MULTIPLE IMPUTATION METHODS FOR STATISTICAL DISCLOSURE CONTROL

by

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To my husband, my parents, aunt Junfu and aunt Yuan

And in loving memory of my grandmother

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Table of Contents

List of Tables

Table

List of Figures

Figure

Chapter I
Introduction

I.1 Statistical disclosure control

The explosion of collection on private data raises concerns about guarding the privacy of survey respondents now more than ever. Statistical disclosure control (SDC) is a class of procedures that deliberately alter data collected by statistical agencies before release to the public, to prevent the identity of survey respondents from being revealed. These methods have increased in importance, with the extensive use of computers and the internet. Inevitably, statistical agencies are confronted with the trade-off between data protection and data utility. The goal of SDC methods is to find a balance for this dilemma, by reducing the risk of disclosure to acceptable levels, while releasing a dataset that provides as much useful information as possible for researchers. One aspect of this is the ability to draw valid statistical inferences from the altered data.

Various SDC techniques have been established to preserve confidentially, including global recoding and local suppression, swapping data values for randomly selected units (Dalenius and Reiss, 1982), or adding random noise (Fuller 1993). These methods involve perturbing and masking of the original data. Though the model-free nature makes them easy to apply, these methods somewhat distort the statistical structure of the data and make analysis difficult for data user.

I.2 Multiple imputation methods of SDC

Rubin (1993) proposes to release fully synthetic data based on multiple imputation (MI) methods. In his proposal, an imputation model is built from the original survey data and data values in the population are imputed by draws from the predictive distribution based on the model. The imputation process is repeated several times and a random sample drawn from each imputed dataset is released to the public. A major attraction of this method is that full protection of confidentiality is achieved, since no actual values from the original data are released. Besides, under well-specified imputation model, valid inference for variant estimands can be obtained with simple combining rules (Raghunathan 2003, Reiter 2002, 2005a). Fully synthetic data also have benefit for data utility, as geographic information for small area can be released, which enables data user to perform analysis in small area. However, model specification is challenging for this method, as it requires building a statistical model for the whole population. Moreover, since the synthetic data need to preserve the same relationship as the original data, the accuracy of the statistical model is crucial to valid inferences from synthetic data, and a mis-specified model leads to distorted results from data users' analyses.

Little (1993) suggests limiting imputation to a set of key variables that contain identification information and releasing partially synthetic data as a mixture of actual and multiply-imputed data values. This method retains the advantage of synthetic data but is more practical than simulating the entire data set, since model mis-specification is less of an issue for simulating certain variables than simulating the entire population. Some other approaches to partial synthesis method are described in Kennickell (1997), Little, Liu and Raghunathan, (2004), and Abowd and Woodcock (2004). Reiter (2003) specifies MI

combining rule for partially synthetic data, with estimate of variance calculated differently from the original formula for missing data in Little and Rubin (2002). Inspired by this approach, this dissertation targets the imputation of a small number (one or two) of variables subject to disclosure limitation.

I.3 Disclosure limitation of extreme values in microdata

A number of confidentiality concerns are raised by extreme values of a variable. For example, in surveys that include income, extremely high income values are considered to have the potential to reveal the identity of respondents. These values are generally referred to as sensitive values and require modification before release to the public. The Health Insurance Portability and Accountability Act (HIPAA) privacy rule also restricts release of all age values over 89 in health survey data. Top-coding is a simple and common SDC method for handling this situation. It prevents disclosure on the basis of extreme values of a variable, by censoring values above a pre-chosen "top-code". For example, in the Survey of Income and Program Participation, the U.S. Census Bureau top-codes monthly income at $8,333 in the 1990-1993 panels, such that all values $8,333 or more are now represented by $8,333.

Data analyst can apply several approaches to analyze top-coded data, such as categorizing the top-coded variable to pool top-coded cases into one category, or treating the top-coded values as the true values. In addition, the data user can treat the extreme values as censored; and calculate estimates (e.g., maximum likelihood estimate) under the assumed statistical model, or apply an imputation method to the top-coded dataset and fill in the censored values. These procedures all have limitations for data user: they more or

less distort data distributions, require complicated custom algorithms, or are sensitive to model assumption about the right tail of the distribution.

Another limitation of top-coding lies in the treatment of high-age individuals in longitudinal datasets, where disclosure limitation is particularly challenging, since information about an individual accumulates with repeated measures over time. Because of the risk of disclosure, ages of very old respondents can often not be released; in particular this is a specific stipulation of HIPAA privacy rule for the release of health data for individuals. Top-coding of individuals beyond a certain age (say 80) is a standard way of dealing with this issue, and it may be adequate for cross-sectional data, since the number of cases affected may be modest. However, this approach seriously limits the ability to do longitudinal analysis, particularly survival analyses with chronological age being a key variable of interest.

This problem arises in the Charleston Heart Study (Nietert *et al.*, 2000), a longitudinal study that collects data over 40 years (1960-2000). For longitudinal data from this study to be included in the data archive at the University of Michigan, individual ages beyond age 80 cannot be disclosed, given the geographic specificity of the respondents. Also, given the longitudinal nature of the data, a top-coding approach would need to be applied to all individuals aged 40 or older in 1960, which makes survival analyses almost impossible.

In this dissertation, I develop MI alternatives to top-coding that allow better inferences for the data user using simple MI combining rules, while preserving the SDC benefits of top-coding. Adjusting the partially synthetic approach to our specific problem, we delete the data values greater than a cutoff point, which is chosen to be smaller than

the top-code to achieve a mixing of sensitive and non-sensitive values, and apply MI to fill in these values. We then release multiple imputed datasets to the public. Data users can apply MI combining rules (Reiter 2003) to obtain valid inferences.

I propose non-parametric and parametric MI methods. The non-parametric method is a hot-deck procedure, where we replace the deleted values with values randomly drawn with replacement from the set of deleted values. The parametric method is Bayesian, and assumes a model for the data, draws model parameters from their posterior distribution and then imputes the deleted values with random draws from the posterior predictive distribution.

This dissertation is organized as follows. Chapter II presents our SDC approaches and describes corresponding methods of inference for a population mean. We compare estimates calculated from our imputed datasets with estimates from the original and top-coded dataset in simulation study and application in the 1995 Chinese household income project. Chapter III provides extension of the MI methods in Chapter II in regression analysis, where the outcome is subject to top-coding and assesses inferences of estimates of regression coefficients. Chapter IV describes SDC approaches for longitudinal data and applies these methods in survival analysis of simulated data and data from the Charleston Heart Study. Chapter V presents conclusions and discusses future work.

Chapter II

Multiple Imputation: An Alternative to Top-coding for Statistical Disclosure Control

Abstract

Top-coding of extreme values of variables like income is a common method of statistical disclosure control, but it creates problems for the data analyst. This article proposes two alternative methods to top-coding for SDC based on multiple imputation (MI). We show in simulation studies that the MI methods provide better inferences of the publicly-released data than top-coding, using straightforward MI methods of analysis, while maintaining good SDC properties. We illustrate the methods on data from the 1995 Chinese household income project.

Keywords: confidentiality, disclosure protection, multiple imputation

II.1 Introduction

Statistical disclosure control (SDC) is a class of procedures that deliberately alter data collected by statistical agencies before release to the public, to prevent the identity of survey respondents from being revealed. These methods have increased in importance, with the extensive use of computers and the internet. The goal of SDC methods is to reduce the risk of disclosure to acceptable levels, while releasing a dataset that provides as much useful information as possible for researchers. One aspect of this is the ability to draw valid statistical inferences from the altered data.

Top-coding is a simple and common SDC method that seeks to prevent disclosure on the basis of extreme values of a variable, by censoring values above a pre-chosen "top-code". For example, in surveys that include income, extremely high income values are considered to be sensitive and have the potential to reveal the identity of respondents. By recoding income values greater than a selected "top-code" value to that value, respondents with very high income have reduced risk of disclosure.

It is left to the analyst to decide how top-coded data are analyzed. One approach is to categorize the variable so that top-coded cases all fall in one category – this is sensible, but precludes analyses that treat the variable as continuous. Another approach is to ignore the fact of top-coding and treat the top-coded values as the truth. This method is straightforward, but clearly the data distribution is distorted and biased estimates will be obtained. A better method is to treat the extreme values as censored. Under an assumed statistical model, maximum likelihood (ML) estimates can be obtained using algorithms such as the Expectation-Maximization (EM) algorithm (Dempster, Laird and Rubin, 1977). This method is model-based, and should yield good inferences if the model is correctly specified. But we expect this method to be quite sensitive to model misspecification, especially when the upper tail of the assumed distribution differs markedly from that of the true distribution. The data users can also apply an imputation method to the top-coded dataset and fill in the censored values. A limitation is that the imputed data fail to reflect imputation uncertainty, and imputations are sensitive to assumptions about the right tail of the distribution. We propose alternatives to top-coding that allow better inferences for the data user using simple multiple imputation (MI) combining rules, while preserving the SDC benefits of top-coding.

Multiple imputation has been proposed as a method of SDC (Little, 1993; Rubin, 1993; Little, Liu and Raghunathan, 2004; Reiter, 2003, 2005a, 2005b). An imputation model is built from the original data and observed values are replaced by draws from the predictive distribution based on the model. The imputation process is repeated several times and the imputed datasets are then released to the public. Applying this approach to our problem, we delete the data values greater than a cutoff point, which is chosen to be smaller than the top-code to achieve a mixing of sensitive and non-sensitive values, and apply MI to fill in these values. We then release multiple imputed datasets to the public. Data users can apply MI combining rules (Reiter 2003) to obtain valid inferences, as described in Section II.3.

We propose non-parametric and parametric MI methods. The non-parametric method is a hot-deck procedure, where we replace the deleted values with values randomly drawn with replacement from the set of deleted values. The parametric method is Bayesian, and assumes a model for the data, draws model parameters from their posterior distribution and then imputes the deleted values with random draws from the posterior predictive distribution.

We compare estimates of the mean of the data from our methods with two estimates from top-coded data. The first, as described previously, is to treat the top-coded values as the true values. The second is to treat those values greater than top-code as censored and apply ML estimation under an assumed model.

We also investigate situations where covariates are present. We use the proposed MI methods to fill in for deleted values without conditioning on covariates. We then perform regression analysis on the imputed dataset and compare regression coefficients

with those from original and top-coded data. Extensions of our methods that condition on covariate data are also outlined.

The rest of this paper is organized as follows. Section II.2 presents SDC approaches, and Section II.3 describes corresponding methods of inference for a population mean. Section II.4 describes a simulation study to evaluate the approaches in Section 3, and Section II.5 applies the methods to data from the 1995 Chinese household income project. Section II.6 considers estimates of regression coefficients for a regression where the outcome is subject to our disclosure control methods. Section II.7 gives conclusions and discusses future work.

II.2 Methods of statistical disclosure control

Let Y denote a survey variable (e.g. income) and suppose that values of Y greater than a particular value y_T are considered too sensitive for release to the public. We consider the following approaches to SDC.

(a) Top-coding. Treat y_T as a top-code value, that is, replace values of Y greater than y_T by y_T. The resulting sample is referred to as "top-coded".

(b) Hot-deck MI (HDMI). Choose a value y_I smaller than y_T. Delete the values of Y greater than y_I and replace them with random draws from the set of deleted values. We choose $y_I < y_T$ to achieve a mixing of sensitive and non-sensitive values. We refer to y_I as the cutoff point.

(c) Parametric MI (PMI). The HDMI method provides disclosure protection by scrambling sensitive and non-sensitive values, but it is arguably limited from the point of view of SDC, since actual sensitive data values are released. The PMI methods address this concern by releasing data simulated from a parametric model. First, values greater

than y_I are deleted, as with HDMI. The model – we consider log-normal model and power-transformed normal model (the power normal model for short) – is fitted to the data. Parameters are drawn from their posterior distribution under the assumed model, and deleted values are imputed with draws from their predictive distribution. See Appendix II.1 for details.

Write the complete data as $Y = (Y_{\text{ret}}, Y_{\text{del}})$, where Y_{ret} denotes the retained values and Y_{del} denotes the deleted values beyond the cut-off. We consider two versions of PMI, labeled PMIC and PMID. For PMIC, we draw the parameter ϕ of the model for the data *Y* from its posterior distribution given the complete data *Y*, that is:

$$\text{PMIC: } \phi^* \sim P(\phi \mid Y).$$

We then draw deleted values from the truncated predictive distribution

$$Y_{\text{del}}^* \sim P(Y \mid Y > y_I, \phi^*).$$

For PMID, we apply the parametric model to the deleted data Y_{del}, and draw ϕ from its posterior distribution given Y_{del}:

$$\text{PMID: } \phi^* \sim P(\phi \mid Y_{\text{del}})$$

The next step is similar to PMIC method, except that we draw deleted values from the non-truncated predictive distribution. PMID is less efficient than PMIC since it models the deleted data and fails to exploit fully the information in *Y* when drawing values of parameters. However, modeling the deleted data only as in PMID provides useful robustness to model misspecification, as we shall see below.

II.3 Methods of inference for the mean

We first consider the properties of these SDC methods for inferences about the mean of a variable *Y* subject to top-coding. Some comments concerning inference for other

parameters are provided in Sections II.6 and II.7. The following estimates and associated standard errors are considered:

(1) Before Deletion (BD): The sample mean of original data $(y_1, y_2, ..., y_n)$ prior to SDC is

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} y_i \ . \tag{1}$$

This estimate is used as a benchmark for comparing SDC methods.

(2) Top-coding (TC): The sample mean of top-coded dataset, namely

$$\hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n} y_{it} \ , \tag{2}$$

where $y_{it} = y_i$ when $y_i < y_T$ and $y_{it} = y_T$ when $y_i \geq y_T$. This approach is obviously biased, and our objective is to improve on it with other methods.

(3) Log-normal ML (LNML): The ML estimate based on the log-normal model, computed by the EM algorithm (Appendix II.2). The log-normal is chosen as a convenient model for right-skewed data, but we emphasize that other models could be considered.

The standard errors for methods (1) – (3) are computed by the bootstrap, with B = 100 bootstrap samples.

The five remaining methods are all based on MI, and create D sets of imputations for values beyond the chosen cut-point y_I; D imputed datasets are thus created, where for the d th imputed dataset $Y^{(d)} = (y_1^{(d)}, y_2^{(d)}, ..., y_n^{(d)})$, where $y_i^{(d)} = y_i$ if $y_i < y_I$ and $y_i^{(d)}$ is the d th MI draw if $y_i \geq y_I$. The MI estimate is then

$$\hat{\theta}_{MI} = \frac{1}{D}\sum_{d=1}^{D} \hat{\theta}^{(d)} \ , \tag{3}$$

where $\hat{\theta}^{(d)}$ is the sample mean of *d* th dataset. The MI estimate of variance is

$$T_{MI} = Var(\hat{\theta}_{MI}) = \bar{W} + B/D, \quad (4)$$

where $\bar{W} = \sum_{d=1}^{D} W^{(d)} / D$ is the average of the within-imputation variances $W^{(d)}$ for imputed dataset *d*, and $B = \sum_{d=1}^{D} (\hat{\theta}^{(d)} - \hat{\theta}_{MI})^2 / (D-1)$ is the between-imputation variance. The formula (4) differs from the original MI formula for missing data (where *B* is multiplied by a factor *(D+1)/D*, see e.g. Little and Rubin, 2002, p86), for reasons discussed in Reiter (2003). Imputations for these MI methods are created as follows:

(4) Hot-deck MI (HDMI): Imputations are drawn randomly with replacement from the set of values beyond the cut-off y_I.

(5) Log-normal MIC (LNMIC): Imputations are posterior predictions from a log-normal model fitted to the complete data before deletion.

(6) Log-normal MID (LNMID): Imputations are posterior predictions from a log-normal model fitted to the deleted data beyond the cut-off.

(7) Power-normal MIC (PNMIC): Imputations are posterior predictions from the power-normal model, the power-transformed normal distribution fitted to the full data before deletion. For convenience the power transformation is estimated by ML, and parameters are drawn from the full-data posterior distribution treating the power transformation as known. An alternative approach is to draw the power from its posterior distribution as well, but we made use of the widely available ML routine box.cox.powers( ) in R (R project, 2007) in our calculations.

(8) Power-normal MID (PNMID): Imputations are posterior predictions from the power-normal model, fitted to the deleted data beyond the cut-off.

II.4 Simulation study

A simulation study was carried out to evaluate and compare the SDC methods in Section II.3. We computed point estimates of means and the corresponding variances and confidence intervals from the imputed datasets, and compared them with those calculated from the original dataset prior to SDC.

II.4.1 Study design

Datasets were generated from the following four distributions, all with mean 1: Exponential (1), gamma (1.25, 0.8), lognormal (-0.2, 0.4) and square-root normal (0.9, 0.19) (variances of these distributions are 1, 0.8, 0.49 and 0.69, respectively). Figure II.1 shows the form of these distributions beyond their approximate upper 10^{th} percentile. For each simulated dataset, we calculated the eight mean estimates and their corresponding variances as discussed in Section II.3. To assess the validity of inferences, we calculated the 95% confidence intervals (CI's) based on the usual normal approximation, and computed the proportion of CI's that contain the true mean. For parametric estimates in Section II.3, the simulated data distributions are allowed to differ from those assumed in the statistical models, in order to provide an assessment of sensitivity to model misspecification.

In our simulations we chose the 95^{th} percentile of the population distribution as the top-code value y_T. Denote by n_S the number of sensitive sample values greater than y_T. We studied two alternative values for the cutoff point y_I: y_{I90}, the value with $2n_S$ larger values in the sample, and y_{I80}, the value with $4n_S$ larger values in the sample. These values correspond approximately to the 90^{th} and 80^{th} percentile values of the

distribution, and for this reason we label the version of a method * that uses cutoff y_{I90} "*90" and the version that uses cutoff y_{I80} "*80".

Clearly the disclosure risk is reduced by increasing the fraction of non-sensitive values that are imputed. A simple measure of the risk of disclosure is the proportion of multiple-imputed values beyond the top-code value y_T. For all the MI methods, this is approximately 50% when the cutoff point is y_{I90}, and approximately 25% when the cutoff point is y_{I80}.

II.4.2 Results

Tables II.1 and II.2 present simulation results for sample sizes 2000 and 200, respectively. Results are based on 500 data sets for each model. We set $B = 100$ for the number of bootstrap samples. For both NPMI and PMI methods, we created $D = 5$ imputed datasets. As expected, TC underestimates the mean and has poor confidence coverage, particularly for the $n = 2000$ sample size where bias is a relatively large component of the RMSE. The HDMI methods (HDMI90 and HDMI80) have minimal bias and close to nominal coverage for all the simulated populations, with small increases in RMSE and CI width compared with the BD estimate. LNML dominates other methods for lognormal data, but has serious bias and very poor confidence coverage for the other data sets, suggesting marked sensitivity to model specification. The LNMIC methods have similar properties, although they are less biased and have somewhat better confidence coverage than LNML when the model is mis-specified. The LNMID methods are much more robust than their LNMIC counterparts, yielding minimal bias and good confidence coverage for all problems simulated.

