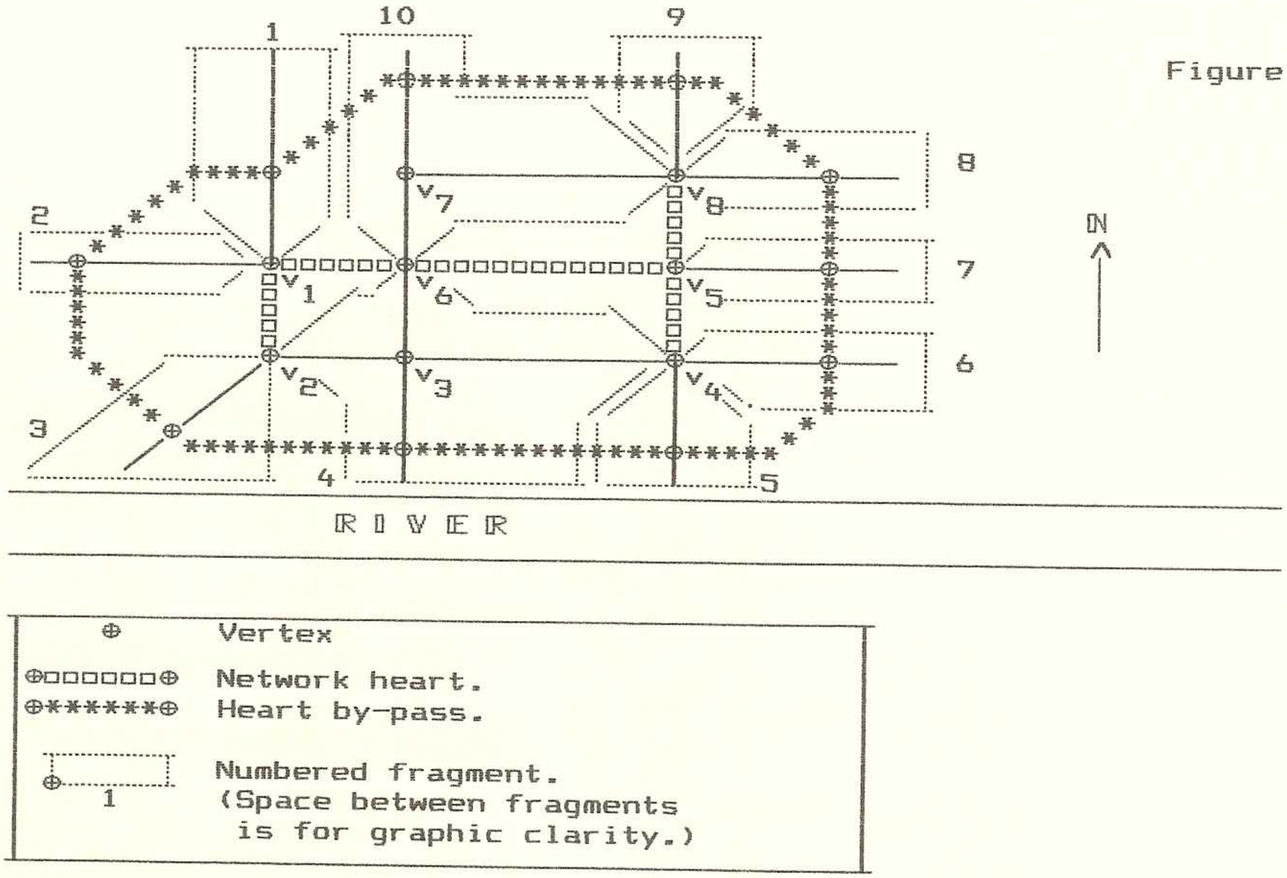


Figure 3

SCENARIO 1: THE EXPRESSWAY SYSTEM OF METROPOLITAN DETROIT

Identification of the heart and of associated fragments of the metropolitan Detroit expressway system might be important in the event that

1) the evacuation of population in a region surrounding the expressway is required, following a crash on the expressway

involving a truck transporting toxic wastes, flammable petroleum products, or nuclear medicine by-products.

2) the evacuation of population is required, following an explosion in a freight train transporting industrial chemicals.

3) the evacuation of population is required, following an airplane crash involving segments of an expressway [7].

The intent is not to suggest that local authoritites discard evacuation plans already in place, but rather to suggest that this graph-theoretical tool might provide additional insight into difficult tactical maneuvers. Nor is it the intent to suggest that the many graph-theoretical algorithms for finding "centers" of supply subnetworks are inadequate to the task [8, 12]; this is simply one approach.

When the graph in Figure 3 is used in conjunction with the dot maps in Figures 4 and 5 (source: [13]), a quick estimate of potentially affected population may be determined for both the residential population as well as for the working population [4]. For example, suppose that an accident occurs at the expressway interchange marked v_5, and that it involves a truck carrying a substance which is toxic when airborne. The accident is severe enough to close the interchange and to force a traffic backup for several miles across the expressway network-heart. The presence of the toxic substance, together with the advance of a low pressure cell from the southwest and the consequent increase in wind speed, dictates evacuation of Fragment 7. One escape route is to move across the expressway in Fragment 7; however, trying to outrun the leading edge of the wind-driven toxic plume would likely not work. Escape to the northwest, orthogonal to the direction of the wind,

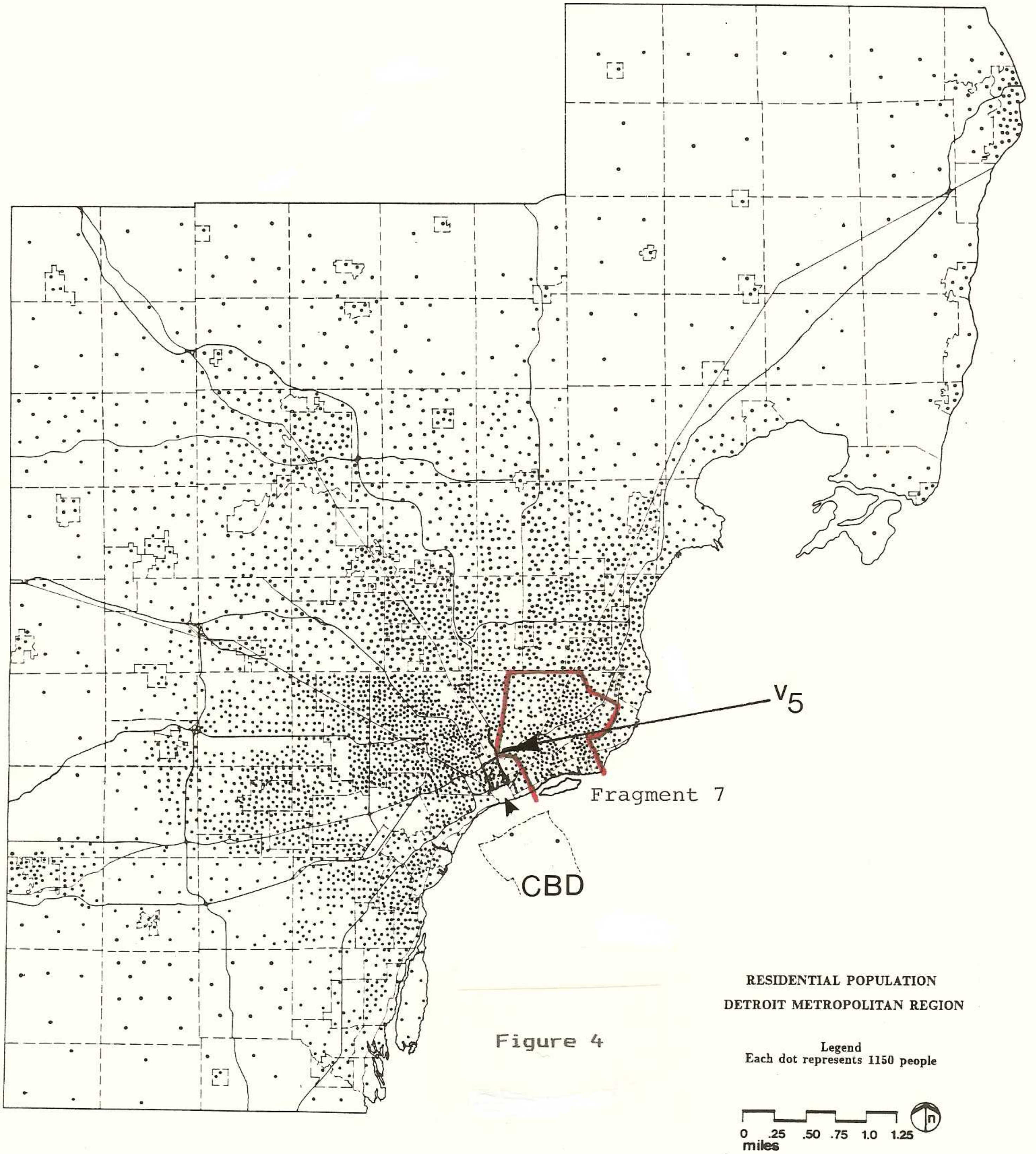
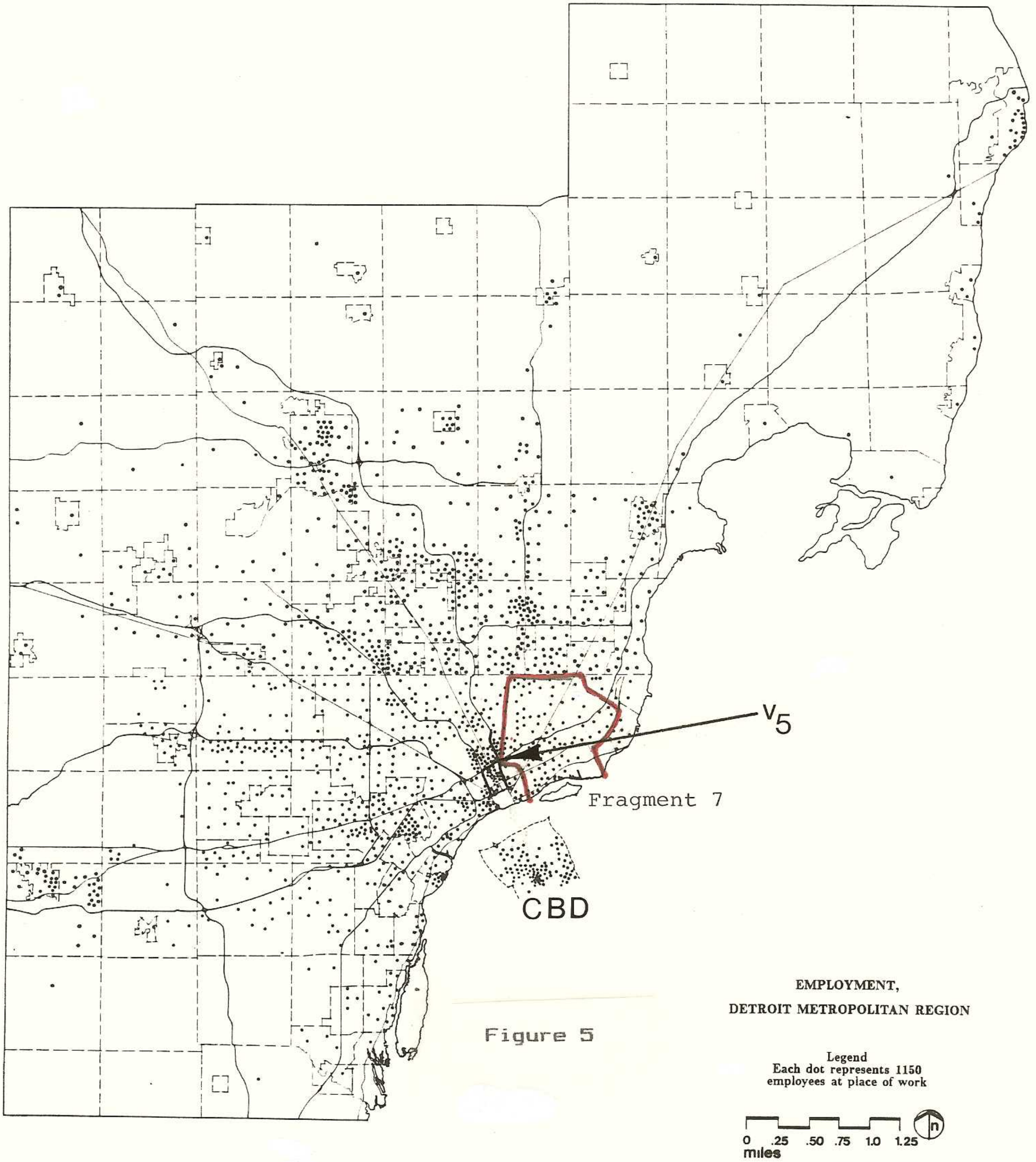


Figure 4

RESIDENTIAL POPULATION
DETROIT METROPOLITAN REGION

Legend
Each dot represents 1150 people

Source: SEMCOG, 1980



EMPLOYMENT,
DETROIT METROPOLITAN REGION

Legend
Each dot represents 1150
employees at place of work

Source: SEMCOG, 1980

Figure 5

appears best. However, because the heart is blocked, quick escape by car across the expressways is not possible for the 345,000 residential (Figure 4) and 121,900 employed (Figure 5) people in this fragment. Here, travel across a by-pass joining fragments would facilitate escape. The starred line in Figure 3 shows the shortest circuit using major surface arteries that links all the fragments; the positions for these correspond to actual streets.

Index of Reliability

As suggested above, one passive means of controlling maximal fragmentation is to construct by-passes beyond the heart that would link fragments at the periphery. The effect is to reduce the number of fragments created through detachment from the heart; by-pass links increase the redundancy of connection of the system by providing extra vertices that need to be disrupted in order to isolate a fragment. They also provide alternate routes for moving population from a fragment or for introducing aid and supplies into a fragment.

A simple index of reliability can be used to characterize the redundancy of connection from each fragment to the heart. The index is defined as the percent connection of the fragment to the heart when one heart-to-fragment node is removed (of course, iteration gives a set of successive measures). For example, Fragment 10 is connected to the heart at the two nodes v_6 and v_8 (Figure 3). If one of these vertices is removed the connection of Fragment 10 to the heart is 50% ((2-1)/2 x 100 = 50%) of what it was. In contrast, Fragment 4 has three heart-to-fragment nodes at v_2, v_4, and v_6; the loss of one of these vertices would result in ((3-1)/12 x 100 =

16.7%) a 16.7% reduction in transmission capability across Fragment 4 (Figure 3).

Conversely, inserting a strategic link between fragments would increase the redundancy of connection. Then percent increase in degree of connectivity may be evaluated using the same Index of Reliability. The percent improvement in reliability may then be used, along with other criteria such as the size of the residential or employed population served by a fragment, as a basis for decisions regarding investments that add redundancy to the system for security purposes.

Planning Potential

Designation of this by-pass route as a high priority emergency route, much as a snow-emergency route, would add increased network redundancy in the expressway system. This might be coupled with other passive defense measures for this network such as focusing limited resources for security on the locations associated with the heart-vertices from which fragements might be easily detached.

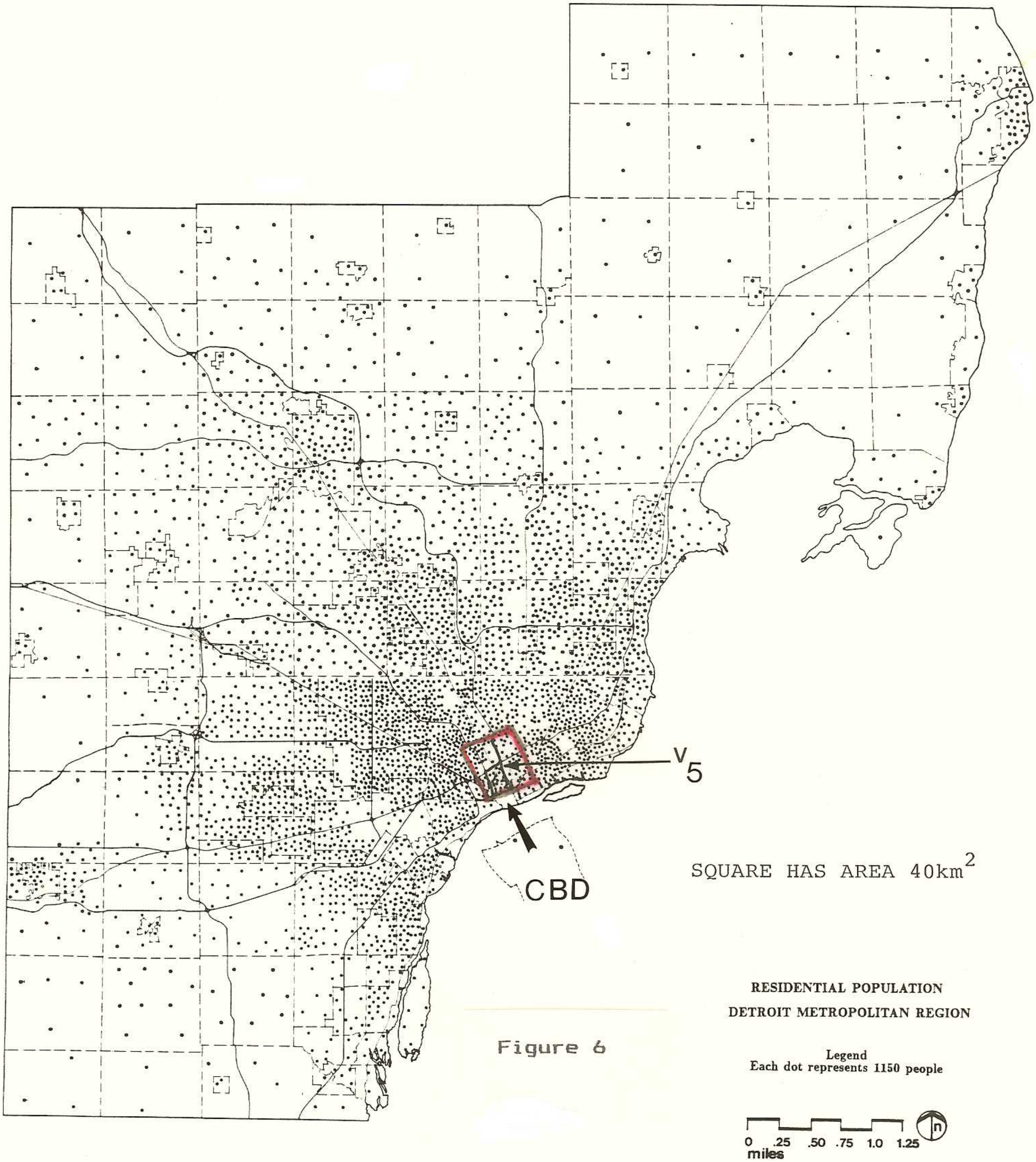
SCENARIO 2: FREIGHT TRAIN DERAILMENT

For the second scenario, actual data, drawn from a 1979 freight train derailment in an industrial section of Mississauga, Ontario, provides quantitative evidence on which to base an hypothetical parallel in Detroit. In mid-1979, following derailment of a Canadian Pacific freight train carrying chemicals, the explosions from tank cars carrying propane gas and the risk of leaks from nearby tank cars loaded with chlorine, forced the evacuation of about 250,000 people from a 40 square-kilometer area surrounding the accident site. The five-day evacuation (November 10-15), together with clean-up, cost about $25 million per day in lost business and wages [5, 10].

