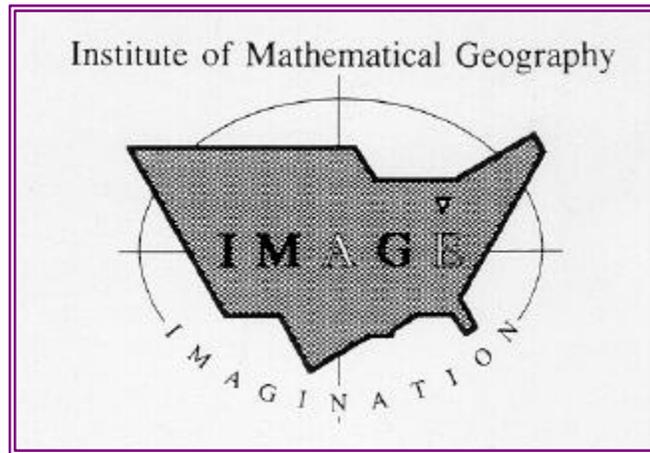


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VOLUME 20

**SOLSTICE VI:
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OF
GEOGRAPHY AND MATHEMATICS**



Ann Arbor, MI
1995

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TABLE OF CONTENTS

1. FIFTH ANNIVERSARY OF SOLSTICE
2. NEW FORMAT FOR SOLSTICE AND NEW TECHNICAL EDITOR.
3. MOTOR VEHICLE TRANSPORT AND GLOBAL CHLIMATE CHANGE:
POLICY SCENARIOS
RICHARD WALLACE
4. EXPOSITORY ARTICLE.
DISCRETE MATHEMATICS AND COUNTING DERANGEMENTS IN BLIND
WINE TASTINGS
JOHN D. NYSTUEN, SANDRA L. ARLINGHAUS, WILLIAM C. ARLINGHAUS
5. INDEX TO VOLUMES I (1990) TO V (1994). [see next issue of Solstice--back of
book]

1.

FIFTH ANNIVERSARY OF SOLSTICE

The current issue marks the beginning of the second half of the first decade of Solstice. Thanks to all who have participated in this project--editors, authors, and readers, alike!

Over the course of the past five years, Solstice has garnered attention from the media. If you know of additional citations, please share them with us for our electronic scrapbook! Thank you.

Science (AAAS) "Online Journals," [Joseph Palca] 29 November 1991, Vol. 254.

Science News "Math for all seasons" Ivars Peterson, Jan. 25, 1992, Vol. 141, No. 4.

Newsletter of the Association of American Geographers, June, 1992.

American Mathematical Monthly, September, 1992.

American Mathematical Monthly, September, 1992.

Harvard Technology Window, 1993.

Graduating Engineering Magazine, 1993.

Earth Surface Processes and Landforms, 18(9), 1993, p. 874.

On Internet, 1994.

Papers in Regional Science: The Journal of the Regional Science Association. "Wide Area Computer Networks and Scholarly Communication in Regional Science." Gunther Maier and Andreas Wildberger.

2.

NEW FORMAT FOR SOLSTICE AND NEW TECHNICAL EDITOR

With this issue, we work to make Solstice available to a wider readership. For the first five years, all articles were typeset using TeX, the typesetting program of Donald Knuth and the American Mathematical Society. Our goal is to continue to provide text that is available to a wide variety of readers; thus, we do transmit directly so that those without Gopher access can read Solstice. Surely one great advantage of e-mail is the ease with which it can deliver information to points remote from its source. We also wish to push the text delivery in the directions of current technology, as well.

Richard Wallace has kindly agreed to serve as Technical Editor of Solstice, working in conjunction with the Editor-in-Chief, to continue to develop innovative presentations that

take advantage of current technology. With this issue, we transmit a separate packet of figures to accompany the single text file. In the future, we hope to have World Wide Web access to Solstice, in addition to the direct delivery via e-mail and continuing archiving on a Gopher. When the mathematics used requires it, we intend to offer that notation within the direct e-mail transmission (as we have in the past).

3.

MOTOR VEHICLE TRANSPORT AND GLOBAL CLIMATE CHANGE:
POLICY SCENARIOS

Richard Wallace

The University of Michigan
Urban, Technological, and Environmental Planning

Driven largely by rapidly increasing atmospheric levels of greenhouse gases, such as carbon dioxide, methane, nitrous oxide, and chlorofluorocarbons (CFCs), the global climate appears to many observers (e.g., Meadows, Meadows, and Randers 1992) to be in a period of change. Computer models suggest that throughout most of the temperate zones of the world this change will take the form of rising temperatures. Motor vehicles, which are powered by fossil-fuel burning engines that emit carbon dioxide and nitrous oxide, contribute significantly to the production of potentially climate altering greenhouse gases. Simply put, carbon dioxide "is an inevitable byproduct of fossil fuel consumption and it streams out of tail pipes in direct proportion to the quantity of fuel burned" (Wilkinson 1993). On average, for every kilogram of standard motor vehicle fuel (gasoline) consumed, three kilograms of carbon dioxide are released into the atmosphere (Faiz 1993). Furthermore, motor vehicles emit other greenhouse gases, too, such as CFCs that leak from air conditioners. Thus, while improved fuel economy can have a mitigating effect on total greenhouse emissions, in general the greater the number of motor vehicles, and the more miles that they are driven, the larger their contribution to global climate change. Therefore, understanding trends in the number of vehicles registered and examining policy options to curb growth in this number can play a significant role in combating global climate change.

Worldwide, transport energy accounts for about 20 percent of total emissions of greenhouse gases (Lashof and Tirpak 1990), but this figure varies from nation to nation. In the OECD nations as a whole, transport contributes between 26 and 31 percent of all greenhouse emissions produced there, while in the U.S. transport accounts for 38 percent of domestic greenhouse emissions (International Energy Agency 1993). In the Less

Developed Countries (LDCs) and in Eastern Europe, largely due to lower rates of car ownership and use, the transport sector currently accounts for a smaller contribution to greenhouse emissions (Faiz 1993), but these nations represent a burgeoning new market for vehicles. As a result, greenhouse emissions from the LDCs are expected to rise in the near future.

By contributing significantly to greenhouse gas emissions and, therefore, to global climate change, the motor vehicle sector plays a role in the general class of phenomena known as population-environment dynamics. This dynamic, as described by Drake (1993), is characterized by transitions from one stable state to another. While the second stable state may be a more or less desirable state than the original condition, and it is often worse (e.g., the transition from forest to desert that is taking place in some regions), Drake argues, and offers supporting data, that the transition phase itself is a period of vulnerability, characterized by the potential for extremely negative outcomes. Transitions, however, also offer opportunities, and positive outcomes can occur, too, especially if appropriate policies are pursued to manage the transition. Thus, while human society and environmental systems both may survive under a new climatic regime, the transition period may prove to be the most dangerous period of all.

The field of population-environment dynamics thus leads us to see not only that transport affects the global environment, but also that transport emissions of greenhouse gases are not purely a function of the number of vehicles, miles driven, fuel use, and emissions technology. Worldwide, the ratio of people to motor vehicles varies considerably from nation to nation. While we might expect this ratio to be highly correlated with GNP per capita, a simple linear regression fails to detect a significant relationship between these two variables. Approaching this relationship geographically (see Figure 1), however, reveals a clear relationship--wealthier nations in general have a lower ratio of vehicles per person.

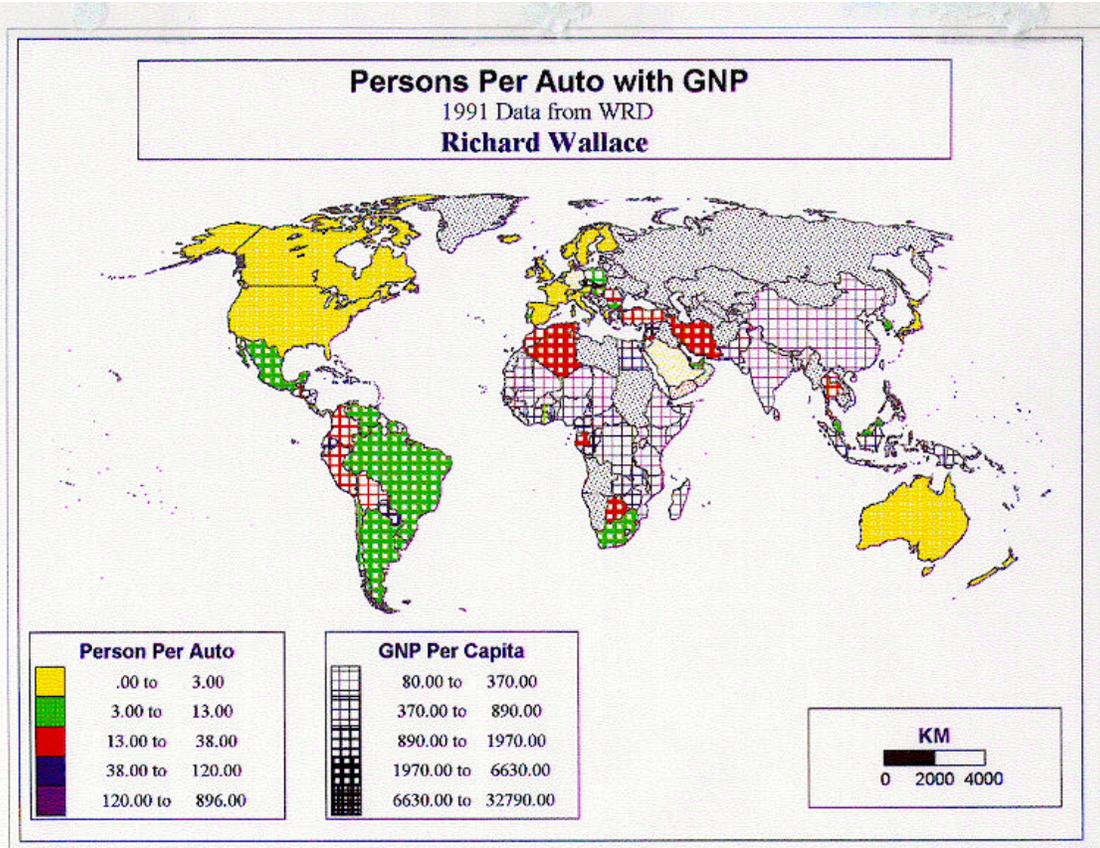


Figure 1.

Population growth, too, can have an effect on the number of vehicles. By examining trends and relationships in and between these technological and social factors, this paper seeks to investigate the efficacy of different policy options on reducing the quantity of greenhouse emissions from the transport sector. This analysis will be performed for six nations that typify the range of transport, economic, and population dynamics across the globe. By examining nations that differ in these key respects, the analysis will illustrate different dimensions of the dynamic and provide policy guidance tailored to the specific circumstances of each nation and beyond to the world community. The six nations examined here, and the patterns that they represent, are listed in Table 1.

Table 1.