The PNMIC methods do consistently well in terms of RMSE. Confidence coverage is close to the nominal value, except for exponential data with n = 2000 where coverage is a little low. This suggests that the power normal model yields good fits to the range of models simulated. The PNMID methods also perform well in terms of bias and confidence coverage, but they are less efficient than the PNMIC methods.

When lowering the cutoff point from y_{I90} to y_{I80}, we observe minor increases in RMSE for HDMI, LNMID and PNMIC, and LNMIC when correctly specified. More substantial increases in RMSE are seen for PNMID, and LNMIC when mis-specified. The losses in efficiency for HDMI80, LNMID80 and PNMIC80 may be acceptable given the increase in disclosure protection.

To provide a visual illustration of the imputation methods under potentially mis-specified models, Figure II.2 shows the original deleted data values and the imputed values from the HDMI and four PMI methods, with cutoff y_{I90}, for one of the simulated square-root normal data sets with n =2000. Note that the mean of the deleted values is 2.78. The HDMI predictions look similar to the deleted values and have a similar mean, 2.80.

The LNMIC predictions are too severely skewed and have some extreme predictions, reflecting the damaging effect on predictions in the tail of applying a mis-specified model to the full data set. The LNMIC predictions average 5.68, a marked overestimate. In contrast, when the lognormal model is correctly specified, the predictions track the deleted values well (data not shown). The LNMID predictions have the shape of a normal distribution, reflecting effects of model misspecification, but their mean, 2.72, matches the mean of the deleted values well.

The PNMIC predictions match the deleted values quite well and have a similar mean (2.87), reflecting that this model is correctly specified, since the power normal model includes the square-root normal as a particular case. The PNMID predictions are more skewed than the deleted values, a reflection that the power normal model does not fit that well when applied to the deleted values; however these predictions average 2.79, very close to the mean of the deleted values.

In summary, we see that for inference about the mean, the HDMI method performs best overall, but has the limitations in terms of SDC noted above. Among the parametric imputations, LNMID has the best performance and it works almost as well as HDMI. In particular it gives good estimates of the mean even when the log-normal model is mis-specified and LNMIC is biased, reflecting the fact that the impact of mis-specification on the mean is limited when the model is fit to the deleted data. (On the other hand this method will work less well for large percentiles under mis-specification, since the imputed distribution in the upper tail is distorted). PNMIC also does quite well, reflecting that the power-normal model fits the simulated distributions well. The PNMID method is satisfactory in terms of bias and confidence coverage, but it is considerably less efficient than PNMIC or LNMIC since it is fitting the larger power-normal model to the small set of deleted values. The risk of disclosure is reduced when we increase the set of value being mixed with the sensitive cases, at the expense of some loss of efficiency of the estimate.

II.5 Application

We applied the above SDC methods to a subset of data from the 1995 Chinese Household Income Project (Riskin et al.2000). This project was designed to measure the personal

income distribution in the People's Republic of China in 1995. Income information on both household and individual were recorded for rural and urban areas. Since SDC was not applied to the released data set, the effectiveness of the various SDC methods can be readily assessed.

II.5.1 Data analysis

We illustrated application of the SDC methods to both urban and rural individual income values. After deletion of missing and zero income values, the urban dataset included 15,983 individuals and the rural dataset had 6,296 individuals. We applied the top-coding, HDMI and PMI methods to the data and compute estimates (1) – (8) described in Section II.3. The power transformation parameter estimated by the R function was 0.13 for the rural data and 0.45 for the urban data.

II.5.2 Results

Table II.3 displays the results from the data analysis. We plot the original deleted data values and the imputed values from PMI and HDMI methods using cutoff point y_{I90} in Figure II.3 and II.4 for urban and rural data, respectively.

Predictably, in both urban and rural cases, TC underestimates the mean and yields an underestimate of standard error because of the reduction in standard deviation from top-coding. HDMI90 provides the estimate of the mean closest to the BD mean, with a 16% increase in standard error. LNML has a large positive bias, indicating sensitivity to the lack of fit of the log-normal model for these data. LNMIC90 is also quite biased, although it performs better than LNML. LNMID90 has negligible bias and a slightly smaller standard error than BD in both urban and rural data. The power-normal model estimates PNMIC90 and PNMID90 also have small bias. For urban data, PNMIC90 has

relative CI widths less than that from BD, which seems anti-conservative; for the rural data it has standard error very similar to BD. PNMID90 shows a slight increase in CI width for urban data but a large increase in CI width for rural data, reflecting difficulties in fitting this complex model to the deleted data. Changing the cutoff point to y_{I80} results in some increases in bias and standard error for LNMIC80 estimates. Estimates from HDMI80, LNMID80 and PNMIC80 are still acceptable, as are PNMID80 estimates in the urban sample. For the rural data, PNMID80 yields an estimate with strikingly large bias and standard error, the result of some very extreme outliers from imputation. It is important to check that the method is not creating extreme outliers as in this illustration.

II.6 Study of SDC methods with covariates

To make the situation more complicated and realistic, we now introduce covariates into our analysis. We use the previous MI methods to impute deleted values, apply a linear regression model to the imputed data set, calculate estimates of regression coefficients and compare them with those from the original data. Since the MI methods do not condition on the covariates, we expect some bias from this procedure; our interest is in the size of the bias and resulting distortions in confidence coverage.

II.6.1 Simulation Study

Datasets were generated from the following two distributions:

High correlation distribution: $\begin{pmatrix} X \\ \log(Y) \end{pmatrix} \sim$ Bivariate Normal $\left(\begin{pmatrix} 38 \\ 8.6 \end{pmatrix}, \begin{pmatrix} 93 & 5 \\ 5 & 0.38 \end{pmatrix} \right)$

Low correlation distribution: $\begin{pmatrix} X \\ \log(Y) \end{pmatrix} \sim$ Bivariate Normal $\left(\begin{pmatrix} 38 \\ 8.6 \end{pmatrix}, \begin{pmatrix} 93 & 2.5 \\ 2.5 & 0.38 \end{pmatrix} \right)$

Here X is considered as independent variable and Y is dependent variable. For each simulated dataset, we applied the SDC methods to impute for deleted values and

performed linear regression of log(Y) on X. We then calculated the estimates of regression coefficient, their corresponding variances and confidence coverage, as we did for the estimates of the mean in Section II.4.

Table II.4 displays results for sample sizes 2000 and 200. For data from the high correlation distribution, TC underestimates the regression coefficient, with large RMSE and very poor confidence coverage. HDMI90 also underestimates the coefficient, as is to be expected since the relationship between the outcome and covariate is attenuated by randomly "shuffling" the values beyond top-code. Nevertheless it is less biased and has better coverage than TC. The other PMI90 methods yield almost the same result as HDMI90. When changing the cutoff point to y_{I80}, all MI methods yield estimates with more bias and RMSE, reduced efficiency and worse confidence coverage. When the data are from the low correlation distribution, all methods have similar properties, but the MI methods have satisfactory properties. This suggests that for more moderately correlated data, the attenuating effect from imputing without conditioning on X is relatively minor. For the smaller sample size of 200, all methods are improved in terms of confidence coverage.

II.6.2 Application in Chinese income data

We also consider the impact of the SDC methods on a multiple regression, estimated on a subset of the urban data in the 1995 Chinese Household Income Project. Our sample included 10,752 individuals and 10 variables, with the logarithm of income treated as the dependent variable. The covariates were age, gender, marital status, education level, occupation, work environment, work intensity, years of work experience and logarithm of hours worked per week. To simplify the analysis, we only investigate the scenario where

the covariates are complete. We applied the top-coding, HDMI and PMI methods to the data, where the PMI methods were applied to the marginal distribution of the dependent variable. We again computed estimates of regression coefficients.

We plot standardized regression coefficients after imputation against those from the original dataset in Figure II.5. We choose HDMI, LNMID and PNMIC as representations of the MI methods and use the 90^{th}, 80^{th}, 60^{th} and 40^{th} percentiles of the outcome variable as cutoff points, to assess the effect of increasingly severe imputation. We observe that with y_{I90}, the regression coefficients from the imputed dataset are very close to those from the dataset before imputation; and imputation with y_{I80} also has a minor effect on the coefficients. This particular case is similar to the low correlation scenario from simulation study. We conclude that in a situation where the outcome and covariates are not strongly associated, the proposed MI methods are robust to the failure of the imputation model to condition on covariates. Lowering cutoff points results in larger deviation from original coefficients, leading to greater attenuation of the relationship between outcome and covariates.

II.7. Discussion

Why should the secondary data analyst prefer our proposed MI methods for SDC to top-coding? First, appropriate treatment of the top-coded data, using methods like maximum likelihood for censored data, requires custom algorithms that are not widely available in standard statistical software; as a result we believe that analysts often treat the top-codes as true values and assume the bias introduced by this will be small. In contrast, MI inferences only require complete-data methods and simple MI combining rules. Second, the MI methods tend to be less sensitive than top-coding to model misspecification, as

seen in our simulation studies. There are two reasons for this – the random draws from the predictive distribution provide variability even if the model is wrong, and the MI's are based on parameter estimates that use information in the original data that is not available in the top-coded data. The data producer is also in a better position to assess and limit model misspecification, since (s)he can compare analyses based on the MI data with analyses based on the original data. In particular, the imputations from the model can be compared with the true values.

For the data producer, MI has the advantage that the balance between disclosure protection and information loss can be controlled by the choice of cut-off and number of MI's released. The use of MI allows imputation uncertainty to be propagated, and the multiple imputations of a particular value enhance disclosure protection by making clear to a potential snooper that these values are not real.

For inference about the mean, the HDMI, PNMIC and LNMID methods were decisively superior to top-coding in our simulations. It is clear that treating the top-coded data as the observed data yields bias, the size of which depends on the fraction of cases top-coded and the extremity of the top-code. The ML methods based on top-coded data are harder to implement for the data user, and are vulnerable to model misspecification. Of our preferred MI methods, the HDMI method produces excellent inferences, but has limitations as an SDC method, since original values in the data set are retained. The PNMIC and LNMID methods both yield good inferences for the mean, with the PNMIC yielding imputations that match well the distribution of the deleted values. The LNMIC method is vulnerable to misspecification, and the PNMID yields good conference coverage but tends to be less efficient than LNMID and PNMIC.

We chose the log-normal and power normal models to illustrate parametric MI, since they are commonly used to model skewed data; they are not universal, and the MI approach could be applied by the data producer with other models that are more suitable for the data at hand. MI based on a model fit to all the data (as in the "C" methods) is efficient, but vulnerable to model misspecification. Hence if this approach is adopted, attention to good model specification is needed – in particular, it is important to check that the distribution of the imputed values in the tail is similar to the distribution of the deleted values.

MI based on a model fitted to the deleted values alone (the "D" methods) involves some loss of efficiency, but is more robust to model misspecification, since the model is being fitted to the data that are being deleted. Here simpler models worked well for the mean, but more refined models may still be needed to get the shape of the distribution in the tail right. We note that while TC is generally inferior, it is better than MI when estimating percentiles below the top-code but above the cutoff point, since the MI methods delete values in this range that are retained by TC.

Our results clearly demonstrate the tradeoff between reducing the risk of disclosure by allowing a larger pool of non-sensitive values for mixing with the sensitive cases, and reduced efficiency of the estimates. The MI technology is very helpful in propagating the increased uncertainty from the disclosure control method, resulting in good confidence coverage.

MI of deleted values should in principle condition on the observed information, and hence a refinement of the proposed methods is to condition the predictive distribution of the deleted values on observed covariates. Our preliminary assessment of inferences

for regression coefficients in Section II.6 confirms that failure to condition on covariates leads to an attenuation of relationships between these covariates and *Y*. The bias was serious for highly correlated covariates and large samples, but in other situations was surprisingly minor. This suggests that when applying the MI method to multivariate data, it may suffice to condition on a relatively small set of covariates that are strongly associated with the variable subject to SDC. A simple way of doing this for a small set of categorical covariates is to apply the methods presented here within strata defined by the covariates, as in the urban and rural strata in the application in Section II.5. More generally, regression-based extensions of the PNMIC and PNMID can be readily defined by including the key covariates in the mean function. We plan to develop and assess these refinements in future work.

We have confined attention here to inferences from top-coding and MI methods; other alternatives to top-coding are also of interest. One such alternative is to add random noise (e.g., normal noise as in Fuller 1993) to the values beyond top-code. This method may yield satisfactory (if less efficient) inferences for the mean, but noise with substantial variance needs to be added to yield reductions of disclosure risk comparable to those of MI, and adding such noise potentially distorts the distribution. Also custom adjustments are needed for inferences about other parameters, such as regression coefficients. Note that if multiple imputes are created by adding noise to the true value, the average of these imputations converges to the true value as the number of imputations increases, an undesirable property from the perspective of disclosure protection. Our MI methods do not have this property: the average of the MI imputed values converges to the conditional mean of the predictive distribution, not the true deleted value. Thus

increasing the number of MI's improves efficiency of inferences without compromising gains in disclosure protection. This is a major attraction of MI as an SDC method.

Acknowledgments

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Table II.1 Inferences about the mean from simulation study, sample size = 2000

Method*	Exponential Data				Gamma Data				Log-normal Data				Square-root-normal Data			
	Bias ($*10^3$)	RMSE** ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)
(1) BD	-2	24	1.00	93.8	-0	19	1.00	96.2	1	16	1.00	94.0	-0	18	1.00	94.4
(2) TC	-51	55	0.84	23.2	-42	45	0.85	30.0	-39	41	0.80	13.6	-33	37	0.89	45.6
(3) LNML	359	363	2.40	0	213	216	1.81	0	1	16	1.01	93.8	823	836	7.99	0
(4) HDMI90	-2	24	1.05	94.8	-0	19	1.05	97.4	1	16	1.09	96.6	-0	19	1.04	95.4
HDMI80	-2	24	1.12	95.8	-0	19	1.10	98.2	1	17	1.14	96.2	-0	18	1.08	96.8
(5) LNMIC90	206	212	2.41	1.0	130	134	1.85	1.0	0	17	1.02	94.8	354	362	4.19	0.6
LNMIC80	317	322	2.80	0	202	206	2.09	0	1	17	1.04	94.4	594	606	5.24	0.2
(6) LNMID90	-2	24	1.00	93.8	-1	19	1.01	95.8	-0	16	1.00	94.4	-1	19	1.01	93.8
LNMID80	-4	24	1.00	93.4	-2	19	1.01	95.8	-1	17	0.99	93.2	-1	19	1.01	94.4
(7) PNMIC90	11	27	1.08	89.6	7	21	1.05	95.2	0	17	1.02	95.0	9	21	1.05	93.0
PNMIC80	14	29	1.10	89.0	9	22	1.07	93.8	1	17	1.03	94.6	15	24	1.07	88.6
(8) PNMID90	2	27	1.18	95.0	2	21	1.15	97.2	0	17	1.15	94.0	1	19	1.08	95.2
PNMID80	21	61	2.29	97.4	14	34	1.72	98.0	5	27	1.65	96.2	8	24	1.40	96.8

* BD = before deletion, TC = top-coded, LNML = Censored ML for lognormal model, HDMI = hot deck MI, LNMIC = lognormal MI fitted to complete data, LNMID = lognormal MI fitted to deleted data, PNMIC = power normal MI fitted to complete data, PNMID = power normal MI fitted to deleted data

** Here “RMSE” refers to root mean squared error. “Rel-wid” refers to “relative width”, which is fraction of 95 CI % width comparing to estimate 1. “Cover” refers to the 95% CI coverage.

Table II.2 Inferences about the mean from simulation study, sample size = 200

Method	**Exponential Data**				**Gamma Data**				**Log-normal Data**				**Square-root-Normal Data**			
	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)	Bias ($*10^3$)	RMSE ($*10^3$)	Rel-wid	Cover (%)
(1) BD	5	71	1.00	94.2	-5	60	1.00	95.2	-6	50	1.00	93.2	-1	55	1.00	94.8
(2) TC	-45	75	0.84	84.6	-47	69	0.86	86.4	-45	60	0.81	77.2	-34	59	0.89	90.4
(3) LNML	384	424	2.56	38.0	207	232	1.80	58.2	-5	50	1.02	95.0	833	961	9.39	40.8
(4) HDMI90	5	72	1.06	96.0	-5	60	1.06	95.4	-6	51	1.08	94.4	-1	55	1.04	96.2
HDMI80	5	71	1.11	96.4	-5	62	1.11	95.8	-5	52	1.14	95.8	-2	55	1.08	97.6
(5) LNMIC90	227	277	2.42	87.4	126	165	1.84	92.2	-7	52	1.03	93.4	364	447	4.17	80.8
LNMIC80	338	395	2.85	70.2	192	232	2.08	78.0	-5	53	1.06	93.6	608	732	5.22	46.8
(6) LNMID90	8	73	1.03	94.8	-4	61	1.03	94.8	-4	51	1.03	95.6	-0	57	1.02	94.4
LNMID80	6	73	1.02	94.4	-6	62	1.02	95.2	-7	52	1.01	94.4	-1	57	1.03	95.6
(7) PNMIC90	18	79	1.09	94.8	0	65	1.05	95.8	-6	51	1.05	95.0	8	58	1.06	95.6
PNMIC80	23	83	1.12	94.6	5	65	1.08	95.8	-4	53	1.07	95.4	17	63	1.09	95.4
(8) PNMID90	15	94	1.22	94.8	4	69	1.23	96.4	-2	55	1.19	95.2	3	60	1.10	96.6
PNMID80	73	407	2.83	96.0	23	222	1.85	95.8	3	67	1.40	95.0	16	112	1.57	96.4

Table II.3 Comparison of mean estimates, 1995 Chinese Household Income Project, Urban and Rural data

Method	Urban data				Rural data			
	Estimate	Fraction (%)	SE	Rel-wid	Estimate	Fraction (%)	SE	Rel-wid
(1) BD	6196	0	36	1.0	2196	0	339	1.0
(2) TC	5895	-4.86	25	0.70	1969	-10.36	25	0.65
(3) LNML	7732	25.8	85	2.38	2675	21.8	59	1.53
(4) HDMI90	6196	-0	41	1.16	2196	0	45	1.16
HDMI80	6196	-0	43	1.19	2197	0.01	47	1.22
(5) LNMIC90	6760	9.10	58	1.61	2512	14.39	70	1.80
LNMIC80	7320	18.14	69	1.92	2653	20.80	77	1.98
(6) LNMID90	6174	-0.35	33	0.92	2179	-0.81	36	0.93
LNMID80	6162	-0.55	32	0.90	2164	-1.46	35	0.90
(7) PNMIC90	6035	-2.60	29	0.80	2205	0.39	39	1.01
PNMIC80	6089	-1.73	30	0.83	2223	1.21	41	1.05
(8) PNMID90	6135	-1.98	37	1.03	2196	-0.02	70	1.80
PNMID80	6108	-1.41	39	1.09	2378	8.26	338	8.74

** Here “SE” refers to standard error of the estimate. “Fraction” refers to fractional deviation from BD mean. “Rel-wid” refers to “relative width”, which is fraction of 95 CI % width comparing to estimate 1.