General Motors "Poletown" plant, which is to manufacture luxury cars, is located just to the northeast of v_5 in Figure 2. Much of the expressway interchange at v_5 is suspended over commercial railroad tracks. Thus, one could imagine the derailment of a freight train, carrying industrial chemicals such as propane gas and chlorine, propelling a fireball high into the air and forcing the closing of the expressway interchange at v_5. Figures 6 and 7 show the 40 km^2 window centered on this site. A count of dots in the maps would suggest the need to evacuate about 134,550 people at their places of work and about 119,600 people at their residences. A look at a Detroit street map shows that this area contains headquarters for General Motors and Burroughs Corporations, Wayne State University, Henry Ford Hospital, Kiefer Hospital, and the Detroit Medical Center, including Wayne State Clinics, Detroit General Hospital, Children's Hospital of Michigan, Harper Hospital, Grace Hospital, and Hutzel Hospital [2]. Casual site observation shows densely populated regions surrounding Wayne State University, as well as a number of nursing homes lining a surface route parallel to an East/West expressway segment in the network heart. The shortest-circuit by-pass in Figure 3 would aid in providing extra looping; however, the eastern edge of the by-pass would fall into the 40 km^2 window, and would point to the obvious need for more than one by-pass linking expressway fragments.

SCENARIO 3: ELEVATED TRAINS AND SUBWAYS

Elevated trains and subways, whether referred to locally as the "El," the "People Mover," or the "Metro," are such that access to the system is possible only at well-marked locations, isolated from



RESIDENTIAL POPULATION
DETROIT METROPOLITAN REGION

Legend
Each dot represents 1150 people

Source: SEMCOG, 1980

Figure 6

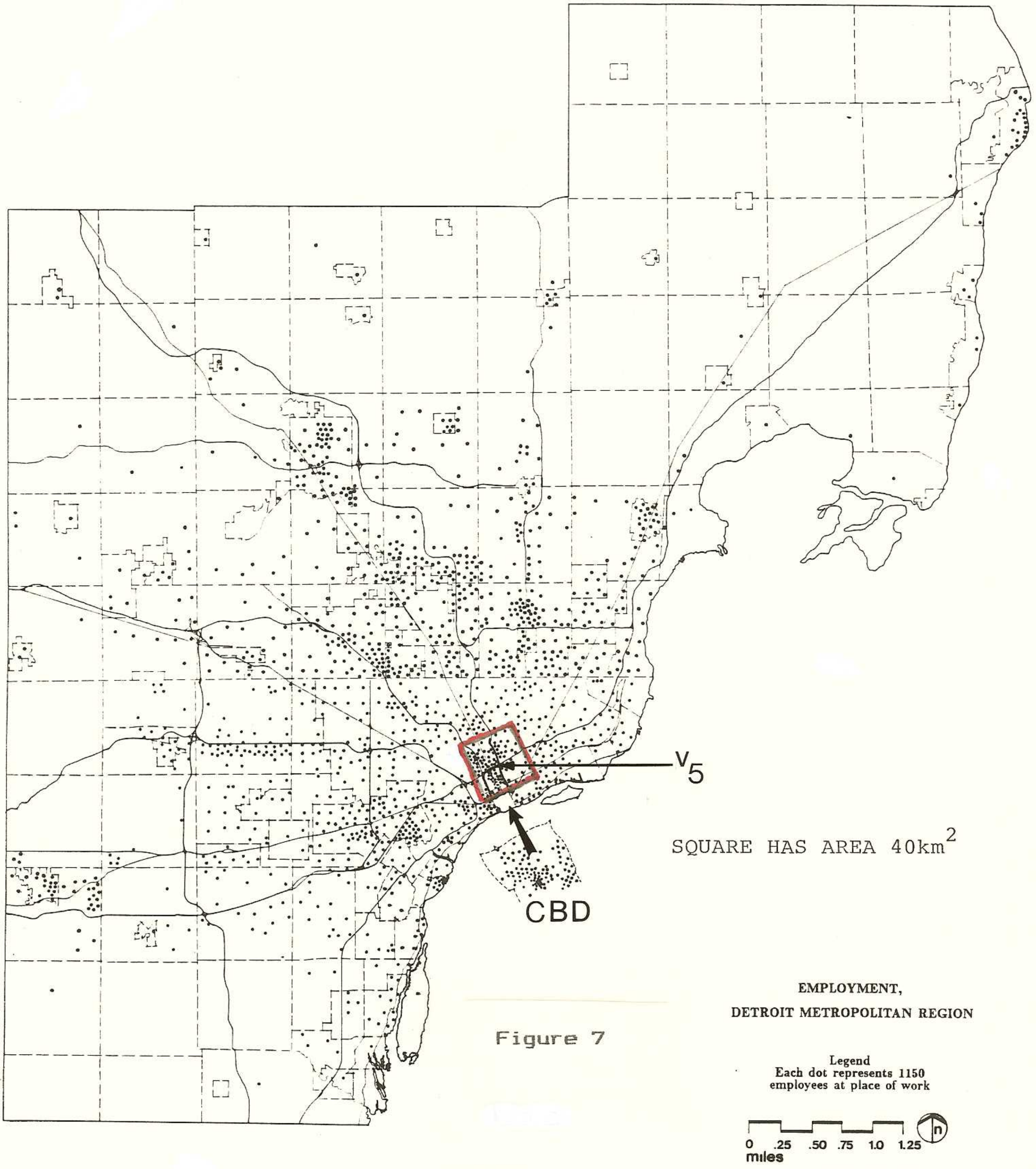


Figure 7

EMPLOYMENT,
DETROIT METROPOLITAN REGION

Legend
Each dot represents 1150
employees at place of work

Source: SEMCOG, 1980

direct casual interaction with other networks and with the underlying population. Urban street-gangs have realized for years the potential that such limited access provides for effective mugging, robbery, and rape. At the scale of the international terrorist, one can imagine the presence of a killer-train loaded with explosives or deadly biological contaminants, roaming the tracks of a large urban mass transit system. When driven by a terrorist to whom the sacrifice of life, in exchange for gaining media exposure, would be an act of heroism, such a train would represent not only an immediate physical danger to the surrounding population but also would present an opportunity for a significant extortion attempt to the terrorist group as a whole.

Equally, the commandeering of an elevated commuter train station with a threat to level it, and many of the surrounding buildings, through an explosion could pose an intimidating threat. Indeed, Japan National Railways suffered a shut down of 23 commuter lines, in November 1985, resulting from the apparent sabotage of the communications and signal system by a left-wing extremist group [1, 6]. The response of the Japanese was to tighten security at all railway stations and other railway installations; no further attacks occured following this increase in security.

To assess the potential for disaster that could arise from situations of this sort, a strategy similar to that employed above could be used. First, determine stations, switches, or signal lights from which network fragments could be detached from a central heart. Second, along potential routes within fragments, encase the network in a "sausage" of diameter sufficient to reflect the area within which risk from such a terrorist extortion attempt is highest (Mandelbrot, Minkowski sausage [9]; Nystuen, boundary dwellers [11]). Then, using dot maps, count the affected population and propose whatever combination of evacuation and security that seems appropriate.

SCENARIO 4: THE POSTAL SYSTEM

Identification of the heart and of associated fragments of the metropolitan Detroit postal system might be important in the event that tracking of a postal bomb, leading to evacuation of the threatened population, is required following a vengeful act by, for example, a laid-off employee.

In systems that are rooted primarily in the institutional rather than primarily in the physical environment, the notion of adjacency, required to characterize the system as a graph, must also rest in the institutional rather than in the physical environment. Thus, in the previous scenarios, edges in the graphs represented physical linkages such as expressway pavement or railroad tracks; here, they would represent administrative linkage within the postal system.

The Zoning Improvement Plan (ZIP) forms the numerical basis of an administrative system for distributing the mails [3]. The first digit of the ZIP Code refers to one of ten geographical areas, nationwide. The remaining numbers within the ZIP Code are as follows:

> the second digit indicates a State, a geographic portion of a heavily populated state, or two or more less populated states within a geographic area; the third digit indicates a major destination area within a State such as a large city post office or a mail concentration point (sectional center) in less

> populated areas; the last two digits indicate either a postal delivery unit (zone) of a large city post office or an individual post office served from a sectional center [15].

Thus, with adjacency derived from the ZIP Code, characterization of the postal system as a graph would follow naturally, as would analysis of that graph in terms of fragments comprised of postal routes detached from postal substations selected by the ZIP (or extended ZIP) Code.

SCENARIO 5: OTHER SYSTEMS

Generally, the style of analysis outlined in these scenarios might be easily extended, conceptually, to a variety of networks. While each of these networks has myriad unique characteristics, all may be represented as abstract graphs. Thus the same sort of general graph theoretic strategy may be applied, independent of individual peculiarities. This strategy consists of

1) identifying the heart of the network;
2) identifying fragments formed downstream from that heart;
3) measuring the affected population, by land-use type, within each fragment (using dot maps).

From there, the use of the unique characteristics of each network aids in making an assessment of conditions under which significant fragments might form in the distribution system.

Electrical Grids

The power grids that provide electricity to urban areas have a high degree of redundancy for back-up; each plant carries no more than a small percentage of grid capacity, so that in the event of plant failure, other plants can electrify the grid. These are grids on which other networks depend, and they are ones in which research on the organization of linkage patterns is well-established [12]. Electrification takes place (virtually) instantaneously; to transmit electricity over distance requires the use of high voltage since, by Kirchhoff's laws, voltage and resistance are inversely related. Thus, high-tension lines linking power plants to substations would be a natural choice for this network heart. Step-downs from the heart, from, say, the level of 110,000 volt lines, across transformers at substations, to 12,000 to 4,000 to 220, to 110 volt lines would suggest a hierarchy of wiring. Fragments that can be detached from transformers would form one set of regions in which to assess risk associated with network disruption. Finally, linkage of the electrical grid to other systems, such as water plants or tornado sirens, creates an institutional hierarchy of decisions as to how much electricity is being sent to different locations in different directions. Thus, the electrical grid might also be represented as a graph formed with institutional decisions as the basis for adjacency.

The Network of Coaxial Cable

At the other extreme of linkage redundancy, the network of coaxial cable used to transmit television signals shows little redundancy when physical bases are used to determine adjacency; the network is a tree-like form. When adjacency is formed on institutional bases, however, there is redundancy, for whenever wires cross the street, from public to private property and back again, a loop is formed. Institutional redundancy of this sort, in a physical network, appears to represent political "red tape" rather than increased facility in transmission.

The Bus System

A bus network is a channeled response to the underlying demand from a scattered urban population to move around freely within transit authority boundaries. Bus stops serve as "boundary dwellers " [11], and as vertices of attachment [14], linking the bus system to the population it serves. Thus, street-gangs which include bus stops that serve as transfer points between distinct bus-routes as part of their "turf", might attempt to threaten a fragment of the bus system's ridership. An effort to overcome such street-gang terrorism might rest in securing critical bus-stops. Another approach is suggested below.

URBAN DESPERADOES

For these there's small hope.
They obey no rule,
Are addicted to dope,
And truant from school.

"Cobras" with their switch blades,
Hooded in the dark,
Attack off-duty maids
In Washington Park.

"Pythons" from the West side
Slith'ring through streets--
Seizing 'cycles to ride--
Choking others' beats.

"Blue Devils" from Hades,
Belt buckles Gillettes,
Slash up rich bookies
Who welshed on bets.

"Vice Lords" drag to Court,
Knowing they offend
The parole board report:
"They are on the mend!"

"Mum Chucks" cast no shadows,
When hijacking a bus;
They throw riders through windows
While hostages fuss.

"Blackstone Rangers" in Blues Bars
Blast beer-barrels for fun;
Some trade guns for guitars--
For them, hope has won.

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TERRAE ANTIPODUM*

> "O, East is East and West is West,
> And never the 'twain shall meet."
>
> Rudyard Kipling.

Maurits Escher provided a celestial view of the antipodal notions of "Heaven and Hell," in "Circle Limit IV" [5], represented in the Euclidean plane using Klein's conformal model of the hyperbolic plane [3, pp. 260-261]. The map in Figure 1, "Terrae Antipodum," represents a terrestrial view of antipodal landmasses in the Euclidean plane using Klein's conformal model of the elliptic plane. Indeed, contemplation of the "antipodes" has been a continuing source of inspiration: as H. G. Wells notes, "It sets one dreaming of the oddest possibilities of intercommunication in the future, of spending an intercalary five minutes on the other side of the world, or being watched in our most secret operations by unsuspected eyes" [9, p. 310].

The cartographer Waldo Tobler has described a scheme in which a pin, poked through a map on a Möbius strip, emerges at its antipodal point [8, p. 486]. Continuing this, a finite number of judiciously selected pin-pricks along the boundary of a region on the strip would approximate the boundary of a region antipodal to the given region, and a cut across the strip would produce in the Euclidean plane a flat, two-sided map. Tobler's procedure describes the construction of a map on a Möbius strip; it does not, however, simultaneously exhibit all land-based points whose antipodal points are also land-based, nor does it examine the non-Euclidean geometric connection. The oceanographer, Athelstan

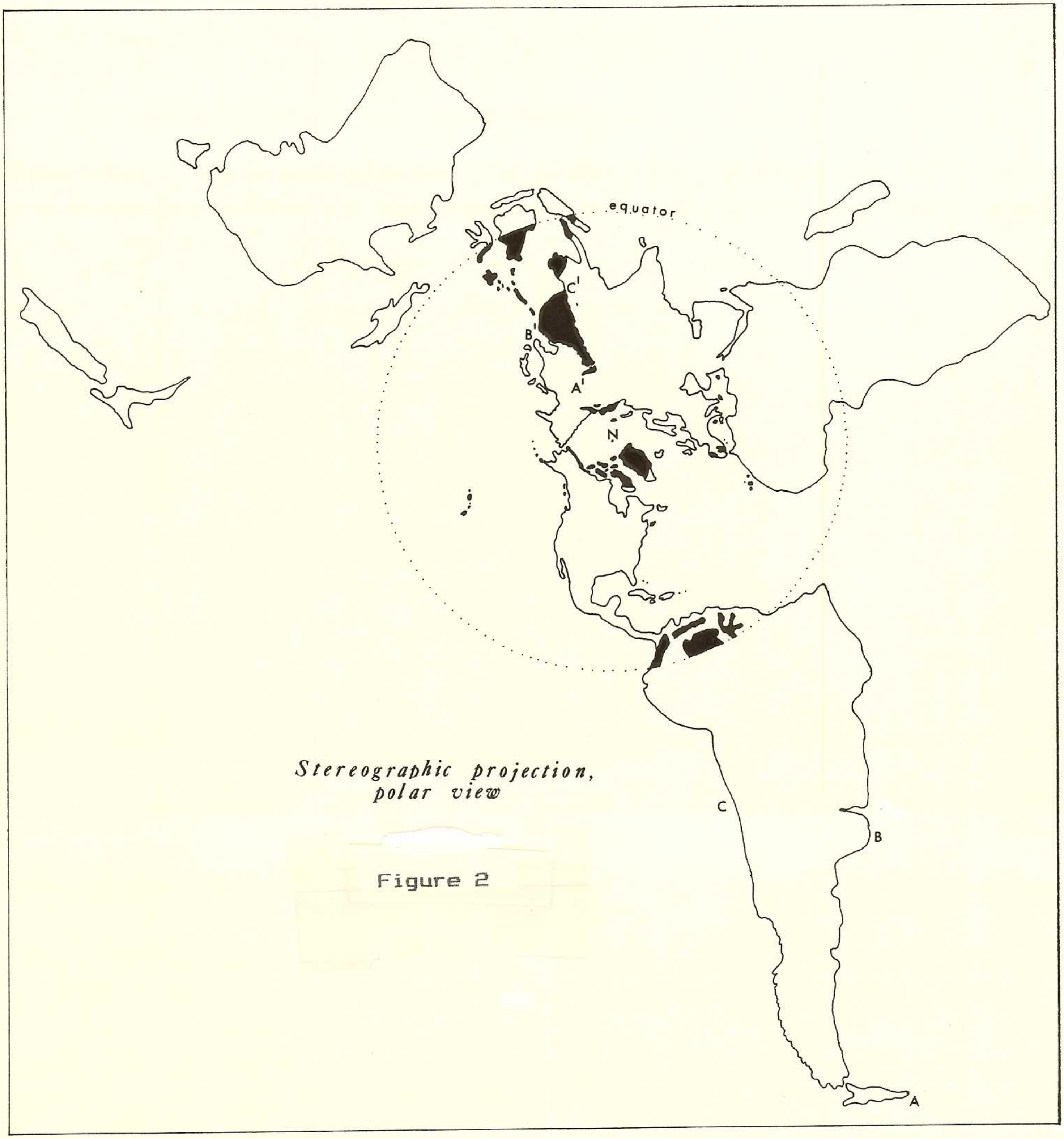
Terrae Antipodum

Figure 1

Shaded regions show landmasses whose antipodal points are on land.

