A LEGIBILITY EQUATION FOR DETERMINING IDEAL VIEWING AREAS IN LECTURE HALLS

by

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ABSTRACT

A LEGIBILITY EQUATION FOR DETERMINING IDEAL VIEWING AREAS IN LECTURE HALLS

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Text presented in modern lecture halls often simultaneously appears on multiple visual media (e.g., blackboard, projection screens, TV) that have different locations, geometries, orientations, and lighting conditions. An ideal viewing area inside which all text is legible to the entire audience is needed for appropriate seating arrangement in lecture hall design. This area has been roughly defined by the architectural guidelines summarized from the experience gained in practice as a fan-shaped plan. For better accuracy and reliability, this ideal viewing area could be calculated using equations that predict the spatial legibility of text viewed from any directions across the lecture hall. Among the 95 legibility equations ever published in the literature, only the Reinwald formula (pre-1980) examines not-perpendicular-to-the-display viewing situations, but it fails to examine all factors that are usually considered. Thus, a new equation is needed.

This study first uses ten assumptions to narrow down the research scope as achromatic text (fonts not examined) with high pixel resolution presented on matte surfaces under uniform and glare-free fluorescent lighting and recognized by subjects aged 20-29 with threshold (just readable) 100% accuracy. Then, this study applies a hypothesis — the solid angle subtended by the legible viewing target (not only text) is a constant at different viewing angles (perpendicular or not) under the same viewing condition — to develop the demanded equation from the existing Howett's equation (1983). This derived equation examines seven critical factors: acuity; viewing distance; viewing angles; visual angle of text; text height, width, and strokewidth; luminance contrast; and target, background, and surrounding luminance. Unfortunately, it does not examine the surrounding luminance of the ambient environment, which may reduce its accuracy.

The constant-solid-angle hypothesis is verified consistent with how retinal images of text activate cones in the centre fovea of an observer's eyes. In addition, this hypothesis is tested in the lighting laboratory at the University of Michigan Transportation Research Institute using legibility data collected from 3 human subjects participating in a pilot experiment and 20 subjects participating in the follow-up main experiment. Both experimental setups abide by the typical viewing conditions surveyed in 38 lecture halls at the University of Michigan. In the main experiment, each subject sits in a fixed chair for about 100 minutes (157 minutes in the pilot experiment) with head on a chin rest to recognize the orientations of letter Es on 16 exchangeable E-chart sheets installed 20 feet away at 16 incident angles. The outcomes show that the constant-solid-angle hypothesis holds when $0° \leqslant \xi \leqslant 65.7°$, but it does not hold when $65.7° < \xi \leqslant 82.8°$ (the largest angle examined in this study). Based on these outcomes, the derived equation is thus accordingly improved.

To test the influence of ambient light on legibility of text, this study develops a second hypothesis from an equation proposed by Moon and Spencer (1945) that calculates the adaptation luminance of viewing environment. This hypothesis claims that ambient light in the viewing environment should have a small influence (less than 9%) on the legibility level of text viewed with constant background luminance and luminance contrast in a glare-free environment. The ambient-light hypothesis is then tested in the same laboratory using 20 human subjects with a modified setup at four different ambient light levels. The collected legibility data show that changing the ambient light levels of the viewing scenario does not affect the threshold legible (with 100% accuracy) heights of letter Es viewed at stable 124.2 cd/m^2 and constant contrast ($C_{\%}$= 97.91).

The validated equation is then improved to expand its examined incident angle ξ from 0°- 82.8° to the entire range of 0°- 90°, by assuming zero legibility distance of text viewed in lecture halls when $82.8° < \xi \leq 90°$, based on two facts. The improved equation is then used as the underlying algorithm for developing a computation-program-aided design method in MatLab. This method allows architects to find an overlapped two-dimensional ideal viewing area of text viewed in modern lecture halls along any viewing plane, such as that parallel to the sloped floor at eye height level. Before this method can be recommended for practice, it is verified using a field experiment carried out in the lecture hall in the Art & Architecture building at the University of Michigan. This method proves accurate and reliable when17 of the 21 subjects choose the predicted seat during the test, three other subjects choose the immediately adjacent seat of the predicted one.

The key outcomes of this study — the derived legibility equation, the preliminary computation program, and the finding that ambient light has a negligible effect on the legibility of text — can help architects and interior designers design new lecture halls or improve existing ones with enhanced legibility, lighting quality, and energy saving. Continuous research studies over the next 5-10 years will first solve the deficiencies in the preliminarily developed computation program, and then overcome the ten assumptions used here to examine more types of real viewing situations in architecture and other fields.

CHAPTER 1

Introduction

Characters have been used to impart information to people ever since they were invented, for example, in ancient Rome, Greece, or China. To serve their purpose, characters must be legible to the intended viewers at a large enough size and contrast. An ideal geometrical relationship also exists between the observers and the visual media on which characters are presented. Basically, observers must not be too far away from the characters to miss details, and too off axis (i.e., off the display normal) to extremely distort the characters thus losing useful information. Finding the ideal viewing distances and viewing angles of characters presented in large spaces will help architects and other professionals determine seating arrangements. The instructional spaces are typically large spaces where characters are commonly presented in the front and recognized by a large number of observers.

In China, the instructional spaces can be traced back to the tutorial space led by Confucius, an ancient Chinese philosopher (551 B.C. – 479 B.C.), as illustrated in Figure 1. Later, four famous instructional spaces called “Shu Yuan” were built from 484 to 1009, which have survived a long history, including (a) Song Yang Shu Yuan (嵩阳书院),

(b) Bai Lu Dong Shu Yuan (白鹿洞书院), (c) Yue Lu Shu Yuan (岳麓书院), and (d) Ying Tian Shu Yuan (应天书院) (Anonymous, 2006, 宋代四大书院简介 (Introduction of the four Shu Yuan in Song dynasty)). Similar instructional spaces could also be found in ancient Rome and Greece. About 200 B.C., the Romans borrowed some aspects of the ancient Greek system of education and began educating their children in school (Crystalinks, n.d., Ancient Roman education, para. 9). In all these ancient instructional spaces, characters were observed by many observers at varying distances and from a variety of viewing angles.

Figure 1. Private tutorial space led by Confucius
(Meilin, 2007, 壁画作品(孔子教学) (fresco (Confucius was tutoring)),
http://xcmeilin.com/jiashownews.asp?id=12)

Instructional spaces to date include, in order of the capacity of seats, seminar rooms (≤19), small standard classrooms (20-49), large classrooms (50-100), small lecture halls (75-149), large lecture halls (150-299), and lecture theatres (≥300) (Sources: Hauf et al., 1961; Duncan, 1966; Kemper, 1979; Allen et al., 1991, 1996;

Niemeyer, 2003; Higher Education General Information Survey (HEGIS); Classification of Instructional Programs (CIP 2000); University of Michigan (http://www.umich.edu/~ofa/Space/RmTyp100.htm); and University of Alaska (http://www.uaf.edu/provost/SPAM/Codes.htm)). The larger spaces — small lecture halls, large lecture halls, and lecture theatres — emerged in the 1960s due to the baby boom. Then, the large increase in enrollments and the shortage of faculties and facilities required larger spaces and the large scale use of visual aids and media (Hauf et al., 1961). According to these sources, small lecture halls, large lecture halls, and lecture theatres share with one another at least five characteristics that distinguish them from other smaller instructional spaces:

1. Large capacity ($\geqslant$75 seats).
2. Teaching-learning activities inside, which are not tied to a specific subject or discipline, where the audience views materials presented in the front space (Hauf et al., 1961).
3. Large scale use of visual aids and media, including blackboards, chalkboards, marker boards, tack boards, projection screens, TV monitors, overhead projectors, slide projectors, media players, and video/data projectors, etc.
4. Fixed and compact seating arrangement. Seats are fixed in tiers in the audience area, with 18 ft^2 per seat for small lecture halls, 16 ft^2 per seat for large lecture halls, and 14 ft^2 per seat for lecture theatres (minimum 12 ft^2 per seat required by code).
5. Fan-shaped, sloped or tiered floor. Small lecture halls may use a flat floor when their capacity is less than 100 seats.

These five characteristics make the three instructional spaces a special group to be examined in this study. For convenience, this study uses “lecture halls” as a general term to cover the small lecture halls, large lecture halls, and lecture theatres. Nowadays, lecture halls have become the primary scene for teaching and conferences.

1.1 Background

In good lecture hall design, architects must satisfy many requirements. A good lecture hall must facilitate the visual perception of material, enhance acoustical performance, provide a pleasant environment (air quality, temperature, and humidity), empower faculty to use visual aids and media, emphasize flexibility, encourage interaction, make technology simple and friendly, expand connectivity with other spaces, and contain costs (Allen, 1991; Niemeyer, 2003). Among these requirements, good viewing conditions is primary, since most of the information audience receives in lecture hall is through visual perception. Then how can favorable viewing conditions in lecture halls be achieved? Since the 1960s, architects have gained some empirical experience in practice, as summarized in Appendix A by Hauf et al. (1961), Duncan (1966), Kemper (1979), Allen et al. (1991, 1996), and Niemeyer (2003).

The guidelines listed in Appendix A have been followed by architects in achieving good viewing conditions in lecture halls since the 1960s. These empirical guidelines, however, have been challenged in modern lecture halls by the upgraded information technologies largely applied in recent decades to meet increasing pedagogical requirements and empower speakers with more capability and flexibility to illustrate their ideas (Niemeyer, 2003). In modern lecture halls, multiple types of visual media with different geometries are commonly installed in the front space at different locations with different mounting heights and orientations, on which materials are often simultaneously presented and observed by the entire audience. Lighting conditions in modern lecture halls are also frequently dimmed at multiple light levels. Under these complicated viewing situations, architects need a more precise and reliable design method than the empirical guidelines for ensuring good viewing conditions in modern lecture halls so that observers sitting in the far and peripheral seats will still have a good view.

Then how can architects ensure good viewing conditions inside modern lecture halls? Theoretically, good viewing conditions can be achieved with (a) good lighting, (b) an ideal viewing area inside which the entire audience can clearly read materials presented on all visual media, (c) legible materials of adequate size and contrast, and (d) good eyesight of the observers. In modern lecture hall design, the audience is always assumed to have average vision (with or without correction). The size and contrast of the viewing materials presented in lecture halls is also assumed to be within a reasonable range and out of control of architects. Therefore, more likely than not, architects may achieve good viewing conditions in lecture halls by providing good lighting conditions and ideal viewing areas inside. Lighting conditions can be conveniently renovated once the lecture hall has been built, but the ideal viewing areas for arranging fixed seats in tiers is usually restricted by the shape and size of the lecture halls. The architects' primary goal must be to determine the ideal viewing areas at the very beginning stage of lecture hall design. This study concerns finding the ideal viewing areas of characters presented inside lecture halls, particularly the modern ones.

What shape must the ideal viewing area take? The answer lies in the spatial legibility of characters viewed across the lecture hall. The spatial legibility of characters means the three-dimensional (3D) distribution of the legibility levels of characters viewed in a space, or their two-dimensional (2D) distribution along a viewing plane. Legibility of text or graphics refers to the observer's decipherability of the spatial mosaic of strokes of text or details of graphics at a specific scenario, to recognize the established forms of these characters and the embedded meanings thereof (Gove et al., 1986). According to Cornog & Rose (1967) and Sanders & McCormick (1993), legibility is the attribute of characters that distinguish each one from others by the features such as stroke width, height-to-width ratio, fonts, form of characters, contrast, and illumination, which determine the speed and accuracy for reading or identifying the characters. In lecture halls, materials are viewed by observers sitting in the peripheral seats in the audience area at the possible range of angles $\pm 0°$- 90°, either horizontally rotated or vertically tilted. Such rotated or tilted characters, according to people's daily experience, are not as legible as those viewed perpendicularly. The larger the incident angle (0°-90°) between the display's normal and observer's sightline, the more difficult it is to recognize the distorted characters. To maintain the legibility level of characters, either their size or contrast, or both, need to be increased when they are viewed not perpendicularly to the display. Alternatively, the observer would have to approach the materials to decrease the viewing distance. Usually, the size and contrast of materials presented in lecture halls does not change with seat locations. Thus, observers at peripheral area have to choose front seats to read tiny materials. The larger the viewing angle, the shorter the viewing distance. At 90°, the viewing distance would have to be zero. Figure 2 (a) illustrates how the viewing distance must decrease to maintain the legibility level of characters viewed at increased viewing angles. If all these viewing positions at different viewing angles are connected, a closed contour is formed, as illustrated in Figure 2 (b), on which characters are viewed at the same legibility level. The area encircled by this contour is then defined as the ideal viewing area for a single material. The material is guaranteed legible to observers sitting inside this ideal viewing area. Any viewing positions inside the contour will have higher legibility levels than those directly on the contour. Any viewing positions closer to the material will have higher legibility levels.

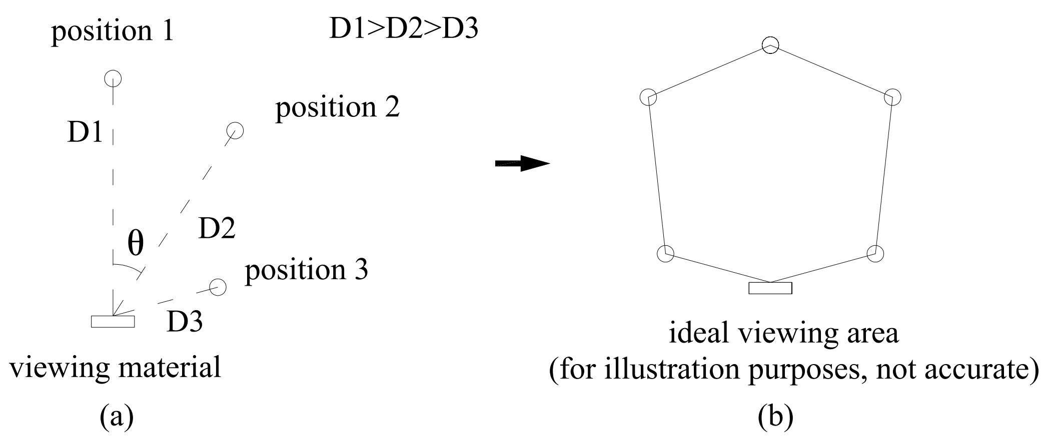


Figure 2. Illustration of the ideal viewing area for reading a single material at the same legibility level (no unit, not to scale)

1.2 Purposes of the Study

This study concerns the development of a new quantitative design method to help architects find the ideal viewing areas of characters presented in modern lecture halls. To enhance accuracy and reliability, this study addresses this issue in light of the spatial legibility of characters viewed not perpendicularly to the observers. The goal of this study is to develop a program-aided design method for architects to find an overlapped ideal viewing area in modern lecture halls for reading materials simultaneously presented on multiple displays, which are installed in the front space at different locations, with different geometries, mounting heights, and orientations. To accomplish this goal, this study will:

1. set up ten assumptions to narrow the research scope
2. identify the gap between the available empirical guidelines, to be summarized in the next chapter, and the required quantitative design method for determining the ideal viewing areas in modern lecture halls
3. bridge this gap by determining the ideal viewing areas in modern lecture halls in light of the spatial legibility of characters presented inside
4. develop and validate a computation-program-aided design method for architects to find the ideal viewing areas in modern lecture halls, and
5. prepare for future research studies to improve this program-aided method to an advanced level.

1.3 Significance of the Study

This study has been needed for a long time. First, architects have rarely used the spatial legibility of characters as a vehicle for determining favorable viewing conditions in lecture halls. Instead, they tend to use the empirical guidelines as summarized in Appendix A, which probably match some rules that could be quantitatively interpreted in light of legibility for good viewing conditions. This study is believed to be the first attempt to use the spatial legibility of characters to find the ideal viewing areas in modern lecture halls. Second, very few studies in the literature predict the spatial legibility of characters viewed not perpendicularly to the observers. This study is also believed to be the first attempt in the past century to extensively examine all critical factors, including viewing angles, viewing distance, lighting levels, contrast, character size, height-to-strokewidth ratio, and observer's acuity, to calculate the spatial legibility of characters. The outcomes of this study will have wide application in practice and stimulate further research.

This study will directly benefit architecture. The approach proposed here to calculate the spatial legibility of characters using legibility equations has never before been introduced to the field. It will benefit architectural design and foster new thinking. Using this approach, architects, interior designers, lighting designers, and other professionals can predict appropriate lighting levels, target sizes, contrasts, display locations, mounting heights, and orientations to create and evaluate good viewing conditions in lecture halls or other large architectural spaces. Such predictions can guide better design with enhanced lighting and legibility, while minimizing energy consumption and decreasing costs. In addition, with this approach, a computation-program-aided design method is developed to find an overlapped ideal viewing area of multiple displays in modern lecture halls. Upon validation, this program-aided method will supplement and even replace the empirical guidelines as summarized in Appendix A with enhanced accuracy, flexibility, and reliability. This method will allow architects, interior designers, lighting designers, and other professionals to predict ideal viewing areas not only in modern lecture halls but also in other large architectural spaces where reading characters is important, such as large commercial interiors, factories, public spaces, libraries, and museums.

The outcomes of this study will also have wide application to other fields, such as traffic and transportation, signs, advertisement, safety and security, electronic displays, where legible characters are crucial. Generally, clientele, designers, and researchers in any field in need of knowledge for predicting the spatial legibility of characters will benefit from this study.

1.4 Framework of the Study

This study overlaps with four fields. First, architects must determine good viewing conditions in modern lecture halls in light of the spatial legibility levels of characters viewed by observers sitting across lecture halls. Second, the legibility of characters has been thoroughly studied in the Human Factors and engineering fields in the past century (summarized in the following chapter), though nearly all of them assumed the characters were viewed perpendicularly to the observers. The spatial legibility levels of characters viewed not perpendicularly to the observers have not been comprehensively examined using all critical factors, such as viewing angles and lighting conditions. Third, many lighting researchers such as Blackwell (1946, 1959, 1972), as cited by CIE, 1981 (CIE 19/1, 2), Adrian (1982), Clear & Berman (1985, 1990, 2001), Rea (1986, 1987), and Veitch & Newsham (1995, 1998), have examined lighting quality for better visual perception of characters. Their outcomes have been widely used in buildings with enhanced lighting. Fourth, there are physical, physiological, and psychobiological fundamentals of the visual discrimination of characters. To examine the ideal viewing areas in modern lecture halls, this study overlaps all four fields of knowledge at one point — the spatial legibility of characters viewed in lecture halls, as shown in Figure 3.

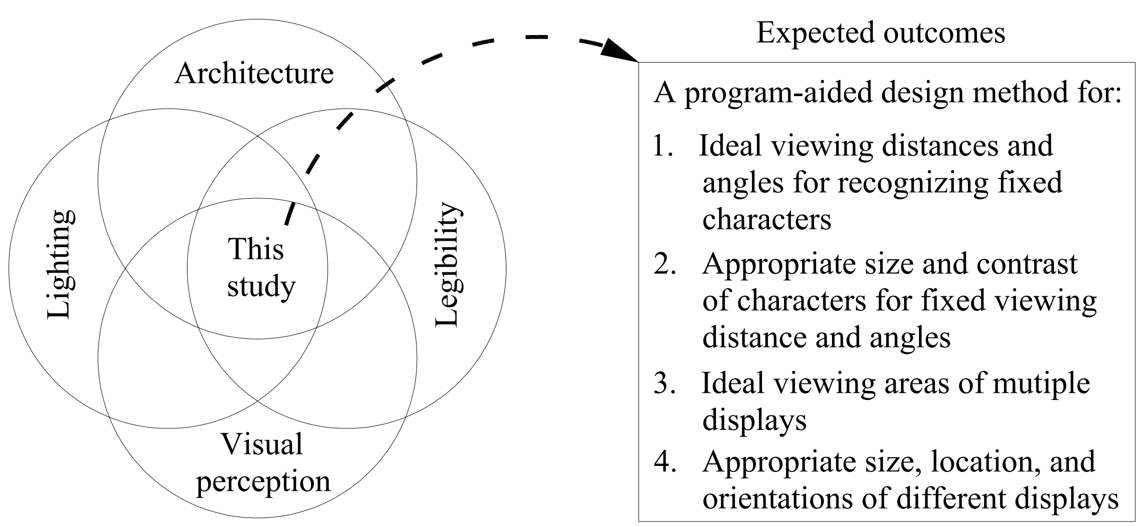


Figure 3. Framework of this study

As predicted, the primary outcome of this study is a computation-program-aided design method for architects to find ideal viewing areas in modern lecture halls. This program-aided method can determine (a) the ideal viewing distances and viewing angles for recognizing fixed characters, (b) the appropriate size and contrast of characters for fixed viewing distance and viewing angles, (c) the ideal viewing areas of multiple displays in lecture halls or other large spaces, and (d) the appropriate size, location, and orientations of different displays installed in buildings or their surroundings.

1.5 Assumptions and Limitations

This study makes 10 assumptions to narrow the research scope, as listed below.

1. For the viewing target, this study examines only text, more specifically, a single letter. Other common viewing targets such as graphics, words, and sentences are not examined. Thus, the spacing between letters within words is also not examined.
2. The font of text will not be examined in this study, but the height-to-strokewidth ratio of letters will be examined.
3. Text is always assumed to be of high pixel resolution, even those presented on projection screens.
4. The color of text is not examined.
5. The visual media where text is presented are assumed to be of ideal diffusive

surface. Specular reflection from TV monitors, etc., is not considered.

6. Target lighting is assumed to be uniform.
7. Viewing situations with glare and light trespass rare in lecture halls are not considered.
8. Only fluorescent T8, daylight, typical in most lecture halls, is examined; other light spectra are not examined.
9. The reading performance of text is assumed to be of threshold (just readable) 100% accuracy. No error or guessing is allowed in this study.
10. The target population is assumed to be 20–29 years of age. Thus, the aging effect on the legibility of text is not examined.

The outcomes of this study cannot be used for graphics, chromatic text, words or sentences, erroneous reading performances, or different age groups, but might be carefully extended to other similar situations besides lecture halls, such as classrooms, where reading text is important. Follow-up research studies are needed to overcome these ten assumptions and extend the outcomes to more general applications in the future.

1.6 Summary

Architects have been seeking better design methods for creating and maintaining favorable viewing conditions in modern lecture halls, which have been lately complicated by the large scale use of visual media with upgraded information technologies. Finding the ideal viewing areas of characters presented on those visual media must be the primary goal if architects are to design good lecture halls. This task cannot be easily accomplished in modern lecture halls using the empirical guidelines architects have followed for decades due to the lack of a legibility index. A quantitative design method is needed to determine ideal viewing areas in light of the spatial legibility of characters viewed by observers sitting across modern lecture halls. Using ten assumptions to narrow the research scope, this study will develop a computation-program-aided design method to predict an overlapped ideal viewing area of text presented on multiple displays installed in the front space of modern lecture halls with different locations, sizes, mounting heights, orientations, and lighting conditions. Future studies are needed to overcome these ten assumptions and improve the program-aided design method to a more advanced level.

CHAPTER 2

Literature Review

Since the emergence of lecture halls in the 1960s, numerous architects, interior designers, and educators, have gradually gained experience finding their appropriate size, shape, and slope angle of the floor for arranging seats for the entire audience. Hauf (1961), Duncan (1966), Conway (1990), and Allen et al. (1991, 1996) have summarized the empirical experience into rules of thumb for defining the ideal viewing areas of text presented in lecture halls. These rules of thumb, as will be detailed later, have been widely accepted in lecture hall design and have proven useful in practice. However, these empirical guidelines lack the spatial legibility of text and cannot calculate the complex viewing situations in modern lecture halls, where text is often viewed simultaneously on multiple displays, using legibility equations. On the other hand, since Erdmann (1898), numerous researchers have thoroughly studied the legibility of Roman characters under a wide range of viewing conditions, as will be reviewed later. Many quantitative studies have proposed equations for predicting legibility. Thus far, unfortunately, architects have rarely used these equations in their practice to design buildings with enhanced legibility. The empirical architectural guidelines for defining the ideal viewing areas in lecture halls, and all legibility equations published ever since, are comprehensively reviewed in this chapter. Both methods have their roots in visual perception. Therefore, the fundamental theories of the visual perception of text are reviewed first.

2.1 Fundamental Theories of the Visual Perception of Text

2.1.1 Visual Perception of Text

The human eye is the organ specialized in visual perception. The structure of the eye includes ocularmotor (e.g., ciliary muscle), optical (e.g., cornea, iris, pupil, lens), and neurological components (e.g., retina, fovea, blind point, optic nerve), as shown in Figure 4. For legibility, text is fixated by the observer on the center fovea of his/her retina and then discriminated by the 50k or so cones (there are no rods in the center fovea of human eye) (Wolken, 1966; Hendee & Wells, 1993; Wandell, 1995; Boff, Kaufman, and Thomas, 1986). Figure 4 illustrates the visual perception of the letter A. Incident light from the letter A passes through the ocular media and then reaches the center fovea to form an inverted image on the retina. During this process, a large amount of light is absorbed and scattered by the cornea, lens, aqueous and vitreous humors inside the eye (Boff & Lincoln, 1988). The remaining incident light then passes to the ganglion cells, amacrine cells, bipolar cells, and horizontal cells, and finally reaches the cones in the center fovea, as shown in Figure 5 (Wolken, 1966). Only 10% of the incident light is left to activate the cones, which fire signals to the cortex nerves in the brain for visual encoding of the letter A (Mouroulis, 1999; Wolken, 1966).

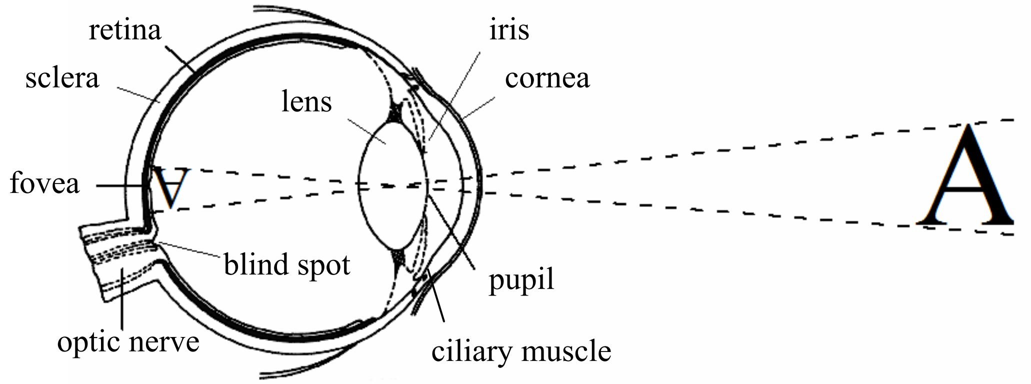


Figure 4. Cross-sectional diagram of the human eye in recognizing the letter A
(Boff & Lincoln, 1988, Figure 1, p. 54)

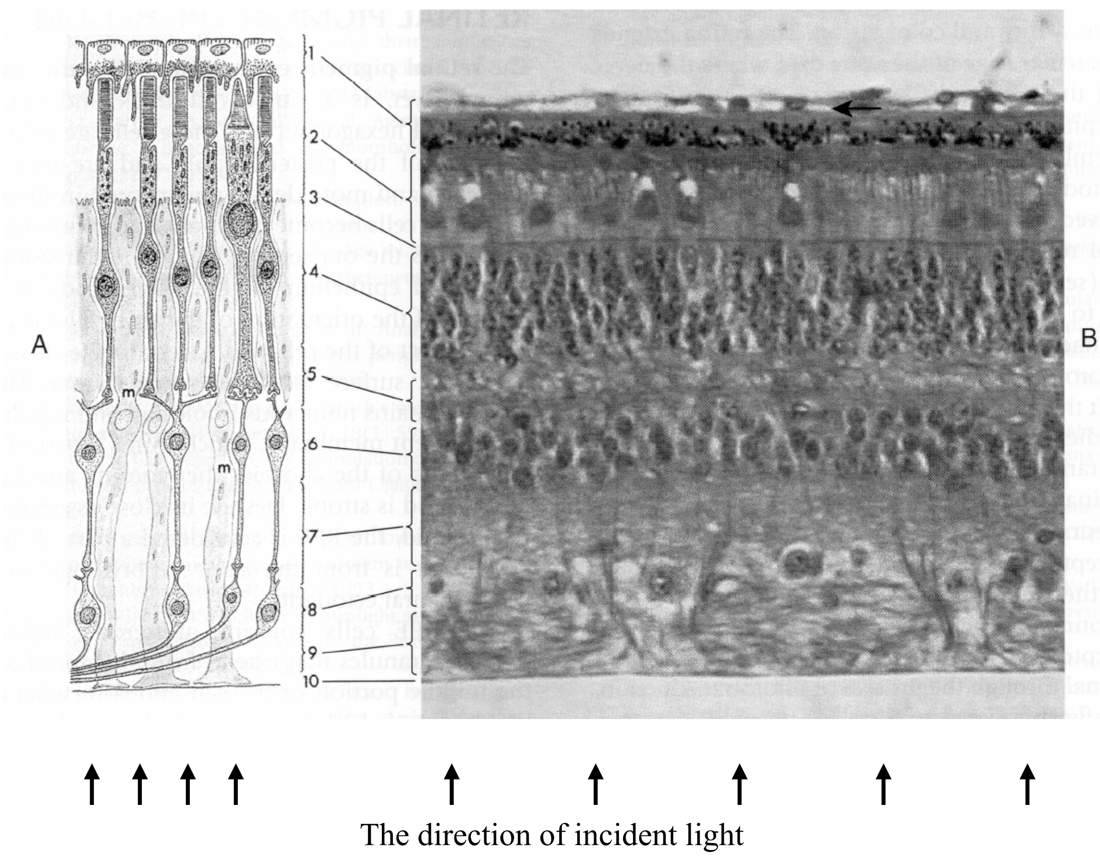


Figure 5. Layers of the retina in center fovea (Remington, 2005, Figure 4-1, p. 56)
1 retinal pigment epithelial layer; 2 photoreceptor layer (cones); 3 external limiting membrane; 4 outer nuclear layer; 5 outer plexiform layer; 6 inner nuclear layer; 7 inner plexiform layer; 8 ganglion cell layer; 9 nerve fiber layer; 10 internal limiting membrane.

How do the activated foveal cones discriminate text with different fonts and sizes? The answer lies in the geometrical characteristics of the cones in the center fovea that has a diameter of 0.5 mm and subtends 1.7° (Wandell, 1995). As illustrated in Figure 6, the foveal cones are very tightly packed and form a two-dimensional triangular array without any strong orientation dependencies (Wandell, 1995). According to Wandell (1995), the peak cone density is 1.6×10^5 per mm^2, the size of the inner segments of cones in the fovea is 2.3 μm, and the intercone spacing is 2.5 μm, so that the minimum discernible visual angle subtended by one cone is 0.5 min arc. This dense representation of the foveal cones suggests that the spatial mosaic of the cones must be very important for the visual encoding (recognition) of text with different fonts and sizes (Wolken, 1966; Hendee & Wells, 1993; Wandell, 1995; Boff et al., 1986).

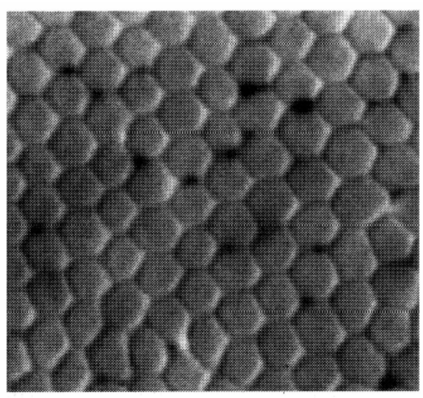


Figure 6. Spatial mosaic of cones in the fovea (Wandell, 1995, Figure 3.4, p. 49)

When viewed, the strokes of the retinal image of text strike the underlying foveal cones and activate them if the strokes are wider than one cone (0.5 min arc) and the incident light is strong enough (minimum 50 –150 quanta striking the cornea is needed for threshold vision (Pirenne, 1967). Legibility of text is eventually determined by the number of activated cones and their spatial distribution in the center fovea. Figure 7 illustrates that different text, or graphics, have a different number and spatial distribution of activated cones, and thus, different patterns for the visual encoding of characters.

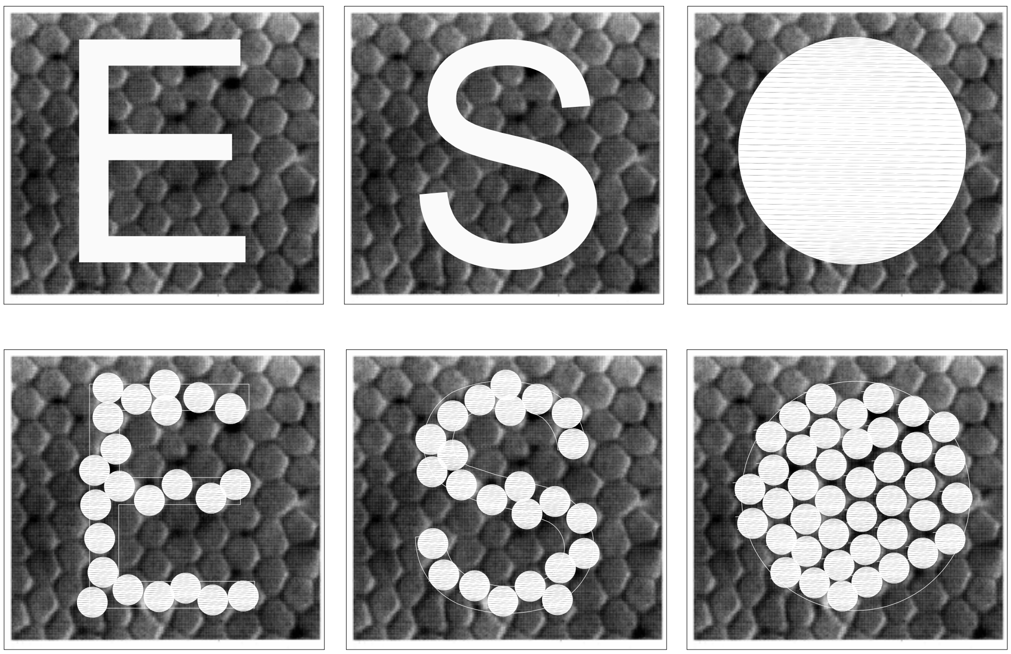


Figure 7. Retinal images of text (E, S) and a disc formed in the fovea with underlying activated cones (bright spots) (Wandell, 1995, Figure 3.4, p. 49).

2.1.2 Factors Affecting Visual Perception of Text in Lecture Halls

Geometric, viewer, and lighting related factors all affect the recognition of text in lecture halls along with the viewing duration. Geometric factors include viewing distance, image size (width, height, strokewidth), and image orientation (perpendicular to the observers or not). Viewer related factors include aberrations and imperfections of human eyes, age, and visual acuity level. Lighting related factors include target and background luminance, ambient light, image luminance contrast, spectrum of lamps, and color contrast. The influence of these critical factors on reading text is expanded on below.

(1) Geometries

The influence of viewing distance and image size on reading text is obvious. Text is better recognized with a larger size, or at a shorter distance. The influence of viewing angles on the legibility of text has yet to be thoroughly examined. According to people's daily experience, text presented on displays not perpendicular to the observers usually has a decreased legibility level compared to that under perpendicular viewing. For example, text presented on a projection screen in lecture halls is often harder to recognize for observers sitting in the peripheral seats than those sitting at the center of the audience area.

(2) Imperfections and refractive errors of the human eye

The human eye is not a perfect viewing system. Besides its inefficiency in transferring light (only 10% reaches cones), the normal human eye varies in terms of some geometrical features (asphericicities, asymmetries, tilts, and decentrations) that "may have marked effects on the ocular aberrations and hence on the retinal image quality" (Mouroulis, 1999, p. 3). Other common abnormal eye problems include refractive errors, chromatic aberration, and neuro-ophthalmological abnormalities. Refractive errors (focusing problems) include myopia (nearsighted), hyperopia (farsighted), astigmatism (multiple foci are formed), and presbyopia (near objects focus behind the retina), which are caused in later life and can be corrected by wearing glasses or contact lens (Boff & Lincoln, 1988; Rea, 2000). Chromatic aberration, commonly called abnormal color vision, is inborn or due to diseases. This research study examines only normal color vision.

(3) Visual acuity

Visual acuity in this research refers to recognition acuity, defined as the ability of the observer to clearly perceive spatial detail, which is equal to the reciprocal of the resolution threshold (Boff et al., 1986). Normal acuity is usually 1 min arc for human eyes (Wandell, 1995). Two versions of notations are used in practice for acuity: decimal or the Snellen fraction. Normal acuity in decimal notation is 1, and 20/20, 6/6, 4/4, or 40/40 as a Snellen fraction (Boff et al., 1986).

The observer’s acuity is not a constant and is affected by many factors. First, acuity improves as the retinal illuminance of text increases due to decreased pupil size, which reduces the effect of the eye's refractive errors (focusing problems), and the decreased receptive field size of foveal cones, which becomes more sensitive to subtle details (Boff et al., 1986; Rea, 2000). However, when glare sources are visible in the viewing field, the influence of veiling luminance on text will reduce its contrast and thus its acuity. Fortunately, glare conditions are rare in lecture halls and are not examined in this study. Second, acuity continues to improve with the background luminance of text, as long as the background size is larger than 0.85° by 1.7° (visual angles) for typical lecture halls (Rea, 2000). Third, at the photopic light level ($\geqslant$3.4 cd/m^2) typical in lecture halls, the highest acuity level is obtained in the center fovea where text is fixated for legibility; other locations on retina have decreased acuity (Boff et al., 1986; Rea, 2000). Fourth, slightly decreased pupil size also enhances acuity, as detailed in the next section. Fifth, text is always viewed in lecture halls with exposure time much longer than the threshold 500 ms, after which visual acuity is maximized (Rea, 2000). Sixth, the accommodation errors when text is viewed at a long distance in lecture halls blur the retinal image, thus decreasing the acuity (Boff et al., 1986). Seventh, acuity also changes considerably over the life span of an individual. An acuity of 1 min arc is approached at 36 months of age and 0.75 min arc during the first 5 years (Boff et al., 1986). Beyond the twenties, acuity decreases (Boff et al., 1986).

(4) Pupil size

Normal pupil diameter is about 2-5 mm at photopic light levels and 3-8 mm for young people (Boff et al., 1986; Rea, 2000). Pupil size varies because the iris constricts and dilates in response to luminances within the field of view (Rea, 2000). Pupil size decreases as the field luminance increases in the range of typical lecture halls, as illustrated in Figure 8, and approaches 2 mm at 6366.0 cd/m^2 (Reeves, 1920). A very small pupil degrades the retinal image by low retinal illumination and diffraction effects, while a very large pupil also degrades the quality of the retinal image by the increased effects of spherical and chromatic aberration (Boff et al., 1986). In between, an optimal pupil size exists to maximize the acuity of the observer, as shown in Figure 9 (Leibowitz, 1952). In addition, older people tend to have smaller pupils under comparable conditions (Rea, 2000).

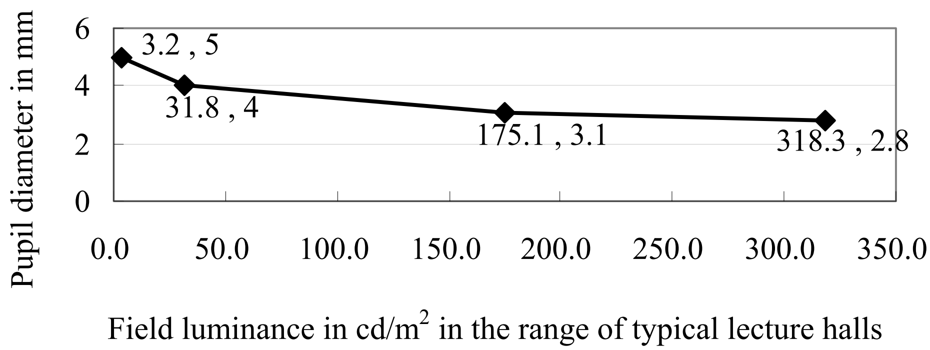


Figure 8. Field luminance vs. pupil size (Reeves, 1920, Table II, p. 39)

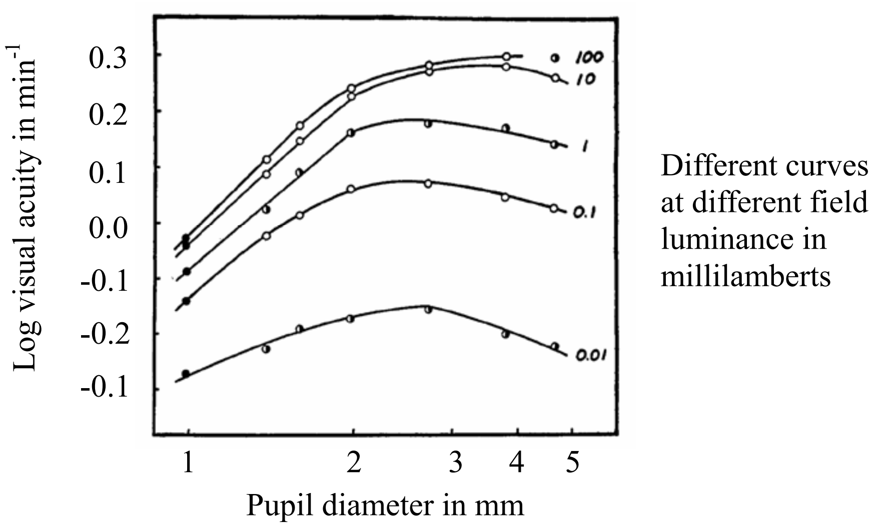


Figure 9. Log visual acuity as a function of log pupil diameter (Leibowitz, 1952, Figure 3, p. 421)

(5) Age

The aging of the observer's eye causes extra errors and decreases the ability to resolve fine details. First, the amplitude of accommodation decreases rapidly with age. By age 45, most people lose the ability for near acuity (presbyopia); by age 60, nearly no accommodation ability remains (Weale, 1992; Rea, 2000). Second, pupil size also increases rapidly in the early years of life and peaks at around age 10, and then slowly decreases to a fixed value around 70 (Weale, 1992, Figure 2.1, p. 48). The constricted pupil size reduces the ocular aperture but enlarges the depth of focus (Weale, 1992). The increased depth of focus somewhat compensates for the lack of focusing ability in the elderly (Rea, 2000). Third, visual acuity increases at early ages and peak in the twenties (approximately 20–29), as shown in Figure 10, then declines sharply in later life (Weale, 1992). Fourth, the optical power of the lens of the human eye in dioptres declines at early ages, and then keeps constant in adults (Weale, 1992). Fifth, the crystalline lens of the human eye yellows progressively with age, thus increasing the absorption and diffusion of short wavelengths (Weale, 1992). Sixth, the axial fluorescence increases with age, while the cornea keeps constant in scattering light throughout life (Weale, 1992). Seventh, the density of photoreceptors in the retina also decreases with age since the photoreceptors are not replaced once lost (Weale, 1992). Although rods are quickly lost between 61 and 82 years of age, the number of cones is constant until the age of 70 or 80, and then declines (Weale, 1992).

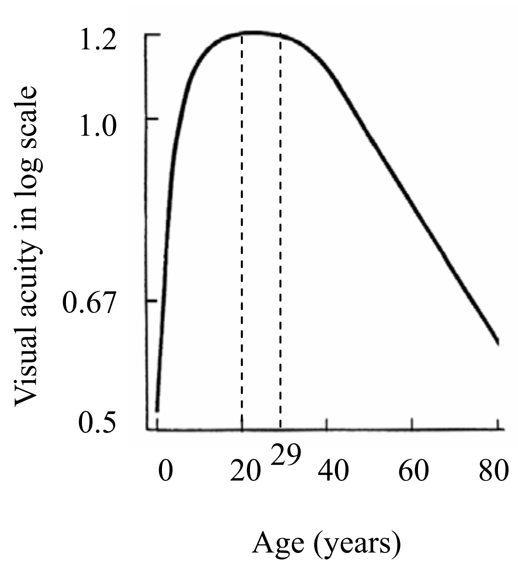


Figure 10. Age-related variation of visual acuity (Weale, 1992, Figure 5.1, p. 228)

(6) Spectrum of light sources

Light spectrum affects reading text in lecture halls due to the spectral sensitivity (sensitivity to different light wavelengths) of three different types of cone: S-, M-, and L-cones in the fovea. Figure 11 shows the 2-degree standard photopic observer developed by CIE (1931) to represent the relative spectral sensitivity of the foveal cones. At the photopic level ($\geqslant$3.4 cd/m^2), the human eye has maximum sensitivity to a target of 555 nm wavelength. The light of other wavelengths will be perceived dimmer; thus, higher intensity is required to produce equal brightness, as depicted in Figure 12 (Boff & Lincoln, 1988).

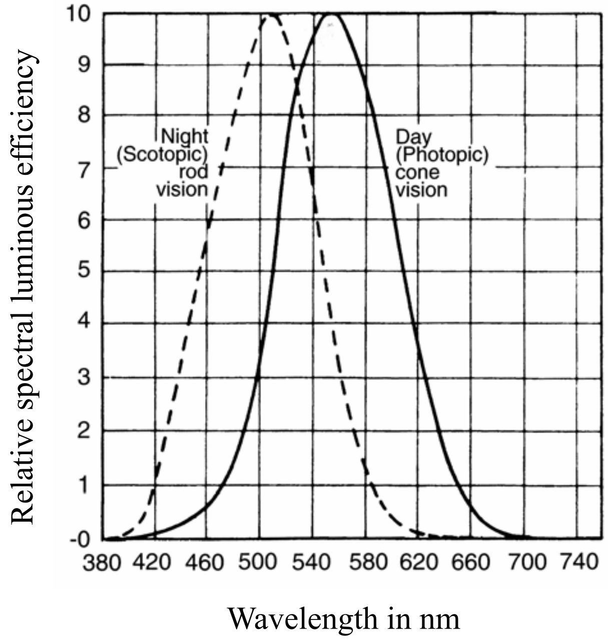


Figure11. CIE Standard Photopic Observer (solid line), representing the relative spectral sensitivity of the cones (Rea, 2000, Figure 3-10, p. 3-6)

The amount of light transmitted by the ocular media in the human eye is also a function of wavelength, as illustrated in Figure 13 (Boff & Lincoln, 1988). A yellowish pigment contained in the fovea affects its sensitivity to different wavelengths (Boff & Lincoln, 1988). However, the influence of the light spectrum on the legibility of text has not yet been thoroughly studied. This study will exclude its influence by examining only one light spectrum — fluorescent T8, daylight, typically used in lecture halls.

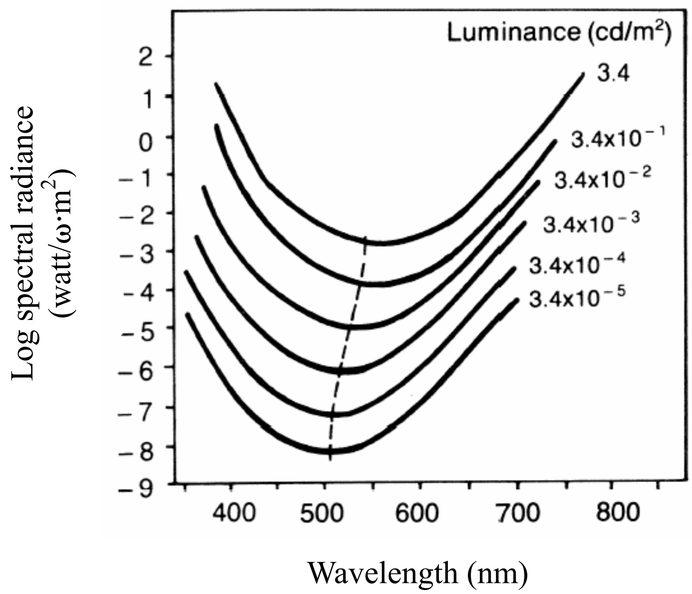


Figure 12. Relative energy level required to produce perceptions of equal brightness for an observer adapted to multiple luminance levels (Boff & Lincoln, 1988, Figure 1, p.124)

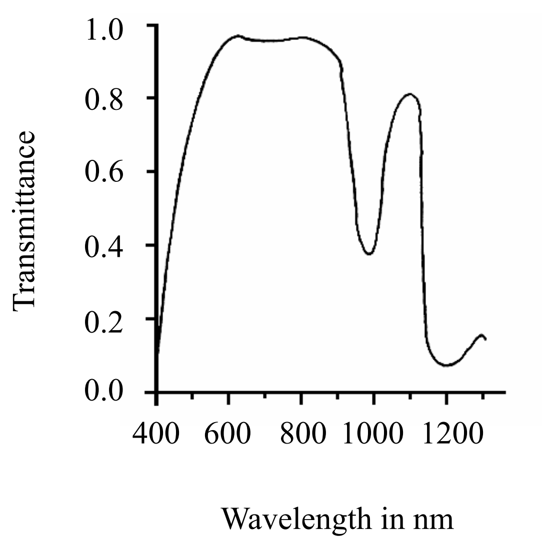


Figure 13. Best current estimates of the amount of light transmitted by the ocular media as a function of wavelength (Boff & Lincoln, 1988, Figure 1, p. 36)

(7) Chromatic contrast

Chromatic contrast (color difference) between text and its background also affects legibility levels. For observers with normal color vision, text with larger chromatic contrast has higher readability[1], while text with a color combination of positive polarity (for example, dark text on light background) is more legible (Wang et al., 2003, cited by Hall & Hanna, 2003; Pastoor, 1990). Then what color combinations have greater legibility? Generally, color combinations with higher luminance contrast will have better legibility, regardless of the specific color combinations (Radl, 1980, and Bruce & Foster, 1982, cited by Hall & Hanna, 2003). Black/white and black/yellow are the most legible color combinations (Luckiesh, 1923, cited by Tinker, 1963; Tinker & Paterson, 1931, and Hackman & Tinker, 1957, cited by Rehe, 1974; Smith, Farquhar and Thomas, 1965; Tinker 1963; MacNeill, 1965, cited by Adams et al., 1988; Adams et al., 1988; Clements-Smith et al., 1993, cited by Nilsson, 1999; and Nilsson, 1999). Other good color combinations include white/green (Luckiesh, 1923; Tinker, 1963; Woods et al., 1970, cited by Adams et al., 1988; Clements-Smith et al., 1993), white/blue (Luckiesh, 1923; Tinker, 1963; Gurney et al., 1977, cited by Adams et al., 1988; Clements-Smith et al., 1993), green/yellow, blue/yellow, black/red (Clements-Smith et al., 1993), and black/green, blue/grey, as well as black/gray (Tinker, 1963). However, to narrow the research scope, this study does not examine color and chromatic contrast. Text presented in this study is always printed black/white or grey/white.

[1] Unlike legibility, readability refers to the recognition of the stylistic and grammatical complexity of prose, which depends more on the spacing of characters and groups of characters, their combination into sentences or other forms, the spacing between lines, and margins than on the specific features of the individual characters (Sanders & McCormick, 1993; Foster, 1980).

(8) Light level and luminance contrast

Lecture halls are typically at the photopic light level ($\geqslant$3.4 cd/m^2), where foveal cones dominate for discriminating text. The luminance contrast of text viewed in lecture halls is usually high to enhance legibility under different light levels. Light level and luminance contrast of text affect its legibility by means of the retinal image, whose intensity and quality determine the number of activated foveal cones and the intensity of signal sent to the brain. On the other hand, the sensitivity of human eye to luminance contrast and its sensitivity to absolute light level are complementary measures (Wandell, 1995). Weber's law (1) predicts that the detectable luminance contrast (the threshold to the absolute light level) is proportional to the intensity of the adapting field (adaptation luminance) (Boff & Lincoln, 1988).

$$I_t = k \times I_a \tag{1}$$

where

I_a = Intensity of the adapting field (adaptation luminance)

I_t = Amount of intensity above I_a to be just detectable (threshold luminance contrast)

k = Constant

According to Weber's law, sensitivity to contrast is greater in low than in high adaptation luminance. The adaptation luminance is dominated by the luminances of the viewing target and its immediate background within 1.5° subtended to the observer's eyes (foveal luminance), is also affected by the surrounding luminances (Moon & Spencer, 1945). Thus, the sensitivity of the observer's eye to text in lecture halls will vary with the lighting conditions inside. For instance, the lowered sensitivity of the human eye at higher adaptation luminance partially contributes to the "washing out" effect of text presented on the projection screen when the previously dimmed lecture halls are lit up.

(9) Ambient light

First, ambient light has proven to be effective nowadays for the visual perception of text mainly through transient adaptation. To view text, the observer's eyes adjust their operating characteristics as a result of the brightnesses within the field of view, that is, the adaptation luminance (Rea, 2000). The surrounding luminance from the ambient environments with an off-axis viewing angle $\geqslant 1.5°$ contributes with small portion to the adaptation luminance (Moon & Spencer, 1945). A lower level of ambient light decreases the adaptation luminance, in turn, according to Weber's law, increasing the contrast sensitivity of the observer's eyes to read text. For example, the general lighting in lecture halls is usually dimmed for better visibility of the projection screens. In this case, however, the better legibility of text is more largely due to the increased contrast of text, since the general lighting (ambient light) in the seating area usually adds light to the projection screens and then washes out the text lit using the projector light. On the other hand, dimming a lecture hall is usually not helpful for increasing the legibility of text written on the blackboard. In the later case, the general lighting in audience area might add some light on the text and thus increase their legibility levels.