Table II.4 Inference for regression coefficient from simulation study

Method	Sample size 2000								Sample size 200							
	High correlation				Low correlation				High correlation				Low correlation			
	Estimate ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Estimate ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Estimate ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Estimate ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
(1) BD	537	8	1.00	94.0	268	13	1.00	93.8	536	24	1.00	94.6	266	41	1.00	94.6
(2) TC	510	28	0.95	5.6	255	19	0.94	76.6	508	39	0.95	73.8	253	43	0.94	92.8
(3) HDMI90	528	12	1.13	82.6	263	14	1.04	94.6	526	27	1.14	95.2	262	41	1.04	95.4
HDMI80	514	25	1.25	26.6	256	18	1.06	86.2	511	37	1.26	91.2	255	43	1.06	95.2
(4) LNMIC90	528	13	1.07	78.2	264	14	1.02	95.0	526	29	1.08	91.2	261	42	1.02	94.
LNMIC80	514	26	1.14	22.2	256	18	1.03	84.2	511	39	1.15	83.4	254	43	1.03	95.2
(5) LNMID90	528	13	1.08	78.2	263	14	1.02	94.2	526	28	1.10	93.2	262	41	1.03	95.0
LNMID80	514	26	1.16	21.2	256	18	1.03	84.6	512	37	1.18	87.2	255	43	1.04	95.6
(6) PNMIC90	528	13	1.06	77.2	264	14	1.01	93.2	526	28	1.08	93.7	262	41	1.02	94.8
PNMIC80	514	25	1.13	23.0	257	18	1.02	85.6	512	38	1.16	86.0	255	43	1.03	94.9
(7) PNMID90	527	13	1.06	72.8	263	15	1.01	91.6	525	28	1.08	93.9	261	41	1.02	95.2
PNMID80	512	27	1.13	14.8	256	18	1.02	83.4	510	38	1.15	83.6	254	43	1.03	94.9

Figure II.1 Tails of the Data Distributions in Simulation Study

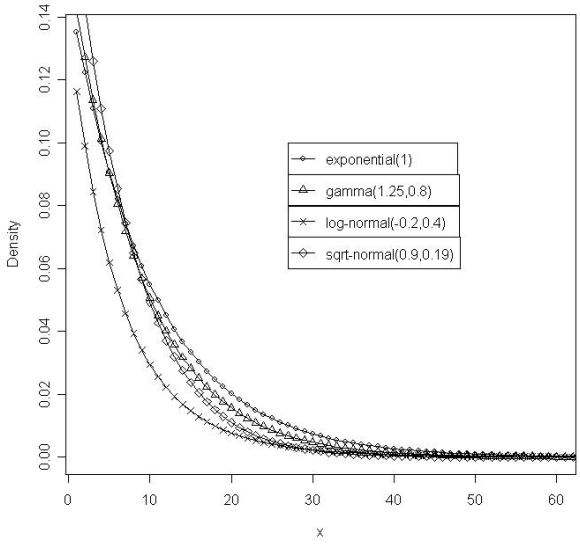


Figure II.2 Deleted and imputed values for square-root-normal data (n=2000) (values greater than 8 are pooled into one category)

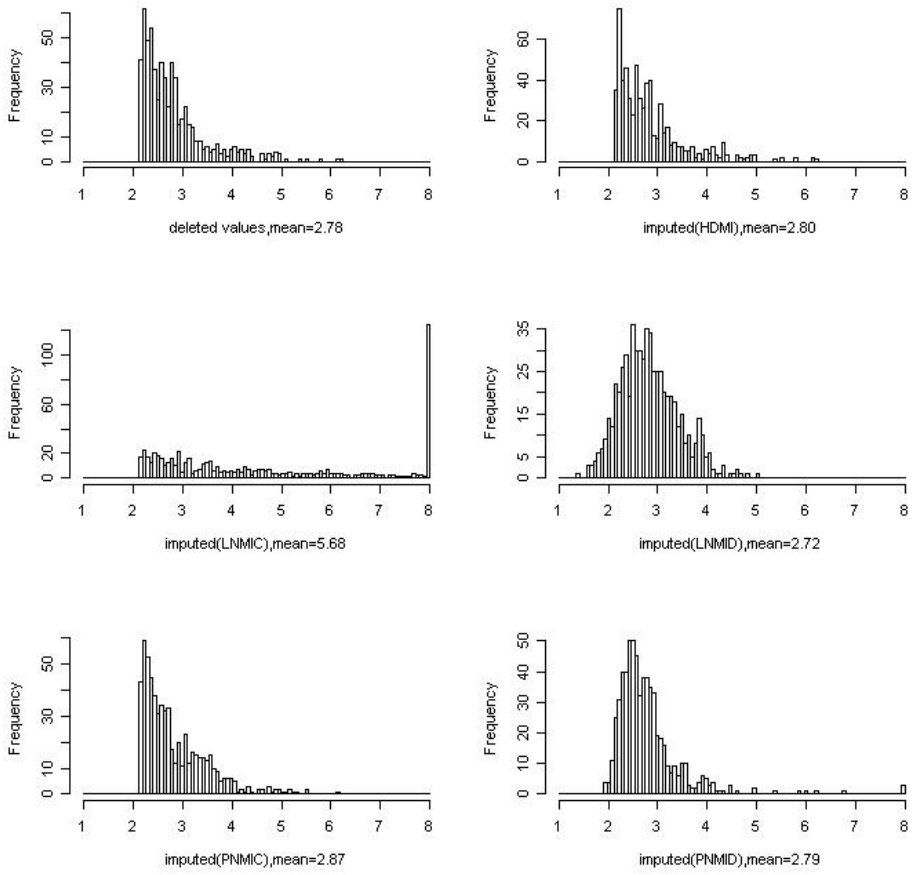


Figure II.3 Deleted and imputed values for 1995 Chinese household income project, urban data (values greater than 85,000 are pooled into one category)

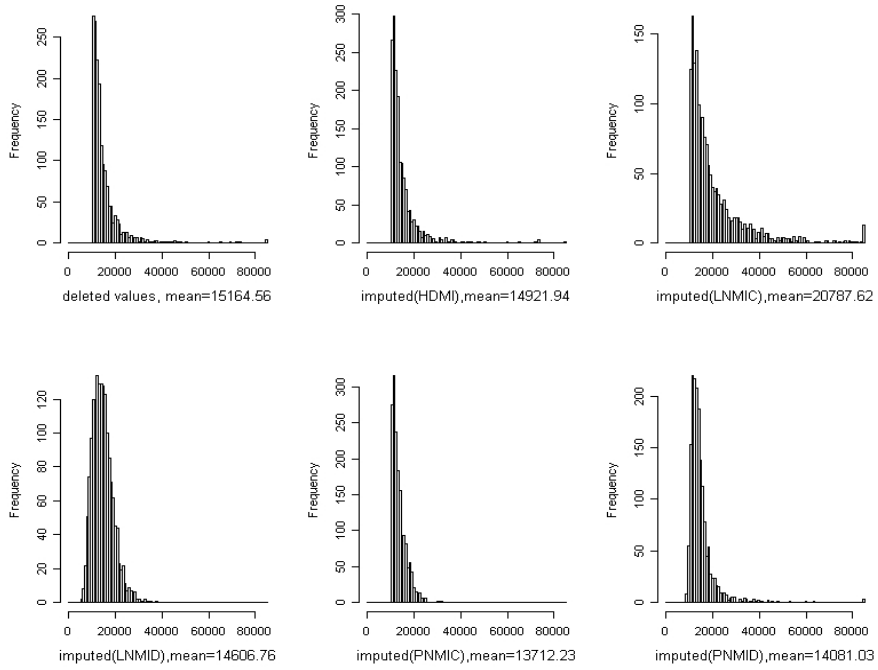


Figure II.4 Deleted and imputed values for 1995 Chinese household income project, rural data (values greater than 60,000 are pooled into one category)

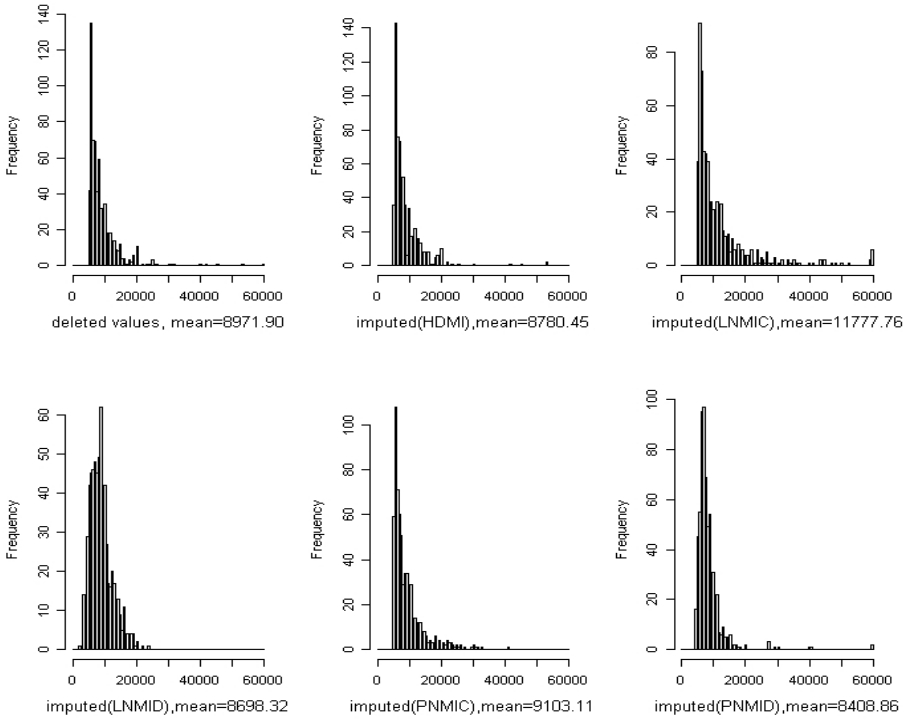
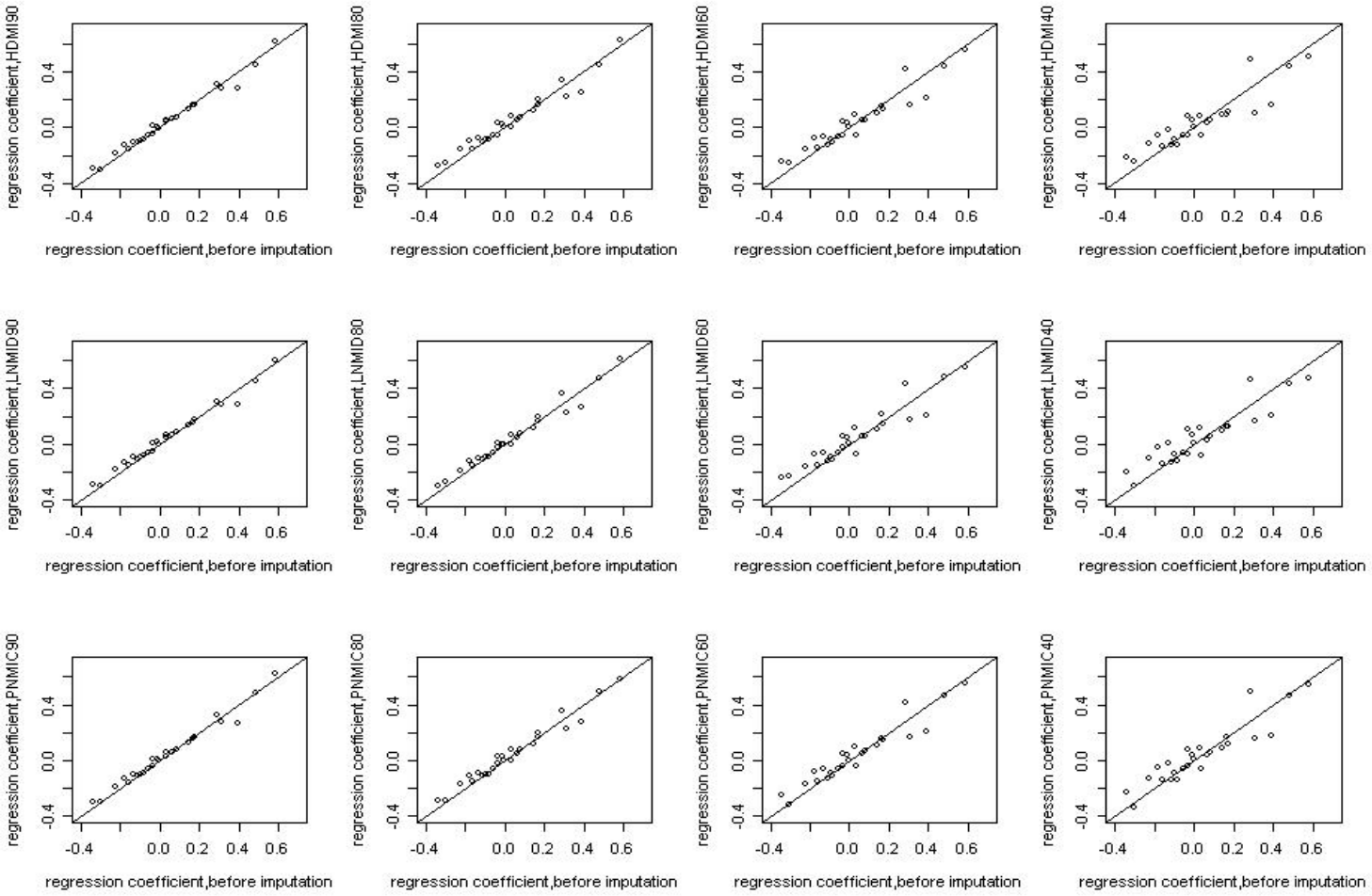


Figure II.5 Standardized regression coefficients, after versus before imputation.
1995 Chinese household income project, urban data. (Top row, HDMI, with cutoff points being 90, 80, 60, 40 percentiles, from left to right. Middle row, LNMID. Bottom row, PNMIC. Line: $y = x$)



Appendix II.1: PMI method for log-normal model and power-transformed normal model

For X from log-normal(μ,σ^2) distribution, $Y=\log(X)\sim N(\mu,\sigma^2)$. If X is from the power-transformed normal (μ,σ^2,λ) distribution with $\lambda\neq 0$, $Y=\left(X^{\lambda}-1\right)/\lambda\sim N(\mu,\sigma^2)$. To apply the PMI method we estimate λ by its ML estimate $\hat{\lambda}$ using the widely available routine box.cox.powers( ) in R (see Fox 2006), and then assume $Y=\left(X^{\hat{\lambda}}-1\right)/\hat{\lambda}\sim N(\mu,\sigma^2)$. (A more principled approach would also simulate λ from its posterior distribution).

Given data $Y=(y_1,...y_n)$ from the $N(\mu,\sigma^2)$ distribution, the posterior distribution of parameters is as follows,

$$\sigma^2\,|\,Y\sim\frac{(n-1)S^2}{\chi^2_{n-1}},\text{ where }S^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2 \qquad \text{(IIA1)}$$

and

$$\mu\,|\,\sigma^2,Y\sim N(\bar{y},\sigma^2/n)\,. \qquad \text{(IIA2)}$$

We draw parameters μ^*,σ^{*2} from their posterior distribution and then draw deleted values for normal data from the predictive distribution

$$Y^*_{\text{del}}\sim N(\mu^*,\sigma^{*2}\,|\,Y>\log y_I)\,. \qquad \text{(IIA3)}$$

We then transform the draws of normal data back to log-normal and power-transformed normal data:

$$\text{log-normal: }X^*_{\text{del}}=\exp(Y^*_{\text{del}}) \qquad \text{(IIA4)}$$

$$\text{power-transformed normal: }X^*_{\text{del}}=\sqrt[\hat{\lambda}]{(\hat{\lambda}Y^*_{\text{del}}+1)} \qquad \text{(IIA5)}$$

Appendix II.2: EM algorithm for log-normal model

If X is log-normal(μ, σ^2), then $Y = \log(X)$ is $N(\mu, \sigma^2)$ and $\mu' = E(X) = \exp(\mu + \sigma^2/2)$.

Let $Y = (y_1, ...y_n)$ be a random sample from $N(\mu, \sigma^2)$, and suppose y_i is treated as missing if and only if $y_i > c$, where c is a known censored value. Without loss of generality, we assume y_i is observed for $i = 1, 2, ..., r$ and missing for $i = r+1, ..., n$. The complete-data likelihood is

$$L(\mu, \sigma \mid Y) \propto \exp\{-n \log \sigma - \sum_{i=1}^{n} y_i^2/(2\sigma^2) - n\mu^2/(2\sigma^2) + \mu \sum_{i=1}^{n} y_i/\sigma^2\}. \tag{IIA6}$$

The complete-data sufficient statistics are

$$S(Y) = (\sum_{i=1}^{n} y_i, \sum_{i=1}^{n} y_i^2). \tag{IIA7}$$

We write $Y = (Y_{\text{obs}}, Y_{\text{del}})$, where Y_{obs} denotes the observed values and Y_{mis} denotes the missing values. Given parameter estimates $\theta^{(t)} = (\mu^{(t)}, \sigma^{(t)})$, the $(t+1)$th iteration of EM method is as follows:

E-step:

$$\begin{aligned} s_0^{(t+1)} &= E(\sum_{i=1}^{n} y_i \mid Y_{\text{obs}}, \theta^{(t)}) \\ &= \sum_{i=1}^{r} y_i + \sum_{i=r+1}^{n} E(y_i \mid y_i > c, \theta^{(t)}) \\ &= \sum_{i=1}^{r} y_i + (n-r) \int_c^{\infty} y \frac{1}{\sqrt{2\pi\sigma^{(t)2}}} \mathrm{e}^{-(y-\mu^{(t)})^2/(2\sigma^{(t)2})}\, dy \left(\int_c^{\infty} \frac{1}{\sqrt{2\pi\sigma^{(t)2}}} \mathrm{e}^{-(y-\mu^{(t)})^2/(2\sigma^{(t)2})}\, dy \right)^{-1} \end{aligned} \tag{IIA8}$$

$$s_1^{(t+1)} = E(\sum_{i=1}^{n} y_i^2 \mid Y_{\text{obs}}, \theta^{(t)})$$

$$= \sum_{i=1}^{r} y_i^2 + (n-r)\int_c^{\infty} y^2 \frac{1}{\sqrt{2\pi\sigma^{(t)2}}} \mathrm{e}^{-(y-\mu^{(t)})^2/(2\sigma^{(t)2})}\, dy \left(\int_c^{\infty} \frac{1}{\sqrt{2\pi\sigma^{(t)2}}} \mathrm{e}^{-(y-\mu^{(t)})^2/(2\sigma^{(t)2})}\, dy \right)^{-1} \tag{IIA9}$$

M-step:

$$\begin{aligned} \mu^{(t+1)} &= s_0^{(t+1)} / n \\ \sigma^{(t+1)2} &= s_1^{(t+1)} / n - s_0^{(t+1)2} / n^2 \end{aligned} \tag{IIA10}$$

Once the sequence of $\theta^{(t)}$ has converged to a stable value ($\tilde{\mu}, \tilde{\sigma}$), we calculate the ML estimate of μ' as

$$\hat{\theta}_4 = \tilde{\mu}' = \exp(\tilde{\mu} + \tilde{\sigma}^2 / 2)\,. \tag{IIA11}$$

Chapter III

Extensions of Multiple Imputation Methods as Disclosure Control Procedure for Multivariate Data

Abstract

Multiple imputation (MI) has been proved to be effective statistical disclosure control (SDC) method for data with extreme values. Previous studies demonstrate MI methods provide better inference of the publicly-released data than the commonly-used top-coding procedure, while maintaining good SDC properties. We propose stratified and regression-based extensions of these MI methods for multivariate analysis. We show in simulation studies that our proposed methods work well in preserving relationship within multivariate data and provide results from regression analysis close to those obtained before imputation. We illustrate the methods on data from the 1995 Chinese household income project.