Spilhaus, describes how to construct a map of antipodes "To show which land is opposite other land...by taking a pair of maps of two hemispheres and putting them back to back with the North Pole covering the South Pole" [7, p. 119]. He also, however, does not touch on the non-Euclidean aspect of this construction.

To fill this gap, Klein's conformal model of the whole sphere on the Euclidean plane, in which the equator appears as an undistorted circle centered at the North (or South) pole, N (or S), with the northern (or southern) hemisphere inside and the southern (or northern) hemisphere outside, is used to construct the base map in Figure 2. In this Figure, the globe is projected stereographically from the South Pole onto the Euclidean plane; with a point added at infinity, as the inverse of N in the equator, parallels of latitude inside (or outside) the equator invert to those outside (or inside) the equator, and the entire map lies in the inversive plane [4, p. 390]. Antipodal points are identified when the Southern hemisphere, transformed by inversion in the equatorial circle followed by rotation through 180° ('negative inversion'), is superimposed on the Northern hemisphere [3, pp. 259-260]. The resulting map then lies in the elliptic plane when duplicate arcs along the equator (as those in Borneo and northern South America) are eliminated by identifying antipodal points on the equatorial boundary [3, p. 260]. This transformation carries parts of New Zealand to Spain, Africa to Hawaii, Antarctica to the Arctic, Indonesia to northern South America, and Argentina to China. In all cases, sense is reversed by the application of inversion, as is illustrated by the reversal



Stereographic projection, polar view

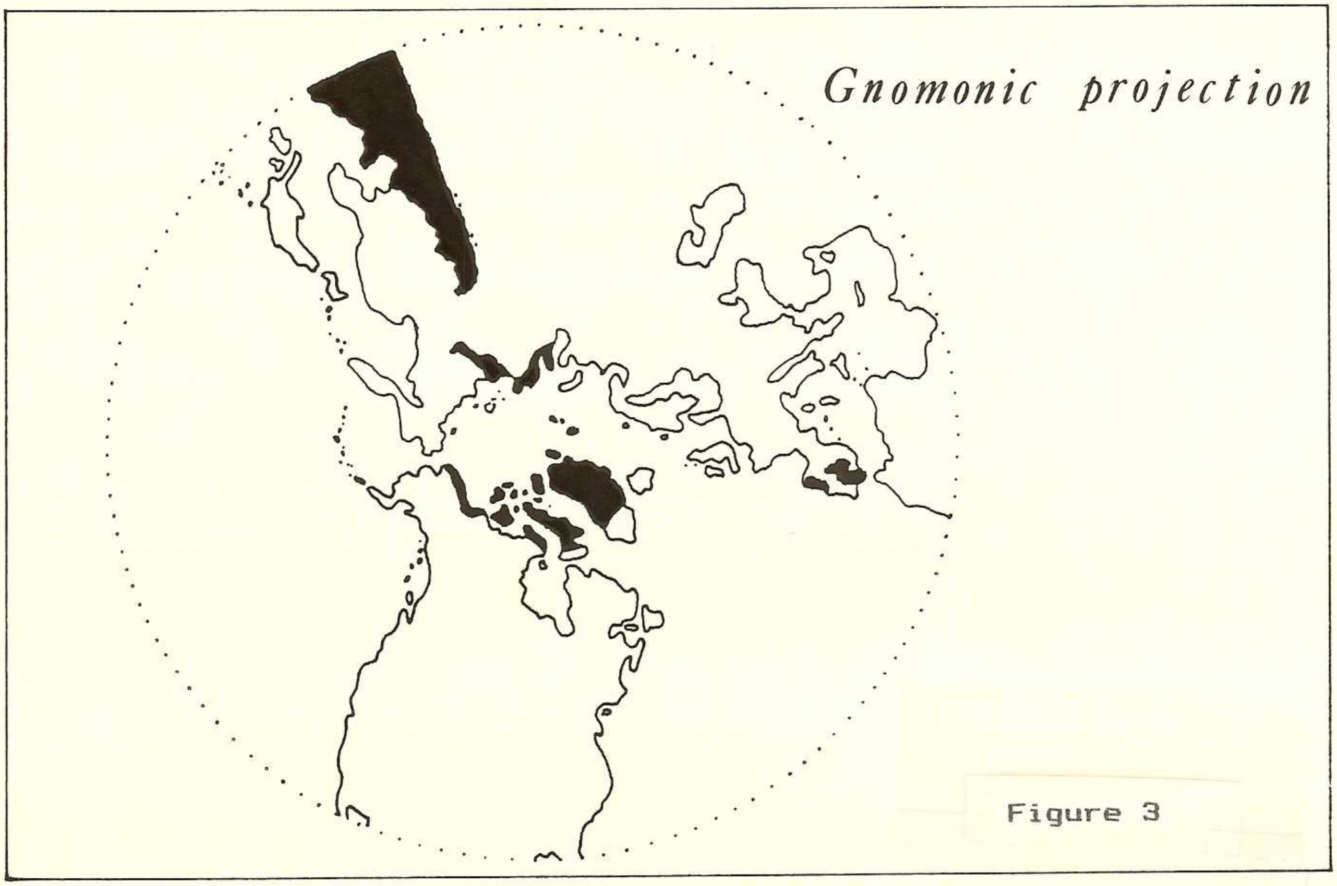
Figure 2

in sense of the labelling of Argentina as ABC and of "antipodal" Argentina (in China) as A'C'B' (Figure 2).

"Terrae Antipodum" (Figure 1) exhibits simultaneously all landmasses whose antipodal points also lie on land; it does so using a non-Euclidean model. The map is conformal, ensuring preservation of angle; however, geodesics are straight lines only along meridians [3, p. 258-259]. Figure 3 shows a version of "Terrae Antipodum" in which all geodesics are straight lines, derived from projecting the globe gnomonically (from its center) onto a plane tangent to the sphere at the North Pole. Lines of projection passing through the sphere's center naturally identify antipodal points on the sphere with a single point in the plane of projection; thus, the resulting map will again lie in the elliptic plane [3, p. 255]. In contrast to Figure 1, Figure 3 displays clearly the Arctic and Antarctic regions superimposed at the map center with highly distorted regions superimposed toward the equator.

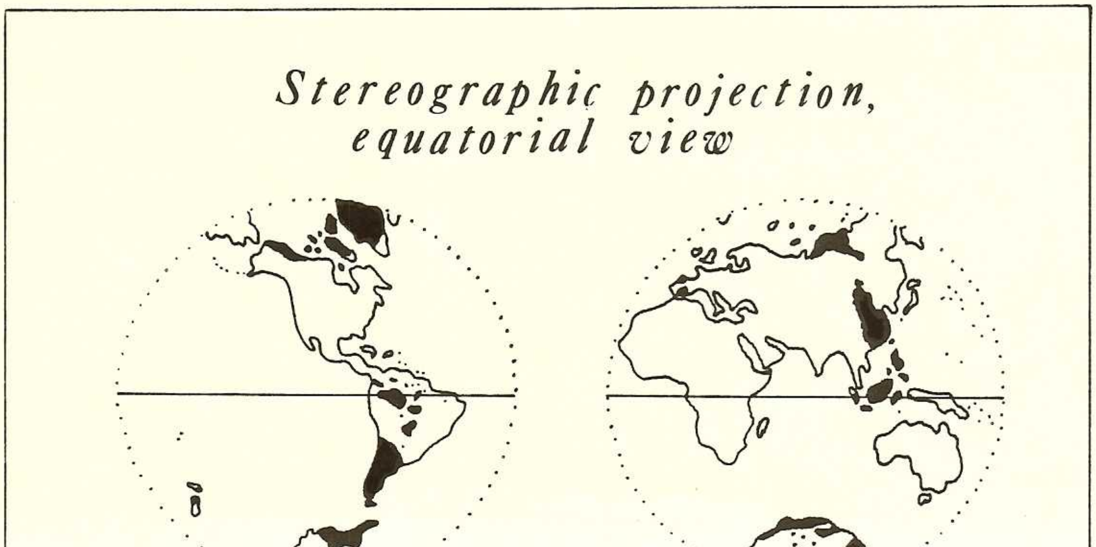
Severe interruption of continental landmasses occurs in both Figures 1 and 3. In geographic application this might present some difficulty; thus, versioins of these constructions using Eastern and Western hemispheres might also prove useful. Two examples of a stereographic version, which interrupts only Arctic and Antarctic regions, appear in Figures 4a and 4b. One would choose either 4a or 4b, depending on the underlying need to use Eastern or Western hemisphere continents as a reference.

In the world of art a version of "Terrae Antipodum," produced on the equal-area "Peters" projection, formed the basis of the



Gnomonic projection

Figure 3



Stereographic projection, equatorial view

(a) (b)

Figure 4

proof that David Barr's global sculpture of a tetrahedron, inscribed in the Earth with one vertex at Easter Island and the other three vertices at various points on land [2], was unique within the set of Platonic solids [1]. Other real-world uses for this map might involve site selection for satellite tracking stations, when the placement of monitoring outposts at antipodal land locations appears critical, as it might in receiving and charting the signal from a weather satellite in polar orbit [6]. Applications of this sort, which exploit the non-Euclidean connection, appear particularly promising; for, like the elliptic plane, the satellite's camera-eye view knows no parallel lines [4, p. 390].

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URBAN INVERSION

Mathematical models of cities often focus on static urban form [4, 6, 7]; it is the purpose of this essay to examine the possibility of using the idea of a mathematical transformation to capture some part of the idea of urbanization--the urban transformation [2, 8, 9]. Common-sense observation of urban process in long-established American cities often reveals the migration of shopping areas from a central business district to satellite suburban malls, the movement of ethnic areas close to an older central business district into suburban enclaves still linked to the CBD by familiar radial routes, or the transfer of an out-dated central industrial belt to more modern headquarters on lower cost, remote, suburban land near beltway transportation. A closer look, over time, often shows this process repeating itself until, eventually, the direction of motion may swing back toward a revitalized CBD in a "renaissance" movement. A view such as this focuses primarily on changing land-use patterns; one could equally focus on changing population patterns as a basis for considering changing urban forms and processes. Whether or not such process is gradual, passing through a sequence of "transitions," or whether it is "catastrophic," jumping across a cusp on an urban surface [9], is an issue that is superimposed on the fundamental notion that what is happening is all a result of a somewhat amorphous urban transformation which, in part at least, is turning the city inside-out.

To characterize this particular way of observing the process of urbanization, a mathematical transformation which literally turns the Euclidean plane inside-out will be employed. (While a non-Euclidean analysis might well prove useful, it is beyond the scope of the present essay [5]). The real-world assumptions involved in applying such a transformation include the following:

a) the facets of the city under consideration can be represented in the plane using Euclidean geometry;

b) the boundary of the city is topologically equivalent (homeomorphic) to a circle;

c) the metric involved in measuring distances between urban places is one that makes sense when embedded in the geometry of the Euclidean plane.

Definition 1

Given a unit circle, in the coordinatized Euclidean plane, centered at the origin. A point (x,y) at a Euclidean distance of r from the origin is said to "invert," with respect to the unit circle, to a point (x',y') at a Euclidean distance of 1/r units from the origin. The origin, itself, inverts to a point at infinity, and the point at infinity inverts to the origin.

Definition 2

The Euclidean plane, with a point at infinity appended, is called the inversive plane.

This mathematical transformation is well-defined, and it has the sought-after intuitive property of turning the plane inside-out with respect to the unit circle. Under this transformation:

a) points inside the circle correspond to points outside the circle;

b) points outside the circle correspond to points inside the circle;

c) the unit circle is invariant;

d) the inside of the unit circle "spreads" to cover everything outside the unit circle;

e) the material outside the unit circle "compacts" to fit within the unit circle (reminscent of a description of a "black hole") [1, 3, 5].

Definition 1 might be applied to a map of an urban area satisfying the prescribed real-world assumptions. (Or, it might be applied to a set of city maps as a classificatory device) A simple uncluttered case, designed to show what happens as a result of using this transformation, is illustrated below. In it, the arbitrary city map (Figure 1)

a) has a boundary that follows the unit circle;

b) has land-use zones corresponding to a simple Burgess model;

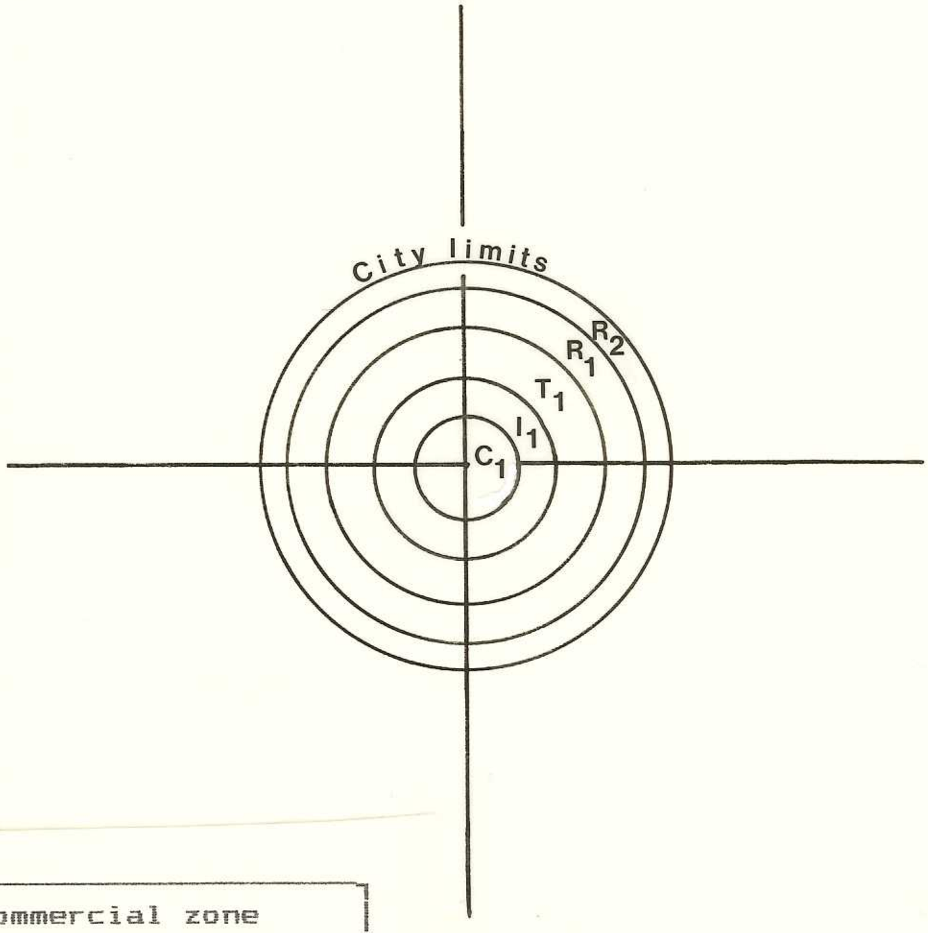
  i) has a commercial zone (Central Business District), C_1 near the circle's center;

  ii) has a ring of industries, I_1, surrounding C_1;

  iii) has a transitional zone, T_1, surrounding I_1;

  iv) has rings of residential land-use, R_1 and R_2 extending from the transition zone to the edge of the city, with wealthier homes in R_2, toward the city boundary.

Figure 1: Burgess concentric ring model of city land-use.



C_1: Commercial zone
I_1: Industrial zone
T_1: Transitional zone
R_1: Residential zone--1
R_2: Residential zone--2

One unit

c) has major radial transport routes emanating from this center;

d) has beltways linking radials.

When the transformation of inversion is applied to this city, the following zones result (Figure 2):

a) C_1 inverts to C_1', far from the city limits;

b) I_1 inverts to I_1', a ring of newer industrial locations;

c) T_1 inverts to a ring, T_1'.

d) R_1 and R_2 invert to R_1' and R_2', where R_2' is bounded by the city limits.

When Figures 1 and 2 are combined, a set of contiguous zones, covering the city proper, as well as suburbia, emerges (Figure 3). When the inversion is considered to have been a transformation over time, one might think of the Burgess model of a 1920's or 1930's Chicago or Detroit projected into a 1950's or 1960's Chicago or Detroit, complete with the rush to tract-housing in suburbia and the attraction of industry and commerce toward land parcels in regions with lower tax bases.