Second, ambient light has an influence on text reading also through pupillary changes. Within the typical range of light levels in lecture halls, as illustrated in Figure 8 before, higher levels of ambient light will decrease pupil size, thus maximizing acuity by enhancing the view depth and upgrading the retinal image due to the lowered diffraction effect. Consequently, when read text presented on externally lit visual media, such as blackboards, books, magazines, but not projection screens, people usually feel more comfortable in their eyes with ambient light on than in total darkness. Of course, as claimed by Leibowitz (1952) in Figure 9 before, an optimal pupil size exists for maximizing the acuity of the observer, depending on the adaptation luminance, which has a partial contribution from ambient light.

(10) Exposure time

Increased spatial and temporal extent can greatly reduce the intensity (quanta or energy per unit area per unit time) that a light must provide to be detected (Boff et al., 1986). The threshold intensity for light detection is inversely proportional to the duration of the viewing target because there is a temporal summation of light energy (Boff & Lincoln, 1988). This relationship is known as Bloch's law (2).

$$IT = k \tag{2}$$

where:

I = Threshold intensity

T = Target duration

k = Constant, equal to the product of the critical duration and the threshold intensity at critical duration (critical intensity)

Up to some critical duration, sensitivity increases (threshold intensity decreases) in inverse proportion to exposure duration, where Bloch's law holds. However, above the critical duration, increasing the length of exposure has no effect (Boff & Lincoln, 1988). The typical value of this critical duration is 20 - 100 ms, which varies with target characteristics and viewing conditions (Boff & Lincoln, 1988). In most situations, text is exposed for a long enough time (> 500ms) in lecture halls after the observer's eyes have adapted to the surrounding level.

2.2 Architectural Guidelines for Determining Ideal Viewing Areas of Lecture Halls

The experiences gained in practice since the 1960s to determine ideal viewing areas in lecture halls were summarized into empirical guidelines by Hauf, Koppes, Green, and Gassman (1961), Duncan (1966), Conway (1990), and Allen et al. (1991). Similar experiences for sport stadiums, sport halls, indoor and outdoor facilities were also standardized by the German Institute for Standardization and National Standards Authority of Ireland. These empirical guidelines and standards have defined the ideal viewing areas of lecture halls because of their ideal plan shape and ideal longitudinal section profile.

2.2.1 Ideal Plan Shape of Lecture Halls

The ideal plan shape of lecture halls is primarily determined by the audience seating area. It is also compromised by other architectural considerations. For good visual perception, the seating area must be coincident with an ideal viewing area of displays mounted in the front space, including blackboards, screens, TV monitors, and so on. Conventionally, this ideal viewing area of displays has been defined as fan-shaped, with ranges of viewing distances and horizontal viewing angles. For instance, the DIN 108 Standard defines this fan-shaped area to view drawn or written black/white slides projected on matte screens as Figure 14, with viewing distance varying between 2 and 6 times of the screen width (w) and a horizontal viewing angle within ± 30° of the center line of screen.

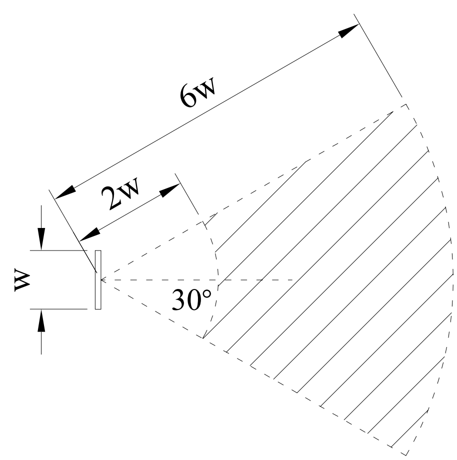


Figure 14. Ideal horizontal viewing area of matte projection screen
(DIN 108, cited by Duncan, 1966)

For the ideal viewing area of general matte projection screens installed in lecture halls, Duncan (1966) recommended the maximum viewing angles on either side of the center line as 30°, the maximum elevation of the eye to the top of the screen as 35°, and the critical angle of depression of the projector as 12°. Likewise, Hauf et al. (1961) defined the fan-shaped ideal viewing area with viewing distance varying between 2w and 6-7w (screen width), horizontal viewing angle ±30°-60°, and maximum angle of elevation 15°. Similarly, Allen et al. (1991) recommended a minimum 1.5w and optimum 2w from the first row of seats to the screen, and a maximum 6w for optical projection, or 4w for electronic projection due to lower (12.5%-25%) resolution. Allen et al. also recommended a maximum 35° from the horizontal subtended by the top of the screen from any seating position. However, some compromises may have to be made in the first few rows of seats to allow sufficient space for chalkboard/marker board and a reasonable screen size, and yet not have the front seats too far from the front of the room (Allen et al., 1991). Conway (1990) suggested that the maximum viewing distance is 6w, while the minimum is 1.5-2w.

Similarly, for the ideal horizontal viewing area of TV monitors used in lecture halls, Hauf et al. (1961) recommended that the viewing distance be between 4w (monitor width) and 12w (14w for less optimum condition), and that the horizontal viewing angle be ±35°-40° (±45° for less optimum condition), with the maximum angle of elevation 15° to the bottom of image (30° for less optimum condition). In addition, Allen et al. (1991) claimed that the farthest viewing distance should be no more than one foot per diagonal inch of the monitor size.

This study has summarized these similar definitions of ideal viewing area in lecture halls into a more general one by integrating all the varying parameters (viewing distances and angles). This general definition will provide a single or overlapped ideal horizontal viewing area for reading text presented on: (a) a single point of a matte screen, (b) a whole matte screen, (c) three screens symmetrically mounted on one plane, (d) three screens symmetrically mounted on three different planes, (e) a single TV monitor, and (f) multiple displays randomly mounted in lecture halls. They are expounded below separately.

2.2.1.1 A Single Point of Matte Screen

For text presented at a single point on a single matte screen in lecture halls, the horizontal ideal viewing area is fan-shaped: viewing distance is 2w-6w (screen width), with a horizontal viewing angle ϕ of $\pm 30°$-$60°$, as illustrated in Figure 15.

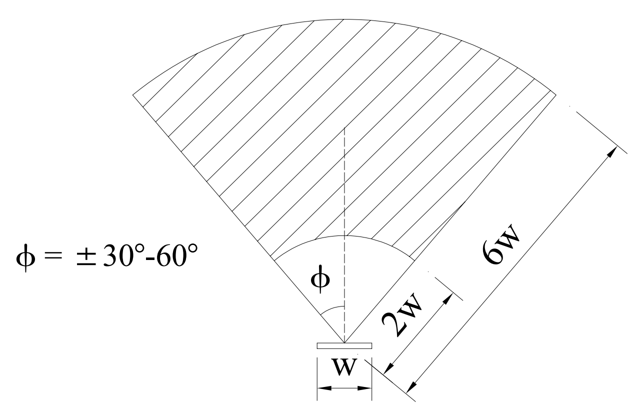


Figure 15. Horizontal ideal viewing area of text presented at a single point on a matte screen (Hauf et al, 1961, the fold-out diagram, after p. 11-14)

2.2.1.2 A Whole Matte Screen

For text presented on a whole matte screen, the ideal viewing area is overlapped at three critical points: middle point, left edge, and right edge, as illustrated in the shaded area in Figure 16. In this overlapped area, the viewing distance is 2w-6w (screen width), while the horizontal viewing angle ϕ is $\pm 30°$-$60°$.

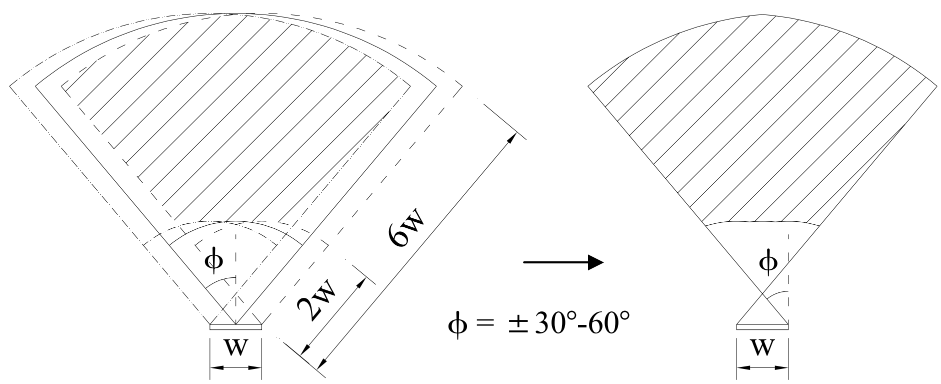


Figure 16. Overlapped ideal viewing area (shadowed area) of a whole single screen (Hauf et al., 1961, the fold-out diagram, after p. 11-14)

2.2.1.3 Three Screens Symmetrically Mounted on One Plane

Multiple screens are often viewed simultaneously in the front space of modern lecture halls. When three screens are symmetrically mounted in the front space of lecture halls on one plane with equal spacing, the shape and size of their overlapped ideal viewing area vary with the viewing angle ϕ and the spacing D between screens, as shown in Figure 17.

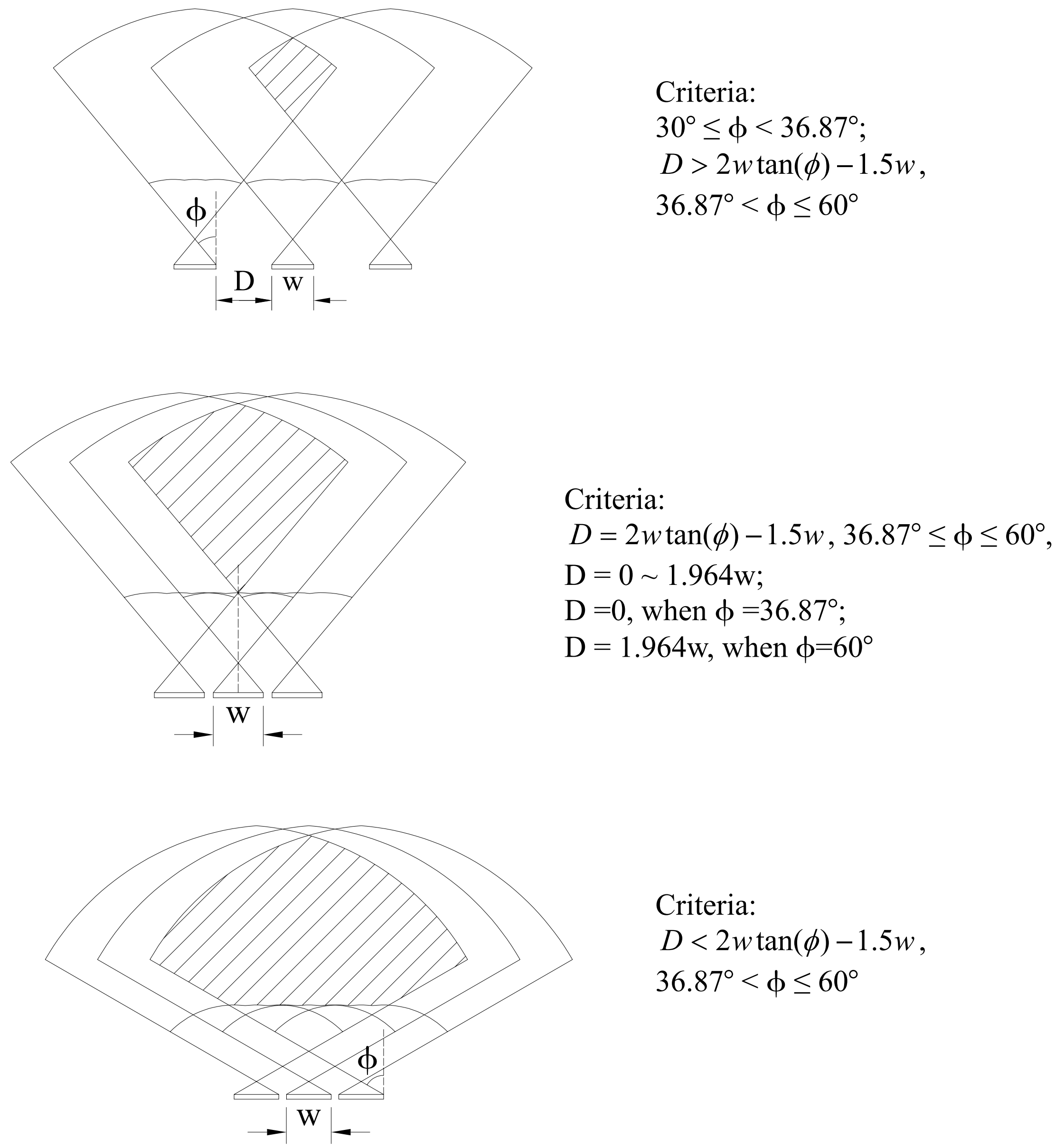
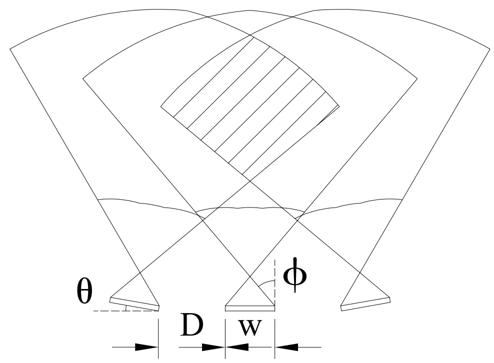


Figure 17. Different shapes of the overlapped horizontal ideal viewing area (shaded) of three matte screens symmetrically mounted on one plane with equal spacing (Hauf et al., 1961, the fold-out diagram, after p. 11-14)

2.2.1.4 Three Screens Symmetrically Mounted on Three Planes

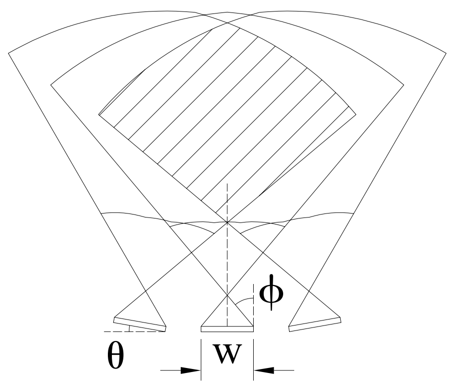
Side screens in the front space of lecture halls are often slightly rotated to face the audience. When three screens are symmetrically mounted on three planes with equal spacing, their overlapped ideal viewing area depends on the viewing angle ϕ, the spacing D between screens, and the rotating angle θ of side screens, as illustrated in Figure 18.



Criteria:

$$D > 2w\tan(\phi+\theta) - \frac{\cos\phi}{\cos(\phi+\theta)}w - 0.5w;$$

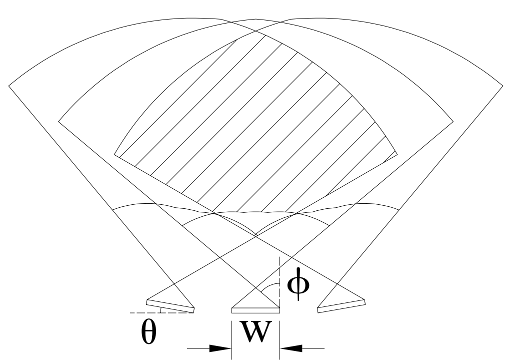
30° < ϕ ≤ 60°; θ =10° or other value



Criteria:

$$D = 2w\tan(\phi+\theta) - \frac{\cos\phi}{\cos(\phi+\theta)}w - 0.5w \ ;$$

30° ≤ ϕ ≤ 60° ; θ =10° or other value



Criteria:

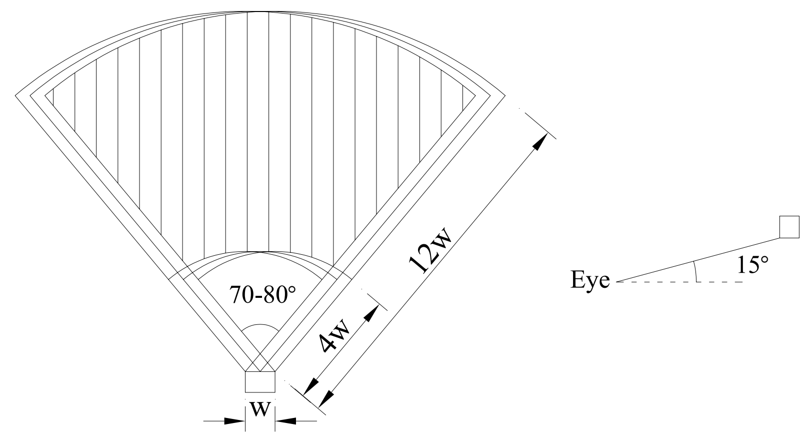
$$D < 2w\tan(\phi+\theta) - \frac{\cos\phi}{\cos(\phi+\theta)}w - 0.5w \ ;$$

30° < ϕ ≤ 60° ; θ =10° or other value

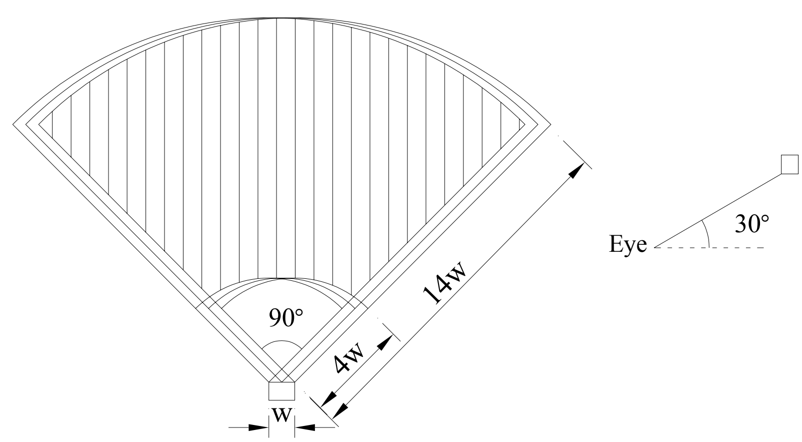
Figure 18. Different shape and size of the horizontal ideal viewing area (shaded) of three matte screens symmetrically mounted on three different planes with equal spacing (Hauf et al., 1961, the fold-out diagram, after p. 11-14)

2.2.1.5 TV Monitor

Due to smaller size, greater brightness, and specular surface of TV monitors, their ideal viewing area has larger dimensions (optimum 4w-12w, or less optimum 4w-14w), but smaller horizontal viewing angles (optimum $\pm 35°$-$40°$, or less optimum $\pm 45°$), as illustrated in Figure 19 (Hauf et al., 1961).



(a) Optimum TV monitor horizontal viewing area with viewing distance 4w-12w, horizontal viewing angle $\pm 35°$-$40°$, and max. elevation angle 15° to the bottom of image



(b) Less optimum TV monitor horizontal viewing area with max. distance may be increased to 14w, horizontal angle to $\pm 45°$, and max. elevation angle to 30°

Figure 19. Ideal viewing area of TV monitors with different mounting heights (Hauf et al., 1961, no Figure number, p. 11-19, 11-20)

2.2.1.6 Multiple Displays Randomly Mounted in Lecture Halls

In reality, observers in lecture halls often need to view multiple displays (e.g., blackboards, tack boards, screens, TV monitors) mounted in different locations, with different geometries, mounting heights, and orientations. Their ideal viewing area could still be determined by overlapping all ideal viewing areas of each display, as illustrated in Figure 20.

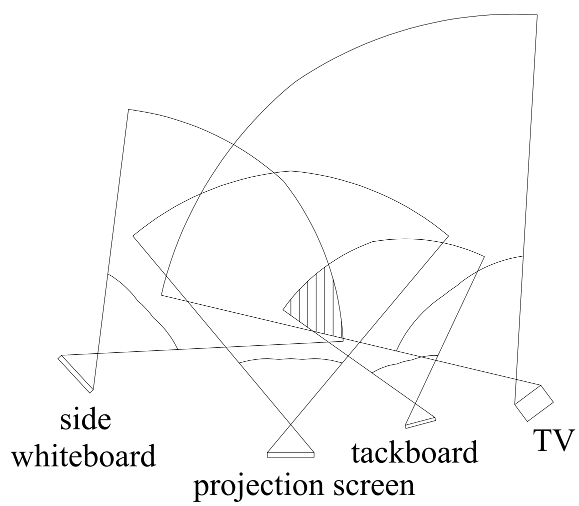


Figure 20. Overlapped ideal viewing area (shaded) of multiple random displays

2.2.1.7 Evaluation of the Plan Shape of Lecture Hall

The ideal viewing areas defined using these empirical guidelines and standards are often compromised by practical considerations when architects determine the plan shape of lecture halls. To evaluate the efficacy of the final plan shape and size of lecture halls in light of ideal viewing, the DIN 108 standard proposed an index η, which is the ratio of usable area to total area, the higher, the better, as shown in (3) (Duncan, 1966). In lecture halls, the usable area refers to the area within the ideal viewing limitations.

$$\eta = \frac{usable\_area}{total\_area} \tag{3}$$

where:

η = Proportion of usable area to total area. The larger, the better.

2.2.2 Ideal Longitudinal Section Profile of Lecture Halls

Lecture halls also have an ideal longitudinal section profile for best viewing materials presented in the front space. The standard DIN 108 recommends the viewing distance range to be between 2H to 6H (H is height of the projection screen) with a vertical viewing angle α within $\pm 30°$ to center line of screen, as illustrated in Figure 21.

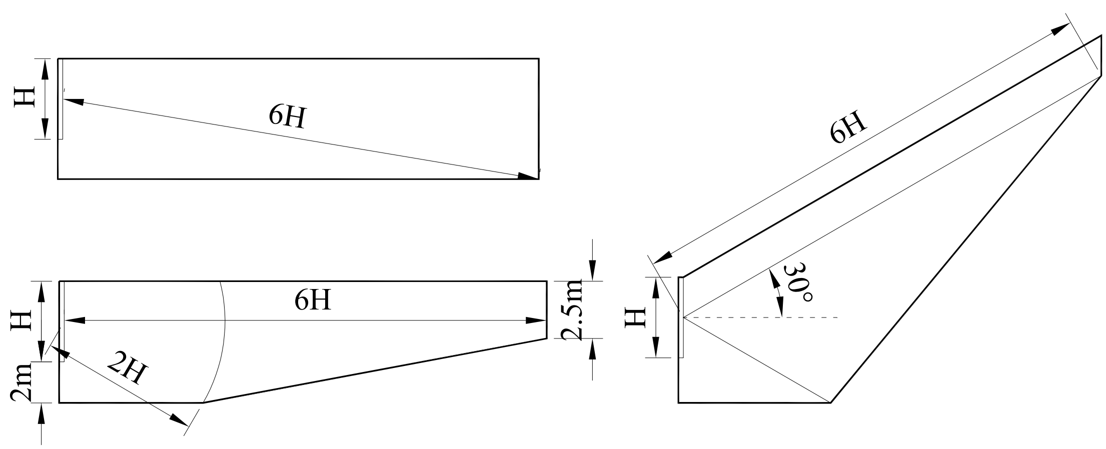


Figure 21. Ideal longitudinal section profiles of lecture halls
(Duncan 1966, Figure 4, p. 18)

For better viewing from the rear seats, the floor of lecture halls with a capacity greater than 100 should be stepped or sloped to some degree (Hauf et al., 1961). Duncan (1966) proposed a mathematical formula (4) for describing the ascending profile for the seat arrangement in lecture halls and other large instructional spaces, as shown in Figure 22.

$$y = \frac{x}{2}\left(Cx^5 - \frac{1}{Cx^5} \right) \tag{4}$$

where:

y = Vertical height to the center point of projection screen

x = Horizontal distance to the center point of screen

C = Constant

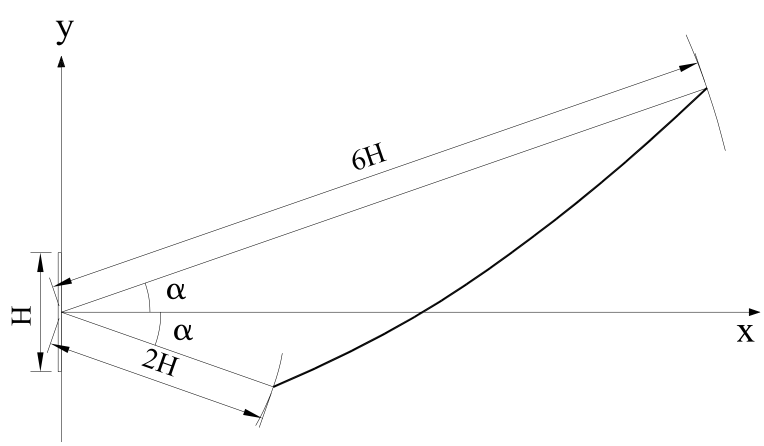
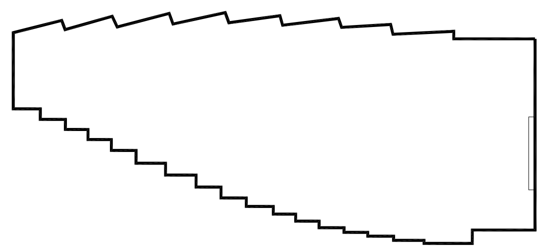
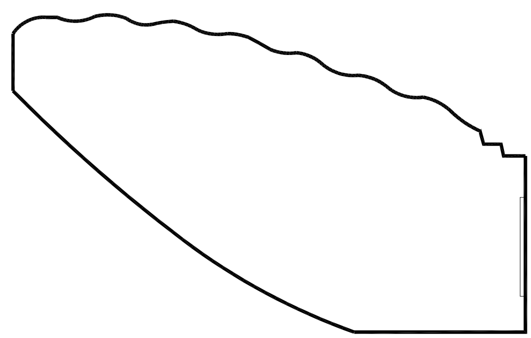


Figure 22. Ascending profile for the seats that provide good viewing for the projection (Duncan, 1966, Figure 9, p. 20)

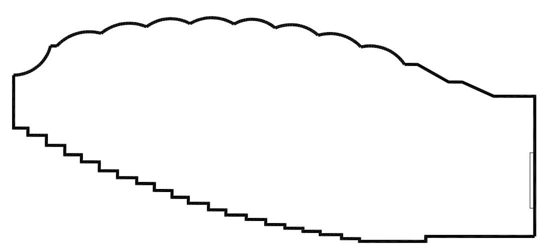
In addition, acoustics also determine the ideal longitudinal section profile of lecture halls. Figure 23 illustrates the typical longitudinal section profiles of lecture halls with a large capacity for obtaining good viewing and acoustical conditions.



(a) Medium-sized lecture halls with stepped profile, volume 1815 m^3 (3.9 m^3 per seat), plan area 360 m^2 (0.7 m^2 per seat) (Duncan, 1966, Figure 12, p. 21)



(b) Section profile for lecture hall with good viewing conditions and favorable projection of sound (Duncan, 1966, Figure 10, p. 21)



(c) Large lecture theatre with stepped profile, volume 6250 m^3 (5.8 m^3 per seat), plan area 800 m^2(0.7 m^2 per seat) (Duncan, 1966, Figure 11, p. 21)

Figure 23. Typical longitudinal section profiles of lecture halls with large capacity to obtain both good viewing conditions and satisfactory acoustical performance (for demonstration only, no dimensions)

In addition, a European Standard ISEN 13200-1-2004 (the English version of German standard DIN 13200-1) has specified that between the eye of a spectator and his focus point P at sport stadiums, sport halls, indoor and outdoor facilities, no constructive obstacle is allowed, as illustrated in Figure 24 and (5) (National Standards Authority of Ireland, ISEN 13200-1-2004, 2004). Although this standard does not cover lecture halls, this rule can still be referred to for sightline construction in lecture halls.

$$D = \frac{a \times B}{(C - 120)} \tag{5}$$

where:

D = Distance recommended from the spectator to the nearest point P of focus (mm)

$A = a + h$, and h = 0 ~ 1000 mm

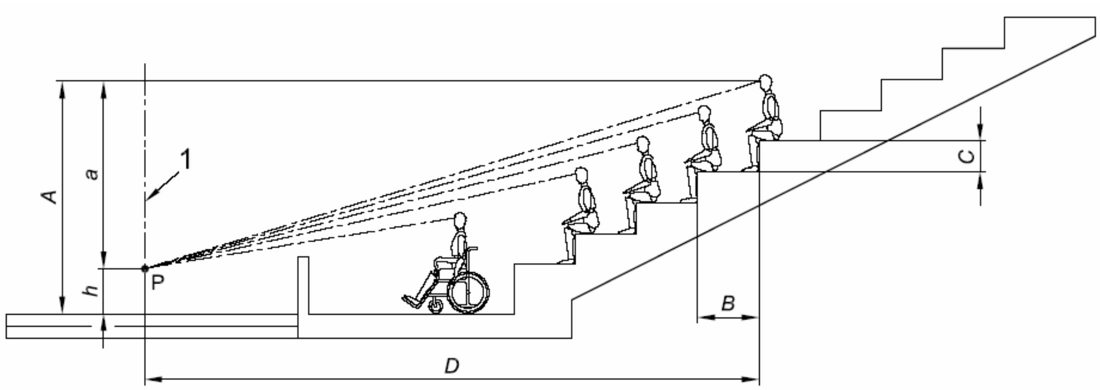


Figure 24. Example of sightline construction
(ISEN 13200-1-2004, Figure 6, p. 14)

2.3 A Review of 95 Legibility Equations

With initial studies appearing over a century ago (e.g., Erdmann, 1898; Scott, 1903; Dearborn, 1906; Dodge, 1907), researchers have thoroughly studied the legibility of Roman characters under a wide range of viewing conditions in such fields as traffic signs, driving interface, electronic displays, instrument panels, safety and security, and wayfinding. Many quantitative studies have proposed equations to predict the legibility of text. This study includes a comprehensive review of the literature on the legibility of Roman characters, to find every equation ever published on that topic in authoritative sources (e.g., books, journal articles, conference papers, and technical reports). A total of 95 equations have been identified and reviewed.

2.3.1 The 95 Legibility Equations

The 95 legibility equations appear in the appendices. Appendix B contains definitions of all terms used in the equations. The actual equations, applicable conditions, units, and other notes are compared in Appendix C. These 95 equations include (1) those that define measures (e.g., visual angle, acuity, legibility index, legibility potential, luminance contrast), or equations to interpret the relationships between these indices, a total of 26 equations, and (2) predictions based on test data in the laboratory or field, a total of 69 regression equations, which have been further categorized into 8 subgroups based on their dependent measure, as shown in Table 1.

In addition, to give readers the whole picture of the historical development of these 95 legibility equations, Figure 25 shows the published years for 77 of the 95 legibility equations for which data was available by 2005. The earliest equations (Equations 86, 87 in Appendix C) appeared in 1925. As shown in Figure 25, research on legibility equations appeared at a fairly stable rate until the late 1960s, after which output increased, with peaks in 1972 (8 equations) and 1976 (9 equations).

Table 1. Classification of 95 legibility equations into 9 categories

Group	Category	Dependent Measure	Number
1	1	Common definitions; relationships between indices	26
2	2	Legible distance; height of text or graphics	22
	3	Visual size; legibility potential	14
	4	Luminance contrast	5
	5	Legibility index	1
	6	Reaction time	10
	7	Exposure or performance time	7
	8	Error rate; percentage of performance	5
	9	Lighting and acuity level	5

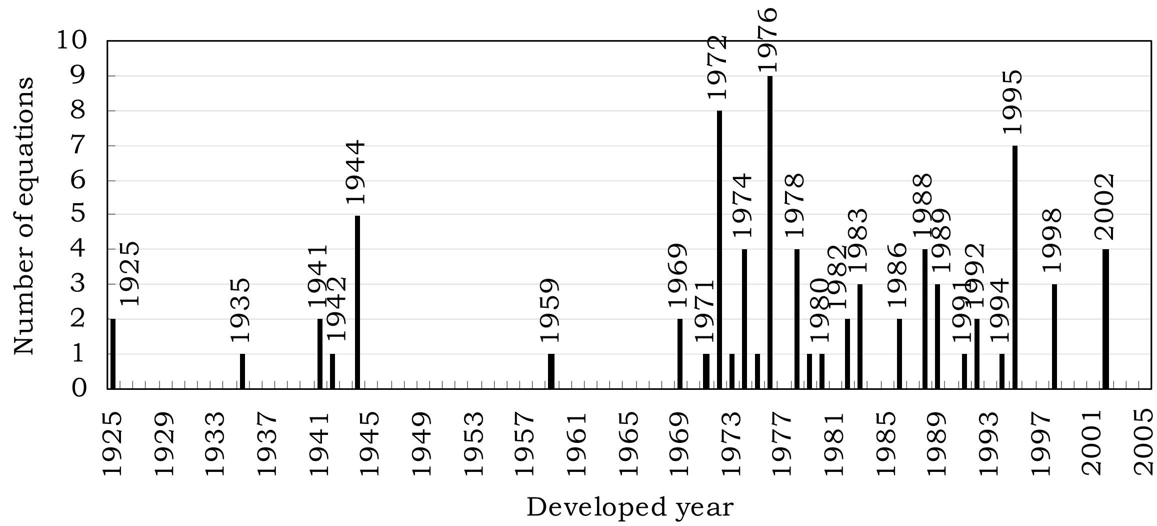


Figure 25. Distribution of 77 legibility equations by year

2.3.2 Critical Variables Explored in the 95 Equations

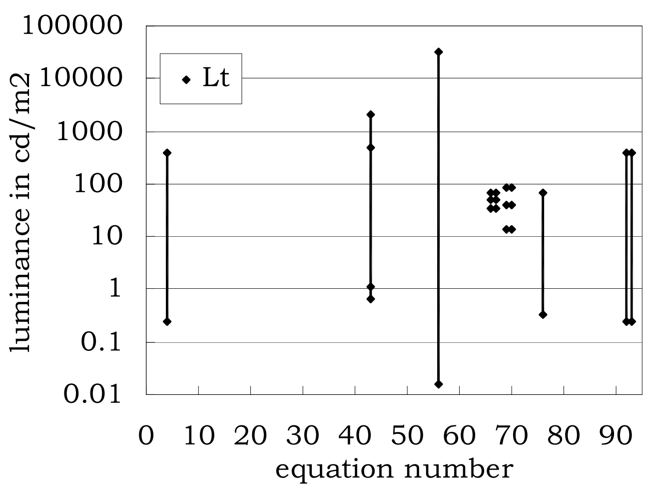
A total of 26 independent factors and 7 dependent factors have been explored in the 95 legibility equations. Among them, 10 critical factors affecting legibility are identified: (a) age and (b) acuity of observer; (c) viewing distance; (d) horizontal and vertical viewing angles; (e) visual angle of text; (f) font; (g) text height, width, and strokewidth; (h) luminance contrast; (i) target luminance, background luminance, and adapting luminance; and (j) color contrast. Theoretically, an ideal legibility equation should holistically examine these 10 critical factors to correctly predict legibility. However, each of the 95 equations has examined at least 2 but at most 7 variables (Equation 85 in Appendix C). Other non-examined factors must be given preset values to delimit the research scope. Values taken for granted, intangible assumptions, and unspecified or not quantified preconditions of non-examined factors, as used by some earlier researchers, would have harmed the validity of the equations they developed.

2.3.3 Explored Range of Viewing Conditions

The ranges of viewing conditions explored in the supporting research studies of the 95 equations, to which they are presumed to apply, are summarized below.

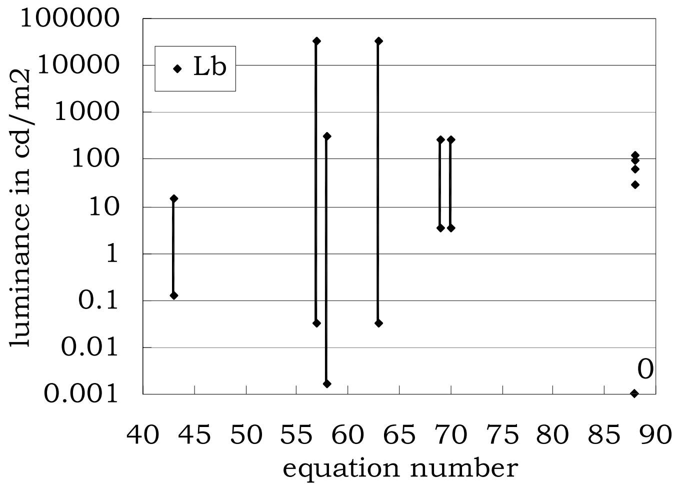
2.3.3.1 Luminance and Illuminance Levels

Good lighting conditions are indispensable for legibility. The supporting research studies of Equations 4, 31-36, 39-40, 43, 56-58, 63, 66, 67, 69-78, 80-85, 88, 91-93, as listed in Appendix C, have quantitatively examined lighting conditions. Figure 26 shows that the target luminance (L_t) explored in these research studies ranges from 0.016 cd/m^2 to 31850 cd/m^2. Likewise, as shown in Figure 27, the explored background luminance (L_b) ranges from 0.0016 cd/m^2 to 31850 cd/m^2. The adapting luminance (L_a) was examined only in Equation 43 (15.42 cd/m^2), and Equations 69, 70 (3.426-34260 cd/m^2).



#	L_t in cd/m^2
4	0.25-400
43	0.65-2055.6, 1.13-479.64
56	0.016-31850
66,67	35, 50, 70
69,70	13.7, 41.11, 82.22
76	0.34-68.52
92,93	0.25-400

Figure 26. Range of target luminance explored in the supporting research of Equations 4, 43, 56, 66, 67, 69, 70, 76, 92, and 93 as listed in Appendix C

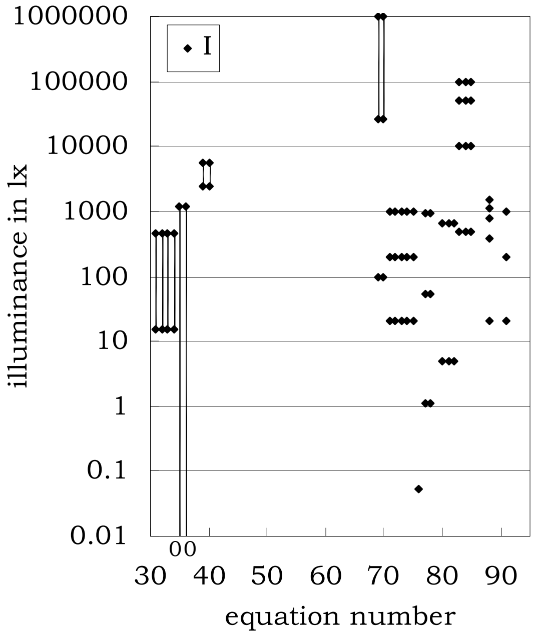


#	L_b in cd/m^2
43	0.127-15.25
57, 63	0.032-31850
58	0.0016-318.5
69, 70	3.426-256.95
88	0, 30, 61, 91, 122

Figure 27. Range of background luminance explored in the supporting research of Equations 43, 57, 58, 63, 69, 70, and 88 as listed in Appendix C

Some research studies explored their lighting conditions in illuminance (I) rather than luminance (L_t, L_b, L_a), probably due to the lack of awareness that luminance rather than illuminance activates the visual perception of text. As shown in Figure 28, illuminance explored in Equations 31-36, 39, 40, 69-78, 80-85, 88 and 91, as listed in Appendix C, ranges from 0 lx (Equations 35, 36) to 1 million lx (Equations 69, 70).

In addition, the supporting research studies of Equations 14, 29, 43, 56, 57, 61, 62, 64, 66-68, 83-85, 89, 90, as shown in Appendix C, have qualitatively described their lighting conditions, as listed in Table 2.



Equation #	I in lx
31, 32, 33, 34	15-450
35, 36	0-1200
39, 40	2500-5500
69, 70	100, 27k-1million
71,72,73, 74, 75	20, 200, 1000
76	0.053
77, 78	1.08, 53.8, 915
80, 81, 82	5, 650
83, 84, 85	500, 10k, 50k, 100k
88	20,390,760,1130,1500
91	20, 200, 1000

Figure 28. Range of illuminance explored in the supporting research of Equations 31-36, 39, 40, 69-78, 80-85, 88 and 91 as listed in Appendix C

Table 2. Qualitative lighting conditions preset in some legibility equations (Green, Goldstein, Zeltner, and Adams, 1988; Forbes, 1969, 1972, 1975; Post, Costanza, and Lippert, 1982; Richardson, 1976)

Equation #	Identification	Lighting conditions	
57	Moon & Spencer, 1944	Target luminance L_t	Black
83,84,85	Snyder & Maddox, 1978		Daylight
89,90,29	Forbes, 1969, 1972	Background luminance L_b	Day, night, summer, winter
66,67	Post et al., 1982		Black
56,64	Shlaer et al., 1942; Moon & Spencer, 1944	Adaptation luminance L_a	Uniform
29,89,90	Forbes, 1969,1972		Dark
14,61,62,68	Richardson, 1976		Low/high beam, daylight
43	Forbes, 1975	Illuminance I	Dark lab, low/high beam outside

In summary, the explored luminance and illuminance levels focus on the magnitude range of 1-1000 (in cd/m^2 or lx). This range covers both mesopic (0.001–3.4 cd/m^2) and photopic ($\geq$3.4 cd/m^2) vision, where a cone-rod breaking effect exists in the transition between them.

2.3.3.2 Luminance Contrast

Luminance contrast has a crucial effect on the legibility of text. Table 3 lists the range of luminance contrast examined in Equations 43, 63, 69, 70, 77, and 78, as listed in Appendix C. In addition, Forbes & Holmes (1939), Kuntz & Sleight (1950), and Zwahlen & Schnell (1995) qualitatively described the luminance contrast in their research studies as positive/negative to develop Equations 49-55 in Appendix C.

Table 3. Explored ranges of contrast in the supporting research (Green et al., 1988; Forbes, 1975; Boreczky, Green, Bos, and Kerst, 1988)

Equation #	Identification	Luminance contrast
63	Hecht, Peskin, and Patt,1935	0.04 - 0.8(C_{min})
43	Forbes,1975	30%,50%,80%; 3.1-25.1(C_r)
69, 70	Rogers, Spiker, and Cicinelli,1986	2, 4, 8(exp.1), 1.2-7.5(exp.2,3)
77, 78	Boreczky, Green, Bos, and Kerst,1988	1.5, 2.4, 20

2.3.3.3 Viewing Angles

Although text in practice is commonly viewed not perpendicular to the viewer, this situation has rarely been examined in the literature. Only Reinwald (pre-1980), Snyder & Maddox (1978), and Payne (1983) examined viewing angles, as shown in Table 4. Snyder & Maddox, and Payne did not propose any equations, but Reinwald developed the famous Reinwald formula (Equation 27 in Appendix C).

Table 4. Explored horizontal and vertical viewing angles (Shurtleff, 1980; Reger, 1989; Green et al., 1988)

Equation #	Identification	Horizontal ϕ	Vertical α
27	Reinwald, before 1980	± 0°-90°	± 0°-90°
83,84,85	Snyder and Maddox, 1978	0°, ±45°	0°, -15°
88	Payne, 1983	0°, 15°, 30°, 45°, 60°	

2.3.3.4 Spectral Effect

Only Moon & Spencer (1944) and Hecht (1935) examined the effect of the light spectrum (incandescent lamps, red light) on legibility and developed Equations 57, 58, 63, as listed in Appendix C. Allen et al. (1966) also examined the spectrum of fluorescent lamps but did not develop any equation. Other HID (High Intensity Discharge) lamps were rarely examined before.

2.3.3.5 Color Contrast

Color contrast has been proven effective on legibility by Richardson (1976), Forbes (1975), Moon & Spencer (1944), and Boreczky et al. (1988), as listed in Table 5. No typical color contrast was used in the supporting research studies of these equations.

Table 5. Explored color contrast in the previous legibility equations

Equation #	Identification	Color contrast (ΔE)
57	Moon & Spencer, 1944	Black/White
43	Forbes, 1975	White/Green; Black/Yellow
14, 61, 62, 68	Richardson, 1976	Black/Orange; Black/Silver; Black/Yellow; Blue/Silver; Red/Silver
77, 78	Boreczky et al., 1988	White/Black, Yellow/Black, White/Blue, Green/Black, Blue-Green/Black

2.3.4 Intended Applications of the 95 Equations

The intended applications of the 95 legibility equations depend on how they were developed. Since Equations 1-25, as listed in Appendix C, are common definitions or mathematically derived, they are believed generally applicable to any field. Equations 37 and 45 in Appendix C (the Bond's rule) are used as a rule of thumb for rough estimation in many viewing situations. In contrast, the application of the 69 regression equations might be constrained to their developing situations. For example, Equation 94 was developed when $1 \leq L_b \leq 1000$ cd/m^2; $10 \leq C_{\%} \leq 90$; $0.2 \leq Ac \leq 2.0$ min^{-1}. Thus, Howett's equation (Equations 47, 48, and 95 in Appendix C), which was derived from Equation 94, would have better applied in those conditions. Appendix C lists the detailed applications of all 69 regression equations. In addition, the majority of the 95 legibility equations deal with suprathreshold performance, and are thus also applicable only in suprathreshold viewing situations. Only Equations 86 and 87 have examined threshold performance; their detailed applicable conditions are listed in Appendix C.

Different viewing media were also used to develop equations, including roadway signs, architectural signs, vehicle instrument panels, dot matrix displays, electronic displays such as LCD, LED, CRT. Each equation is supposed to be used for the same type of visual media on which it was developed. For example, Equations 4, 5, 39-44, 49-55, 61, 62, 65, 79, 89, 90, 92, and 93 in Appendix C were developed based on roadway signs, highway signs, and traffic signs, and should thus be applied in traffic situations. Table 6 lists the equations with their viewing media. Some equations might be carefully extended to similar visual media used to develop them. In terms of viewing angles, only Equations 18-25, 66-67, and 88, as listed in Appendix C, are applicable to common not-perpendicular-to-the-display viewing situations. Among them, the Reinwald formula (Equation 27 in Appendix C) is the only one to date that examines the effect of both horizontal and vertical viewing angles on legibility distance. However, the Reinwald formula does not examine other critical factors that are usually considered, such as lighting and contrast, acuity of observers. Its application is thus limited in practice.

Table 6. Different types of viewing media examined in supporting research

Equation #	Viewing media
4, 5, 39-44, 49-55, 61, 62, 65, 79, 89, 90, 92, 93	Roadway signs, traffic signs, reflective signs
26, 29, 37	Outdoor signs
6, 35, 36, 71-78, 91	Alphanumeric information signs, instrument panels, numerical speedometers
41, 79-81, 83-85	Matrix pixels, dot matrix
30-34, 70, 80-82, 88	Electronic displays: LCD, CRT
45	Newsprints, magazine ads, letterheads, etc. in room
63, 64	Large contiguous surfaces with uniform surround luminance
23, 24	Uniform color spaces
66, 67	Colored patches on video displays
56, 57, 60	Landolt rings
14, 58, 59	Bars
38	Neon advertising
42	Blur techniques of symbol highway signs

The majority of the 95 legibility equations have quantitatively specified their viewing conditions where they are supposed to be applied. However, Equations 13, 30-36, 40, 56, 57, 63, 64, as listed in Appendix C, have only intangibly described their viewing condition as optimum, excellent, preferred, or no-error. Such vague viewing conditions must be used as the preconditions to apply these equations in practice to ensure they are applicable. Moreover, nearly all of the 95 equations assume materials are viewed without error. Thus, they are not applicable for examining the error rates of imperfect viewing performance. Only Equations 59 and 60, as listed in Appendix C, can be used for this purpose.

2.3.5 Available Equations to Predict Spatial Legibility of Text

The Reinwald formula (6) is the only one to date to predict the spatial legibility of text. However, since the Reinwald formula does not examine critical factors such as geometries, target and background luminance, contrast, acuity of observers, color, its accurate prediction of legible text in practice is handicapped, particularly in cases that include contrast, illumination level, and letter geometry.

$$D = \begin{cases} D_0(\cos\phi)^{2/3} & \textit{horizontal} \\ D_0(\cos\alpha)^{1/2} & \textit{vertical} \end{cases} \tag{6}$$

where:

D = Viewing distance from observer to the display viewed at angles

D_0 = Viewing distance from observer to the display viewed perpendicularly

ϕ = Horizontal viewing angle

α = Vertical viewing angle

In addition, among the 95 legibility equations, Howett's equation (7) is probably one of the best equations for predicting the legibility of text viewed perpendicularly, for two reasons. First, this equation examines the maximum number of parameters, including geometries, background luminance, and contrast of text, and the Snellen eyesight of the observer. Second, Howett's equation (1983) was mathematically derived from Kaneko's equation (8) (Kaneko and Ito, 1978, cited by Howett, 1983).

$$H = 4.1 \times 10^{-4} \times D \cdot \left(\frac{H}{Sw} \right) \cdot S_d \cdot \left(L_b \right)^{-0.213} \cdot C_{\%}{}^{-0.532} \tag{7}$$

where:

H = Character height

D = Legibility distance

Sw = Strokewidth of character, or detail of graphics

S_d = Denominator in the Snellen ratio

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

$$A_c = 0.06298 L_b{}^{0.213} C_{\%}{}^{0.532} \tag{8}$$

where:

A_c = Visual acuity, the reciprocal of the finest legible detail

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

Neither Kaneko's equation nor Howett's equation examines the ambient light of the viewing environments, even though ambient light has been proven effective on visual perception. In addition, Howett's equation examines only single letter, A~Z, and is thus not applicable to words or sentences. Furthermore, the aging effect on the legibility of text is believed to be indirectly evaluated in Howett's equation through the parameter of acuity level, since aged people usually have decreased eyesight. Using this parameter, Howett's equation is able to examine all levels of eyesight.

2.4 Summary

Since the emergence of lecture halls in the 1960s, architects have had to accurately and conveniently define ideal viewing areas in order to appropriately arrange seats in the audience area. The empirical experience gained in practice have been summarized into architectural guidelines for architects to define the ideal viewing area as a fan-shaped plan and the optimum shapes of longitudinal sections. Although these empirical guidelines are convenient to use and have proven useful in practice, they have never incorporated a scientific and quantitative index, which is the spatial legibility of text. Ideal viewing areas in modern lecture halls could be precisely found by using legibility equations to calculate the spatial legibility of text presented on multiple displays mounted at different locations with different geometries, mounting heights, orientations, and lighting conditions.

Through a comprehensive review of the literature on the legibility of Roman characters, all of the 95 legibility equations ever published have been identified and reviewed. Ten critical factors for determining the legibility levels of text viewed in lecture halls have been identified as (a) age and (b) acuity of observer; (c) viewing distance; (d) horizontal and vertical viewing angles; (e) visual angle of text; (f) font; (g) text height, width, and strokewidth; (h) luminance contrast; (i) target luminance, background luminance, and adapting luminance; and (j) color contrast. By reviewing the 95 legibility equations, also in light of the fundamentals of visual perception of text, a good legibility equation for calculating the ideal viewing areas in lecture halls should be able to predict the legible size or distance of text viewed at different angles, lighting levels, by examining all critical factors.

Among the 95 equations, only the Reinwald formula can be used to predict the legible distances of text viewed from different viewing angles. Unfortunately, this equation does not examine other critical factors that are usually examined, and thus is insufficient for determining the ideal viewing areas in lecture halls. On the other hand, Howett's equation, which was mathematically derived from Kaneko's equation, examines the most number of critical factors, a total of 6, but not viewing angles.

CHAPTER 3

Research Problems and Steps to Their Solution

The goal of this study is to provide solutions for two principal research demands. First, architects need to determine ideal viewing areas in modern lecture halls, where observers can read text simultaneously presented on multiple visual media mounted at different locations with different geometries, orientations, and lighting conditions. Such complicated viewing situations require that architects ensure that every single seat in the audience area, particularly those in the back row and off axis, has a very good view (with enough legibility of the text presented on all displays). However, as reviewed in Chapter 2, the conventional guidelines that architects have been using for decades cannot guarantee that because the guidelines were developed based not on a legibility index but on experience, making them too imprecise for finding an overlapped ideal viewing area in modern lecture halls. Therefore, a new reliable and quantitative design method is necessary for better accuracy and flexibility.

Second, the expected quantitative design method for determining ideal viewing areas should be undertaken in light of the spatial legibility of text viewed in modern lecture halls, calculated using legibility equations. As reviewed in Chapter 2, a total of 95 legibility equations have been commonly used in the human factors and engineering fields. Thus far, however, architects have rarely used them to design buildings with enhanced legibility, especially lecture halls, although "spatial legibility in architecture" as a topic for wayfinding has been dealt with for years (e.g., Watanabe, A., 1999, A Study on the Spatial Legibility in Architecture by Wayfinding Experiment, which won the AIJ (Architectural Institute of Japan) Prizes 1999). To meet these two research demands, this study will develop a preliminary computation-program-aided design method for solving three major research problems identified in the literature review of Chapter 2, as detailed below.

3.1 Research Problems

3.1.1 Problem 1: Architects Lack a Quantitative Method for Finding Ideal Viewing Areas

Research problem 1: Architects lack a quantitative method for precisely defining ideal viewing areas in modern lecture halls, taking into account the spatial legibility of text.

What is an ideal viewing area of text presented in lecture halls and how should it be defined? When text is viewed at different angles by observers sitting on a viewing plane, for example, horizontal or vertical, a two-dimensional ideal viewing area of text is shaped by the gradually decreased legibility distance (the distance from the observer's eye to the legible text) at increasing viewing angles $\pm 0° - 90°$, as illustrated in Figure 29.