Keywords: confidentiality, disclosure protection, multiple imputation

III.1 Introduction

Statistical disclosure control (SDC) is a class of procedures that deliberately alter data collected by statistical agencies before release to the public, to prevent the identity of survey respondents from being revealed. These methods have increased in importance, with the extensive use of computers and the internet. The goal of SDC methods is to reduce the risk of disclosure to acceptable levels, while releasing a dataset that provides

as much useful information as possible for researchers. One aspect of this is the ability to draw valid statistical inferences from the altered data.

A great number of confidentiality concerns are raised by extreme values of variable. For example, in surveys that include income, extremely high income values are considered to have the potential to reveal the identity of respondents. Top-coding is a simple SDC procedure in this situation. A "top-code" is defined, and values greater than the top-code are recoded to that value. Top-coding is easy to implement, and widely used in surveys.

We have proposed multiple imputation as an alternative to top-coding for disclosure limitation (An and Little, 2007a). Data values greater than a cutoff point, which is chosen to be smaller than the top-code, are deleted. These values are replaced either by random draws from the set of deleted values (the hot-deck procedure), or by draws from the posterior predictive distribution based on the imputation model (the Bayesian procedure). The imputation process is repeated several times and the imputed datasets are then released to the public. Inferences can be calculated with MI combining rules (Reiter 2003). An and Little (2007a) show that MI methods provide better inferences than top-coding, while maintaining good SDC properties.

An and Little (2007a) focus mainly on inference for a population mean, yet most uses of publicly-released data files concern multivariate analysis. That paper also shows that in situation where the outcome variable is subject to top-coding, failure of the imputation model to condition on covariates leads to attenuation of relationships between outcome and covariates. The goal of this article is to propose extensions of MI methods

for multivariate data that preserve the associations between variables and yield valid estimate of regression coefficients.

We propose two extensions, stratified MI and regression-based MI. For the stratified method, we calculate predicted values of the outcome variable from regression model and create strata based on the predicted values. We then apply previous MI methods within each stratum to fill in deleted values. The regression method is based on a regression of the outcome on the set of fully observed covariates. We condition the predictive distribution of the deleted values on covariates for imputation, by including the covariates in the mean function of the outcome.

We compare estimates of regression coefficients from our methods with estimates from the original data, and with two estimates from top-coded data. The first treats the top-coded values as the true values. The second treats values greater than top-code as censored, and bases inferences on a model fitted to the censored data.

The rest of this paper is organized as follows. Section III.2 presents our SDC approaches and extensions. Section III.3 describes corresponding methods of inference for regression coefficients. Section III.4 describes a simulation study to evaluate the approaches in Section III.3, and Section III.5 applies the methods to data from the 1995 Chinese household income project. Section III.6 concludes with discussion.

III.2 Methods of statistical disclosure control

Let Y denote a survey variable (e.g. income) and suppose that values of Y greater than a particular value y_T are considered too sensitive for release to the public. Let X denote a set of fully observed variables that are not subject to disclosure limitation methods. Our goal is to develop SDC methods that preserve relationship between Y and the X's .

III.2.1 Previous SDC methods

For inference about the marginal mean of *Y* without covariates, An and Little (2007a) distinguish the following methods.

(A) Top-coding. Treat y_T as a top-code value, that is, replace values of Y greater than y_T by y_T. The resulting sample is referred to as "top-coded".

(B) Hot-deck MI (HDMI). Choose a value y_I smaller than y_T. Delete the values of Y greater than y_I and replace them with random draws from the set of deleted values. We choose $y_I < y_T$ to achieve a mixing of sensitive and non-sensitive values. We refer to y_I as the cutoff point.

(C) Parametric MI (PMI). The HDMI method is arguably limited from the point of view of SDC, since actual sensitive data values are released. The PMI methods address this concern by releasing data simulated from a parametric model. As with HDMI, we delete values greater than y_I. Fit a statistical model (e.g. lognormal model) to the data. Parameters are drawn from their posterior distribution under the assumed model, and deleted values are imputed with draws from their predictive distribution.

Write the complete data as $Y = (Y_{\text{ret}}, Y_{\text{del}})$, where Y_{ret} denotes the retained values and Y_{del} denotes the deleted values beyond the cut-off. We consider two versions of PMI, labeled as PMIC and PMID. For PMIC, we draw the parameter ϕ of the model for the data *Y* from its posterior distribution given the complete data *Y*. For PMID, we apply the parametric model to the deleted data Y_{del}, and draw ϕ from its posterior distribution given Y_{del}. For inference about a population mean, PMID is less efficient than PMIC because it models the deleted data and fails to exploit fully the information in *Y* when

drawing values of parameters. However, modeling the deleted data only as in PMID provides useful robustness to model misspecification, since the model is being fitted to the data that are being deleted. See An and Little (2007a) for more details.

III.2.2 Extensions of MI methods for multivariate data

The methods in Section III.2.1 do not condition on covariates and potentially attenuate relationships between the variables. We propose methods that condition imputation of deleted values on the observed X's. From this section we refer to (Y, X) as the complete data prior to SDC; and refer to the deleted values of Y and their corresponding values of X's as the deleted data.

(a) Stratified HDMI method. Assign the deleted data into strata based on predicted values of Y from regression of Y on X. Apply HDMI within each stratum to impute for deleted values.

(b) Stratified PMI method. Again create strata based on predicted values of Y. For PMIC methods, we stratify the complete data. For PMID methods, we stratify the deleted data as in (a). We then apply statistical models to the values of Y in each stratum and impute deleted values with draws from predictive distribution.

(c) Regression PMI method. Instead of fitting models to the marginal distribution of variable Y, we include covariates in the mean function of the model for Y. We draw parameters from their posterior distribution under the assumed model, and draw deleted values from predictive distribution. We fit the model to the complete data (for PMIC method) and the deleted data (for PMID). See Appendix III.1 for details for log-normal and power-transformed-normal model.

(d) Regression MI method based on top-coded data set. Fit a statistical (e.g., log-normal) model to the data with values of *Y* below the top-code. We obtain draws of parameter using a Gibbs sampler (Little and Rubin, 2002), and impute deleted values with draws from predictive distribution.

The stratified and regression versions of HDMI and PMI methods in (a)-(c) will be later referred to as "S*" and "R*" methods, respectively.

III.3 Methods of inference

We study the properties of these SDC methods for inferences about regression coefficient with *Y* being outcome (or covariate). The regression model is fitted to the dataset before and after imputation. The following estimates and associated standard errors are considered:

(1) Before Deletion (BD) – the estimate of regression coefficient calculated from original data prior to SDC. This estimate is used as a benchmark for comparing SDC methods.

(2) Top-coding (TC) – the estimate of regression coefficient from the top-coded sample, where we treat the top-coded values as the true values.

The standard errors for methods BD and TC are computed by the bootstrap, with $B = 100$ bootstrap samples.

(3) Log-normal MI from top-coded data (LNMIT) – the estimates from *D* imputed datasets, where we draw imputations for values beyond the top-code from the posterior distributions with a log-normal model fitted to the top-coded data. The MI estimate is calculated using the standard MI combining rule for missing data (Little and Rubin, 2002). In particular, the MI estimate of variance from this method is calculated as

$$T_{MI} = Var(\hat{\theta}_{MI}) = \overline{W} + B * (D+1)/D . \quad (1)$$

This is different from the calculation of variance estimate for the rest of MI methods (see below), because parameters are drawn from their posterior distribution given the top-coded data, rather than their posterior distribution given the complete data (An & Little, 2007).

The remaining MI methods create *D* sets of imputations for values beyond the chosen cut-point y_I, with the *d* th imputed dataset $Y^{(d)} = (y_1^{(d)}, y_2^{(d)}, ..., y_n^{(d)})$, where $y_i^{(d)} = y_i$ if $y_i < y_I$ and $y_i^{(d)}$ is the *d* th MI draw if $y_i \geq y_I$. The MI estimate is then

$$\hat{\theta}_{MI} = \frac{1}{D}\sum_{d=1}^{D}\hat{\theta}^{(d)} \quad , \tag{2}$$

where $\hat{\theta}^{(d)}$ is the coefficient estimate from regression of the *d* th dataset. The MI estimate of variance is

$$T_{MI} = Var(\hat{\theta}_{MI}) = \bar{W} + B/D, \tag{3}$$

where $\bar{W} = \sum_{d=1}^{D} W^{(d)}/D$ is the average of the within-imputation variances $W^{(d)}$ for imputed dataset *d*, and $B = \sum_{d=1}^{D}(\hat{\theta}^{(d)} - \hat{\theta}_{MI})^2/(D-1)$ is the between-imputation variance (Reiter, 2003).

Methods (4)-(8) all create strata based on predictions from a regression model of *Y* on *X*, and then apply an unconditional method within each stratum. Imputations for these methods are created as follows (details are described in Section III.2.2).

(4) Stratified Hot-deck MI (SHDMI) – imputations are drawn randomly with replacement from the set of values beyond the cut-off y_I.

(5) Stratified Log-normal MIC (SLNMIC) – imputations are posterior predictions from a log-normal model fitted to the complete data before deletion.

(6) Stratified Log-normal MID (SLNMID) – imputations are posterior predictions from a log-normal model fitted to the deleted data beyond the cut-off.

(7) Stratified Power-normal MIC (SPNMIC) – imputations are posterior predictions from a power-transformed normal model fitted to the full data before deletion. For convenience the power transformation is estimated by ML, and parameters are drawn from the full-data posterior distribution treating the power transformation as known.

(8) Stratified Power-normal MID (SPNMID) – imputations are posterior predictions from the power-normal model, fitted to the deleted data beyond the cut-off.

Methods (9)-(12) are based on predictions from a regression model that includes the covariates linearly in the mean structure of the model. Details of these methods are described in Appendix III.1. Imputations for these methods are created in a similar manner as their counterparts of stratified methods.

(9) Regression Log-normal MIC (RLNMIC)

(10) Regression Log-normal MID (RLNMID)

(11) Regression Power-normal MIC (RPNMIC)

(12) Regression Power-normal MID (RPNMID)

III.4 Simulation study

A simulation study was carried out to evaluate and compare the SDC methods in Section III.3. We computed estimates of regression coefficients, the corresponding variances and confidence intervals from the imputed datasets, and compared them with those calculated from the original dataset prior to SDC.

III.4.1 Study design

Datasets were generated from the following two distributions. For each distribution, we simulated data where the covariates are strongly or weakly correlated.

<u>Data distribution 1:</u>

- When *X1* and *X2* are strongly correlated,

X1 ~ Normal (0, 1); *X2*|*X1* ~ Normal (0.9**X1*, 0.19); *X3*|*X1*, *X2* ~ Normal (0.2**X1*+*X2*, 0.16)

- When *X1* and *X2* are weakly correlated,

X1 ~ Normal (0, 1); *X2*|*X1* ~ Normal (0.3**X1*, 0.91); *X3*|*X1*, *X2* ~ Normal (0.2**X1*+*X2*, 0.13)

Data distribution 2:

- When *X1* and *X2* are strongly correlated,

X1~ Normal (0, 1); *X2*|*X1*~ Normal (0.9**X1*, 0.19); *X3*|*X1*, *X2*~ Normal (*X1*+*X2*, 0.42)

- When *X1* and *X2* are weakly correlated,

X1 ~ Normal (0,1); *X2*|*X1* ~ Normal (0.3**X1*, 0.91); *X*3|*X1*, *X2* ~ Normal (*X1*+*X2*, 0.29)

Here *X3* is logarithm of variable *Y* subject to disclosure control. For regression purpose we treated *X3* as dependent variable and *X1* and *X2* as independent variables. Data distributions 1 and 2 have different proportions of contribution from the two covariates. To assess sensitivity of SDC methods to model misspecification, we also investigate situation where *X3* was generated from a different distribution with the same mean function.

For each simulated dataset, we applied top-coding, stratified and regression MI methods to impute the deleted values of *Y* and performed linear regression on imputed dataset. We then calculated estimates of regression coefficients, the corresponding variances, 95% confidence intervals (CI's) based on normal approximation and the coverage of confidence intervals. For comparison, we also considered MI methods that failed to condition on the covariates (referred to as unconditional methods).

In our simulations we chose the 95th percentile of the population distribution as the top-code value y_T. Denote by n_S the number of sensitive sample values greater than y_T. We studied two alternative values for the cutoff point y_I: y_{I90}, the value with $2n_S$ larger values in the sample, and y_{I80}, the value with $4n_S$ larger values in the sample. These values correspond approximately to the 90th and 80th percentiles of the distribution, and for this reason we label the version of a method * that uses cutoff y_{I90} "*90" and the version that uses cutoff y_{I80} "*80".

Clearly the disclosure risk is reduced by increasing the fraction of non-sensitive values that are imputed. A simple measure of the risk of disclosure is the proportion of multiple-imputed values beyond the top-code value y_T. For all the MI methods, this is approximately 50% when the cutoff point is y_{I90}, and approximately 25% when the cutoff point is y_{I80}.

III.4.2 Results

Unless specified otherwise, the results from simulation are based on 500 data sets generated from data distribution 1, with sample sizes 2000. We set B = 100 for the number of bootstrap samples. For MI methods, we created D = 5 imputed datasets for values beyond y_{I90}. For stratified MI methods, we created strata with stratum size around 40.

Table III.1 and III.2 show estimates of regression coefficients for *X1*, *X2*, and the intercept term, when *X1* and *X2* are strongly correlated and weakly correlated, respectively. Results are calculated from top-coding, unconditional and conditional MI methods. TC in Table III.1 underestimates the regression coefficients for both covariates.

The estimates of the coefficient of *X2* have larger bias and less coverage, since top-coding the outcome results in greater attenuation of the relationship between outcome and covariate when they are more associated. TC also provides underestimates of the intercept term with poor coverage, suggesting inadequate estimation of the marginal mean of *X3*. When *X1* and *X2* are weakly correlated, TC estimates for *X1* and *X2* have reduced coverage. The impact is more severe with *X2*, suggesting that the attenuation effect has been reduced by the high correlation between the covariates.

All unconditional MI methods behave similarly and underestimate coefficients of both covariates, with larger bias for estimate of the coefficient of *X2*. Though most of the estimates have acceptable confidence coverage (except that estimates of the coefficient of *X2* have low coverage when *X1* and *X2* are not strongly associated), it is worth noticing that these estimates have a 20-30% increase (or 30-40% in some cases) in CI width compared with BD. As a result, some over coverage is observed for the intercept term.

Stratified HDMI produces negligible bias and close to nominal coverage for all three estimates. SLNMID and SPNMID methods also work quite well, with small increases in RMSE and CI width compared to BD estimates. Estimates from SLNMIC and SPNMIC methods have good confidence coverage, though they tend to be more biased and less efficient than those from stratified HD and PMID methods. There is a minor increase in bias for estimate of the coefficient of *X2* from all MI methods, as for the TC method. Results in Table III.2 show some loss of efficiency in the estimate of coefficient of *X2*. We observe that increasing number of strata results in better inference, especially for the S-PMIC method (result not shown).

All regression PMI methods yield inferences close to those before deletion. LNMIT works almost as well as the R-PMI methods; and appears to be a reasonable approach to the analysis of the top-coded dataset. LNMIT and RPNMID have slightly less efficient estimates of the coefficient of *X2* when *X1* and *X2* are weakly correlated. Regression PMI methods (especially RLNMIC and RPNMIC) are more efficient than stratified PMI methods, and produce less bias for the coefficient of *X2*. Overall, estimates from stratified and regression methods are less biased and more efficient than those from unconditional MI methods.

When the data are from the second distribution with *X1* and *X2* contributing evenly in regression (Table III.3 and III.4), we observe similar properties of stratified and regression methods as from the first data distribution, except that here estimates of the coefficients of *X1* and *X2* have very similar inferential properties.

For the smaller sample size of 500 (Table III.5 and III.6), estimates from the stratified methods have larger RMSE and relative CI width. Regression methods also result in larger RMSE and RPNMID shows some increases in CI width; but in general they produce better inferences than stratified methods.

When changing the cutoff point from y_{I90} to y_{I80} (Table III.7 and III.8), stratified HDMI almost has same performance. SLNMID and SPNMID methods have minor increases in bias, RMSE and CI width. More substantial increases are seen with SLNMIC and SPNMIC. In situation where there is low correlation between two covariates, these two methods do not provide full coverage. Results from all regression methods remain somewhat unchanged, whereas RPNMID yields less efficient estimates. Unlike stratified and regression methods, lowering cutoff point for unconditional MI methods results in

larger bias and RMSE, and major increase in CI width. Estimates of coefficient of *X1* still have satisfactory coverage, while for intercept some over coverage occurs. With *X2* the estimates of the coefficient have low coverage, which gets worse when *X1* and *X2* are weakly correlated.

Table III.9 and III.10 display results in situation where *X3* was generated from an exponential distribution instead of normal distribution, to evaluate method performance when model is mis-specified for the outcome. TC again underestimates the regression coefficients for *X1* and *X2*, yielding serious bias and low coverage for estimate of the coefficient of *X2*. Estimate of intercept is even more biased and has worse coverage. All TC estimates have 20% less of CI width than BD estimates. Unconditional HDMI and LNMID, as well as PN methods for strongly correlated covariates, yield satisfactory results, though they are in general more biased and less efficient than stratified and regression methods. Among stratified MI methods, SHDMI and SLNMID have the best performances. They work consistently well and produce estimates with minimal bias and good coverage. SLNMIC method has very similar properties as TC, though it is somewhat less biased and has better confidence coverage. Stratified PNMIC and PNMID methods have larger bias than SLNMID, otherwise they work quite well. For regression methods, estimates from LNMIT have sizable bias and reduced CI width, and have acceptable coverage except for the intercept term. RLNMIC has even worse performance than LNMIT, and has lower coverage for estimate of the coefficient of *X2*, as *X2* associates more with the outcome. RLNMID works best with inferences close to before deletion, and seems to be robust to model misspecification of *X3*. RPNMIC is more biased than RLNMID but also works well. RPNMID produces satisfactory results for *X1*

and *X2*, whereas for intercept it is more biased and does not provide full coverage (even unconditional PNMID method has better results in this case). In a word, regression based MI methods are no better than the stratified versions of these methods.