At a finer level of resolution, one might look within the zone of transition, to find an ethnic grouping of immigrants (E_1). Under inversion, this region maps to E_1', an area more distant from the CBD, but along the same transport radial leading to places of work (Figure 3). This might suggest, as one example, the movement of Detroit-area Poles from centrally-located Hamtramck, north along rail lines serving auto plants, to suburban Warren, Michigan.

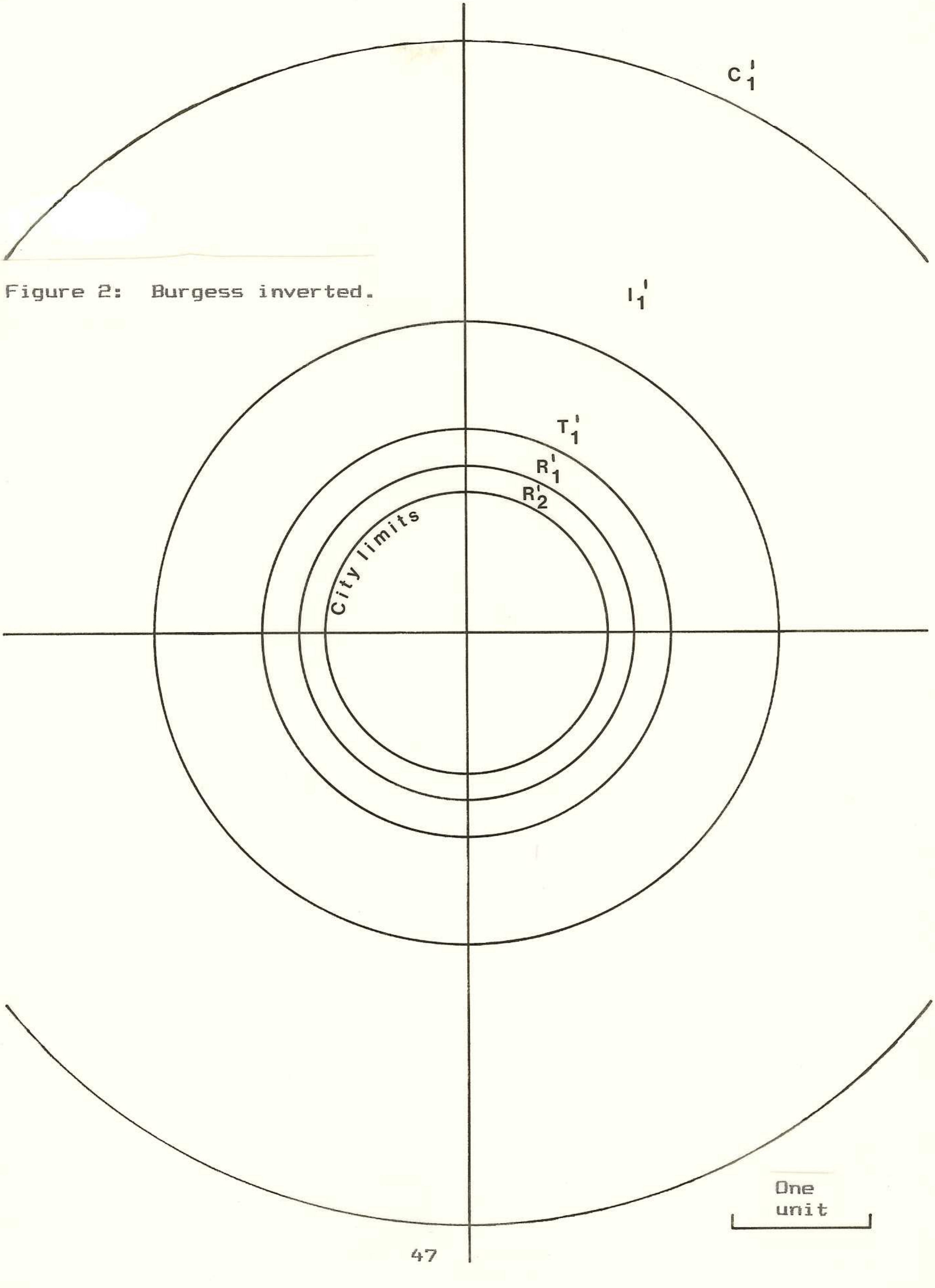
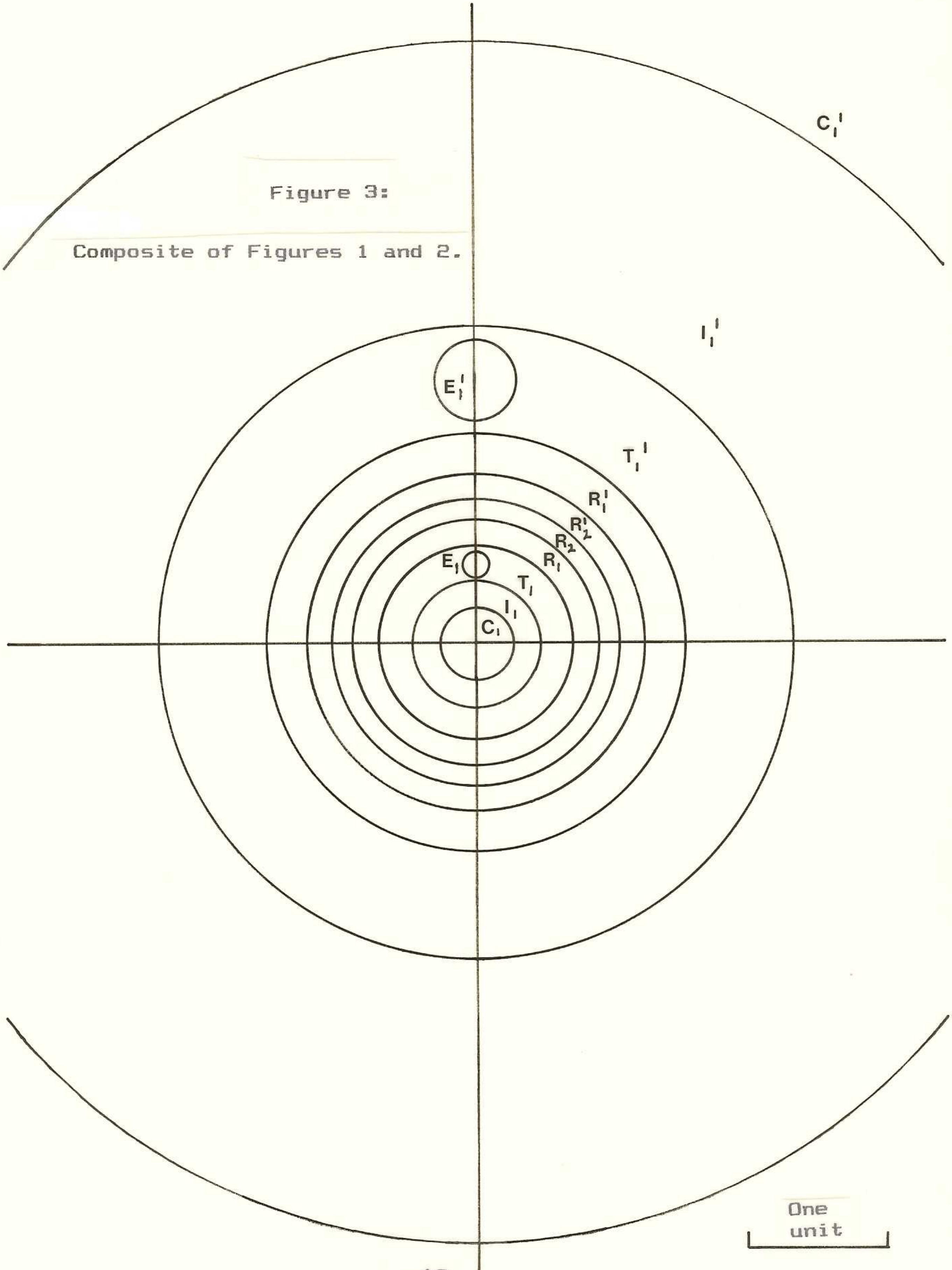


Figure 2: Burgess inverted.

Figure 3:

Composite of Figures 1 and 2.



With the continuing progression of time, one expects to need to apply this transformation not only to the central city of the mid-twentieth century, but also to apply it to the set of smaller cities that grow up as a result of migration to the suburbs. Suppose there are n such cities $U_1, \ldots, U_n$. Once each has been inverted, the area not in any of (outside of) $U_1, \ldots, U_n$ is partitioned into the sets $\{R_{2,1}', R_{2,2}', \ldots, R_{2,n}'\}$, $\{R_{1,1}', R_{1,2}', \ldots, R_{1,n}'\}$, $\{T_1, \ldots, T_n\}$, $\{I_1', \ldots, I_n'\}$, and $\{C_1', \ldots, C_n'\}$ (Figure 4). The closed area determined by $\bigcap_{i=1}^{n} C_i'$ might represent a site of future commercial development C_{n+1}', and the union of the industrial, transitional, and residential rings might represent the rings surrounding this "new" central city (Figure 5). Thus, the old CBD acquires the status of "suburb" and, over time, might itself become yet another "new" central city, or, when the inversion is completed by mapping the "outside" into the "inside," even possibly a "Renaissance" center.

Using the geometry of the inversive plane it was possible to mathematically characterize one view, over time, of urbanization. Specifically, this transformation is merely a simple case of a linear fractional transformation, mapping points from a complex plane z to a complex plane w. Points in these planes are located with respect to a "real" or x-axis and an "imaginary" or y-axis (Argand diagram). The general form for a linear fractional transformation from the z-plane to the w-plane is given by $w=(az + b)/(cz + d)$, where $ad - bc \neq 0$ and a, b, c, d, are complex constants. Inversion may be expressed as a linear fractional

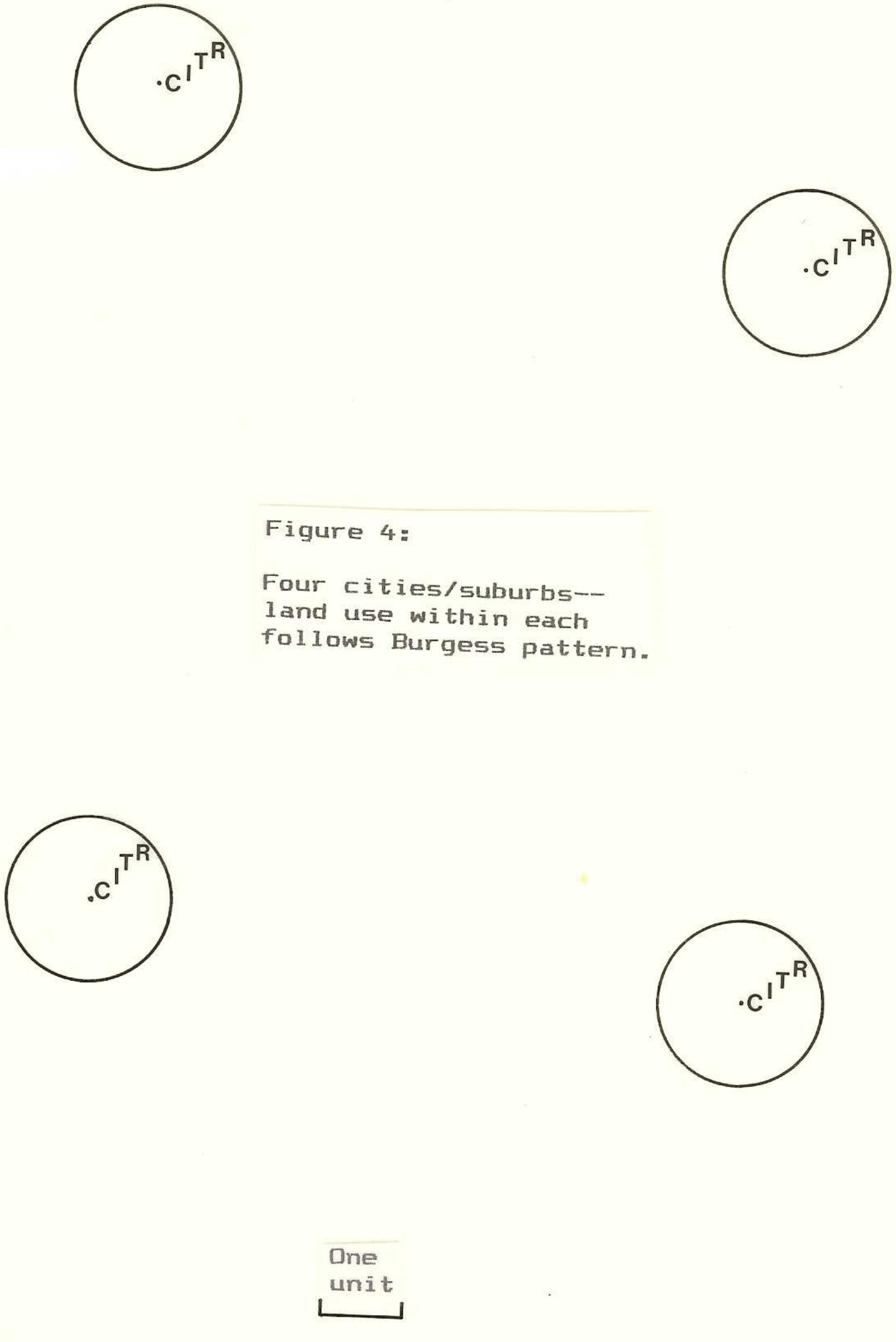


Figure 4:

Four cities/suburbs-- land use within each follows Burgess pattern.

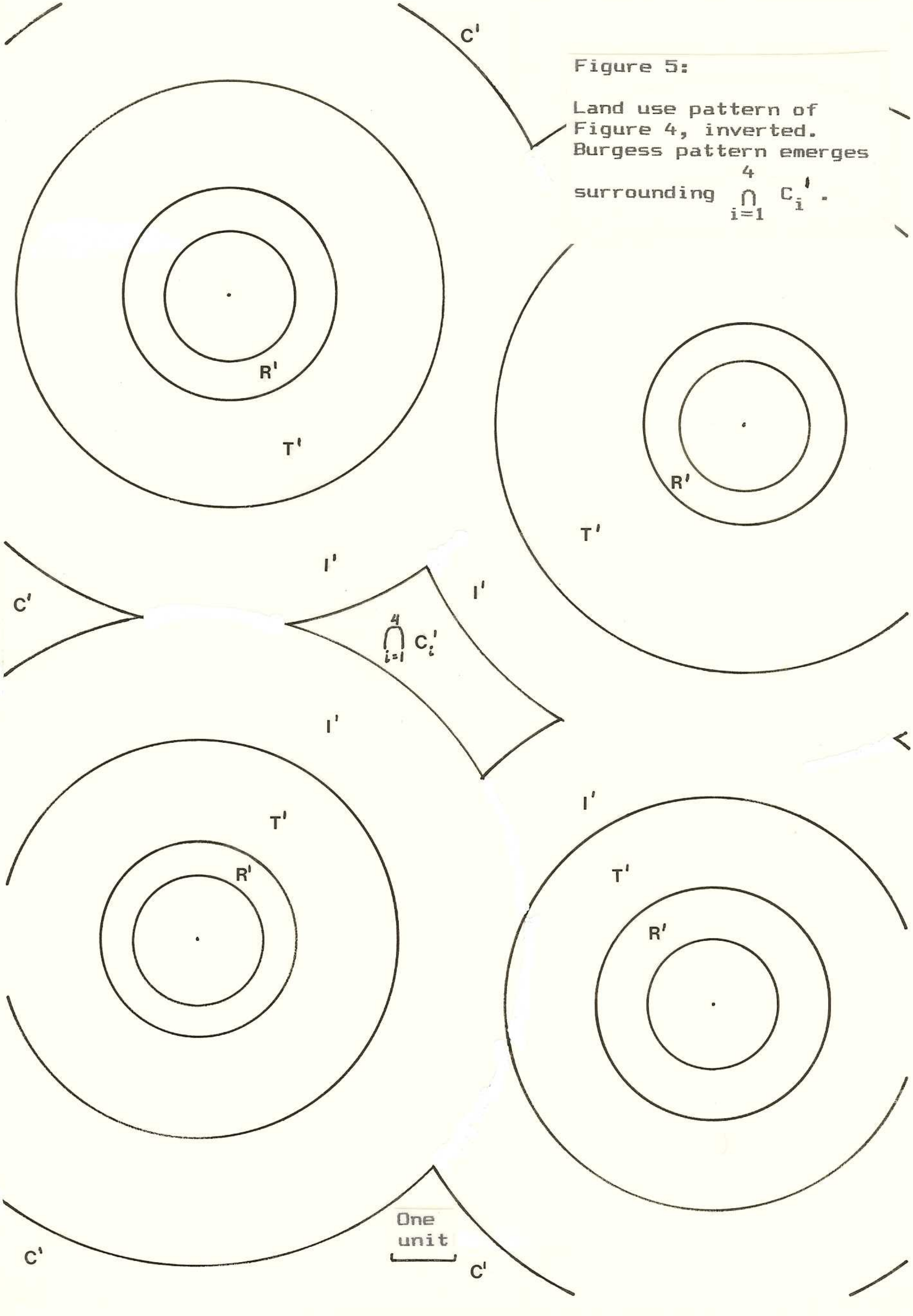


Figure 5:

Land use pattern of Figure 4, inverted. Burgess pattern emerges surrounding $\bigcap_{i=1}^{4} C_i'$.

transformation in the form $w = (0 \cdot z + 1)/(1 \cdot z + 0) = 1/z$, $ad-bc = 0 - 1 = -1$ [1, 3, 5]. This simple transformation, when applied to a straightforward Burgess model, not only preserved elements of this basic structure, but also embodied elements of Hoyt's radial sector model, and Harris and Ullman's multiple nuclei model, when it was applied over time. It also extended these to account for the natural process of revitalization of declining areas. In a broader context, when this transformation is viewed as but one case of the infinite number of possible linear fractional transformations of the complex plane, one wonders what broad vistas await the geographic explorer of transformations of the complex plane.