Nation
Pattern

United States
Wealthy, high use of autos and energy

Japan
Wealthy, more emphasis on public transit and
more energy efficient

Hungary
Former Eastern Block, dirty cars

India
Less developed, low car ownership, booming
population

Mexico
Latin American, Industrializing

South Korea
Asian Newly Industrialized Country (NIC)

While no rigorous attempt will be made to justify this categorization, a few observations backed by data collected by the World Resources Institute (WRD) and the American Automobile Manufacturers Association (AAMA) provides it some legitimacy. First, vehicle technology varies substantially between the most highly industrialized nations and the rest of the world. The typical vehicle manufactured in Eastern Europe and the LDCs is only about half as fuel efficient as typical OECD-manufactured vehicles (Faiz 1993). Second, an examination of trends in the number of registered vehicles in Hungary, India, Mexico, and South Korea (see Figure 2) reveals a clear distinction between them, with the two industrializing nations (South Korea and Mexico) showing an especially steep growth curve, India showing a slower rate of growth, and Hungary showing little growth at all. Based on this evidence, these six nations do appear to represent distinct patterns.

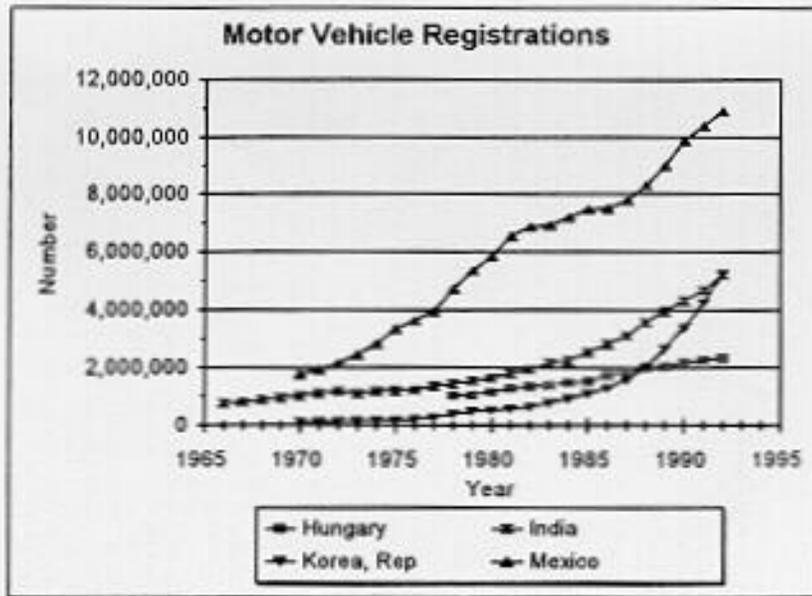


Figure 2. Source: AAMA.

Figure 2. Motor Vehicle Registrations in the six nations considered.

Patterns of Vehicle Use

Across the six nations listed in Table 1, reliance on motor vehicles for transportation varies greatly. In 1992, for example, India and South Korea each had roughly the same number of motor vehicles registered, but India's population was nearly twenty times that of South Korea. On the other hand, of these six nations, India, which had by far the highest ratio of people to vehicles in 1992, experienced the largest absolute drop in this figure over the last twenty years (see Figure 3, which is drawn to log scale so that all six nations may be viewed on one graph). Thus, while the U.S. and Japan currently are the largest contributors to greenhouse emissions from the transportation sector, population and consumption trends suggest that other nations will account for an ever-increasing percentage of transportation's contribution to greenhouse gas emissions in the future.

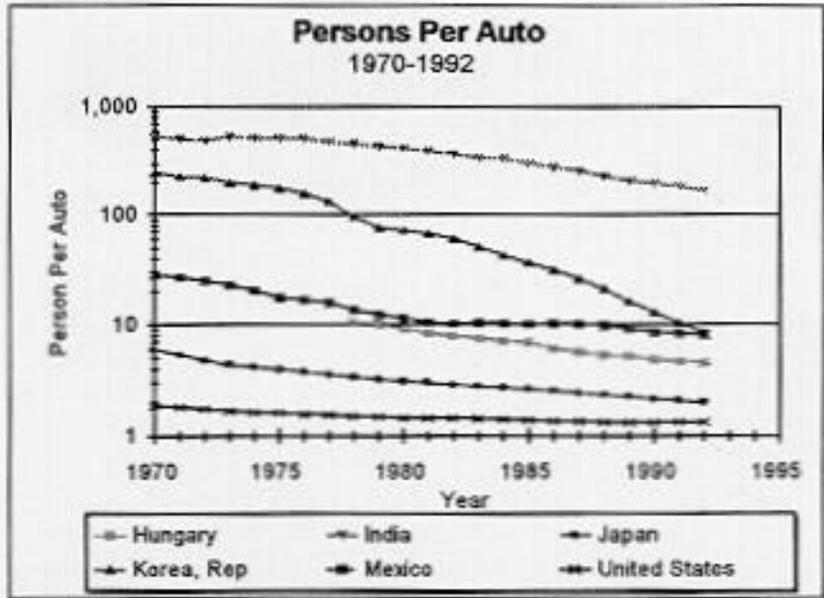


Figure 3. Sources: WRD and AAMA.

Figure 3 shows that all six nations, with the exception of the U.S., still are experiencing a decline in the ratio of people to vehicles. What is driving this trend, however, varies from nation to nation. In some cases, rising consumption, as measured by GNP growth, appears to be the driving factor, while in others increasing urbanization as measured by urban population, appears to be the culprit. Epitomizing these two dynamics are Japan, South Korea, and Hungary. In Japan (see Figure 4), the increase in vehicle registrations appears to have been driven largely by increases in population and urbanization in the early post-war years, with more recent gains appearing to be more associated with increased GNP. By comparison, South Korea (see Figure 5), displays a relationship between increased vehicle registrations and a rising GNP, with population appearing to have little effect. Finally, Hungary (see Figure 6) displays a combination of the two effects, with both urban population growth (despite a steady total population) and GNP growth associated with increased vehicle registrations during the study period.

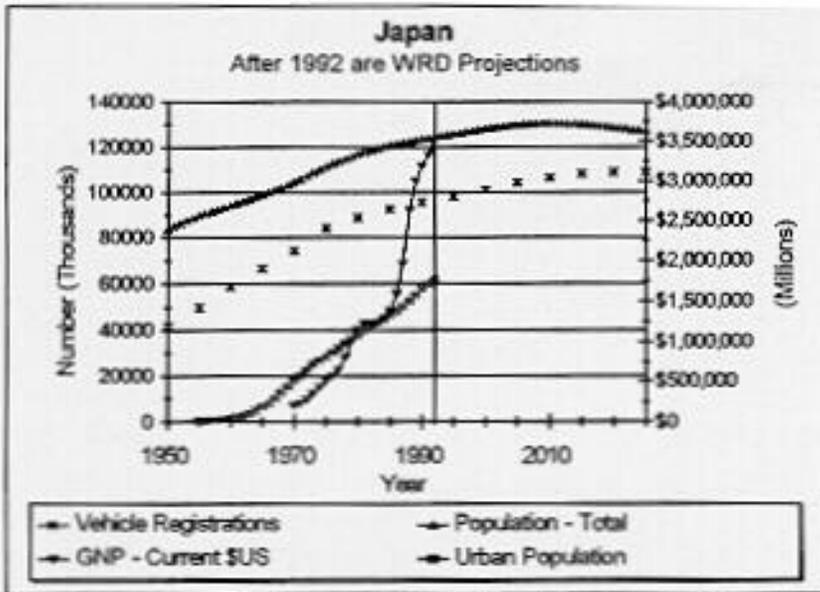


Figure 4. Sources: WRD and AAMA. Note that GNP is tied to the right-hand axis.

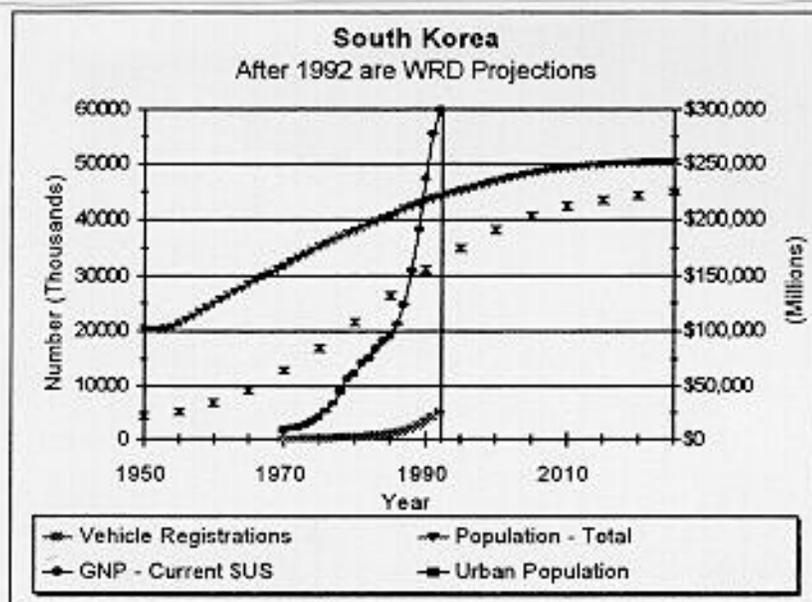


Figure 5. Sources: WRD and AAMA. Note that GNP is tied to the right-hand axis.

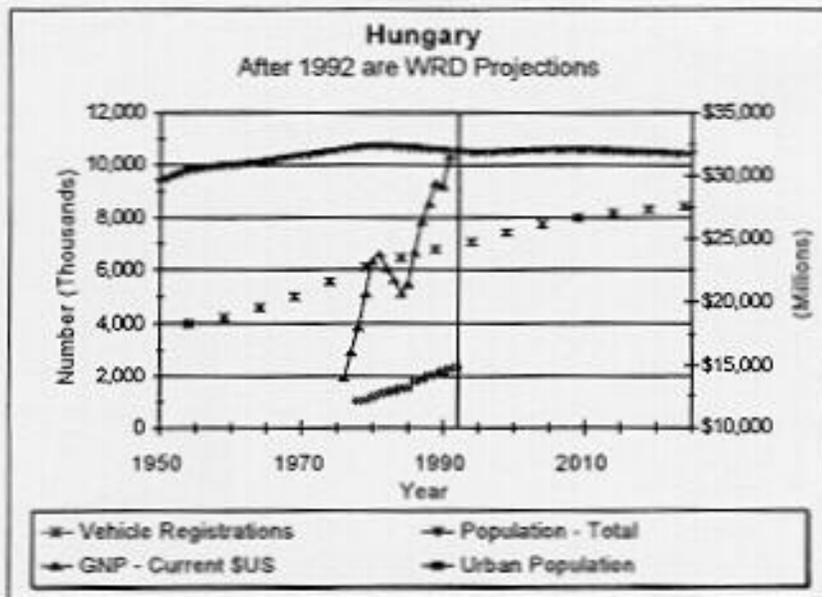


Figure 6. Sources: WRD and AAMA. Note that GNP is tied to the right-hand axis.