In summary, stratified HDMI and PMID methods perform well overall. Stratified PMIC methods are less satisfactory in some situations, indicating that stratification on deleted data is adequate and efficient. Among regression methods, RLNMID has the best performance. RPNMIC also works quite well. RPNMID produces satisfactory inferences under correct model; and with incorrect model it yields biased estimates for the marginal mean of the outcome. LNMIT only imputes values beyond top-code, which may be one reason for its close performance as other MI methods. LNMIT and S/RLNMIC methods are all sensitive to model misspecification. LNMIT has less impact with tail of distribution being mis-specified, due to the fact that it conditions only on values below top-code. This could also explain why LNMIT works almost as well as other R-PMI methods under correct model, as fewer values are being imputed. On the other hand, LNMIT presents higher risk of disclosure than other MI methods.

III.4.3 Results from regression of *X1* on *X2* and imputed *X3*

We further investigate the impact of SDC methods on regressions where the sensitive variable subject to top-coding is a covariate. We applied previous SDC approaches to impute for deleted values of *X3* as before. We then regressed *X1* on *X2* and *X3* and computed coefficients from regression.

Simulation setting is the same as described in Section III.4.1. Table III.11 and III.12 present results from situation where *X1* is strongly and weakly correlated with *X2*, respectively. TC results in biased estimates with poor confidence coverage.

Unconditional MI methods provide poor results for coefficients of *X2* and *X3*. Estimates from these methods have serious bias and much lower coverage than TC estimates.

Table III.11 shows SHDMI has minimal bias and confidence coverage close to before deletion. SLNMID and SPNMID methods work nearly as well. SLNMIC produces good estimate for intercept; and estimates of the coefficients of *X2* and *X3* have less CI width and less coverage than BD, yet they behave better than TC estimates. SPNMIC yields estimate with similar inferences to those from the SLNMIC method. When *X1* and *X2* are weakly correlated (Table III.12), SHDMI maintains same properties except for some minor increase in bias and RMSE. All estimates of the coefficients of *X2* and *X3* from stratified PMI methods have larger bias and lower coverage than in Table III.11; especially with MIC methods.

All regression methods yield estimates with good inferences. Result from LNMIT method is close to those from RLNMIC and RPNMIC methods. RPNMID has slightly higher bias especially when correlation between *X1* and *X2* is weak. Overall, these methods have reduced bias and RMSE comparing with their stratified counterparts. We conclude that in situations where imputations are carried out on a covariate, regression MI methods are obviously advantageous to stratified methods for inference about regression coefficient; and they definitely outdo unconditional methods.

III.5 Application

We also consider the properties of the SDC methods on a multiple regression, estimated on a subset of the urban data in the 1995 Chinese Household Income Project (Riskin et al.2000). This project was designed to measure the personal income distribution in the People's Republic of China in 1995. Income information on both household and

individual were recorded for rural and urban areas. This dataset is a good example to assess the effectiveness of the various SDC methods, since SDC was not applied to the released dataset.

III.5.1 Data analysis

Our sample included 10,752 individuals and 10 variables, with the logarithm of income treated as the dependent variable. The covariates involved were age, gender, marital status, education level, occupation, work environment, work intensity, years of work experience and logarithm of hours worked per week. To simplify the analysis, we only investigate the situation where the covariates are complete.

We applied the stratified and regression HDMI and PMI methods to the data as previously described and computed estimates of regression coefficients from imputed dataset. As in simulation study, we also calculated estimates from the unconditional MI methods (i.e., imputation does not condition on covariates) for comparison.

III.5.2 Results

We plot estimates of the standardized regression coefficients after imputation against those from the original dataset (Fig. III.1-III.3). We choose HDMI, LNMID and PNMIC as representations of the MI methods and use the 90^{th}, 80^{th}, 60^{th} and 40^{th} percentiles of the outcome variable as cutoff points, to assess the effect of increasingly severe imputation.

Figure III.1 shows the result from unconditional imputation. We observe that with y_{I90}, the regression coefficients from the imputed dataset are quite close to those from the dataset before imputation; and imputation with y_{I80} also has a minor effect on the coefficients. Lower cutoff points result in larger deviation from original coefficients. Figure III.2 displays result from stratified MI methods. Imputations with y_{I90}, as well

as y_{I80}, yield regression coefficients very close to those from original data. Coefficients computed from RLNMID and RPNMIC (Figure III.3) present very similar properties as in Figure III.2. Comparing to Figure III.1, coefficients from Figure III.2 and Figure III.3 show some minor improvements, especially with lower cutoff point. But overall, they are not much different from those in Figure III.1.

This particular case is similar to the scenario from simulation study where the outcome and covariates have low correlation (as the case with *X1*). We conclude that in such situation, the unconditional MI methods are robust to the failure of the imputation model to condition on covariates. Lowering cutoff points results in larger deviation from original coefficients, leading to greater attenuation of the relationship between outcome and covariates. This impact is less severe with stratified and regression methods.

III.6 Discussion

When applying the MI method to multivariate data, we should condition the predictive distribution of the deleted values on observed covariates. Our previous assessment of inferences for regression coefficients from unconditional MI methods confirms that failure to condition on covariates leads to an attenuation of relationships between outcome and covariates. In simple situation where a small set of categorical covariates associate strongly with the outcome, it may suffice to apply the MI methods within strata defined by these covariates. We base our stratified method on this idea and consider more general application with presence of continuous covariates. Since we are interested in preserving association between outcome and covariates, we define strata with the predicted values from regression.

Of our proposed methods, the stratified methods are easy to apply and involve only a limited amount of computation. The regression-based methods are potentially more efficient, but a bit more complicated computationally. As for method performance, these stratified and regression extensions of MI methods are in general superior to top-coding and unconditional MI methods for inference about regression coefficient. It is clear that treating the top-coded data as the observed data yields bias, the size of which depends on the fraction of cases top-coded and the extremity of the top-code. The LNMIT method based on top-coded data works quite well under correct model, but is vulnerable to model misspecification. Regression LNMID has the best performance and yields results close to before deletion. SHDMI, SLNMID and RPNMIC methods also produce good inferences. RPNMID method works well except when estimating the marginal mean of outcome, with mis-specified model. SPNMIC and SPNMID methods work well when the outcome is subject to SDC. When the imputations are performed on a covariate, they (SPNMIC in particular) yield less satisfactory results. Both stratified and regression versions of LNMIC method are vulnerable to misspecification.

We chose the log-normal and power normal models to illustrate parametric MI, since they are commonly used to model skewed data; they are not universal, and the MI approach could applied by the data producer with other models that are more suitable for the data at hand.

We have confined attention here to inferences from top-coding and MI methods; other alternatives to top-coding are also of interest. One such alternative is to add random noise (e.g., normal noise as in Fuller 1993) to the values beyond top-code. This method may yield satisfactory (if less efficient) inferences for the mean, but noise with

substantial variance needs to be added to yield reductions of disclosure risk comparable to those of MI, and adding such noise potentially distorts the distribution. Also custom adjustments are needed for inferences about other parameters, such as regression coefficients. Note that if multiple imputes are created by adding noise to the true value, the average of these imputations converges to the true value as the number of imputations increases, an undesirable property from the perspective of disclosure protection. Our MI methods do not have this property: the average of the MI imputed values converges to the conditional mean of the predictive distribution, not the true deleted value. Thus increasing the number of MI's improves efficiency of inferences without compromising gains in disclosure protection. This is a major attraction of MI as an SDC method.

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Table III.1 Inference of regression coefficients from simulation study, when X1 and X2 are strongly correlated

Method	Bias ($*10^4$)	RMSE** ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	-1	211	1	94	3	210	1	94.2	4	87	1	95.4
TC	-102	236	1.02	91.8	-499	542	1.04	33.4	-257	272	1.01	17.8
LNMIT	-1	213	1.02	95	6	214	1.03	95	4	87	1.02	95.4
HDMI90	-32	229	1.24	96.2	-170	280	1.26	93.4	4	87	1.24	98.8
SHDMI90	-3	215	1.03	94.2	-13	213	1.03	93.8	4	86	1.02	96
LNMIC90	-33	227	1.24	97	-163	281	1.27	93.6	8	95	1.24	98.6
SLNMIC90	-7	215	1.07	94.8	-40	219	1.08	94.6	17	92	1.07	95.6
RLNMIC90	-1	214	1.01	94	6	214	1.02	93.4	6	90	1.01	94.4
LNMID90	-34	227	1.24	96.6	-167	277	1.29	94.2	5	90	1.3	98.8
SLNMID90	-5	218	1.04	93.8	-13	215	1.04	94.4	3	88	1.04	95.2
RLNMID90	-3	211	1.02	95.4	6	211	1.02	94	3	87	1.02	96
PNMIC90	-36	227	1.24	97.6	-162	278	1.27	93	7	93	1.24	98.6
SPNMIC90	-8	217	1.08	94.8	-44	221	1.08	95	15	89	1.08	96.6
RPNMIC90	0	213	1.01	94	4	212	1.02	94.6	6	89	1.01	94.4
PNMID90	-41	230	1.23	96.4	-194	296	1.27	91.8	-15	90	1.29	98.6
SPNMID90	-3	216	1.04	93.6	-8	217	1.04	93.8	6	88	1.04	96
RPNMID90	2	214	1.03	94.4	-2	213	1.04	94.4	4	87	1.03	96.8

** Here “RMSE” refers to root mean squared error. “Rel-wid” refers to “relative width”, which is fraction of 95 CI % width comparing to estimate 1. “Cover” refers to the 95% CI coverage.

Table III.2 Inference of regression coefficients from simulation study, when X1 and X2 are weakly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	2	86	1	96.2	1	91	1	93.8	4	79	1	94.8
TC	-100	133	1.02	78.2	-498	509	1.13	0.2	-225	240	1.01	19.8
LNMIT	1	87	1.02	94.8	1	93	1.09	93.8	4	80	1.03	95.4
HDMI90	-34	96	1.22	96.4	-166	193	1.33	74.8	4	79	1.22	98.8
SHDMI90	-1	86	1.02	95.4	-13	91	1.05	94	4	79	1.03	95.8
LNMIC90	-33	96	1.23	96.8	-158	197	1.36	73.2	9	85	1.24	97.2
SLNMIC90	-6	86	1.07	96.6	-37	97	1.11	94	17	81	1.08	96.6
RLNMIC90	2	87	1.01	94	4	92	1.04	93	6	80	1.01	95.4
LNMID90	-33	96	1.23	96.4	-163	192	1.46	78	5	82	1.29	97.8
SLNMID90	-1	86	1.03	94.6	-10	93	1.08	94.4	5	80	1.04	96
RLNMID90	1	86	1.02	94.8	-0	89	1.05	94.2	3	80	1.02	95.8
PNMIC90	-33	97	1.22	97	-161	195	1.35	72.2	7	83	1.23	98.4
SPNMIC90	-8	86	1.07	96.4	-44	101	1.13	93.4	14	81	1.08	96.2
RPNMIC90	3	87	1.01	94.2	4	92	1.04	92.4	6	80	1.01	95.2
PNMID90	-39	100	1.22	95.6	-193	218	1.43	69.6	-13	83	1.28	98.4
SPNMID90	-1	86	1.04	95.2	-11	93	1.08	93.6	5	80	1.04	96.2
RPNMID90	2	87	1.03	95.6	1	92	1.08	93.8	5	80	1.03	95.8

Table III.3 Inference of regression coefficients from simulation study, when X1 and X2 are strongly correlated, data distribution 2

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	2	348	1	93.8	2	346	1	93.2	6	143	1	95.4
TC	-498	613	1.02	67.8	-506	613	1.03	67.2	-423	449	1.01	17.8
LNMIT	-3	350	1.03	94.4	4	347	1.03	95.2	6	145	1.02	95.2
SHDMI90	-14	352	1.03	94.6	-14	348	1.03	94	6	142	1.02	95.4
SLNMIC90	-37	353	1.08	95.4	-41	351	1.08	94.8	29	150	1.07	96
RLNMIC90	-2	354	1.02	93.8	9	354	1.02	94.4	9	148	1.01	94.8
SLNMID90	-14	356	1.04	94	-10	351	1.04	94.2	8	147	1.04	94.8
RLNMID90	3	352	1.03	94	3	350	1.03	93.6	7	144	1.02	95.4
SPNMIC90	-48	355	1.08	94.4	-46	351	1.08	96.2	22	146	1.08	97
RPNMIC90	-2	350	1.02	93.8	6	354	1.02	94	7	145	1.01	95.4
SPNMID90	-9	356	1.04	93.6	-8	355	1.04	95.2	10	143	1.04	96
RPNMID90	-1	354	1.04	93.8	3	349	1.04	94.8	7	145	1.03	96.2

Table III.4 Inference of regression coefficients from simulation study, when X1 and X2 are weakly correlated, data distribution 2

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	3	128	1	95	2	131	1	93.6	6	118	1	95.6
TC	-502	521	1.08	5.4	-499	517	1.08	4	-349	370	1.02	18.2
LNMIT	1	133	1.05	94	2	134	1.05	95.2	6	119	1.03	95.6
SHDMI90	-13	133	1.03	94.4	-11	137	1.04	93.8	6	118	1.03	95.8
SLNMIC90	-38	138	1.08	94.8	-38	139	1.09	95.2	25	122	1.08	96.4
RLNMIC90	3	135	1.02	93.4	5	137	1.02	93.2	8	123	1.02	94.6
SLNMID90	-16	135	1.05	94.8	-14	138	1.06	94.4	4	120	1.04	95.4
RLNMID90	3	128	1.03	95.4	1	132	1.03	93.4	9	118	1.02	95.8
SPNMIC90	-47	141	1.09	95.6	-46	144	1.1	94	19	123	1.09	95.6
RPNMIC90	5	131	1.02	94	5	134	1.02	93.2	9	119	1.02	95.4
SPNMID90	-9	134	1.05	95.4	-6	137	1.06	93.6	9	120	1.05	96.4
RPNMID90	2	133	1.05	95.8	-1	134	1.06	95.2	6	119	1.04	96

Table III.5 Inference of regression coefficients from simulation study, when X1 and X2 are strongly correlated, n = 500

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	-3	425	1	93.2	-0	422	1	92.6	4	185	1	93.2
TC	-89	437	1.02	92.8	-513	674	1.04	76.4	-255	320	1.02	68.4
LNMIT	-2	429	1.03	93.8	1	432	1.04	94	4	186	1.03	93.4
SHDMI90	-2	447	1.04	92.4	-16	447	1.04	93.6	3	185	1.04	93.8
SLNMIC90	2	444	1.15	95.8	-34	444	1.16	95.8	71	197	1.16	94.8
RLNMIC90	-7	457	1.03	90.8	12	450	1.03	91.6	10	191	1.03	92.8
SLNMID90	-3	458	1.13	95.2	-20	455	1.14	95.2	-0	190	1.14	95.2
RLNMID90	-3	441	1.05	93	-1	437	1.05	94.2	3	188	1.04	94.4
SPNMIC90	-3	439	1.16	96.2	-31	440	1.17	95.4	71	194	1.17	94.8
RPNMIC90	-6	452	1.03	91.8	3	447	1.04	92.4	5	190	1.03	93
SPNMID90	-0	452	1.13	95	-24	455	1.14	95.4	-4	190	1.14	94.4
RPNMID90	-2	440	1.07	93.8	18	442	1.08	93	12	190	1.07	95.2

Table III.6 Inference of regression coefficients from simulation study, when X1 and X2 are weakly correlated, n = 500

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	-1	176	1	93	-1	176	1	93	4	168	1	93.6
TC	-107	206	1.02	89.8	-500	538	1.14	25	-233	291	1.02	70.2
LNMIT	-0	180	1.03	94.8	5	188	1.09	93.4	6	168	1.03	93.6
SHDMI90	-6	185	1.03	92.6	-18	185	1.05	93.4	1	168	1.03	94
SLNMIC90	-10	183	1.15	96	-33	197	1.2	94.6	60	184	1.15	95
RLNMIC90	-0	189	1.02	93.2	8	190	1.05	91	10	173	1.02	93
SLNMID90	-2	188	1.13	94.2	-12	198	1.2	93.8	4	175	1.14	94.8
RLNMID90	-2	178	1.04	94	-1	187	1.07	93.2	4	168	1.04	93.6
SPNMIC90	-7	185	1.16	95.6	-27	190	1.22	96	66	184	1.17	95.2
RPNMIC90	1	186	1.02	93	9	188	1.05	92.8	11	172	1.03	94.2
SPNMID90	-4	188	1.12	94.4	-20	200	1.2	94	-2	171	1.13	96.8
RPNMID90	-0	184	1.07	94.6	15	196	1.13	93.6	10	169	1.07	95

Table III.7 Inference of regression coefficients from simulation study with cutoff point y_{I80}, when X1 and X2 are strongly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	-1	211	1	94	3	210	1	94.2	4	87	1	95.4
TC	-102	236	1.02	91.8	-499	542	1.04	33.4	-257	272	1.01	17.8
LNMIT	-1	213	1.02	95	2	213	1.03	94.4	4	86	1.02	95.4
HDMI80	-90	263	1.53	98.2	-434	498	1.55	77.6	5	88	1.52	99.6
SHDMI80	-5	224	1.04	93.4	-10	221	1.04	93.8	5	88	1.03	95.8
LNMIC80	-90	261	1.53	97.4	-429	495	1.56	77.4	9	105	1.58	98.6
SLNMIC80	-15	218	1.12	96	-88	233	1.13	93.4	18	90	1.13	97
RLNMIC80	-4	219	1.02	93.6	6	217	1.03	94.2	5	92	1.02	93.6
LNMID80	-89	261	1.53	97.4	-439	501	1.58	77.8	4	96	1.67	99.8
SLNMID80	-6	222	1.05	94.6	-7	224	1.06	93.2	5	90	1.06	95.4
RLNMID80	-0	216	1.04	93.6	1	215	1.04	93.6	3	87	1.03	94.8
PNMIC80	-91	263	1.53	97	-431	497	1.57	77.4	8	100	1.58	99.4
SPNMIC80	-15	217	1.12	96	-86	229	1.13	96.6	20	91	1.13	96.8
RPNMIC80	-2	218	1.02	93.4	5	217	1.03	93.2	5	88	1.02	94.6
PNMID80	-98	260	1.53	97.6	-472	528	1.58	72.2	-28	100	1.66	99.2
SPNMID80	-4	220	1.05	94.4	-3	220	1.06	94.6	8	91	1.06	95.4
RPNMID80	-1	216	1.06	94	19	222	1.07	93.6	9	90	1.06	96