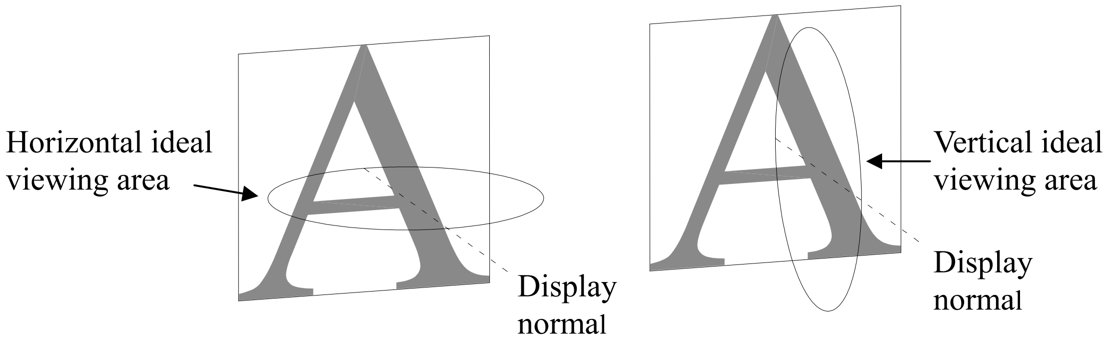


Figure 29. Horizontal and vertical ideal viewing area of text
(for illustrative purposes, not to scale, no units)

In reality, however, text is commonly viewed by observers randomly sitting in a three-dimensional space at various incident angles between the display normal and the sightline of the observer, rather than strictly on a plane. The incident angle in 3D viewing is jointly determined by both horizontal and vertical viewing angles. For such not-perpendicular-to-the-display viewing of a single text, there is a 3D ideal viewing sphere, as illustrated in Figure 30. This ideal viewing sphere is actually the spatial distribution of the legibility distances of single text along three dimensions. Although the 3D ideal viewing sphere gives architects a good feeling about its size and shape, it is rather impractical for architects to use directly in the drawings of lecture hall design. Architects tend to prefer an ideal viewing area along the viewing plane at eye height level parallel to the sloped floor for seating arrangements. Architects may also demand ideal viewing areas along other viewing planes, for example, the vertical plane for the ideal section profile or the horizontal plane for the ideal plan shape of modern lecture halls. Geometrically, such an ideal viewing area along a specified viewing plane can be obtained by slicing the 3D ideal viewing sphere with the viewing plane, as illustrated in Figure 31.

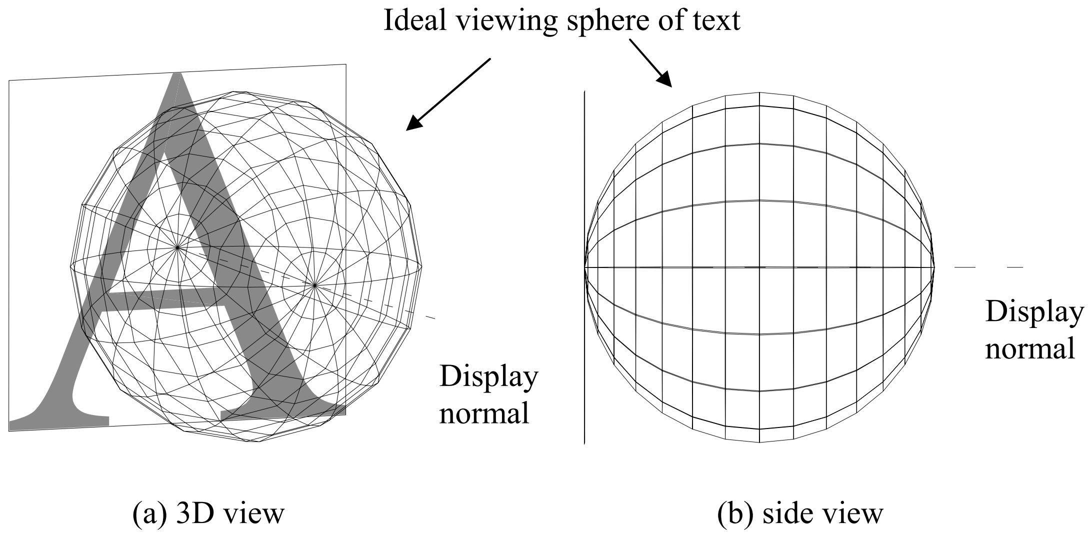


Figure 30. Three-dimensional ideal viewing sphere of a single text
(for illustrative purposes, not to scale, no units)

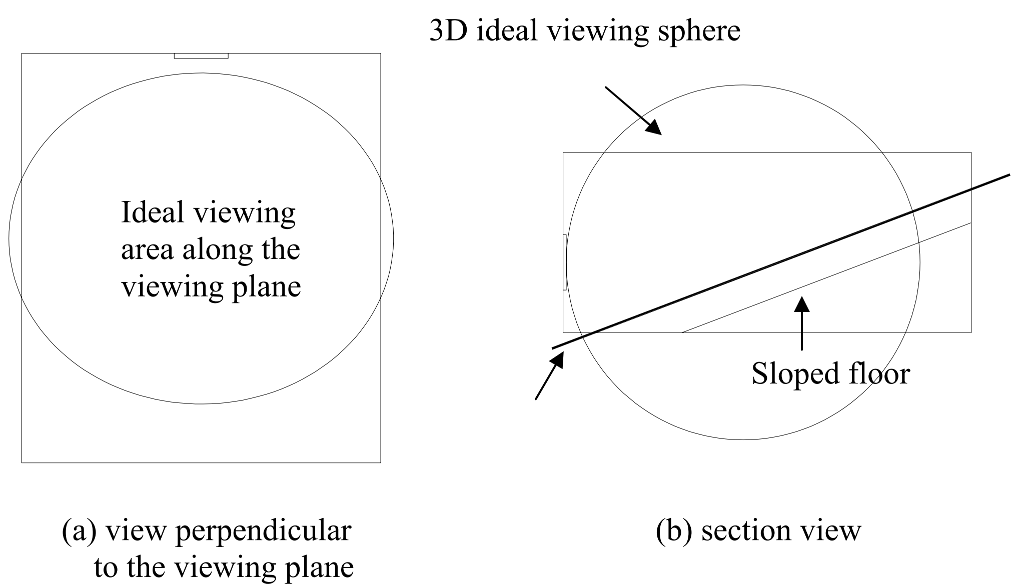


Figure 31. Ideal viewing area of a single text along the viewing plane at the observer's eye height parallel to the sloped floor in lecture hall, obtained by slicing the 3D ideal viewing sphere with this viewing plane
(for illustrative purposes, not to scale, no units)

For the same text viewed under the same viewing conditions, in contrast to its constant 3D ideal viewing sphere, the shape and size of its 2D ideal viewing area varies with the location and orientation of the viewing plane. In lecture hall design, fortunately, architects usually require that the ideal viewing area be along the viewing plane parallel to the sloped floor at the observer's eye height, as shown in Figure 31.

Then how can this ideal viewing area in lecture halls be determined? This ideal viewing area is shaped by slicing the 3D ideal viewing sphere of text with the specified viewing plane at the observer's eye height parallel to the sloped floor, as shown in Figure 31. Apparently, this task goes beyond the conventional architectural guidelines summarized in Chapter 2.2, for three reasons. First, these empirical guidelines are not based on the spatial legibility of text but on experience, and are thus not quantitative and sufficiently precise. For example, these guidelines give the same legibility distance of a single text viewed at angles $\pm 30°$-$60°$, as shown in Figure 15, which is clearly different from the prediction using legibility equations, such as the Reinwald formula, (6) previously. Second, these guidelines assume the viewing plane is either horizontal or vertical; the viewing plane parallel to the sloped floor at eye height level as specified in lecture halls is not considered. Third, except for the screen width, viewing distance, and viewing angles, these guidelines do not consider other critical factors that are usually considered for legibility, e.g., lighting, acuity. The inability of these guidelines is more obvious in modern lecture halls where text is presented on multiple visual media having different locations, geometries, orientations, and lighting conditions. Therefore, architects need a quantitative and reliable method for precisely defining ideal viewing areas of modern lecture halls, in light of the spatial legibility of text. This quantitative method should use an equation as the underlying algorithm that predicts the spatial legibility of text from all critical factors that are usually considered.

3.1.2 Problem 2: No Equation Predicts Spatial Legibility of Text from 7 Factors

Research problem 2: The equation that predicts the spatial legibility of text from all seven critical factors is not available in the literature.

Recall from Chapter 2, there are ten critical factors that could be examined by the demanded legibility equation to predict the ideal viewing areas of text in lecture halls. Restricted by the ten assumptions used in this study, seven critical factors still remain, including (a) acuity of observer; (b) viewing distance; (c) viewing angles; (d) visual angle of text; (e) text height, width, and strokewidth; (f) luminance contrast; and (g) target luminance, background luminance, and surrounding luminance. However, no legibility equation has ever appeared in the literature to predict the spatial legibility of text based on all seven critical factors.

As thoroughly reviewed in Chapter 2, the majority of these legibility researchers assumed in their studies that materials were viewed perpendicularly. Only Reinwald (pre-1980) and Payne (1983), as cited by Shurtleff, 1980; Reger, 1989; and Green et al., 1988, examined the common not-perpendicular-to-the-display viewing conditions. Among the 95 legibility equations, the Reinwald formula, (6) previously, is the only one that examines the spatial legibility of text. This equation examines only legibility distance and horizontal and vertical viewing angles; other critical factors, such as luminances, contrast, text size, are not considered. Therefore, its predictability of the spatial legibility of text in modern lecture halls is handicapped, where all seven critical factors need to be holistically examined for accuracy. Consequently, a new equation is needed to predict the spatial legibility of text based on all seven critical factors.

3.1.3 Problem 3: Ambient Light Is Not Examined In the Derived Equation

Research problem 3: The derived equation for predicting the spatial legibility of text fails to examine the surrounding luminance of the ambient environments.

The required equation for predicting the spatial legibility of text based on all seven critical factors will be derived later in this study based on the existing Howett's equation (1983), as (7) previously. However, Howett's equation does not examine the surrounding luminance (L_s) of the ambient environments, even though ambient light has been proven effective nowadays on visual perception, as reviewed in Chapter 2. Nevertheless, it is not yet clear whether the surrounding luminance (L_s) has a large effect on the legibility of text, when its background luminance and luminance contrast remain constant. If the surrounding luminance affects legibility of text, then its absence in the equation to be derived might seriously harm its accurate prediction of the spatial legibility of text viewed in modern lecture halls. Otherwise, its absence is tolerable.

3.2 Steps to Solve These Problems

Based on the ten assumptions, this study has four steps to take in solving the three research problems.

Step 1: Survey of lecture halls. This study will first carry out a field survey of a total of 38 lecture halls at the University of Michigan. The purpose is to find the typical viewing conditions in lecture halls where the next 3 steps will be carried out within those conditions, to enhance their external validity in lecture halls. Variables to be measured in the field include (a) maximum viewing distance, (b) maximum horizontal and vertical viewing angles, and (c) typical range of background luminance of visual media and surrounding luminance of the adjacent viewing environments.

Step 2: Derivation of a new legibility equation for text and its verification. Based on a hypothesis, this study will then derive the required new equation that predicts the spatial legibility of text from the seven critical factors, except surrounding luminance, from the existing Howett's equation (1983). The following task is to verify the hypothesis using (a) fundamental theories how retinal images of text activate cones in the fovea of the viewer's eyes, and (b) legibility data collected from human subjects participating in a pilot experiment followed by a main experiment to be carried out in a lighting laboratory at the University of Michigan Transportation Research Institute (UMTRI).

Step 3: Testing ambient light effect on legibility. Theoretically, ambient light may contribute only little to the adaptation luminance of text viewed in lecture halls. Thus, the surrounding luminance might have little effect on the legibility of text when the background luminance and contrast of the text are kept constant. This is the second hypothesis used in this study. This hypothesis will be tested using legibility data collected from human subjects participating in a third laboratory experiment to be carried out at UMTRI.

Step 4: Development of a computation program and its application in lecture halls. After the derived equation that predicts the spatial legibility of text from all seven critical factors has been verified, it will then be used to develop a computation program in MatLab. This program will calculate an overlapped ideal viewing area of text simultaneously presented on multiple displays, which are mounted at different locations, with different geometries, orientations, and lighting conditions, along any viewing plane. This program can be used in any fields, such as architecture, transportation, advertisement, electronic displays, where reading text at different viewing angles is important. This program is then applied in modern lecture halls to develop a computation-program-aided design method for architects to determine an overlapped ideal viewing area of multiple displays at the observer's eye height level along the sloped floor. To verify the external validity of this method, a field experiment will be carried out using human subjects in the lecture hall in the Art & Architecture building at the University of Michigan.

3.3 Summary

Using ten assumptions, this study will solve three problems, including:

1. Architects lack a quantitative and reliable method to precisely determine ideal viewing areas of lecture halls, in light of the spatial legibility of text.
2. The equation that predicts the spatial legibility of text based on all seven critical factors is not available in the literature. A new equation is needed.
3. The equation to be derived from the existing Howett's equation (1983), using a hypothesis, does not examine the surrounding luminance of the ambient environment. The influence of the surrounding luminance on legibility of text is not clear, however, when the background luminance and contrast of text viewed remain unchanged.

This study will use four steps to develop a computation-program-aided design method for architects to determine an overlapped ideal viewing area of text presented on multiple displays installed in modern lecture halls.

Step 1: Field survey of a total of 38 lecture halls at the University of Michigan, to find the typical viewing conditions in lecture halls where the next 3 steps will be carried out under those conditions.

Step 2: Derivation of a new legibility equation that predicts the spatial legibility of text based on seven critical factors, except surrounding luminance, from the existing Howett's equation (1983), and verification of the hypothesis used for this derivation, using fundamental theories and laboratory experiments.

Step 3: Testing the ambient light effect on legibility using a third laboratory experiment.

Step 4: Development of a computation program in MatLab, and its application in lecture halls, to calculate an overlapped ideal viewing area of text simultaneously presented on multiple displays, and verification of this computation-program-aided design method using a field experiment.

CHAPTER 4

Step 1: Survey of Lecture Halls

As the first step in solving these research problems, this study measures the viewing conditions of text presented in a total of 38 lecture halls at the University of Michigan. The purpose of this survey is to determine a boundary of typical viewing conditions of text presented in lecture halls, including (a) the maximum viewing distance D_{max} from the observer's eye to the displays, (b) the maximum horizontal viewing angle ϕ_{max} and the maximum vertical viewing angle α_{max}, which are then used to calculate the maximum incident angle ξ_{max} between the observer's visual line and the display normal, (c) the typical range of background luminance (L_b) of text, that is, surface luminance of the visual media, and (d) the typical range of surrounding luminance (L_s) of text. To strengthen their outcomes, the viewing conditions in all experiments to be carried out later must match these surveyed ranges.

4.1 Lecture Halls Selected for Survey

The surveyed 38 lecture halls are carefully chosen from the list on the University of Michigan Postsecondary Education Facilities Inventory & Classification Manual, using five standards listed below:

1. Large capacity ($\geqslant$75 seats). Specifically, small lecture halls have 75-149 seats. Large lecture halls have 150-299 seats. Lecture theatres have $\geqslant$300 seats.
2. Teaching-learning activities inside that are not tied to a specific subject or discipline, where the audience focuses their vision on some focal points (visual materials) in the front space.
3. At least two visual media are used inside, including blackboard, chalkboard, marker board, tack board, projection screens, or TV monitor.
4. Fixed and compact seating arrangement.
5. Sloped or tiered floor, or flat floor (in small lecture halls).

Appendix D lists the 38 lecture halls in terms of their capacity, room area in square feet, campus location, number of visual media, and year built. By capacity, the 38 lecture halls include 16 small lecture halls, 15 large lecture halls, and seven lecture theatres, as shown in Appendix D. Among the 38 lecture halls, 24 have two visual media installed, most are blackboard/whiteboard and projection screen, while nine lecture halls use three visual media, and five lecture halls use four media. The years in which 35 of the 38 surveyed lecture halls were built are known. Among them, two lecture halls were built before 1900, 12 lecture halls were built from 1915 to 1958, while 20 lecture halls were built from 1960 to 1991; only one lecture hall was built after 2000, in 2004. Therefore, the ranges of viewing conditions surveyed in the 38 lecture halls are sufficient for this study to find the general viewing conditions in typical lecture halls.

4.2 Procedure and Equipments

Four steps are used to field measure the typical viewing conditions of text presented in the 38 lecture halls.

Step 1. Find the number and type of visual media installed in the lecture hall, which will determine how many different instructional methods, and correspondingly, different viewing conditions, might have been used.

Step 2. Measure the maximum viewing distance D_{max} from the most off-axis back row seat to the far end of the visual media using a 300 ft fiber glass tape, as shown in Figure 32. Then take pictures from this seat location to demonstrate the real view of these displays at the maximum viewing distance.

Step 3. Find the most off-axis seat(s) where text presented on the far end edge of the visual media are viewed at the maximum incident angle ξ_{max}, and then measure the geometrical relationships between this seat(s) and the visual media, to calculate the maximum horizontal viewing angle ϕ_{max} and the maximum vertical viewing angle α_{max}. In addition to the 300 ft fiber glass tape, a 50 ft sonic laser tape is used to facilitate measuring heights, as shown in Figure 32. Finally, take pictures at this seat(s) location to show the real off-axis view of displays at the maximum viewing angle ξ_{max}.

Figure 32. A 300 ft fiber glass tape and a 50 ft sonic laser tape to measure distance, and a recently calibrated Minolta LS-100 luminance meter to measure the brightness

Step 4. Turn on different lighting conditions used for different instructional modes, one by one (e.g., blackboard mode versus projection screen mode). Under each of these lighting conditions, measure the minimum and maximum surface luminance of the visual media, that is, the background luminance (L_b) of text presented, using a recently calibrated Minolta LS-100 luminance meter, as illustrated in Figure 32, by following the measurement pattern illustrated in Figure 33. Then measure the minimum and maximum surrounding luminance (L_s) of text using another measurement pattern shown in Figure 34. Finally, take pictures of the lecture hall interior under different viewing conditions.

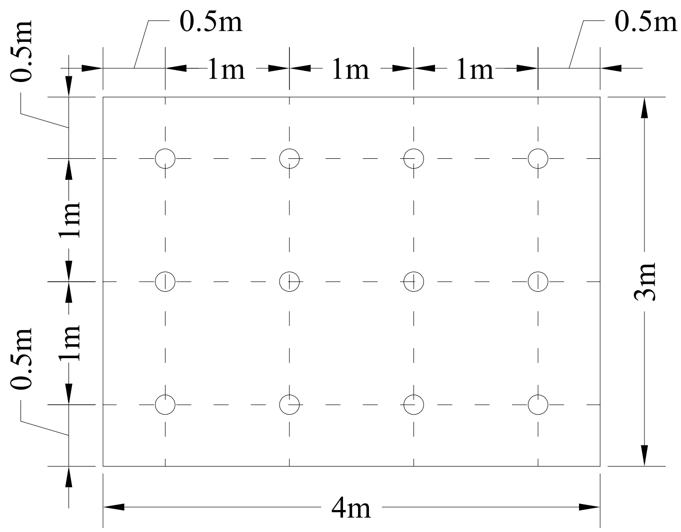


Figure 33. Example pattern for measuring the background luminance (L_b) of text presented on a display that is 4 m wide by 3 m high

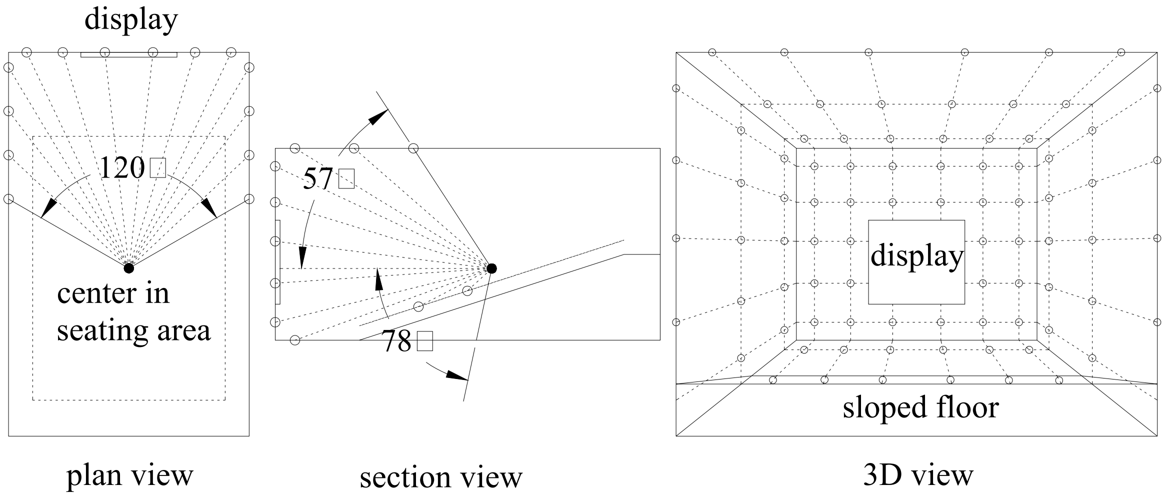


Figure 34. Example pattern for measuring the surrounding luminance (L_s) of text presented on a display that is 4 m wide by 3.5 m high, measuring points are distributed inside the viewing field (120°×135°) of the observer's binocular eyes

The survey usually takes 30 to 60 minutes per lecture hall when it is unoccupied. Unfortunately, most of the 38 lecture halls are occupied most of the day, thus, requiring a much longer time for surveying each one.

4.3 Viewing Conditions Surveyed in the 38 Lecture Halls

The surveyed data, including (a) the maximum horizontal viewing angle ϕ_{max}, (b) the maximum vertical viewing angle α_{max}, (c) the maximum incident angle ξ_{max} calculated from ϕ_{max} and α_{max} using $\cos\xi_{max} = \cos\phi_{max} \cdot \cos\alpha_{max}$, (d) the maximum viewing distance D_{max} from the observer's eye to the displays, (e) the minimum and maximum background luminance (L_b) of text, and (f) the minimum and maximum surrounding luminance (L_s) of text, are summarized in Appendix E.

Figure 35 illustrates the surveyed maximum viewing distance D_{max} in the 38 lecture halls, at a range of 9.96 m – 25.74 m with a mean value of 15.50 m. Ideally, the field experiment (full scale) to be carried out later should not overrun this range of maximum viewing distance. However, the maximum viewing distance in the scaled laboratory experiments to be carried out later with size-reduced simulated visual media need not stick to this range (9.96 m – 25.74 m with mean 15.50 m), since the maximum viewing distance D_{max} largely depends on the size of the visual media that allows the largest size of text to be presented on.

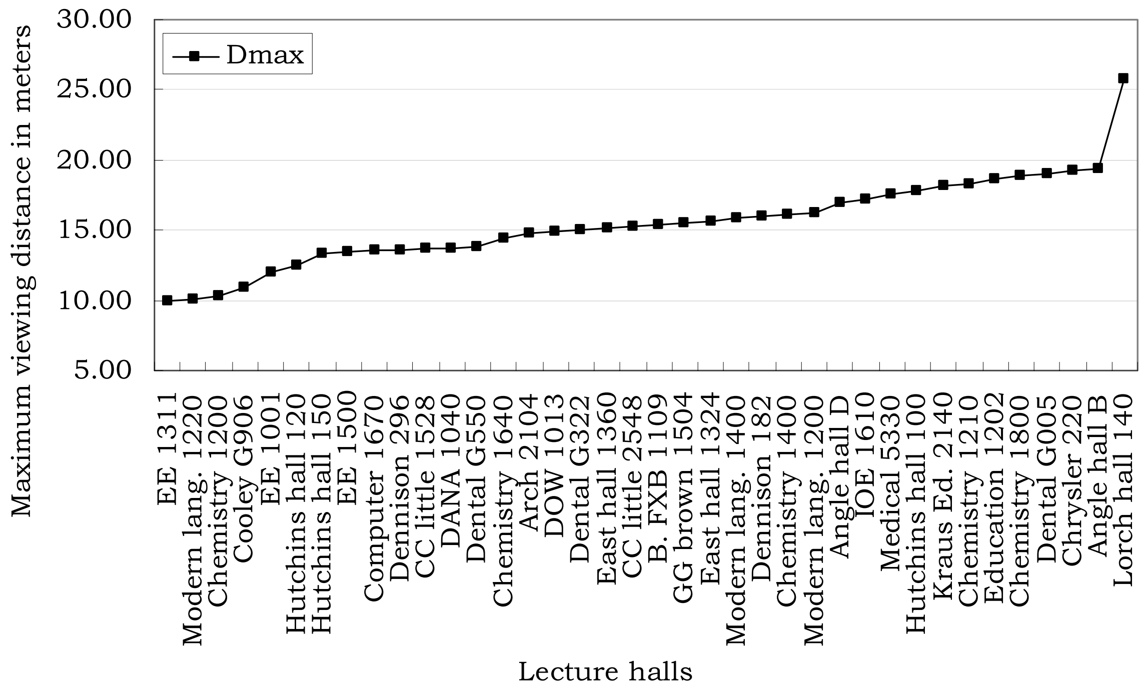


Figure 35. Surveyed maximum viewing distance D_{max} in all 38 lecture halls, at a range 9.96 m -25.74 m with an average 15.50 m

For viewing angles, Figure 36 illustrates the maximum horizontal viewing angle ϕ_{max} and the maximum vertical viewing angle α_{max}, which are calculated using the geometrical relationships measured in the 38 lecture halls. As shown in Figure 36, the maximum horizontal viewing angle ϕ_{max} ranges from 43.5° to 80.4° with a mean value of 64.1°, while the maximum vertical viewing angle α_{max} ranges from 22.2° to 68.5°, with a mean value of 43.6°. Therefore, observers in the 38 lecture halls usually have wider horizontal viewing fields than vertical ones. Ideally, the laboratory and field experiments to be carried out later should have their viewing angles (1) $\alpha_{max} < \phi_{max}$, and (2) fit in the ranges of $43.5° \leq \phi_{max} \leq 80.4°$ and $22.2° \leq \alpha_{max} \leq 68.5°$. In addition, for the maximum incident angle ξ_{max}, calculated using $\cos\xi_{max} = \cos\phi_{max} \cdot \cos\alpha_{max}$, Figure 36 shows its range in the 38 lecture halls as $48.3° \leq \xi_{max} \leq 86.5°$, with a mean value of 71.6°.

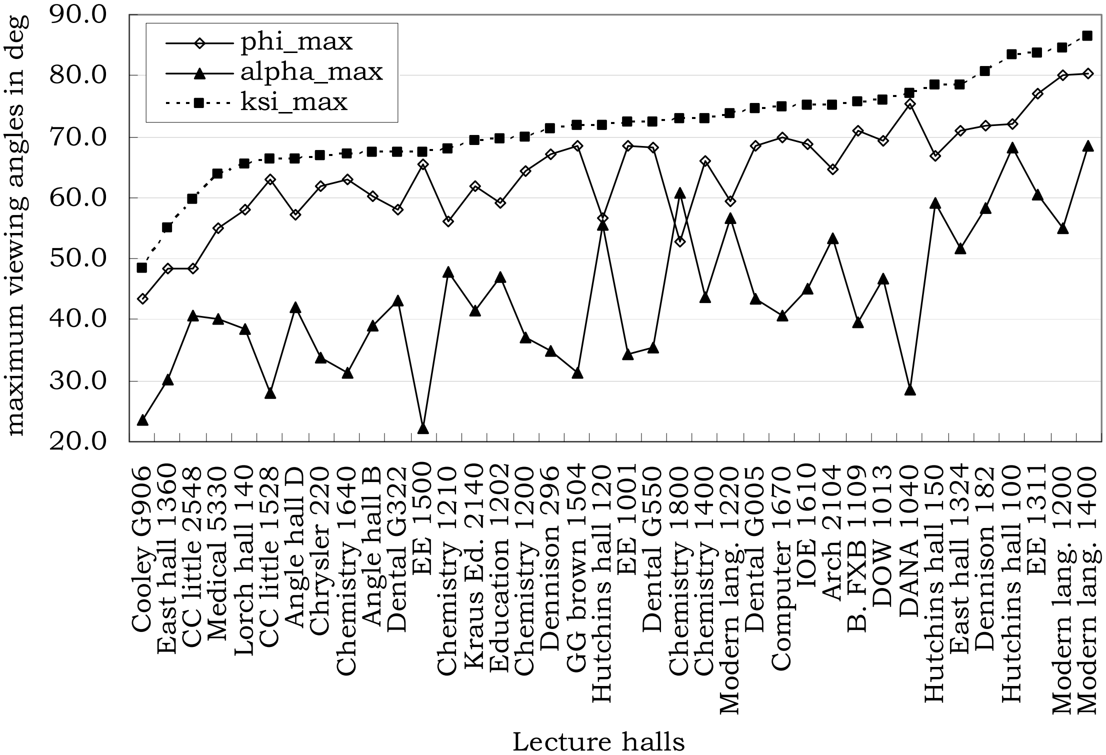


Figure 36. Surveyed maximum horizontal viewing angle ϕ_{max}, maximum vertical viewing angle α_{max}, and maximum incident angle ξ_{max}, $\cos\xi_{max} = \cos\phi_{max} \cdot \cos\alpha_{max}$, in all 38 lecture halls, usually $\alpha_{max} < \phi_{max}$, $48.3° \le \xi_{max} \le 86.5°$, with a mean value of 71.6°

Figure 37 lists the measured minimum and maximum background luminances L_b of text presented on all visual media in the 38 lecture halls, except for the projection screens, due to the inaccessibility of the coded projectors in most of the lecture halls (32 of 38). Based on Figure 37, the range of background luminance L_b of text presented in typical lecture halls is 2.81 cd/m^2 ± 4.73 cd/m^2 $\le L_b \le$ 86.00 cd/m^2 ±102.28 cd/m^2. Fortunately, the typical brightness of projection screens used in practice also falls in this range for two facts. First, the standard ANSI/SMPTE 196M-2003 for the indoor theatre and review room projection-screen luminance and viewing conditions requires a nominal screen luminance as 55 cd/m^2 with a range of 41 cd/m^2 to 75 cd/m^2 allowed and a minimum 34 cd/m^2 for theatres, with 55 cd/m^2 ± 7 cd/m^2 for review rooms (Society of

Motion Picture and Television Engineers, 2003). Second, as listed in Table 7, the surface brightness of projection screens measured in 6 lecture halls also falls in the range $2.81\ cd/m^2 \pm 4.73\ cd/m^2 \leq L_b \leq 86.00\ cd/m^2 \pm 102.28\ cd/m^2$.

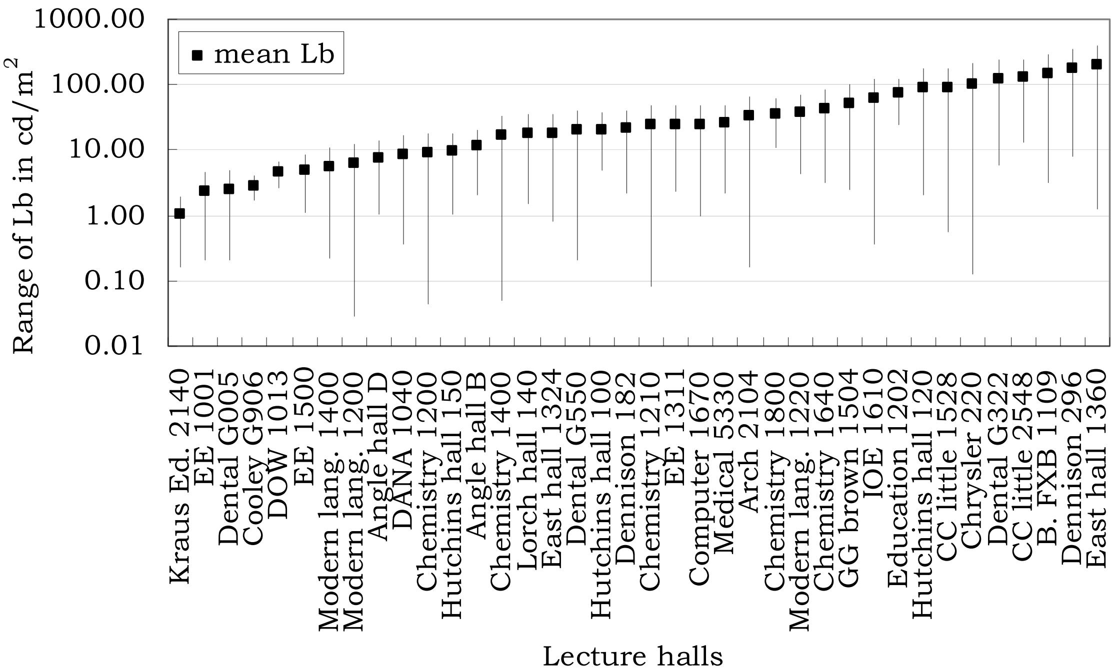


Figure 37. Range of background luminance L_b of text presented on all visual media in the 38 lecture halls except for projection screens

Table 7. Surface luminance of projection screens measured in 6 lecture halls

L_b in cd/m^2	Lecture halls					
	Chemistry 1210	Chemistry 140	Education 1202 (overhead projector)	EE1311	Hutchins Hall 120 (overhead projector)	Lorch Hall 140
min	29.24	19.5	59.95	33.38	39.41	29.47
max	48.17	33.27	104.6	47.72	173.8	34.45
mean	39.01	26.38	84.42	43.43	87.38	32.55

Figure 38 illustrates the range of surrounding luminance L_s measured at each point of the pattern shown in Figure 34 in the 38 lecture halls. The L_s in the 38 lecture halls has a range of 1.50 cd/m^2 ± 3.15 cd/m^2 ≤ Ls ≤ 77.87 cd/m^2 ±76.78 cd/m^2, which is then used to guide the experiments to be carried out later in this study.

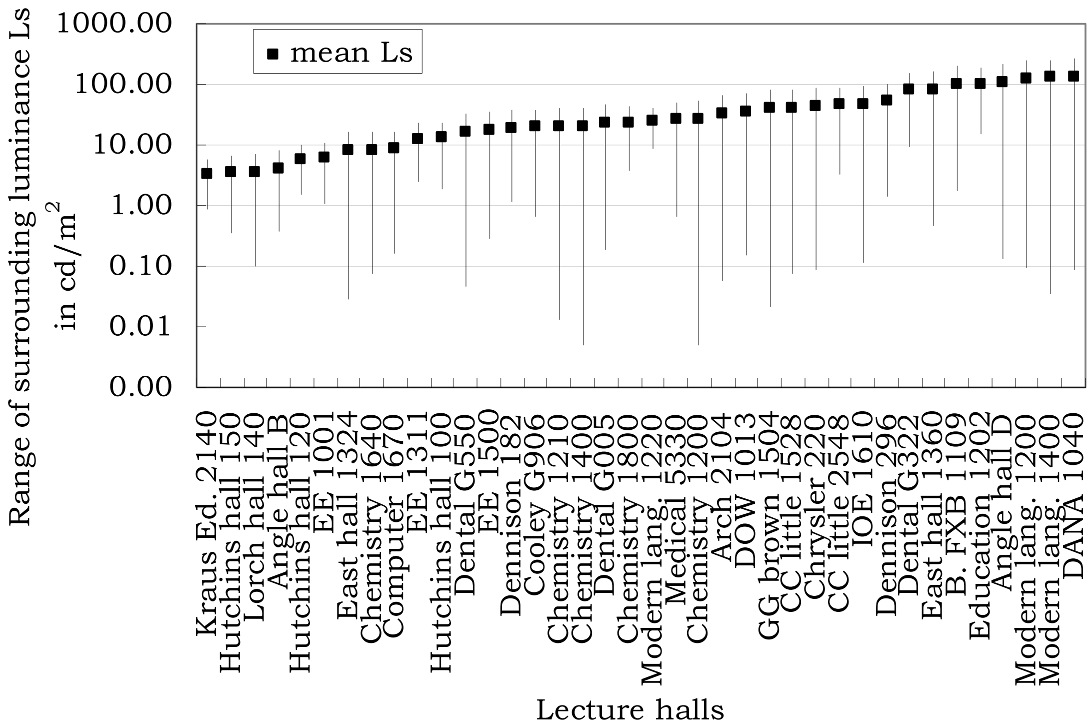


Figure 38. Range of surrounding luminance L_s measured in the 38 lecture halls

4.4 Summary

Based on the field measurements in the 38 lecture halls at the University of Michigan, the typical viewing conditions of common lecture halls are found as:

1. For the maximum viewing distance, 9.96 m $\leq D_{max} \leq$ 25.74 m, with mean 15.50m.
2. For the horizontal and vertical viewing angles, $\alpha_{max} < \phi_{max}$, 43.5° $\leq \phi_{max} \leq$ 80.4° and 22.2° $\leq \alpha_{max} \leq$ 68.5°; for the incident angle, 48.3° $\leq \xi_{max} \leq$ 86.5°, with mean 71.6°.
3. The range 2.81 cd/m^2 ± 4.73 cd/m^2 $\leq L_b \leq$ 86.00 cd/m^2 ±102.28 cd/m^2 is for the background luminance of text.
4. For the surrounding luminance measured at each point following the measuring pattern shown in Figure 34, 1.50 cd/m^2 ± 3.15 cd/m^2 $\leq L_s \leq$ 77.87 cd/m^2 ±76.78 cd/m^2.

These typical viewing conditions should be abided by in the next steps of this study to carefully design the experiments by matching their viewing conditions to these ranges.

CHAPTER 5

Step 2: Derivation of an Equation to Predict the Spatial Legibility of Text and Its Verification

Based on a hypothesis, this study derives a new equation to predict the spatial legibility of text based on seven critical factors, including (a) acuity of observer, (b) viewing distance, (c) viewing angles, (d) visual angle subtended by text, (e) text height, width, and strokewidth, (f) luminance contrast, and (g) target luminance and background luminance. This hypothesis is then verified using (a) fundamental theories of how retinal images of text activate cones in the fovea of a viewer's eyes, and (b) laboratory data from human subjects.

5.1 Derivation of an Equation to Predict the Spatial Legibility of Text

This equation utilizes the definition of solid angle ω subtended by any characters to the observer's eyes. The solid angle is defined as (9), and is illustrated in Figure 39. In Figure 39, an area A at viewing distance D away subtends a solid angle ω to the eye located at point P, with an incident angle ξ between the sightline OP and the normal axis of the area A.

$$\omega = \frac{A}{D^2}\cos\xi \tag{9}$$

where:

ω = Solid angle subtended by the character to the observer's eyes

ξ = Incident angle, the angle between the display normal and observer's sightline

A = Normal character area

D = Viewing distance

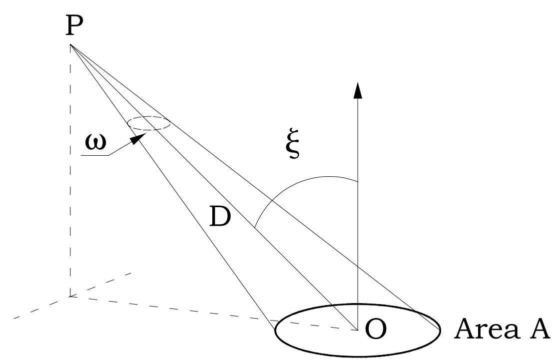


Figure 39. Definition of solid angle ω (Rea, 2000, Figure 9-1, p. 9-2)

The normal character area (an notated as A), as used in (9), is measured differently for letters or graphics. The spatial distribution (orientations and spacing) of strokes of letters A~Z determines which letter it is. Even for the same letter, its spatial distribution of strokes also varies with its font. Thus, for the legibility of letters, the spatial mosaics of strokes are as crucial as their size. This is also true for some symbols with established forms (e.g., ⩾, /, ∞, ×, ÷). Therefore, the normal area (A) of letters or symbols should be defined as the product of width multiplied by height to count the orientations and spacing of strokes or details, as shown in Figures 40 (a) and 40 (b). In practice, geometries of text have often been measured using height and width. On the other hand, for random graphics without established forms, the normal area (A) is defined as the summation of areas of all details, as shown in Figure 40 (c).

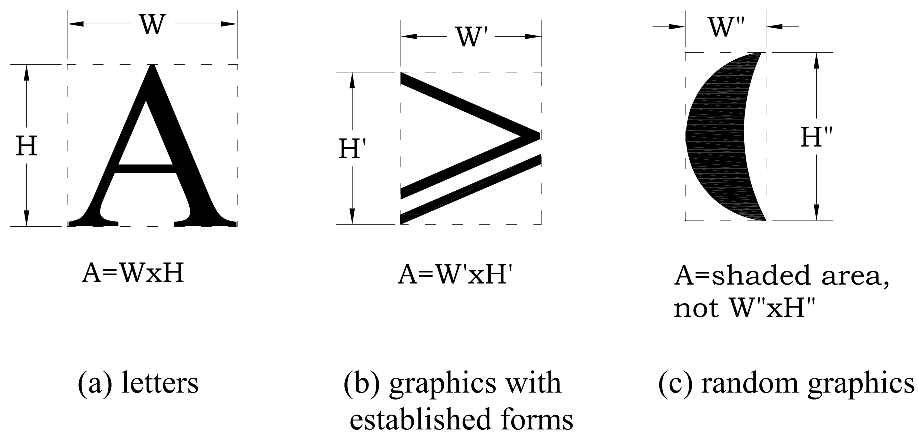


Figure 40. Different measurements of normal area A: text (A=W × H), graphics with established forms (A=W′ × H′), and random graphics (A=summation of all details)

Restricted by the ten assumptions, only text (single letters) is examined here. The normal area of text is expressed in (10).

$$A = W \cdot H \tag{10}$$

where:

A = Normal text area

W= Normal text width

H = Normal text height

Based on the geometrical relationship shown in Figure 41, the incident angle ξ, which is the angle between display normal (OA) and observer's sightline (OP), can be substituted with the horizontal viewing angles ϕ and the vertical angle α, using (11).

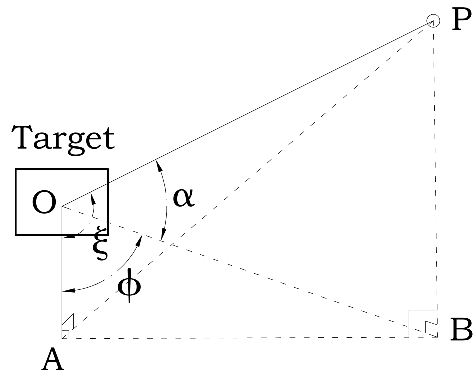
$$\cos\xi = \cos\phi\cos\alpha \tag{11}$$

where:

ξ = Incident angle

ϕ = Horizontal viewing angle

α = Vertical viewing angle



$\cos\xi = \cos\phi\cos\alpha$

Incident angle ξ = <AOP;

Horizontal viewing angle ϕ = <AOB;

Vertical viewing angle α = <BOP;

cos<AOP=OA/OP;

cos<AOB=OA/OB;

cos<BOP=OB/OP

Figure 41. Off-the-display-axis viewing of target and the incident angle ξ
(Target is located at O, observer's eye is at P, OA is display normal, B is an assistant point, angle <OAP = <OAB = <ABP = <OBP = 90°)

Substitute (10) and (11) into (9), and we get (12):

$$\omega = \frac{A}{D^2}\cos\xi = \frac{W\cdot H}{D^2}\cos\phi\cos\alpha \tag{12}$$

where:

ω = Solid angle subtended by the legible text to the observer's eyes

A = Normal text area

W = Normal text width

H = Normal text height

D = Viewing distance

ξ = Incident angle

ϕ = Horizontal viewing angle

α = Vertical viewing angle

When text is viewed perpendicular to the observer, $\phi = 0$, $\alpha = 0$, the solid angle ω subtended by the text is calculated using (13).

$$\omega_0 = \frac{W \cdot H}{D_0^{\,2}} \tag{13}$$

where:

ω_0 = Solid angle subtended by text viewed perpendicular to the display

W = Normal text width

H = Normal text height

D_0 = Viewing distance when text is perpendicular to the observer

This study then uses a hypothesis to derive the target equation to predict the spatial legibility of text, which is detailed below and illustrated in Figure 42.

Constant-solid-angle hypothesis. The solid angle ω subtended by the legible viewing target (not only text) is a constant at different viewing angles (perpendicular or not) under the same viewing condition, that is, with the same target viewed by the same observer at the same recognition performance (threshold 100% accurate) under the same lighting conditions, but with different viewing distances at different viewing angles.

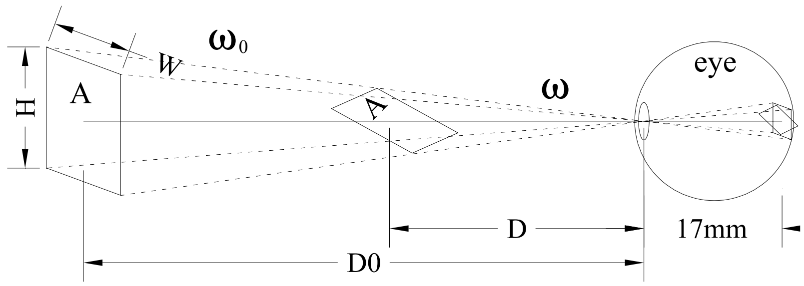


Figure 42. Illustration of the constant-solid-angle hypothesis
(Same target A with normal height H and width W, viewed either perpendicular or not at different distances, with the same threshold of 100% accurate recognition performance)

According to the constant-solid-angle hypothesis, the solid angle ω_0 subtended by the target A viewed perpendicularly at distance D_0, as shown in Figure 42, equals the solid angle ω subtended by the same target A viewed not perpendicularly at decreased distance D. The target A forms two upside down retinal images on the center fovea of the observer's eye, when viewed at viewing distances D_0 and D, respectively. These two retinal images have different shapes but an equal area, thus subtending equal solid angles to the effective center of the eye's optics at 17 mm from the retina (Wandell, 1995).

Based on the constant-solid-angle hypothesis and Figure 42, we gets (14):

$$\omega = \frac{W \cdot H}{D^2} \cos\phi \cos\alpha = \frac{W \cdot H}{D_0^{\ 2}} = \omega_0 \qquad (14)$$

where:

ω = Solid angle subtended by text viewed not perpendicularly

ω_0 = Solid angle subtended by the same text viewed perpendicularly at the same recognition performance (threshold 100% accurate)

W = Normal text width (the same text viewed perpendicularly or not)

H = Normal text height (the same text viewed perpendicularly or not)

D = Legibility distance when text is viewed not perpendicular to the observer

D_0 = Legibility distance when text is viewed perpendicularly, $D_0 > D$

ϕ = Horizontal viewing angle

α = Vertical viewing angle

Equation (14) was further derived as (15):

$$D = D_0 \sqrt{\cos\phi \cos\alpha} \tag{15}$$

where:

D = Legibility distance when text is viewed not perpendicularly

D_0 = Legibility distance when text is viewed perpendicularly

ϕ = Horizontal viewing angle

α = Vertical viewing angle

Equation (15) describes how the legibility distance (D) of text viewed not perpendicular to the observer varies with the viewing angles from the original legibility distance (D_0) when text is viewed perpendicularly. Therefore, using (15), the equation to predict the spatial legibility of text viewed not perpendicularly can be derived from an existing equation that predicts the legibility of text viewed perpendicularly. Among the 95 legibility equations ever published, Howett's equation, (7) previously, can serve this purpose best because (a) Howett's equation predicts the legibility of letters, (b) it has the maximum number of critical factors examined, including geometries, background luminance, contrast, and the Snellen eyesight of the observer, and (c) Howett's equation was also mathematically derived from Kaneko's equation (1978), as thoroughly reviewed in Chapter 2. Howett's equation can be re-expressed as (16).

$$D_0 = 2443.5 \cdot H \cdot \left(\frac{H}{Sw}\right)^{-1} \cdot S_d^{\ -1} \cdot L_b^{\ 0.213} \cdot C_{\%}^{\ 0.532} \tag{16}$$

Where:

H = Normal character height

D_0 = Legibility distance when text is viewed perpendicularly

Sw = Strokewidth of text

H/Sw = Height-to-strokewidth-ratio of text

S_d = Denominator in the Snellen ratio of observer's eyesight

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

Substitute (16) into (15), to get (17). Equation (18) is a different expression of (17).

$$D = 2443.5 \cdot H \cdot \left(\frac{H}{Sw}\right)^{-1} \cdot S_d^{\ -1} \cdot L_b^{\ 0.213} \cdot C_{\%}^{\ 0.532} \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \tag{17}$$

$$H = 4.1\times 10^{-4} \cdot D \cdot \left(\frac{H}{Sw}\right) \cdot S_d \cdot L_b^{\ -0.213} \cdot C_{\%}^{\ -0.532} \cdot (\cos\phi)^{-0.5} \cdot (\cos\alpha)^{-0.5} \tag{18}$$

where:

D = Legibility distance of text viewed not perpendicular to the display

H = Normal text height

Sw = Strokewidth of text

H/Sw = Height-to-strokewidth-ratio of text

S_d = Denominator in the Snellen ratio of observer's eyesight

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ϕ = Horizontal viewing angle

α = Vertical viewing angle

Equation (17) or (18) is the target equation for predicting the spatial legibility of text from all seven critical factors that must be examined in this study. Unfortunately, (17) or (18) does not consider (a) text width and (b) ambient light of the viewing environments. The absence of text width in (17) or (18) derives from the fact that Howett's equation deals only with strokewidth. This neglect might not be a problem in practice since what is usually required is text height rather than text width. In any case, text width can be easily calculated as long as the height-to-width ratio of different fonts is given. The effect of the missing factor of ambient light in (17) or (18) will be checked later using legibility data collected from human subjects in the UMTRI laboratory. Outcomes will then be used to improve the derived equation. However, before (17) or (18) can be recommended to architectural practice for predicting the spatial legibility of text, the constant-solid-angle hypothesis must be proven using fundamental theories of visual perception and legibility data collected from human subjects in the UMTRI laboratory.

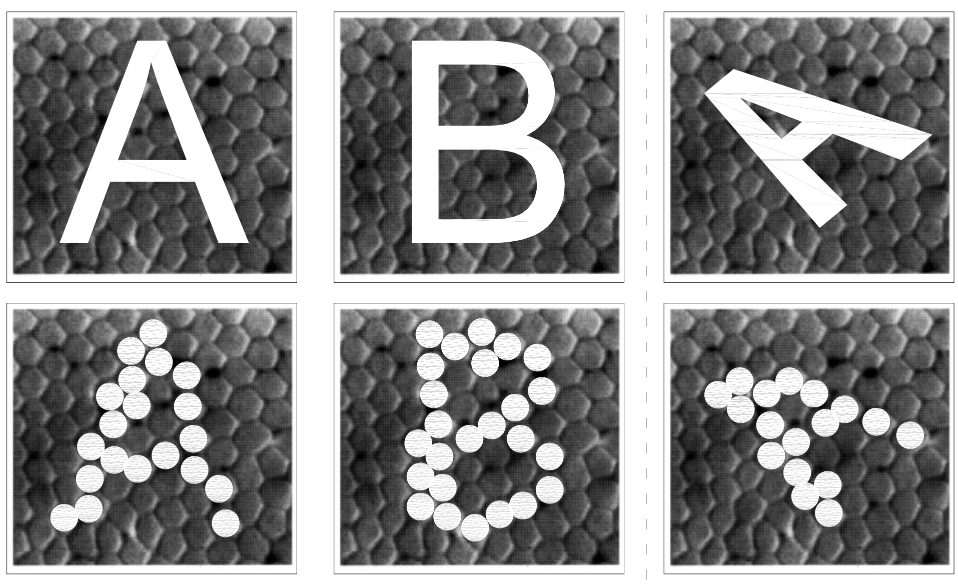
Equation (17) or (18) can be traced back to Kaneko's equation, (8) in Chapter 2, which was developed when $1 \leqslant L_b \leqslant 1000$ cd/m^2; $10 \leqslant C_{\%} \leqslant 90$; $0.2 \leqslant Ac \leqslant 2.0$ min^{-1}. Thus, (17) or (18) would better hold under those viewing conditions. In lecture halls, the audience usually has an acuity level $0.2 \leqslant Ac \leqslant 2.0\ min^{-1}$ with normal $1.0\ min^{-1}$. The background luminance L_b of text presented on visual media in lecture halls is also commonly $1 \leqslant L_b \leqslant 1000\ cd/m^2$ ($2.81\ cd/m^2 \pm 4.73\ cd/m^2 \leq L_b \leq 86.00\ cd/m^2 \pm 102.28\ cd/m^2$ as surveyed). In addition, text presented in lecture halls often has high contrast for better legibility. Therefore, (17) or (18) can likely be used in lecture halls, except for text with contrast percent $C_{\%} > 90$. In this exceptional situation, further research is needed to investigate the predictability of the derived equation (17) or (18).

5.2 Verification of the Derived Equation

The constant-solid-angle hypothesis is then verified using two different approaches: (a) fundamental theories as to how retinal images of text activate cones in the fovea of viewer’s eyes, and (b) legibility data collected from human subjects participating in a pilot experiment and a main experiment carried out in the laboratory.