Policy Options

As described by Meadows, Meadows, and Randers (1992), environmental impacts can be viewed as a function of population, consumption patterns, and the state of technology. These variables also appear within the policy options available to reduce the contribution of the transportation sector to greenhouse gas emissions. Among these are: (1) increased fuel efficiency, (2) reliance on alternative fuels, (3) reliance on public transportation and other travel behavior approaches, (4) consumption limits, and (5) population-control policies. While these approaches range from technological fixes to changes in societal norms, even the technology-based approaches demand a corresponding societal component. Increasing fuel efficiency, for example, requires the political will to raise minimum standards, and increasing use of public transportation requires alteration of travel behavior. If we are to explore the likely effectiveness of each of these policies, we must first understand how and what each contributes to the reduction in emissions of greenhouse gases and, where possible, gauge how this dynamic might play out in the near future--not an easy undertaking.

Given a rather large body of research and literature in the field, gauging future technical abilities is perhaps the simplest forecasting task. DeLuchi (1993) has estimated the reduction in greenhouse gas emissions from a variety of alternative fuel sources. His findings indicate a broad range of outcomes depending on the source of the alternative power. Electric vehicles powered by coal-burning plants, for example, can be expected to lead to an increase in the amount of greenhouse emissions compared to a standard gasoline- or diesel-burning vehicle. Emissions reductions, however, would be realized from a variety of alternative fuels, including solar-powered electric, compressed natural gas, methanol (from wood), and ethanol (from several sources). The International Energy Agency (IEA; 1993) produced similar findings, also adding liquid hydrogen to the list of alternative fuels that would reduce greenhouse emissions. Using middle estimates from the latter source, anywhere from a 25 to 50 percent reduction in emissions appears feasible. Recent technological breakthroughs in the manufacture of photovoltaic panels, which suggest the possibility of producing solar energy at very competitive market rates by next year, promise to make this figure even higher (Myerson 1994).

The IEA also studied potential improvements to fuel economy and found that by 2006 a 10 to 20 percent improvement is feasible for OECD nations. This estimate is conservative and is based only on marginal improvements to currently employed vehicle technologies and materials. Other researchers, however, have cast aside such industry-bound restraints and discussed what could be done even today using technical inputs from beyond the traditional steel-centered perspective of the global auto industry. Typifying this approach, Lovins and Lovins (1994) tout the revolutionary potential of a new car design known as an ultralight hybrid. These light-weight vehicles, manufactured from high-tech composite materials such as carbon fiber, and powered by a combination of liquid fuel and a battery-powered flywheel that captures and stores energy now lost during braking, offer a tenfold improvement in fuel economy over today's typical car. These vehicles also offer equal or better safety, mostly because, pound for pound, these ultralight materials possess far superior crash resistance than does steel. As the Lovins's recognize, bringing such vehicles to market will require a major restructuring of the auto industry, but large auto companies that resist making these changes may soon be leapfrogged by other

manufacturers, such as former defense contractors, more experienced with these new materials.

Public transportation provides another alternative to reliance on motor vehicles. Although increased urbanization seems to be a factor associated with increased vehicle registrations, public transportation works best in urbanized areas. Which effect seems to dominate may be a consequence of urban density. As Newman and Kenworthy (1991) showed in their study of 32 of the world's major cities, per capita consumption of gasoline can vary considerably, even across affluent nations. They found that, generally, in high-density cities such as Tokyo, per capita gasoline consumption is far less (about 1/6) than in relatively low-density U.S. cities. Much of this difference can be explained by examining the rate of transit use between these cities. The average resident of Tokyo takes 472 transit trips per year, while the typical New Yorker takes 58 and a Detroitter only 17; vehicle ownership rates may be similar in Japan and the U.S., but vehicle use is lower in high density areas. Given the relationship between fuel consumption and greenhouse emissions, less vehicle use results in less greenhouse emissions.

Consumption limits represent perhaps the most difficult policy option to address. While draconian laws certainly hold out the promise of being effective in reducing reliance on motor vehicles, such approaches carry high social costs and appear incompatible with dominant social and political standards in much of the world. Fortunately, policies exist that achieve the goal of reduced consumption without serious intrusions on individual liberties. On the regulatory side, for example, implementation and enforcement of vehicle occupancy regulations can reduce the preponderance of singly-occupied vehicles. On the market-based side, high fuel taxes provide economic incentives for people to choose non-vehicular travel modes, as does the elimination of free parking in employment centers (Wachs 1981).

Finally, in those nations experiencing rapid population growth, policies aimed at slowing or reducing population growth may have some promise in reducing their transportation sector's contribution to greenhouse emissions. On the other hand, it may be that controlling population will increase wealth in these nations, thereby increasing demand for motor vehicles. Either way, India, with its large population of would-be

motorists, has an enormous potential to contribute to greenhouse emissions, and the ratio of people to motor vehicles in India has been falling.

Tailoring Policy to Population-Environment Conditions

Given differences in the forces influencing increased motor vehicle registrations, and therefore increased greenhouse emissions, the range of potentially effective policy options available to each nation also is likely to be different. Each of these policy options thus is best suited to a particular set of political, technological, economic, and societal conditions. A policy that promises to be effective in one country, therefore, may be inappropriate for another. Population-control measures, for example, appear to be an unlikely candidate to reduce greenhouse emissions from the transportation sector in nations such as Hungary that are experiencing little or no population growth. The task at hand, therefore, is to match policies to nations in the most effective manner. Doing so requires forecasting growth in the motor vehicle population, particularly the fossil-fuel powered motor-vehicle population, within the societal context of each nation.

To start this analysis, let us begin with the number one emitter of greenhouse gases via motor vehicles--the United States. Because the U.S. has so many more motor vehicles than any other nation, reducing emissions in the U.S. alone would signal progress toward worldwide reduction. As can be seen in Figure 7, the U.S. is undergoing a steep rise in GNP, along with a moderate rise in both total and urban population. The rate of increase in the number of registered automobiles, however, appears to be declining. The already enormous number of vehicles registered in the U.S. points to the need of reducing either the per-capita use of each vehicle or the amount of emissions per vehicle or both. This state of high wealth and an abundance of motor vehicles suggests pursuing one or more of the technological approaches listed above--California's mandate that two percent of vehicles sold be zero-emissions vehicles by 1998 (and 10 percent by 2005) is a good example--along with some modifications to travel behavior, such as increasing the importance of public transportation. Based on these policy pursuits, future vehicle registrations for the U.S. can be projected.

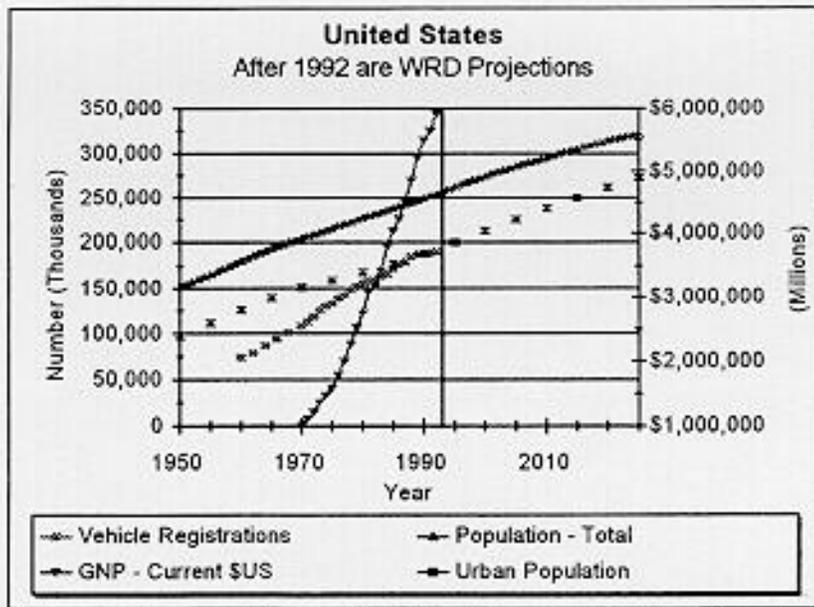


Figure 7. Sources: WRD and AAMA. Note that GNP is tied to the right-hand axis.

To begin forecasting, we first need to find a good fit to the current trend in vehicle registrations. From Figure 7, the growth in the number of motor vehicles in the U.S. appears to be S-shaped, suggesting a logistic (or other similar) function. Assuming a maximum value of 225,000,000 cars by 2025, an S-shaped curve can be fit successfully to the data. Furthermore, following California's lead, we can mandate that one percent of all vehicles registered be zero emissions by 1998. Next, we can go a step further and assume that the zero-emissions portion of the vehicle fleet will increase by one percent each year thereafter. As shown in Figure 8, this scenario implies a gradual decrease in the number of fossil-fuel powered vehicles, with this number falling below current levels by 2005 and continuing down to 160 million (more than 30 million below the current number) by 2025. While this decrease is modest, combining it with increased fuel economy in remaining fossil-fuel-powered vehicles, and at worst, no increase in vehicle miles traveled per vehicle, could multiply the result by a factor of ten or so. In effect, this means the equivalent of about 16 million of today's motor vehicles--a figure not seen in the U.S. since the mid-1920s.

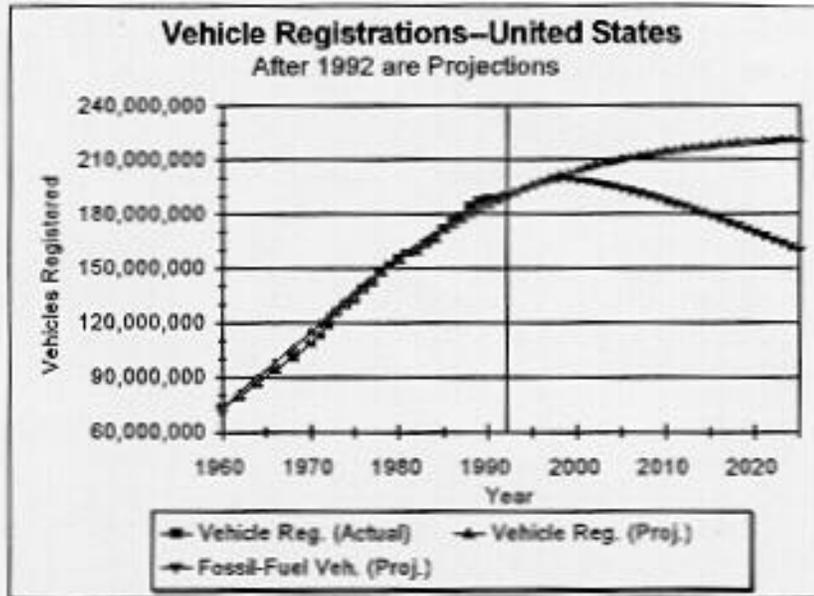


Figure 8. Source: AAMA.