Table III.8 Inference of regression coefficients from simulation study with cutoff point y_{I80}, when X1 and X2 are weakly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	2	86	1	96.2	1	91	1	93.8	4	79	1	94.8
TC	-100	133	1.02	78.2	-498	509	1.13	0.2	-225	240	1.01	19.8
LNMIT	1	86	1.02	95.2	1	92	1.09	93.4	4	80	1.03	96
HDMI80	-85	131	1.5	94.4	-435	447	1.6	6.6	4	80	1.5	99.6
SHDMI80	-2	87	1.03	94.4	-14	92	1.06	94.2	4	80	1.03	96
LNMIC80	-85	133	1.5	95.4	-429	448	1.69	14	9	94	1.55	99.2
SLNMIC80	-16	88	1.12	96.4	-86	127	1.17	86.8	17	84	1.13	96.6
RLNMIC80	2	88	1.02	94.8	5	95	1.05	93.2	7	82	1.03	95.2
LNMID80	-88	133	1.5	95.4	-440	454	1.8	14.4	1	88	1.64	99.6
SLNMID80	-1	88	1.05	94.8	-15	94	1.11	95	4	82	1.06	95.6
RLNMID80	3	88	1.03	94	1	94	1.07	94	4	80	1.04	95.4
PNMIC80	-84	133	1.49	94.8	-431	447	1.7	11.6	7	89	1.55	99.6
SPNMIC80	-17	89	1.12	96.6	-83	123	1.18	89	19	83	1.13	96.6
RPNMIC80	3	88	1.02	94	2	93	1.05	93.6	6	83	1.02	94.2
PNMID80	-93	137	1.5	93.6	-469	483	1.79	10.2	-22	91	1.63	99
SPNMID80	-1	88	1.05	94.2	-7	92	1.11	94.4	7	82	1.07	95.8
RPNMID80	3	89	1.06	95.4	12	94	1.13	94.8	8	82	1.06	95.2

Table III.9 Inference of regression coefficients from simulation study from incorrect model, when X1 and X2 are strongly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	37	3229	1	94.6	-93	3224	1	93.8	-69	1412	1	92.8
TC	-393	2703	0.82	93	-1964	3308	0.82	86.4	-3369	3637	0.82	18.2
LNMIT	-326	3748	0.88	94.2	-1527	4020	0.88	92.6	-3116	3610	0.88	49.6
HDMI90	-238	3124	1.08	96	-876	3233	1.08	95.4	-80	1427	1.08	95.4
SHDMI90	-57	3383	1.02	92.6	-61	3380	1.02	94.4	-67	1424	1.02	94.8
LNMIC90	-463	2691	0.84	93.6	-2148	3405	0.84	86	-3584	3830	0.84	15.2
SLNMIC90	-391	2750	0.86	93.6	-1796	3268	0.86	88.8	-2967	3275	0.87	30.6
RLNMIC90	-440	3654	0.84	93.8	-2012	4154	0.84	89	-4561	4903	0.84	18.6
LNMID90	-270	3074	1.08	96.6	-830	3186	1.08	96.2	-81	1434	1.14	95.6
SLNMID90	-72	3380	1.03	92.6	-48	3335	1.03	94.2	-62	1470	1.05	92.8
RLNMID90	45	4324	1.03	95	-88	4321	1.03	94.4	-103	1874	1.03	94
PNMIC90	-273	3005	1.07	96.8	-1059	3196	1.07	95.4	-624	1593	1.07	92.2
SPNMIC90	-218	3088	1.04	95.4	-874	3187	1.03	95	-275	1510	1.04	93.2
RPNMIC90	-187	4158	1.02	94.8	-935	4258	1.02	93.6	-546	2007	1.02	92.4
PNMID90	-271	3043	1.03	96	-1073	3216	1.02	95	-654	1582	1.06	91.6
SPNMID90	-89	3290	1	93.2	-286	3271	0.99	94.2	-523	1508	1	92.2
RPNMID90	-151	4014	0.97	95.2	-720	4062	0.97	94.6	-1135	2157	0.97	88.6

Table III.10 Inference of regression coefficients from simulation study from incorrect model, when X1 and X2 are weakly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X3 on X1, X2	**X1**				**X2**				**Intercept**			
BD	-43	1438	1	93.6	-46	1556	1	94.6	-68	1399	1	93.8
TC	-432	1289	0.82	92.8	-1941	2342	0.8	61.2	-3300	3568	0.83	20.2
LNMIT	-362	1712	0.88	94.6	-1501	2285	0.87	83.8	-3089	3581	0.88	49.6
HDMI90	-227	1387	1.08	97.2	-896	1751	1.07	92.6	-78	1411	1.08	95.4
SHDMI90	-72	1505	1.02	94.8	-79	1515	1	93.4	-68	1413	1.02	94.6
LNMIC90	-474	1290	0.84	94.4	-2134	2505	0.84	59	-3487	3739	0.85	17
SLNMIC90	-417	1298	0.86	94	-1824	2241	0.85	65.6	-2953	3260	0.87	31.2
RLNMIC90	-470	1677	0.84	93.6	-2001	2616	0.83	74.6	-4515	4865	0.84	18.4
LNMID90	-217	1424	1.08	97	-882	1763	1.07	92.8	-75	1424	1.14	95.8
SLNMID90	-72	1519	1.03	95.2	-82	1519	1.02	94.6	-69	1423	1.05	94.4
RLNMID90	-99	1903	1.03	95.4	-30	2009	1.02	94.4	-91	1890	1.03	93.8
PNMIC90	-273	1417	1.07	97.4	-1019	1813	1.07	90.2	-486	1534	1.08	91.8
SPNMIC90	-235	1410	1.03	97.2	-868	1699	1.02	91.4	-216	1490	1.04	93.8
RPNMIC90	-230	1865	1.02	95.6	-863	2099	1.02	93.2	-455	1994	1.03	91.2
PNMID90	-262	1393	1.02	97	-1081	1836	1.01	89.6	-625	1545	1.06	92
SPNMID90	-111	1483	0.99	94.6	-278	1522	0.98	93.2	-491	1520	1.01	91.8
RPNMID90	-193	1798	0.97	95.8	-694	1976	0.97	93.2	-1105	2120	0.98	89.4

Table III.11 Inference of regression coefficients from simulation study, when X1 and X2 are strongly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X1 on X2, X3	**X2**				**X3**				**Intercept**			
BD	2	295	1	94.2	1	239	1	93.6	0	94	1	96.2
TC	331	438	0.97	78.8	-169	294	1	89.6	56	110	1	90.4
LNMIT	3	298	1.02	94.6	0	241	1.02	94.8	-0	93	1.01	96
HDMI90	684	733	0.97	30.2	-547	586	0.98	32.4	-0	93	1.01	96.2
SHDMI90	75	305	1	93.6	-58	245	1	94.2	-1	93	1.01	96
LNMIC90	685	730	0.97	30.6	-548	584	0.98	32.8	-1	93	1.01	95.8
SLNMIC90	290	401	0.97	84.4	-234	324	0.97	82.8	-3	93	1.01	96.2
RLNMIC90	2	301	1.01	94.2	1	244	1.01	94.8	-1	94	1.01	95.8
LNMID90	689	737	0.97	28	-551	590	0.98	31.4	-0	93	1.02	96.2
SLNMID90	107	314	1	92.4	-84	252	1	93.4	-0	93	1.01	96
RLNMID90	10	295	1.02	94.6	-5	238	1.02	95.2	-0	93	1.01	95.8
PNMIC90	689	735	0.97	27.8	-551	589	0.98	31.4	-1	93	1.01	96.4
SPNMIC90	307	415	0.97	82.4	-245	335	0.97	82.2	-3	93	1.01	96.2
RPNMIC90	1	302	1.01	94.2	1	243	1.01	95.2	-1	93	1.01	96
PNMID90	681	730	1	34	-538	580	1.02	36.8	3	93	1.02	96
SPNMID90	112	312	1	92.8	-90	253	1	93	-1	93	1.01	96.2
RPNMID90	58	297	1.01	95	-46	241	1.02	94.8	-0	94	1.01	96.2

Table III.12 Inference of regression coefficients from simulation study, when X1 and X2 are weakly correlated

Method	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
Regression of X1 on X2, X3	**X2**				**X3**				**Intercept**			
BD	-26	511	1	95.2	19	451	1	95.6	4	182	1	95.2
TC	1128	1245	1.04	48.8	-548	726	1.05	79.8	250	311	1.01	74.4
LNMIT	-23	513	1.02	96.2	16	452	1.02	95.6	3	184	1.01	95.8
HDMI90	2611	2667	1.06	0.4	-2110	2163	1.1	1.4	4	182	1.05	96.8
SHDMI90	226	558	1	95	-203	492	1.01	94.2	4	183	1.01	95.2
LNMIC90	2631	2678	1.06	0.2	-2135	2179	1.1	0.6	-1	185	1.05	96.8
SLNMIC90	991	1111	0.99	55.2	-901	1004	1.02	50	-9	183	1.02	95.6
RLNMIC90	-25	510	1.01	96	16	450	1.01	95.8	1	184	1.01	95
LNMID90	2621	2678	1.06	0.2	-2122	2176	1.08	1	2	184	1.06	96.6
SLNMID90	326	607	1.01	91.4	-300	542	1.01	91.4	2	182	1.01	95.6
RLNMID90	-4	514	1.01	96	0	453	1.01	95.8	4	184	1.01	95.6
PNMIC90	2618	2668	1.06	0.2	-2122	2168	1.1	1	1	183	1.05	96.4
SPNMIC90	1061	1180	0.99	48.4	-961	1064	1.02	45.2	-6	184	1.02	95.6
RPNMIC90	-27	518	1.01	96	17	457	1.01	96.2	1	184	1.01	94.6
PNMID90	2592	2649	1.16	0.4	-2066	2122	1.24	3	19	184	1.06	97.4
SPNMID90	363	638	1.02	89.8	-335	569	1.02	90.4	3	184	1.02	95.6
RPNMID90	188	560	1.03	94	-182	502	1.03	95.2	3	183	1.01	95.4

Figure III.1 Standardized regression coefficients, after versus before unconditional imputation.
1995 Chinese household income project, urban data. (Top row, HDMI, with cutoff points being 90, 80, 60, 40 percentiles, from left to right. Middle row, LNMID. Bottom row, PNMIC. Line: $y = x$)

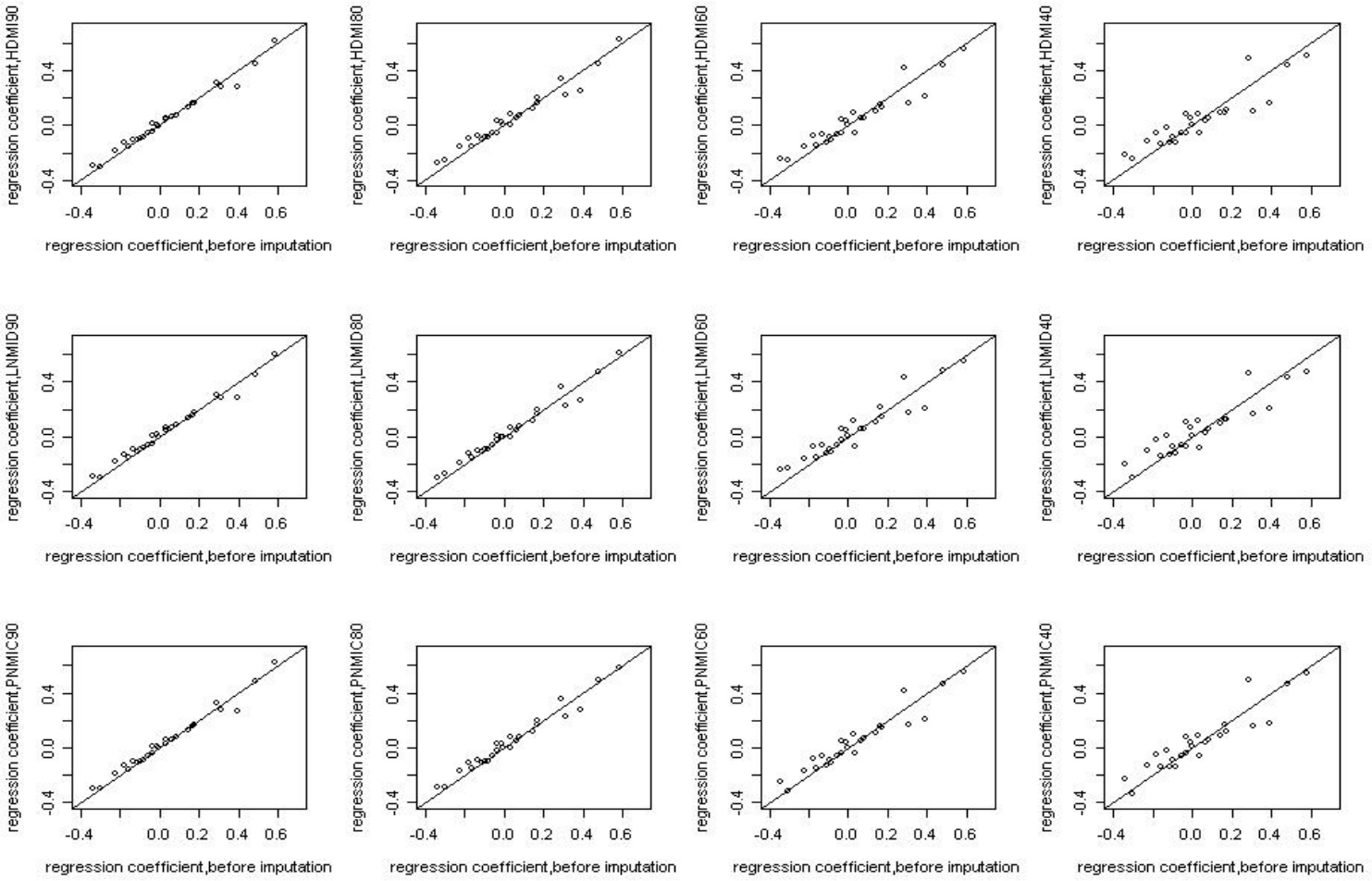


Figure III.2 Standardized regression coefficients, after versus before stratified imputation.
1995 Chinese household income project, urban data. (Top row, SHDMI, with cutoff points being 90, 80, 60, 40 percentiles, from left to right. Middle row, SLNMID. Bottom row, SPNMIC. Line: y = x)

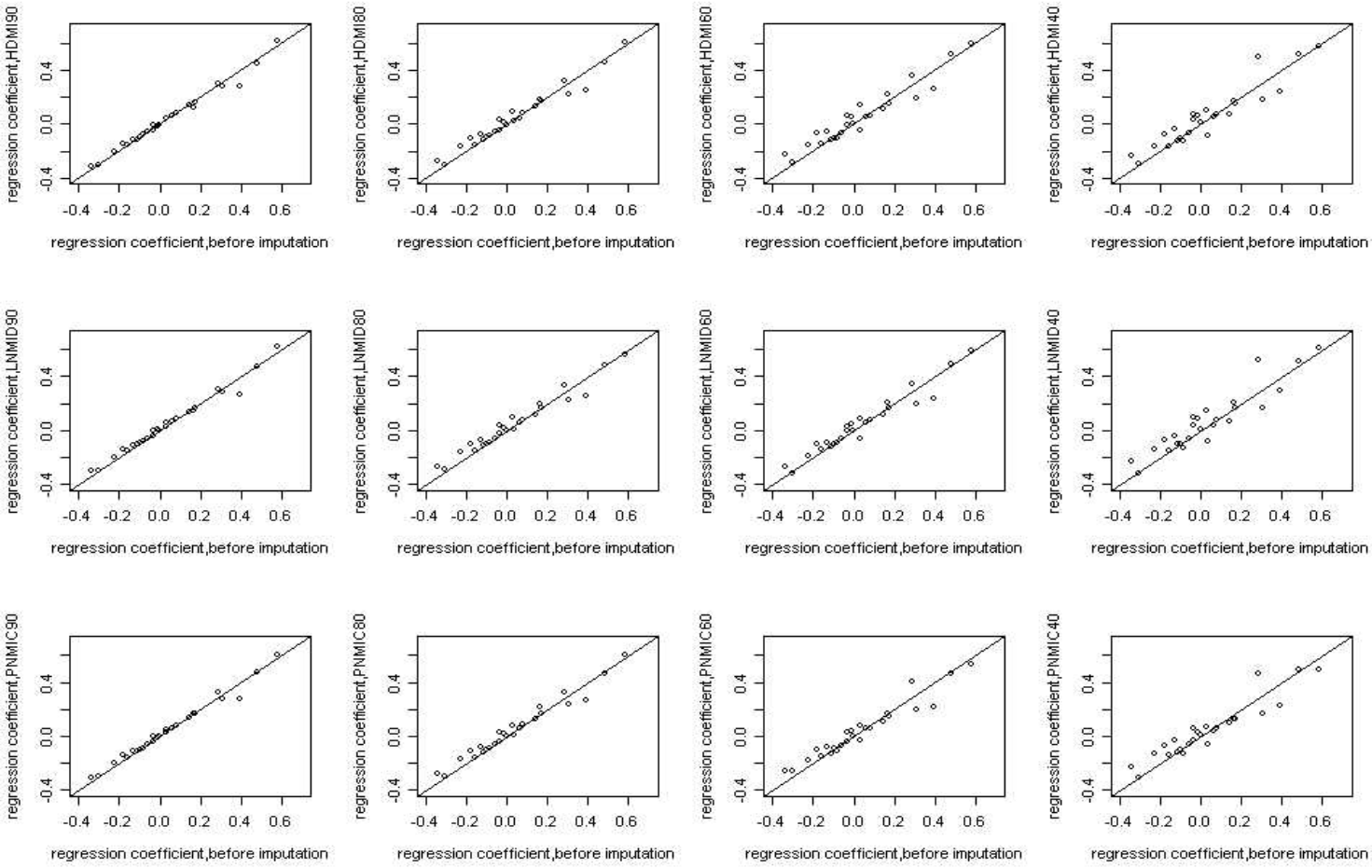
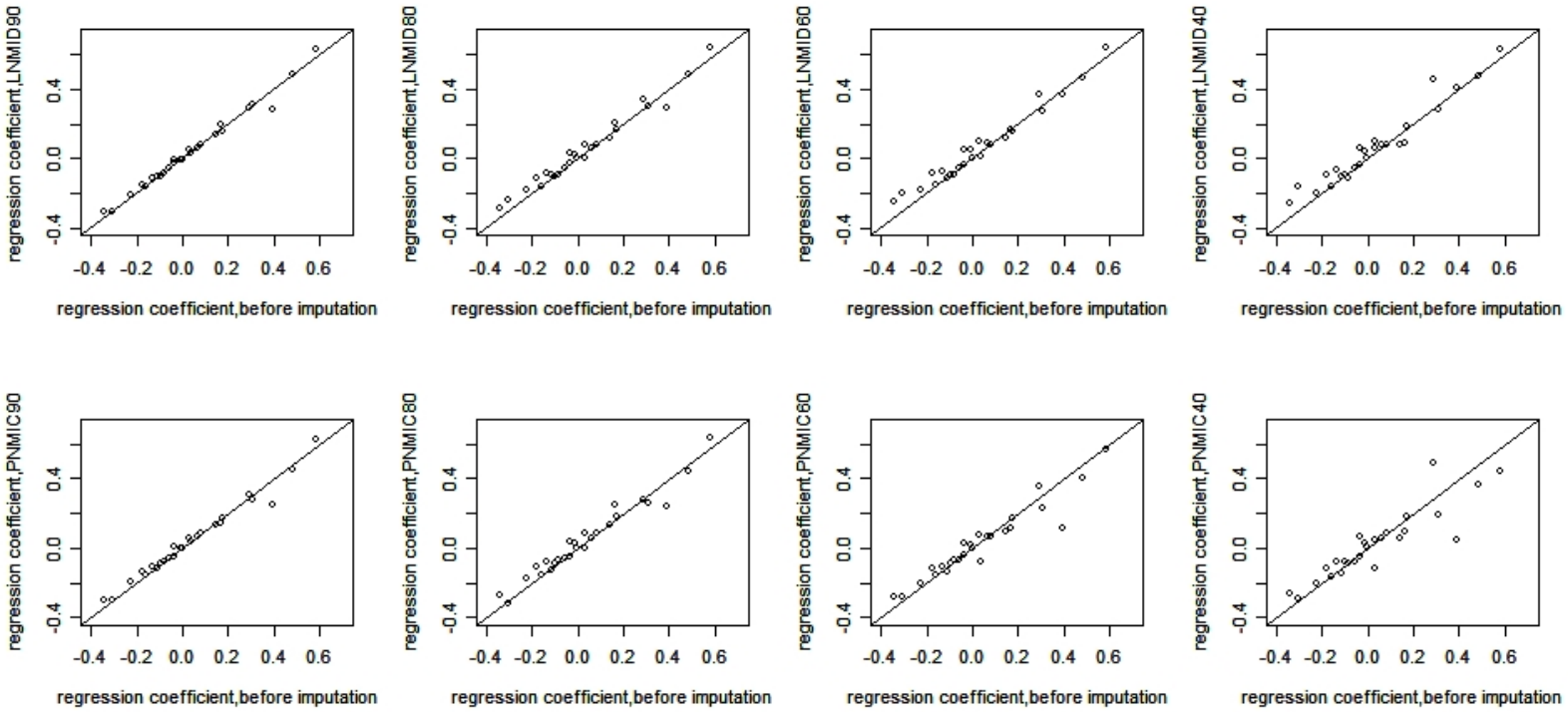