5.2.1 Physiological and Photochemical Foundation

The constant-solid-angle hypothesis is consistent with how retinal images of characters activate cones in the center fovea of an observer’s eyes. When characters (text or graphics) are viewed, either perpendicularly or not, they form a retinal image in the center fovea of a viewer’s eyes, which then activates the underlying foveal cones to fire signals to the cortex nerves in the brain. The legibility level of characters viewed is eventually determined by (a) the spatial distribution of the activated cones on the retina, and (b) the number of the activated foveal cones and the strength of signals they fired. Different characters have different shapes and structures of their strokes or details, and thus, a different spatial mosaic of activated foveal cones when viewed. For example, as illustrated in Figure 43 (a), the spatial mosaic of activated foveal cones of the letter A is different from that of the letter B, both viewed perpendicularly. The viewing angle also affects the spatial mosaic of the activated foveal cones. When the target is viewed not perpendicularly, its projected image perpendicular to the viewing line, rather than itself, forms the retinal image that is distorted as a result of the viewing angle. Correspondingly, the spatial mosaic of activated foveal cones is also distorted, as shown in Figure 43 (b).



spatial mosaic of retinal image A and its activated cones

spatial mosaic of retinal image B and its activated cones

spatial mosaic of distorted retinal image A due to viewing angles and its activated cones

(a) viewed perpendicularly

(b) viewed at angles

Figure 43. Spatial mosaic of the retinal images of letter A and B, when viewed perpendicularly or not, as well as their underlying activated cones (Wandell, 1995, Figure 3.4, p. 49)

While the spatial mosaic of the activated foveal cones determines which character it is, the number of the activated foveal cones and the strength of signals fired by them to the neurons jointly determine the legibility levels of characters viewed (how clear and sharp the character is), because of the one-to-many relationship between the foveal cones and optic-tract neurons (Wandell, 1995). In addition, since foveal cones are very tightly packed as a triangle lattice without any strong orientation dependencies, and the cone threshold is very low 3.18×10^{-4} cd/m^2 (cone will not be activated to fire a signal until the incoming light intensity reaches this threshold value), the number of activated foveal cones is in proportion to the area of the retinal image of legible characters at mesopic and photopic light levels (Boff & Lincoln, 1988). The area of the retinal image is solely determined by the geometrical relationships between the target and the observer's eye, including viewing distance, target size, location, and orientations (viewing angles). The strength of the signal fired by the activated foveal cones depends on the observer's acuity, age, and lighting conditions of the viewing environment (e.g., reflectance, luminance contrast, target or background luminance, surrounding luminance, lighting uniformity, possible glare or light trespass, color difference, spectrum of lamps). Therefore, four practical ways to increase the legibility levels of text (or graphics) include:

1. Increase the viewer's eyesight by wearing glasses or contact lens.
2. Increase the lighting conditions in the visual environment to enhance the quality of the retinal image and thus increase the strength of the fired signals.
3. Increase the target size or decrease the viewing distance to increase the area of the retinal image and have more foveal cones activated.

4. Decrease the viewing angles, with perpendicular viewing having the maximum area of the foveal cones activated, as illustrated in Figure 43.

Inside the observer's eye, as illustrated in Figure 44, the area of the retinal image of text (or graphics) viewed can be measured using solid angle ω' subtended by this area to the effective center of the eye's optics at an average 17 mm (Wandell, 1995). Geometrically, this solid angle ω' equals the solid angle ω subtended by the target character outside the observer's eye. Thus, the solid angle ω can be used to measure the area of the retinal image. In addition, the solid angle ω can be used to assess the legibility levels of characters viewed by the same observer under the same lighting conditions such that the foveal cones fire signals at a stable rate of strength. Specifically, as illustrated in Figure 44, if two targets A1 (perpendicular to the observer) and A2 (not perpendicular to the observer) are viewed by the same observer at different viewing distances D1 and D2, such that they subtend equal solid angles to the observer's eye, $\omega' = \omega 1 = \omega 2$, their retinal images have an equivalent area ($A1' = A2'$) but different shapes. If these two targets are viewed under the same lighting conditions, they will have equivalent legibility levels due to the same number of activated foveal cones and the stable signal firing rate of strength. In contrast, if target A1 would like to have the same legibility levels as target A2, assessed using the same recognition performance (e.g., threshold 100% accurate), they shall subtend equivalent solid angles to the same observer's eye ($\omega 1 = \omega 2$), when viewed under the same viewing condition.

$$\omega 1 = \omega 2 = \omega' \qquad A1' = A2'$$

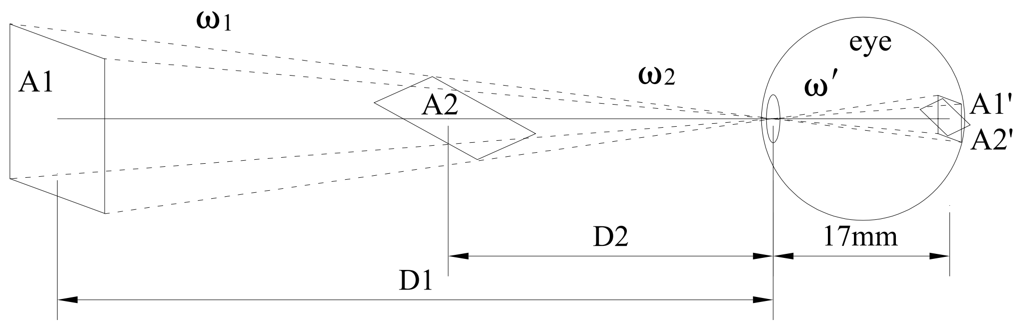


Figure 44. Solid angles subtended by targets with different orientations viewed at different distances, and those subtended by their retinal images

Figure 44 also illustrates the relationship between viewing distances and orientations of the viewing targets to subtend an equal solid angle to the observer's eye. In Figure 44, the rotated and/or tilted target A2 has a decreased projected area perpendicular to the observer's eye. Thus, target A2 should be viewed at decreased viewing distance D2 to have an equivalent solid angle as that subtended by target A1 at distance D1. In practice, the decreased projected target area is usually due to three changes: (a) decreased size when viewed perpendicularly, (b) rotated or tilted target at viewing angles, or (c) both. After these changes, if viewing distances are appropriately decreased, observers can still have equivalent recognition performance (e.g., threshold 100% accuracy) due to an equivalent area of retinal images, and thus, viewing targets still have an equivalent solid angle ω subtended to the observer's eye.

Figure 45 illustrates the equivalent area of retinal images of a disk, thus, an equivalent solid angle subtended, when it is viewed (a) perpendicularly with normal size at the original distance, (b) perpendicularly with decreased normal size at a decreased distance, (c) not perpendicularly with the original normal size but at a decreased distance, and (d) not perpendicularly with a decreased normal size at a decreased distance.

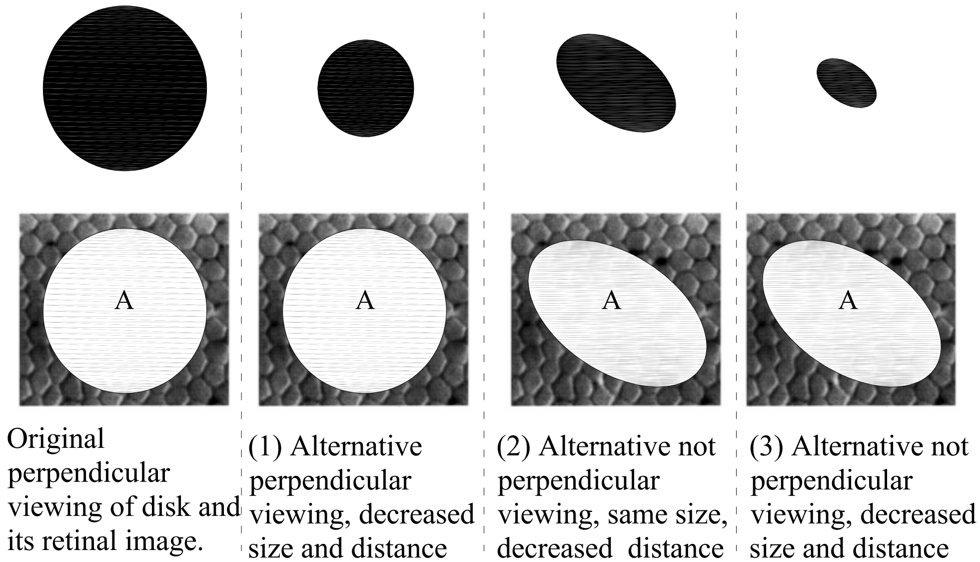


Figure 45. Constant area of retinal images of a legible disk viewed with different orientations or different normal size at different viewing distances

Therefore, no matter how the target character (not only text) changes in size, orientation, and viewing distance, as long as it is viewed by the same observer at the same recognition performance (e.g., threshold 100% accurate) under the same lighting conditions, the equivalent area of retinal images of the target guarantees that the solid angle ω subtended by the legible viewing target is a constant at different viewing angles (perpendicular or not) under the same viewing condition. This is what has been claimed by the constant-solid-angle hypothesis. This hypothesis assumes that the distortion of text viewed at extremely large angles does not degrade its recognition. This assumption might be incorrect thus needs further verification from the laboratory data.

5.2.2 Pilot Experiment

The constant-solid-angle hypothesis is then tested in the lighting laboratory at the University of Michigan Transportation Research Institute (UMTRI) using legibility data collected from human subjects. A pilot experiment is first planned to (a) verify that the equipment for the main experiment works properly, (b) verify that the experiment can be conducted in the time allotted and that the data are reliable, (c) preliminarily verify that the constant-solid-angle hypothesis holds using three human subjects, and (d) preliminarily check that ambient light has little influence on legibility.

5.2.2.1 Laboratory Settings and Installation of Facilities

The pilot experiment is carried out in room 338 at the University of Michigan Transportation Research Institute (UMTRI), as shown in Figure 46. This rectangular lighting laboratory has black walls and ceiling to reduce light reflectance, a white floor that needs coverage to prevent reflectance glare, and unshielded ceiling lamps that probably have a direct glare on the observer's eyes.

Figure 46. Lighting laboratory at UMTRI

The experimental set up is illustrated in Figure 47. Exchangeable viewing materials are presented on the opalescent surface of a specially designed visual medium — a dimmable fluorescent light box (T8, daylight) that is painted black and mounted on a movable base put on a desk at one end of the laboratory. The dimmable fixture can be rotated and tilted to provide different viewing angles between the display normal and the observer's sightline. The materials are recognized by subjects sitting in a chair at 6.1 meters (20 feet) away with their chins on a chin rest (to fix the viewing distance) at different viewing angles but the same recognition performance (threshold 100% accurate).

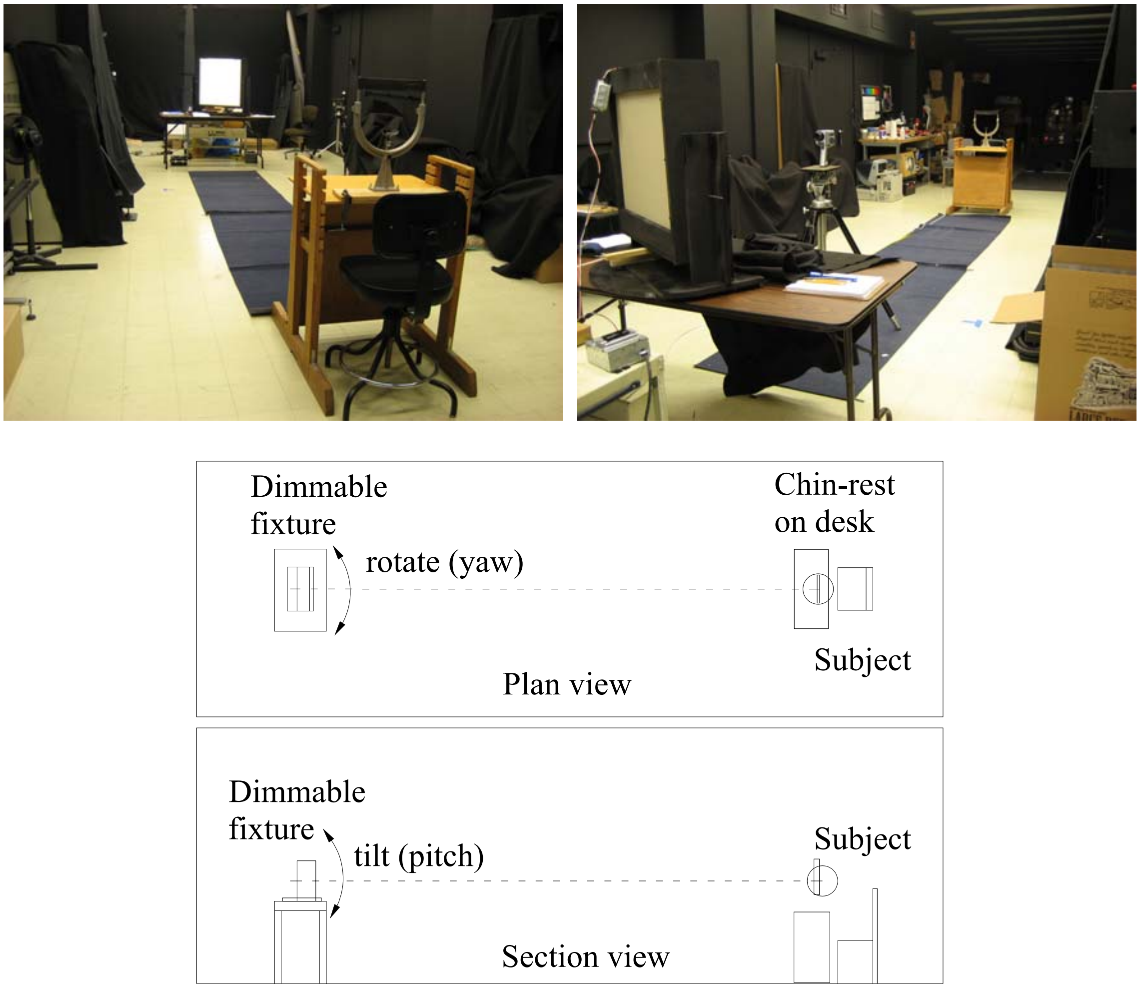
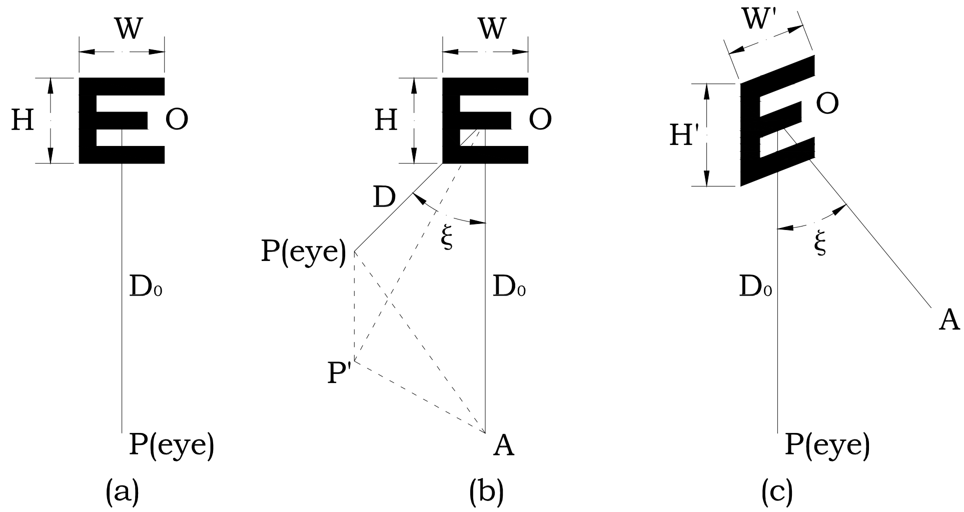


Figure 47. Experimental settings in the laboratory at UMTRI

Note that this experimental set up uses variable sizes of materials viewed at different viewing angles but a constant viewing distance for better experimental arrangement rather than the same material viewed at different viewing angles and variable viewing distances, as claimed by the constant-solid-angle hypothesis. Such modification of the experimental set-up will not affect the verification of the hypothesis since the solid angle ω subtended by the viewing materials, and thus the inverted retinal image formed on the observer's center fovea, are exactly the same when the materials are viewed of either way. This idea is illustrated and verified in Figure 48.



$$\omega = \frac{W \cdot H}{D_0^{\ 2}} \qquad \omega = \frac{W \cdot H}{D^2}\cos\xi \qquad \omega = \frac{W' \cdot H'}{D_0^{\ 2}}\cos\xi$$

Figure 48. Illustration and verification of the experimental set up matches that claimed by the constant-solid-angle hypothesis

Figure 48 (a) shows a letter E of width W and height H located at point O is perpendicularly viewed by an observer P at D_0 away. Figure 48 (b) shows the same letter E viewed not perpendicular to the display by the same observer P, who has moved left and up, and closer to letter E, at decreased viewing distance D and at an incident angle ξ between the display normal OA and the visual line OP. Figure 48 (c) shows the rotated and tilted letter E of an increased size (width W' and height H', in locked aspect ratio), as viewed by the same observer P at the same incident angle ξ who stays in the original position. For any value of the incident angle ξ (0° ≤ ξ ≤ 90°), as long as $\frac{D}{D_0} = \frac{W}{W'} = \sqrt{\cos\xi}$, an equivalent solid angle ω is subtended by the viewing target letter E to the observer's eye in the three viewing scenarios shown in Figure 48, such that an equivalent (in both shape and size) retinal image is formed on the centre fovea of the observer's eyes. For example, given $\xi = 45°$, then $D = 0.84D_0$, $W' = 1.19W$.

A close look at the dimmable fixture is shown in Figure 49. This fixture is lit by two fluorescent T8 tubes (daylight). Its surface luminance (L_b) can be dimmed from 1150 cd/m^2 to 16 cd/m^2 (1% dimming), with a mean uniformity of 89.5% (min/max) at 5 different light levels (22.2, 63.5, 117.0, 485.7, and 1150 cd/m^2). This fixture can be horizontally rotated at 7 angles ($\phi = 0°, 15°, 30°, 45°, 60°, 75°$, and $85°$) and vertically tilted at 7 angles ($\alpha = 0°, 17°, 31.5°, 46.5°, 61°, 75°$, and $85°$). The pilot experiment examines four horizontal angles ($\phi = 0°, 30°, 60°, 75°$) and four vertical angles ($\alpha = 0°, 31.5°, 61°, 75°$), for a total of $4 \times 4 = 16$ incident angles ξ, calculated using $\cos\xi = \cos\phi\cos\alpha$, as listed in Table 8. Additionally, by dimming the surface luminance of the fixture to a constant 187.5 cd/m^2 for all 16 incident angles in the pilot experiment, this fixture has a simulated ideal diffusive surface whose surface brightness is independent of viewing angles.

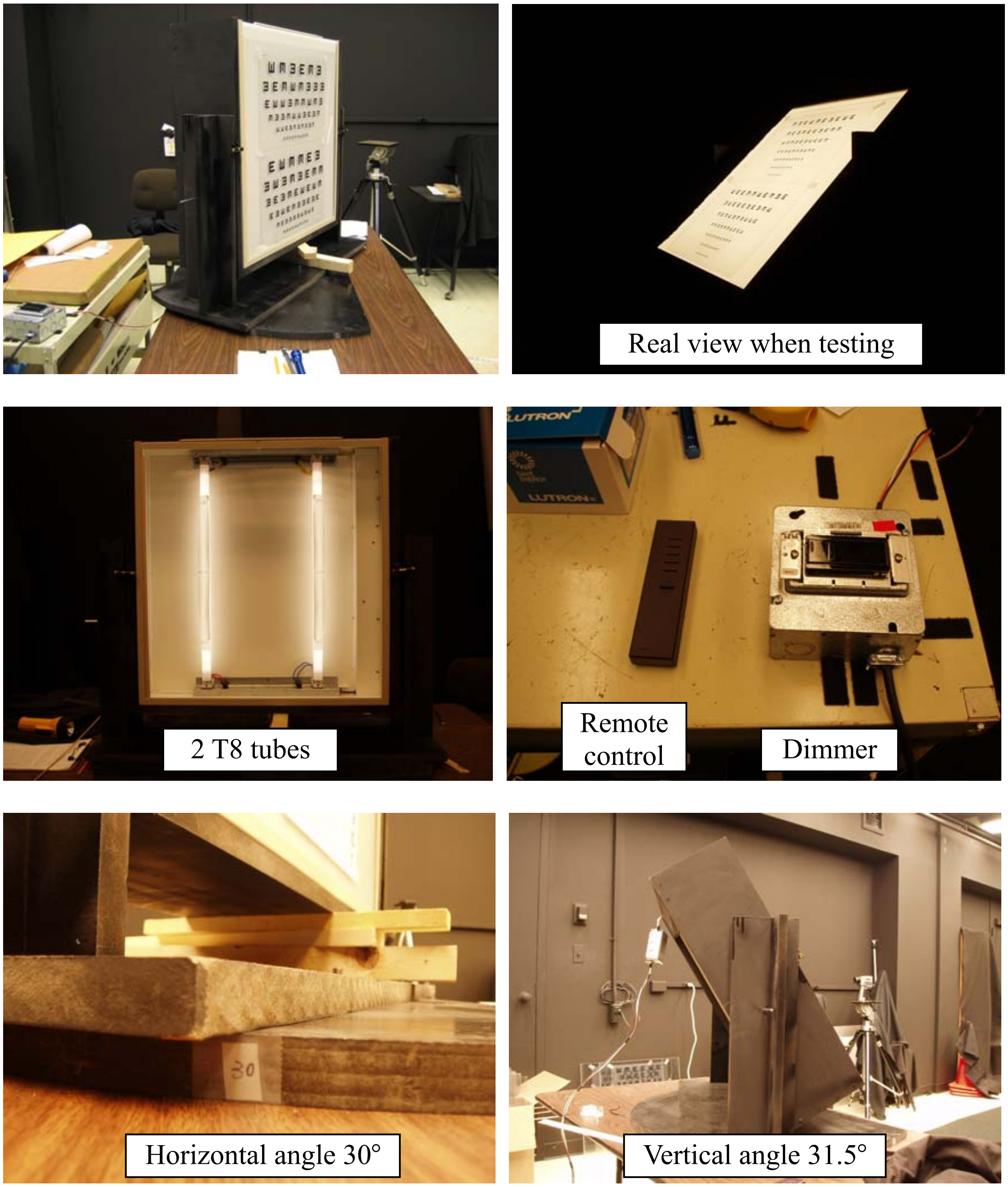


Figure 49. Dimmable fluorescent fixture (two T8 tubes, daylight) with simulated ideal diffusive surface, and 7 horizontal and 7 vertical viewing angles

Table 8. Total of 16 incident angles ξ examined in the pilot experiment (calculated using $\cos\xi = \cos\phi\cos\alpha$)

Incident angle ξ in deg	Horz. angle ϕ in deg (yaw)	Vert. angle α in deg (pitch)
0	0	0
30	30	0
31.5	0	31.5
42.4	30	31.5
60	60	0
61	0	61
64.8	60	31.5
65.2	30	61
75	75	0
75	0	75
76	60	61
77	30	75
77.3	75	31.5
82.6	60	75
82.8	75	61
86.2	75	75

In addition, the Minolta LS-100 luminance meter is used to measure luminance in the pilot experiment, and later in the main experiment, the field experiment, and other test scenarios. Figure 50 illustrates this meter in the laboratory when mounted on a tripod for better manipulation. In addition, the 300ft fiber glass measuring tape and the 50ft sonic laser tape have been used in both laboratory and field experiments for measuring distances. A piece of black cloth was used to cover the desk on which the fixture is placed, as shown in Figure 50. A wood pointer is also used to help the observers locate the target text when reading the materials.

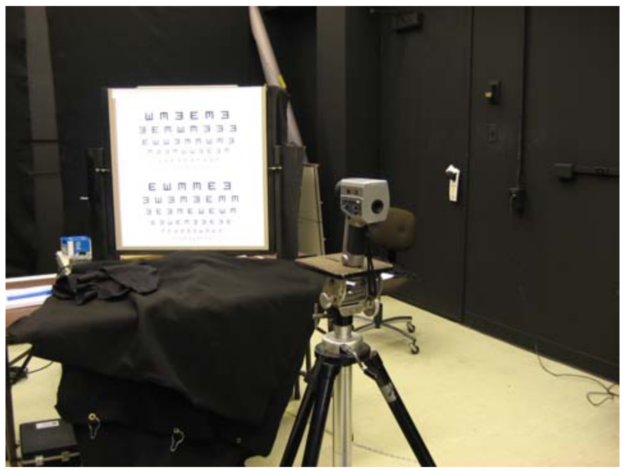


Figure 50. Luminance meter mounted on a tripod in the pilot experiment

5.2.2.2 Viewing Materials

The derived equation in this study predicts the legible height of letters if the height-to-strokewidth ration (H/Sw) is known, which in fact is defined for each font. Therefore, to simplify the experimental design, the viewing materials used in the pilot and main experiments are letters with a fixed height-to-strokewidth ratio. Among the letters A-Z, some letters are more easily recognizable than others by the same observer under the exact same viewing condition, as shown in Figure 51 (Zwahlen & Schnell, 1999).

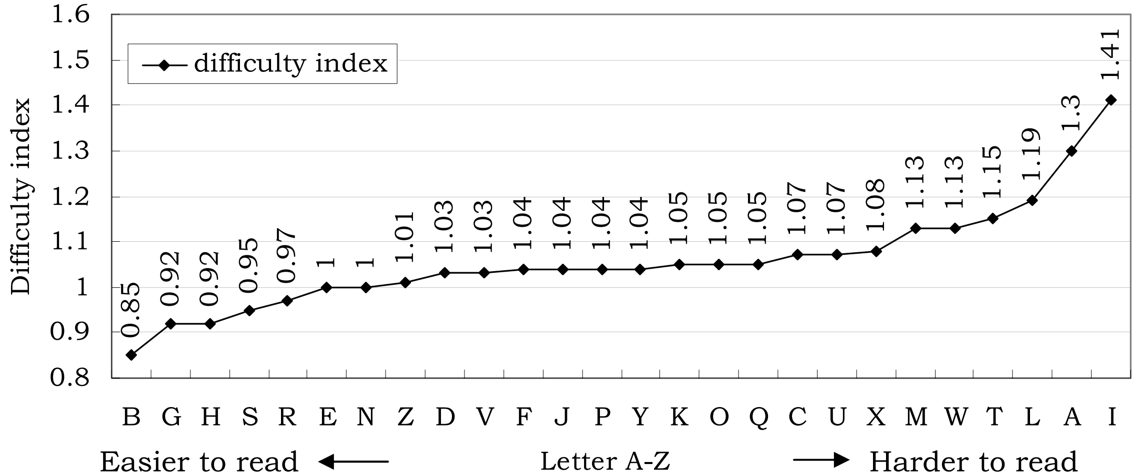


Figure 51. Index of legibility difficulty for standard highway alphabet letters A-Z (Zwahlen & Schnell, 1999, Figure 2, p. 144)

In this study, to exclude the individual distraction of different letters on legibility, a single letter is preferred to a group of letters as viewing materials. The eventual choice is the letter E because of its popularity in practice, with a height-to-width ratio H/W = 1, and a height-to-strokewidth ratio H/Sw = 5. Seven lines of letter Es with gradually increasing sizes, random orientations, and positive contrast of black/white constitute an E-chart, which is printed on letter or A3 size transparencies. These E-charts are the viewing materials used in this study. To double check the observer's reading performance, usually two E-charts with identical ranges of letter E heights but different orientations are attached side by side on one clear acrylic sheet and tested together in the laboratory, as shown in Figure 52. These exchangeable E-chart sheets are then attached to the simulated ideal diffusive surface of the dimmable fixture and read by observers sitting 6.1 m away. At a single test in this pilot experiment, one E-chart sheet (each sheet has two E-charts side by side) is viewed by the observer with one of three eyesight levels (20/20, 20/16, or 20/12.5) at one incident angle (total 16 incident angles examined) subtended between the display normal and the sightline of the observer. Because there are 3 levels of eyesight and 16 incident angles tested in the pilot experiment, there are $3 \times 16 = 48$ tests. Thus, a total of 49 E-chart sheets are used in the pilot experiment (2 sheets are used together for the largest incident angle $\xi = 86.2°$, when $\phi = 75°$ and $\alpha = 75°$), as shown in Figure 52.

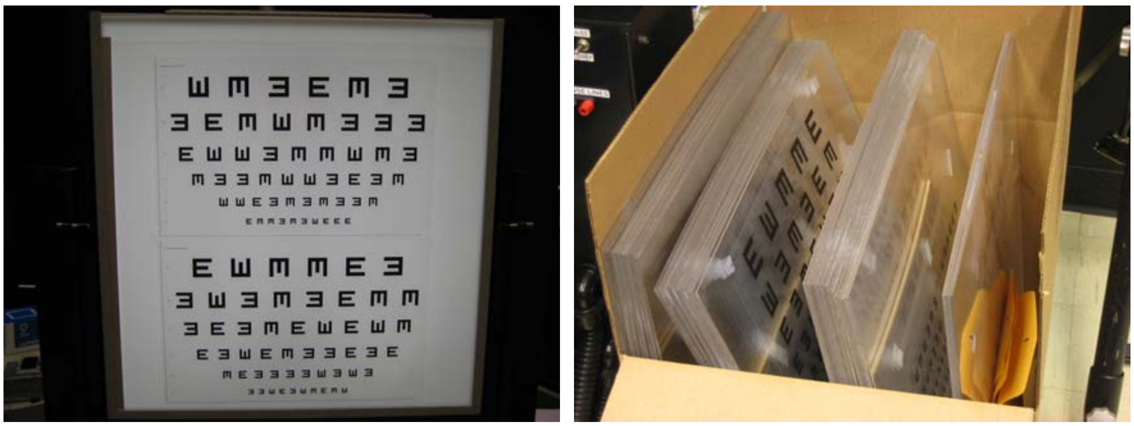


Figure 52. A total of 49 E-chart sheets used as viewing materials in the pilot experiment

The E-charts are developed from the minimum angle of resolution (MAR) used in the British standard BS 4274-1:2003. Using MAR, the height of letter Es in the middle line of E-charts is predictable, which is threshold legible (100% accurate) to observers with any one of the three eyesight levels (20/20, 20/16, or 20/12.5). However, the actual observed threshold legible height of letter Es in the laboratory might diff from the predicted value assigned to the middle line due to individual differences in an observer's eyes, age, viewing angles, and other hidden factors. Thus, a range of heights of letter Es arrayed in 7 total lines, with the predicted base value in the middle and a minimum graduation added or subtracted in the other 6 lines, as illustrated on the E-charts in Figure 52, is provided for all tests in the pilot and the follow-up main experiment.

There are a total of five steps in developing these E-charts, as detailed below.

Step 1. This study calculates the threshold legible (100% accurate) strokewidth of letter Es viewed perpendicularly to the observer at 6.1m (20ft), using (19). As shown in Figure 53, this retinal area subtends the minimum angle of resolution (MAR, in min arc, 2D) to the effective center of the eye's optics. The retinal image of this stroke strikes a critical number of foveal cones to fire signals to the brain needed for sharp recognition. A smaller stroke will not activate the foveal cones and thus not be legible.

$$Sw = 0.000291 \cdot MAR \cdot D \quad (19)$$

where:

Sw = Threshold legible (100% accurate) strokewidth of the letter E, in mm

MAR = Minimum angle of resolution of the observer's eye, in min arc.

MAR = 1 min arc for eyesight 20/20, 0.8 for 20/16, 0.63 for 20/12.5.

D = Viewing distance, at a constant 6100 mm

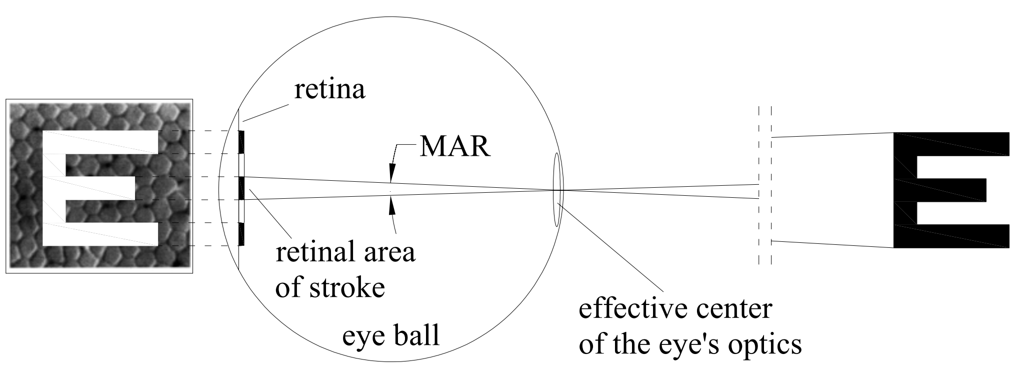


Figure 53. Minimum angle of resolution (MAR) subtended by the threshold legible stroke to the effective center of the eye's optics

Step 2. This study calculates the threshold legible (100% accurate) height of letter Es in the middle line of the E-charts viewed perpendicularly by the same observer, using (20), since the height-to-strokewidth ratio of letter Es equals 5 (H/Sw = 5).

$$H = 0.00145 \cdot MAR \cdot D \tag{20}$$

where:

H = Threshold legible (100% accurate) height of letter E, in mm

MAR = Minimum angle of resolution of the observer's eye, in min arc.

MAR = 1 min arc for eyesight 20/20, 0.8 for 20/16, 0.63 for 20/12.5.

D = Viewing distance, at constant 6100 mm

Step 3. The minimum graduation of heights (increases or decreases) of letter Es in the other 6 lines on the E-charts viewed perpendicularly by the same observer is determined in Step 3. Reflecting the discrimination power of the observer's eyes, the minimum angle of resolution (MAR) varies with the observer's acuity, age, and lighting conditions of the environment, but is independent of the geometries of the viewing materials and viewing angles. Thus, at a constant lighting condition in the pilot experiment with an equivalent background luminance level (187.5 cd/m^2), MAR is stable for E-charts viewed by the same observer at all 16 incident angles. Consequently, the threshold legible strokewidth calculated using (19) is actually the minimum graduation of heights of letter Es that can be recognized with 100% accuracy. Using this strokewidth as equal linear steps, the backup threshold legible heights of letter Es in the other 6 lines on the E-charts can be calculated using (21).

$$H' = H \pm n \cdot Sw \tag{21}$$

where:

H = Threshold legible (100% accurate) height of letter E in the middle line, in mm

H' = Threshold legible height of letter Es in the other 6 lines on the E-charts

Sw = Threshold legible strokewidth of the letter E, in mm

n = Natural number 1, 2, 3

Step 4. This study then calculates the ranges of heights of letter Es on the E-charts viewed perpendicular to the observers with each of the three different eyesight levels (20/20, 20/16. 20/12.5), as shown in Table 9.

Table 9. Heights of 7 lines of letter Es on E-charts viewed perpendicularly

7 lines of Es	Eyesight	20/20	20/16	20/12.5
	MAR	1 minarc	0.8	0.63
	Threshold Sw	1.78mm	1.42	1.12
Line 1, top	H + 3Sw	14.17mm	11.34	8.93
Line 2	H + 2Sw	12.4mm	9.92	7.81
Line 3	H + Sw	10.62mm	8.5	6.69
Line 4, middle	H	8.85mm	7.08	5.57
Line 5	H − Sw	7.07mm	5.66	4.45
Line 6	H − 2Sw	5.29mm	4.24	3.34
Line 7, bottom	H − 3Sw	3.52mm	2.82	2.22

Step 5. In this step, the ranges of heights of letter Es on the E-charts viewed at other non-zero incident angles can be predicted. In light of the derived legibility equation (18), the threshold legible (100% accurate) height of letter Es viewed not perpendicularly can be calculated using (22). The results are listed respectively in Appendix F for the three eyesight levels (20/20, 20/16, 20/12.5), which are the geometries for making E-charts. If the observed threshold legible heights (100% accuracy) in the pilot and main experiment match the predicted heights at the middle lines of all E-charts, the constant-solid-angle hypothesis is verified.

$$H' = H \cdot (\cos\phi)^{-0.5} \cdot (\cos\alpha)^{-0.5} \tag{22}$$

where:

H = Height of 7 lines of letter Es viewed perpendicularly, as shown in Table 9

H' = Height of 7 lines of letter Es viewed not perpendicularly

ϕ = horizontal viewing angle, ϕ = 0°, 30°, 60°, 75°

α = vertical viewing angle, α = 0°, 31.5°, 61°, 75°

5.2.2.3 Lighting Conditions

In the pilot experiment, target lighting is provided by the dimmable fixture, with a constant surface luminance of 187.5 cd/m^2 at all 16 incident angles (perpendicular or not) to simulate the ideal diffusive surface. In addition, to preliminarily check the influence of ambient light on the legibility of text at all 16 incident angles, two levels of ambient light are provided: (1) zero ambient light, (2) ambient light (value is measured later in the laboratory) provided by T12 lamps mounted on the ground behind the dimmable fixture, as shown in Figure 54. At either ambient light level, the background luminance (L_b) of the E-charts, namely the surface luminance of the dimmable fixture, has been dimmed to a constant 187.5 cd/m^2 at different viewing angles. To prevent the reflectance glare, as shown in Figure 54, dark blue carpets are placed on the floor between the observer and the fixture. Unfortunately, the carpets are too narrow and thus do not serve this purpose very well, a situation that is improved in the main experiment.

(a) zero ambient light (b) ambient light provided by T12 lamps

Figure 54. Two levels of ambient light provided in the pilot experiment
when text is viewed perpendicularly

In terms of the luminance contrast of letter Es, the positive contrast of black/white commonly used in lecture halls is adopted in the pilot and the main experiments. In AutoCAD, the white is in RGB (255, 255, 255) with luminance scale 100 (not in cd/m^2), while the black is in RGB (0, 0, 0) with scale 0, as illustrated in Figure 55.

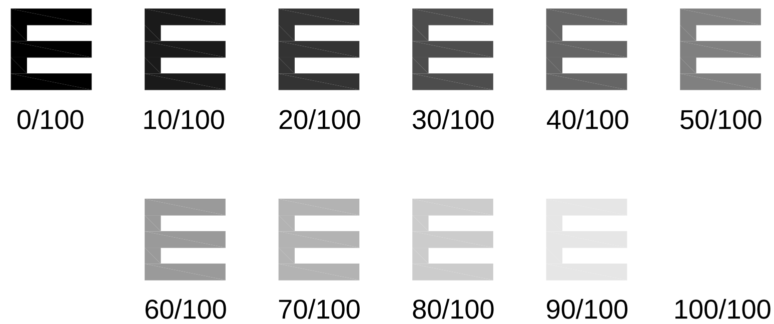


Figure 55. Letter Es with different contrast adjusted from 0-100 in AutoCAD

The percent luminance contrast of a letter E on a sample E-chart attached on the surface of the dimmable fixture viewed perpendicularly in the laboratory has a mean value of 97.9 ($C_{\%}$) when measured at five levels of background luminance (1200, 505.3, 123.1, 68.1, and 23.8 cd/m^2). Theoretically, the luminance contrast of target letter Es is independent of the light levels and viewing angles. To double check, the measured luminance contrast of the letter E on the sample E-chart has a mean value of 98.5 ($C_{\%}$) with a uniformity ratio of 0.99 (min/max) at 7 viewing angles (ϕ = 0°, 15°, 30°, 45°, 60°, 75°, and 85°, while α = 0°) and 14 background luminance levels (488.6 – 34.76 cd/m^2). Therefore, as long as the E-charts are printed black/white (0/100) in AutoCAD, this study assumes that the luminance contrast ($C_{\%}$) of the letter Es remains approximately 97.9 ~ 98.5 (usually 97.9 is used), at different light levels and viewing angles, as tested in the pilot and the main experiments.

Recall that Kaneko's equation, which is the origin of the derived legibility equation in this study, was developed with $10 \leqslant C_{\%} \leqslant 90$. The percent luminance contrast (97.9) tested in the pilot experiment does not fall in this range. However, the contrast (97.9) selected in this study represents the common viewing situation in lecture halls. It is used only as a sample to verify the hypothesis that is supposed to hold for the whole range of luminance contrasts $0 \leqslant C_{\%} \leqslant 100$.

5.2.2.4 Subjects

The participants in the pilot experiment, as well as the follow-up main experiment, must be 20-29 years old, with binocular eyesight 20/20 or better (20/20, 20/16, or 20/12.5), with or without glasses, and normal color vision. According to the curve of the age-related variation of visual acuity shown in Figure 10 previously, at age 20-29, people have the maximum acuity levels in their life span. Although color is not examined in this study, the requirement of normal color vision is intended to avoid the possible but unknown negative effect of abnormal color vision on the legibility of letter Es printed on black/white. The recruited potential subjects are strictly screened upon arrival to find those meeting the requirements. As shown in Figure 56, one purchased Snellen chart and two self-made eyesight E-charts based on the British standard BS 4274-1:2003 are used for this purpose. In terms of the sample size of the pilot experiment, four subjects are recruited and three of them have participated in the pilot experiment; one failed the requirements.

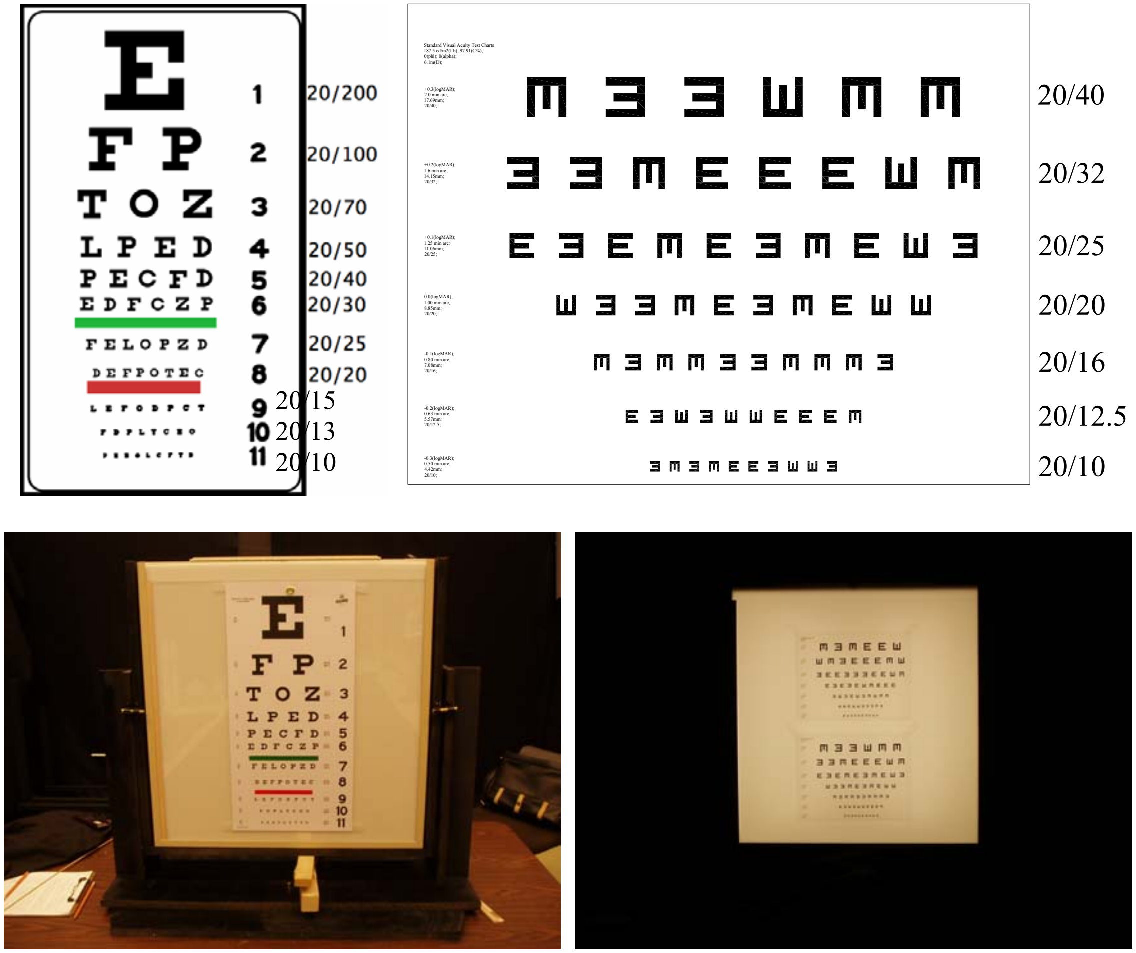


Figure 56. Snellen chart and eyesight E-chart used to screen potential subjects according to the requirements (drawings are not at the same scale)

Similar to the previous E-chart making, the eyesight E-chart is developed using the previous (19) and (20), based on the minimum angle of resolution (MAR) for different eyesight levels provided in the BS 4274-1:2003. The calculated geometries are shown in Table 10. All eyesight E-charts are viewed perpendicularly to the observer at 187.5 cd/m^2.

Table 10. MAR and geometries of eyesight E-chart

Eyesight	MAR	Viewing distance (D)	Strokewidth (Sw)	Height (H)
20/40	2	6100 mm	3.55 mm	17.69 mm
20/32	1.6	6100	2.84	14.15
20/25	1.25	6100	2.22	11.06
20/20	1	6100	1.78	8.85
20/16	0.8	6100	1.42	7.08
20/12.5	0.63	6100	1.12	5.57
20/10	0.5	6100	0.89	4.42

5.2.2.5 Factors Examined

To test the constant-solid-angle hypothesis, the solid angle subtended by the legible letter Es viewed at each of a total of 16 incident angles must be examined in the laboratory. In actuality, a solid angle is rarely measured in practice. Instead, it is calculated from the threshold legible height (H) of letter Es measured at a constant viewing distance (6.1m) but at different incident angles (ξ), using (23), which is developed from (12) previously. One more factor also examined in this pilot experiment is the surrounding luminance (L_s) to preliminarily check the effect of ambient light on the legibility of text. To enhance the internal validity of this study, other non-examined factors all have preset values as listed in Table 11.

$$\omega = \frac{H^2}{6100^2}\cos\xi \tag{23}$$

where:

ω = Solid angle subtended by the legible text to the observer's eyes

H = Normal text height

ξ = Incident angle

Table 11. Preset values of non-examined factors in the pilot and main experiments

Non-examined factors	Preset values
Color contrast	Black/white
Average luminance contrast ($C_{\%}$)	97.9
Constant height-to-width ratio	1
Height-to-strokewidth ratio	5
Legibility distance	6.1 m, constant
Background luminance (L_b)	187.5 cd/m^2, constant
Recognition performance	Threshold legible with 100% accuracy
Recognition time	⩾500 ms
Spectrum of fluorescent	T8, daylight
Subjects age	20-29
Subject eyesight	20/20 or better, with or without glasses
Subject color vision	Normal

5.2.2.6 Experimental Tests, Procedures, and Duration

In the pilot experiment, there are a total of 32 experimental tests divided into 4 sessions, as shown in Table 12. Among all 32 tests, the background luminance (L_b) of the E-charts remains a constant 187.5 cd/m^2 by dimming the fluorescent fixture. These 32 tests are carried out in 10 steps in the laboratory. The duration of the experiment is about 157 minutes, or roughly 2.5 hours.

Table 12. A total of 32 tests in the pilot experiment

Tests	Sessions	Incident angle ξ in deg	Vert. angle α in deg (pitch)	Horz. angle ϕ in deg (yaw)	Ambient light level
1	1	0	0	0	zero
2		0	0	0	T12 lamps
3		30	0	30	zero
4		30	0	30	T12 lamps
5		60	0	60	zero
6		60	0	60	T12 lamps
7		75	0	75	zero
8		75	0	75	T12 lamps
9	2	31.5	31.5	0	zero
10		31.5	31.5	0	T12 lamps
11		42.4	31.5	30	zero
12		42.4	31.5	30	T12 lamps

Table 12 (continued)

Tests	Sessions	Incident angle ξ in deg	Vert. angle α in deg (pitch)	Horz. angle ϕ in deg (yaw)	Ambient light level
13	2	64.8	31.5	60	zero
14		64.8	31.5	60	T12 lamps
15		77.3	31.5	75	zero
16		77.3	31.5	75	T12 lamps
17	3	61	61	0	zero
18		61	61	0	T12 lamps
19		65.2	61	30	zero
20		65.2	61	30	T12 lamps
21		76	61	60	zero
22		76	61	60	T12 lamps
23		82.8	61	75	zero
24		82.8	61	75	T12 lamps
25	4	75	75	0	zero
26		75	75	0	T12 lamps
27		77	75	30	zero
28		77	75	30	T12 lamps
29		82.6	75	60	zero
30		82.6	75	60	T12 lamps
31		86.2	75	75	zero
32		86.2	75	75	T12 lamps

The 10 steps are listed below.

(1) Prepare for eyesight test and the first session. Before the scheduled potential subject comes to the laboratory, the experimenter has dimmed the surface luminance of the fluorescent fixture to a constant 187.5 cd/m^2 for all 4 incident angles at the first session with a vertical viewing angle $\alpha = 0°$, using the remote control (maximum 4 different light levels can be preset and recalled). The equipment prepared for the first trial (incident angle $\xi = 0°$) is also used to test the subjects' eyesight.

(2) Screen subjects upon arrival, 10 minutes. The subject is screened upon arrival to meet the requirements. To test the subject's acuity, the Snellen chart is first used under the general lighting of all ceiling lamps in the laboratory; then two self-made eyesight E-charts are used at a background luminance of 187.5 cd/m^2 ($\geqslant$ 120 cd/m^2 as required by the British standard BS 4274-1:2003). The experimenter asks whether the subject has abnormal color vision and what his/her age is. Only qualified subjects continue.

(3) Explain, sign consent form, 5 minutes. The experimenter explains the pilot experiment to the subject and answers any questions. After approval, the experiment is videotaped. The camcorder is mounted on a tripod placed behind the subject at the other end of the laboratory. Refusing to be videotaped does not make the subject illegible to participate. After explanation, the subject signs the consent form.

(4) Carry out the first session, 28 minutes. The first session includes 8 tests at 4 different viewing angles and 2 levels of ambient light, as shown in Table 12. The average duration of each test is 1.83 min, including 20 seconds to prepare (exchanging E charts and adjusting the viewing angles for different tests) and 10 seconds to record the data. Before each test starts, the subject has 2 min (appropriate due to the small fluctuation in the lighting conditions) to adapt their eyes to the light environment.

(5) Prepare for the second session, 10 minutes. The subject then has a 10-minute break out of the laboratory while the experimenter prepares for the second session by dimming the surface luminance of the fluorescent fixture to 187.5 cd/m^2 at 4 viewing angles to be examined during the second session.

(6) Carry out the second session, 28 minutes. Similar to the first session.

(7) Prepare for the third session, 10 minutes. Likewise, the subject has another 10-minute break while the experimenter prepares for the third session.

(8) Carry out the third session, 28 minutes.

(9) Prepare for the fourth session, 10 minutes.

(10) Carry out the fourth session, 28 minutes.

5.2.2.7 Data analysis

For all 32 tests, as illustrated in Figure 57, the mean background luminance of the E-charts is 187.5 cd/m^2 at a statistical significance level of 0.94 (very probably true), with a standard deviation of 5.5 cd/m^2. Figure 58 shows the threshold legible heights of letter Es viewed at an eyesight level of either 20/12.5 or 20/20 at 16 incident angles under two levels of ambient light (zero, or that provided by T12 lamps). Figure 59 illustrates the solid angles subtended by those threshold legible letter Es collected in Figure 58 at 16 incident angles. In both figures, the legibility data are sorted by incident angles increasing from zero to the largest viewing angle.