For Japan, the other economic powerhouse in the analysis, technological solutions also are appealing. As can be seen in Figure 4, however, growth in motor vehicle registrations in Japan continues at what appears to be a linear rate (with $r^2=0.99$ when beginning with the year 1960, this trend is fit better by a straight line than either an exponential or logistic curve), driven by a growing GNP. If this linear trend continues, Japan will have about 125 million registered motor vehicles by 2025 (see Figure 9). Assuming a similar pattern of switching to zero-emissions vehicles as was forecast for the U.S., we find that the number of fossil fuel vehicles does begin to assume an S-shaped curve. Like the U.S., Japan would appear to have the technical and economic means to also pursue fuel-economy improvements, too. Again, another tenfold increase in effectiveness is possible, meaning the equivalent of a little more than 9 million of today's cars by 2025. As discussed earlier, Japan also displays less reliance on the cars that are registered. If this pattern also continues, then the effectiveness of Japanese efforts to reduce greenhouse emissions can be further enhanced.

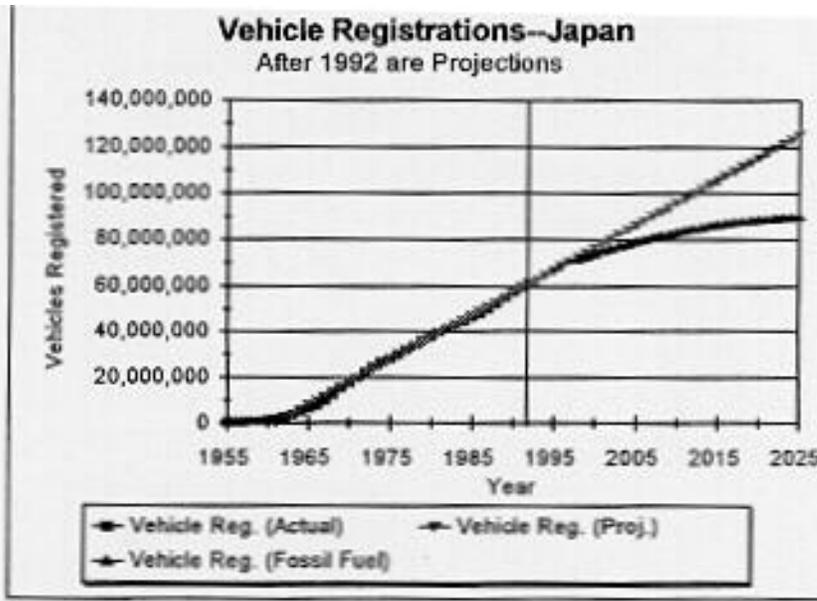


Figure 9. Source: AAMA.

In Hungary, as we saw in Figure 3, the population growth rate is close to zero. Nonetheless, the urban population continues to grow, as does the number of registered vehicles. Currently, however, Hungary has the fewest registered vehicles of all six nations included in this study, but many of these cars are relatively high polluting and inefficient vehicles based on the engine technology typical of Eastern European nations (Michelberger 1991). To a large extent, then, Hungary's contribution to greenhouse emissions is related to poor vehicle technology. Thus the ongoing replacement of its fleet of older, outdated vehicles with newer, Western-style vehicles offers a sure path to fewer overall emissions. This dynamic, however, competes with growth in the size of the vehicle fleet. This growth is fit well by both a linear and a logistic curve (with a maximum value of 5 million), with these two not diverging by much until around 2010 (see Figure 10). This result suggests that Hungarian transportation policy stands able to influence future growth in vehicles: Hungary can choose to maintain and improve alternate modes (especially public transit) or accept the consequences of a vehicle-centered society. Either way, however, Hungary will continue to have the smallest vehicle fleet in this study and can be expected to make only a small contribution to greenhouse emissions.

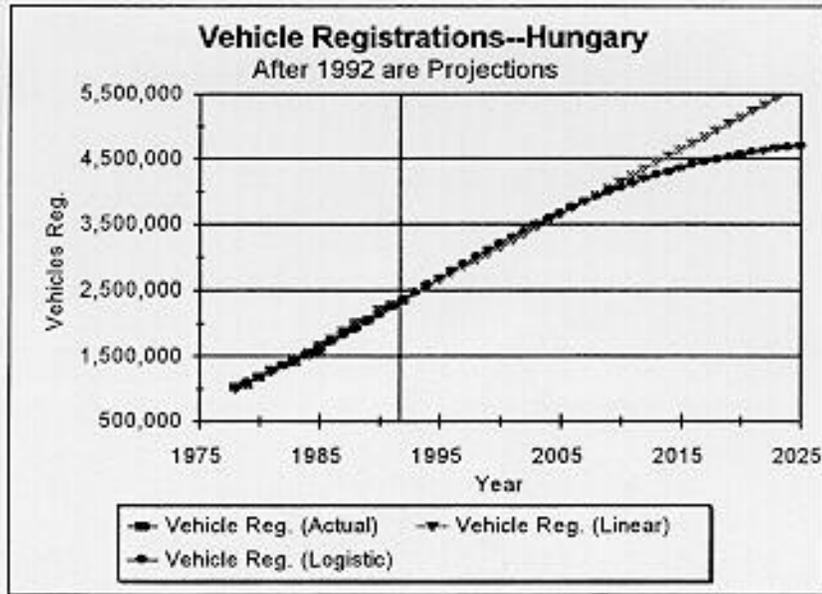


Figure 10. Source: AAMA.

India, which has yet to see its population growth rate begin declining, has been projected to soon overtake China as the world's most populous nation (see Figure 11). Relative to this enormous population, however, India has a very low number of registered vehicles. In the near term, the rate of growth in the vehicle population, however, is exponential, regardless of whether an exponential or logistic function (with a maximum value of either 140 or 200 million) is fit to the actual data (see Figure 12). Thus India's growth rate in this area may not be boundless, but current trends suggest that India eventually will meet or surpass the number of motor vehicles currently registered in the U.S.

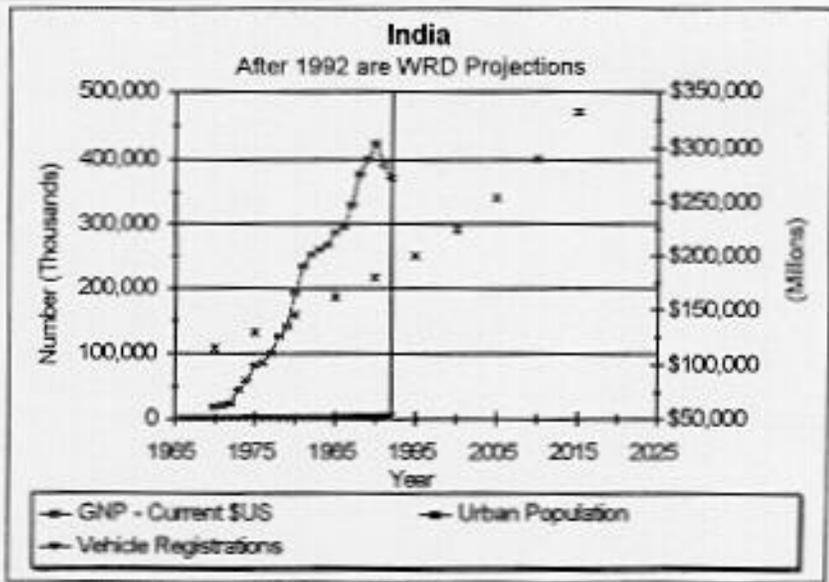


Figure 11. Sources: WRD and AAMA.

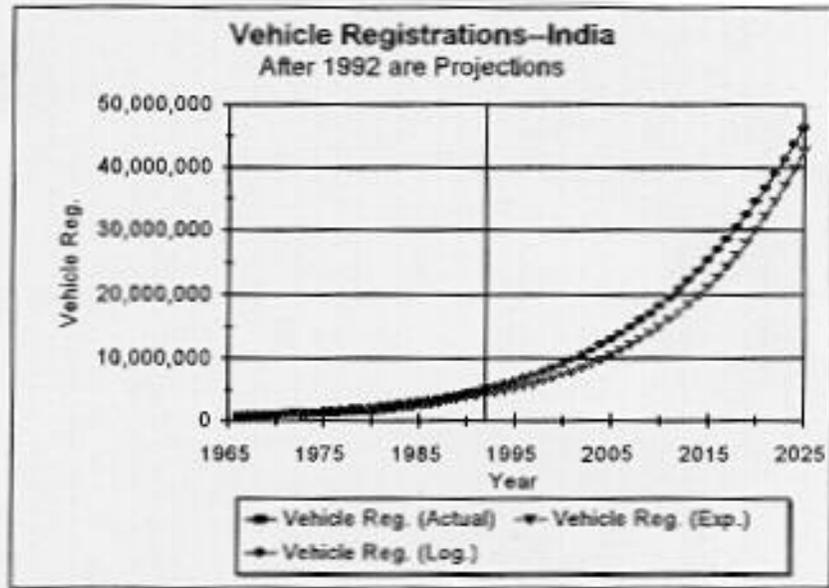


Figure 12. Source: AAMA.

Given this growth in vehicle registrations, India can be expected to experience increased negative consequences of fossil-fuel based travel, including worsening congestion and pollution, and to dramatically increase its contribution to greenhouse

emissions from the transportation sector. The level at which this growth is bounded, along with which technologies and policies India applies to mitigate these problems (e.g., alternative fuels, emissions controls, demand management), will determine the severity of future outcomes and India's contribution to greenhouse emissions. Unfortunately, however, India does not appear to have the economic strength to address this problem with technological approaches, save what may accrue naturally from advancing vehicle technology. Developing infrastructures for alternative fuels, for example, would appear unlikely in the near future. These leaves India with only one option from the above list of policies for addressing greenhouse gas emissions from transportation--travel behavior measures. In this regard, India already relies predominantly on non-motorized forms of personal transit in rural areas and on high vehicle-occupancy rates in urban areas. Restricting access to vehicle purchases--perhaps unpopular with the growing middle class--and financial disincentives, such as a high fuel tax, for vehicle travel both are options worth considering.

Excluding the U.S. and Japan, Mexico has far more registered vehicles than the other nations in this study and, as Figure 1 shows, trails only OECD nations and Australia in having a low people to vehicle ratio. As in India, in Mexico both the population and the motor-vehicle fleet are growing quickly, but Mexico possesses a much stronger economy than does India. As can be seen in Figure 13, Mexico's GNP has shown a strong increase--despite a brief setback in the early 1980s--over the last 25 years; meanwhile, both its total and urban population have grown steadily. During the period of economic decline in the early 1980s, motor vehicle registrations also stagnated, but did not decline. This suggests a strong and tight temporal relationship in Mexico between GNP and vehicle registrations ($r^2=0.87$ for a simple linear model since 1970 for a relationship that would seem to demand a time lag). Therefore, if Mexico's GNP continues to rise--perhaps as a result of NAFTA--then vehicle registrations also should soar, barring a policy intervention.



Figure 13. Sources: WRD and AAMA.

In many ways, the situation in South Korea closely resembles that in Mexico: GNP, vehicle registrations, population, and urban population all are rising--most of these rapidly. In South Korea, however, the economy did not decline in the early 1980s and the rate of population growth began declining shortly after 1950, while Mexico is just now beginning to see its rate of population growth decline. As a result, motor vehicle registrations have grown exponentially in South Korea with no break since 1970. Clearly, however, this growth cannot be unbounded, because it were it would lead to nearly one billion registered vehicles by 2025--about 200 cars per person! Therefore, we must assume a logistic growth rate, bounded at about 50 million, or one per person (see Figure 14).