Figure III.3 Standardized regression coefficients, after versus before regression-based imputation.
1995 Chinese household income project, urban data. (Top row, RLNMID, with cutoff points being 90, 80, 60, 40 percentiles, from left to right. Bottom row, RPNMIC. Line: *y = x)*



Appendix III.1: Regression-based parametric MI methods for log-normal model and power-transformed normal model

As described in the paper, let *Y* denote the variable subject to disclosure limitation and *X* denote the covariate matrix. Let *Z* be a normal variable transformed from *Y*. To be specific, if *Y* is from a log-normal distribution, let $Z = \log(Y)$. If *Y* is from a power-transformed-normal distribution with $\lambda \neq 0$, let $Z = (Y^{\lambda} - 1)/\lambda$. Here we estimate λ by its ML estimate $\hat{\lambda}$ using the widely available routine boxcox ( ) in R (see Fox(2006)) and then assume that $Z = (Y^{\hat{\lambda}} - 1)/\hat{\lambda}$.

Let X_i denote the vector of covariates for the *i* th observation,

$$Z_i \mid X_i \sim N(\sum_j x_{ij}\beta_j, \sigma^2). \tag{IIIA1}$$

Write $Z = (Z_{ret}, Z_{del})$, without loss of generality, assume $Z_{ret} = (z_1, ..., z_r)$ and $Z_{del} = (z_{r+1}, ..., z_n)$.

For PMIC method, the posterior distribution of parameters is

$$\sigma^{*2} \mid Z \sim \frac{(n-p)\hat{\sigma}^2}{\chi^2_{n-p}} \tag{IIIA2}$$

and

$$\beta^* \mid \sigma^{*2}, Z \sim MVN(\hat{\beta}, (X^T X)^{-1}\sigma^{*2}), \tag{IIIA3}$$

where

$$\hat{\beta} = (X^T X)^{-1} X^T Z \tag{IIIA4}$$

$$\hat{\sigma}^2 = \frac{\sum_1^n (z_i - \sum_j x_{ij}\hat{\beta}_j)^2}{n-p}. \tag{IIIA5}$$

We draw parameters β^* and σ^{*2} from their posterior distribution and draw deleted values for normal data from the predictive distribution

$$Z^*_{del(i)} \mid X_i \sim N(\sum_j x_{ij}\beta_j^*, \sigma^{*2} \mid Z > z_I), i = r+1,...,n, \quad \text{(IIIA6)}$$

where $z_I = \log(y_I)$ for log-normal distribution; or $z_I = (y_I^{\lambda} - 1)/\lambda$ for power-normal distribution.

We then transform the draws of normal data back to log-normal,

$$Y^*_{del(i)} = \exp(Z^*_{del(i)}), \quad \text{(IIIA7)}$$

and power-transformed normal data,

$$Y^*_{del(i)} = \sqrt[\hat{\lambda}]{(\hat{\lambda}Z^*_{del(i)} + 1)}. \quad \text{(IIIA8)}$$

For PMID method the calculations are quite similar as above, except that the model is fitted to the deleted data instead of the complete data.

Chapter IV

A Multiple Imputation Approach to Disclosure Limitation for High-age Individuals in Longitudinal Studies

Abstract

Disclosure limitation is an important consideration in the release of public use data sets. It is particularly challenging for longitudinal data sets, since information about an individual accumulates with repeated measures over time. Despite the challenges, research on disclosure limitation methods for longitudinal data has been very limited. We consider here problems created by high ages in cohort studies. Because of the risk of disclosure, ages of very old respondents can often not be released; in particular this is a specific stipulation of the Health Insurance Portability and Accountability Act (HIPAA) for the release of health data for individuals. Top-coding of individuals beyond a certain age is a standard way of dealing with this issue, and it may be adequate for cross-sectional data, given that a modest number of cases are likely to be affected. However, this approach has severe limitations in longitudinal studies, when individuals have been in the study for many years. We propose and evaluate an alternative to top-coding for this situation based on multiple imputation (MI). This MI method is applied to a survival analysis of simulated data and data from the Charleston Heart Study (CHS), and is shown to work well in preserving the relationship between hazard and covariates.

Keywords: confidentiality, disclosure protection, longitudinal data, multiple imputation, survival analysis

IV.1 Introduction

Statistical disclosure control is a class of procedures that deliberately alter data collected by statistical agencies before release to the public, to prevent the identity of survey respondents from being revealed. These methods have increased in importance, with the extensive use of computers and the internet. The goal of SDC methods is to reduce the risk of disclosure to acceptable levels, while releasing a dataset that provides as much useful information as possible for researchers. One aspect of this is the ability to draw valid statistical inferences from the altered data.

Top-coding is a simple and common SDC method that seeks to prevent disclosure on the basis of extreme values of a variable, by censoring values above a pre-chosen "top-code". For example, in surveys that include income, extremely high income values are considered to be sensitive and to have the potential to reveal the identity of respondents. By recoding income values greater than a selected "top-code" value to that value, respondents with very high income have reduced risk of disclosure.

It is left to the analyst to decide how top-coded data are analyzed. One approach is to categorize the variable so that top-coded cases all fall in one category – this is sensible, but precludes analyses that treat the variable as continuous. Another approach is to ignore the fact of top-coding and treat the top-coded values as the truth. This method is straightforward, but clearly the data distribution is distorted and biased estimates will be obtained. A better method is to treat the extreme values as censored. Under an assumed statistical model, maximum likelihood (ML) estimates can be obtained using algorithms

such as the Expectation-Maximization (EM) algorithm (Dempster, Laird and Rubin, 1977). This method is model-based, and should yield good inferences if the model is correctly specified. But we expect this method to be quite sensitive to model misspecification, especially when the upper tail of the assumed distribution differs markedly from that of the true distribution. The data users can also apply an imputation method to the top-coded dataset and fill in the censored values. A limitation is that the imputed data fail to reflect imputation uncertainty, and imputations are sensitive to assumptions about the right tail of the distribution. An and Little (2007a) propose an alternative to top-coding based on multiple imputation (MI), which allows valid inferences to be created based on applying multiple imputation combining rules described by Reiter (2003), while preserving the SDC benefits of top-coding; for other discussions of MI in the disclosure control setting, see Little (1993); Rubin (1993); Little, Liu and Raghunathan (2004); Reiter(2005a, 2005b). The methods in An and Little (2007a) are extended to handle covariate information in An and Little (2007b).

We propose here MI for disclosure control in the context of the treatment of age in longitudinal data sets. Because of the risk of disclosure, ages of very old respondents can often not be released; in particular this is a specific stipulation of HIPAA regulations for the release of health data for individuals. Top-coding of individuals beyond a certain age (say 80) is a standard way of dealing with this issue, and it may be adequate for cross-sectional data, since the number of cases affected may be modest. However, this approach has severe limitations in longitudinal studies, when individuals have been in the study for many years; for example, consider an individual in a 40-year longitudinal study, who enters the study at age 42 at time t and is still in the study at age 82 at time t+40. The

age at time t+40 cannot simply be replaced by a top code of 80, since age at time t+40 can be inferred by simply adding 40 to the age at time t. A strict application of top-coding would replace all individuals aged 40 or older at time t by a top code of 40, but this strategy seriously limits the ability to do longitudinal analysis, particularly survival analyses where chronological age is a key variable of interest. In particular, since age at entry is a marker for cohorts, differences in outcomes between cohorts aged 40 or greater at entry can no longer be estimated, since these cohorts are all top-coded to the same value.

This problem arises in the Charleston Heart Study (Nietert *et al.*, 2000), a longitudinal study that collects data over 40 years (1960-2000). The study was originally conducted to understand the natural aging process in a community-based cohort. The data include baseline characteristics such as age, race, gender, occupation, education; as well as death information for respondents. For longitudinal data from this study to be included in the data archive at the University of Michigan, individual ages beyond age 80 cannot be disclosed because of HIPAA regulation, given the geographic specificity of the respondents. Also, given the longitudinal nature of the data, a top-coding approach would need to be applied to all individuals aged 40 or older in 1960, which has the limitation discussed above.

The goal of this research is to develop MI methods that suffice to limit disclosure risk and preserve the relationship between hazard and covariates in survival analysis. We propose a non-parametric MI method, specifically a stratified hot-deck procedure, where we create strata and draw deleted ages with replacement from each stratum. Our method

concerns MI of two age variables – entry age and final age (age at death or age at last contact).

To assess the proposed method, we apply a proportional hazard (PH) model to the multiply-imputed datasets, calculate estimates of regression coefficients for putative risk factors, and compare these estimates, and corresponding estimates from top-coded data, with estimates from the PH model applied to the original data prior to SDC. We also present simulation studies where data are simulated according to a known survival model, and inferences for parameters of this model are compared with the true values.

The rest of this paper is organized as follows. Section IV.2 presents our SDC approaches for longitudinal data and describes corresponding methods of inference for regression coefficients. Section IV.3 describes a simulation study to evaluate the approaches in Section IV.2, and Section IV.4 applies the methods to CHS data. Section IV.5 gives discussion and future work.

IV.2 Methods

IV.2.1 SDC methods for longitudinal data

An and Little (2007a) propose SDC methods for a single variable with extreme values. In this paper, we investigate a more complicated situation with longitudinal data, where two age variables are subject to top-coding.

Let Y_{end} denote participants' age at the end of study (referred to as final age) and Y_{start} denote their entry age. Let C be the censoring indicator. Let L represent the length of study and S denote time of survival. Individuals with $S \geq L$ are treated as censored (C = 1), and otherwise died (C = 0). We consider individuals with values of Y_{end} greater than a particular value y_0 to be at risk of disclosure, and refer to these individuals as sensitive

cases. Thus values of Y_{end} and Y_{start} of the sensitive cases are treated as sensitive values. We consider the following approaches to SDC.

(a) Top-coding. Replace values of Y_{end} greater than y_0 by y_0 and replace values of Y_{start} greater than $y_0 - L$ by $y_0 - L$. The resulting dataset is referred to as "top-coded" data.

(b) Hot-deck MI (HDMI). Classify sensitive and non-sensitive values into strata, to be defined below. Then delete the values of Y_{end}, Y_{start}, and C for sensitive cases and replace them with random draws from the set of deleted values in the same stratum. Our stratified HDMI method is similar to the approach described in An and Little (2007b), where we assign the deleted data into strata based on predicted values of either age variables from regression on other variables, and apply HDMI within each stratum to impute deleted values. The following choices of strata are considered here:

(i) HD1. Strata are defined by predicted values of the logarithm of hazard computed from the proportional hazard model.

(ii) HD2. Strata are defined by predicted values of entry-age, from the regression of entry-age on other variables involved.

(iii) HD3. We develop a two-way stratification, where strata are defined by both predicted values of the logarithm of hazard, and predicted values of entry-age.

(iv) HD4. Stratification depends on the value of C. For individuals that are censored, strata are defined by predicted values of entry-age; and for those not censored, strata are defined by both predicted values of the logarithm of hazard and predicted values of entry-age.

(v) HD5. We directly apply HDMI method without stratification, for comparison with the stratified methods.

Note that for methods HD1 – HD3, we delete values of Y_{end}, Y_{start}, and C of sensitive cases and jointly impute these values. HD4 retains values of C and imputes Y_{end} and Y_{start} only.

It is worth mentioning that for above stratified methods, we perform regression only on the deleted cases to obtain predicted values. We also consider an alternative way of stratification, where we perform regression on the complete data, and then stratify the sensitive cases for imputation. Results from these methods are briefly described in Section IV.3.

IV.2.2 Methods of inference

We consider the properties of the SDC methods for inferences about the regression coefficient, where a PH model is fitted to the dataset before and after imputation. The following estimates and associated standard errors are considered:

(1) Before Deletion (BD) – the estimates of regression coefficients calculated from original data prior to SDC, used as a benchmark for comparing SDC methods.

(2) Top-coding (TC) – the estimates of regression coefficients calculated from top-coded dataset.

The standard errors for methods (1) and (2) are computed by the bootstrap.

The five remaining methods HD1 – HD5 are as described in Section IV.2.1, yielding D MI datasets. The MI estimate is calculated as

$$\hat{\theta}_{MI} = \frac{1}{D}\sum_{d=1}^{D}\hat{\theta}^{(d)}, \quad (1)$$

where $\hat{\theta}^{(d)}$ is the parameter estimate from d th data set. The MI estimate of variance is

$$T_{MI} = Var(\hat{\theta}_{MI}) = \bar{W} + B/D, \quad (2)$$

where $\bar{W} = \sum_{d=1}^{D} W^{(d)} / D$ is the average of the within-imputation variances $W^{(d)}$ for imputed data set *d*, and $B = \sum_{d=1}^{D} (\hat{\theta}^{(d)} - \hat{\theta}_{MI})^2 / (D-1)$ is the between-imputation variance. The formula (2) differs from the original MI formula for missing data (where *B* is multiplied by a factor *(D*+1*)/D*, see e.g. Little and Rubin, 2002, p86), for reasons discussed in Reiter (2003).

IV.3 Simulation study

A simulation study was carried out to evaluate the top-coding and MI methods in Section IV.2. We computed estimates of regression coefficients, their corresponding variances and confidence intervals from the imputed and top-coded datasets, and compared them with those calculated from the original dataset prior to SDC.

IV.3.1 Study design

For simplicity we simulated survival data with just two binary covariates, representing gender (male and female) and entry age (say 30 - 40 and 40 - 50). Datasets were simulated from multinomial distribution in four categories defined by these variables. Values of entry-age were generated from uniform distribution. Survival times (in years) were generated from piece-wise exponential distributions with hazard rates specified in Table IV.1 and IV.2. An individual was treated as censored if (s)he survived more than 40 years from age at entry. We investigated the following three scenarios.

Scenario I Distributions of entry age do not depend on gender; both male and female have same entry-age distributions.

Scenario II Distributions of entry age are different for males and females.

Scenario III Distributions of entry age are the same for males and females, and there is interaction between entry age and gender.

In this study we considered individuals with final age greater than or equal to 75 years to be at risk of disclosure, and refer to these individuals as sensitive cases. For each simulated dataset, we applied the stratified HDMI methods to both final age and entry age variables for sensitive cases as described in Section IV.2. We also applied the top-coding method, with top-code being 75 for final age and 35 for entry age (as the length of study is 40 years). We then calculated estimates of regression coefficients from the PH model, the corresponding variances of the estimates, as well as 95% confidence intervals (CI's) based on normal approximation, and the confidence coverage of these intervals.

IV.3.2 Results

Simulation results are based on 500 datasets of sample size 2000. We set the number of bootstraps B to be 100 for calculating standard errors of BD and TC estimates; and create $D = 5$ imputed datasets. For stratified HDMI methods, we create strata with stratum size around 25.

Table IV.3 presents results from scenario I, where distributions of entry-age are the same for male and female. TC yields estimate of regression coefficient with serious bias and RMSE, and zero confidence coverage for the entry-age variable. As for gender, TC estimate has relatively better properties, yet it still has sizable bias and low coverage. All stratified HDMI methods produce quite satisfactory results for the entry-age variable, with negligible bias and confidence coverage close to before deletion. HD5 also work well in terms of bias and coverage, but it is somewhat less efficient than the stratified HD methods. HD4 method works best for gender variable, yielding estimate of regression coefficient with minimal bias and good confidence coverage. Estimates from other HD methods are also acceptable, though they are in general more biased and have less

coverage. When male and female have different entry-age distributions as in scenario II (Table IV.4), most methods behave similarly as in the first scenario, except that HD3 yields larger bias, RMSE and less coverage for estimate of the regression coefficient of gender. In fact, it has even worse results than TC method.

Table IV.5 displays results from scenario III, where there is interaction between the age and gender variables. TC yields estimates with considerable bias and poor coverage for regression coefficients of age, gender and the interaction between these two variables. Among stratified HD methods, HD4 has the best performance and yields estimates with good inferences for both variables and the age-gender interaction. HD2 also has satisfactory results for all three terms, though it is more biased than HD4. Estimates from HD1 and HD3 methods have similar properties as from HD2, except that they have less sufficient coverage for the interaction term. Estimates from HD5 have larger bias and less confidence coverage than those from the stratified HD methods.