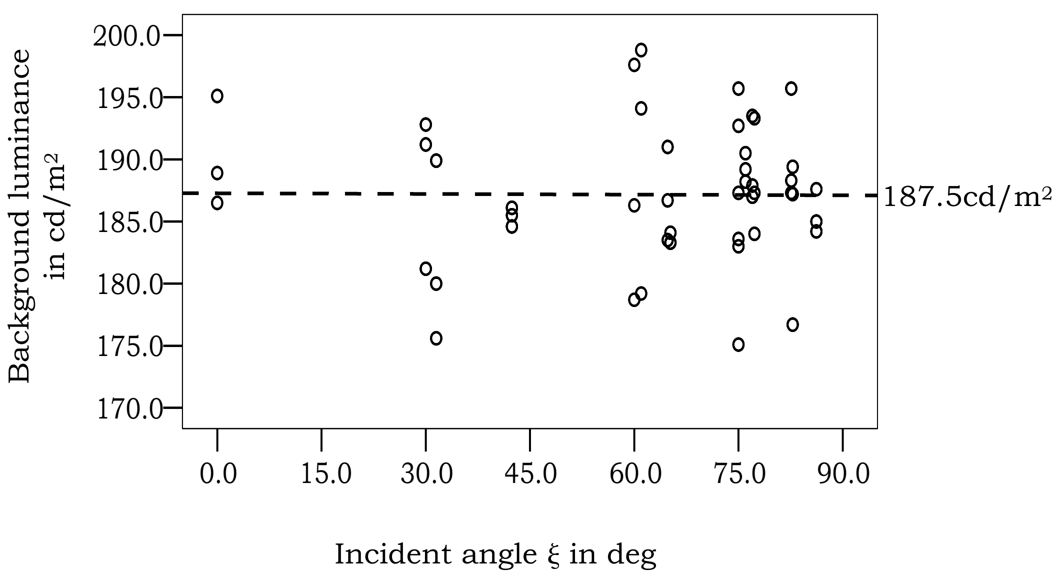


Figure 57. Background luminance of letter Es viewed at different incident angles under two different levels of ambient light (zero, or that by T12 lamps), at mean 187.5 cd/m^2 with standard deviation 5.5 cd/m^2

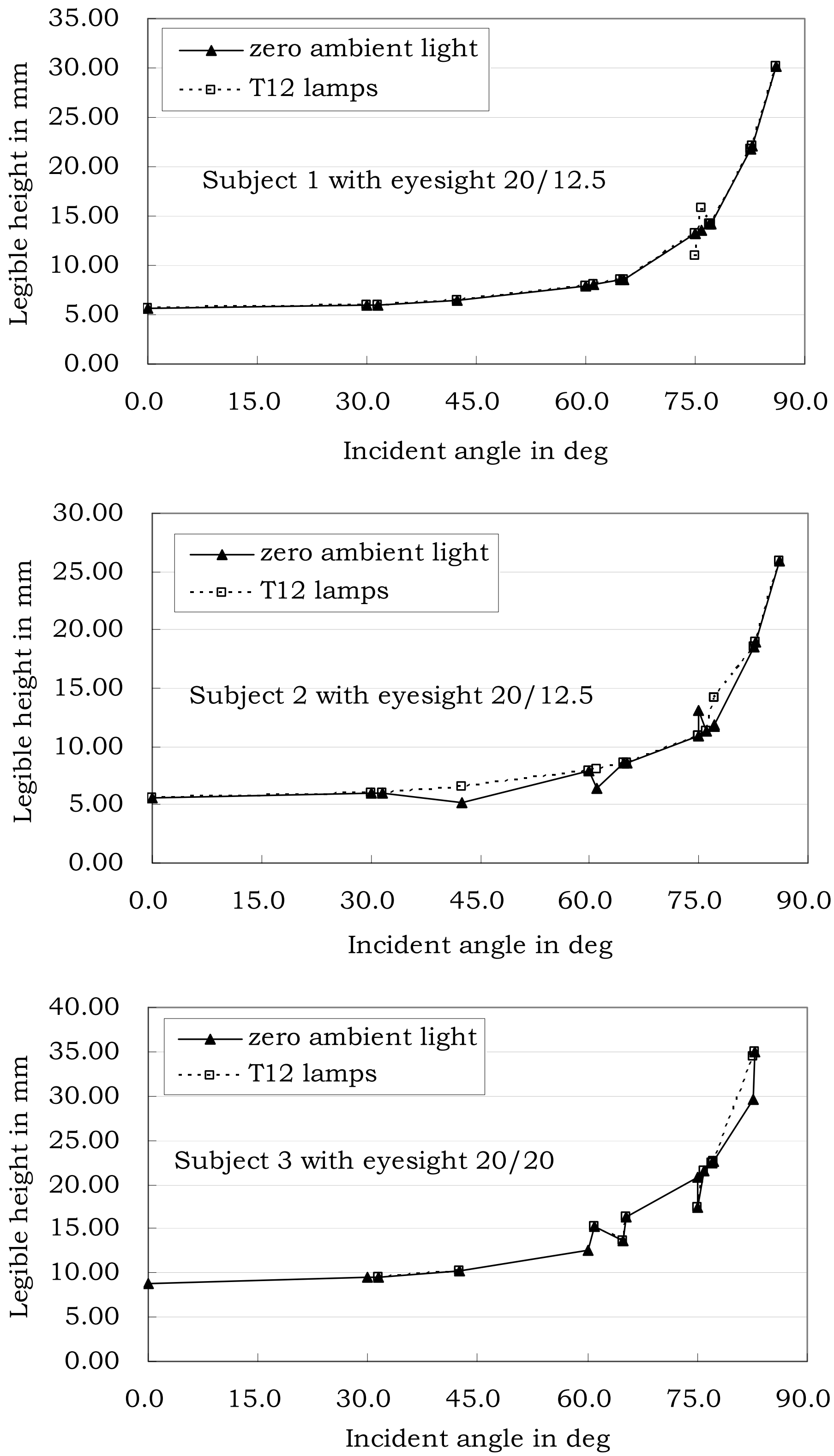


Figure 58. Threshold legible height of letter Es viewed at different incident angles under two different levels of ambient light (zero, or that by T12 lamps)

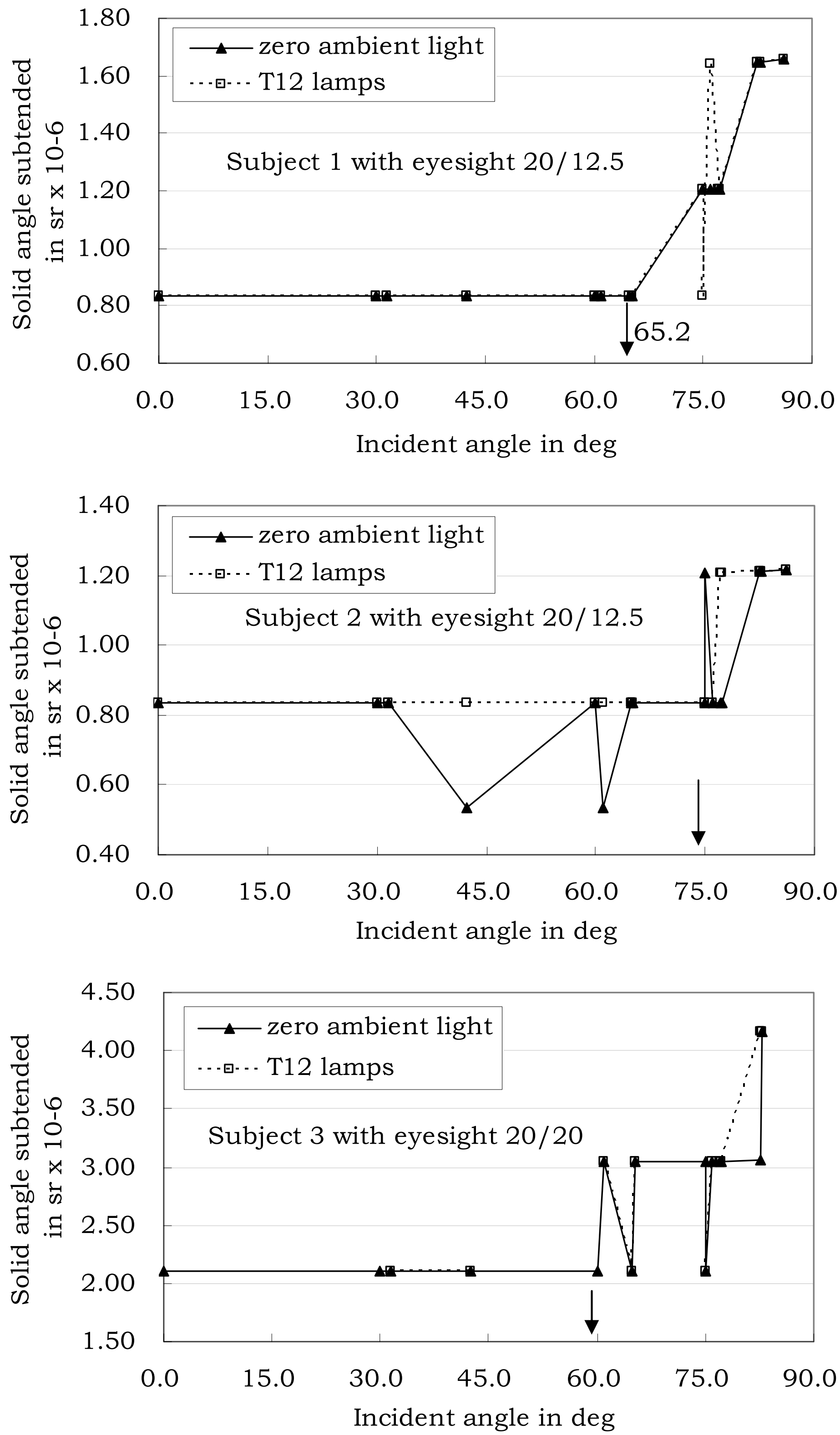


Figure 59. Solid angles subtended by the threshold legible letter Es viewed at different incident angles under two different levels of ambient light (zero, or that by T12 lamps)

5.2.2.8 Preliminary Findings and Expected Improvements

Preliminary finding (1): The constant-solid-angle hypothesis probably holds when $0° \leq \xi \leq 66.7°$. The solid angle subtended by the threshold legible letter Es remains almost constant when the incident angle ξ ranges from 0° to approximately 66.7°. The angle 66.7° is averaged using three critical incident angles ((65.2°+75°+60°)/3=66.7°) at the three breaking points as indicated by arrows in Figure 59, where the constant-solid-angle hypothesis no longer holds. When incident angle ξ is larger than 66.7° till 90°, the hypothesis does not hold. When $66.7° < \xi \leq 90°$, larger characters have been recognized with threshold legibility (100% accuracy). Such larger characters subtend larger solid angles to the observer's eyes. The larger the incident angle ξ between 66.7° and 90°, the larger the solid angle subtended by the threshold legible letter Es. This might be due to: (a) glare caused by the high bright fringe of the fixture surface on the far end facing the observer; (b) extremely distorted letter Es that are inconsistent with the observer's reading habits and are thus harder to recognize; (c) eye fatigue of the observer at the later stage of experiment when larger viewing angles are tested.

Preliminary finding (2): Ambient light probably has a very limited effect on the legibility of text as long as the background luminance remains constant. As long as the background luminance remains constant, as illustrated in Figure 58, the legible heights of letter Es viewed under zero ambient light or T12 lamps are almost the same. However, the limited change might be due to the fact that the ambient light level provided by T12 lamps in the pilot study is too low, as illustrated in Figure 54 (b). Therefore, the ambient light will be enhanced in the main experiment by turning on the ceiling lamps of the laboratory. The absence of surrounding luminance in the derived legibility equation might well be tolerable for its accurate predictability of legibility at different ambient lights.

The pilot experiment also suggests that several improvements need to be made in the follow-up main experiment.

1. The background luminance of the E-charts will be changed from 187.5 cd/m^2 to 120 cd/m^2 to more closely match the one surveyed in the 38 lecture halls at the University of Michigan
   (2.81 cd/m^2 ± 4.73 cd/m^2 $\leq L_b \leq$ 86.00 cd/m^2 ±102.28 cd/m^2) and meet the requirement of threshold background luminance (⩾120 cd/m^2) for testing eyesight according to the British standard BS 4274-1:2003.
2. In terms of viewing angles, a reduced range of four vertical angles (α = 0°, 31.5°, 46.5°, 61°) will be used in the main experiment to replace the four angles used in the pilot experiment (α = 0°, 31.5°, 61°, 75°) to match the actual range of maximum vertical viewing angles (mean 43.6° with standard deviation of 11.8°) surveyed in the 38 lecture halls. Note that the wider vertical viewing angles were purposely used in the pilot experiment to extend the experimental conditions.
3. As indicated in Figure 59, the apparent inconsistent performance of the three subjects during the pilot experiment suggests stricter screening of potential subjects to find their true acuity level to increase their reading performance. Subject 3 performed worse than the other two subjects largely due to the

experimenter's haste in measuring her acuity level. Her actual eyesight might be 20/25 rather than 20/20. Therefore, more eyesight charts should be used and a longer time should be taken in the follow-up main experiment to double check the potential subject's eyesight.

4. A black cloth should cover the bright fringe of the dimmable fixture surface on the far end facing the observer to prevent direct glare. Likewise, wider coverage of the white floor between the fixture and the observer are needed in order to reduce the reflective glare from the fixture to the observer's eye.
5. The order of viewing angles tested in all trials should be randomly arranged to avoid possible fatigue of the observer's eyes at larger incident angles, to distinguish its influence on the reading performance of text from that of larger incident angles.

5.2.3 Main Experiment

The main experiment carried out in the same laboratory uses 20 subjects with different eyesight levels (20/20, 20/16, or 20/12.5) to verify the constant-solid-angle hypothesis under zero ambient light level. Zero ambient light is used to (a) exclude its unverified effect on the legibility of text, (b) prevent reflective glare from the illuminated back half of the laboratory or highlighted ceiling reflected on the opalescent surface of the dimmable fixture and direct glare from the ceiling lamps in front of the observers, and (c) avoid the addition of ambient light on the E-charts, which might wash out the image contrast, degrade the retinal image, and produce reflectance glare when the E-charts are viewed at large viewing angles.

5.2.3.1 Improvements in the Experimental Settings

The revised experimental set up is shown in Figure 60, with the full ceiling lights of the laboratory turned off during the experiment. Wider dark cloth has replaced the previous narrow carpet to cover whiter floor between the fixture and the observer. At large viewing angles, the bright fringe of the dimmable fixture surface on the far end facing the observer is also covered to prevent direct glare, as shown in Figure 61. The new range of 16 incident angles ξ is shown in Table 13, which is more evenly distributed. The surface luminance of the fixture (L_b) is dimmed to a constant 120 cd/m^2 at all 16 viewing angles.

Figure 60. Revised experimental settings in the main experiment

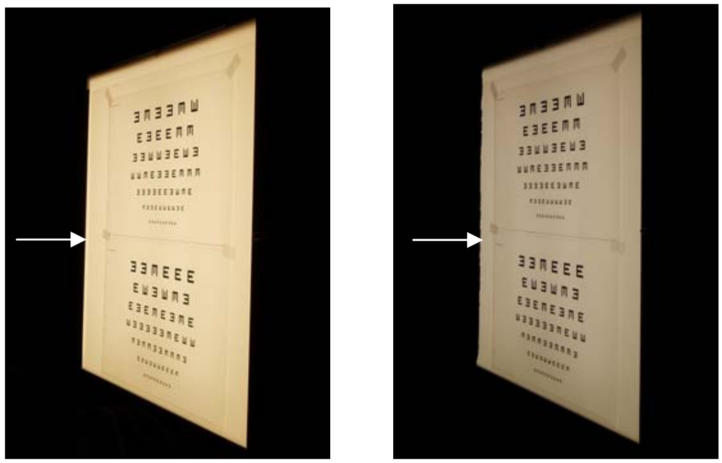


Figure 61. No coverage (left) and coverage (right) of the bright fringe of the dimmable fixture surface, using a dark cloth strip

Table 13. Total 16 incident angles ξ examined in the main experiment

Horz. angle ϕ in deg (yaw)	Vert. angle α in deg (pitch)	Incident angle ξ in deg
0	0	0.0
30	0	30.0
0	31.5	31.5
30	31.5	42.4
0	46.5	46.5
30	46.5	53.4
60	0	60.0
0	61	61.0
60	31.5	64.8
30	61	65.2
60	46.5	69.9
75	0	75.0
60	61	76.0
75	31.5	77.3
75	46.5	79.7
75	61	82.8

The requirement for 20 subjects is based on the fact that only one factor — the solid angle subtended by the threshold legible letter Es at 16 viewing angles — is examined, while the legible heights of letter Es at 16 viewing angles are actually recorded in the laboratory. After consultation with Dr. Brenda Gillespie of the Center for Statistical Consulting and Research (CSCAR) at the University of Michigan, it is determined that a sample size of 20 will enable the main experiment (a binomial test: height versus viewing angle) with a nominal 0.05 one-sided significance level to have 93% power to detect the difference between the null hypothesis proportion (π_0 of 0.99) and the alternative proportion (π_A of 0.80). This main experiment recruits 42 subjects; 22 are screened by the requirements but only 20 qualified subjects participate in the experiment.

The E-charts used in this main experiment are updated with the new range of 16 viewing angles as listed in Table 13. The corresponding ranges of heights of 7 lines of letter Es for different eyesight levels (20/20, 20/16, 20/12.5) at these 16 viewing angles are listed in Appendix F. Likewise, a total of 48 E-chart sheets are used in the main experiment for 3 levels of eyesight at 16 viewing angles.

5.2.3.2 Viewing Scenarios of 16 Tests

Figure 62 illustrates the viewing scenarios of 16 tests in the laboratory with a constant background luminance (L_b) of 120 cd/m^2, an image contrast ($C_{\%}$) of 97.9, and zero ambient light. Due to unavoidable fluctuations when manually dimming fixture surface luminance, 120.7 cd/m^2 has actually been observed as the average background luminance in a total of 320 tests for 20 subjects, with 16 tests each ($20 \times 16 = 320$), as illustrated in Figure 63. The experimenter collects the heights (H) of the threshold legible letter Es recognized with 100% accuracy by the 20 subjects sitting 6.1 m away at 16 viewing angles under a constant lighting condition (image background luminance L_b=120.7 cd/m^2, image contrast C%=97.9, and zero ambient light). Solid angles subtended by these legible letter Es are then calculated from these heights using (23) previously to verify the constant-solid-angle hypothesis.

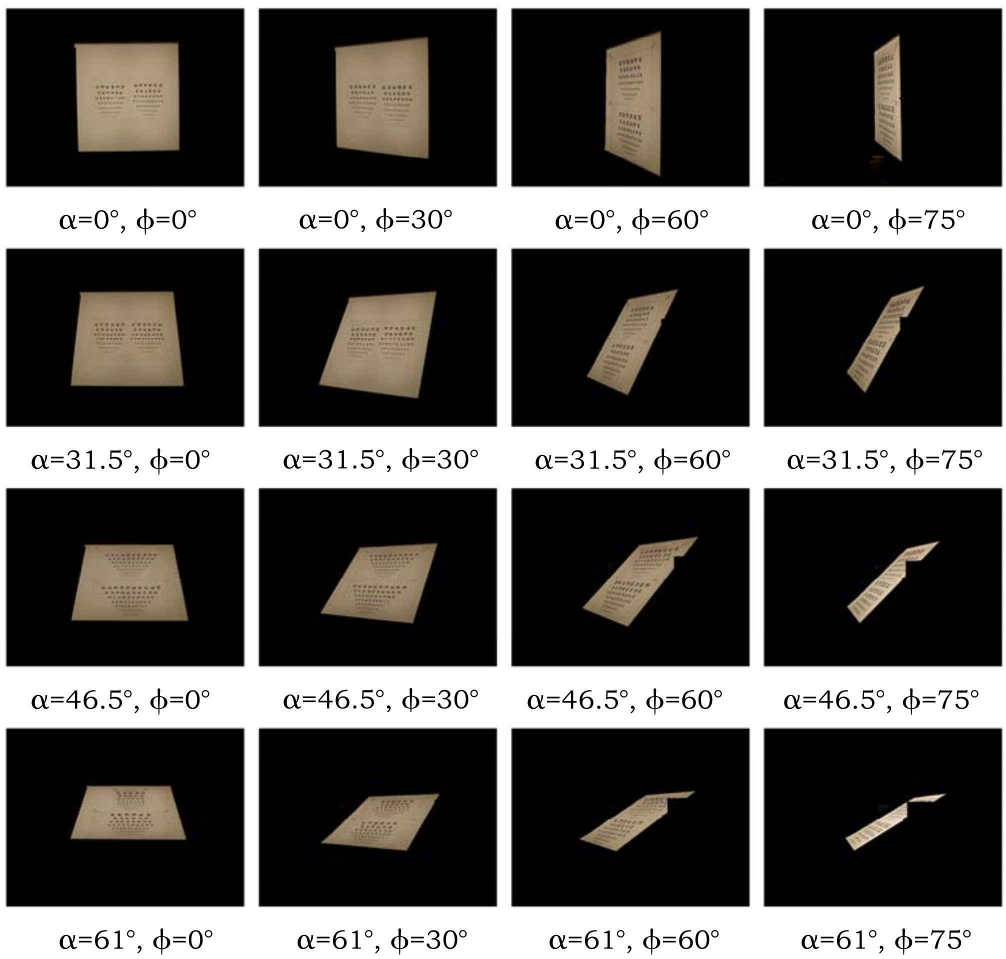


Figure 62. Viewing scenarios of 16 tests in the laboratory with constant background luminance L_b=120.7 cd/m^2, image contrast $C_{\%}$= 97.9, and zero ambient light

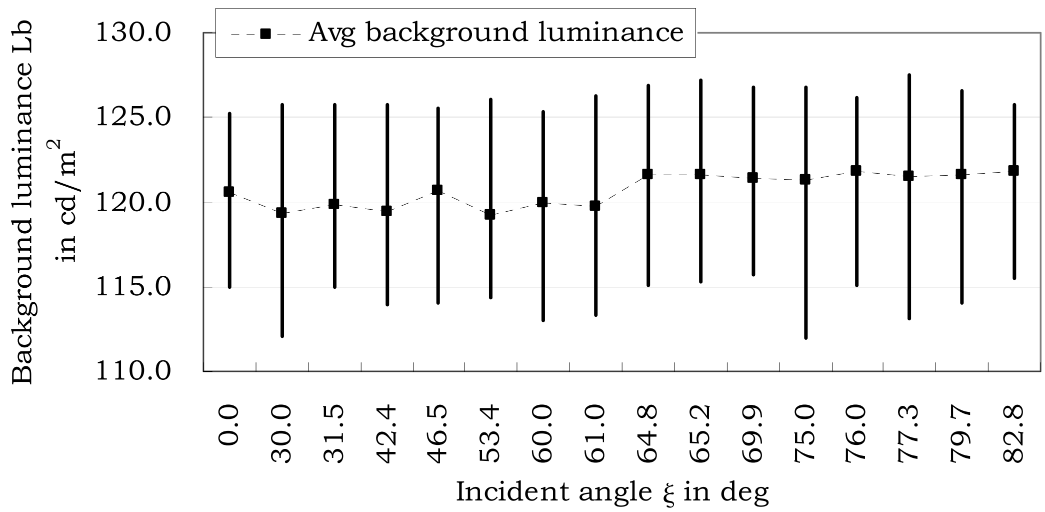


Figure 63. Mean background luminances of E-charts viewed at each of 16 tests

5.2.3.3 Experimental Procedures and Random Tests

Table 14 lists the procedures and duration for carrying out the 16 tests, which are divided into 4 sessions in this main experiment. The duration of the main experiment is approximately 100 minutes. Except for the first session, which is carried out first and has a fixed order for its 4 tests (ξ = 0°, 30°, 60°, 75°), as shown in Figure 64, the other three sessions and the order of the 12 tests are randomly arranged to counteract the negative effect of eye fatigue on the reading performance of text at larger incident angles.

Table 14. Procedure and duration of the main experiment

Order	Procedures	Minutes
1	Prepare for eyesight test and the first session	n/a
2	Screen subjects	10
3	Paperwork, explain	5

Table 14 (continued)

Order	Procedures					Minutes
	Trial	Session	ϕ in deg	α in deg	ξ in deg	
4	1	1	0	0	0.0	10
	2		30	0	30.0	
	3		60	0	60.0	
	4		75	0	75.0	
5	Dimming fixture, subject has break					15
6	Random	2	0	31.5	31.5	10
			30	31.5	42.4	
			60	31.5	64.8	
			75	31.5	77.3	
7	Dimming fixture, subject has break					15
8	Random	3	0	46.5	46.5	10
			30	46.5	53.4	
			60	46.5	69.9	
			75	46.5	79.7	
9	Dimming fixture, subject has break					15
10	Random	4	0	61	61.0	10
			30	61	65.2	
			60	61	76.0	
			75	61	82.8	
					Total	100

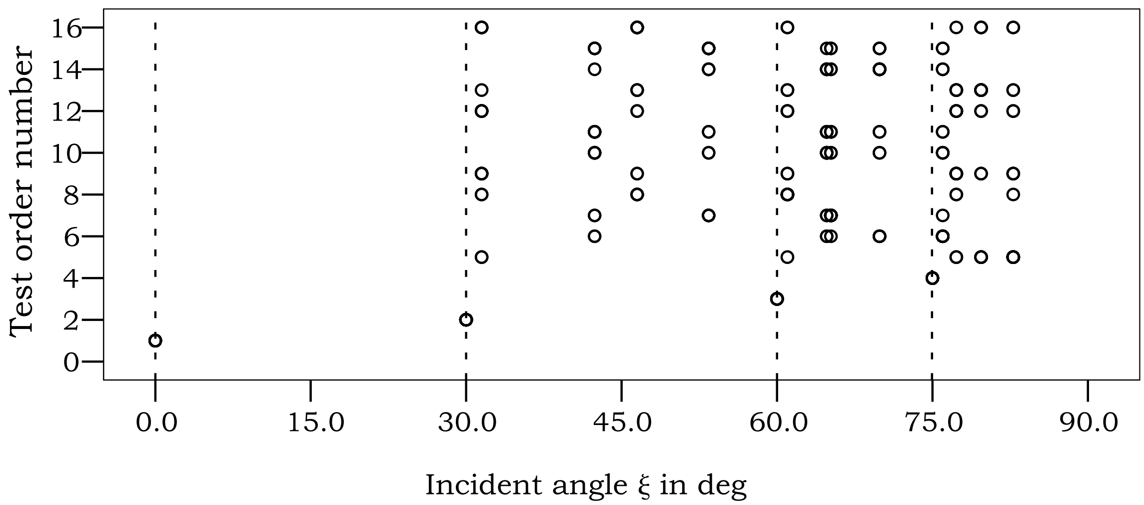
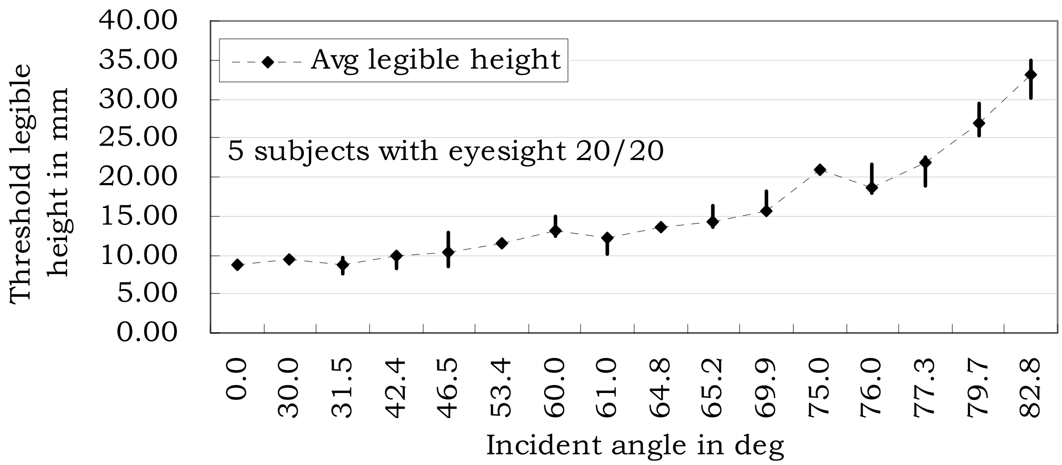
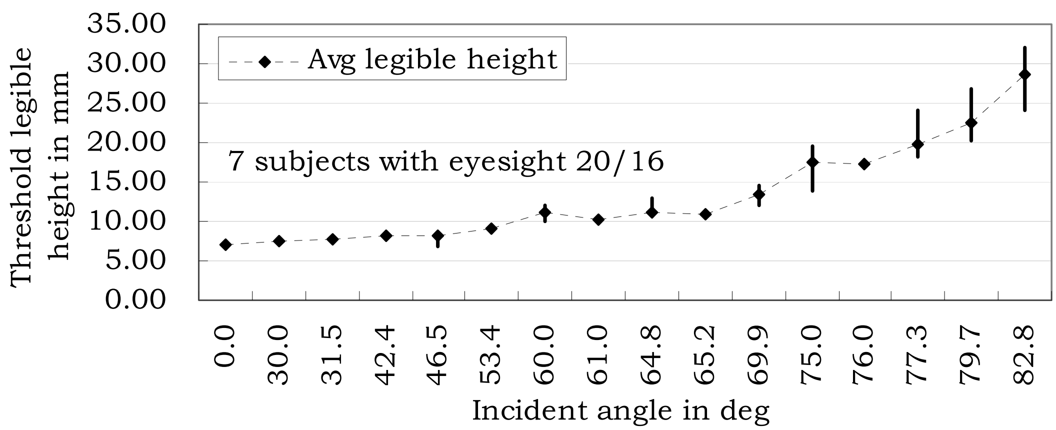


Figure 64. Random order of 12 tests in the main experiment in the later three sessions, and the fixed order of the four tests (0°, 30°, 60°, 75°) in the first session, as indicated by the dash line

5.2.3.4 Data Analysis

Figure 65 lists the threshold legible heights of letter Es recognized at each incident angle with 100% accuracy separately by 5 subjects with eyesight 20/20, by 7 subjects with eyesight 20/16, and by 8 subjects with eyesight 20/12.5, a total of 20 subjects. Figure 66 lists the calculated solid angles subtended by those threshold legible letter Es.





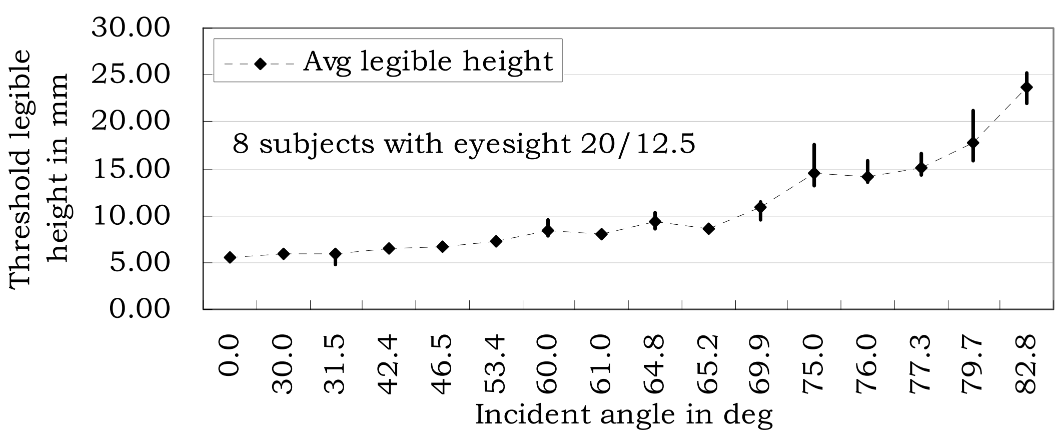


Figure 65. Threshold legible heights of letter Es recognized at 16 incident angles by subjects at different eyesight levels (20/20, 20/16, 20/12.5)

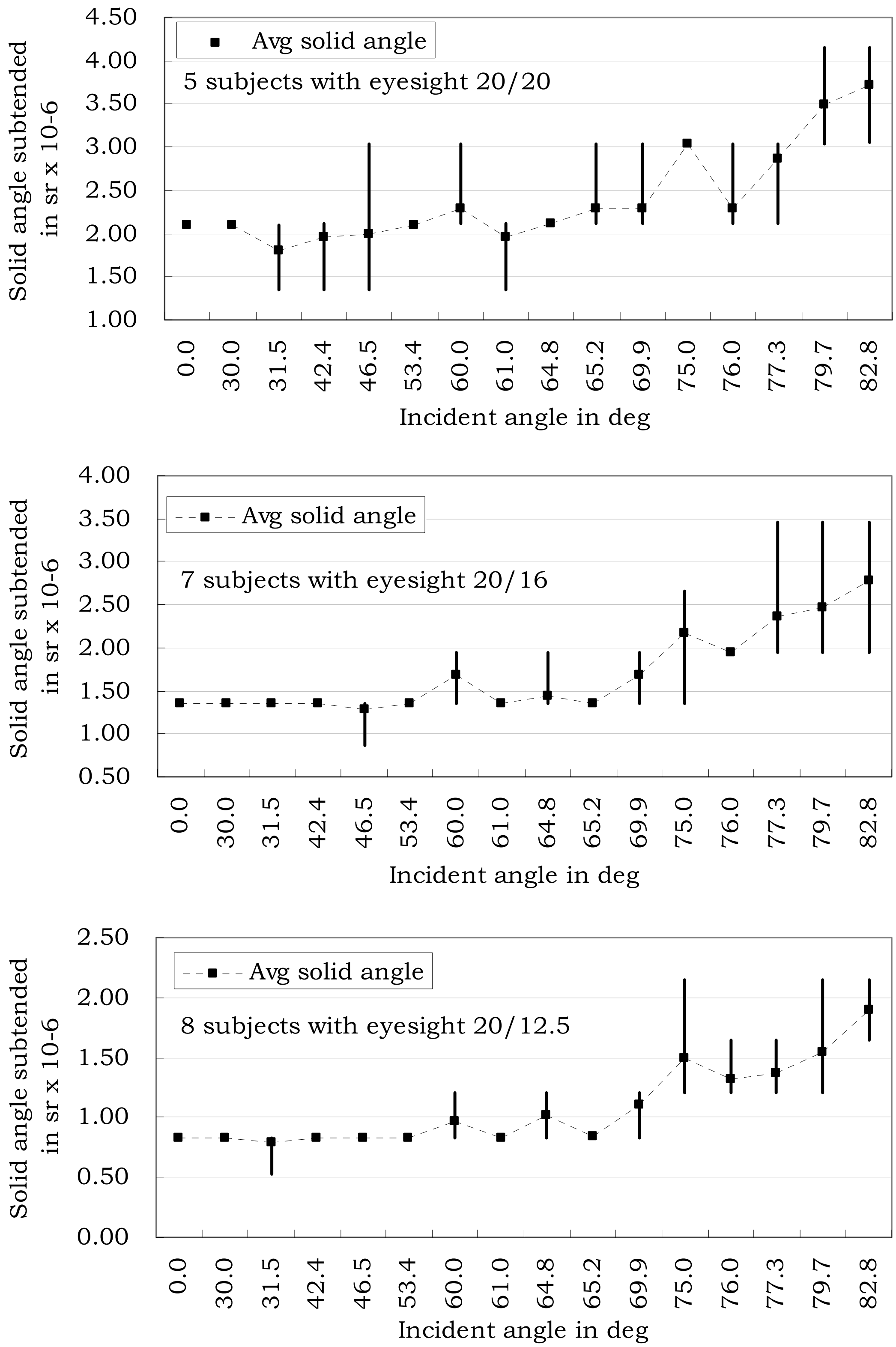


Figure 66. Calculated solid angles subtended by the threshold legible letter Es recognized at 16 incident angles by 20 subjects at three eyesight levels

To test the constant-solid-angle hypothesis, the legibility data collected from 20 subjects need a ratio of the observed legibility data in the laboratory to the theoretically predicted ones based on the constant-solid-angle hypothesis. This ratio must be independent of the three eyesight levels (20/20, 20/16, 20/12.5). If the required ratios calculated for all 16 viewing angles all equal 1, the constant-solid-angle hypothesis is thus validated.

In the main experiment, two correlated ratios are used.

$H_{observed}$ / $H_{predicted}$. Ratio of the observed threshold legible heights of letter Es in the laboratory to those of the predicted letter Es that subtend a constant solid angle for 16 viewing angles for all three eyesight levels, as shown in Figure 67.

$\omega_{observed}$ / $\omega_{predicted}$. Ratio of the solid angles ω subtended by the observed threshold legible letter Es for all 16 incident angles to the predicted constant solid angle based on the constant-solid-angle hypothesis, as shown in Figure 68.

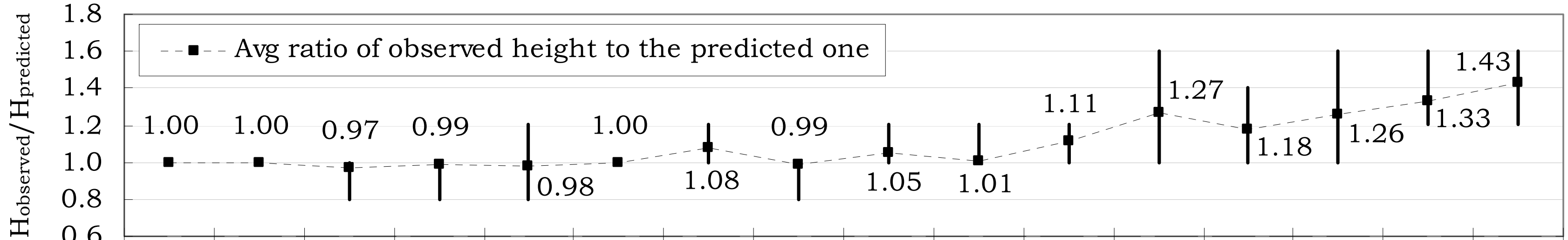
According to (23), these two correlated ratios are in the relationship as in (24).

$$\left(\frac{H_{observed}}{H_{predicted}}\right)^2 = \frac{\omega_{observed}}{\omega_{predicted}} \qquad (24)$$

where

H = Threshold legible height of letter Es

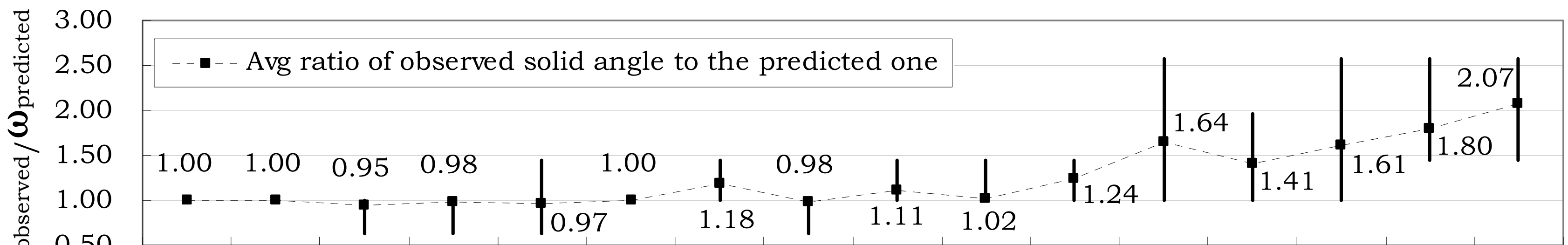
ω = Solid angles subtended by the threshold legible letter Es



ξ	0.0	30.0	31.5	42.4	46.5	53.4	60.0	61.0	64.8	65.2	69.9	75.0	76.0	77.3	79.7	82.8
ϕ	0	30	0	30	0	30	60	0	60	30	60	75	60	75	75	75
α	0	0	31.5	31.5	46.5	46.5	0	61	31.5	61	46.5	0	61	31.5	46.5	61

Incident angle (ξ), and its horizontal (ϕ) and vertical portion (α) in deg

Figure 67. Ratios of the observed threshold legible heights of letter Es to the predicted ones versus 16 incident angles



ξ	0.0	30.0	31.5	42.4	46.5	53.4	60.0	61.0	64.8	65.2	69.9	75.0	76.0	77.3	79.7	82.8
ϕ	0	30	0	30	0	30	60	0	60	30	60	75	60	75	75	75
α	0	0	31.5	31.5	46.5	46.5	0	61	31.5	61	46.5	0	61	31.5	46.5	61

Incident angle (ξ), and its horizontal (ϕ) and vertical portion (α) in deg

Figure 68. Ratios of the solid angles subtended by these observed threshold legible letter Es to the predicted ones to hold the constant-solid-angle hypothesis versus 16 incident angles

Table 15 lists the predicted threshold legible heights of letter Es viewed at 16 incident angles and their subtended constant solid angle based on the constant-solid-angle hypothesis, respectively for three eyesight levels (20/20, 20/16, 20/12.5). The predicted constant solid angle for all 16 incident angles is determined in the laboratory as the ones subtended by the threshold letter Es viewed at zero incident angle.

Figure 69 illustrates the scattergram of the average $H_{observed}/H_{predicted}$ ratios of the threshold legible heights of letter Es collected from the 20 subjects at each of the 16 incident angles, while Figure 70 illustrates the scattergram of the average $\omega_{observed}/\omega_{predicted}$ ratios versus 16 incident angles. Such average ratios can counteract the individual differences of the subjects' eyes and age and provide 93% power for detecting the constant-solid-angle hypothesis in the main experiment with a nominal 0.05 one-sided significance level.

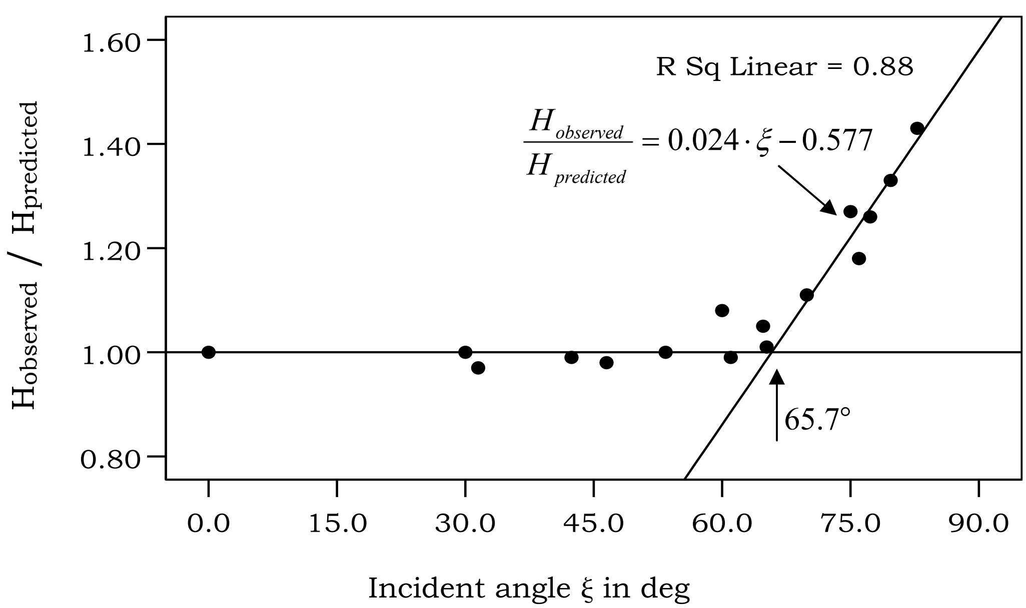


Figure 69. Simple scattergram of $H_{observed}/H_{predicted}$ ratios versus 16 incident angles

Table 15. Predicted threshold legible heights of letter Es at 16 incident angles and the predicted solid angles subtended by these threshold legible letter Es at three eyesight levels to hold the constant-solid-angle hypothesis

α	ϕ	ξ	For eyesight 20/20		For eyesight 20/16		For eyesight 20/12.5	
Vert. angles in deg	Horz. angles in deg	Incident angles in deg	Threshold legible height in mm	Subtended solid angle in sr $\times 10^{-6}$	Threshold legible height in mm	Subtended solid angle in sr $\times 10^{-6}$	Threshold legible height in mm	Subtended solid angle in sr $\times 10^{-6}$
0	0	0.0	8.85	2.10	7.08	1.35	5.57	0.83
0	30	30.0	9.51	2.10	7.61	1.35	5.99	0.83
31.5	0	31.5	9.58	2.10	7.67	1.35	6.03	0.83
31.5	30	42.4	10.30	2.10	8.24	1.35	6.48	0.83
46.5	0	46.5	10.67	2.10	8.53	1.35	6.71	0.83
46.5	30	53.4	11.46	2.10	9.17	1.35	7.21	0.83
0	60	60.0	12.52	2.10	10.01	1.35	7.88	0.83
61	0	61.0	12.71	2.10	10.17	1.35	8.00	0.83
31.5	60	64.8	13.55	2.10	10.84	1.35	8.53	0.83
61	30	65.2	13.66	2.10	10.93	1.35	8.60	0.83
46.5	60	69.9	15.09	2.10	12.07	1.35	9.49	0.83
0	75	75.0	17.40	2.10	13.92	1.35	10.95	0.83
61	60	76.0	17.98	2.10	14.38	1.35	11.31	0.83
31.5	75	77.3	18.84	2.10	15.07	1.35	11.86	0.83
46.5	75	79.7	20.97	2.10	16.77	1.35	13.20	0.83
61	75	82.8	24.98	2.10	19.99	1.35	15.72	0.83

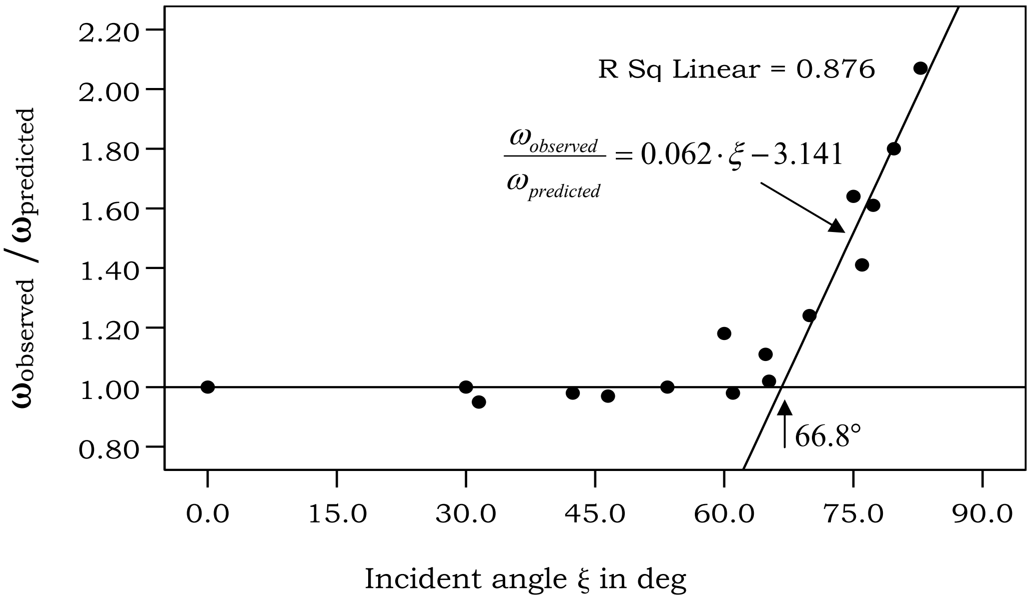


Figure 70. Simple scattergram of the $\omega_{observed}/\omega_{predicted}$ ratios versus 16 incident angles

As shown in Figure 69, the trendline of the average $H_{observed}/H_{predicted}$ ratios approximately form a horizontal line at 1 ($H_{observed}/H_{predicted} = 1$) with fluctuations less than 0.1 when incident angle ξ is smaller than a critical value around 65°. From this critical value till 82.8°, which is the maximum viewing angle tested in the laboratory, the average $H_{observed}/H_{predicted}$ ratios increase linearly with the incident angle, following (25) as regressed in SPSS with $R^2 = 0.88$. The breaking point is at the critical value, which has been calculated to be 65.7° when (25) equals 1.

$$\frac{H_{observed}}{H_{predicted}} = 0.024 \cdot \xi - 0.577 \tag{25}$$

where

$H_{observed}/H_{predicted}$ = Ratio of the observed threshold legible height of letter Es in the experiment to the predicted one based on the constant-solid-angle hypothesis.

ξ = Incident angle, 65.7° < ξ ≤ 82.8° (the largest viewing angle tested in the laboratory)

In terms of the statistical significance of the horizontal line ($H_{observed}/H_{predicted} = 1$) when 0° ≤ ξ ≤ 65.7°, note that in the main experiment, as well as in other experiments using E-charts as viewing materials, the minimum increase of $H_{observed}/H_{predicted}$ ratio is 0.2, equals the ratio of the threshold legible strokewidth of letter Es to its height. Therefore, since the fluctuations of the observed average $H_{observed}/H_{predicted}$ ratios are less than half the minimum increase (0.2) when 0° ≤ ξ ≤ 65.7°, the $H_{observed}/H_{predicted} = 1$ is assumed to have 93% power provided by the sample size of 20 in the main experiment to interpret the real legibility data when incident angle ξ ≤ 65.7°.

Likewise, as shown in Figure 70, the average $\omega_{observed}/\omega_{predicted}$ ratios remain at 1 ($\omega_{observed}/\omega_{predicted} = 1$) with 93% power (N = 20), when incident angle 0°≤ ξ ≤ 66.8°. The average $\omega_{observed}/\omega_{predicted}$ ratios increase linearly using (26) with $R^2 = 0.876$, when incident angle 66.8°< ξ ≤ 82.8°.

$$\frac{\omega_{observed}}{\omega_{predicted}} = 0.062 \cdot \xi - 3.141 \qquad (26)$$

where

$\omega_{observed}/\omega_{predicted}$ = Ratio of the solid angle subtended by those observed threshold legible letter Es to the predicted constant one based on the constant-solid-angle hypothesis.

ξ = Incident angle, 66.8° < ξ ≤ 82.8°

5.2.3.5 Outcomes

1. The constant-solid-angle hypothesis holds when $0° \leqslant \xi \leqslant 65.7°$; it does not hold when $65.7° < \xi \leqslant 82.8°$ (the largest incident angle examined in the main experiment). For viewing angles $82.8° < \xi \leqslant 90°$, which are rare in lecture halls, the constant-solid-angle hypothesis most likely does not hold either, but further experimentation is needed to verify. The reasons for choosing 65.7° as shown in Figure 69 as the critical angle rather than 66.8° as shown in Figure 70 include: (a) the smaller critical angle 65.7° is more conservative and thus more reliable; (b) the threshold legible height rather than the solid angle is normally measured in practice; thus the 65.7° associated with height should be used; (c) the derived legibility equation in this study will be improved later in light of the legible height of letter Es, rather than the subtended solid angle; thus, the 65.7° associated with the threshold legible height is more appropriate.
2. The negative effect of extremely large horizontal viewing angles ($\phi = 60°, 75°$) against the observer's reading habit explains the jumping points (higher than the adjacent before and after points) at angles $\xi = 60°, 64.8°, 75°$, as shown in Figures 67, 68, 69, or 70.
3. Under the same viewing condition, the threshold legible height of letter Es is determined by the subjects' eyesight level, as well as affected by individual differences such as age and light scattering characteristics (astigmatism) inside eyes, which explain the data span (errors) at most of the viewing angles, as shown in Figures 65, 66, 67, or 68.

4. The slightly improved reading performance (with average $H_{observed}/H_{predicted} < 1$) at angles ξ = 31.5°, 42.4°, 46.5°, 61°, as shown in Figure 67, is probably due to the rewarding effect of the decreased pupil size that enhances the visual depth as a result of the slight amount of ambient light, which results from the light reflected from the white surfaces of the laboratory ceiling lamps that have actually been turned off when the dimming fixture is tilted up.

5.3 Improvement of the Derived Equation

Based on the the pilot and main laboratory experiments, the constant-solid-angle hypothesis holds only when the incident angle $0° \leqslant \xi \leqslant 65.7°$; it does not hold when $65.7° < \xi \leqslant 82.8°$, or possibly even to 90°. Therefore, the derived legibility equation in this study, (17) or (18) previously, should be improved to match the observed legibility data collected in the laboratory so that it can better predict the reading performance of text in reality. Improvement requires two steps. First, re-express (25) as (27).

$$H_{observed} = H_{predicted} \cdot (0.024\xi - 0.577) \tag{27}$$

where

$H_{observed}$ = Observed threshold legible height of letter Es in practice

$H_{predicted}$ = Predicted threshold legible height which holds the constant-solid-angle hypothesis, which is calculated using (17) previously

ξ = Incident angle, 65.7° < ξ ⩽ 82.8°

Second, substitute (27) into (17) when the incident angle 65.7° < ξ ⩽ 82.8°, getting (28), which is the improved legibility equation. For incident angles between 82.8° and 90°, which have not yet been tested in the laboratory, (28) becomes inapplicable. Fortunately, text is rarely viewed at such extremely large viewing angles between 82.8° and 90° in lecture halls or other viewing scenarios.

$$D = \begin{cases} 2443.5 \cdot H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{\ -1} \cdot L_b^{\ 0.213} \cdot C_{\%}^{\ 0.532} \cdot (\cos\xi)^{0.5} & 0^0 \le \xi \le 65.7^0 \\ 2443.5 \cdot H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{\ -1} \cdot L_b^{\ 0.213} \cdot C_{\%}^{\ 0.532} \cdot (\cos\xi)^{0.5} \cdot (0.024\xi - 0.577)^{-1} & 65.7^0 < \xi \le 82.8^0 \end{cases} \tag{28}$$

where:

D = Legibility distance when text is viewed not perpendicular to the observer

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of observer's acuity level

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ξ = Incident angle between the display normal and the sightline of observer

5.4 Summary

A new equation for predicting the spatial legibility of text from seven critical factors is derived from the existing Howett's equation (1983), based on a constant-solid-angle hypothesis, as

$$H = 4.1\times10^{-4}\cdot D\cdot\left(\frac{H}{Sw}\right)\cdot S_d\cdot L_b^{-0.213}\cdot C_{\%}^{-0.532}\cdot\left(\cos\phi\right)^{-0.5}\cdot\left(\cos\alpha\right)^{-0.5}$$

(D = Legibility distance of text viewed not perpendicular to the display, H = Normal text height, H/Sw = Height-to-strokewidth-ratio of text, S_d = Denominator in the Snellen ratio of observer's eyesight, L_b = Background luminance, $C_{\%}$ = Luminance contrast percent, ϕ = Horizontal viewing angle, α = Vertical viewing angle). This hypothesis is then verified consistent with how retinal images of text activate cones in the centre fovea of an observer's eyes. In addition, this hypothesis is tested in the lighting laboratory at the University of Michigan Transportation Research Institute (UMTRI) using legibility data collected from 3 human subjects participating in a pilot experiment and 20 subjects participating in the follow-up main experiment. The outcomes show that the constant-solid-angle hypothesis holds when the incident angle $0° \leqslant \xi \leqslant 65.7°$, which is the viewing angle between the display normal and the sightline of the observer, calculated using $\cos\xi = \cos\phi\cos\alpha$, but does not hold when $65.7° < \xi \leqslant 82.8°$ (the largest incident angle examined in the main experiment). Consequently, the legibility equation is improved as:

$$D = \begin{cases} 2443.5\cdot H\cdot\left(\frac{H}{Sw}\right)^{-1}\cdot S_d^{-1}\cdot L_b^{0.213}\cdot C_{\%}^{0.532}\cdot\left(\cos\xi\right)^{0.5} & 0^0 \le \xi \le 65.7^0 \\ 2443.5\cdot H\cdot\left(\frac{H}{Sw}\right)^{-1}\cdot S_d^{-1}\cdot L_b^{0.213}\cdot C_{\%}^{0.532}\cdot\left(\cos\xi\right)^{0.5}\cdot\left(0.024\xi - 0.577\right)^{-1} & 65.7^0 < \xi \le 82.8^0 \end{cases}$$

CHAPTER 6

Step 3: Testing Ambient Light Effect on Legibility of Text

The verified and improved legibility equation (28) in Chapter 5 does not examine the factor of ambient light. However, as described in Chapter 2, the ambient light can affect the quality of the retinal image of text viewed. Therefore, its effect on the legibility of text should be examined.

6.1 Theoretical Foundation for the Ambient-Light Hypothesis

Based on the model proposed by Moon and Spencer (1945) for calculating the adaptation luminance, this study has developed an ambient-light hypothesis.

Ambient-light hypothesis. Without glare sources in the periphery (beyond 1° or 1.5° visual angle) of an observer's field of view, ambient light in the viewing environment should have a small influence (less than 9%) on the legibility level of text when viewed with constant background luminance and luminance contrast.