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FRACTALS: CONSTRUCTIONS, SPECULATIONS, AND CONCEPTS*

INTRODUCTION

Fractal geometry is a tool which may be used to assess the extent to which a geometric figure fills the space which contains it. Abstractly, this tool is not new within the realm of pure mathematics; however, the recent computerized characterizations of this complicated mathematical process have brought it to a position of widespread prominence within the sciences [14, 18, 19].

It is the object of this paper to display in detail one use of fractals in geography, so that the reader unfamiliar with fractals might gain exposure to the associated constructions. This will be followed by speculations as to some other real-world settings for the application of fractals and will conclude with an identification of some concepts in geography which might lend themselves to the application of mathematical ideas based in theoretical frameworks parallel to those from fractal geometry [13, 14, 15].

CONSTRUCTIONS: A FRACTAL VIEW OF THE K=4 CENTRAL PLACE HIERARCHY

The construction that follows shows how to use a simple template, a "Fractal-Dactyl," to generate as many levels as desired of a K=4 central place hierarchy. Iterative use of the template, applied initially to an arbitrary hexagon, will generate the complete K=4 central place geometry as a fractal construction [1].

Given an arbitrary hexagon, with vertices labelled 1, 2, 3, 4, 5. 6 (Figure 1). When a template, which includes a shape scaled to fit the side of the hexagon (1,2,3,4,5,6) (Figure 2.a), is applied to that hexagon in such a way that each edge of the hexagon is replaced by a shape from the template, a more complicated form results (Figure 3) [1].

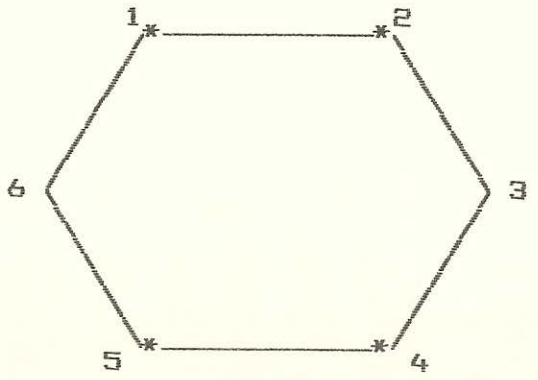
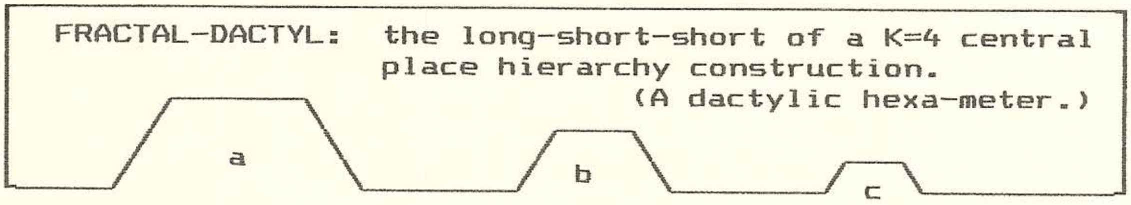


FIGURE 1



FRACTAL-DACTYL: the long-short-short of a K=4 central place hierarchy construction.
(A dactylic hexa-meter.)

FIGURE 2: a fractal template

Specifically, when successive replacement of sides occurs in such a way that the template shape appears alternately "inside" and "outside" of the original hexagon, the correct spacing and orientation will be completely determined for two superimposed central place hexagonal nets, adjacent within the K=4 central place hierarchy (Figure 4) [1].

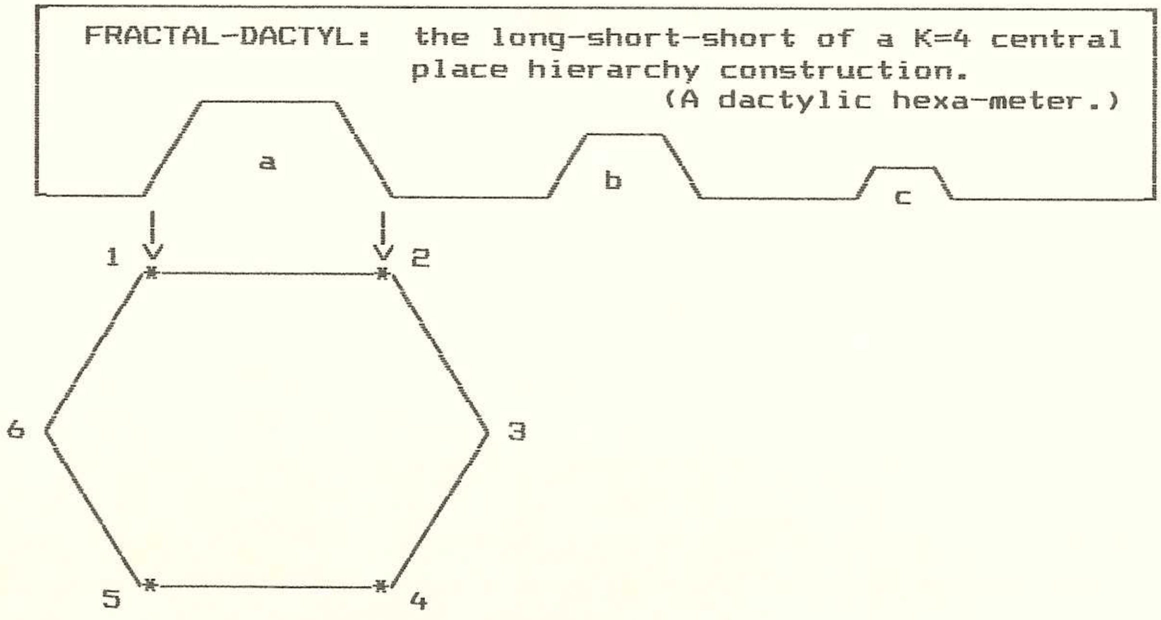


FIGURE 3: template applied to an edge of a hexagon

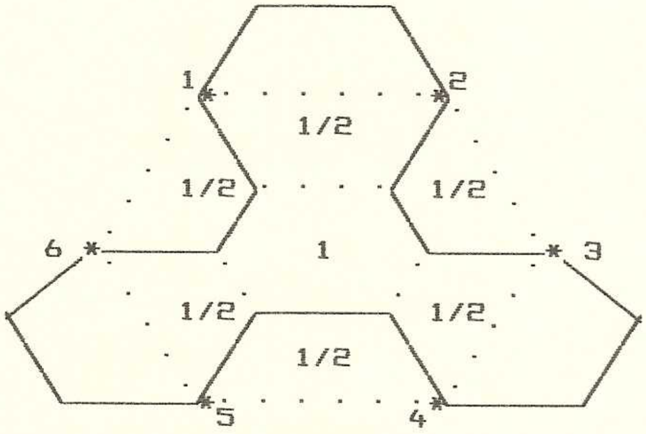


FIGURE 4: The hexagon (1,2,3,4,5,6) contains the equivalent of 1+(6·(1/2))=4 unit hexagonal areas derived from using the fractal template.

Hexagonal nets in the K=4 hierarchy, of successively finer mesh, may be generated through repeated application of a template in the manner demonstrated above. When the second shape along the template (Figure 3.b) is applied to each of the 18 sides in Figure 4 (alternating "in" and "out"), the next level in the K=4 hierarchy is obtained (Figure 5). A tracing paper copy of the

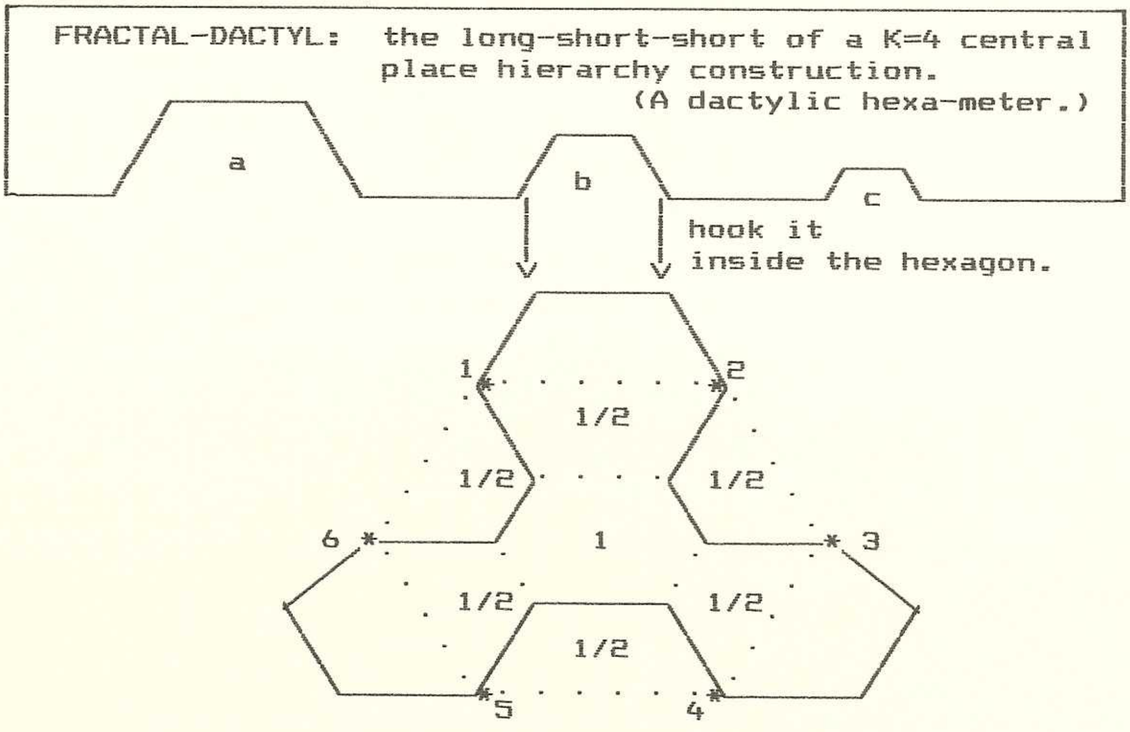


FIGURE 5

"Fractal-dactyl" applied iteratively through several stages, represented along the template as "a," "b," and "c," would reinforce the process in the mind of a newcomer to fractal constructions.

When this procedure is repeated, *ad infinitum*, the central place net "fills" an amount of the plane, calculated by Mandelbrot's formula as: the fractional dimension, F, of the fractal, $F = (\log 3)/(\log 4^{1/2}) = 1.585$ [14]. The value of "3" reflects the number of edges in a single template shape, independent of the scale of that shape; the value "4" reflects the number of equivalent "small" hexagons within a "large" hexagon in adjacent levels of the hierarchy (again, independent of scale).

The value of 1.585 as a fractional dimension for the entire hierarchy means that this network "fills" more of a plane region than does a network of lower fractional dimension, and less than does one of higher fractional dimension. A straight line has fractional dimension "1" and a plane has fractional dimension "2" [14]. This twisted configuration of line segments "fills" more of the plane than does a one-dimensional straight line segment, yet there are spaces between line segments so that it is not two-dimensional; hence, a dimension for it between the Euclidean dimensions of one and two. Therefore, fractional dimensions provide specific indices for assessing the extent to which geometric networks can penetrate underlying service areas [3].

SPECULATIONS ON APPLICATIONS OF FRACTAL GEOMETRY

One way to describe the extent to which figures fill space is in the repeated application of a single shape to a given shape, at a variety of scales. A sequence of such applications produces a single iterative procedure from which an entire hierarchy of geometric forms follows. This notion, which transcends scale problems within a hierarchy capable of geometric representation, is called "self-similarity" by Mandelbrot [14]. Because many real-world phenomena do lend themselves to such representation, a multitude of material has appeared following Mandelbrot's attractive graphical representations, residing in 20th century computers, of topological monsters rooted in 19th century mathematical ideas.

A recent article by Goodchild and Mark lists, and provides valuable comments on, a selection of published works that employ fractal geometry used in a variety of geographical settings: from the theoretical to the empirical. The bibliography in their article shows, as does any other reasonably comprehensive attempt to accumulate references on applications of fractal methods, a filling of the spaces between early fractal articles that accelerates through time with the swiftness of the photographic zoom-techniques used in the computer generation of fractals, themselves [10].

Fractalogue

1. In the case of a forest, one might expect a crenulated "fractal surface" to describe an aerial view of the forest, derived from, say, Thiessen polygons based on precipitation regions. Clearly, one would expect also to have other data, such as, for example, those pertaining to underlying soil type. This sort of description might then have specific applications for location within the forest. Fire is a self-similar phenomenon, and if a forest fire were viewed as a "fractal surface," in opposition to the forest "fractal surface," global criteria for locating a net of ranger stations, or for other global fire-fighting tactics, along the twisting boundary separating forest and fire fractals might follow [2--Fleet].

2. Another approach is to characterize the forest using self-similarity (one basis of fractal generation) in terms of majority cover. Thus, when the forest is mapped at a relatively global scale into regions of "trees" and "meadows" a set of

homogeneous blocks of cover appears; at a more local scale there is repetition of pattern with overlapping boundary areas--continuing these observations across a number of scales would single out the most "heterogeneous" regions. Linking these with park entrances and exits would produce a dendritic network along which the "scenic" value would be high (if exposure of humans to a wide variety of species and communities be desirable) and could be quantified using fractal dimension as a single value. Of course, those regions away from the network would be protected from visitors; those which always remain outside this overlap along boundaries would be wilderness areas achieving the highest protection values. Again, the fractal dimension, seems a likely candidate for producing a single quantitative index to describe the relative levels of protection from visitor contact afforded to various regions of the forest,and to evaluate scenic vs. protected value. With a finite number of cover types, interdigitated through scale change along boundaries, a network linked to the outside along these boundaries would maximize exposure of visitors to the variety the forest has to offer, would maximize isolation of homogeneous regions from visitor contact, and would minimize damage from that contact by minimizing the possibility of human destruction of homogeneous communities [2, esp., Nystuen, Fleet].

3. The same sort of analysis might work for an exotic (or a pest) in the forest. Again, characterize regions of self-similarity by majority cover and look for regions of overlap that arise through shift in scale, in order to locate regions of high mixing of an exotic with indigenous plants. As in the previous case, one might wish to introduce a natural enemy along an associated dendritic network [2, esp., Nystuen, Fleet].

4. In a soil profile with a fluctuating surface temperature and a steady base temperature, fractal geometry might be used to measure:

a) the transfer of heat, across a set of discrete soil particles, through self-similar horizons--heat radiates through a predictable, continuous, process. In transit, through the profile, it passes through layers of discrete particles of variable texture [17, 2];

b) the spacing between particles in an undisturbed profile, assuming some general knowledge of the horizon, such as finer particles are near the surface of the profile and coarser ones are near the base.

5. Fractal geometry might be used to outline spatial criteria for network sites. An initial stage with constricted flow across a network, might be acted on by a generator to model the extent of increase in transfer across the network, through self-similar stages which see the widening of trunk-lines, until diffuse penetration of space is achieved by the network [2, 16].

6. In an agricultural setting, the San Francisco Bay/Sacramento River system displays self-similarity in that the stream feeding into Suisun Bay has the same shape as the larger system. What are the implications of this in assessing extent of estuary action [2]?

7. Skylines in some small cities might have the same general shape as do some skylines in larger cities. What insight does such self-similarity provide in assessing land-use patterns in the central business districts of an entire set of such cities?

Because applications of this sort transcend considerations of scale, through the use of self-similarity, immediate theoretical opportunities appear in purely spatial problems possessing some sort of self-similar pattern. When fractal constructions are derived from computerized simulations, rather than as deductive synthetic geometric forms, applications range across a broad spectrum of empirical geographical problems. Again, because these simulations are free from scale problems, they are suited to understanding the dynamics of geographical processes and have been used in that mode in physical settings, in terrain analysis, as well as in urban settings, in land-use pattern analysis [9; 4, 5, 6].

Recent issues of more general science periodicals, such as *Scientific American* and *Science*, contain a wide variety of articles displaying fractal applications of an empirical nature; in doing so they suggest directions for theoretical extensions of fractal-like analysis which in turn feed into more empirical applications and concepts [8, 16].

CONCEPTS: THEORETICAL EXTENSIONS

Fractal geometry measures, generally speaking, the extent to which geometric forms "fill" the Euclidean space that contains them. A figure composed of straight line segments which twist and bend, "fills" more of the plane than does a straight line segment itself; as seen in the construction above, such a figure has fractional dimension between the Euclidean dimensions "1" and "2." Topological methods yield a formula for calculating this fractional dimension; indeed, within the mathematical literature, material from point-set topology and from the theory of functions of a complex variable interact to produce a formal picture of space-filling ideas as complex dynamical systems.