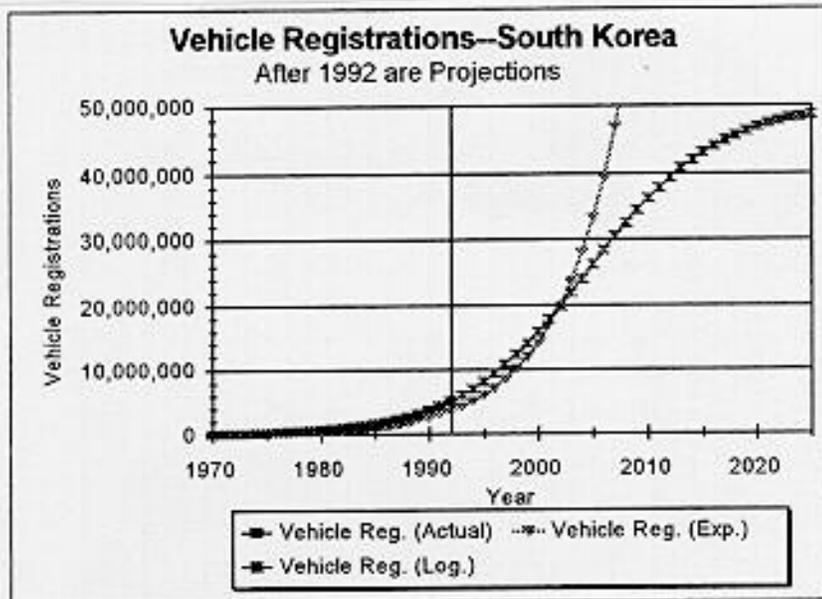


Figure 14. Source: AAMA.

This curve fit indicates a nearly tenfold increase in South Korea's motor vehicle fleet, which can be expected to increase South Korea's contribution to greenhouse gas emissions. Given South Korea's strong economic position and world class domestic vehicle production facilities, however, South Korea, like the U.S. and Japan, stands in a good position to pursue technology as one policy approach for mitigating the effects of a growing vehicle fleet. Furthermore, given that most of South Korea's population is urban, the maintenance of public transportation and the restriction of parking in city areas offer parallel options aimed at the behavioral side of the equation.

Discussion

Overall, all nations in this study displayed a propensity toward more motor vehicle registrations and, hence, increased greenhouse emissions. Furthermore, all save Hungary, either already are or soon will be large contributors on a global scale. Analysis of trend data, however, also shows that all five nations faced with this problem have available policy options that will allow them to mitigate the problem, perhaps even leading to a net decline in greenhouse emissions from the motor vehicle sector. As expected, the likely solutions that emerged from the data vary from nation to nation.

In general, the nations with high GNPs, domestic auto industries, and low rates of population growth seem best poised to pursue technologically based remedies, including transitioning to alternative fuels and mandating improved fuel economy. Such nations also appear able to benefit from changes in travel behavior. While numerous approaches to reducing the reliance on vehicular travel have been proposed, including following the European lead of internalizing some of the social costs of driving into car and fuel prices, none have been greeted enthusiastically in the U.S. In Japan and South Korea, where such policies do exist, car ownership is associated with status, prestige, and other difficult-to-overcome attributes. Two related possibilities that have some promise in all three nations are road pricing and congestion pricing. These two policies are both aimed at capturing some of the social costs of driving by introducing per-trip tolls based on mileage, time of travel, or both, thereby increasing the marginal cost of a vehicle trip (Gomez-Ibanez 1992). Congestion pricing already has succeeded in reducing traffic volume in places such as Singapore, and California soon will unveil a congestion pricing scheme in the Anaheim-to-Riverside corridor.

In Hungary and the rest of Eastern Europe, typified by little or no population growth and outdated vehicle technology, technological advances again appear to be the best path to reduced greenhouse emissions, but the relevant technologies are different than what was prescribed for the U.S. and Japan. Rather than expecting a transition to ultralights and super-efficient cars of tomorrow, Hungary and other Eastern European nations can be served well simply by allowing market forces to engender a transition from 1970s to 1990s technology, while at the same time taking steps to protect against the development of an autocentric culture. In this regard, following the lead of their Western-European neighbors and internalizing the social costs of driving should provide an ameliorative influence.

For third world nations, characterized here by India, technological solutions appear to be out of reach due to economic factors. Although the transportation sector in such nations currently makes a relatively small contribution to global greenhouse emissions, large populations and the hope of future economic development leave these nations poised to dramatically increase their vehicle use and greenhouse emissions. In these nations, a

policy emphasis on travel behavior patterns appears to be most appropriate. If pursued, such a policy may allow such nations to avoid developing an emissions problem in the first place.

Finally, for nations such as Mexico caught squarely in the middle of larger and more unstable population and economic transitions, the motor-vehicle future is far more difficult to predict. Compared to South Korea--a developing nation nearing the end of its development transition and on a par economically with the developed world--Mexico has yet to enter an exponential growth phase in motor vehicle registrations (a simple linear model better fits vehicle registrations than does an exponential curve). Probably, Mexico and similar nations have the possibility of pursuing all policies available--bringing the latest technology into their vehicle fleet when possible, promoting public transit, while at the same time bringing population growth under control--to avoid entering this vehicle growth spurt. Therefore, these nations have the opportunity to approach or match the OECD nations in economic development without falling prey to at least one of the ills of life in the industrialized world--over reliance on motor vehicles.

Thus, the transition period presents nations with two paths, and the selection of one or the other is to a large extent within the control of policy. One path--chosen explicitly or implicitly by South Korea--closely follows the Western world and leads to over-reliance on motor vehicles and increased environmental degradation on a global scale. The other path, less traveled and partially unexplored, appears to lead to economic development that is more friendly to the global environment. As described by chaos theory (see, e.g., Prigogine and Stengers 1984), periods of uncertainty may quickly give rise to irreversible outcomes. Therefore, the selection of a path to follow is crucial, and evidence from the industrialized world suggests that choosing the motor-vehicle dominated path is costly to the global environment and may dictate an undesirable future from which there may be no escape.

Conclusion

The ultimate effects of greenhouse gases on the global climate remain uncertain. Even more uncertain is how these global changes will play out at local and regional scales.

Some still argue that global climate change is either not occurring or not dangerous even if it is occurring. Others, of course, predict dire consequences if global climate change is not halted (Brown, et al. 1994). Following Drake, however, we must not neglect the possibility that the period during which the climate is changing, and not the resultant state of the future global climate, is the period most deserving of close scrutiny and remedial action. That period is now. By following some or all of the policy recommendations discussed in this paper, we may yet be able to slow the pace of change, which should serve to mitigate the negative consequences of a transitional period. Furthermore, by not delaying our policy response, we allow scientific research on global climate change to find more answers before we needlessly condemn future generations to a changed global climate of our making.

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4. EXPOSITORY ARTICLE

DISCRETE MATHEMATICS AND COUNTING DERANGEMENTS IN BLIND WINE TASTINGS

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The statistician Fisher explained the mathematical basis for the field of "Design of Experiments" in an elegant essay couched in the context of the mathematics of a Lady tasting tea (Fisher, in Newman 1956; Fisher 1971). In Fisher's text, the problem is to analyze completely the likelihood that the Lady can determine whether milk was added to the tea or tea added to milk. Problems associated with the tasting of wines have a number of obvious similarities to Fisher's tea-tasting scenario. We offer an analysis of this related problem, set in the context of Nystuen's wine tasting club. To begin, a brief background of the rules of that club seems in order; indeed, it is often the case that the application is forced to fit the mathematics in order to illustrate the abstract. Here, it is the real-world context that guides the mathematics selected.

Wine Tasting Strategy

The Grand Crew wine club of Ann Arbor has been blind-tasting wines monthly for years. In a blind tasting, several wines are offered with their identity hidden. Not only are labels covered, but the entire bottle is covered as well because the shape and color of the bottle provides some clues as to the identity of the wine. The wines are labeled 1 through n in the order presented. Six to eight wines are tasted at a sitting. Members sip the wines and score each on a scale from 1 to 20, using a scoring method suggested by the American

Wine Association. The wines are judged on the basis of quality and individual taster preference. The evening's host is in charge of choosing and presenting the wines. Usually wines of a single variety but from different vineyards, wineries, prices, or distributors are tasted. Two sheets of paper are provided to each taster. One is a blank table with a row for each wine numbered 1 through n in the order presented. The columns on this sheet provide space for comments, the individual's numerical ratings of the wines, average ratings of the group, and the range in scores for each wine. One column is reserved for the member's guess as to the identity of the wine. The second sheet contains information about each wine to be used to match the wines tasted. On this sheet the wines are labeled a, b, c, and so forth, along with information on age, winery, negotiant, and price. The tasters try to match the identity of the wine with their individual rating on sheet 1.

The wines are listed in unknown order on the second sheet. The tasters make their decisions by matching the letter identification with the numerical order of presentation. On rare occasions one or more members correctly identifies every wine. More often two or more wines are mislabeled, and quite often the identities seem hopelessly scrambled. Guessing at random would seem just as effective. The question then arises, "what are the chances of getting one, two, more, or all correct by chance alone?" Discrete mathematics and the algebra of derangements provides the answer to this question.

Probabilities are a matter of counting. In what proportion does a particular combination of correct and incorrect identifications occur purely at random out of all possible combinations? The denominator in this proportion is a count of all possible arrangements and the numerator is a count of all possible ways a particular event occurs, such as one right, all the rest wrong. The denominator is easily determined. If one has five things any of the five might be chosen first; there remain four things any of which might be chosen next. The process continues until the last stage in which only one can be chosen. Thus, there are $5*4*3*2*1=120$ ways to arrange five bottles of wine--the customary notation for this product is $5!$ (read five factorial). This notation extends to arbitrarily large positive integers in the obvious way; $0!$ is defined to be 1. The factorial of a number grows rapidly with an increase in the size of the number; thus, $7!=5040$ while $8!=40,320$.

The numerator of the proportion sought is not found as easily. Consider the case of a blind tasting of three bottles of wine. Suppose the first one is correctly identified; the remaining two outcomes must be both right or both wrong. It is not possible to identify two wines correctly and the third one incorrectly. Table 1 illustrates all possible patterns of identification for three bottles of wine, a, b, and c, with bottle "a" presented first, bottle "b" presented second, and bottle "c" presented third. As this table indicates, there is only one arrangement in which all are correct, two arrangements with none correct and three arrangements with two correct. There are, of course, no arrangements with exactly one correct.

Table 1. All possible arrangements of three items, a, b, and c.
Number of matches and non-matches to the arrangement abc.

	Matches	Non-matches
abc	3	0
acb	1	2
bac	1	2
bca	0	3
cab	0	3
cba	1	2

When all possible outcomes, shown in Table 1, are enumerated, it is an easy matter to calculate the probability of each type of event-- to obtain the probability, divide each outcome from Table 1 by 3!, the number of total possible arrangements. Table 2 shows the probability of each outcome: P(0) denotes none right, P(1) denotes exactly one right, and so forth. The sum of all probabilities adds to 1.00, as it should.