This hypothesis is developed based on the adaptation luminance. Adaptation luminance refers to the average luminance of those objects and surfaces in the immediate vicinity of the observer, including the luminance of the observer's fixation point, which covers approximately one visual degree (1.5° by some authors), and the surrounding luminances (Matković, 1997). Moon and Spencer (1945) proposed an equation to calculate the adaptation luminance, as (29). According to (29), an observer's eyes primarily adapt to the luminance of their fixation point (1°, visual angle), while the surrounding luminance beyond 1° in the viewing field also contributes to the adaptation luminance (Matković, 1997).

$$L_a = 0.913 \cdot L_f + \frac{K}{\pi} \int_{\theta > \theta_f} \int \frac{L(\theta, \phi)}{\theta^2} \cdot \cos(\theta) \cdot \sin(\theta) d\theta d\phi \qquad (29)$$

where:

L_a = Adaptation luminance in cd/m^2

L_f = Average foveal luminance in cd/m^2

$L(\theta, \phi)$ = Surrounding luminance in the direction (θ, ϕ), as shown in Figure 71

θ_f = Foveal half angle, 0.5°

K = 0.0096

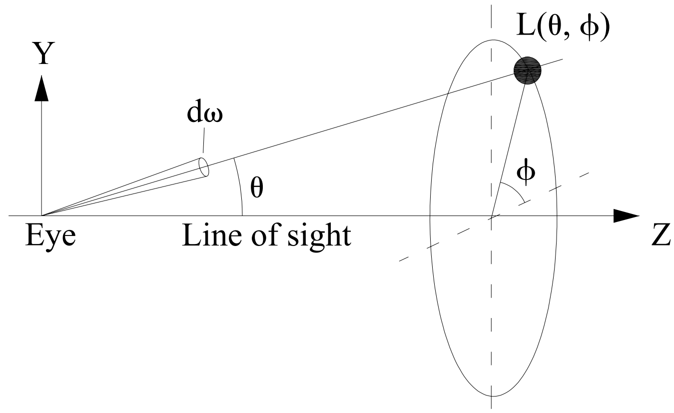


Figure 71. Surrounding luminance in the field of view
(Moon & Spencer, 1943, Figure 1, p. 445)

As claimed by Matković (1997), it is obvious from (29) that the surrounding luminances located in the periphery (beyond 1° or 1.5°) of the observer's field of view contribute less than 9% to the adaptation luminance, which is dominated by the foveal luminance. If there are no bright sources at the periphery, this influence will be negligible (Matković, 1997). If there are some glare sources at the periphery of the viewing field, they reduce contrast visibility because light scattered in the lens obscures the fovea, thus, substantially lowering the legibility of text. Therefore, the veiling luminance (L_v) should be taken into consideration in this glare situation by using (30), as proposed by IESNA (IESNA, 2000, RP-8-00, p. 23). Since this study does not examine glare and light trespass, (29) is more appropriate than (30) for examining the influence of ambient light on the legibility of text.

$$L_a = L_b + L_v \tag{30}$$

where:

L_a = Adaptation luminance

L_b = Background luminance of target

L_v = Veiling luminance, if glare sources are visible at the periphery.

Based on (29), the adaptation luminance of letter Es viewed in the laboratory is dominated by the immediate background luminance of the E-charts, which has been fixed at 187.5 cd/m^2 in the pilot experiment and 120.7 cd/m^2 in the main experiment, while the ambient light should contribute little (less than 9%). According to Weber's law, (1) previously, adaptation luminance dominates the discrimination sensitivities of the observer's eyes to the details of text viewed, as well as the pupillary changes that affect the view depth and quality of the retinal image. Therefore, with constant background luminance of the E-charts in the laboratory, the discrimination sensitivities and pupillary diameter of an observer's eyes would remain almost constant within a very small range of fluctuation, less than 9%, due to the variation of ambient light. Consequently, the ambient light would have a small effect (less than 9%) on the legibility of text when viewed with constant background luminance and luminance contrast in this study, as claimed by the ambient-light hypothesis.

6.2 Laboratory Experiment to Test the Ambient-Light Hypothesis

A third laboratory experiment is designed to test the ambient-light hypothesis, using legibility data collected from 20 human subjects in the same lighting laboratory at the University of Michigan Transportation Research Institute (UMTRI). Restricted by the 10 assumptions used in this study, also aiming to focus on key variables, this experiment examines the influence of ambient light on the legibility of text viewed only perpendicularly to the observers.

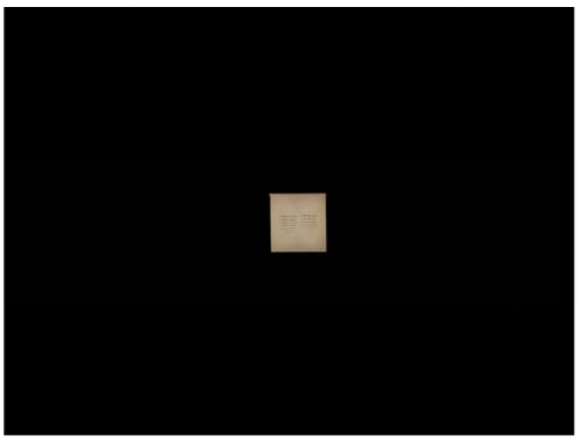
6.2.1 Laboratory Settings

Three modifications to the set up from the previous experiments are made, as shown in Figure 72. First, the previous black background wall is now covered with an off-white canvas drop cloth to provide more significant surrounding luminance when changing the ambient light. Second, diffusive white sheets of paper of letter-size are attached behind the ceiling lamps to prevent direct glare to the observer's eyes and provide more uniform lighting on the background drop cloth. Third, two floor standing fixtures with two T12 lamps each are mounted behind the dimmable fixture on both sides. Subjects sit 6.1m away (20 ft) and recognize a total of 12 E-chart sheets perpendicularly presented at different ambient light levels.

Figure 72. Modified laboratory settings to test the ambient-light hypothesis

6.2.2 Ambient Light

A total of four levels of ambient light are provided in the experiment to examine its effect on the legible size of text viewed perpendicularly to the observers, as shown in Figure 73, including (a) zero ambient light; (b) ambient light provided by the rear half laboratory ceiling fluorescent lamps behind the subject; (c) ambient light provided by T12 lamps mounted behind the fixture on both sides; and (d) ambient light provided by the front half laboratory ceiling fluorescent lamps.



Zero ambient light

Rear lab ceiling lighting

T12 lamps

Front lab ceiling lighting

Figure 73. Four levels of ambient light provided in the experiment

During the course of the experiment, when changing ambient light levels, by dimming the fluorescent fixture, the background luminance (L_b) of viewing materials remains at 124.2 cd/m^2 with a standard deviation of 0.66 cd/m^2 for all tests. The luminance contrast of letter Es is also kept constant 97.9 ($C_{\%}$).

6.2.3 Subjects

The 20 subjects participating in this experiment are also required to be 20-29 years of age, with binocular eyesight of 20/20 or better (20/16, or 20/12.5), with or without glasses, and normal color vision, to avoid the negative effect of age and color deficient eyes. A total of 22 potential subjects are screened upon arrival: one fails the eyesight requirement (20/25); a second subject with a superior eyesight level (> 20/10) has participated in this experiment to satisfy the experimenter, though his data is not included in the main analysis.

6.2.4 Factors Examined

The only dependent factor examined in this experiment is the legible size of letter Es (height H) recorded for each test of the experiment. The only independent factor is the surrounding luminance (L_s) at the periphery of the viewing field provided by each of the four levels of ambient light. All other factors are fixed values. For example, the letter Es on E-charts are printed black/white, with an average luminance contrast of 97.9 ($C_{\%}$). The incident angle remains zero for perpendicular viewing. The legibility distance is a constant 6.1m (20 ft). The background luminances (L_b) are 124.2 cd/m^2 with a standard deviation of 0.66 cd/m^2 at different ambient light levels. The recognition performance is at threshold legibility with 100% accuracy. Only the spectrum of fluorescent T8, daylight, is examined.

6.2.5 Experimental Procedure and Duration

As shown in Table 16, the experiment has four steps and is completed by each subject in approximately 30 minutes. Each subject is paid $10. The subject who has failed the vision requirement is paid $5.

Table 16. Steps to carry out the experiment and duration

Step #	Activities	Time (min)
1	Prepare for eyesight test and four experimental tests	Not counted
2	Screen subjects upon arrival	5
3	Explain and sign consent form	5
4	Carry out four tests (legible height of Es is recorded at each test)	20
	Total	30

The four steps to carry out this experiment are explained below.

Step 1: Prepare for eyesight test and four experimental tests. Before each subject arrives, the experimenter manually dims the surface luminance of the fluorescent fixture to 124.2 cd/m^2 with a standard deviation of 0.66 cd/m^2 at four different ambient light levels, to prepare for the eyesight test and the four experimental tests afterwards.

Step 2: Screen subjects upon arrival. Each subject is screened upon arrival to determine whether they meet the requirements. Under the laboratory full ceiling lighting, one purchased Snellen chart is first used; then two self-made eyesight E-charts are used at an average background luminance 124.2 cd/m^2. Only the qualified subjects continue to the next steps.

Step 3: Explain and sign consent form. The experimenter then explains the procedure to the subject, answers any questions the subject might have, and then asks the subject to sign the consent form. In addition, the experimenter asks for the subject's approval to videotape the whole experiment. Refusing to be videotaped does not rule out the subject's eligibility to participate in the experiment.

Step 4: Carry out the 4 tests. The experimenter then starts to test the subject's reading performance. The legible sizes of letter Es at four tests with different ambient light levels are tested. To smooth the transient adaptation of the subject's eyes, the order of the four tests is preset from the highest (front laboratory lighting) to the lowest (no ambient light). Between tests, the subject has a 5-minute break to allow his/her eyes to fully adapt to the ambient light. For each test, the threshold legible height of letter Es with 100% accuracy is recorded.

6.2.6 Data analysis

The range of background luminances of E-charts viewed during the four tests carried out by each of the 21 subjects (with one outlier) is shown in Figure 74. Based on Figure 74, all materials are viewed at a mean 124.2 cdm^2 with a standard deviation of 0.66 cd/m^2 in this experiment for all tests for 21 subjects (subject #21 is with superior vision > 20/10).

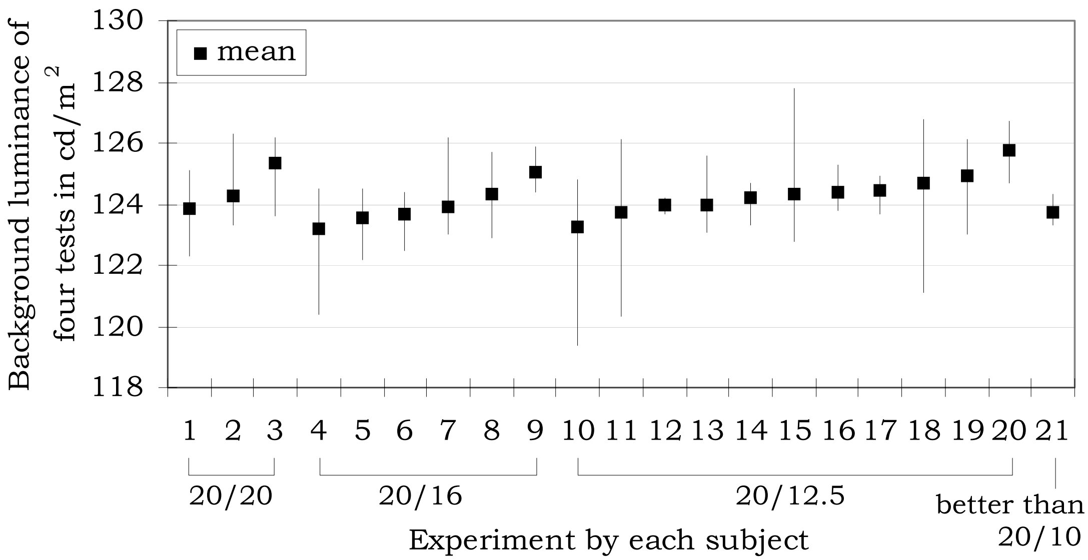


Figure 74. Range of background luminance of four tests in the experiment by each subject at different eyesight levels (20/20, 20/16, or 20/12.5), at mean 124.2 cd/m^2 with standard deviation of 0.66 cd/m^2

The almost constant background luminance of all viewing materials for the four tests (mean 124.2 cd/m^2 with standard deviation of 0.66 cd/m^2) dominates the adaptation luminance of the environment (laboratory) to the observer's eye. To calculate the actual contribution of the four levels of ambient light to the adaptation luminance, (31) is used in this study, which is derived from (29) previously. This study uses the numerical method to approximate (31), after measuring the surface luminances of the drop cloth (divided into small grids 3 ft $\times$3 ft), right and left black walls (3 ft $\times$6 ft), ceiling (4.5 ft $\times$ 5 ft), and floor (6 ft $\times$5 ft). The calculated contributions of the surrounding luminances (located beyond 1° or 1.5° of the field of view) of the background drop cloth, left black wall, right black wall, ceiling, and floor to the adaptation luminance of text viewed perpendicularly to the observers in this experiment are listed in Table 17.

$$L_{ambient} = \frac{K}{\pi}\int_{\theta>\theta_f}\int \frac{L(\theta,\phi)}{\theta^2}\cdot\cos(\theta)\cdot\sin(\theta)d\theta d\phi \qquad (31)$$

where:

$L_{ambient}$ = Contribution of ambient light to the adaptation luminance

$L(\theta, \phi)$ = Surrounding luminance in the direction (θ, ϕ), as shown in Figure 71

θ_f = Foveal half angle, 0.5°

K = 0.0096

Table 17. Measured surface luminances in cd/m² and their contribution ($L_{ambient}$) to the adaptation luminance

Surfaces	Luminance	Front lab ceiling lighting	T12 lamps	Rear lab ceiling lighting	Zero ambient light
Back drop cloth	Mean measured	122.42	45.10	2.03	0
	Min measured	3.42	0.65	0.27	0
	Max measured	381.90	108.40	5.38	0
	$L_{ambient}$ calculated	3.53	1.97	0.08	0
Left dark wall	Mean	6.38	1.26	0.17	0
	Min	1.92	0.00	0.05	0
	Max	16.08	9.99	0.43	0
	$L_{ambient}$	0.03	0.00	0.00	0
Right dark wall	Mean	7.71	1.36	0.19	0
	Min	0.91	0.00	0.04	0
	Max	36.05	12.03	1.05	0
	$L_{ambient}$	0.03	0.01	0.00	0
Ceiling	Mean	107.92	5.44	1.14	0
	Min	2.06	0.02	0.07	0
	Max	273.40	47.74	3.99	0
	$L_{ambient}$	0.52	0.02	0.01	0
Floor	Mean	45.56	3.33	1.13	0
	Min	1.75	0.00	0.15	0
	Max	115.40	32.88	5.52	0
	$L_{ambient}$	0.38	0.02	0.01	0

In addition, Table 18 illustrates (a) the contribution of ambient light ($L_{ambient}$) to the adaptation luminance, (b) the foveal luminance (L_f), which is the mean background luminance of the fixture surface, (c) the adaptation luminance (L_a), calculated using (32) that is derived from (29) and (31), and (d) the percentage $L_{ambient}/L_a$, at each level of ambient light in this experiment.

$$L_a = 0.913 \cdot L_f + \sum L_{ambient} \tag{32}$$

where:

L_a = Adaptation luminance

$L_{ambient}$ = Contribution of ambient light to the adaptation luminance

L_f = Foveal luminance (the mean background luminance 124.2cd/m^2)

Table 18. Calculated adaptation luminances and the contribution of ambient light in the whole surrounding environment at each test

Four different ambient light levels	$L_{ambient}$ in cd/m^2	L_f in cd/m^2	L_a in cd/m^2	$L_{ambient}/L_a$
Front lab ceiling lighting	4.48	124.20	117.88	3.80%
T12 lamps	2.02	124.20	115.42	1.75%
Rear lab ceiling lighting	0.10	124.20	113.49	0.08%
Zero ambient light	0.00	124.20	113.39	0%

In sum, of all the surrounding surfaces, the background drop cloth makes the maximum contribution of its surrounding luminance to the adaptation luminance (e.g., 3.53 cd/m^2 for front laboratory ceiling lighting), though the contribution is still very small (3.0%). Those from the dark side walls and dark floor approach zero. Thus, the contribution of the surrounding environment at each ambient light level to the adaptation luminance is very small, with a maximum of only 3.80% even under the bright front laboratory ceiling lighting.

The threshold legible heights of letter Es recorded at three different eyesight levels (20/20, 20/16, 20/12.5) and four levels of ambient light by all 21 subjects (one outlier, with eyesight >20/10) are listed in Figure 75. As shown in Figure 75, the three subjects with acuity 20/20 have a constant threshold legible height of 8.85mm of text viewed at all four ambient light levels. For the six subjects with acuity 20/16, five have the same threshold legible height of text (7.08mm) at all four ambient light levels, while one has his threshold legible height as 7.08mm at three ambient light levels but 8.50mm under the rear laboratory ceiling lighting. Ten of 11 subjects with acuity 20/12.5 have an equivalent threshold legible height of 5.57mm at all four ambient light levels, while one subject has better reading performance (4.45mm) in darkness. The subject with super acuity (> 20/10) maintains the threshold legible height at 3.33mm for all four ambient light levels.

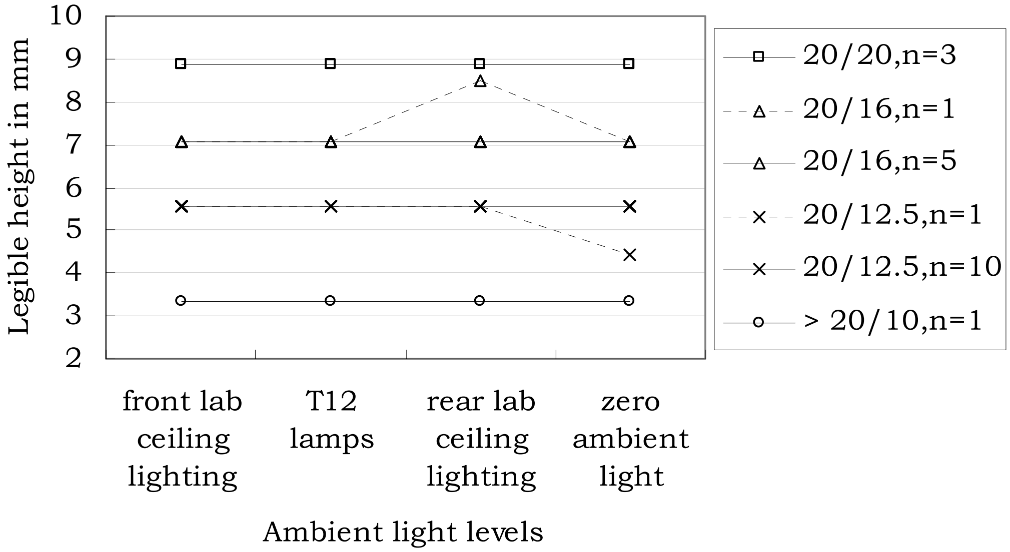


Figure 75. Legible heights of letter Es at different ambient light levels

6.2.7 Conclusions

Two conclusions can be drawn based on the data analysis:

1. When the background luminance (foveal luminance) of text remains constant, changing the ambient light levels of the viewing scenario does not change the reading performance of text. Over the range examined, as shown in Table 17, ambient light has a very small effect on the legibility of text.
2. The adaptation luminance of text is determined primarily by the foveal luminance. The contribution of ambient luminance to the adaptation is very small, as long as no glare sources are visible at the periphery of the field of view.

6.3 Summary

The influence of ambient light on the legibility of text is tested to validate the legibility equation, (28) in Chapter 5, which does not examine ambient light as a factor. First, a hypothesis (ambient-light hypothesis) is theoretically developed from a model proposed by Moon and Spencer (1945) to calculate the adaptation luminance. This hypothesis claims that ambient light in the viewing environment should have a small influence (less than 9%) on the legibility level of text viewed with constant background luminance and luminance contrast in a glare-free environment. This hypothesis is then tested in the laboratory using 20 human subjects at four different ambient light levels, including (a) zero ambient light, (b) ambient light provided by the rear half laboratory ceiling fluorescent lamps behind the subject, (c) ambient light provided by T12 lamps mounted behind the fixture on both sides, and (d) ambient light provided by the front half laboratory ceiling fluorescent lamps. Legibility data collected from the 20 subjects show that when the background luminance of text remains at 124.2 cd/m^2 with a standard deviation of 0.66 cd/m^2, changing the ambient light levels of the viewing scenario does not affect the threshold legible heights of letter Es (with 100% accuracy). Therefore, ambient light has a negligible effect on the legibility of text.

CHAPTER 7

Step 4: A Computation Program and Its Application in Lecture Halls

Thus far, this study has derived, verified, and improved the legibility equation, (28) previously, to predict the spatial legibility of text viewed at incident angles $0° \leq \xi \leq 82.8°$. The absence of ambient light as a factor examined in (28) has also been shown to not influence the accurate prediction of the legibility of text. Equation (28) is therefore used as the underlying algorithm for a computation program to be developed in this chapter.

7.1 Development of a Computation Program

Equation (28) has wide applications in many fields where the legibility of text is concerned. Generally, it can be used to determine for observers of varied ages and eyesight levels:

1. ideal viewing distances and viewing angles for recognizing given text with fixed geometries, contrast, font, etc., under different lighting conditions.
2. appropriate size, contrast, and font of text presented at a fixed viewing

distance and viewing angles, under various lighting conditions.

3. three-dimensional (3D) ideal viewing space, or 2D ideal viewing areas along a specific viewing plane of text presented on a single or multiple displays, inside of which text is guaranteed legible with 100% accuracy.
4. appropriate size, location, and orientations of different displays installed in buildings or their surroundings on which text is presented under multiform viewing conditions.

This study adopts (28) to predict the spatial distribution of many consecutive viewing spots from which text presented in lecture halls on a single or multiple displays is viewed at an equivalent legibility level — threshold (just readable) with 100% accuracy. Using (28), these viewing spots can be accurately located on x-y-z coordinates by calculating their legibility distances at different viewing angles. Geometrically, these viewing spots distributed in three dimensions confine a 3D ideal viewing space. Viewing spots located right on the surface of this space have a threshold legibility level (just legible with 100% accuracy) of text. Inside the 3D ideal viewing space, the closer to the text the viewing spots are, the more legible the text is.

Although the 3D ideal viewing space of text directly shows its geometry and shape, the spatial distribution of those viewing spots located actually on a specific viewing plane where observers usually locate, such as that parallel to the sloped floor in lecture halls at eye height level, is probably more useful in practice. Such distribution of viewing spots along the specific viewing plane defines a 2D ideal viewing area inside of which text is guaranteed legible to the observers. This 2D ideal viewing area of text is helpful in design activities, particularly for drawings. For example, the 2D ideal viewing area of text presented in the front of lecture halls along the viewing plane parallel to the sloped floor at the observer's eye height should be coincident to the seating area for ideal seat arrangement. Thus, this study will develop a computation program in MatLab to facilitate finding such a 2D ideal viewing area of text. Since the program will determine the 2D ideal viewing area of text based on the 3D ideal viewing space, the 3D ideal viewing space of text is thus examined first.

7.1.1 Algorithm of the Computation Program

To compute a 3D ideal viewing space of text, an underlying algorithm is needed to locate the critical viewing spots on the x-y-z coordinates at incident angles $0° \leqslant \xi \leqslant 90°$. Equation (17) previously serves as the prototype algorithm for this purpose, which is re-expressed on x-y-z coordinates as (33), by considering the general viewing situations where text presented at the original point O' (x_0, y_0, z_0) with initial orientation $(\Delta\phi, \Delta\alpha)$ is recognized at viewing spot P (x, y, z) with orientation (ϕ, α) to the original point O'. In MatLab, (33) can identify all the critical viewing spots located actually on the surface of a 3D ideal viewing space from which text is viewed at a threshold legibility level with 100% accuracy.

$$
\begin{cases}
x = x_0 + D_0 \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{1.5} \cdot \sin(\phi + \Delta\phi) \\
y = y_0 + D_0 \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot (\cos\alpha \cdot \cos(\phi + \Delta\phi) \cdot \cos(\Delta\alpha) - \sin\alpha \cdot \sin(\Delta\alpha)) \\
z = z_0 + D_0 \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot (\cos\alpha \cdot \cos(\phi + \Delta\phi) \cdot \sin(\Delta\alpha) + \sin\alpha \cdot \cos(\Delta\alpha)) \\
D_0 = 2443.5 \times H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{\ -1} \cdot (L_b)^{0.213} \cdot C_{\%}^{\ 0.532}
\end{cases}
\tag{33}
$$

where:

x = Legibility distance of text projected on x coordinate

y = Legibility distance of text projected on y coordinate

z = Legibility distance of text projected on z coordinate

D_0 = Legibility distance when text is viewed at zero incident angle $\xi = 0°$

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of observer's eyesight

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ϕ = Horizontal viewing angle, $-90° \leqslant \phi \leqslant 90°$

α = Vertical viewing angle, $-90° \leqslant \alpha \leqslant 90°$

$\Delta\phi$ = Initial horizontally rotated angle of the visual media, positive for clockwise

$\Delta\alpha$ = Initial vertically tilted angle of the visual media, positive for clockwise

Although (17) has been improved to (28), it does not examine the incident angles $82.8° < \xi \leqslant 90°$, which will be examined in the future. To compute a complete 3D ideal viewing space of text, (28) is then improved to (34) to expand its examined viewing angles from 0°- 82.8° to the entire range of 0°- 90°, by assuming zero legibility distance when the incident angle is beyond 82.8° till 90°.

$$D = \begin{cases} 2443.5H\left(\dfrac{H}{Sw}\right)^{-1} S_d^{\ -1} L_b^{\ 0.213} C_{\%}^{\ 0.532} (\cos\xi)^{0.5} & 0^0 \le \xi \le 65.7^0 \\ 2443.5H\left(\dfrac{H}{Sw}\right)^{-1} S_d^{\ -1} L_b^{\ 0.213} C_{\%}^{\ 0.532} (\cos\xi)^{0.5} (0.024\xi - 0.577)^{-1} & 65.7^0 < \xi \le 82.8^0 \\ 0 & 82.8^0 < \xi \le 90^0 \end{cases} \tag{34}$$

where:

D = Legibility distance when text is viewed at any incident angle 0°≤ξ≤90°

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of observer's acuity level

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ξ = Incident angle between the display normal and the sightline of observer

Such improvement is based on two facts. First, in reality, text is rarely viewed at incident angles 82.8°<ξ≤90°. Also, such an extremely distorted viewing of text should be purposely avoided in practice for better legibility. According to the lecture hall survey, only four lecture halls have their maximum viewing angles of visual media larger than the critical angle 82.8°, as shown in Appendix E, including the Modern Language Building 1200 (84.4°) and 1400 (86.5°), EE 1311 (83.7°), and Hutchins Hall 100 (83.5°).

Second, legibility distance for recognizing the same text viewed at an equivalent legibility level decreases rapidly at incident angles 82.8°<ξ≤ 90° that the assumption of zero legibility distance at this range of incident angle ξ is appropriate because: (a) the shape of the 3D ideal viewing space of text predicted using (34) remains almost unchanged compared to that computed using (33), as illustrated in Figure 76 and 77 later, and (b) zero legibility distance improves the 3D ideal viewing space of text in a conservative manner by enhancing the legibility levels of text viewed at any incident angles 82.8°<ξ≤90°.

Accordingly, (33) is updated with the improved algorithm (34), and is re-expressed on x-y-z coordinates as (35).

$$
\begin{cases}
x = x_0 + D_0' \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{1.5} \cdot \sin(\phi + \Delta\phi) \\
y = y_0 + D_0' \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot (\cos\alpha\cos(\phi+\Delta\phi)\cos(\Delta\alpha) - \sin\alpha\sin(\Delta\alpha)) \\
z = z_0 + D_0' \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot (\cos\alpha\cos(\phi+\Delta\phi)\sin(\Delta\alpha) + \sin\alpha\cos(\Delta\alpha)) \\
D_0' = 2443.5 \times H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{-1} \cdot L_b^{0.213} \cdot C_{\%}^{0.532} \quad 0^\circ \le \xi \le 65.7^\circ \\
D_0' = 2443.5 \times H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{-1} \cdot L_b^{0.213} \cdot C_{\%}^{0.532} \cdot (0.024\xi - 0.577)^{-1} \quad 65.7^\circ < \xi \le 82.8^\circ \\
D_0' = 0 \quad 82.8^\circ < \xi \le 90^\circ
\end{cases}
\tag{35}
$$

where:

x = Legibility distance of text projected on x coordinate

y = Legibility distance of text projected on y coordinate

z = Legibility distance of text projected on z coordinate

D_0' = Modified legibility distance of text viewed at zero incident angle ξ =0°, which remains constant for 0°≤ξ≤65.7°, then is modified with the incident angle ξ for the extremely distorted viewing situation when 65.7°<ξ≤82.8°, and is assumed to approach zero when 82.8°<ξ≤90°

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of observer's eyesight

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ξ = Incident angle between the display normal and the sightline of the observer, 0°≤ξ≤90°, $\cos\xi = \cos\phi\cos\alpha$

ϕ = Horizontal viewing angle, -90°≤ϕ≤90°

α = Vertical viewing angle, -90°≤α≤90°

$\Delta\phi$ = Initial horizontally rotated angle of the visual media, positive for clockwise

$\Delta\alpha$ = Initial vertically tilted angle of the visual media, positive for clockwise

7.1.2 Shape of the 3D Ideal Viewing Space of Text

Using (35), the shape of the 3D ideal viewing space of text can be plotted in MatLab. According to (35), the shape of the 3D ideal viewing spaces of different text — which have different geometries, contrasts, fonts, etc., and are presented on different visual media with different locations, mounting heights, initial orientation (Δϕ, Δα), and viewed under different lighting conditions — should be homomorphous to each other, but with different sizes, orientations, and locations. This study adopts a simple viewing situation to plot an example 3D ideal viewing space of text in MatLab. In this sample viewing scenario, a single letter E (H = 8.85mm, H/Sw = 5, $C_{\%}$ = 97.9, L_b = 120cd/m^2) is presented at the origin point O ($x_0 = 0$, $y_0 = 0$, $z_0 = 0$) with normal orientation ($\Delta\phi = 0$, $\Delta\alpha = 0$), and recognized by a young observer with 20/20 eyesight. By assigning this sample viewing situation to (35), this study derives (36) as the algorithm for plotting the 3D ideal viewing space of the sample letter E, as illustrated in Figure 76.

$$\begin{cases} x = D_0' \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{1.5} \cdot \sin\phi \\ y = D_0' \cdot (\cos\phi)^{1.5} \cdot (\cos\alpha)^{1.5} \\ z = D_0' \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot \sin\alpha \\ D_0' = 6.87 \quad 0^\circ \le \xi \le 65.7^\circ \\ D_0' = 6.87 \times (0.024\xi - 0.577)^{-1} \quad 65.7^\circ < \xi \le 82.8^\circ \\ D_0' = 0 \quad 82.8^\circ < \xi \le 90^\circ \end{cases} \tag{36}$$

where:

x = Legibility distance of text projected on x coordinate, in meters

y = Legibility distance of text projected on y coordinate, in meters

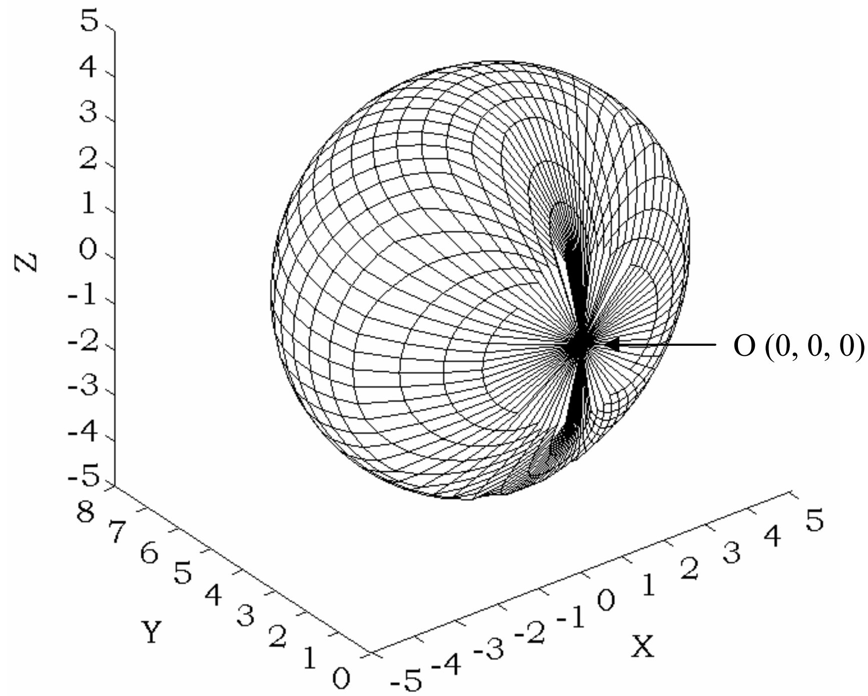
z = Legibility distance of text projected on z coordinate, in meters

D_0' = Modified legibility distance of text viewed at zero incident angle $\xi = 0°$

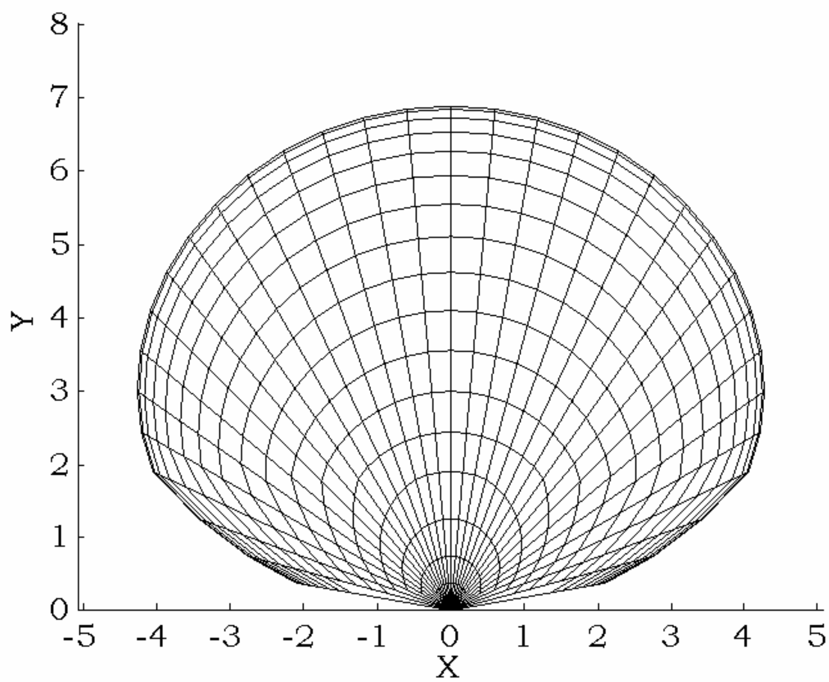
ϕ = Horizontal viewing angle, $-90° \leqslant \phi \leqslant 90°$

α = Vertical viewing angle, $-90° \leqslant \alpha \leqslant 90°$

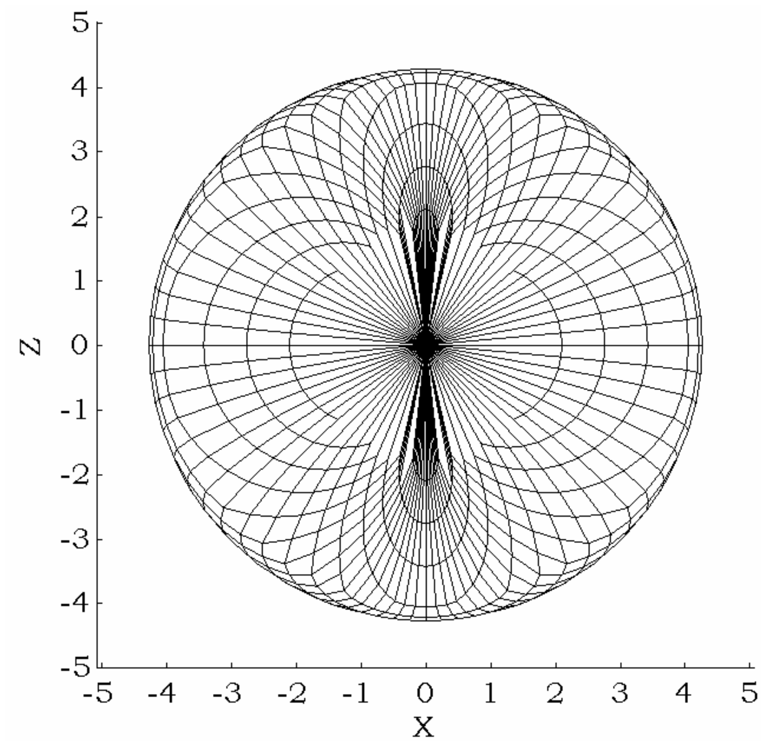
ξ = Incident angle between the display normal and the sightline of the observer, $0° \leqslant \xi \leqslant 90°$, $\cos\xi = \cos\phi\cos\alpha$



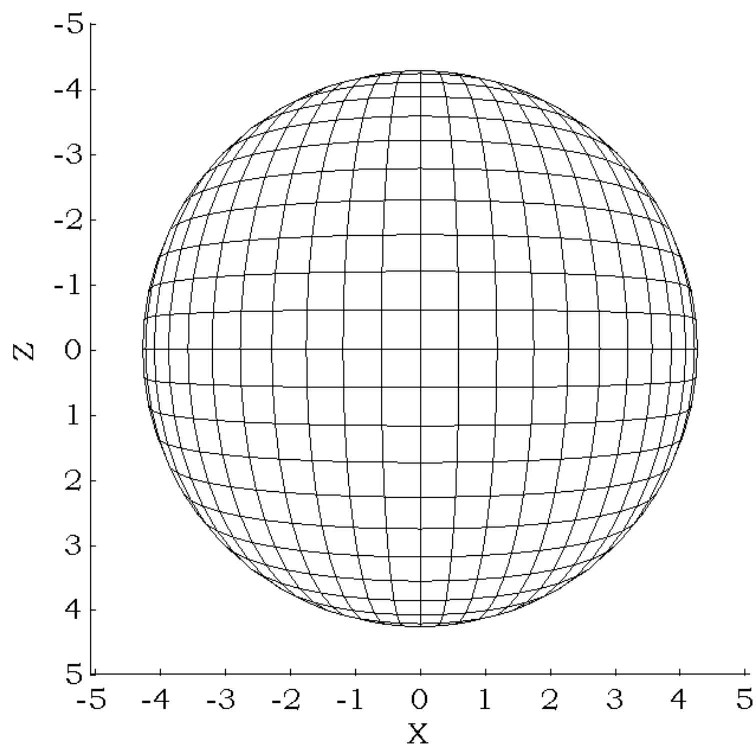
(a) X-Y-Z view of the 3D ideal viewing space of the sample letter E



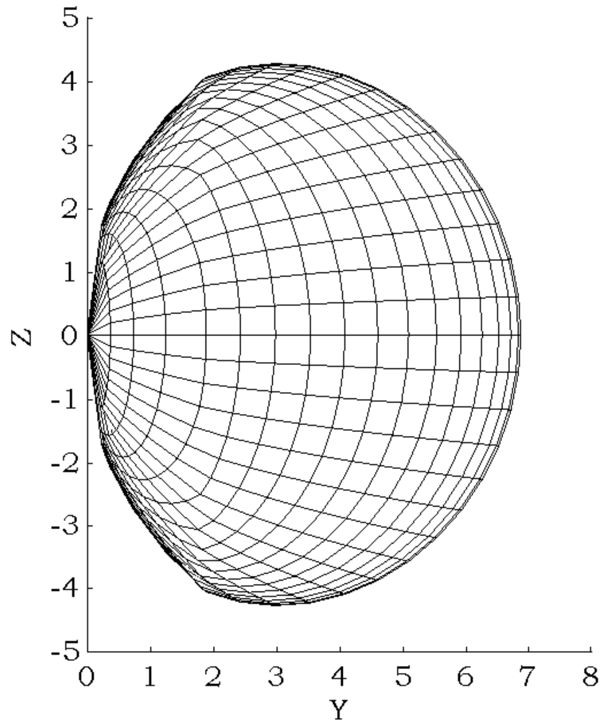
(b) X-Y view of the 3D ideal viewing space of the sample letter E



(c) X-Z view of the 3D ideal viewing space of the sample letter E



(d) Alternative X-Z view of the 3D ideal viewing space of the sample letter E



(e) Y-Z view of the 3D ideal viewing space of the sample letter E

Figure 76. The 3D ideal viewing space of a single letter E (H=8.85mm, H/Sw=5, $C_{\%}$=97.9, L_b=120cd/m^2) presented at O ($x_0 = 0$, $y_0 = 0$, $z_0 = 0$) with normal orientation ($\Delta\phi$=0, $\Delta\alpha$=0), and recognized by a young observer with 20/20 eyesight

The next step is to show the small differences between the shapes of the 3D ideal viewing space of the sample letter E computed using algorithms before and after the assumption of zero legibility distance at incident angles 82.8°<ξ≤90°. The shape of the 3D ideal viewing space of the same letter E predicted using (37), which is derived from (33), is plotted in MatLab, as shown in Figure 77. After comparing Figure 76 and 77, particularly their X-Y and Y-Z view, only a small difference can be discerned on the immediate portions near the origin point O (0, 0, 0) where the letter E is presented with incident angle 65.7°<ξ≤90°. Such small differences sustain the previous assumption of zero legibility distance when incident angle ξ ranges beyond 82.8° till 90° for enhanced legibility without sacrificing the accurate prediction of the 3D ideal viewing space of text.

$$
\begin{cases}
x = D_0 \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{1.5} \cdot \sin\phi \\
y = D_0 \cdot (\cos\phi)^{1.5} \cdot (\cos\alpha)^{1.5} \\
z = D_0 \cdot (\cos\phi)^{0.5} \cdot (\cos\alpha)^{0.5} \cdot \sin\alpha \\
D_0 = 6.87
\end{cases}
\tag{37}
$$

where:

x = Legibility distance of text projected on x coordinate, in meters

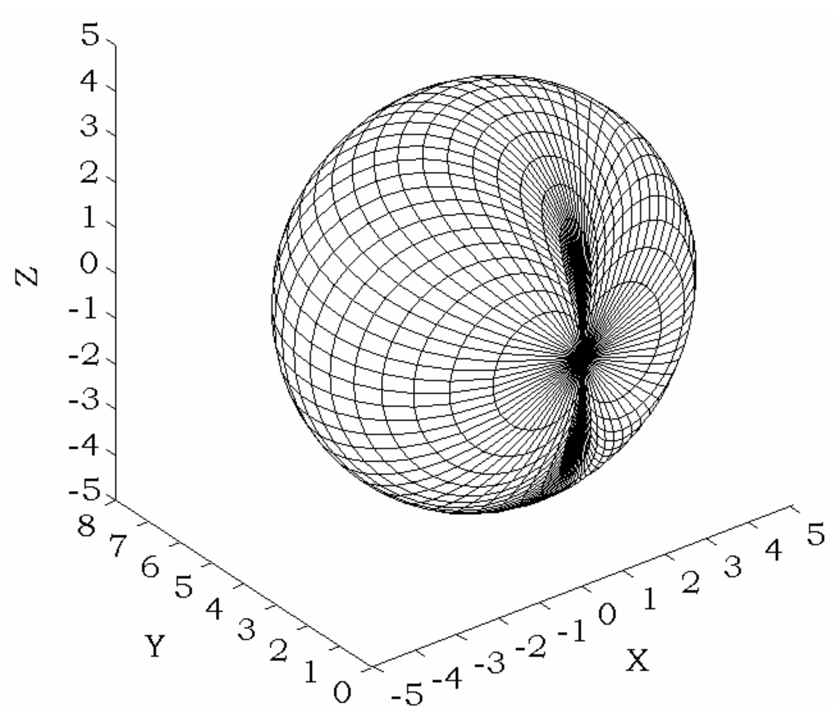
y = Legibility distance of text projected on y coordinate, in meters

z = Legibility distance of text projected on z coordinate, in meters

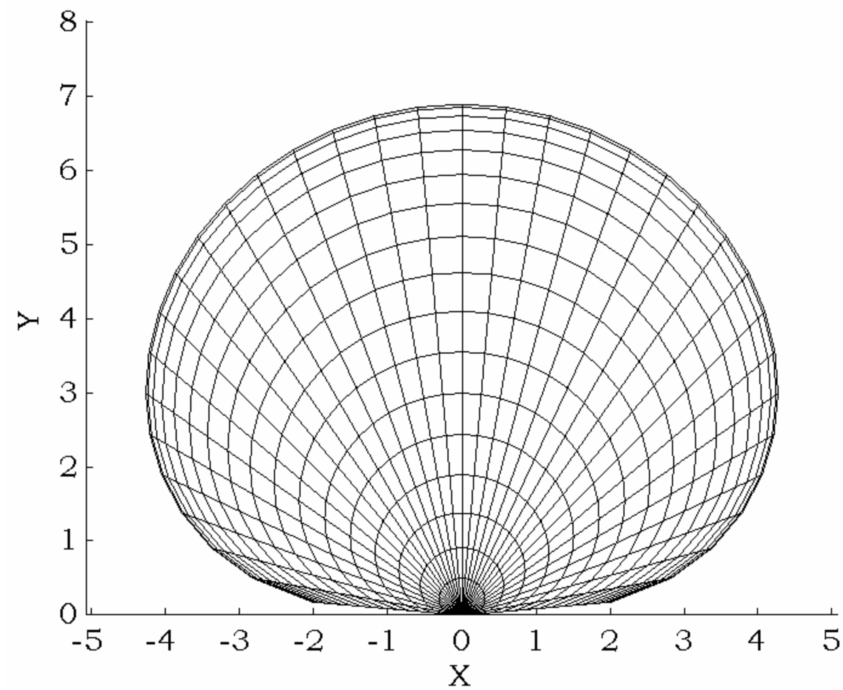
D_0 = Legibility distance of text viewed at zero incident angle $\xi = 0°$

ϕ = Horizontal viewing angle, $-90° \leqslant \phi \leqslant 90°$

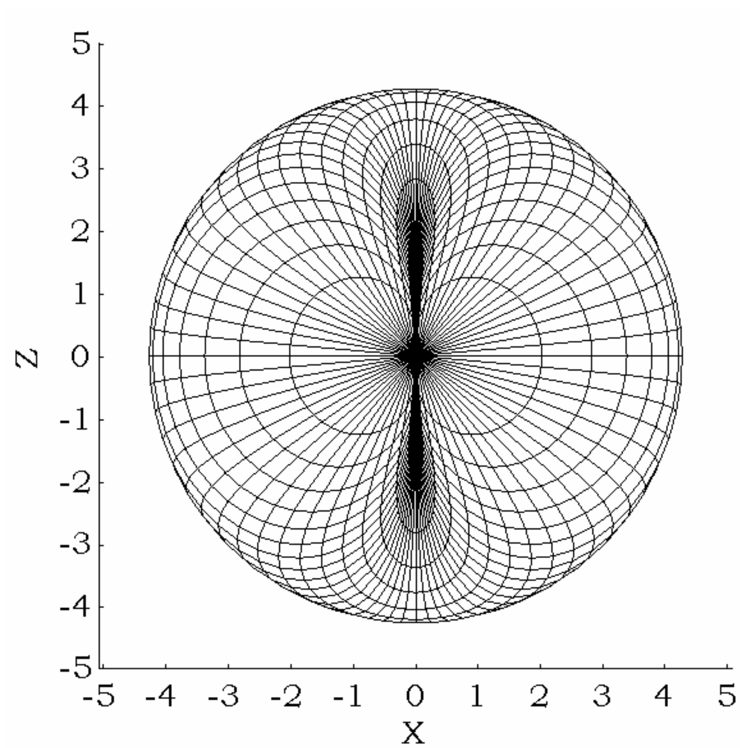
α = Vertical viewing angle, $-90° \leqslant \alpha \leqslant 90°$



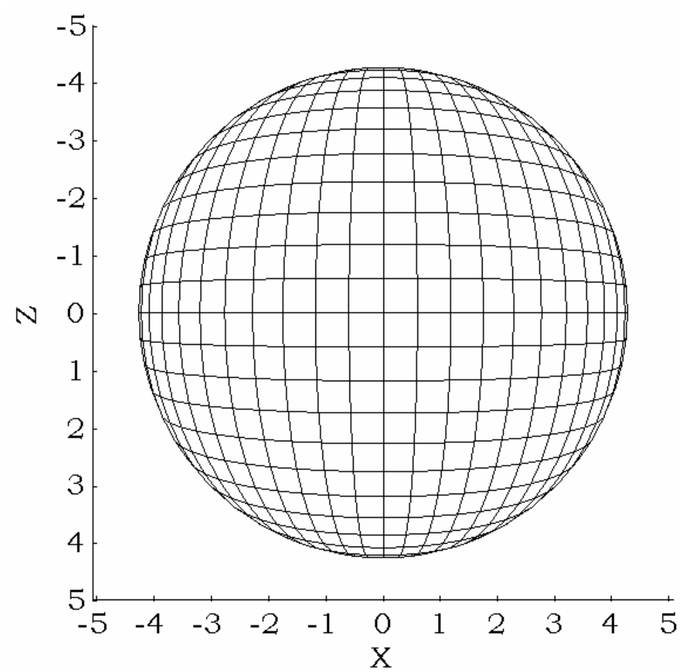
(a) X-Y-Z view of the 3D ideal viewing space of the same letter E plotted using (37), which is the algorithm before the improvement



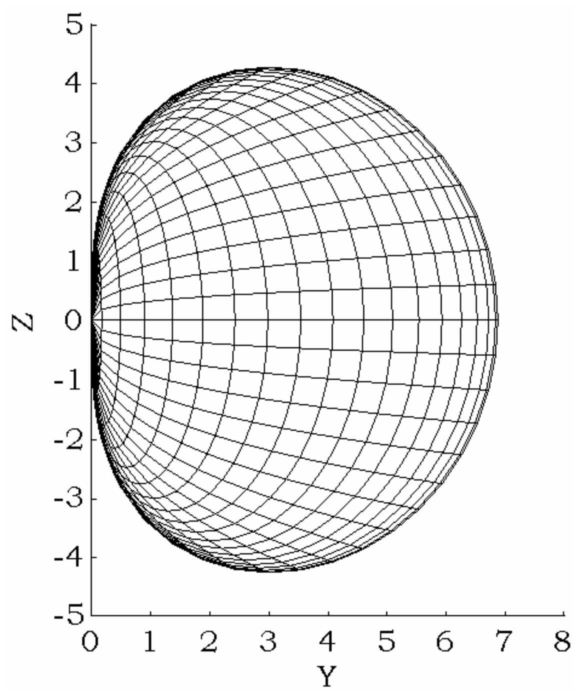
(b) X-Y view of the 3D ideal viewing space of the same letter E plotted using (37)



(c) X-Z view of the 3D ideal viewing space of the same letter E plotted using (37)



(d) Alternative X-Z view of the 3D ideal viewing space of the same letter E plotted using (37)



(e) Y-Z view of the 3D ideal viewing space of the same letter E plotted using (37)

Figure 77. For comparison to the one plotted using (36), shape of the 3D ideal viewing space of the same letter E plotted using (37) before improvement of the algorithm

7.1.3 2D Ideal Viewing Area of Text Viewed along a Viewing Plane

Theoretically, the 2D ideal viewing area of text viewed in practice along a specific viewing plane is geometrically shaped by slicing the 3D ideal viewing space using this plane. Specifically, the section view (y-z coordinates) of the 3D ideal viewing space of text is formed by slicing it with a vertical viewing plane, and plan view (x-y coordinates) with a horizontal plane. However, in practice, the viewing plane is often tilted, similar to the one parallel to the sloped floor of the lecture halls at the observer's eye height, which slices the 3D ideal viewing space of text and forms a sloped 2D ideal viewing area. Then how can the 2D ideal viewing area of text viewed along any viewing plane as desired in practice be found, including but not limited to the plan and section view? The straightforward solution is to find an algorithm to describe the required 2D ideal viewing area of text on x-y-z coordinates, and then plot this area using this algorithm in a mathematical software program such as Graph or MatLab. This solution is useful for finding the plan and section view of the 3D ideal viewing space of text viewed in the simplified situations where text is presented with a normal orientation ($\Delta\phi = 0$, $\Delta\alpha = 0$). For instance, based on the unimproved algorithm (37), this study generates algorithms (38) and (39) to describe the plan and section views, by assuming $\alpha = 0$ and $\phi = 0$ respectively, of the 3D ideal viewing space of the previous single letter E (H = 8.85mm, H/Sw = 5, $C_{\%} = 97.9$, $L_b = 120$ cd/m^2, located at the origin point O ($x_0 = 0$, $y_0 = 0$, $z_0 = 0$) with normal orientation ($\Delta\phi=0$, $\Delta\alpha=0$), and recognized by a young observer with 20/20 eyesight). Using (38) and (39), respectively, the plan and section views of the 3D ideal viewing space of the single letter E are plotted in Graph, as illustrated in Figure 78.