We also applied the alternative stratified method described in Section IV.2.1, where we obtained predicted values from regression on the complete data, and then stratified the sensitive cases for imputation. Estimates from these methods (not shown) are more biased and have less confidence coverage compared to the methods above. This suggests that when a regression model is fitted to the data that are being deleted, it makes the method more robust to model mis-specification and yield better result (see Section IV.5 for more discussion).

In summary, HD4 performs best under all circumstances. Other stratified HD methods yield estimates of regression coefficient with good inferential properties for the entry-age variable. These methods also provide satisfactory results for gender, except for

HD3 in scenario II. With presence of interaction between age and gender, estimates for the interaction term from HD1 and HD3 methods do not have sufficient coverage. HD5 tends to be slightly less efficient than the stratified HD methods, but it works surprisingly well in the first two scenarios, indicating stratification may not be necessary in such data setting. For more complicated situation (scenario III), it yields biased estimates with low confidence coverage.

IV.4 Application in Charleston Heart Study data

We chose a subset of the CHS data and studied the relationship between hazard rate and certain risk factors. Since an intact data file prior to disclosure control was available to us, the effectiveness of our SDC methods can be readily assessed.

IV.4.1 Primary data analysis

After deletion of missing values and recoding on some variables, our sample included 1344 individuals, of which 303 survived the study. The variables involved were entry-age, final-age, censoring indicator, race/gender, education level, current cigarette smoking status, history of myocardial infraction (MI), history of diabetes, history of hypertension, electro-cardiographic interpretation (EKG), living place between age 20 to 65 and body mass index (BMI). For the PH regression model, final-age instead of survival time was treated as the time-scale variable.

To examine effects of our chosen risk factors, we applied the PH model to the dataset prior to SDC. Table IV.6 displays results from the regression. All factors have significant effect on participant's hazard ratio except BMI and entry-age (overall). Comparing to individuals that enter the study between 35 and 40 years old, those with entry-age greater than 50 have about a 30% increase in risk of death. White females tend

to have 34% less of risk than white males. Achieving education after high school reduces hazard by 30% comparing to non-high school education. Smoking cigarette increases death risk by 76%. Participants with definite history of myocardial infraction have twice the risk of death as those without a history. History of diabetes as well as EKG problems increases the hazard by over 50%, while history of hypertension increases risk of death by 17%. Rural residents have 25 % less of hazard than urban residents. Most of these coefficients are in the expected direction.

IV.4.2 Results from SDC methods

As described earlier, variables subject to disclosure limitation are entry-age and final-age variables. Respondents with final-age greater than or equal to 80 years are considered to be sensitive cases, which intuitively leads to top-code values of 40 for entry-age and 80 for final-age. For this dataset, top-coding the age variables has great impact on the analysis, since the entry-age variable is recoded into only two categories (40 or below 40), in contrast to the five categories for entry-age in the original data. We applied HDMI methods to the data and computed estimates of regression coefficient from a PH model.

Table IV.7 shows results from original, top-coded and imputed datasets based on 500 replications. Predictably, TC considerably alters the relationship between hazard and covariates and yields estimates of the regression coefficients with serious bias, especially for the entry-age variable. Of the stratified HDMI methods, HD3 and HD4 yield estimates of coefficients of entry-age close to those from BD. HD1 provides better estimates of regression coefficients than other methods for the gender variable. For the rest of covariates, none of the stratified HD methods seems to have an obvious advantage, with HD2 being slightly inferior. HD5 has less satisfactory results, though it still yields

better estimates than TC for some covariates. Overall, the stratified HD methods all work better than top-coding in preserving the relationship between risk of death and the covariates on this dataset.

IV.5 Discussion

Longitudinal data raise particular confidential concerns with potentially extensive longitudinal information gathered over time. We consider a specific application concerning disclosure risk caused by some participants attaining high ages because of prolonged participation in a longitudinal study, as in the Charleston Heart Study. One of the authors (McNally) has the responsibility to prepare a public use version of this data set at the Data Archive at Michigan that meets HIPAA regulations. As discussed earlier, the standard approach of top-coding age has severe limitations in this longitudinal setting, especially for survival analyses with age being a key variable of interest. We develop MI-based SDC methods for this particular data setting. Similar to the methods in An and Little (2007b), our proposed MI methods are based on stratification, with strata defined by the predicted values of the age variables from a regression model.

Regarding the longitudinal nature of dataset in this study, we have focused on inference about regression coefficients from Cox's proportional hazard model. As expected, top-coding method yields seriously biased estimate especially for the entry-age variable. Among our stratified HDMI methods, HD4 has the best performance and yields results close to before deletion in simulation studies. The other stratified methods also work well overall, except that sometimes they do not quite attain the nominal confidence coverage. When there are fewer censored cases, as with the CHS data (number of censored cases is one fourth the total sample size), HD4 does not have obvious advantage

over other methods, though it still yields satisfactory results. The no-stratification method HD5 works almost as well as stratified HD methods in simple data settings. In situations with more covariates and a larger number of sensitive cases, it yields biased estimates with low confidence coverage.

An and Little (2007a) present two versions of MI methods, the “C” method which is based on a model fitted to the complete data; and the “D” method based on a model fitted to the deleted values alone. The “D” method is somewhat less efficient than the “C” method, but it is more robust to model misspecification, since the model is fitted to the data that are being deleted.

Similarly, we develop two alternatives in this study. The first method calculates predicted values from regression on the deleted data; and the second one utilizes the complete data for regression. Results show the first method yield estimates with better inferential properties. This finding supports the justification in An and Little (2007a), as regression on deleted data tends to be more robust to model mis-specification.

Our stratified HDMI methods produce excellent inferences, but they arguably have the limitation as SDC methods that original values in the dataset are retained, although not attached to the right records. Moreover, we have confined attention to individuals with high age values. The whole field of SDC methods raised by other variables (e.g. geographic) in longitudinal health data like the CHS data remains rather unexplored. We plan to rise to these challenges and develop suitable SDC methods in future work.

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Table IV.1 Hazard rate for simulation study, scenario I and II

	Age at death					
	30-40	40-50	50-60	60-70	70-80	80+
Category 1 Male Entry-age 31-40	0.003	0.005	0.011	0.04	0.06	0.1
Category 2 Female 31-40 Entry-age 31~40	0.024	0.004	0.0088	0.032	0.048	0.08
Category 3 Male 41-50 Entry-age 41~50		0.0075	0.0165	0.06	0.09	0.15
Category 4 Female 41-50 Entry-age 41~50		0.006	0.0132	0.048	0.072	0.12

Table IV.2 Hazard rate for simulation study, scenario III

	Age at death					
	30-40	40-50	50-60	60-70	70-80	80+
Category 1 (0,0) Male 31-40	0.003	0.005	0.011	0.04	0.06	0.1
Category 2 (1,0) Female 31-40	0.003	0.005	0.011	0.04	0.06	0.1
Category 3 (0,1) Male 41-50		0.0075	0.0165	0.06	0.09	0.15
Category 4 (1,1) Female 41-50		0.006	0.0132	0.048	0.072	0.12

Table IV.3 Simulation study scenario I: inferences of regression coefficients from PH model

method	Entry-age (40~50)				Gender (female)			
	Bias ($*10^4$)	RMSE** ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
BD	38	570	1	95.2	-38	582	1	92.6
TC	11501	11513	0.94	0	486	746	0.99	84.8
HD1	8	574	1.01	94.6	183	623	1.01	93
HD2	25	571	1.01	95.4	257	622	1.01	91.8
HD3	7	569	1.01	95.2	276	645	1.01	91.2
HD4	36	573	1.01	94.8	-17	585	1	93.6
HD5	7	581	1.03	94.2	325	648	1.01	91

Table IV.4 Simulation study scenario II: inferences of regression coefficients from PH model

method	Entry-age (40~50)				Gender (female)			
	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-wid	Cover (%)
BD	36	583	1	93.6	-15	580	1	93.6
TC	11463	11475	0.94	0	486	737	0.99	83.8
HD1	6	578	1.01	93.8	204	609	1.01	93.2
HD2	13	582	1.01	93.8	346	652	1.01	91.2
HD3	13	582	1.01	93.4	560	884	1.01	78.6
HD4	30	581	1.01	93.6	-7	577	1.01	94.2
HD5	96	599	1.03	93.6	225	588	1.02	94.2

** Here “RMSE” refers to root mean squared error. “Rel-wid” refers to “relative width”, which is fraction of 95 CI % width comparing to estimate 1. “Cover” refers to the 95% CI coverage.

Table IV.5 Simulation study scenario III: inferences of regression coefficients from PH model

method	Entry-age (40~50)				Gender (female)				Interaction			
	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-width	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-width	Cover (%)	Bias ($*10^4$)	RMSE ($*10^4$)	Rel-width	Cover (%)
BD	28	781	1	94.2	-39	810	1	94.4	13	1094	1	95
TC	10383	10411	0.95	0	-710	1129	1.07	84.6	2423	2646	0.97	38.6
HD1	-217	836	1.01	92.8	-128	839	1.01	93	501	1277	1.01	90.2
HD2	-244	803	1.02	94.8	-53	803	1	93.8	568	1166	1.01	93.6
HD3	-241	823	1.01	94	-123	850	1.01	92.8	550	1298	1.01	89.4
HD4	-20	760	1.01	96.4	-67	798	1	94.6	104	1070	1.01	95.4
HD5	-706	985	1.04	88.8	-437	854	1.01	91	1452	1646	1.03	81.4

Table IV.6 Estimates of regression coefficients from PH model, original CHS data

	Parameter Estimate (*10^4)	Standard Error (*10^4)	Pr > Chisq.	Hazard Raito
Entry-age 1 (40~44)	1977	1128	0.08	1.22
Entry-age 2 (45~49)	1814	1151	0.1	1.2
Entry-age 3 (50~59)	2786	1072	0.009	1.32
Entry-age 4 (60+)	2878	1242	0.02	1.33
Race/Gender 2 (white woman)	-4171	955	<0.0001	0.66
Race/Gender 3 (black man)	-241	949	0.8	0.98
Race/Gender 4 (black woman)	-1870	1031	0.07	0.83
Education 1 (some high school)	-1100	832	0.2	0.9
Education 2 (after high school)	-3761	1000	0.0002	0.69
Current cigarette smoking 1 (Yes)	5677	701	<0.0001	1.76
History of MI 1 (possible)	3741	3416	0.3	1.45
History of MI 2 (definite)	6949	1889	0.0002	2
History of diabetes 1 (Yes)	4330	1602	0.007	1.54
History of hypertension 1 (Yes)	1547	750	0.04	1.17
EKG 1 (with problem)	4644	947	<0.0001	1.59
Living place 20~65 2 (rural)	-2947	1028	0.004	0.75
Living place 20~65 3 (mix of rural and urban)	-1361	1467	0.4	0.87
BMI	28	74	0.7	1

Table IV.7 Estimates of regression coefficients from PH model, CHS data after SDC

	Estimate (SE) (*10^4)						
	BD	TC	HD1	HD2	HD3	HD4	HD5
Entry-age 1 (40~44)	1992 (1154)	Entry-age 1 (<40) -792 (975)	2597 (1164)	2129 (1155)	1962 (1157)	1977 (1152)	1801 (1173)
Entry-age 2 (45~49)	1815 (1153)		1817 (1181)	2429 (1173)	1872 (1180)	1999 (1178)	2269 (1187)
Entry-age 3 (50~59)	2711 (1056)		1640 (1097)	2658 (1094)	2371 (1098)	2706 (1090)	2638 (1095)
Entry-age 4 (60+)	2799 (1254)		2393 (1240)	3446 (1268)	2922 (1262)	3099 (1262)	3716 (1230)
Race/Gender 2 (white woman)	-4200 (913)	-3813 (1189)	-4724 (1002)	-2667 (953)	-3798 (979)	-3971 (965)	-2177 (960)
Race/Gender 3 (black man)	-205 (1004)	982 (1142)	-248 (966)	723 (960)	54 (975)	16 (963)	845 (971)
Race/Gender 4 (black woman)	-1876 (1073)	-1734 (1346)	-1984 (1036)	-1596 (1055)	-1771 (1054)	-1660 (1054)	-1267 (1043)
Education 1 (some high school)	-1127 (829)	-1347 (1029)	-996 (841)	-1108 (843)	-1224 (843)	-1257 (846)	-924 (847)
Education 2 (after high school)	-3806 (963)	-4958 (1257)	-3559 (1024)	-3081 (1003)	-3721 (1027)	-3793 (1013)	-3290 (1025)
Current cigarette smoking 1 (Yes)	5785 (718)	7328 (891)	5763 (714)	5463 (709)	5874 (724)	5596 (711)	4875 (706)
History of MI 1 (possible)	4211 (4548)	5360 (6113)	3397 (3515)	2702 (3483)	2467 (3516)	2946 (3599)	3863 (3552)
History of MI 2 (definite)	7080 (1936)	5622 (2766)	4678 (1980)	3392 (2027)	5029 (1979)	5280 (1954)	4716 (2017)
History of diabetes 1 (Yes)	4616 (2158)	6234 (2189)	4013 (1681)	3426 (1685)	3695 (1676)	4414 (1674)	4744 (1677)
History of hypertension 1 (Yes)	1637 (840)	2581 (977)	2006 (775)	1976 (769)	1877 (778)	1678 (777)	1823 (778)
EKG 1 (with problem)	4754 (1091)	4717 (1197)	4129 (982)	2421 (992)	3936 (982)	3754 (974)	3327 (992)
Living place 20~65 2 (rural)	-3042 (1029)	-3719 (1299)	-3297 (1054)	-2741 (1040)	-3189 (1058)	-3162 (1047)	-2522 (1039)
Living place 20~65 3 (mix of rural and urban)	-1296 (1887)	-594 (1969)	-1375 (1545)	-397 (1474)	-1239 (1500)	-559 (1480)	-410 (1519)
BMI	28 (81)	20 (98)	57 (76)	40 (76)	61 (76)	13 (75)	10 (76)

Chapter V

Conclusion and Discussion

Statistical disclosure control is a field with increasing attention and interest nowadays. Though progress has been made in implementing a variety of SDC techniques, these methods are not totally satisfactory in providing sufficient protection while reducing information loss. In this dissertation I propose both non-parametric and parametric MI methods for disclosure limitation problems caused by extreme values of variable.

In Chapter II, I describe an approach to SDC of extreme values based on multiple imputation of values beyond a cut-off. I illustrate the performance of these methods for inference about the mean of a variable subject to SDC, by simulations and application to data from the Chinese income project. We conclude that our hot-deck MI method, as well as the MI methods with log-normal model fitted to the deleted data, and with power-normal model fitted to the complete data, are decisively superior to top-coding in our simulations. They all produce excellent inferences for the mean, with the method based on power-normal model yielding imputations that match well the distribution of the deleted values. The “D” method based on power-normal model also yields good conference coverage but tends to be less efficient than the former methods; and the

method based on log-normal model fitted to the complete data is vulnerable to model misspecification. I further introduce covariates into the analysis and assess impact of the SDC methods on a regression where outcome is subject to top-coding. Our results prove that when applying the MI method to multivariate data, we should condition the predictive distribution of the deleted values on observed covariates, as failure to condition on covariates leads to an attenuation of relationships between outcome and covariates. I address this situation in Chapter III, by proposing stratified and regression-based extensions of our MI methods.

The regression-based methods are potentially more efficient, but a bit more complicated computationally than stratified methods. As for method performance, the stratified and regression extensions of MI methods are in general superior to top-coding and unconditional MI methods for inference about regression coefficient. Regression method with log-normal model fitted to the deleted data has the best performance and yield results close to before deletion. Stratified hot-deck method and the "D" method based on log-normal model, and regression method with power-normal model fitted to the complete data also produce good inferences. Regression method with power-normal model fitted to the deleted data works well except when estimating the marginal mean of outcome, with mis-specified model. Stratified MI methods based on power-normal model work well when the outcome is subject to SDC. When the imputations are performed on a covariate, they yield less satisfactory results. Both stratified and regression methods with log-normal model fitted to the complete data vulnerable to misspecification.

Longitudinal data raise particular confidential concerns with potentially extensive longitudinal information gathered over time, yet research on SDC method for longitudinal study is very limited. In Chapter IV I consider a specific application concerning disclosure risk caused by some participants attaining high ages because of prolonged participation in a longitudinal study, and develop nonparametric, stratified MI methods for this particular data setting.

I have focused on inference about regression coefficients from Cox's proportional hazard model. Among our stratified hot-deck MI methods, the method that retains the censoring indicator (HD4) has the best performance and yields results close to before deletion in simulation studies. The other stratified methods also work well overall, except that sometimes they do not quite attain the nominal confidence coverage. The no-stratification method works almost as well as stratified HD methods in simple data settings. In situations with more covariates and a larger number of sensitive cases, it yields biased estimates with low confidence coverage.

In this dissertation I present two different versions of parametric MI methods, the "C" method which is based on a model fitted to the complete data; and the "D" method based on a model fitted to the deleted values alone. The "C" method is efficient, but vulnerable to model misspecification. The "D" method involves some loss of efficiency, but is more robust to model misspecification, since the model is being fitted to the data that are being deleted. This finding is further confirmed in Chapter IV, with two alternative stratification methods. The first method calculates predicted values from

regression on the deleted data; and the second one utilizes the complete data for regression. Results show the first method yields estimates with better inferential properties, since regression on deleted data tends to be more robust to model mis-specification.

Our MI methods have the following advantages over the standard approach, top-coding. First, appropriate treatment of the top-coded data, using methods like maximum likelihood for censored data, requires custom algorithms that are not widely available in standard statistical software. In contrast, MI inferences only require complete-data methods and simple MI combining rules. Second, the MI methods tend to be less sensitive than top-coding to model mis-specification, as seen in our simulation studies. For the data producer, MI has the advantage that the balance between disclosure protection and information loss can be controlled by the choice of cut-off and number of MI's released. The use of MI allows imputation uncertainty to be propagated, and the multiple imputations of a particular value enhance disclosure protection by making clear to a potential snooper that these values are not real.

Overall, our proposed MI methods for SDC are relatively easy to implement, and yield valid inferences close to those from the data before deletion in the situations investigated. Thus, we expect these methods will prove valuable to practitioners.

On the other hand, the research in this dissertation is limited to a single variable that needs disclosure protection and considered inference of the marginal mean of a variable, or regression coefficient from a regression model. Future work should

investigate our SDC methods in multivariate analysis involving a set of variables that are subject to disclosure limitation procedure.

Moreover, I have confined attention to the comparison of our methods with top-coding. Other alternatives to top-coding, such as adding random noise to the values beyond top-code are also of interest. More simulation studies that compare our MI methods with these alternatives would be of interest.

Finally, my research of disclosure limitation methods for longitudinal data has been limited to individuals with high age values. The whole field of SDC methods raised by other variables (e.g. geographic) in longitudinal health data remains rather unexplored. I also plan to consider other possible confidential concerns for longitudinal data and develop suitable SDC methods for these problems.

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