Table 2. Probability of a correct labeling.

$$P(0) = 2/6 = 0.33$$

$$P(1) = 3/6 = 0.50$$

$$P(2) = 0/6 = 0.00$$

$$P(3) = 1/6 = 0.17$$

A total enumeration approach to finding the probabilities is satisfactory for introductory purposes and for very small samples. Even for six, seven, or eight wines at a single tasting it is, however, not satisfactory; Table 1 would expand to 720, 5040, or 40,320 columns for each of those cases. Clearly more clever and mathematically elegant ways of counting, rather than brute force listings, are required. In this latter regard, one is reminded of the story of Gauss who, as a young child, astounded his German schoolteacher with an instant result for what the teacher had planned as a tedious exercise. The teacher, in order to keep his students busy, told them to add all the numbers from 1 to 100. Gauss immediately wrote the answer on his slate. He had apparently discovered for himself that the sum, S , of the first n positive integers is given by the recursive relationship $S=(n(n+1)/2)$. Thus, all he had to do was multiply 50 by 101 to obtain the answer: an elegant solution to an otherwise tedious problem. It was the more mature Gauss and later Laplace that would do pioneering work in the Theory of Errors of Observation which in turn would serve as a significant part of the base for applications of mathematics and statistics (in Design of Experiments) in the Scientific Method.

Derangements

For our problem, we need a way to count the number of times a taster can get all the wines right, one wine right and all the others wrong, two wines right and all the others wrong, and so forth. To convert the tedious, brute force task of listing permutations and combinations for this problem, to a more tractable situation, we employ the concept of "derangement," that will eliminate, notationally, combinations that we do not wish to consider.

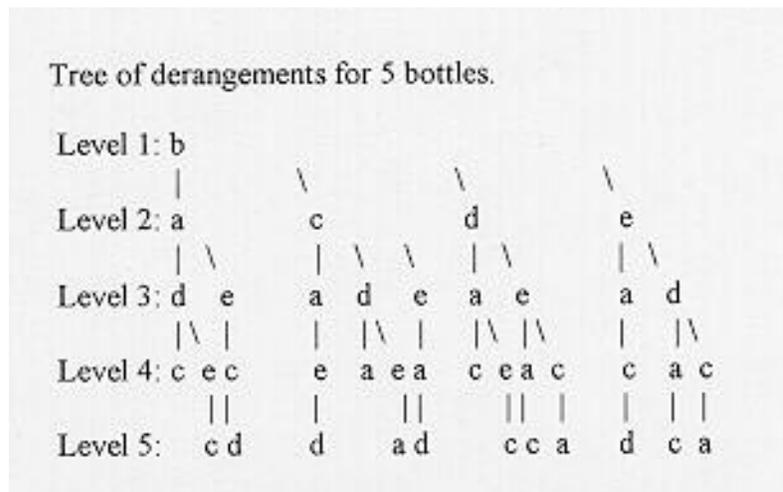
A "derangement" is a permutation of objects that leaves no object in its original position (Rosen 1986; Michaels and Rosen 1991). The permutation $badec$ is a derangement of $abcde$ because no letter is left in its original position. However, $baedc$ is not a derangement of $abcde$ because this permutation leaves d fixed. Thus, the number of times a wine taster gets all the wrong answers in tasting n bottles is the number of derangements of n numbers, $D(n)$, divided by $n!$: $D(n)/n!$. The value of $D(n)$ is calculated as a product of $n!$ and a series of terms of alternating plus and minus signs:

$$D(n)=n!(1-1/1!+1/2!-1/3!+1/4!-1/5!+\dots+((-1)^n/n!).$$

Readers wishing more detail concerning this formula might refer to Rosen (1988); for the present, we continue to consider the use of derangements.

In order to see how derangements can be enumerated visually, we construct the following tree of possibilities for arrangements of 5 letters which do not match the natural order of abcde. On the first level, the natural choice is a--so choose some other letter instead. The second level would be b in the natural order so choose all others, instead, and continue the process until all possibilities have been exhausted. Following each path through the tree will give all possible derangements beginning with the letter b--there are 11 such routes. Thus, there are 11*4 derangements.

Tree of derangements for 5 bottles.



Indeed, when there are five wines, $D(n)=5!(1/2!-1/3!+1/4!-1/5!)=5!/2!-5!/3!+5!/4!-5!/5!=60-20+5-1=44$.

What is of particular significance is that derangements focus only on wrong guesses: because a non-wrong guess is a correct guess, it is possible to focus only on one world. The Law of the Excluded Middle, in which any statement is "true" or "false"--with no middle partial truth admitted, is the basis for this and for most mathematical assessments

of real-world situations. It is therefore important to use the tools appropriately, on segments of the real-world situation in which one can discern "black" from "white."

Derangements and Probability in Random Guesses

In the case of the five wine example, the number of ways of choosing (for example) three correctly out of five is the combination of five things taken three at a time: $C(5,3)=5!/2!3!=10$. Exhausting all possible combinations reflects an expected connection with the binomial theorem--these values are the coefficients of $(x+y)^5$.

$$C(5,0)=1$$

$$C(5,1)=5$$

$$C(5,2)=10$$

$$C(5,3)=10$$

$$C(5,4)=5$$

$$C(5,5)=1.$$

The total number of right/wrong combinations is therefore 2^5 or 32. Notice, though, that the pattern within each grouping is disregarded; to discover the finer pattern, of how right/wrong guesses are arranged we need permutations. To limit the number of permutations necessary to consider, we investigate the derangements.

If we can count derangements, we can now address the question of how many times a taster, guessing randomly, gets exactly one wine correct. The answer is simply the number of ways one bottle can be chosen from n bottles times the number of derangements of the other $(n-1)$ bottles of wine. When this value is divided by $n!$, the probability $P(1)$ of guessing exactly one wine correctly is the result. That probability is:

$$P(1)=(n!/1!(n-1)!)*D(n-1)/n!$$

This idea generalizes in a natural manner so that the probability of choosing exactly k wines correctly is given as:

$$P(k)=(n!/k!(n-k)!)*D(n-k)/n!$$

Table 3 displays all the probabilities for outcomes in blind tastings in which random choices are made in situations for which from 2 to 8 wines are offered by the evening's host. Notice that there is less than a one percent chance of guessing all wines correctly by chance alone whenever the host offers five or more wines in the evening's selection. Evidently, some knowledge of wines is displayed by a taster who accomplishes this feat

with any regularity. On the other hand, one could expect, by chance alone, to guess none of the wines correctly about 37 percent of the time, independent of the number of wines offered for tasting. The same situation holds for guessing exactly one wine correctly.

The reason that this is so, as readers familiar with infinite series will note, is that the alternating series contained in the parenthetical expression in the formula for counting derangements is precisely $1/e$, where e is the base of natural logarithms (a transcendental number of value approximately 2.71828). That is, $e^x = 1 + x/1! + (x^2)/2! + (x^3)/3! + \dots$ so that when $x = -1$, then e^{-1} , or $1/e$, is precisely the parenthetical expression in the formula for $D(n)$. The larger the value of n , the closer the approximation to $1/e = 0.3678797$. In a blind tasting with an infinite number of bottles of wine, random choices will result in approximately a 0.368 probability that all will be in error!

Table 3. Probability of correctly matching K wines from tasting a total of n wines

K	n							
	2	3	4	5	6	7	8	
0	.5000	.3333	.3750	.3667	.3681	.3679	.3679	
1	.0000	.5000	.3333	.3750	.3667	.3681	.3679	
2		.0000	.2500	.1667	.1875	.1833	.1840	
3		.1667	.0000	.0833	.0555	.0625	.0611	
4			.0417	.0000	.0208	.0139	.0156	
5				.0083	.0000	.0042	.0028	
6					.0014	.0000	.0007	
7						.0002	.0000	
8							.0000	

Table 3 suggests some rules of thumb about how well a taster has done. In a normal-sized tasting of six, seven, or eight wines, identifying at least five of them correctly occurs less than 1% by chance alone. Identifying four correctly happens by chance about 2 percent (or less) of the time. However, identifying three correctly occurs by chance from near five to six percent of the time: in every 16 to 18 tastings. Usually there are ten to twelve tasters at a sitting in this one club. None to one member at a sitting rates to guess three

wines correctly by chance alone; the group usually does substantially better than this, suggesting some expertise in identifying the wines.

The Principle of Inclusion and Exclusion: The Basis for Counting.

The expression for counting derangements, as a product of $n!$ and a truncated series for $1/e$, has some interesting properties, most notably perhaps the alternating plus and minus signs preceding terms of the series. This alternation occurs because the principle of inclusion and exclusion has been used as the basis for the counting.

Readers versed in elementary set theory, Boolean algebra, or symbolic logic, are familiar with the idea of including the intersection, and then subtracting it out, in order to count the number of elements in intersecting sets. This idea, in this context, was clearly familiar to Augustus DeMorgan in the late nineteenth century. Indeed, in a wider context, it dates back to the time of Eratosthenes of Alexandria and his sieve for determining which numbers are prime: those that are multiples of numbers early in the ordering of positive integers are excluded. Only those numbers not excluded have divisors of only themselves and 1, and so are exactly the set included as prime numbers.

The following example illustrates how inclusion and exclusion is used in counting derangements; the reader interested in the general proof is referred to Rosen (1988). It is easy to visualize cases when n is small using Venn diagrams--thus, the linkage between inclusion/exclusion, set theory, and derangements becomes clear.

Consider for example, a tasting of two wines. Let a be the event that the first wine is correctly identified; let b be the event that the second wine is correctly identified. Draw a rectangle on a sheet of paper and within the rectangle draw two intersecting circles, a and b --a familiar Venn diagram. The content of the rectangle is the universe of discourse. The content of circle a is the set of all events that the first wine is correctly identified (either alone or with another), denoted $N(a)$. The content of circle b is the set of all events that the second wine is correctly identified, denoted $N(b)$. The intersection of the two circles has content ab , the set of all events in which both the first wine and the second wine are correctly identified, denoted $N(ab)$. The set of all derangements is the content of that area of the rectangle outside the two circles. The content of the two circles is the sum of the

content of the first circle plus the sum of the content of the second circle: $N(a)+N(b)$. This sum however includes $N(ab)$ in the first term and also $N(ab)$ in the second term; thus, $N(ab)$ must be excluded from the sum to get an accurate count of the content of the union of the two circles--hence inclusion and exclusion. The accurate count of one or more wines correct is thus given as $N(a)+N(b)-N(ab)$. The case for three circles is more complicated to visualize but can be enumerated carefully as a set of three two-circle problems. With values greater than 3, visualization in this manner becomes impossible and one must rely on extension of the notation and visualization in the world of language rather than in the world of pictures--both subsets of "the world of mathematics." Indeed, geographers interested in spatial statistics should be familiar with this issue in using the statistical forms to capture what becomes increasingly too complex to map.