Plan view (x-y coordinates, α=0)

$$\begin{cases} x = 6.87 \cdot (\cos\phi)^{0.5} \cdot \sin\phi \\ y = 6.87 \cdot (\cos\phi)^{1.5} \end{cases} \tag{38}$$

Section view (y-z coordinates, ϕ=0)

$$\begin{cases} y = 6.87 \cdot (\cos\alpha)^{1.5} \\ z = 6.87 \cdot (\cos\alpha)^{0.5} \cdot \sin\alpha \end{cases} \tag{39}$$

where:

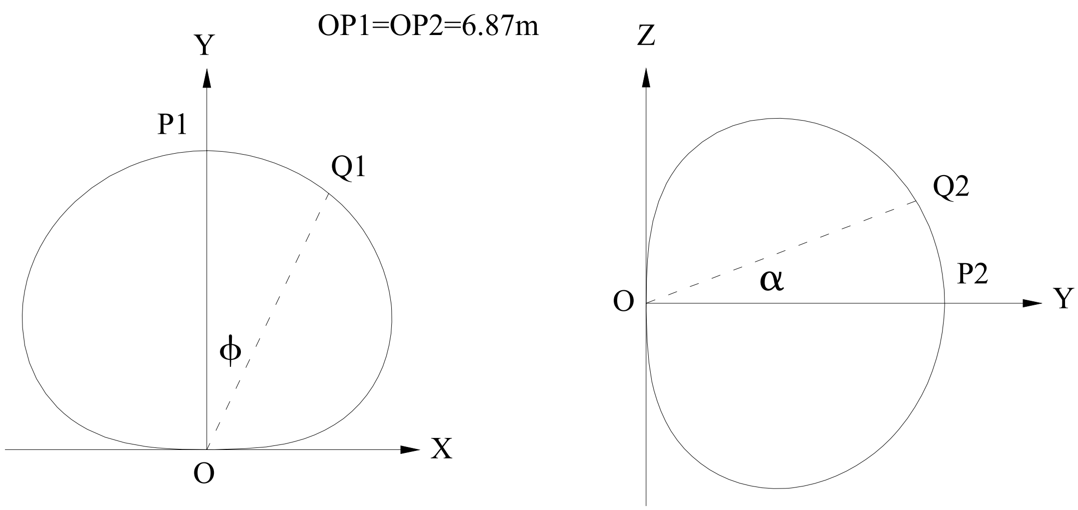
x = Legibility distance of text projected on x coordinate, in meters

y = Legibility distance of text projected on y coordinate, in meters

z = Legibility distance of text projected on z coordinate, in meters

ϕ = Horizontal viewing angle, $-90° \leqslant \phi \leqslant 90°$

α = Vertical viewing angle, $-90° \leqslant \alpha \leqslant 90°$



(a) Plan View

(b) Section View

Figure 78. Plan and section views of the 3D ideal viewing space of the single letter E (H = 8.85mm, H/Sw = 5, $C_{\%}$ = 97.9, L_b = 120cd/m^2), presented at the origin point O (x_0 = 0, y_0 = 0, z_0 = 0) with normal orientation (Δϕ=0, Δα=0), and recognized by a young observer with 20/20 eyesight, as shown in Figure 77 previously

Similarly, based on the improved algorithm (36), this study generates (40) and (41) to describe the plan and section views, respectively, of the single letter E, and plot Figure 79 in MatLab.

Plan view (x-y coordinates, α=0)

$$\begin{cases} x = D_0' \cdot (\cos\phi)^{0.5} \cdot \sin\phi \\ y = D_0' \cdot (\cos\phi)^{1.5} \\ D_0' = 6.87 \quad 0^\circ \le \xi \le 65.7^\circ \\ D_0' = 6.87 \times (0.024\xi - 0.577)^{-1} \quad 65.7^\circ < \xi \le 82.8^\circ \\ D_0' = 0 \quad 82.8^\circ < \xi \le 90^\circ \end{cases} \tag{40}$$

Section view (y-z coordinates, ϕ=0)

$$\begin{cases} y = D_0' \cdot (\cos\alpha)^{1.5} \\ z = D_0' \cdot (\cos\alpha)^{0.5} \cdot \sin\alpha \\ D_0' = 6.87 \quad 0^\circ \le \xi \le 65.7^\circ \\ D_0' = 6.87 \times (0.024\xi - 0.577)^{-1} \quad 65.7^\circ < \xi \le 82.8^\circ \\ D_0' = 0 \quad 82.8^\circ < \xi \le 90^\circ \end{cases} \tag{41}$$

where:

x = Legibility distance of text projected on x coordinate, in meters

y = Legibility distance of text projected on y coordinate, in meters

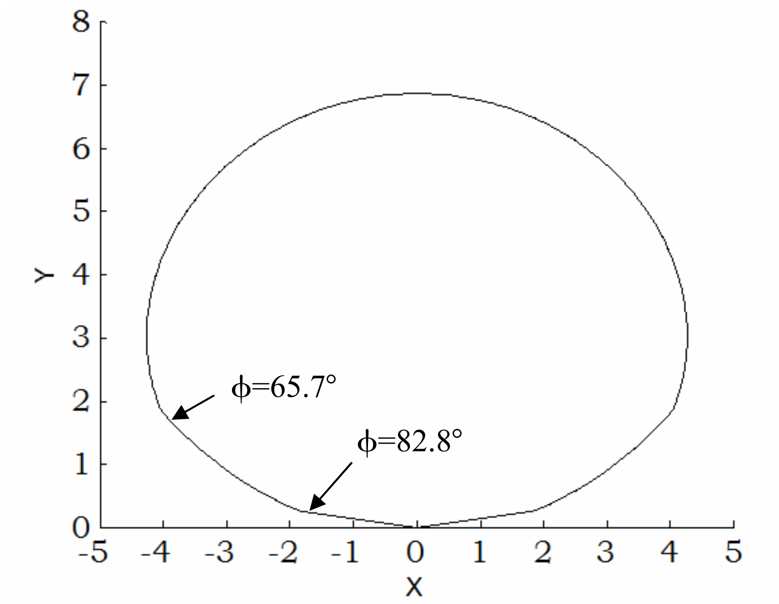
z = Legibility distance of text projected on z coordinate, in meters

D_0' = Modified legibility distance of text viewed at zero incident angle ξ =0°

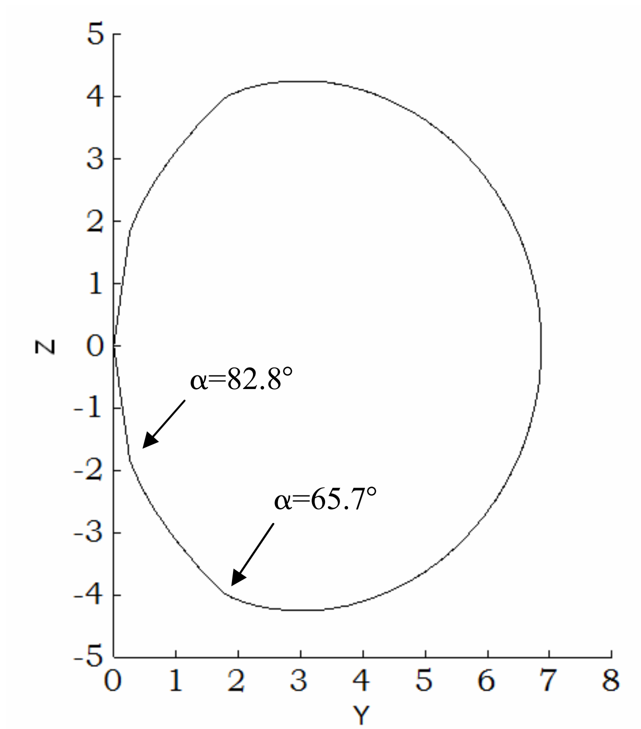
ϕ = Horizontal viewing angle, -90°≤ϕ≤90°

α = Vertical viewing angle, -90°≤α≤90°

ξ = Incident angle between the display normal and the sightline of the observer, 0°≤ξ≤90°, $\cos\xi = \cos\phi\cos\alpha$



(a) Plan view (α=0, unit in meters)



(b) Section view (ϕ=0, unit in meters)

Figure 79. Plan and section views of the 3D ideal viewing space of the single letter E (H = 8.85 mm, H/Sw = 5, $C_{\%}$ = 97.9, L_b = 120 cd/m^2) presented at the origin point O (x_0 = 0, y_0 = 0, z_0 = 0) with normal orientation ($\Delta\phi$=0, $\Delta\alpha$=0), and recognized by a young observer with 20/20 eyesight, as shown in Figure 76 previously

Comparing Figure 78 and Figure 79, their small differences further sustain the previous assumption of zero legibility distance at incident angles $82.8° < \xi \leqslant 90°$, thus validating the improved algorithm (36).

However, when the general viewing situation is considered where text is presented at the original point O' (x_0, y_0, z_0) with initial orientation $(\Delta\phi, \Delta\alpha)$ and recognized by observers with different eyesight levels, it is difficult to generate a prototype algorithm in light of (35) previously to describe the 2D ideal viewing area of text along any tilted viewing plane, and then plot it out in Graph or MatLab. Therefore, finding the 2D ideal viewing area of text viewed in general situations should be facilitated by a computation program, which calculates and plots the 2D ideal viewing area by geometrically slicing the specified viewing plane (horizontal, vertical, or tilted) across the 3D ideal viewing space of text presented in any viewing situations. This study has developed this computation program in MatLab.

7.1.4 Computation Program to Find the 2D Ideal Viewing Area

The computation program developed in MatLab has used (35) previously as the underlying algorithm. The program code appears in Appendix G. In daily life, text is often simultaneously presented on a wide variety of displays: instrument panels, TV monitors, computers screens, blackboards, projection screens, billboards, warning placards, architectural or roadway signs, maps, books, magazines, etc. Considering such popular multitask viewing conditions in practice, this program calculates an overlapped 2D ideal viewing area of text presented on multiple visual media installed with different geometries, locations, mounting heights, and orientations, under different lighting conditions, and viewed by observers located along any viewing plane, similar to that parallel to the sloped floor of lecture halls at eye height level.

As preliminarily developed in this study, this program makes three presumptions.

1. Text presented on multiple visual media is viewed by an identical observer each time to compute a 2D ideal viewing area. For a mass audience, like that in lecture halls, their average eyesight level is used in the computation program, that is, usually 20/20, which is the normal eyesight level of the population. Better eyesight levels might also be used to predict smaller 2D ideal viewing areas with enhanced legibility.
2. Text presented on different visual media is assumed to have different geometries (height, height-to-strokewidth ratio) and be under different lighting conditions (background luminance, luminance contrast percent), while text presented on the same visual medium is assumed to be identical in their geometries and under uniform lighting conditions. In practice, however, even on the same visual medium, text might be of different sizes and the lighting might be uneven. In such cases, the visual medium can be divided into several uniform pieces and then individually calculated in the program.
3. Thus far, the visual media is assumed to be rectangular in shape and without depth. The overlapped 2D ideal viewing area of text presented on a single

rectangular display is computed using nine critical points, as illustrated in Figure 80. Most visual media used in practice are rectangular in shape. For those not rectangular, similar calculating points can be found for computing their overlapped ideal viewing areas.

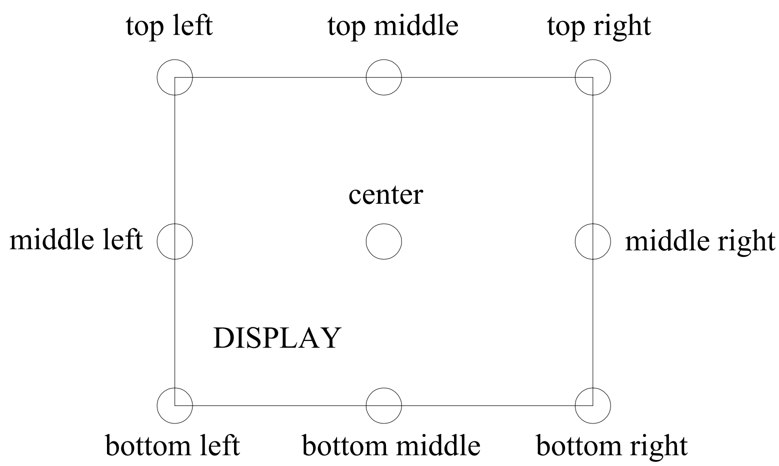


Figure 80. Nine points where text is presented to calculate the overlapped 2D ideal viewing area of a single visual medium

The flow of this computation program is listed below.

1. Input the number of visual media
2. Input the observer's eyesight level
3. Input the text geometries (height, height-to-strokewidth ratio), and the lighting conditions (background luminance, luminance contrast percent) where text is presented on each visual media
4. Calculate the on-axis legibility distance of text presented on each visual media
5. Input the geometries (heights, widths, locations on x-y-z coordinates) and initial orientations (Δϕ, Δα) of each visual media

6. Find the x-y-z coordinates of the nine calculating points on each media
7. Define the viewing plane where the observer's eyes are located
8. Draw the 3D ideal viewing spaces of text presented on each calculating point on each visual media, and then slice them all with the specified viewing plane
9. Plot the overlapped 2D ideal viewing area of text presented on all visual media
10. Display the parameters of the 2D ideal viewing area.

7.2 Architectural Application of the Computation Program in Lecture Halls

The developed computation program can be used in any field where reading text is important, such as architecture, wayfinding, driving safety, transportation, manufacturing, advertising, and exhibitions. This study uses it to facilitate modern lecture hall design, where architects need to precisely know the ideal shape and size of the audience area for arranging seats along the flat or sloped floor. The required overlapped ideal viewing area of text presented on multiple displays inside modern lecture halls can be determined using this program, by slicing the viewing plane parallel to the sloped floor of lecture halls at eye height level through all the 3D ideal viewing spaces of text presented on each single display. For instance, given 3 visual media installed in the front of a sample lecture hall with sloped floor, including a whiteboard, a middle projection screen, and a side tack board, and viewed by a standard observer with 20/20 eyesight sitting along the slopped floor. Size of the sample lecture hall is 16m (width) × 20m (length) × 8m (height), with the original point O(0m, 0m, 0m) preset to the bottom center of the front wall, as shown in Figure 81. The sloped angle of the floor is θ=18°. The distance in y-coordinate from the original point O to the start edge of the sloped viewing plane is 3.5m. The eye height of the seated observers on the floor is 1.2m. Table 19 lists all the geometries of the visual materials presented inside the lecture, along with their lighting conditions.

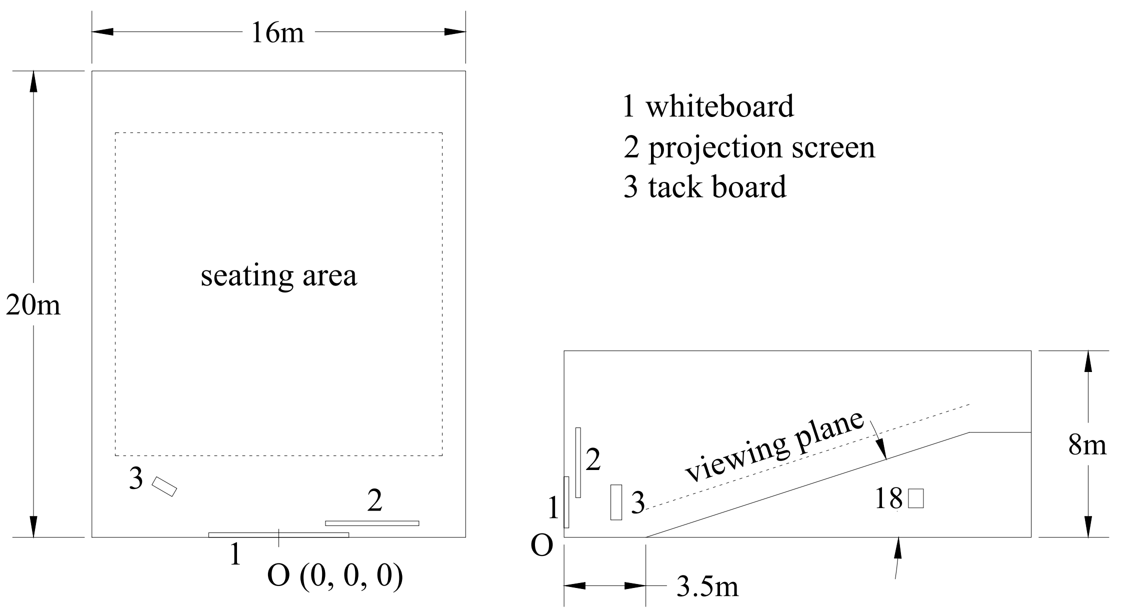
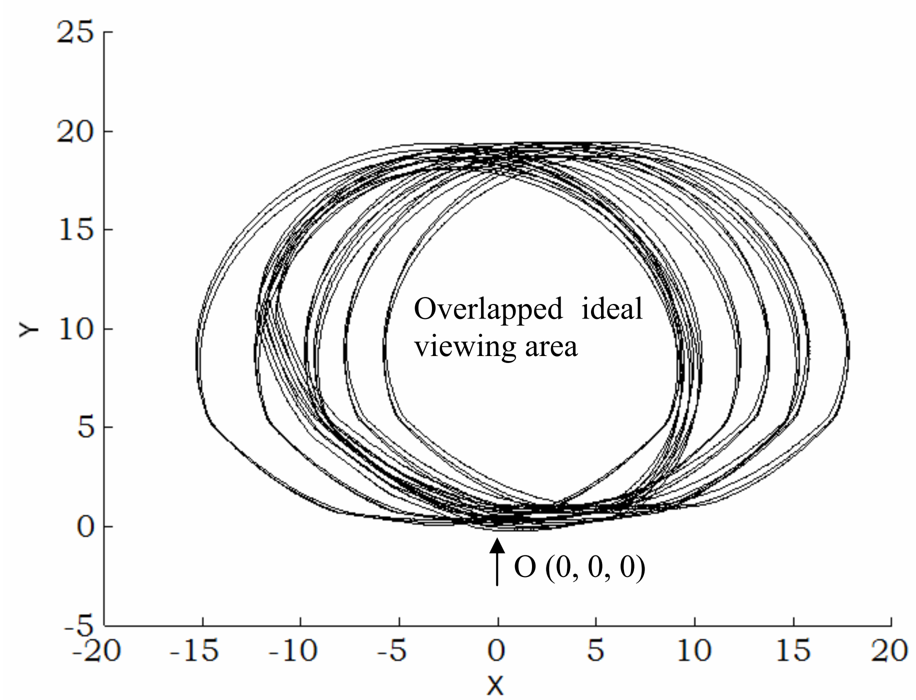


Figure 81. Sample lecture hall inside of which text is presented on three visual media and viewed by observers located along the viewing plane parallel to the sloped floor

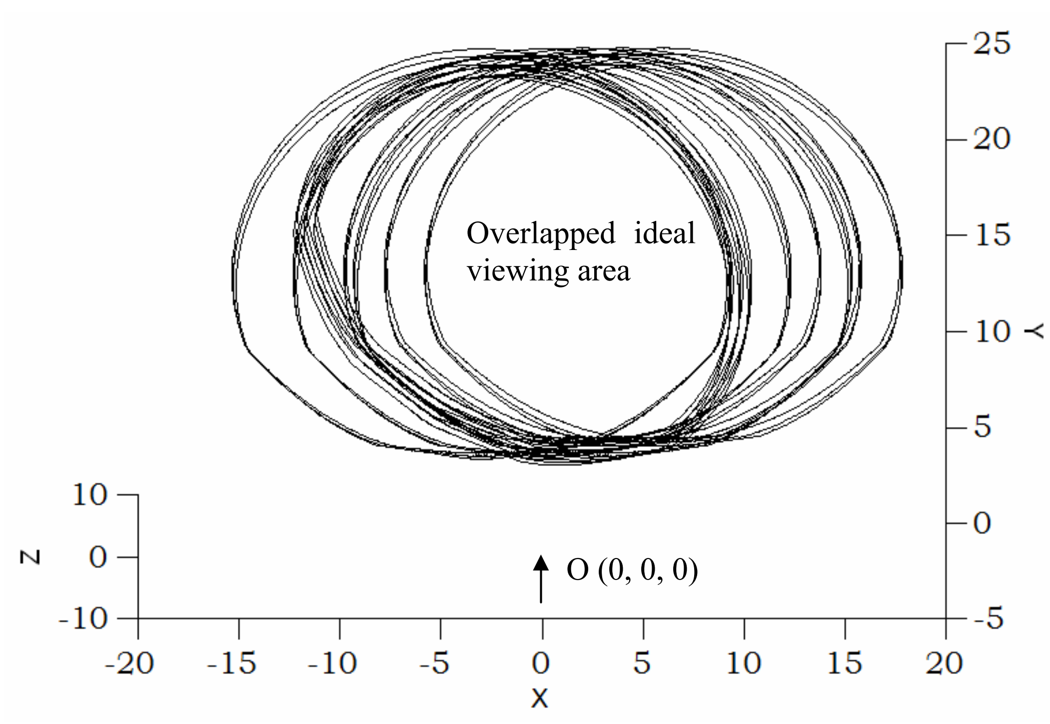
Table 19. Geometries of three viewing materials

Geometries	Whiteboard	Projection screen	Tack board
Text height	40mm	40mm	40mm
Height-to-strokewidth ratio	5	4.8	5.5
Background luminance L_b	65cd/m^2	55cd/m^2	70cd/m^2
Luminance contrast percent	93%	85%	90%
Size of visual media	6m (width) × 2.2m (height)	4m (width) × 3m (height)	1m (width) × 1.5m (height)
Original point of media	(0m, 0m, 1.5m)	(4m, 0.5m, 3.2m)	(-5m, 2m, 1.5m)
Initial orientation	$\Delta\phi=0°$, $\Delta\alpha=0°$	$\Delta\phi=0°$, $\Delta\alpha=0°$	$\Delta\phi=30°$, $\Delta\alpha=15°$

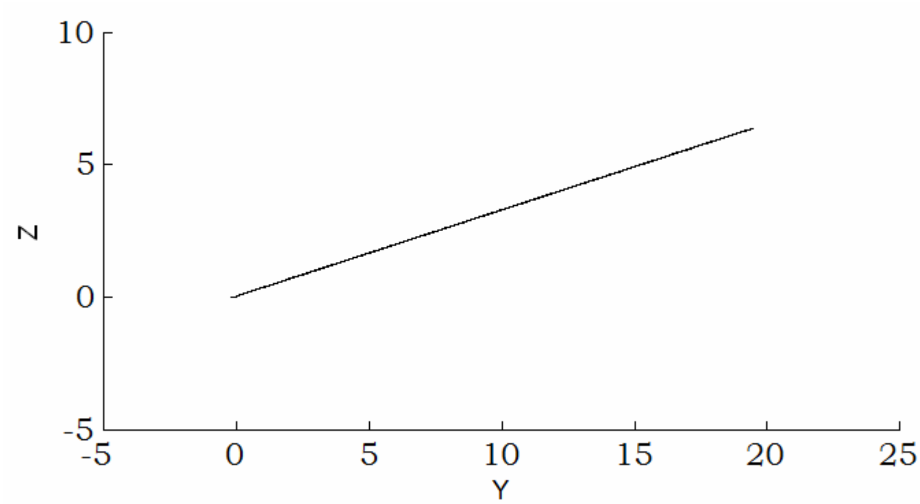
After running the program using all these inputs, the predicted overlapped 2D ideal viewing area is shown in Figure 82. This ideal viewing area should be fit into the audience area shown in Figure 81 previously, as illustrated in Figure 83, to arrange seats from which text simultaneously presented on all these visual media is guaranteed legible.



(a) X-Y plan view of the predicted overlapped 2D ideal viewing area of text (unit in meters)

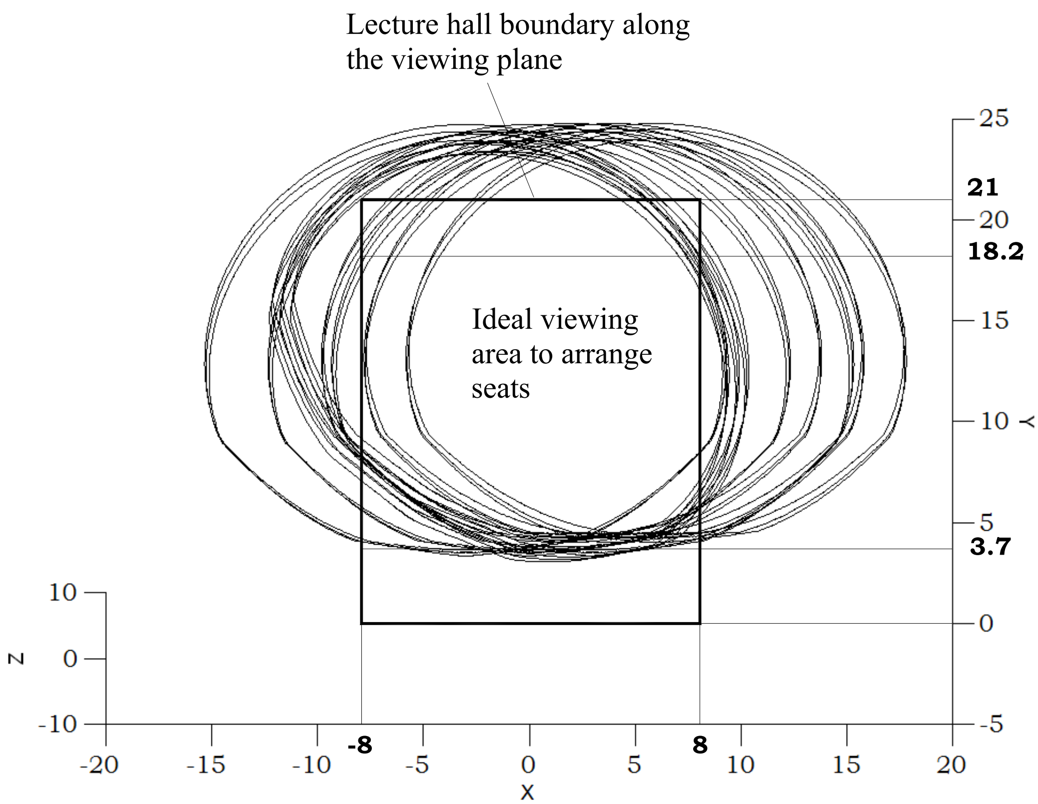


(b) X-Y-Z view of the predicted overlapped 2D ideal viewing area of text exactly on the viewing plane, which is parallel to the sloped floor at eye height level of 1.2m (unit in meters)

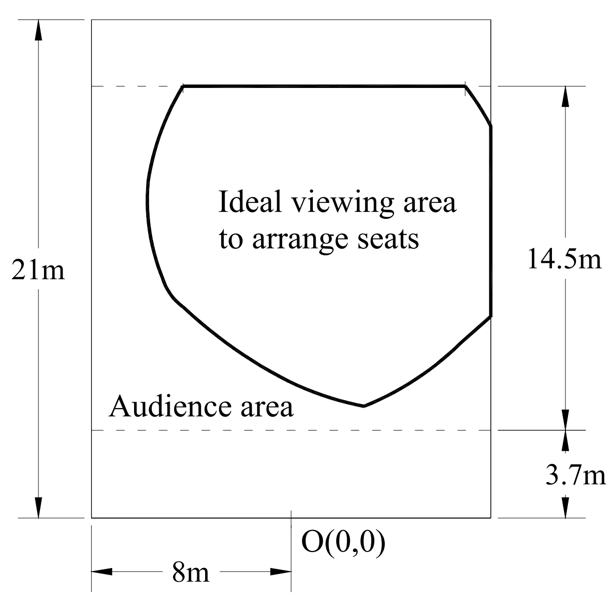


(c) Y-Z section view of the predicted overlapped 2D ideal viewing area of text
(unit in meters)

Figure 82. Predicted overlapped 2D ideal viewing area of text presented on whiteboard, projection screen, and tack board, simultaneously viewed by an observer with 20/20 eyesight who sits along the sloped floor with his eyes at 1.2m above the floor



(a) Process to determine the ideal viewing area for arranging seats



(b) Determined ideal viewing area to arrange seats (unit in meters)

Figure 83. Ideal viewing area used to arrange seats in the sample lecture hall, viewed along the plane parallel to the sloped floor at eye height 1.2 m

7.3 Verification of the Computation-Program-Aided Design Method

The previous example, as illustrated in Figure 82 and Figure 83, has demonstrated the applicability and the procedure for using the computation-program-aided design method in lecture hall design to determine the ideal viewing area, inside of which observers can clearly read text simultaneously presented on multiple displays with 100% accuracy. Before this innovative design method is recommended to architects in practice, the external validity of this preliminarily developed computation program to be used in lecture halls is verified using a field experiment carried out in the lecture hall in the Art & Architecture Building. This field experiment collected legibility data from 21 human subjects who read viewing materials presented at three different locations in the lecture hall.

7.3.1 Subject Requirements and Screening

Subjects (N = 21) must be 20-29 years of age, with exactly binocular eyesight 20/12.5 (with or without glasses), and normal color vision. This experiment uses 20/12.5 eyesight rather than 20/20, that is, the normal acuity level of the population, because:
(a) 20/12.5 eyesight is used just as a sample to test the computation-program-aided design method; the experimental results could be tested by other eyesight levels as well,
(b) much more subjects recruited in previous experiments have 20/12.5 eyesight than

20/20, and (c) observers with 20/12.5 eyesight have less common abnormal eye problems, therefore, more reliable results. All participants are strictly screened upon arrival using two self-made eyesight E charts based on the British standard BS 4274-1:2003, as shown in Figure 56 previously. These charts are mounted on the whiteboard in the front of the lecture hall, and lit with a table lamp to have uniform background luminance of at least 120 cd/m^2. Potential participants sit 20 ft away from the eyesight charts, with their gaze line perpendicular to the charts. To double check, all have their eyesight tested using two charts. In addition, potential participants are also asked if they have abnormal color vision. Since viewing targets in this experiment are achromatic letter Es, the requirement of normal color vision in this experiment is to preclude any potential negative effect of chromatic aberration on achromatic text.

7.3.2 Field Experiment Settings

The lecture hall (room 2104) in the Art & Architecture building is carefully chosen to do the test. The viewing materials include 3 E-charts (four lines of letter Es with the same height but random orientations, different from those used in previous experiment) printed on matte paper and presented in the lecture hall at different locations with different orientations, as illustrated in Figure 84. Subjects are asked to find the right seat(s) location to clearly read all three E-charts at the same time with threshold 100% accuracy.

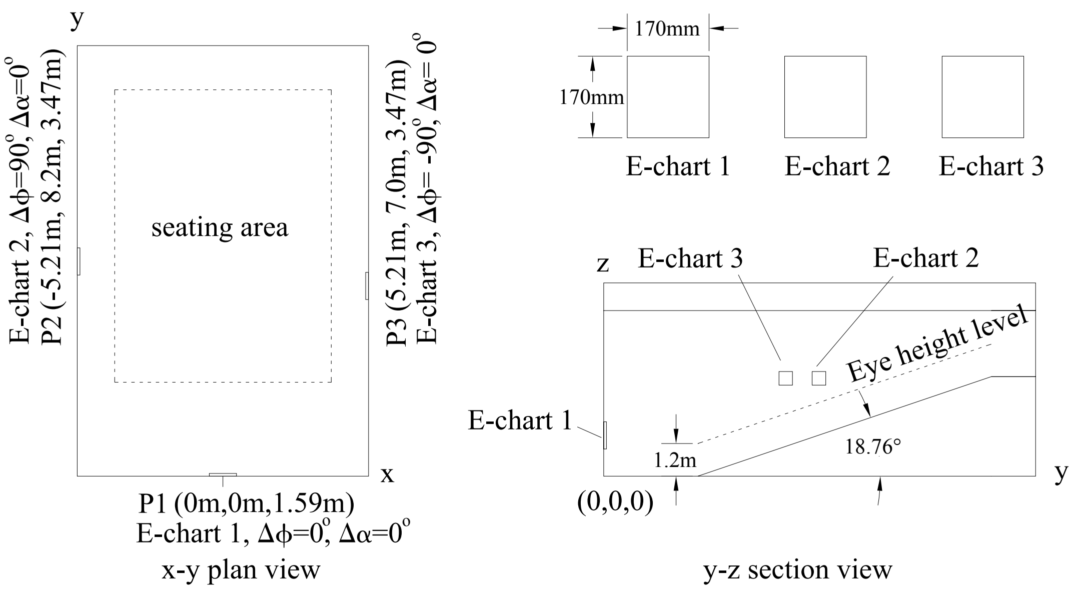


Figure 84. Experimental settings in the lecture hall

For the location of the viewing materials, as shown in Figure 84, E-chart 1 is installed on the whiteboard in the front of lecture hall and centered at point P1 (0m, 0m, 1.59m) with initial orientation (Δϕ=0°, Δα=0°). E-chart 2 is installed on the west side wall and centered at point P2 (-5.21m, 8.2m, 3.47m) with initial orientation (Δϕ=90°, Δα=0°). E-chart 3 is installed on the east side wall and centered at point P3 (5.21m, 7.0m, 3.47m) with initial orientation (Δϕ= -90°, Δα=0°). As illustrated in Figure 85, different E-charts have different letter size (6.6mm, 8.7mm, and 5.2mm) and different contrast (90%, 86%, and 53.3%). These letter Es are printed on matte paper. Each E-chart, including letter Es and margins, is 170mm wide by 170mm high, and subtends about 1.5° to the observer's eyes within the center foveal area when viewed at about 6.7 meters (Moon & Spencer, 1943). Excluding the margins, the actual distributed area of all letter Es is 125.4mm wide by 85.6mm high in E-chart 1, 147.9mm by 93.3mm in E-chart 2, and 98.8mm by 76.8mm in E-chart 3.

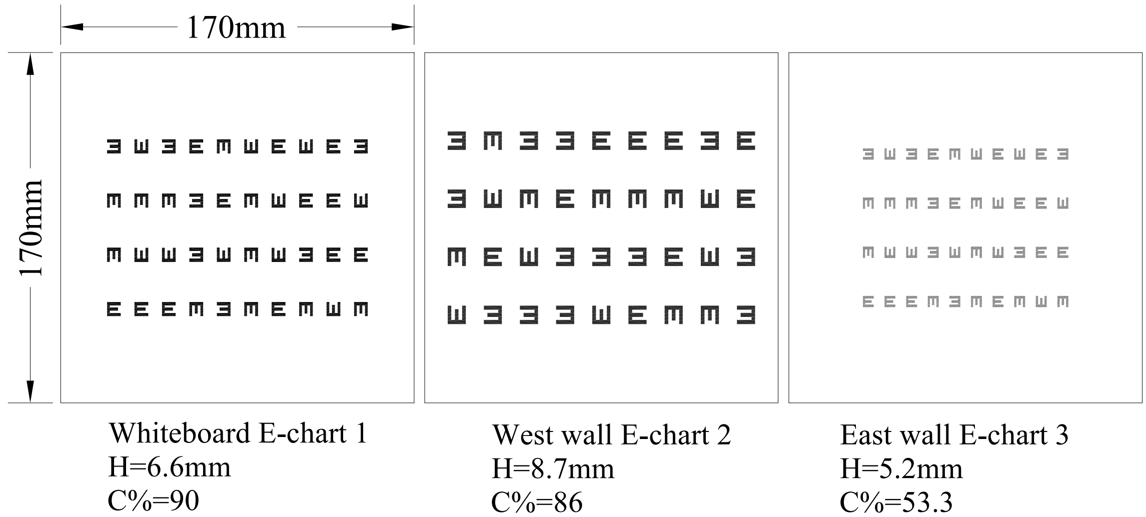
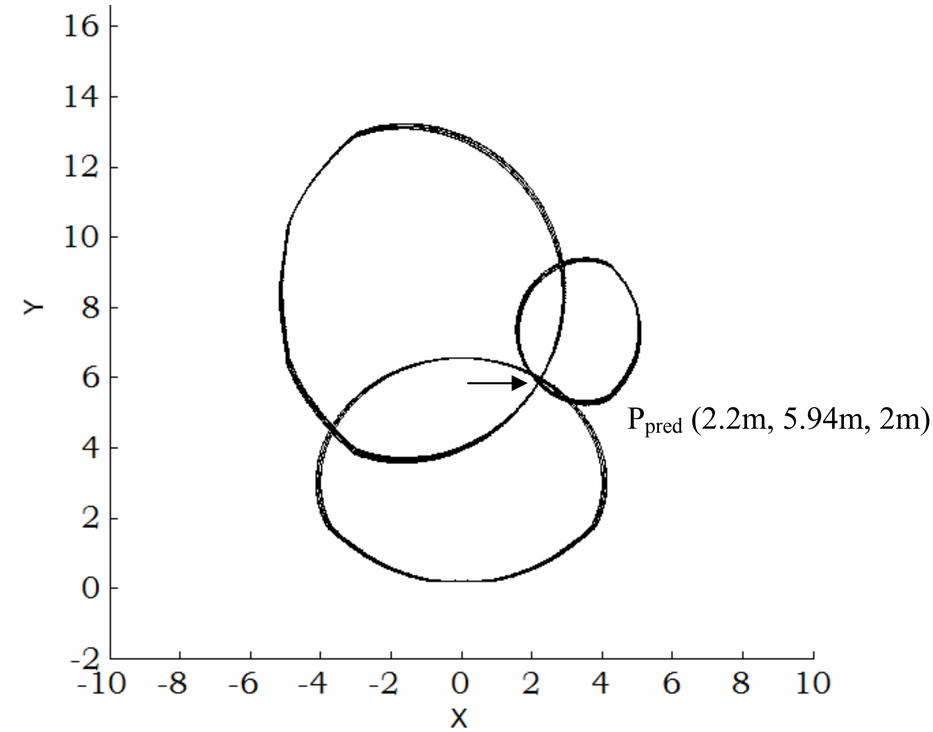


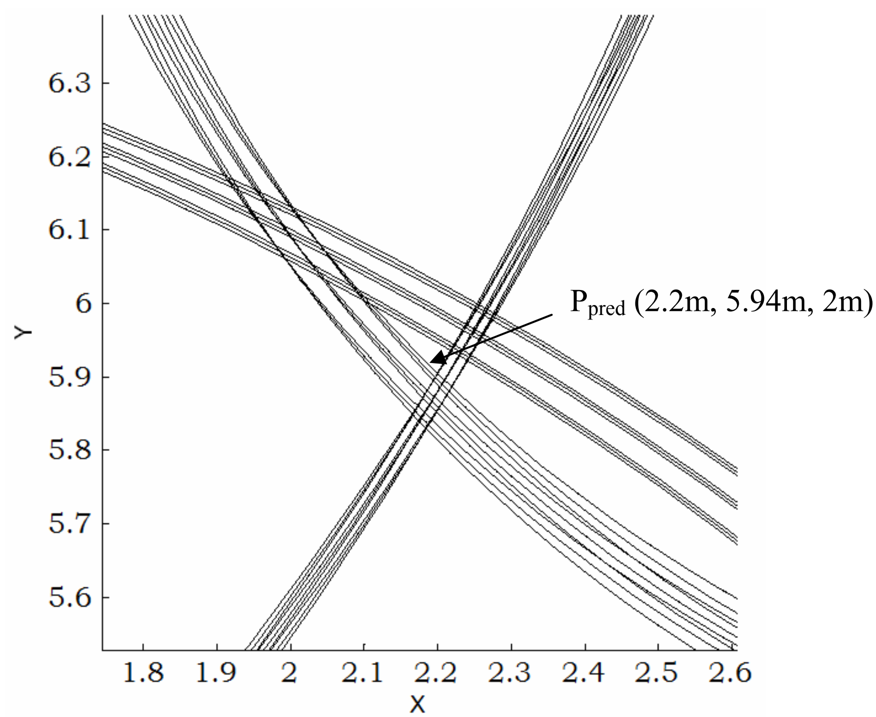
Figure 85. E-charts as the viewing materials

7.3.3 Predicted Ideal Seat Location

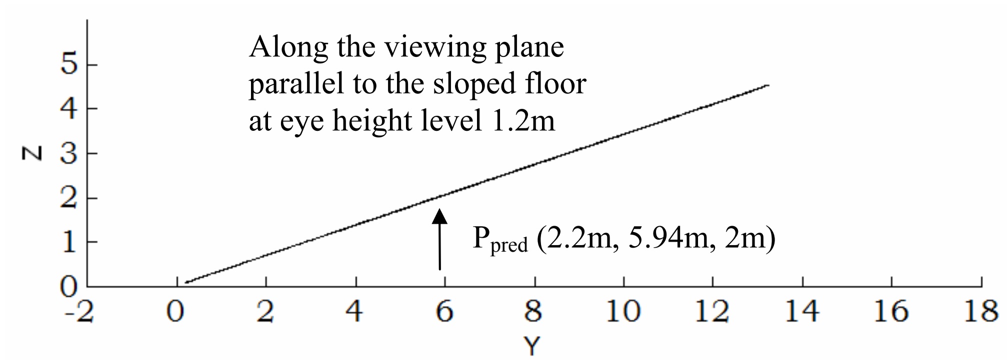
The ideal seat location where subjects can simultaneously read all three E-charts with 100% accuracy is then predicted using the computation-program-aided design method as the overlapped small area shown in Figure 86, which is located at the center point P_{Pred} (2.2m, 5.94m, 2m) with size about 0.2m by 0.2m. This predicted area is so small that it can be occupied within only one seat. Following field measurement in the lecture hall, the predicted location is actually the fourth fixed seat counting from the east side wall in the second row. In other words, as predicted using the computation-program-aided design method, sitting only in this seat can the subjects with 20/12.5 acuity clearly read all three E-charts.



(a) Predicted ideal seat location at the XY plan view (unit: meters)



(b) Zoom in of predicted ideal seat location (unit: meters)



(c) YZ section view of the ideal seat location (unit: meters)

Figure 86. Predicted ideal seat location in the lecture hall as the overlapped small area

In the field experiment, the actually observed seat locations P_{ob} where subjects are able to recognize all three E-charts with threshold 100% accuracy are recorded, and then compared to the predicted seat location P_{pred} (2.2m, 5.94m, 2m). Theoretically, if these two locations are coincident at a good statistical significance level, the application of this computation-program-aided design method in lecture halls is then proven, and can thus be recommended to architects in their design practice.

7.3.4 Experimental Procedure and Duration

The field experiment takes about 20 minutes for each subject. The 10 steps to carry out this field experiment are:

1. Subject recruitment. A total of 28 potential subjects are recruited using emails, posters, and flyers.

2. Experiment installation. The three E-charts are mounted in the lecture halls at their preset locations and orientations.
3. Screening subjects. All 28 subjects are screened upon arrival; 7 fail the requirements. The 21 subjects with eyesight 20/12.5 and normal color vision participate in this experiment.
4. Subjects sign the consent form once the experimenter explains the procedures and answers any questions.
5. Before the experiment starts, subjects sit in the lecture hall for 5 minutes to adapt their eyes to the preset light levels.
6. The experimenter demonstrates the selection pattern for subjects to choose an ideal seat location in lecture halls to simultaneously read three E-charts with 100% accuracy.
7. Experiment starts. The experimenter asks the subjects to find the ideal seat(s) location where he/she is able to clearly read all three E-charts with 100% accuracy. In this experiment, at least 1, at most 4, on the average 3 times have been tried by each subject to find the final ideal seat location. During this process, the experimenter double checks the reading performance of the subjects.
8. The final seat location the subject finds is then recorded for each subject.
9. Subjects are paid and dismissed.
10. Data analysis.

7.3.5 Factors Examined

Only one parameter is recorded in this field experiment — seat location, which is the combination of viewing distances and viewing angles. Other non-examined factors all have preset values. Only one light level is examined — the typical fluorescent lighting conditions in the lecture hall for blackboard teaching and/or class discussion, with both blackboard lighting and audience area lighting. Subjects are actually 20-28 years old (20-29 as required), mostly 22~23, and all have the same Snellen acuity level of 20/12.5. The viewing materials of the E-charts are printed black/white on matte paper. Letter Es have constant height-to-strokewidth ratio of 5, but different heights (6.6mm, 8.7mm, 5.2mm) and different contrast (90%, 86%, and 53.3%) for E-charts 1, 2, and 3, respectively.

7.3.6 Data Analysis and Results

The distribution of the observed ideal seat locations are illustrated in Figure 87, which is the XY plan view of the lecture hall tested. The predicted ideal seat location is lightly shadowed as the fourth fixed seat counting from the east side wall in the second row. Of the 21 subjects, 17 (81% of 21) choose the predicted ideal seat location during the test, while 3 other subjects (14% of 21) choose the immediately adjacent one, the fifth fixed seat counting from the east wall in the second row. Only one subject (5% of 21) insists that he can see all three E-charts the best in the sixth seat counting from the east wall in the second row.

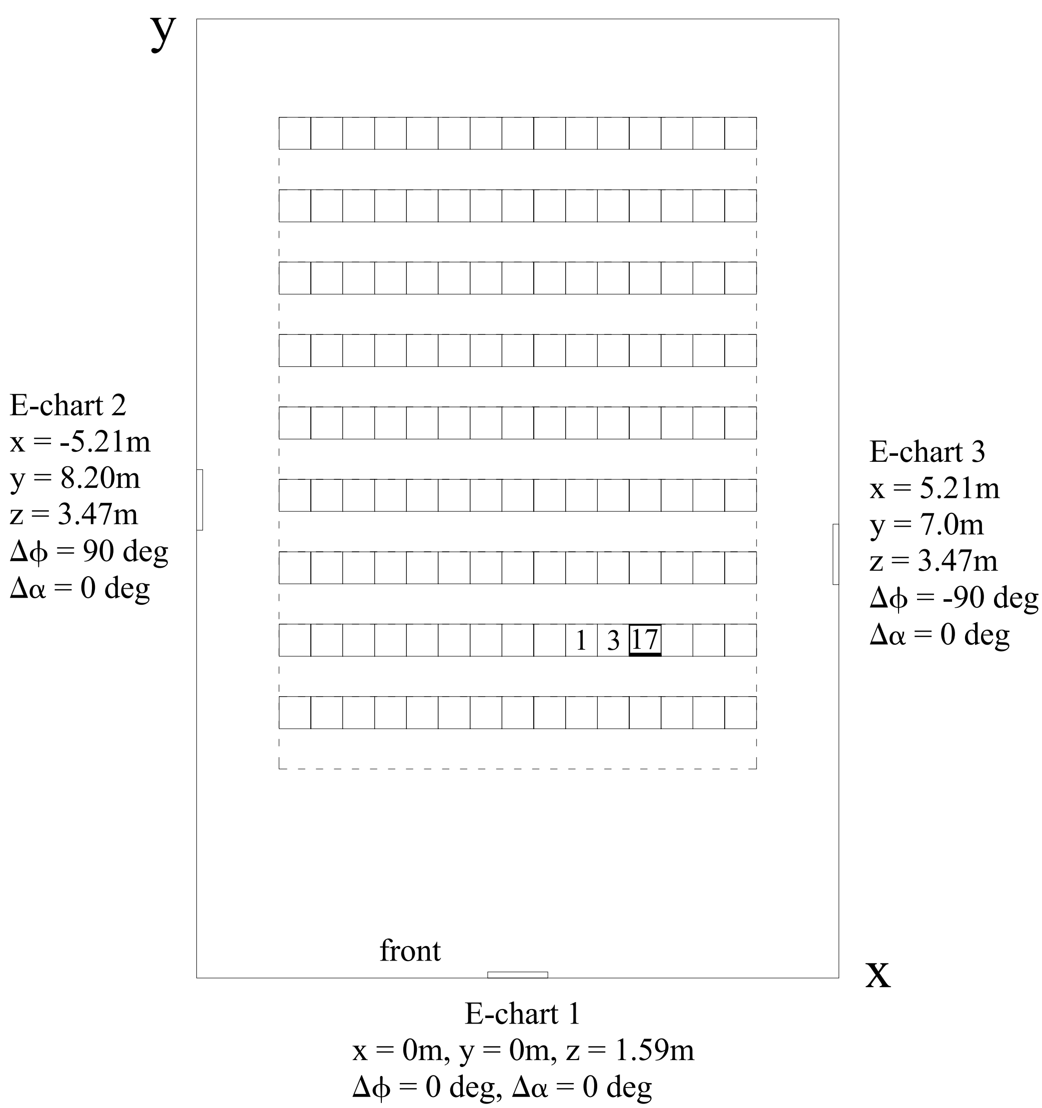


Figure 87. Observed ideal seat locations chosen by the 21 subjects

To statistically verify the computation-program-aided design method using the outcomes, this study calculates the *p*-value if the null hypothesis (H_0: $p \leq 0.5$) is true, that is, less than 50% of the audience (i.e., not a majority of the population) sitting in lecture halls will choose the actual seat predicted by the design method. The calculated *p*-value equals 0.0023 ($z = 2.84$) for the outcome of the experiment that 17 of 21 subjects have chosen the predicted seat. Therefore, the outcome (17 of 21 subjects choose the predicted seat) is considered to be very statistically significant to reject the null hypothesis (H_0: $p \leq 0.5$) even at the significance level of 0.01 (usually $\alpha = 0.05$ in conventional criteria). In addition, given an error allowance of one seat, which might still be acceptable in lecture hall design for determining the ideal viewing area, 20 of 21 subjects have chosen the predicted seat or its immediately adjacent one in the experiment. For a stricter null hypothesis H_0: $p \leq 0.8$, that is, less than 80% of the audience sitting in lecture halls will choose the predicted seat or its immediately adjacent one, the calculated p-value equals 0.003 ($z = 2.75$) for the outcome that 20 of 21 subjects choose the predicted seat or its immediately adjacent one. According to the p-value of 0.003, the outcome (20 of 21 subjects choose the right seats) is very statistically significant at the level of 0.05 to reject the stricter null hypothesis (H_0: $p \leq 0.8$). Consequently, the computation-program-aided design method has proven to be an accurate and fully reliable tool for predicting the overlapped ideal viewing area of text viewed in modern lecture halls.

7.4 Summary

After the derived and verified equation (28) is further improved to (34) to predict the legibility levels of text viewed at any incident angles $0° \leq \xi \leq 90°$, (34) is then used as the underlying algorithm to develop a computation-program-aided design method in MatLab. This program-aided method is specifically used to find an overlapped 2D ideal viewing area of text presented on multiple visual media installed with different geometries, locations, mounting heights, and orientations, under different lighting conditions and viewed by observers with different eyesight levels located along any viewing plane, similar to that parallel to the sloped floor in modern lecture halls at eye height level. This program-aided method can be used in many fields where simultaneously reading text presented on multiple displays is very important for functions such as navigation, well-being, productivity, safety, and security. Its application in modern lecture hall design is to help architects find ideal viewing areas of text for arranging seats in the audience area. Before this computation-program-aided design method can be recommended to architects in practice, its external validity is verified using a field experiment carried out in the lecture hall in the Art & Architecture Building. This experiment has proven that the program-aided method is accurate and reliable with the outcomes that 17 of the 21 subjects choose the predicted ideal seat location during the test, while 3 other subjects choose the immediately adjacent seat of the predicted one.

CHAPTER 8

Conclusions

8.1 Key Outcomes of the Study

In summary, this research study presents three key outcomes that have been proven using (a) fundamental theories of visual recognition and legibility of text, and (b) legibility data collected from human subjects in the laboratory and field experiments.

Key outcome 1. The equation below, also (28) previously, for predicting the legibility levels of text presented on matte surfaces of various visual media under uniform fluorescent target lighting without glare, and recognized by young observers (aged 20-29) at incident angles 0° - 82.8° with a threshold (just readable) 100% accuracy.

$$D = \begin{cases} 2443.5 \cdot H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{-1} \cdot L_b^{0.213} \cdot C_{\%}^{0.532} \cdot (\cos\xi)^{0.5} & 0^0 \le \xi \le 65.7^0 \\ 2443.5 \cdot H \cdot \left(\dfrac{H}{Sw}\right)^{-1} \cdot S_d^{-1} \cdot L_b^{0.213} \cdot C_{\%}^{0.532} \cdot (\cos\xi)^{0.5} \cdot (0.024\xi - 0.577)^{-1} & 65.7^0 < \xi \le 82.8^0 \end{cases}$$

where:

D = Legibility distance when text is viewed not perpendicular to the observer

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of the observer's acuity level

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ξ = Incident angle between the display normal and the sightline of the observer

This legibility equation examines seven critical factors, including the viewing angle, which is rarely examined in the literature. It can predict legibility distance, legible height of text, height-to-strokewidth ratio of text, observer's eyesight in the Snellen fraction, background luminance of text (surface luminance of display), luminance contrast of text, and incident angle between 0° and 82.8°. This equation cannot be used, however, for extremely distorted text viewed beyond 82.8° till 90°. In addition, this equation would better hold within viewing conditions $1 \leqslant L_b \leqslant 1000$ cd/m^2; $10 \leqslant C_{\%} \leqslant 90$; $0.2 \leqslant Ac \leqslant 2.0$ min^{-1}, which can be traced back to Kaneko's equation. In general, this new legibility equation is applicable in multiple fields where reading text is important, such as instruction and presentation, traffic and transportation, navigation and wayfinding, advertising, safety and security, when the restricted viewing conditions are met.

Key Outcome 2. Adaptation luminance of text is primarily determined by the foveal luminance, with very small contribution from the ambient luminance, given no glare sources are visible at the periphery of the viewing field. When the background luminance of text (that is, surface luminance of displays) remains constant, changing the ambient light levels of the viewing scenario does not change the reading performance of text. Therefore, ambient light has a negligible effect on the legibility of text.

This conclusion seems to go against people's common experience of turning on the ambient light to comfort their eyes when reading text. It actually does not, though, for two reasons. First, ambient light used in daily life usually has a small portion reflected to the display surface and increases the background luminance of the text viewed, thus enhancing its legibility. Second, ambient light might have a positive psychological effect, which has not yet been proven, to comfort the observer's eyes by balancing the brightness distribution in the viewing field. In this study, many subjects tested in the laboratory experiments have reported such feelings to the experimenter when the ambient light is turned on and the display surface luminance is dimmed to a constant level. Therefore, the key outcome 2 can be useful for enhancing the legibility of text viewed in various fields to save energy, but only if the comfort of observer's eyes is not sacrificed. Further research is required.