What remains to do, in extending theory, appears to be two-fold. First, apply fractal methods in a variety of abstractly similar, but not identical, settings as suggested in the "Fractalogue" above. Second, step back and remember, at a deeper theoretical level, that fractal methods are but one small part of a mixture of complex geometry and point set topology. From this vantage point, one might consider the application of mathematical tilings of the plane to be useful in characterizing geometric properties of sets of contiguous geographic regions [11]. Both tilings and fractals involve the use of successive placements of a geometric shape in a mathematical space. Where tilings differ from fractals is that tilings emphasize the content of spatial pieces within boundaries while fractals emphasize the processes underlying the placement of those pieces. From this viewpoint, when the K=4 hierarchy is viewed as a tiling it is the hexagonal shape of the market area that is emphasized [7, 12]; when it is viewed as a fractal it is the nested hierarchy of market areas that is emphasized. In this particular case, the fractal view is the one best suited to the actual real-world situation. Generally, fractals and tilings are complementary; if one abstract structure is not tailored to fit the fundamental spatial characteristics of a real-world problem, try the other. This interlocking of fractal/tiling methodology brings the scale problem back into play; this time, however, at the level of choosing a particular methodology rather than at the level of applying one.

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SOLAR WOKS*

"Die Strahlen der Sonne
vertreiben die Nacht."

Wolfgang Amadeus Mozart,
"Die Zauberflöte."

INTRODUCTION

The wok (Chinese cooking vessel) and the parabolic solar collector both concentrate energy, thereby generating high temperatures at a focal point. Such concentration is a consequence of the mathematical definition of the parabola as the locus of points equidistant from a line, the directrix, and a point, the focus. For, it follows from this definition that parallel lines, perpendicular to the directrix, bounce off the parabola to converge at the parabola's focus. Woks and parabolic solar collectors are unlike, however, because the source of energy for the wok is on the side of the parabola opposite from the focus, but, for the parabolic solar collector it is on the same side. Other types of solar collectors, such as the flat plate collector, can make use of diffuse radiation not available to focusing collectors. Abstractly viewed, solar collectors therefore either focus solar energy or intercept it without focusing it; only focusing collectors will be dealt with here. The forcing of solar energy toward the focus of a parabolic collector is possible because fundamental geometric properties of paraboloids apply to solar energy as it arrives at the earth's surface as a set of "parallel" lines from a source "infinitely" far away.

It seems natural, therefore, to consider the possibility of using parabolic solar collectors in which the energy source is opposite the focus, as a kind of a "solar wok". Two classes of concern arise , for a solar wok, surrounding the associated transfer in orientation, with respect to the focus, of a traditional parabolic solar collector: firstly, what sorts of corresponding mathematical transformations are involved; and, secondly, what sorts of real-world applications, that fit with theory, are there for solar woks.

THE BASIS FOR A WOK GEOMETRY: SOLAR AFFINE TRANSFORMATIONS

The viewpoint of Felix Klein, in developing a geometry as a set of transformations from one 'space' to another, is adopted here [7, pp. 342, 640]. The definitions will be cast in notation suited to yielding theorems that might concern structures and properties that remain invariant under these transformations.

In this case, the 'spaces' to be linked by transformations will be geographical regions or spaces on, and near, the surface of the earth. The transformation set will be comprised of a set of *affine* transformations [3; 2, p. 289].

Definition 1

An *affine* transformation, T, of one space into another is such that T transforms straight lines into straight lines and parallel lines into parallel lines but may alter distance between points and angles between lines.

To specify solar radiation as an affine transformation to be applied to geographic spaces, consider as motivation the following example. A cube, balanced on one vertex on the earth's surface, casts a variety of shapes of shadows as the position of the sun in the sky changes throughout the day. At times the shadow might be a square, at other times an almost-regular hexagon, while still at others, a centrally-symmetric hexagon (Figure 1). All of these shadows are affine images of the cube when it is assumed that solar radiation enters the earth's atmosphere as a set of parallel lines. Under this assumption, the straight edges of the cube are mapped to straight edges of the shadow (a local view of the shadow assumes it to be flat) and parallel lines are mapped to parallel lines (although angles between adjacent edges change, of course). Here, incoming solar radiation serves, locally, as an affine transformation of the balanced cube, a geographic space *near* the surface of the earth, to its shadow, a geographic space *on* the surface of the earth. Under this solar affine transformation, the image of a real-world object is its shadow.

More generally, the solar affine transformation, S, will map the set, X, of all objects, inclined at an angle to the earth's surface, into a set, E, representing the earth's surface. Objects which make an angle of zero degrees with the earth's surface (such as residents of "Flatland" [7]) cast no shadow; the set of all such objects might be viewed as the identity element of X--when it is combined with other objects it has no effect on the angle these objects make with the earth.

To verify that S is a transformation requires that S be well-defined [5]. That is, that an element of X correspond to exactly one element of E, or in other words, that a real-world object correspond to exactly one shadow on the earth. Because two different objects cannot occupy the same volume of space, they cannot cast the same shadow when a single point of light, located at an infinite distance (as is the sun in this case), is the

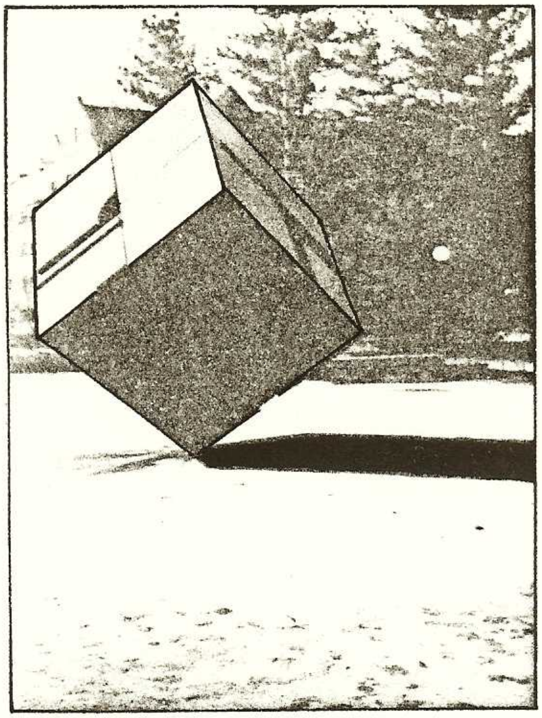
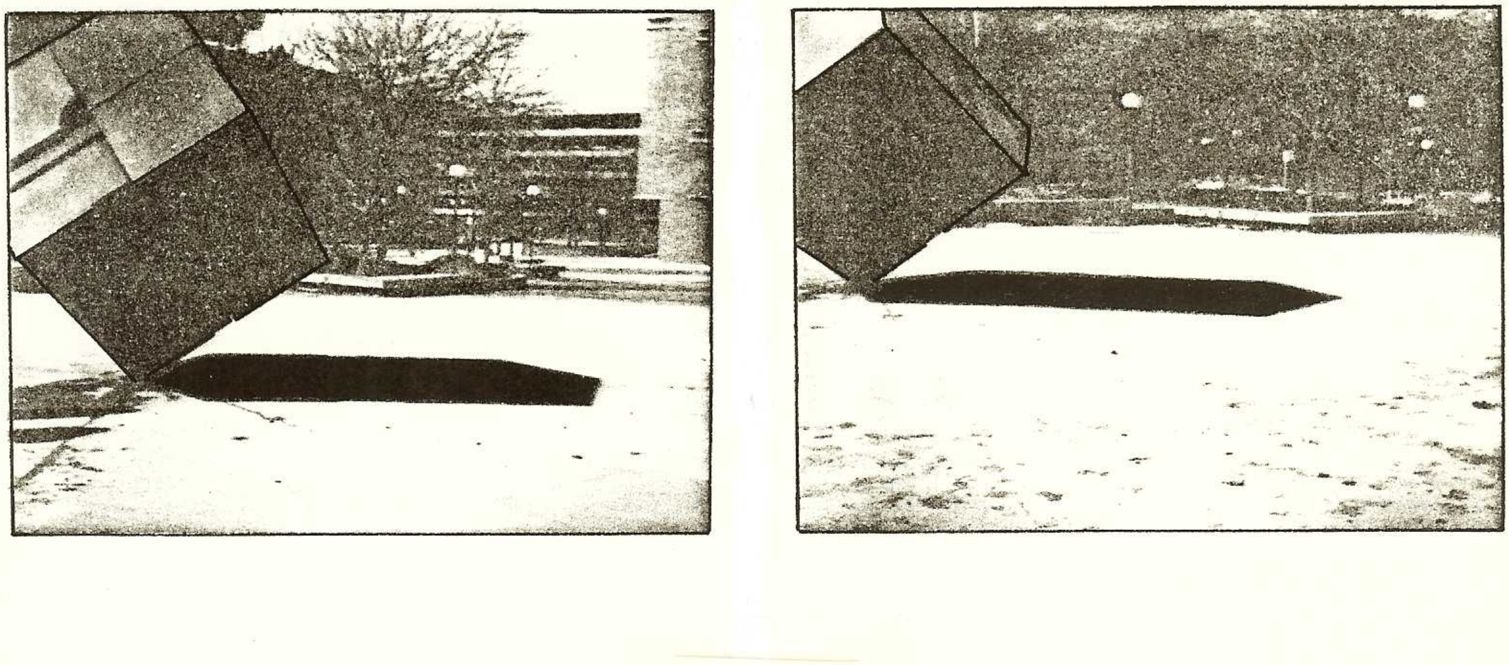


Figure 1: This sculpture occupies "Regents' Plaza" on The University of Michigan campus in Ann Arbor. A similar sculpture, by Isamu Noguchi, is near the Marine Midland Bank building in New York City. Notice the shapes of the shadows of the cube.



source. Therefore, S is well-defined on the non-identity objects of X. On the identity, there is the absence of the possibility of shadow from objects near the earth, which corresponds in E to the idea of a region of the earth that is already blackened by a total eclipse of the sun (in daylight hours) or of the moon (at night). Thus, S is well-defined.

Here, incoming solar radiation serves as an affine transformation, S, of a geographic space, X, *near* the surface of the earth, to its shadow, a geographic space, E, *on* the surface of the earth. Under this solar affine transformation, S, the image of a real-world object is its shadow on the surface of the earth.

If the spaces, X and E, can be endowed with some sort of (algebraic) structure, then a body of theory might develop in "parallel" to that involving morphisms linking algebraic objects [6]. For example, within such a "parallel" structure, one might ask [6]:

1. how to specify S, in terms of binary operations on X and E, so that S is a morphism-- that is, if '$\square$' is a binary operation on X and '$\blacksquare$' is a binary operation on E, $S:(X,\square) \longrightarrow (E,\blacksquare)$ is a morphism if S is a function from X to E and if the first operation is lifted to the second in such a way that $S(x \square y) = (Sx) \blacksquare (Sy)$ for all x,y in X;

2. When S is a morphism, and the kernel of S, $\ker S = \{x \in X \mid Sx = id_E\}$, is, for example, the set of objects in X that correspond under S to the set of eclipses (or the set of flat objects in X), under what conditions is there a morphism $\tilde{S}$ such that $\tilde{S} = S \circ (proj_S)$ (i.e., such that the following diagram commutes).

$$\begin{tikzcd} X \arrow[r, "\mathrm{proj}_S"] \arrow[rd, "S"'] & X/(\ker S) \arrow[d, dashed, "?"', "\tilde{S}"] \\ & E \end{tikzcd}$$

(The set X/(ker S) is a quotient set, comprised of the set of all equivalence classes on X with respect to some equivalence relation; the notation 'proj_S' refers to the 'projection' of X on X/(ker S) which assigns an equivalence class to each $x \in X$.)

3. Is $\tilde{S}$ in (2) above necessarily unique, when it exists?

Once an underlying transformational structure has been found to link two spaces, and has been developed along some theoretical mathematical path such as that suggested above, further mathematical alignment, in terms of finding sets of theorems, might progress. The approach of aligning physical/natural and mathematical definitions as a road to uncovering structural relationships concerning one or the other (or both) is not unusual. When the notion of 'straight line' was aligned with 'the path light travels along', in Poincaré's model of the non-Euclidean geometry called hyperbolic geometry, it became clear that so-called parallel light rays could, in fact, be viewed to diverge [4]. It is an open question to discover what implications viewing solar radiation as an affine transformation of geographic spaces might have in arenas as far-flung as the hydrologic cycle and urban bus networks.

REAL-WORLD APPLICATIONS OF SOLAR WOKS

Real-world applications of solar woks, that fit with the mathematically theoretical alignment based on solar affine transformations, range across the scale of human activities from those which are agricultural to those which are urban. Independent of the specific nature of the application, however, there remains the ever-present concern as to what sort of impact the introduction of different technology might have on underlying human cultural traditions. Such issues are beyond the scope of this material; instead, ideas of how solar woks might be used are presented at an abstract level.

Any use of a solar wok will involve, necessarily, local redirection of the sun's rays through focusing them in a parabolic collector. Because the sun and the parabolic focus are on opposite sides of the collector, the solar wok will function in more the way a tent does than in the way a traditional parabolic solar collector does. The tent image [8] suggests the notion of suspending solar woks as a means of distributing their focused energy. The extent to which such woks generate heat will of course depend on the latitude at which they are placed, and the local climatic conditions, including typical cloud-cover.

1. The physical structure of the woks, themselves, would need to respond to a number of concerns. The following list includes a set of properties which it would desirable for wok material to have.

    a. It should conduct heat very well.

    b. It should be relatively lightweight--perhaps be some form of metallic Mylar [8].

c. It should be flexible enough to bend into a parabolic shape, yet rigid enough to retain that shape.

d. It should resist weathering--perhaps be coated with a thin film of suitable material [9].

e. It should resist high winds--perhaps be porous.

2. Solar Woks might be used to raise, albeit slightly, the temperature of urban sidewalks (or streets) in the winter in order to assist with snow removal. Solar woks suspended, in a concave-down position, from light poles already in place would redirect heat from the sun, broadcasting energy over sidewalks or streets. The resultant melt-regions would form a central-place pattern (on a perfectly flat surface with no interference from wind or from pedestrian or vehicle traffic). The use of the associated central-place notions of hierarchy and levels of hexagonal nets, positioned in a variety of orientations with respect to one another, might suggest general positions for new woks to fill gaps in the pattern or to intensify the heat generated by the woks in the present pattern of light pole spacing.

Definition 2 [3, pp.53-57]

A Dirichlet region about a point W is a region within a lattice of points, every point of which is closer to W than to any other lattice point.

One general geometric form of a Dirichlet region is a centrally symmetric hexagon [3]. Thus, the extension of results derived from central place theory, to sets of contiguous Dirichlet regions, might project geographically-derived central-place insight into the broader geometric realm of theorems concerning Dirichlet regions.

To supplement the broadcasting of solar energy from a set of woks suspended from sessile elements of an urban lattice, solar woks might also be hung from moving objects. Buses, commuter trains, trolley buses, and other mass transit vehicles might be well-suited to this task.

3. Large solar woks, in concave-down position, might be used as covers for bus stops [8]. Not only would they serve as giant umbrellas to shield passengers from rain and snow, but they would serve, as well, to focus heat. (The size of such woks would depend on various physical laws, such as the law of conversation of energy.) When suitably constructed, they could be easily converted, perhaps by inverting them, to reflect heat away from passengers in the summer. Naturally it would be desirable to have control of them monitored from a secure central administrative area for relative ease of use by transit authority officials and so that abuse to the system might be kept at a minimum.