Retrospect

These classical ideas, whether cast in the number theoretic context of prime numbers, in the discrete mathematics context of inclusion and exclusion, or in the set theoretic context of intersections, served once again, when cast in the context of derangements and the counting of incorrectness, to permit a clever solution to a complicated, uncontrived, real-world problem. What this sort of analysis offers is a challenge to look at the world in different ways: from the use of classical theoretical material in new real world situations, to the development of new theoretical material which can foster further theoretical exploration and application.

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TABLE OF CONTENTS

1. MISSION STATEMENT AND BEST E-ADDRESS
2. SOLSTICE ARCHIVES
3. PUBLICATION INFORMATION
4. ELEMENTS OF SPATIAL PLANNING: THEORY. PART I.
SANDRA L. ARLINGHAUS
5. MAPBANK: AN ATLAS OF ON-LINE BASE MAPS
SANDRA L. ARLINGHAUS
WITH ZIPPED SAMPLE ATTACHED
6. INTERNATIONAL SOCIETY OF SPATIAL SCIENCES
7. INDEX TO VOLUMES I (1990) TO V (1994); VOL. VI, NO. 1.
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4. ELEMENTS OF SPATIAL PLANNING: THEORY. PART I.**

SANDRA L. ARLINGHAUS

One reason that planning of any sort is a difficult process is that it involves altering natural boundaries to fit human needs and desires. While it may not be "nice to fool Mother Nature" the act of planning may be predicated on such an attempt, especially when the balance between human and environmental needs is tipped strongly toward the human side. At a very general level, planning how to use the Earth's surface involves what space to use and when to use it. The "what" issues are those that involve spatial planning; they typically involve the concept of scale. The "when" issues involve temporal planning; they typically involve the concept of sequence.

Particular spatial issues might address whether or not boundaries of a parcel of land are clearly designated with respect to one's neighbors; whether or not a proposed land use is consistent with the general character of a larger region; or whether or not a developer's site plans give sufficient attention to natural features. Temporal issues might address the long range and the short range view of a traffic circulation pattern; the sequence, in years, in which lands are to be annexed to a city; or the length of time trees need to have lived in order to be designated landmark trees. When one considers that budget concerns often function as an underlying factor that can help to sway this balance, the fragility of the art of planning becomes apparent.

One way to view complicated issues is to consider them at an abstract level in order to understand the logic that links them. The two-valued system of logic on which much of mathematics is based offers one structure that exposes logical connections. When using this structure in conjunction with real-world settings, which often defy the Law of the Excluded Middle, one generally has a number of difficult decisions to make; it is in the act of making these decisions that thoughts can become clearer.

WATERSHED PRINCIPLE

The preservation of natural features is an issue that can be a developer's nightmare, just as development can be the bete noir of the environmentalist. When man-made boundaries are superimposed on the natural environment, there is often little correspondence between the two partitions of space. Abstractly it is not surprising, therefore, that individuals using one way to partition space will be at loggerheads with those using a different partition of space.

When the topography of a region is altered, it is necessarily the case that the natural features on that surface are also altered. Considering the contrapositive of this statement, a logical equivalent, leads to the idea that the preservation of natural features is dependent on the preservation of topography. When this idea is coupled with the notion that the fundamental topographic unit is the drainage basin or watershed (Leopold, Wolman, and Miller, *Fluvial Processes in Geomorphology*), the following principle emerges.

Watershed Principle.

If the preservation of natural features depends upon the preservation of topography and if the fundamental topographic unit is the watershed, then the preservation of natural features depends upon the watershed.

If one accepts this Principle, then it may well be a small step to the following Corollaries.

Corollary 1.

When environmental concerns are involved, the drainage basin should be the fundamental planning unit.

Corollary 2.

When the drainage basin is the fundamental planning unit, the partition of wetlands and other elements of the drainage network, by man-made planning unit boundaries, is not possible.

Decisions as to the impact a proposed development project will have on a wetland are facilitated by having the entire wetland contained within the legal boundaries of the parcel; using the drainage basin as the fundamental planning unit ensures that such set-theoretic containment will be the case. Issues involving the welfare of the entire watershed also become tractable under such an alignment: neighbors become neighbors with respect to the drainage pattern rather than with respect to superimposed human boundaries. Indeed, what my neighbor does three miles upstream from me may have far more impact on my land than does the action of a neighbor 100 feet away who is in a different drainage basin. Current technology (Geographic Information Systems, for example) might make it possible to alter the inventory of lands to create suitable, substantial changes, along these or along other lines, in legal definitions. The use of technological capability to make legal definitions correspond more closely to natural definitions can lead to the resolution of conflicts: the closer the fit between natural and man-made boundaries the fewer the disagreements.

MINIMAX PRINCIPLE

The basic idea behind the Watershed Principle might be captured as one that minimizes damage to the environment and maximizes satisfaction of human needs and desires. Viewed more broadly, the Watershed Principle might be recast as a MiniMax Principle which can then be recast downstream abstractly, in a number of other more specific forms (such as the Watershed Principle).

MiniMax Principle

An optimal plan is one which minimizes alteration of existing entities and maximizes the common good.

Highly general principles, such as this one, demand attention to definitional matters: what is meant by "common good" or how might one measure "alteration." These are difficult

problems: one advantage to an abstract view is to bring important and difficult issues into focus.

EARTH-SUN RELATIONS: GEOGRAPHIC COORDINATES AND TIME ZONES.

One case in which the fit between natural and man-made boundaries is done in a style consistent with the Minimax Principle is the spatial layout of reckoning time (thus, time becomes transformed in a "meta" fashion into space). Much of the developed world measures the passage of time by the position of Earth relative to our Sun. One unit of time, the year, corresponds roughly to one revolution of the Earth around the Sun. Another, smaller, unit of time, the day, corresponds roughly to the rotation of the Earth on its axis--the man-made boundaries in both cases are set by the natural planetary motions in space.

When planetary motions do not permit any further refinement of the day into even smaller units, we subdivide the day into hours. When the partition of the day into 24 hours is put into correspondence with the grid system based on latitude and longitude, one hour corresponds to fifteen degrees of longitude. Fifteen degrees of longitude corresponds to a central angle of fifteen degrees intercepted along the Equatorial great circle. Thus, 24 man-made time zones of 15 degrees of longitude each envelop the Earth--man-made boundaries again follow (although a bit indirectly) from natural boundaries. The Earth becomes a "clockwork orange" of 24 sections, each 1 hour wide, with boundaries along meridians spaced 15 degrees apart. Across oceans, this alignment of time-zone and longitude may reasonably have boundaries along meridians; interior to a continent, however, human needs and desires may reasonably prevail, making it prudent to bend the natural alignment for the common good.

** The author wishes to thank her colleagues on the City of Ann Arbor Planning Commission and in the Planning Department of the City of Ann Arbor. The challenge and stimulation fostered by this lively Commission helped to generate this viewpoint.

5. MAPBANK: AN ATLAS OF ON-LINE BASE MAPS

SANDRA L. ARLINGHAUS

The attached file contains a Mollweide equal area projection of the world made from Geographic Information System (GIS) technology. A map from a vector GIS (MapInfo for Windows) was copied onto the Windows clipboard and then pasted into Windows Paintbrush. It was then zipped using PKZIP 204G. Readers of Solstice can download this raster file from the attachments to Solstice. Other (free) maps are available from the MapBank on the WebSite <http://www-personal.umich.edu/~sarhaus/iss>.

When these downloaded maps are put into Windows Paintbrush (or other software) and are projected from the computer screen onto the wall (using a data-show and overhead projector, or some such) their resolution is of about the same quality as that of the original on-screen display in the GIS. Thus, wall-maps can be carried around on diskette. This strategy offers an easy way for university professors and pre-collegiate teachers alike to give lectures with maps tailored to their needs--from base maps for simple place-name recognition, to maps showing voting patterns by party in presidential elections, to maps showing global vehicle registration patterns, to detailed topographic maps. Naturally, the first in the MapBank series of maps offered for this style of communication are base maps.

6. INTERNATIONAL SOCIETY OF SPATIAL SCIENCES

July 18, 1995, the International Society of Spatial Sciences (ISSS--I-triple-S) was founded as a division of the non-profit Community Systems Foundation of Ann Arbor, MI. This primarily electronic society has as board members:

Sandra L. Arlinghaus (founder), W. C. Arlinghaus, M. L. Bird, B. R. Burkhalter, W. D. Drake, F. L. Goodman, F. Harary, J. A. Licata, A. L. Loeb, K. E. Longstreth, J. D. Nystuen, W. R. Tobler. To follow its activities, browse the under-construction WebSite <http://www-personal.umich.edu/~sarhaus/iss>

with direct links to material of the American Geographical Society
and the Thunen Society.

internet: sarhaus@umich.edu

The focus of this new society is to place in a core position those sciences, of spatial character, that are often relegated to the periphery within academic institutional structure. Such sciences are, to name only a few, geology, geography, and astronomy. In moving along this continuum from the center of the Earth to the outer reaches of the universe, one might imagine a whole host of sciences that could also be included (from oceanography to atmospheric science to regional science). Thus, ISSS offers a platform from which individuals, institutions, and professional societies devoted to some aspect of spatial science might further their interests.

The previous article announces one of the projects of ISSS. For the past two years, during the developmental stages of ISSS, a continuing project has been the development of a MapBank. This is a bank composed of maps made by students; most of the maps are thematic maps made to supplement student term papers or as maps to be used in the classroom by students of Education. For teachers to use the MapBank, free of charge, they must make a deposit of an electronic map. Currently the MapBank numbers more than 100 electronic maps. Look for thematic maps to appear in future issues of *Solstice* and on the WebSite of ISSS. There are currently ten base maps of the world on the ISSS MapBank WebSite: <http://www-personal.umich.edu/~sarhaus/iss>.

7. INDEX TO VOLUMES I (1990) TO V (1994); VOL. VI, NO. 1.

ZIPPED FILE ATTACHED TO THIS FILE; ZIPPED USING PKZIP 204G

Volume VI, Number 1, June, 1995.

Fifth Anniversary of Solstice

New format for Solstice and new Technical Editor

Richard Wallace. Motor Vehicle Transport and Global Climate Change: Policy Scenarios.

Expository Article. Discrete Mathematics and Counting Derangements in Blind Wine Tastings.

Sandra L. Arlinghaus, William C. Arlinghaus, John D. Nystuen

Volume V, No. 2, Winter, 1994.

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary: The Paris Metro: Is its Graph Planar?

Planar graphs; The Paris Metro; Planarity and the Metro; Significance of lack of planarity.

Sandra Lach Arlinghaus: Interruption!

Classical interruption in mapping; Abstract variants on interruption and mapping; The utility of considering various mapping surfaces--GIS; Future directions.

Reprint of Michael F. Dacey: Imperfections in the Uniform Plane. Forewords by John D. Nystuen.

Original (1964) Nystuen Foreword; Current (1994) Nystuen Foreword; The Christaller spatial model; A model of the imperfect plane; The disturbance effect; Uniform random disturbance; Definition of the basic model; Point to point order distances; Locus to point order distances; Summary description of pattern; Comparison of map pattern; Theoretical model; Point to point order distances; Locus to point order distances; Summary description of pattern; Comparison of map pattern; Theoretical order distances; Analysis of the pattern of urban places in Iowa; Almost periodic disturbance model; Lattice parameters; Disturbance variables; Scale variables; Comparison of M(2) and Iowa; Evaluation; Tables.

Sandra L. Arlinghaus: Construction Zone: The Brakenridge-MacLaurin Construction.

William D. Drake: Population Environment Dynamics: Course and Monograph--descriptive material.

Volume V, No. 1, Summer, 1994.

Virginia Ainslie and Jack Licate: Getting Infrastructure Built. Cleveland infrastructure team shares secrets of success;

What difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; How can local communities translate these successes to their own settings?

Frank E. Barmore: Center Here; Center There; Center, Center Everywhere.

Abstract; Introduction; Definition of geographic center; Geographic center of a curved surface; Geographic center of Wisconsin; Geographic center of the conterminous U.S.; Geographic center of the U.S.; Summary and recommendations; Appendix A: Calculation of Wisconsin's geographic center; Appendix B: Calculation of the geographical center of the conterminous U.S.; References.

Barton R. Burkhalter: Equal-Area Venn Diagrams of Two Circles: Their Use with Real-World Data

General problem; Definition of the two-circle problem; Analytic strategy; Derivation of $B\%$ and $AB\%$ as a function of $r(B)$ and $d(AB)$.

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary, John D. Nystuen. Los Angeles, 1994 -- A Spatial Scientific Study.

Los Angeles, 1994; Policy implications; References; Tables and complicated figures.

Volume IV, No. 2, Winter, 1993.

William D. Drake, S. Pak, I. Tarwotjo, Muhilal, J. Gorstein, R. Tilden. Villages in Transition: Elevated Risk of Micronutrient Deficiency.

Abstract; Moving from traditional to modern village life: risks during transition; Testing for elevated risks in transition villages; Testing for risk overlap within the health sector; Conclusions and policy implications

Volume IV, No. 1, Summer, 1993.

Sandra L. Arlinghaus and Richard H. Zander: Electronic Journals: Observations Based on Actual Trials, 1987-Present.

Abstract; Content issues; Production issues; Archival issues; References

John D. Nystuen: Wilderness As Place.

Visual paradoxes; Wilderness defined; Conflict or synthesis; Wilderness as place; Suggested readings; Sources; Visual illusion authors.

Frank E. Barmore: The Earth Isn't Flat. And It Isn't Round Either:

Some Significant and Little Known Effects of the Earth's Ellipsoidal Shape.

Abstract; Introduction; The Qibla problem; The geographic center; The center of population; Appendix; References.

Sandra L. Arlinghaus: Micro-cell Hex-nets?

Introduction; Lattices: Microcell hex-nets; References

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary:

Sum Graphs and Geographic Information.

Abstract; Sum graphs; Sum graph unification: construction; Cartographic application of sum graph unification; Sum graph unification: theory; Logarithmic sum graphs; Reversed sum graphs; Augmented reversed logarithmic sum graphs; Cartographic application of ARL sum graphs; Summary.

Volume III, No. 2, Winter, 1992.

Frank Harary: What Are Mathematical Models and What Should They Be? What are they?

Two worlds: abstract and empirical; Two worlds: two levels; Two levels: derivation and selection; Research schema; Sketches of discovery; What should they be?

Frank E. Barmore: Where Are We? Comments on the Concept of Center of Population.

Introduction; Preliminary remarks; Census Bureau center of population formulae; Census Bureau center of population description; Agreement between description and formulae; Proposed definition of the center of population; Summary; Appendix A; Appendix B; References.

Sandra L. Arlinghaus and John D. Nystuen: The Pelt of the Earth: An Essay on Reactive Diffusion.

Pattern formation: global views; Pattern formation: local views; References cited; Literature of apparent related interest.

Volume III, No. 1, Summer, 1992.

Harry L. Stern: Computing Areas of Regions with Discretely Defined Boundaries.

Introduction; General formulation; The plane; The sphere; Numerical examples and remarks; Appendix--Fortran program.

Sandra L. Arlinghaus, John D. Nystuen, Michael J. Woldenberg: The Quadratic World of Kinematic Waves.

Volume II, No. 2, Winter, 1991.

Reprint of Saunders Mac Lane: Proof, Truth, and Confusion, The Nora and Edward Ryerson Lecture at The University of Chicago in 1982.

The fit of ideas; Truth and proof; Ideas and theorems; Sets and functions; Confusion via surveys; Cost-benefit and regression; Projection, extrapolation, and risk; Fuzzy sets and fuzzy thoughts; Compromise is confusing.

Robert F. Austin: Digital Maps and Data Bases: Aesthetics versus accuracy.

Introduction; Basic issues; Map production; Digital maps; Computerized data bases; User community.

Volume II, No. 1, Summer, 1991.

Sandra L. Arlinghaus, David Barr, John D. Nystuen:
The Spatial Shadow: Light and Dark -- Whole and Part.

This account of some of the projects of sculptor David Barr attempts to place them in a formal systematic, spatial setting based on the postulates of the science of space of William Kingdon Clifford (reprinted in Solstice, Vol. I, No. 1.).

Sandra L. Arlinghaus: Construction Zone--The Logistic Curve.

Educational feature--Lectures on Spatial Theory.

Volume I, No. 2, Winter, 1990.

John D. Nystuen: A City of Strangers: Spatial Aspects of Alienation in the Detroit Metropolitan Region.

This paper examines the urban shift from "people space" to "machine space" (see R. Horvath, Geographical Review, April, 1974) in the Detroit metropolitan regions of 1974. As with Clifford's Postulates, reprinted in the last issue of Solstice, note the timely quality of many of the observations.

Sandra Lach Arlinghaus: Scale and Dimension: Their Logical Harmony.

Linkage between scale and dimension is made using the Fallacy of Division and the Fallacy of Composition in a fractal setting.

Sandra Lach Arlinghaus: Parallels Between Parallels.

The earth's sun introduces a symmetry in the perception of its trajectory in the sky that naturally partitions the earth's surface into zones of affine and hyperbolic geometry. The affine zones, with single geometric parallels, are located north and south of the geographic parallels. The hyperbolic zone, with multiple geometric parallels, is located between the geographic tropical parallels. Evidence of this geometric partition is suggested in the geographic environment--in the design of houses and of gameboards.

Sandra L. Arlinghaus, William C. Arlinghaus, and John D. Nystuen: The Hedetniemi Matrix Sum: A Real-world Application.

In a recent paper, we presented an algorithm for finding the shortest distance between any two nodes in a network of n nodes when given only distances between adjacent nodes (Arlinghaus, Arlinghaus, Nystuen, *Geographical Analysis*, 1990). In that previous research, we applied the algorithm to the generalized road network graph surrounding San Francisco Bay. Here, we examine consequent changes in matrix entries when the underlying adjacency pattern of the road network was altered by the 1989 earthquake that closed the San Francisco--Oakland Bay Bridge.

Sandra Lach Arlinghaus: Fractal Geometry of Infinite Pixel Sequences: "Super-definition" Resolution?

Comparison of space-filling qualities of square and hexagonal pixels.

Sandra Lach Arlinghaus: Construction Zone--Feigenbaum's number; a triangular coordinatization of the Euclidean plane; A three-axis coordinatization of the plane.

Volume I, No. 1, Summer, 1990.

Reprint of William Kingdon Clifford: Postulates of the Science of Space.

This reprint of a portion of Clifford's lectures to the Royal Institution in the 1870s suggests many geographic topics of concern in the last half of the twentieth century. Look for connections to boundary issues, to scale problems, to self-similarity and fractals, and to non-Euclidean geometries (from those based on denial of Euclid's parallel postulate to those based on a sort of mechanical "polishing"). What else did, or might, this classic essay foreshadow?

Sandra Lach Arlinghaus: Beyond the Fractal.

The fractal notion of self-similarity is useful for characterizing change in scale; the reason fractals are effective in the geometry of central place theory is because that geometry is hierarchical in nature. Thus, a natural place to look for other connections of this sort is to other geographical concepts that are also hierarchical. Within this fractal context, this article examines the case of spatial diffusion.

When the idea of diffusion is extended to see "adopters" of an innovation as "attractors" of new adopters, a Julia set is introduced as a possible axis against which to measure one class of geographic phenomena. Beyond the fractal context, fractal concepts, such as "compression" and "space-filling" are considered in a broader graph-theoretic setting.

William C. Arlinghaus: Groups, Graphs, and God.

Sandra L. Arlinghaus: Theorem Museum--Desargues's Two Triangle Theorem from projective geometry.

Construction Zone--centrally symmetric hexagons.

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Electronic copy of Solstice, as well as of selected other monographs, is available on the web page of the Institute of Mathematical Geography (current electronic address available in various search engines).

1. Arlinghaus and Nystuen. *Mathematical Geography and Global Art: The Mathematics of David Barr's Four Corners Project*. 1985. 78 pp.
2. Arlinghaus. *Down the Mail Tubes: The Pressured Postal Era, 1853-1984*. 1985. 79 pp.
3. Arlinghaus. *Essays on Mathematical Geography*. 1986. 167 pp.
4. Austin. *A Historical Gazetteer of South East Asia*. 1986. 118 pp.
5. Arlinghaus. *Essays on Mathematical Geography, II*. 1987. 101pp.
6. Hanjoul, Beguin, and Thill. *Theoretical Market Areas under Euclidean Distance*. 1988. 162 pp.
7. Tinkler. *Nystuen-Dacey Nodal Analysis*. 1988. 115pp.
8. Fonseca. *Urban Rank-Size Hierarchy: A Mathematical Interpretation*. 1988. 85pp.
9. Arlinghaus. *An Atlas of Steiner Networks*. 1989. 84pp.
10. Girffith. *Simulating $K=3$ Christaller Central Place Structures: An Algorithm Using A Constant Elasticity of Substitution Consumption Function*. 1989. 103pp.
11. Arlinghaus and Nystuen. *Environmental Effects on Bus Durability*. 1990. 104pp.
12. Griffith, et al. *Spatial Statistics: Past, Present, and Future*. 1990. 398pp.
13. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume I*. 1990.
14. Arlinghaus. *Essays on Mathematical Geography, III*. 1991. 52pp.
15. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics, Volume II*. 1991.
16. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume III*. 1992.
17. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume IV*. 1993.
18. Gordon. *The Cause of Location of Roads in Maryland: A Study in Cartographic Logic*. 1995. 109pp.
19. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume V*. 1994.
20. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume VI*. 1995.
21. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume VII*. 1996.
22. Arlinghaus et al. *Solstice: An Electronic Journal of Geography and Mathematics. Volume VIII*. 1997.

OTHER PUBLICATIONS, PRODUCED ON-DEMAND.

- Philbrick, Allen K. *This Human World*. Reprint.
- Kolars, John F. and Nystuen, John D. *Human Geography*. Reprint.
- Nystuen, John D. et al. *Michigan Inter-University Community of Mathematical Geographers. Complete Papers*. Reprint.
- Griffith, Daniel A. Discussion Paper #1. *Spatial Regression Analysis on the PC*. 84pp.