4. A large solar wok, in concave-down position, might be hung over a large clear-plastic concave-up wok, of catenary (graph of the hyperbolic cosine) cross-section, as a "hydrologic accelerator" based on natural forces (Figure 2). Gravity sends water down the mountain swiftly, and evaporation and condensation, speeded through focusing solar energy, pulls water back up the mountain. Suppose that this double-wok is suspended, perhaps on telescoping poles to facilitate its removal during a rainstorm, above a mountain. The solar wok focuses heat above the

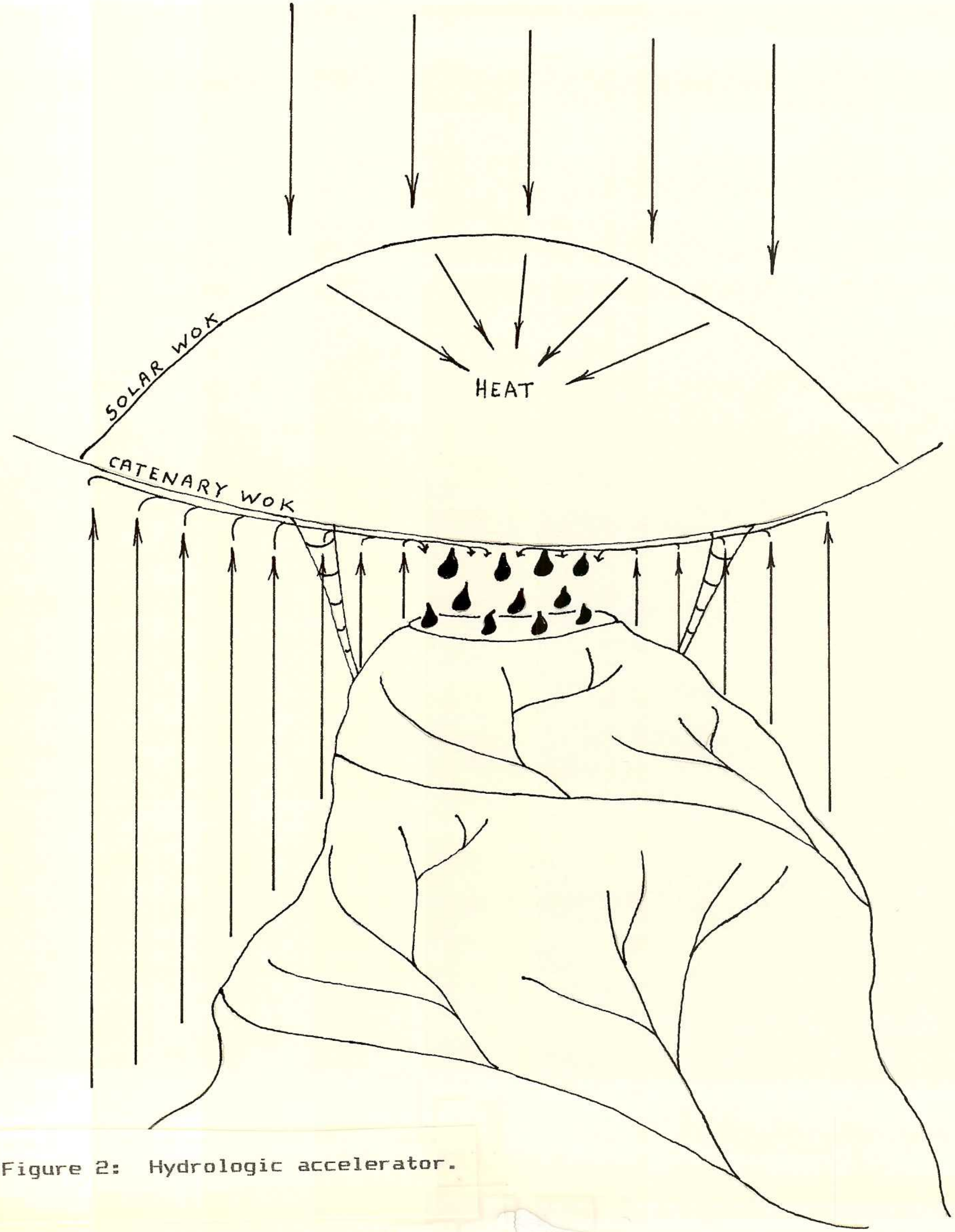


Figure 2: Hydrologic accelerator.

catenary-wok; when condensation of stream water occurs on the catenary, recirculation of the water, throughout the hydrologic cycle, is hastened. Use of the catenary shape ensures that water condensing on this clear plastic sheet, as a result of evaporation, will slide to the lowest point and drop into a reservoir which might be built on the mountain top. (The dynamics of air flow under the double-wok might be similar to that of the circulation of global air masses in Hadley cells [8].) When enough water accumulates, run it down the mountain, past a hydroelectric plant near the base; then, recycle it. While this arrangement might speed up water movement within this 'small' closed system, it does not add water within the entire system.

5. Solar woks might have possible uses in satellite broadcasting of solar energy. The tracking, orientation, and coatings required for such a project might have terrestrial parallels.

6. The production of animal protein generally requires large spaces of land. The breeding of fresh-water shrimp is land intensive, but requires ponds of constant temperature [1]. Solar woks might provide a cheap source of energy to supplement a traditional heating source in keeping water temperature constant [1]. If ponds can be stacked in a "skyscraper," the "hydrologic accelerator" described above might be employed to circulate water through the system.

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A PNEUMATIC POSTAL PLAN: THE CHAMBERED INTERCHANGE AND ZIPPR CODE*

INTRODUCTION

Congestion within urban networks appears to be at a level that threatens continuing network efficiency, whether such networks arise in response to the commuting needs or to the communicating needs of the underlying population. The United States Postal Service exhibits this difficulty: rate increases for network use occur more closely spaced in time than previously, yet the quality of service seems unchanged. At the local level, movement of the mail relies largely on trucks, cars, and postmen while at the interregional or national level, movement of the mail within the numerical ZIP (Zoning Improvement Plan) Code hierarchy depends mainly on airplanes and interstate tractor trailers. Evidence of congestion at the local urban scale is supplied by the recent increase from a five digit to a nine digit ZIP Code, in which the extra digits represent increased local refinement in delivery. Evidence of congestion at the interregional or national scale is suggested by the presence of numerous, competitive, private messenger services assuring overnight delivery of letters and packages between remote points in the continental United States.

Beginning in the mid-nineteenth century in Western Europe and in the late nineteenth century in the United States of America, the contemporary level of congestion within urban postal systems was considered sufficiently severe to threaten quality and efficiency of service [1]. Postal officials attempted to remedy contemporary congestion in these systems by supplementing urban surface mail routes with a network of pneumatic mail tubes installed urderground, or separated in other ways, from the established surface route systems [6].

Pneumatic technology was attractive as supplementary technology primarily because it permitted continuous transmission of materials and constant availability of the system with no pile-ups. (Also during this period, department stores used pneumatic tubes to send cash from one store location to another, postal checking services used pneumatic tubes to send cash from secure locations to areas serving the public, and an amusement park in Chicago had a ride in which people were propelled through pneumatic tubes [1].) Pneumatic postal networks pre-dated the widespread use of automobiles and the telephone system; indeed, as the automobile and networks associated with it became dominant, pneumatic tube technology did not keep pace, and frequently pneumatic postal networks were abandoned [5]. More current uses of this technology are in evidence in a wide range of applications: garbage in selected Russian apartment complexes is carried from individual apartments to disposal sites through a pneumatic system [4]; serum and various other medical supplies are dispatched from one hospital location to another in some U.S. hospitals; banks use pneumatic delivery for the local transmission of cash to tellers at drive-through windows, and some libraries with closed-stacks have, in the recent past, used pneumatic tubes to send book-orders, and sometimes books, through layers of stack floors [3]. What is common to all, is the use of this technology when 1) efficiency and security of local delivery is required; and, 2) the transmission of actual documents or goods, rather than a computerized replication of them, is required.

During the late 19th and early 20th century, the pneumatic tube was seen as a controversial, but technologically sophisticated, device. The proposal that follows attempts to project that radically "modern" idea of 1900, into a spatial framework suitable for application of current and future technology.

SPATIAL MECHANICS OF THE PROPOSED NETWORK

In handling some of the volume of local mail normally moved by truck, a pneumatic postal network might supplement the usual manner of mail distribution and therefore speed up local delivery within a postal sectional center by reducing surface congestion. General evidence to support an approach which combines fixed-position with movable carriers, may be garnered from the oil industry which appears to use efficiently both movable carriers, such as trucks and ships, as well as fixed carriers, such as pipeline, to distribute its product. (Indeed, one commercial oil company carries the slogan "Pipeline on wheels" on the sides of its trucks.)

In early pneumatic postal installations, the tubes used to conduct the channels of air had rigid form. Pressurized air can shape the form of its container as it does, at any given instant, in a toy balloon; previous rigid shaping is unnecessary. To remove this redundancy in form, the envisioned pneumatic network would be composed of tubes, made of non-stretchable flexible plastic, that are to be inflated by the channel of air carrying the mail (not unlike the air-inflated dome on the football stadium of the Detroit Lions in Pontiac, Michigan). Flexible plastic tubing would be cheaper than traditional rigid, preformed metal tubing; large amounts of it could be transported easily for its flexibility would permit it to be flattened out and rolled up on giant spools. It would be significantly lighter in weight per unit than metal tubing, and therefore could be used in a wide variety of situations. Placement of such tubing along telephone lines and along channels of limited access, such as expressways and canals, would expose the system physically, and would provide relative ease and cheapness of initial installation and continuing repair.

Another means of reducing initial costs, and one that permits direct connection of places not available through most usual transportation routes, is to suspend the flexible air-filled plastic network in a large body of water. In loose analogy with the human circulatory system, water pressure could be used to shut down parts of the system not in use, as long as valves had been installed at or near bifurcation points; use of water pressure would also reduce continuing energy costs. Then spatial freedom to arrange tubes optimally with respect to construction cost would be largely dependent on minimizing total length of tubing used, and marginally dependent on physical site characteristics such as seasonal variation in water temperature, location of surface shipping channels, and interference from vegetation and animal life.

If the network were assembled in segments and sealed, in zipper fashion, it could be changed when necessary, without extensive repair and demolition costs. Segments of the system that are used heavily could be removed easily and replaced regularly. Expansion of pneumatic trunk lines to accommodate an increase in volume of flow (resulting from a shift in central importance of a particular location) could be achieved by increasing tube radius with an additional longitudinal section of tubing zippered into the original tube.

The Chambered Interchange

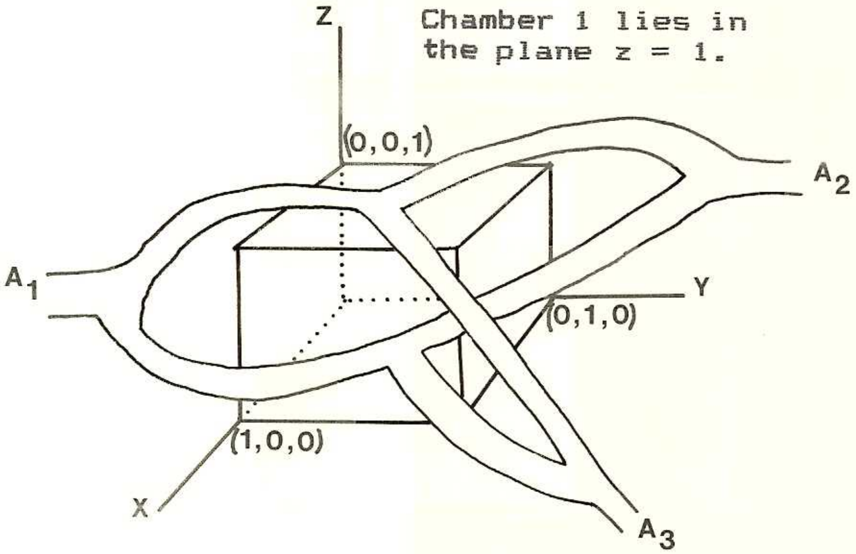
Networks of any sort are composed basically of linear elements and of interchange elements which permit the shunting of flow from one set of linear elements to another. In order to permit bidirectional flow in the proposed pneumatic network, a medial wall might be inserted in both linear and interchange elements (presumably it is cheaper to construct tubing with a medial wall than it is to place two separate tubes next to each other). Partitioning of this sort assures that no message collision, between opposing streams of flow, can occur in a linear element. Withn an interchange element with medial walls, collisions could occur at an infinite number of positions; assurance that message collisions between opposing flows could not occur would require that only one half of the available paths be accessible at each interchange--clearly this would not be a desirable situation. Further extensive partitioning of the interchange element or nesting of tubing within it might lead to a solution to the problem. However, another approach, based on interchange elements with simple medial walls (which retains the relative low-cost benefit associated with simplicity of construction) is presented below.

As a carrier enters an interchange, it is shunted to a tube on the left or to one on the right; let a zero represent movement to the right, and a one, movement to the left. Suppose two interchange elements with medial walls are placed vertically above one another and are joined to appropriate linear elements (Figure 1). This forms a chambered interchange, with the upper chamber lying in the plane $z = 1$ and the lower chamber in the plane $z = 0$. The joining points, or points of entry to the interchange, are labelled A_1, A_2, and A_3. The proposed chambered interchange, using binary precoding of messages, would work as follows.

Suppose a message approaches A_1; if it is to be routed to A_3, the tube to the right of A_1, then it will be sent through the lower level, or the chamber in $z = 0$. If it is to pass into the linear element attached at A_2, it would be sent through the upper level, or the chamber in $z = 1$ (Figure 1). Such a system permits bidirectional movement and, abstractly, continuous flow through the system part of the time. Collisions between opposing streams of flow can no longer occur at an infinite number of positions within the interchange; the positions at which collisions can occur has been reduced to three--at points of entry, A_1, A_2, A_3.

For example, suppose that a carrier coded 1 enters the upper chamber at A_1 and is sent to A_2. At the same time, a carrier coded 0 enters the lower chamber at A_3 and is sent to A_2. Then the carriers from A_1 and A_3 could reach A_2 simultaneously and collide

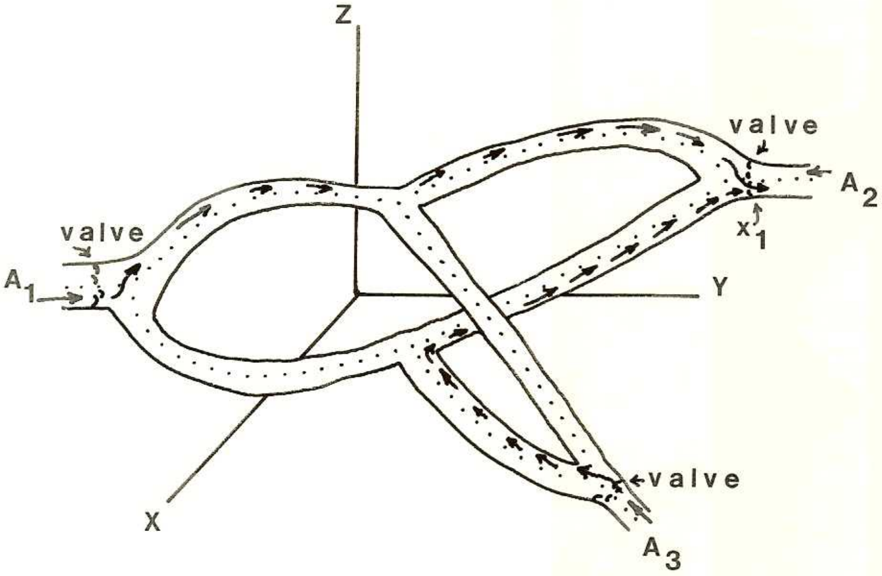
Figure 1: Chambered interchange--view of position of chambers.



Chamber 1 lies in
the plane z = 1.

Chamber 0 lies in
the plane z = 0.

Figure 2: Chambered interchange--view of flow through it.



upon entering the linear element attached at A_2 (Figure 2). In this example, both carriers were to pass through the point of attachment of the chambered interchange at A_2; a closer look at A_2 shows that in fact the collision would be forced to take place at x_1, where the segments of the upper and the lower chambers hook into the portion of the linear tube conveying carriers away from A_2 (Figure 3).

To avoid difficulties with carrier collisions such as those described above, suppose that the idea of carrier collisions is admitted as an acceptable possibility, so long as such collisions do not interfere with the functioning of the system. To realize this idea, suppose that a pneumatic carrier is formed from two concave disc-like structures that are made of resilient material. The concave structures are placed back-to-back and are connected; the messages are placed between the central depressions. The general appearance of the carrier is similar to that of a red corpuscle (Figure 4); when these carriers collide, they simply bounce off each other and continue through the system. The diameter of such a carrier is to be slightly greater than one-half the tube diameter. Thus, colliding carriers cannot clog the tubes, stop the air flow, and trap other carriers to form a clot. In addition, air can pass beyond one carrier and provide thrust to those downstream from the air source. (Attachment of streamlining structure might help to focus air after it has passed a given carrier.) With the carrier-diameter greater than one-half the tube-diameter, carriers must move single-file through the tube, although they may tend to travel in chains, as do red corpuscles.

Figure 3: Detail of attachment of a chambered interchange to a linear element. Notation employed suggests an enlargement of Figure 2 at vertex A_2.

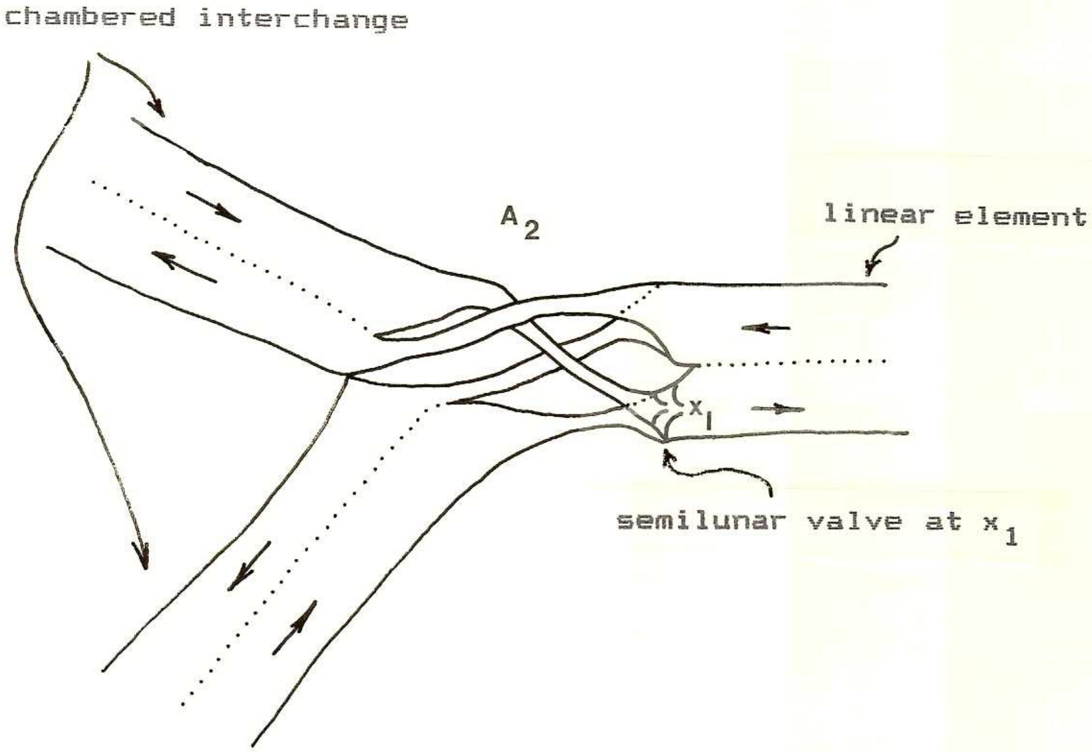
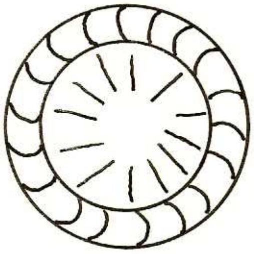


Figure 4: Resilient, corpuscular, pneumatic carrier.



Cross-section of a circular pneumatic carrier.

To prevent the backup of the carrier-chains into the heart-like chambered interchange, semilunar valves might be installed at points with potential for collision, such as x_1 (Figure 3). Lines of material that support this valve structure might follow geodesics of the valve surface, as do muscle fibers in human heart valves [2].

The chambered interchange overcomes problems of collision between opposing streams of message flow through reduction, from an infinite number to three, of the number of positions where collisions can occur; collisions at these points can be tolerated when resilient carriers, of the sort described above, are used in conjunction with slight additions (valves) to the chambered interchange. Together with appropriate scheduling of tube use, such as binary precoding of messages indicating routing through a sequence of chambered interchanges, this scheme could provide a method to permit bidirectional use of the system with a relatively large number of messages within the system at any given time.

ZIPPR Code

A sequence of binary digits might be used to precode the actual path of a carrier through the pneumatic system. It could assign direction through each chambered interchange, in the manner described above, in the sequence of chambered interchanges to be encountered on the trip from origin to destination. A set of chambered interchanges and associated linear tubing, forming a small pneumatic network, might be hooked into the extended ZIP Code to supplement local delivery of the mail. Thus, a Pneumatic Routing (PR) zone could be a subregion of a ZIP Code area consisting of one dominant post office together with the set of houses and businesses it serves. Boundaries induced by the partitioning of ZIP Code areas into Pneumatic Routing zones might be drawn along existing surface network patterns in a manner consistent with specifications of the Postal Manual. (The benefit of embedding pneumatic tubes in previously existing channels in the urban landscape is clear. Directness and rights of access to such channels would be vital to minimizing initial as well as continuing network costs in a heavily built up urban area.) Once such boundaries have been drawn, the extended ZIP Code, extended still further by a Pneumatic Routing Code, could assign to each household or business-office a unique ZIPPR Code. Subscribers to pneumatic postal service might transmit messages themselves--not merely their content--across town, simply by pressing an appropriate sequence of binary numerals, in much the way that users of cable television and personal computers have direct access to a variety of networks outside their homes. (Education of the public concerning non-abusive use of such a system might be a significant task.) Such a system could function in addition to the regular Postal Service delivery schedule, or as do private couriers (at the interregional and national scale), it could compete with the delivery system provided by the Postal Service at the local scale.

SPECULATION BEYOND CURRENT TECHNOLOGY

Rigidity was characteristic of previous pneumatic systems; substitution of flexible tubes for rigid tubes led to a pneumatic network with expanded potential. Abandonment of inelastic material for tubing elements might permit more extensive expansion of this potential, and might foreshadow the creation of networks similar to the ones described below.

Soft plastic contact lenses are apparently capable of altering their form in response to changes in eye fluid. Thus, one might envision a flexible plastic pneumatic network with sufficient elasticity to respond to changes in the landscape that contains it. An appropriate first attempt at such extension of analogy might be to form soft plastic networks under water that respond to pressure changes in accord with the volume of mail forced through the tubing. Further extension might see elastic flexible plastic pneumatic networks expanding and shrinking according to elasticity of demand for network use. Eventually one might be led to the idea of a network that responds not only to changes in the physical or economic landscape, but also is capable of growth, regeneration of parts, and abandonment of useless parts. New branches might grow to service recent pockets of growth while branches associated with abandoned centers shrink and cut themselves off from the system; they might then be removed and placed in a suitable environment where, again, they would be capable of growth.

This growth might take place along paths of least resistance to formation of a network of minimal total length, which in the Euclidean case (for example), would be along lines of a Steiner network. Formal techniques for describing and predicting such growth would be essential to monitoring and controlling these networks. For it can be demonstrated (via Steiner transformations [*]) that such a network would have the potential to choke an entire region through infinite regeneration.

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*Based, in part, on material from the author's unpublished Ph.D. dissertation, "On Geographical Network Location Theory," Department of Geography, The University of Michigan, 1977.

ENDPIECE

A child in a tire-swing observes, from a variety of perspectives, the variation in pattern on the earth below. So too the theoretical mathematical-geographer, within the security of the logical shell of pure mathematics, can swing across a variety of fields. The realization of mathematical theory as geographical theory will require a sensitive back-and-forth combination of instinct in the selection of mathematical tools and precision in the knowledge of how to apply them to the earth. Such application of theoretical mathematics, is an art; attempts to communicate art reveal only one, or several, aspects of the underlying abstract form. Fusing pure mathematics with geography reveals the magnificence behind this form of creation. When feelings for this beauty and power remain--in fact abound--future swing-trips will yield deeper ties of mind and heart, thereby issuing a challenge to communicate the mass of ideas that springs forth.

INSTITUTE OF MATHEMATICAL GEOGRAPHY (IMaGe)
2790 BRIARCLIFF
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A. Einstein

MONOGRAPH SERIES

Exclusive of shipping and handling; prices listed and payable in U.S. funds on a U.S. bank, only.

1. Sandra L. Arlinghaus and John D. Nystuen. *Mathematical Geography and Global Art: the Mathematics of David Barr's "Four Corners Project,"* 1986.

This monograph contains Nystuen's calculations, actually used by Barr to position his abstract tetrahedral sculpture within the earth. Placement of the sculpture vertices in Easter Island, South Africa, Greenland, and Indonesia was chronicled in film by The Archives of American Art for The Smithsonian Institution. In addition to the archival material, this monograph also contains Arlinghaus's solutions to broader theoretical questions–was Barr's choice of a tetrahedron unique within his initial constraints, and, within the set of Platonic solids?

2. Sandra L. Arlinghaus. *Down the Mail Tubes: the Pressured Postal Era, 1853-1984*, 1986.

The history of the pneumatic post, in Europe and in the United States, is examined for the lessons it might offer to the technological scenes of the late twentieth century. As Sylvia L. Thrupp, Alice Freeman Palmer Professor Emeritus of History, The University of Michigan, commented in her review of this work "Such brief comment does far less than justice to the intelligence and the stimulating quality of the author's writing, or to the breadth of her reading. The detail of her accounts of the interest of American private enterprise, in New York and other large cities on this continent, in pushing for construction of large tubes in systems to be leased to the government, brings out contrast between American and European views of how the new technology should be managed. This and many other sections of the monograph will set readers on new tracks of thought."

3. Sandra L. Arlinghaus. *Essays on Mathematical Geography*, 1986.

A collection of essays intended to show the range of power in applying pure mathematics to human systems. There are two types of essay: those which employ traditional mathematical proof, and those which do not. As mathematical proof may itself be regarded as art, the former style of essay might represent "traditional" art, and the latter, "surrealist" art. Essay titles are: "The well-tempered map projection," "Antipodal graphs," "Analogue clocks," "Steiner transformations," "Concavity and urban settlement patterns," "Measuring the vertical city," "Fad and permanence in human systems," "Topological exploration in geography," "A space for thought," and "Chaos in human systems–the Heine-Borel Theorem."

4. Robert F. Austin, *A Historical Gazetteer of Southeast Asia*, 1986.

Dr. Austin's Gazetteer draws geographic coordinates of Southeast Asian place-names together with references to these place-names as they have appeared in historical and literary documents. This book is of obvious use to historians and to historical geographers specializing in Southeast Asia. At a deeper level, it might serve as a valuable source in establishing place-name linkages which have remained previously unnoticed, in documents describing trade or other communications connections, because of variation in place-name nomenclature.

5. Sandra L. Arlinghaus, *Essays on Mathematical Geography–II*, 1987.

Written in the same format as IMaGe Monograph #3, that seeks to use "pure" mathematics in real-world settings, this volume contains the following material: "Frontispiece–the Atlantic Drainage Tree," "Getting a Handel on Water-Graphs," "Terror in Transit: A Graph Theoretic Approach to the Passive Defense of Urban Networks," "Terrae Antipodum," "Urban Inversion," "Fractals: Constructions, Speculations, and Concepts," "Solar Woks," "A Pneumatic Postal Plan: The Chambered Interchange and ZIPPR Code," "Endpiece."

6. Pierre Hanjoul, Hubert Beguin, and Jean-Claude Thill, *Theoretical Market Areas Under Euclidean Distance*, 1988. (English language text; Abstracts written in French and in English.)

Though already initiated by Rau in 1841, the economic theory of the shape of two-dimensional market areas has long remained concerned with a representation of transportation costs as linear in distance. In the general gravity model, to which the theory also applies, this corresponds to a decreasing exponential function of distance deterrence. Other transportation cost and distance deterrence functions also appear in the literature, however. They have not always been considered from the viewpoint of the shape of the market areas they generate, and their disparity asks the question whether other types of functions would not be worth being investigated. There is thus a need for a general theory of market areas: the present work aims at filling this gap, in the case of a duopoly competing inside the Euclidean plane endowed with Euclidean distance.

(Bien qu'ébauchée par Rau dès 1841, la théorie économique de la forme des aires de marché planaires s'est longtemps contentée de l'hypothèse de coûts de transport proportionnels à la distance. Dans le modèle gravitaire généralisé, auquel on peut étendre cette théorie, ceci correspond au choix d'une exponentielle décroissante comme fonction de dissuasion de la distance. D'autres fonctions de coût de transport ou de dissuasion de la distance apparaissent cependant dans la littérature. La forme des aires de marché qu'elles engendrent n'a pas toujours été étudiée ; par ailleurs, leur variété amène à se demander si d'autres fonctions encore ne mériteraient pas d'être examinées. Il paraît donc utile de disposer d'une théorie générale des aires de marché : ce à quoi s'attache ce travail en cas de duopole, dans le cadre du plan euclidien muni d'une distance euclidienne.)

7. Keith J. Tinkler, Editor, *Nystuen—Dacey Nodal Analysis*, 1988.

Professor Tinkler's volume displays the use of this graph theoretical tool in geography, from the original Nystuen—Dacey article, to a bibliography of uses, to original uses by Tinkler. Some reprinted material is included, but by far the larger part is of previously unpublished material. (Unless otherwise noted, all items listed below are previously unpublished.) Contents: " 'Foreward' " by Nystuen, 1988; "Preface" by Tinkler, 1988; "Statistics for Nystuen—Dacey Nodal Analysis," by Tinkler, 1979; Review of Nodal Analysis literature by Tinkler (pre–1979, reprinted with permission; post—1979, new as of 1988); FORTRAN program listing for Nodal Analysis by Tinkler; "A graph theory interpretation of nodal regions" by John D. Nystuen and Michael F. Dacey, reprinted with permission, 1961; Nystuen—Dacey data concerning telephone flows in Washington and Missouri, 1958, 1959 with comment by Nystuen, 1988; "The expected distribution of nodality in random (p, q) graphs and multigraphs," by Tinkler, 1976.

8. James W. Fonseca, *The Urban Rank-size Hierarchy: A Mathematical Interpretation*, 1989.

The urban rank–size hierarchy can be characterized as an equiangular spiral of the form $r = ae^{\theta \cot \alpha}$. An equiangular spiral can also be constructed from a Fibonacci sequence. The urban rank–size hierarchy is thus shown to mirror the properties derived from Fibonacci characteristics such as rank–additive properties. A new method of structuring the urban rank–size hierarchy is explored which essentially parallels that of the traditional rank–size hierarchy below rank 11. Above rank 11 this method may help explain the frequently noted concavity of the rank–size distribution at the upper levels. The research suggests that the simple rank–size rule with the exponent equal to 1 is not merely a special case, but rather a theoretically justified norm against which deviant cases may be measured. The spiral distribution model allows conceptualization of a new view of the urban rank–size hierarchy in which the three largest cities share functions in a Fibonacci hierarchy.

9. Sandra L. Arlinghaus, *An Atlas of Steiner Networks*, 1989.

A Steiner network is a tree of minimum total length joining a prescribed, finite, number of locations; often new locations are introduced into the prescribed set to determine the minimum tree. This Atlas explains the mathematical detail behind the Steiner construction for prescribed sets of n locations and displays the steps, visually, in a series of Figures. The proof of the Steiner construction is by mathematical induction, and enough steps in the early part of the induction are displayed completely that the reader who is well–trained in Euclidean geometry, and familiar with concepts from graph theory and elementary number theory, should be able to replicate the constructions for full as well as for degenerate Steiner trees.

10. Daniel A. Griffith, *Simulating $K = 3$ Christaller Central Place Structures: An Algorithm Using A Constant Elasticity of Substitution Consumption Function*, 1989.

An algorithm is presented that uses BASICA or GWBASIC on IBM compatible machines. This algorithm simulates Christaller $K = 3$ central place structures, for a four–level hierarchy. It is based upon earlier published work by the author. A description of the spatial theory, mathematics, and sample output runs appears in the monograph. A digital version is available from the author, free of charge, upon request; this request must be accompanied by a 5.5–inch formatted diskette. This algorithm has been developed for use in Social Science classroom laboratory situations, and is designed to (a) cultivate a deeper understanding of central place theory, (b) allow parameters of a central place system to be altered and then graphic and tabular results attributable to these changes viewed, without experiencing the tedium of massive calculations, and (c) help promote a better comprehension of the complex role distance plays in the space–economy. The algorithm also should facilitate intensive numerical research on central place structures; it is expected that even the sample simulation results will reveal interesting insights into abstract central place theory.

The background spatial theory concerns demand and competition in the space–economy; both linear and non–linear spatial demand functions are discussed. The mathematics is concerned with (a) integration of non–linear spatial demand cones on a continuous demand surface, using a constant elasticity of substitution consumption function, (b) solving for roots of polynomials, (c) numerical approximations to integration and root extraction, and (d) multinomial discriminant function classification of commodities into central place hierarchy levels. Sample output is presented for contrived data sets, constructed from artificial and empirical information, with the wide range of all possible central place structures being generated. These examples should facilitate implementation testing. Students are able to vary single or multiple parameters of the problem, permitting a study of how certain changes manifest themselves within the context of a theoretical central place structure. Hierarchical classification criteria may be changed, demand elasticities may or may not vary and can take on a wide range of non–negative values, the uniform transport cost may be set at any positive level, assorted fixed costs and variable costs may be introduced, again within a rich range of non–negative possibilities, and the number of commodities can be altered. Directions for algorithm execution are summarized. An ASCII version of the algorithm, written directly from GWBASIC, is included in an appendix; hence, it is free of typing errors.

11. Sandra L. Arlinghaus and John D. Nystuen, *Environmental Effects on Bus Durability*, 1990.

This monograph draws on the authors' previous publications on "Climatic" and "Terrain" effects on bus durability. Material on these two topics is selected, and reprinted, from three published papers that appeared in the *Transportation Research Record* and in the *Geographical Review*. New material concerning "congestion" effects is examined at the national level, to determine "dense," "intermediate," and "sparse" classes of congestion, and at the local level of congestion in Ann Arbor (as suggestive of how one might use local data). This material is drawn together in a single volume, along with a summary of the consequences of all three effects simultaneously, in order to suggest direction for more highly automated studies that should follow naturally with the release of the 1990 U. S. Census data.

12. Daniel A. Griffith, Editor. *Spatial Statistics: Past, Present, and Future*, 1990.

Proceedings of a Symposium of the same name held at Syracuse University in Summer, 1989. Content includes a Preface by Griffith and the following papers:

Brian Ripley, "Gibbsian interaction models";
J. Keith Ord, "Statistical methods for point pattern data";
Luc Anselin, "What is special about spatial data";
Robert P. Haining, "Models in human geography:
    problems in specifying, estimating, and validating models for spatial data";
R. J. Martin, "The role of spatial statistics in geographic modelling";
Daniel Wartenberg, "Exploratory spatial analyses: outliers, leverage points, and influence functions";
J. H. P. Paelinck, "Some new estimators in spatial econometrics";
Daniel A. Griffith, "A numerical simplification for estimating parameters of spatial autoregressive models";
Kanti V. Mardia "Maximum likelihood estimation for spatial models";
Ashish Sen, "Distribution of spatial correlation statistics";

Sylvia Richardson, "Some remarks on the testing of association between spatial processes";
Graham J. G. Upton, "Information from regional data";
Patrick Doreian, "Network autocorrelation models: problems and prospects."

Each chapter is preceded by an "Editor's Preface" and followed by a Discussion and, in some cases, by an author's Rejoinder to the Discussion.

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ABOUT THE AUTHOR

Sandra Arlinghaus is Director of the Institute of Mathematical Geography. She is a graduate of Vassar College, in Mathematics, and has done graduate work in Mathematics at The University of Chicago, The University of Toronto, and Wayne State University, and earned a Ph.D. in Geography from the University of Michigan. She has held positions in universities in Mathematics as well as in Geography Departments. Most of her research centers on applications of pure mathematics to real-world problems; she has published in geography and psychology journals as well as in the IMaGe Monograph Series, has given papers at Mathematics and at Geography meetings, and has been invited to lecture at The University of Chicago and The University of Michigan. Her hobbies include cooking for friends and family, designing crossword puzzles and playing with geometric puzzles, and listening to the music of Mozart.