Key Outcome 3. The computation-program-aided design method for architects to predict an overlapped 2D ideal viewing area of text presented in modern lecture halls on multiple visual media installed with different geometries, locations, mounting heights, and orientations, under different lighting conditions and viewed by observers located along any viewing plane, similar to that parallel to the sloped floor of lecture halls at eye height level. The program code appears in Appendix G.

This program-aided design method presumes: (a) identical young observer (aged 20-29) each time to compute a 2D ideal viewing area, (b) identical geometries and uniform lighting conditions of text presented on the minimum calculating unit of display, (c) rectangular visual media without depth, (d) a threshold (just readable) 100% accuracy of reading performance. For a mass audience, their acuity level is usually chosen to be 20/20, which is the normal eyesight level of population. In addition to lecture halls, this method can also be used in other scenarios with multitask viewing conditions of text. However, this program-aided design method cannot be used for chromatic text, observers in other age groups, specular displays, viewing situations with glare and light trespass, or a light spectrum other than fluorescent lamps.

8.2 Architectural Implications of the Key Outcomes and Their Limitations

The key outcomes of this study will help architects, interior designers, lighting designers, and other professionals design new lecture halls or improve existing ones with enhanced legibility, lighting quality, and energy savings. The innovative design methods—the derived legibility equation and the preliminary computation program—will benefit architectural design and foster new thinking in creating and maintaining legible environments. The applications of these key outcomes for lecture hall design are detailed in the following five areas.

(1) Calculating ideal viewing distances or angles for recognizing fixed text

Fixed text presented in lecture halls, such as signage and placards, usually have known—either assigned or measured—sizes (e.g., height H, strokewidth Sw) and lighting conditions (e.g., background luminance L_b, luminance contrast percent $C_{\%}$). For an observer with known acuity, the ideal viewing distance D to recognize the fixed text at a given incident angle ξ with threshold 100% accuracy can be calculated directly using the equation developed from this study, (28) previously. In lecture hall design, the entire audience is usually the primary concern rather than the individual; therefore, the standard Snellen acuity level 20/20 of the population is recommended for calculating the ideal viewing distance. On the other hand, when ideal viewing angles are desired for fixed text viewed at a known distance, they are calculated using (42) re-expressed from (28).

$$
\begin{cases}
\xi = \arccos\left[1.68\times10^{-7}\cdot D^2\cdot H^{-2}\cdot\left(\dfrac{H}{Sw}\right)^2\cdot {S_d}^2\cdot {L_b}^{-0.426}\cdot {C_\%}^{-1.064}\right] & 0^\circ \le \xi \le 65.7^\circ \\
\dfrac{\cos\xi}{(0.024\xi-0.577)^2} = 1.68\times10^{-7}\cdot D^2\cdot H^{-2}\cdot\left(\dfrac{H}{Sw}\right)^2\cdot {S_d}^2\cdot {L_b}^{-0.426}\cdot {C_\%}^{-1.064} & 65.7^\circ < \xi \le 82.8^\circ
\end{cases}
\tag{42}
$$

where:

ξ = Incident angle between the display normal and the sightline of the observer

D = Legibility distance when text is viewed not perpendicular to the observer

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of the observer's acuity level

L_b = Background luminance

$C_\%$ = Luminance contrast percent

The predicted ideal viewing distances and viewing angles of fixed text will allow architects and interior designers to arrange them more efficiently in lecture halls. However, equation (28) or (42) cannot be used for incident angles beyond 82.8° until 90°. In addition, its application in lecture halls is restricted to text presented with background luminance $1 \leqslant L_b \leqslant 1000$ cd/m^2 and luminance contrast percent $10 \leqslant C_\% \leqslant 90$, and viewed by observers with acuity $0.2 \leqslant Ac \leqslant 2.0$ min^{-1}. This study has found that, inside typical lecture halls, background luminance L_b of text usually falls in the range of 1 – 1000 cd/m^2, while observers' acuity Ac falls in the range of 0.2 – 2.0 min^{-1}. The luminance contrast percent $C_\%$ of text, however, may fall in the range of 90 – 100, particularly for text in black/white.

(2) Calculating the legible size of text viewed at fixed distances and angles

For creating legible environments, architects and interior designers need to know the threshold legible (just readable with 100% accuracy) size of text viewed at known distance D, incident angle ξ, and lighting conditions. In lecture hall design, text is often presented on fixed displays installed at different locations with different mounting heights and orientations. The viewing distance D and viewing angle ξ of each display are determined if the observer is located in the audience area. The background luminance L_b and luminance contrast percent $C_{\%}$ of text are either assigned or measurable before or after the lighting design and installation. Therefore, the threshold legible strokewidth Sw of text viewed by an observer with specific acuity or by the entire audience with standard 20/20 acuity can be calculated using (43), re-expressed from (28) previously.

$$Sw = \begin{cases} 4.1\times10^{-4} \cdot D \cdot S_d \cdot L_b{}^{-0.213} \cdot C_{\%}{}^{-0.532} \cdot (\cos\xi)^{-0.5} & 0^\circ \le \xi \le 65.7^\circ \\ 4.1\times10^{-4} \cdot D \cdot S_d \cdot L_b{}^{-0.213} \cdot C_{\%}{}^{-0.532} \cdot (\cos\xi)^{-0.5} \cdot (0.024\xi - 0.577) & 65.7^\circ < \xi \le 82.8^\circ \end{cases} \tag{43}$$

where:

Sw = Threshold legible strokewidth of text

D = Legibility distance when text is viewed not perpendicular to the observer

S_d = Denominator in the Snellen ratio of the observer's acuity level

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

ξ = Incident angle between the display normal and the sightline of the observer

With predicted threshold legible strokewidth *Sw*, the threshold legible height *H* and width *W* of the text can be easily calculated using the height-to-strokewidth ratio *H/Sw* and height-to-width ratio *H/W*, which are determined by text font. However, extremely distorted incident angles $82.8° < \xi \leqslant 90°$, which are rare in lecture halls, are not examined in (43). Likewise, the unexamined luminance contrast percent $90 < C_{\%} \leqslant 100$ in (43) may pose problems for predicting the threshold legible size of text in black/white.

(3) Calculating the threshold lighting conditions for energy savings

Among all functional and aesthetical considerations for lighting lecture halls, creating legible yet energy efficient environments is the top one. In practice, architects and lighting designers often seek energy efficient lighting solutions that will not sacrifice good viewing conditions inside lecture halls. This study has provided them a quantitative and reliable method. For text with assigned or measured size (e.g., *H*, *H/Sw*) to be legible when viewed at known distances *D* and angles ξ in lecture halls, the minimum required background luminance L_b and luminance contrast percent $C_{\%}$ can be calculated using (44) for energy savings without sacrificing legibility. The calculated mathematical product of L_b and $C_{\%}$ of the threshold legible text can provide architects the bottom line for energy savings in lecture hall design. For external validity of this method, the standard acuity 20/20 of the population is recommended for the calculation rather than any individual acuity levels.

$$L_b^{\;0.213} \cdot C_{\%}^{\;0.532} = \begin{cases} 4.1\times10^{-4} \cdot D \cdot H^{-1} \cdot \left(\dfrac{H}{Sw}\right) \cdot S_d \cdot (\cos\xi)^{-0.5} & 0^\circ \le \xi \le 65.7^\circ \\ 4.1\times10^{-4} \cdot D \cdot H^{-1} \cdot \left(\dfrac{H}{Sw}\right) \cdot S_d \cdot (\cos\xi)^{-0.5} \cdot (0.024\xi - 0.577) & 65.7^\circ < \xi \le 90^\circ \end{cases} \tag{44}$$

where:

L_b = Background luminance

$C_{\%}$ = Luminance contrast percent

D = Legibility distance when text is viewed not perpendicular to the observer

H = Normal text height

Sw = Strokewidth of text

S_d = Denominator in the Snellen ratio of the observer's acuity level

ξ = Incident angle between the display normal and the sightline of the observer

This method calculates the mathematical product of background luminance L_b and luminance contrast percent $C_{\%}$. Thus, L_b could be lowered for more energy savings by increasing $C_{\%}$ of text to remain at the same legibility level. However, the unexamined luminance contrast percent $90 < C_{\%} \leqslant 100$ in (44) might harm the potential of this method for energy savings when text is presented in black/white at the highest contrast.

Furthermore, the finding that ambient light has a negligible effect on legibility can be used for additional energy savings by dimming the ambient light in lecture halls while keeping the background luminance of text constant. However, it is unknown whether dimming the ambient light to total darkness will sacrifice observer's vision comfort.

(4) Calculating the ideal viewing areas of multiple fixed displays

As demonstrated in Chapter 7.2, architects can use the program-aided design method to calculate ideal viewing areas of multiple displays installed in lecture halls for better seating arrangement, more reasonable shape of lecture halls, and more efficient allocation of interior spaces. Ten inputs are required from architects to run this preliminary program and output the overlapped drawing including:

1. Number of visual media
2. Height and width of each visual media in meters
3. x-y-z coordinates of the center point of visual media in meters
4. Initial horizontal and vertical viewing angles of the visual media in degrees
5. The denominator of observer's Snellen eyesight
6. Height and height-to-strokewidth ratio of text to be viewed in mm
7. Background luminance and luminance contrast percent of the text in cd/m^2
8. The angle of the sloped viewing plane in degrees
9. The y-coordinate distance of the sloped viewing plane from original point to the start edge in meters
10. The height of observers' eyes on the sloped floor in meters

These required values might be easily measured or assigned for fixed visual media installed in lecture halls when the viewing plane on which observers' eyes are located is known. For acuity level, the population's standard acuity of 20/20 or better is recommended. This preliminary program usually takes up to 10 minutes to calculate for 3 visual media. However, when the number of visual media is larger than 4 or 5, this preliminary program begins to consume too much computer memory and takes hours to output the drawing.

(5) Determining appropriate sizes, locations, and orientations of visual media

In lecture hall design, one of the primary tasks of architects and interior designers is the arrangement of visual media in the front space. By adjusting the 10 inputs as listed previously, the program can help them find the appropriate locations, sizes, orientations, and mounting heights of different visual media. The adjustment is a back-and-forth process. Architects may need to run the preliminary program many times before achieving a satisfactory result, which could take an uncomfortably long time. To expedite the adjustment, there are three guidelines architects might follow.

1. Find out the adjustable and nonadjustable inputs. Not all of the 10 inputs are adjustable in every lecture hall. Nonadjustable inputs are less dependent on the visual media, such as the number of visual media, observer's acuity level and eye height, text font (for height-to-strokewidth ratio), the location and sloped angle of the viewing plane.
2. Determine reasonable ranges of each input before the adjustment; then set up the most favorable and the worst viewing conditions within these ranges.
3. Begin with the worst viewing conditions and adjust only one input each time by half of the remaining adjustable range of each step.

8.3 Recommendations for Future Research

Continuous research studies over the next 5-10 years will first solve the deficiencies in the preliminarily developed computation program, and then overcome the ten assumptions used here to examine more types of real viewing situations to read not only text but also graphics in architecture and other fields. Subtasks include:

1. Investigate the predictability of the derived equation, (17) or (18) previously, for text with a luminance contrast percent $C_{\%} > 90$. Then examine the spatial legibility of text viewed extremely off axis at incident angles beyond 82.8° till 90°, to improve the derived equation, (34) previously. In addition, find the calculating points for visual media not in rectangular shape to compute their overlapped ideal viewing areas. All the outcomes are then used to improve the computation program developed here.
2. Investigate the spatial legibility of different viewing targets, including different fonts, words, graphics, chromatic characters, and Asian characters (e.g., Chinese, Japanese, Korean).
3. Examine the spatial legibility of text and graphics presented on non-diffusive surfaces, including, for example, computer screens, TV monitors, and projection screens.
4. Study the spatial legibility of text and graphics under unfavorable lighting conditions, including, for example, nonuniform target lighting, glare, and the spectrum of lamps other than fluorescent ones.

5. Analyze the aging effect on the spatial legibility of text and graphics.
6. Test the influence of imperfect reading performance, other than the threshold (just readable) 100% accuracy, on the legibility levels of characters, when error rate or guessing is allowed.
7. Investigate the possible positive psychological effect of comforting the observer’s eyes by balancing the brightness distribution in the viewing field, and then quantify this effect, together with the developed legibility equation, on legibility enhancement and energy savings.
8. Use these new outcomes to develop an advanced software program based on the computation program developed here. This software can be used by architects or professionals in other fields to determine for observers of varying ages and eyesight levels (a) ideal viewing distance and viewing angles for recognizing fixed characters, (b) appropriate size, contrast, font, and color of characters for fixed viewing distance and viewing angles, (c) ideal viewing areas of multiple displays in large spaces, and (d) appropriate size, location, and orientations of different displays installed in buildings or their surroundings.
9. Redefine the Legibility Index (LI') in light of the solid angle subtended by the characters viewed (this portion of the work has already been carried out in another related research study carried out by this author), and then develop a practical method and a corresponding legibility meter for assessing the spatial legibility of text and graphics, based on the redefined Legibility Index. As we continue to test and popularize the Legibility Index (LI') in practice, we plan

to develop a "Standard Legibility Index (LI_S)" to assess the legibility levels of standard viewing characters recognized in different scenarios by a standard observer with 20/20 eyesight, explore the principles and potentials of a legibility meter in a lighting laboratory, develop a tentative legibility meter in the laboratory, and if successful, improve the legibility meter in laboratory testing and practical application.

APPENDICES

APPENDIX A

Guidelines for Achieving Favorable Viewing Conditions in Lecture Halls (Hauf et al, 1961; Duncan, 1966; Kemper, 1979; Allen et al, 1991 & 1996; and Niemeyer, 2003)

Aspects	Guidelines for achieving favorable viewing conditions
Shape of lecture halls	Rectangular small lecture halls have an ideal ratio of 1.5 (depth) × 1 (width). For large lecture halls, a fan-shape or a semicircular shape is preferred. Lecture halls with a capacity over 100 need sloped or tiered floors and staggered seating to improve sightlines. The sloped floors should be no more than a 1/12 ratio. For flat-floor lecture halls with capacity over 100, a platform in the front of the room is considered.
Layout of the front space	Architects should cover the front space of lecture halls with boards and screens but not overlap them. No protrusions of the front wall into the room are allowed. Adequate space in the front center should be reserved to accommodate overhead projectors, screens, etc., or walking spaces for the pacing speaker, and open space for presentations, displays, and experiments. Architects should calculate audience sightlines to make sure all boards and screens can be seen from top to bottom.
Projection screens	Front projection screens are recommended over rear projection screens for higher resolution, better color fidelity, and better contrast ratios. Matte screen is preferred to glass-beaded and lenticular screens for wider viewing angles. Multiple screens are preferred than one very large screen for more flexibility and reduced obstruction of the writing board, with a minimum of 6′ chalkboard remaining exposed. In addition, lecture halls with capacity over 200 need two or more 10′, 12′, or 14′ motorized screens. Screen size should be determined for the maximum viewing distance within the room. Screen size 6′ high is required for maximum viewing distance 35-40′. Likewise, 6.75′(H) screen for 40-45′maximum viewing distance (D_{max}), 7.5′(H) for 45-50′(D_{max}), 8.25′(H) for 50-55′(D_{max}), 9′(H) for 55-60′(D_{max}), and 10.5′(H) for 60-70′(D_{max}).

Appendix A (continued)

Aspects	Guidelines for achieving favorable viewing conditions
Viewing materials	Projected images must be large enough; text must be at least 12 point to be readable in the back row.
Lighting conditions	Four lighting zones are needed for lecture halls: (a) audience seating area, (b) front presentation area, (c) center of board/screen, and (d) both sides of board. No light trespass (no more than 3-5 fc) on the screen is allowed. During the course of lecture hall presentation, 40-50 fc is required for audience interaction. Normally 60-70 fc is required at the writing area, reduced to 5-10 fc when dimming. Use separate pairs of front lectern spotlights. Lights for the lower chalkboard and upper chalkboard/screen should be separated. Projection screen should be mounted to clear any chalkboard or marker board lights. Surface reflectances are 80% or higher for ceilings, 50-70% for walls, 20-40% for floors, and 25-45% for desktops.

APPENDIX B

Glossary of All Signs Used in This Study, Particularly the 95 Legibility Equations Listed in Appendix C

ΔE = Color contrast

ΔL^* = Color difference at L^* coordinates of CIELUV

Δu^* = Color difference at u^* coordinates of CIELUV

Δv^* = Color difference at v^* coordinates of CIELUV

ϕ = Horizontal viewing angle, $\pm 0°$-$90°$

α = Vertical viewing angle, $\pm 0°$-$90°$

ξ = Incident angle between display normal and the viewing line, 0°-90°

ω = Solid angle

A = Target area

A_c = Visual acuity

A_g = Age

A_{horz} = Horizontal area

A_{LC} = Letter copy area in a sign design

A_S = Snellen visual acuity

A_V = Vistech visual acuity

A_{vert} = Vertical area

C = Luminance contrast

C_a = Absolute luminance contrast

C_r = Luminance ratio

C_m = Luminance modulation

C_{min} = Minimum perceptible contrast

C_∞ = Minimum perceptible contrast for target luminance L_t approaching infinity

Appendix B (continued)

C_{th} = Threshold contrast of luminance

$C_{\%}$ = Luminance contrast percent

d = Size of graphic details

dL' = Color difference at L' coordinates of CIELAB

du' = Color difference at u' coordinates of CIELAB

dv' = Color difference at v' coordinates of CIELAB

D = Viewing distance

D_L = Legible distance

D_0 = Viewing distance at normal angle (0 degree)

D_e = No-error viewing distance

D_{ER}= Expected recognition or clear sight distance

D_{min} = Minimum required visibility distance

D_{op} = Optimum viewing distance

D_p = Preferred viewing distance

E = Error rates

f_h = Horizontal fundamental spatial frequency

f_v = Vertical fundamental spatial frequency

G = Acceptable glare

H = Character height

H' = Character height for all upper case letters

H_r = Required letter size

H_{min} = Minimum legible character height

H/Sw = Height to strokewidth ratio

I_t = Target illuminance,

I_a = Ambient illuminance level

Appendix B (continued)

L_a = Adapting luminance

k = Constants, k_1, k_2, …

L = Luminance level

L_a = Adaptation luminance

L_b = Background luminance

L_g = Greater luminance

L_l = Lesser luminance

L_{max} = Maximum luminance

L_{min} = Minimum luminance

L_s = Surrounding luminance

L_t = Target luminance

LI = Legibility index

LI_{th} = Threshold legibility index

LI_p = Preferred legibility index

LP = Legibility potential

NSp = Negative space

P = Percentage of performance

r = Height-to-width ratio of character

RS = Reading speed

RT = Response time

S = General off-axis viewing target size

Sp = Spacing between characters within word

S_d = Denominator in the Snellen ratio

Sw = Strokewidth of the character

s/p = Scotopic to photopic output ratio

Appendix B (continued)

T = Exposure time or performance time

T_{glance} = Glance time

T_{long} = Long exposure time

T_p = Perception time

T_R = Reading time

T_S = Searching time

v = Visual angle of strokewidth or details

V = Visual angle

VI = Visibility index as the visibility meter reading in density units

V_{min} = Minimum perceptible visual angle

W = Character width

y = Fraction of total answers that were correct

APPENDIX C
Comparison of Existing 95 Legibility Equations

Category 1 Common definitions and mathematically derived equations				
#	Equations	Identification	Applicable conditions	Notes
1	$V = \arctan(H/D)$	Visual angle subtended by whole characters	Viewing materials are assumed perpendicular to the viewer, that is, on-axis viewing	V in radian, H, D in same unit
2	$v = \arctan(d/D)$	Visual angle subtended by graphic details		v in radian, d, D in same unit
3	$v = \arctan(Sw/D)$	Visual angle subtended by strokes of text		v in radian, Sw, D in same unit
4	$V = 3438(H/D)$	Holick & Carlson, 2002, visual angle subtended by roadway sign	Roadway signs viewed on axis	V in min arc; H, D in same unit
5	$v = 3438(Sw/D)$	Holick & Carlson, 2002, visual angle subtended by the strokes of roadway sign		V in min arc; Sw, D in same unit
6	$A_c = 1/v_{\min} = D/3438\,d$	Colomb, Hubert, Carta, Bry, & Dore-Picard, 1991, visual acuity	Alphanumeric information signs viewed on axis	Ac in min^{-1}; v_{min} in min arc; D, d in same unit
7	$LI = D/H$	USSC,1998, Legibility Index of characters	Characters viewed on axis	LI in ft/in,D in ft, H in in
8	$LI = D/Sw$	Legibility Index of strokes of text	Strokes of text viewed on axis	LI in ft/in, D in ft, Sw in in
9	$LI = D/d$	Legibility Index for grapic details	Details of graphics viewed on axis	LI in ft/in, D in ft, d in in
10	$LI = 12\cot(V)$	LI related to visual angle subtended by characters	Characters viewed on axis	LI in ft/in, V in radian
11	$LI = 12\cot(v)$	LI related to visual angle subtended by graphic details or text strokes	Graphic details or text strokes viewed on axis	LI in ft/in, V in radian
12	$LI_{th} = D_{\min}/H_r$	Mace, 1988, threshold Legibility Index	On axis, threhold viewing conditions	LI_{th} in ft/in, D_{min} in ft, H_r in in
13	$LI_p = D_p/H$	Preferred Legibility Index	On axis, optimum viewing conditions	LI_p in ft/in, D_p in ft, H in in
14	$LP = d/D$	Richardson, 1976, legibility potential	Bar or graphic details viewed on axis	d, D in same unit, LP no unit
15	$LP = Sw/D$	LP of text strokes, based on Richardson, 1976	Text strokes viewed on axis	Sw, D in same unit, LP no unit
16	$LP = H/D$	LP of characters, based on Richardson, 1976	Characters viewed on axis	H, D in same unit, LP no unit

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
17	$LP = \frac{1}{12\,LI} = \tan V = \tan v$	LP versus LI, V, and v	Characters, graphic details, text strokes viewed on axis	LP no unit, LI in ft/in
18	$C_a = \lvert (L_t - L_b)/L_b \rvert$	Absolute luminance contrast	Any conditions	
19	$C = (L_g - L_l)/L_g$	Luminance contrast		
20	$C_r = L_g / L_l$	Luminance ratio		Cr $\geqslant$ 1
21	$C_m = (L_{max} - L_{min})/(L_{max} + L_{min})$	Definition of modulation	Periodic patterns with max.and min. in each cycle	Cm $\leqslant$ 1
22	$C_{\%} = (L_g - L_l)/L_g \times 100$	Percent luminance contrast	Any conditions	$C_{\%} = 100\,C$
23	$\Delta E = \left[(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2\right]^{1/2}$	CIE, 1976, CIELUV color contrast	Any uniform color space	
24	$\Delta E(Yu'v') = \left[(155\,C)^2 + (367\,du')^2 + (167\,dv')^2\right]^{1/2}$	CIE, 1976, CIELAB color contrast		Color contrast ΔE vs. luminance contrast C
25	$P = 1 - E$	Performance to error rate	Any condition	P, E $\leqslant$ 1
26	$NSp \geq \begin{cases} 2A_{LC} & one\_line \\ 2.25\,A_{LC} & multiple\_lines \end{cases}$	USSC, 1998, negative space versus the letter copy area	Characters in sign design when viewed on axis	A_{LC} is the letter copy area in sign design
Category 2 Equations for predicting viewing/legibility distances, or legible heights of characters				
#	Equations	Identification	Applicable conditions	Notes
27	$D = \begin{cases} D_0(\cos\phi)^{2/3} & horizontal \\ D_0(\cos\alpha)^{1/2} & vertical \end{cases}$	Reinwald formula, the only existing off-axis legibility equation; cited by Reger (1989) and Shurtleff (1980)	Off-axis viewing with the same reading performance	No other critical factors are examined except for viewing distance and off-axis angles
28	$D = 25.4/V$	Jacobs, 1973, cited by Bryant (1982)	Length of sign face is standardized as 25.4mm	D in meter, V in milliradians
29	$D = \frac{C_{\%(SB)} + C_{\%(LS)}}{2} \times D_{ER}$	Forbes, 1972, outdoor signs	D_{ER} = smaller one of expected recognition distance or clear sight distance	Expected recognition distance = small dimension of sign in ft $\times$ 1200
30	$D_{op} = \frac{H}{0.00291}$	Fletcher 1972, cited by Duncan & Konz, 1974	Optimum viewing distance, with 20/20 acuity, no-error readability, on LCD/LED	Character subtended with 10 min arc; D, H in inches

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
31	$D_e = 2.81 + 2.14H$	Duncan & Konz, 1974, LCD/LED viewing	No-error viewing distance, w/o strokewidth examined	Ambient illumination varies from 1.5 fc to 45 fc; De, Dp in ft, H in mm
32	$D_p = -1.16 + 0.82H$		Preferred viewing distance, w/o strokewidth examined	
33	$D_e = 7.98 + 2.28H - 87.64(Sw/H)$		No-error viewing distance, with Sw/H examined	
34	$D_p = 0.75 + 0.87H - 32.30(Sw/H)$		Preferred viewing distance, with Sw/H examined	
35	$D_e = 666.7H - 34.6(H/Sw) + 233.3$	Duncan & Konz, 1976, intrument panels, cited by Green et al (1988)	No-error viewing	Characters presented on instrument panels; De, Dp, H in cm
36	$D_p = 263.2H - 10.1(H/Sw) + 22.7$		Preferred viewing	
37	$D_L = 50H$	Adler & Straub, 1971, sign height vs.viewing distance	Commonly accepted for daylight operations	Luminance ⩾ 5 fl required; D_L in ft, H in in
38	$D_L = 500H$	Gleeson neon advertising	Neon light for advertising	D_L, H in meter
39	$D_L = 116.227 + 1236.1(Sw/H) - 3370(Sw/H)^2$	Uhlaner, 1941, highway signs	Highway signs, daylight, character height 3 inches	For roadway signs with height of 3 inches only; D_L in ft, Sw and H in in, v in min arc
40	$D_L = 116.227 + 247.265v - 134.925v^2$		Highway signs, daylight, no error vision	
41	$D_L = -0.97 + 7.05(W/H) - 2.32(W/H)^2$	Collins & Hall, 1992	Light reflecting matrix pixles variable message road signs	D_L in meter; W/H ratio is from 0.46-1.0
42	$D_L = 326 - 21.8 \times BRT$	Schieber, 1994	Blur techniques; symbol on highway signs	BRT = blur recognition threshold; D_L in meter
43	$D_L(T_{long}) = 1.5 \times D_L(T_{glance})$	Forbes, 1975, exposure time vs.legibility; traffic signs	High luminance = "glance", low luminance = "long"	D_L (max) = 55 ft/in
44	$H' = 1.15H$	USSC,1998, traffic signs	All upper case letters viewed on axis	15% increase of letter height to be legible
45	$H = 0.007 \times D$	Smith,1979, characters, cited by Green et al (1988)	News prints etc. read on axis in a room; lighting effect on legibility ignored	Visual size ⩽ 7 deg; D, H in same unit
46	$H = 0.0022D + k_1 + k_2$ (1)	Peters & Adams, 1959, cited by Green et al (1988)	On-axis viewing; H, D in inches	k1: illumination and viewing conditions, k2: importance level

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
47	$H = (H/Sw) \times 1.45 \times 10^{-5} \times A_S \times D$	Howett, 1983, NBS method to predict character height	Derived from equation 94; $1 \leqslant Lb \leqslant 1000$ cd/m^2; $10 \leqslant C\% \leqslant 90$; $0.2 \leqslant Ac \leqslant 2.0$ min^{-1}	D, H and Sw in the same unit
48	$H = 4.1 \times 10^{-4} \times D \cdot \left(\frac{H}{Sw}\right) \cdot S_d \cdot (L_b)^{-0.213} \cdot C_{\%}^{-0.532}$			
Category 3 Equations for predicting visual size & legibility potential				
#	Equations	Identification	Applicable conditions	Notes
49	$V = k - 35.63(W/H) + 21.37(W/H)^2$ (2)	Zwahlen et al., 1995, textual information presented on traffic signs, license plate, etc. to calculate the visual angle as function of either character height, width,strokewidth, ratios, or spacing dimension. Second-order polynomial least-quares function were established. V in min of arc.	Avg. W/H, positive contrast	k = constant
50	$V = k_1 - 1.26(W/H) + 1.93(W/H)^2$ (3)		Avg. W/H, negative contrast	k1, k2= constant
51	$V = k_2 - 14.03(W/H) + 7.89(W/H)^2$ (4)			
52	$V = k - 49.76(Sp/H) + 88.74(Sp/H)^2$ (5)		Avg. Sp/H, positive contrast	k is constant for different experiments
53	$V = k - 9.87(Sp/H) + 11.7(Sp/H)^2$ (6)		Avg. Sp/H, negative contrast	
54	$V = 4.57 - 23.92(Sw/H) + 138.25(Sw/H)^2$		Sw/H ratio, positive contrast	
55	$V = 6.89 - 31.21(Sw/H) + 85.94(Sw/H)^2$		Sw/H ratio, negative contrast	

Footnotes (1) – (6)

(1) k1 = 0.06 for illumination > 1.0 fc, favorable reading conditions
0.16 for illumination > 1.0 fc, unfavorable conditions or illumination < 1.0 fc, favorable conditions
0.26 for illumination < 1.0 fc, unfavorable conditions

k2 = 0.075 for emergency labels, counters, scales, legend lights
0.0 for other (unimportant) panel markings

(2) $k = \begin{cases} 19.11 & \textit{Solomon}, 1956 \\ 18.38 & \textit{Forbes}, 1951 \\ 19.36 & \textit{Mace}, 1993 \\ 21.11 & \textit{Zwahlen}, 1991 \end{cases}$

(3) $k_1 = \begin{cases} 10 & \textit{Zwahlen}, 1994/1991 \\ 9.26 & \textit{Mace}, 1993 \end{cases}$

(4) $k_2 = \begin{cases} 10.3 & \textit{Berger}, 1948/1950 \\ 9.26 & \textit{Forbes}, 1939 \end{cases}$

(5) $k = \begin{cases} 15.65 & \textit{Zwahlen}, 1991 \\ 11.65 & \textit{Solomon}, 1956, \textit{letterC} \\ 10.51 & \textit{Baerwald}, 1960 \\ 11.24 & \textit{Solomon}, 1956, \textit{letterE} \end{cases}$

(6) $k = \begin{cases} 11.05 & \textit{Zwahlen}, 1991 \\ 8.51 & \textit{Baerwald}, 1960 \end{cases}$

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
56	$V_{\min} = (\frac{V_\infty}{L_b}) \times (k_2 + L_b^{1/n})^n$	Shlaer, Smith, & Chase, 1942, minimum visual angle, cited by Moon & Spencer (1944)	Excellent vision, uniform background, variant ambient, Landolt type targets, 56.25% correct response	V_∞ is the visual angle as Lb approaches infinity; V in radian, Lb in millilambert
57	$V_{\min} = (\frac{118.3 \times 10^{-6}}{L_b + L_a}) \times (0.412 + (L_b + L_a)^{1/3})^3$	Moon & Spencer, 1944, minimum visual angle, cited by Green et al. (1988)	Excellent vision, large uniform surround luminance, Landolt type targets	V in radians, Lb + La in millilambert; 56.25% correct response
58	$V_{\min} = (\frac{1.95 \times 10^{-6}}{L_b + L_a}) \times (1.433 + (L_b + L_a)^{1/2})^2$		Single-bar test targets; modified from equation 57	V in radians, Lb + La in millilambert
59	$V = \left(\frac{V_\infty}{L_a + L_b}\right) \times [\frac{(y - 1/8)}{(1-y)}]^{1/k} \times (A_2 + (L_a + L_b)^{1/n})^n$		Single-bar targets. To predict V at different reading performance (percent correct)	V_∞ is the visual angle as La+Lb approaches infinity. A2, k are constants; V in radians
60	$V = (\frac{118.3 \times 10^{-6}}{L_b + L_a}) \times \left(\frac{(y - 1/8)}{(1-y)}\right)^{0.01} \times \left(0.412 + (L_b + L_a)^{1/3}\right)^3$		Landolt-type targets; y is the fraction of total answers that were correct	Modified from equation 59; V in radians, Lb+La in millilambert
61	$LP = 0.280 - 0.037 \ln(L_g) + 0.0037\, L_g$	Richardson, 1976	Reflective sign; Lg = the greater of background and legend luminance	Two variable model
62	$LP = 0.286 - 0.023 \ln(L_g)$			One variable model
	Category 4 Equations for predicting luminance contrast			
#	Equations	Identification	Applicable conditions	Notes
63	$C_{\min} = \left(\frac{C_\infty}{L_t}\right) \times \left(k_1 + L_t^{1/n}\right)^n$	Hecht, Peskin, & Patt, 1935, min. percptible contrast; cited by Green et al. (1988)	Large contiguous surfaces with uniform surround luminance	C_∞ = min. perceptible contrast for Lt approaching infinity
64	$C_{\min} = (\frac{0.0123}{L_t}) \times (0.808 + L_t^{1/2})^2$	Moon & Spencer, 1944, min. perceptible contrast; cited by Green et al. (1988)	Uniform surround, 10-20° comparison surfaces, > 0.1s exposure time	Based on equation 63 by fitting the experimental data
65	$C_{th} = k_1 \times \frac{0.0923}{k_2} \times \left[\left(\frac{k_3}{k_4 \cdot L_b}\right)^{0.4} + 1\right]^{2.5}$ (7)	Farber, 1988, contrast threshold model based on Blackwell's work; cited by Green et al. (1988)	Objects on road illuminated by automible headlights; Lb in cd/m^2	ks depend on the age, target visual size, transmittance, and background luminance

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
66	${C_m}^2 = 0.0226 + 3.9778(dL^2) + 0.135(du^2) + 0.065(dv^2)$	Post, Costanza, & Lippert, 1982, luminance contrast and color contrast; Colored pataches on video displays	Only luminance difference	Achromatic contrast is represented by luminance modulation
67	${C_m}^2 = 0.1053 + 2.7746(dL^2) + 0.1824(du^2) + 0.0914(dv^2)$		Luminance, hue, and saturation difference	

Footnote (7)

k1 = target size factor, $k_1 = \begin{cases} 3 \times 0.37 \log_2 d & d \le 10 \\ 0.106 - 0.0006\,d & d > 10 \end{cases}$ when d is target diameter (min of arc)

$$k_2 = \left(\left(\frac{k_3}{100\,k_4} \right)^{0.4} + 1 \right)^{2.5}$$

k3 = the effect of age on slope of RCS function of luminance, $k_3 = 0.59 - 0.6235 \log d - s$ when s = adjustment parameter due to age

K4, $\log k_4 = \begin{cases} 0 & 20 < A_g \le 30 \\ 0.01053\left(A_g - 30\right) & 30 < A_g \le 44 \\ 0.1474 - 0.0134\left(A_g - 44\right) & 44 < A_g \le 64 \\ 0.4154 - 0.0175\left(A_g - 64\right) & 64 < A_g \le 80 \end{cases}$

$$s = \begin{cases} 0 & 20 < A_g \le 44 \\ 0.00406\left(A_g - 44\right) & 44 < A_g \le 64 \\ 0.0812 + 0.00667\left(A_g - 64\right) & 64 < A_g \le 80 \end{cases}$$

Category 5 Equations for predicting legibility index

#	Equations	Identification	Applicable conditions	Notes
68	$LI = \dfrac{1}{12\tan(5v)} \approx \dfrac{1}{60\,LP}$	Richardson, 1976, detail angle related to legibility index of character	Critical detail subtends roughly 1/5 of the angle subtended by character	Minimum angular resolution v≈V/5 as rule of thumb; LI in ft/in

Category 6 Equations for predicting reaction time

#	Equations	Identification	Applicable conditions	Notes
69	$RT = k\left(L_a\right)^a \left(L_b\right)^{-b} \left(C_r\right)^{-c}$	Rogers, Spiker & Cicinelli, 1986, response time; self-luminous displays in aircraft cockpits; cited by Green et al. (1988)	General formula	Constants to be solved in future studies
70	$RT = \begin{cases} 100{C_r}^{-1} & I_a = 3.4cd/m^2 \\ 31{C_r}^{-2} & I_a = 34.3cd/m^2 \\ 8{C_r}^{-2} & I_a = 257cd/m^2 \end{cases}$		Response time as function of display parameters, CRT under different adaptation levels	RT in sec; three experiemnts in series were conducted

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
71	$T_p = (\frac{1.75}{VI^2}) - (\frac{0.81}{VI}) + 0.42$	Sauter & Kerchaert, 1972, cited by Green et al., 1988	Text on automobile instrument panels, on-axis viewing at normal angles; visibility index measured by Luckiesh-Moss visibility meter; Tp and RT in sec,VI is reading in density unit	Preliminary findings
72	$RT = (\frac{1.58}{VI^2}) - (\frac{0.55}{VI}) + 0.56$			
73	$RT = (\frac{-0.547}{VI^3}) + (\frac{3.38}{VI^2}) - (\frac{7.052}{VI}) + 5.665$			Alternative version of equation 72 to better fit exterme data points
74	$RT = \frac{-2.996}{\log(VI)} + 1.527$			Alternative version of equation 72 with a logarithmic relationship
75	$RT = (\frac{4.39}{\log(VI^2)}) - (\frac{3.793}{\log(VI)}) + 1.415$			Quadratic version of equation 74 to fit better
76	$RT = 5.82 - 13.03H - 0.70\log(L_t) + 2.94 / C_r$	Mourant & Langolf, 1976, visual search, regressed by Green et al., 1988	Text on automobile instrument panels	RT in sec, H in in, L in fl
77	$RT = 1054 - 320(k_A) + 1050(\frac{1}{H}) + 202(k_L) + 89.6(\frac{1}{\ln(C_r)})$ $- 9.58(\ln(I_t)) + 4538(\frac{1}{H^2})$	Boreczky, Green, Bos, & Kerst, 1988, response time to instrument clusters and distance targets; assumed a working knowledge of chromaticity and ΔE as a measure of chrominance (color difference)	Response time on numerical speedometer, age is considerred; Each unit ΔE (9-106) increase resulted in a 2.1 ms decrease in RT	RT in ms; k_A = age group (1 for old, 2 for young); H in mm (5-19mm); k_L = location (1 for center, 2 for sides); C is from 1.5:1 to 20:1; I in lux (1.08 to 915 lux)
78	$RT = 574 + 1050(\frac{1}{H}) + 202(k_L) + 89.6(\frac{1}{\ln(C_r)})$ $- 9.58(\ln(I_t)) + 4538(\frac{1}{H^2})$		Response time on numerical speeddometer, if age is not a factor	

Category 7 Equations for predicting exposure time or performance time

#	Equations	Identification	Applicable conditions	Notes
79	$T = 527 - 5388\left(\frac{Sw}{H}\right) + 22457\left(\frac{Sw}{H}\right)^2 + 26.7\left(\frac{W}{H}\right)$	Collins & Hall, 1992, exposure time	Reflecting matrix pixles, variable message road signs	T in ms, h is pixel height as a proportion of letter height
80	$T_R = 1.43 + 0.23\left(f_v^{\,2} - 14\right) + 3.64\left(A_{horz}\right) + 0.221\left(A_{vert}\right) - 4.825 \log_{10}{}^{A_{horz}}$	Snyder & Maddox, 1978, CRT reading and searching time, cited by Green et al., 1988; Simulated characters from three electronic displays were shown on a high-resolution CRT; element size in mm, illumination in lux, viewing distance in cm/inch, exposure time in msec	Dot matrix characters on electronic display, CRT, reading task	Tinker reading task model; most sensitive to dot element design; TR in sec
81	$T_S = 0.78 + 0.024\left(f_v^{\,2} - 14\right) + 2.72 \log_{10}\left(\frac{f_h}{C_m}\right) + 0.193\, A_{vert}$		Dot matrix characters on electronic display, CRT, menu search task	Menu search task model; Ts in sec
82	$T_S = -48.5 - 138.49 \log_{10}{}^{f_h} + 192.89 \log_{10}{}^{f_v} - 0.642\, A_{horz} - 0.734\left(f_h^{\,2} - 14\right) + 0.982\left(f_v^{\,2} - 14\right) - 0.043\left(\frac{f_h}{C_m}\right)$		Random characters on electronic display, CRT, random search task	Random search task model; Ts in sec
83	$T_R = 5.74 + 0.3111\, f_h + 2.379\, C_m + 4.369 \log_{10}{}^{\left(\frac{f_h}{C_m}\right)} - 14.973 \log_{10}{}^{f_h} + 1.112 \log_{10}{}^{A_{vert}}$	Derived equations from Snyder & Maddox (1978), cited by Reger, 1989 as in Green et al.'s review (1988)	Information transfer associated with dot-matrix addressed displays, variables based on spatial frequency and MTF measurements	Tinker speed of reading task
84	$T_S = 7.27 + 0.027\left(\frac{f_h}{C_m}\right) + 2.159 \log_{10}\left(\frac{f_h}{C_m}\right) + 5.916 \log_{10}{}^{f_v} - 0.339\, A_{vert} - 0.054\left(f_c - f_v\right) + 5.487 \log_{10}{}^{A_{vert}}$			Menu search task
85	$RS = 0.40266 - 0.19609\, D + 0.04309\left(\frac{Sw}{H}\right) + 0.00931\left(\Delta E\right) - 0.52871\, C_m + 0.01239\, Sw + 0.01269\left(De\right) - 2.18773\, C_m + 0.00642\, A$	Derived equations from Snyder & Maddox (1978), cited by Reger, 1989 as in Green et al.'s review (1988)	Information transfer associated with dot-matrix addressed displays, variables based on spatial frequency and MTF measurements	Reading speed; D in m; ΔE = calculated color contrast using lippert's equation (1986)

Appendix C (continued)

#	Equations	Identification	Applicable conditions	Notes
Category 8 Equations for predicting error rate or percentage of performance				
86	$E = 120 \times 0.836^{V}$	American Medical Association, 1925, cited by Luckiesh & Moss, 1937	Threshold stimulus, for whole characters	From the book "the science of seeing"
87	$E = 120 \times 0.836^{v}$		Threshold stimulus, for details	
88	$E = 1.52 + 0.02L_b - 1.40V + 0.02\phi - 0.0006I_a$	Payne, 1983, error rate, cited by Green et al., 1988	Characters on four-digit, seven-segment, reflective LCDs	V in deg (0.025-1.34); ϕ is 0-60 deg; L in cd/m^2 (0-122), I in lux
89	$P = \frac{C_{r(S_iB_i)} + C_{r(L_iS_i)}}{\sum_i^n (C_{r(SB)} + C_{r(LS)})} \times AR_{LS} \times SF \times 100$	Forbes, 1969, error rate for highway signs	Cr (SB) is sign to background brightness ratio; Cr (LS) is legend to sign brightness ratio; C% (SB) is sign-to-background percent contrast; C% (LS) is letter-to-sign percent contrast; ARLS is legend-to-sign-area ratio, expressed as percent of largest ratio; SF is size factor	
90	$P = \frac{C_{\%(S_iB_i)} + C_{\%(L_iS_i)}}{\sum_i^n (C_{\%(SB)} + C_{\%(LS)})} \times AR_{LS} \times SF \times 100$			
Category 9 Equations related with lighting and acuity level				
#	Equations	Identification	Applicable conditions	Notes
91	$VI = 0.47 \times \log(I_t) - (1.12 \times A_c) + 1.38$	Sauter & Kerchaert, 1972; cited by Green et al., 1988	Aged people, instrumental panel	VI in density units, I in fc, Ac in minarc^{-1}
92	$\log_{10} L_t = -0.874 + 0.02161(A_g) + 0.01525(A_S) + 0.03339(A_V) - 0.0537(t) + 0.0149(H) - 0.743(V)$	Holick & Carlson, 2002, luminance required for legibility	Overhead sign illuminated by headlights, no glare sources	t is the trial number
93	$\log_{10} L_t = -0.545 + \frac{A_g}{46.27} + \frac{A_S}{20.56} - 212.9\left(\frac{Sw}{D}\right)$			Lt in cd/m^2
94	$A_c = 0.06298 L_b^{0.213} C_{\%}^{0.532}$	Naneko & Ito, 1978, Japan	$1 \leqslant$ Lb $\leqslant 1000$ cd/m^2; $10 \leqslant$ C% $\leqslant 90$; $0.2 \leqslant$ Ac $\leqslant 2.0$ min^{-1}	Howett's equation was deduced from Naneko and Ito's equation; Lb in cd/m^2
95	$A_S = Sd \times (\frac{85}{L_b})^{0.213} \times (\frac{90}{C})^{0.532}$	Howett, 1983, Snellen acuity		

APPENDIX D

Total of 38 Surveyed Lecture Halls

Building	Rmnbr	Capacity	Rmsqrft	Campus location	Media #	Year	Number
Cooley Mortimer E Memorial	G906	80	1525	North Campus	2	1953	16
Dental and W K Kellogg Institute	G550	98	1787	Central Campus	2	1940	
Dental and W K Kellogg Institute	G322	100	2023	Central Campus	2	1940	
Electrical Eng & Computer Sci Bldg	1311	108	1366	North Campus	2	1986	
Dennison David M Building	296	110	2035	Central Campus	2	1963	
Electrical Eng & Computer Sci Bldg	1001	110	1177	North Campus	2	1986	
Hutchins Hall	120	111	1591	Central Campus	2	1933	
Modern Languages Building	1220	118	1264	Central Campus	4	1972	
Little Clarence Cook Science Bldg	1528	120	2462	Central Campus	2	1960	
Chemistry & Dow Willard H Lab	1640	120	1762	Central Campus	3	1989	
Computer Science & Eng. Bldg	1670	120	1981	North Campus	2	2004	
Electrical Eng & Computer Sci Bldg	1500	130	1660	North Campus	2	1986	
Chemistry & Dow Willard H Lab	1200	130	1346	Central Campus	2	1989	
Little Clarence Cook Science Bldg	2548	133	1565	Central Campus	2	1960	
Industrial & Operations Engin Bldg	1610	144	2670	North Campus	2	1963	
East Hall	1360	145	1926	Central Campus	3	1883	

Appendix D (continued)

Building	Rmnbr	Capacity	Rmsqrft	Campus location	Media #	Year	Number
Brown George Granger Memorial Lab	1504	150	2759	North Campus	2	1958	15
Bagnoud Francois-Xavier Bldg	1109	150	2307	North Campus	3	1991	
Art & Architecture Building	2104	154	1769	North Campus	2	1974	
Dow Herbert H Building	1013	164	2792	North Campus	2	1982	
Hutchins Hall	150	172	2741	Central Campus	2	1933	
Angell Hall Auditoriums	D	178	2666	Central Campus	3	1952	
Dana Samuel Trask Building	1040	198	2391	Central Campus	4	?	
Chemistry & Dow Willard H Lab	1400	206	2001	Central Campus	2	1989	
Medical Science Unit I	5330	220	3353	Medical Campus	3	1958	
Chrysler Cntr Cont Engineer Ed	220	234	2947	North Campus	3	1967	
Education, School of	1202	236	2237	Central Campus	2	?	
Chemistry & Dow Willard H Lab	1210	250	2429	Central Campus	2	1989	
Dental and W K Kellogg Institute	G005	280	2711	Central Campus	2	1940	
Angell Hall Auditoriums	B	284	3400	Central Campus	2	1952	
Dennison David M Building	182	286	2779	Central Campus	4	1963	

Appendix D (continued)

Building	Rmnbr	Capacity	Rmsqrft	Campus location	Media #	Year	Number
East Hall	1324	315	3007	Central Campus	2	1883	
Modern Languages Building	1400	318	3976	Central Campus	3	1972	
Hutchins Hall	100	372	4163	Central Campus	3	1933	
Lorch Hall	140	374	3238	Central Campus	3	?	7
Kraus Edward Henry Building	2140	429	3490	Central Campus	2	1915	
Chemistry & Dow Willard H Lab	1800	500	6110	Central Campus	4	1989	
Modern Languages Building	1200	551	5450	Central Campus	4	1972	

APPENDIX E

Surveyed Viewing Conditions of Text Presented in the 38 Lecture Halls

Lecture halls	ϕ_{max} in deg	α_{max} in deg	ξ_{max} in deg	D_{max} in meters	L_b in cd/m^2		Ls in cd/m^2	
					min	max	min	max
Cooley G906	43.5	23.6	48.3	10.91	1.70	3.94	0.65	38.61
East Hall 1360	48.3	30.1	54.9	15.20	1.28	398.70	0.45	163.50
CC Little 2548	48.3	40.6	59.7	15.21	13.22	240.20	3.26	87.98
Medical 5330	54.9	40.0	63.9	17.58	2.16	49.62	0.66	51.10
Lorch Hall 140	57.9	38.4	65.4	25.74	1.53	34.45	0.10	7.09
CC Little 1528	62.9	27.9	66.3	13.64	0.58	179.10	0.08	83.13
Angle Hall D	57.2	42.1	66.3	16.95	1.01	14.34	0.13	219.80
Chrysler 220	61.8	33.8	66.9	19.27	0.13	208.50	0.09	87.94
Chemistry 1640	62.9	31.4	67.1	14.48	3.21	83.76	0.08	16.55
Angle Hall B	60.2	39.0	67.3	19.40	2.09	20.68	0.37	7.91
Dental G322	58.1	43.2	67.3	14.98	5.86	244.10	9.58	153.10
EE 1500	65.6	22.2	67.5	13.41	1.07	8.78	0.28	33.93

Appendix E (continued)

Lecture halls	ϕ_{max} in deg	α_{max} in deg	ξ_{max} in deg	D_{max} in meters	L_b in cd/m^2		Ls in cd/m^2	
					min	max	min	max
Chemistry 1210	56.0	47.9	68.0	18.29	0.08	48.17	0.01	39.38
Kraus Ed. 2140	61.8	41.4	69.2	18.13	0.16	1.89	0.84	5.70
Education 1202	59.2	47.0	69.6	18.65	24.97	119.50	15.58	186.80
Chemistry 1200	64.4	37.2	69.9	10.33	0.04	17.55	0.01	53.81
Dennison 296	67.1	34.8	71.4	13.57	7.95	343.90	1.37	103.10
GG Brown 1504	68.5	31.4	71.8	15.56	2.54	101.00	0.02	78.87
Hutchins Hall 120	56.6	55.5	71.8	12.54	2.03	173.80	1.57	10.24
EE 1001	68.5	34.4	72.4	11.98	0.20	4.55	1.08	10.82
Dental G550	68.3	35.3	72.4	13.83	0.21	40.35	0.05	33.22
Chemistry 1800	52.7	60.9	72.9	18.90	11.05	60.42	3.64	42.18
Chemistry 1400	66.0	43.7	72.9	16.13	0.05	33.27	0.01	41.34
Modern Lang. 1220	59.5	56.6	73.8	10.04	4.33	71.36	8.55	39.81
Dental G005	68.6	43.4	74.6	18.98	0.20	4.83	0.19	45.48
Computer 1670	69.9	40.8	74.9	13.52	0.95	49.26	0.17	16.64
IOE 1610	68.7	45.0	75.1	17.17	0.37	124.20	0.12	93.91
Arch 2104	64.7	53.3	75.2	14.76	0.16	65.26	0.06	65.00

Appendix E (continued)

Lecture halls	ϕ_{max} in deg	α_{max} in deg	ξ_{max} in deg	D_{max} in meters	L_b in cd/m^2		Ls in cd/m^2	
					min	max	min	max
B. FXB 1109	71.1	39.6	75.6	15.38	3.15	297.20	1.72	195.10
DOW 1013	69.3	46.8	76.0	14.95	2.56	6.72	0.15	70.54
DANA 1040	75.3	28.5	77.1	13.71	0.36	16.88	0.09	267.60
Hutchins Hall 150	66.8	59.2	78.4	13.38	1.04	17.99	0.35	6.60
East Hall 1324	71.1	51.6	78.4	15.68	0.83	35.95	0.03	16.09
Dennison 182	71.8	58.3	80.6	16.00	2.25	40.48	1.17	37.04
Hutchins Hall 100	72.1	68.2	83.5	17.82	4.94	36.59	1.89	23.69
EE 1311	77.0	60.6	83.7	9.96	2.32	47.72	2.42	22.40
Modern Lang. 1200	80.2	54.9	84.4	16.29	0.03	12.30	0.09	247.30
Modern Lang. 1400	80.4	68.5	86.5	15.90	0.22	10.79	0.04	255.90

APPENDIX F

Predicted Ranges of Height of 7 Lines of Letter Es on E-Charts Viewed by Observers with Eyesight 20/20, 20/16, and 20/12.5 at a Total of 25 Incident Angles

Table a. Predicted Heights for Eyesight 20/20

Viewing angles		Height of letter Es viewed not perpendicularly, in mm						
Horz. ϕ	Vert. α	H'-3Sw	H'-2Sw	H'-Sw	H'	H'+Sw	H'+2Sw	H'+3Sw
0 deg	0 deg	3.51	5.29	7.07	8.85	10.63	12.41	14.19
0	31.5	3.80	5.73	7.66	9.58	11.51	13.44	15.37
0	46.5	4.23	6.38	8.52	10.67	12.81	14.96	17.10
0	61	5.04	7.60	10.15	12.71	15.27	17.82	20.38
0	75	6.90	10.40	13.90	17.40	20.89	24.39	27.89
30	0	3.77	5.68	7.60	9.51	11.42	13.34	15.25
30	31.5	4.08	6.16	8.23	10.30	12.37	14.44	16.51
30	46.5	4.55	6.85	9.16	11.46	13.77	16.07	18.38
30	61	5.42	8.16	10.91	13.66	16.41	19.15	21.90
30	75	7.41	11.17	14.93	18.69	22.45	26.21	29.97
45	0	4.17	6.29	8.41	10.52	12.64	14.76	16.87
45	31.5	4.52	6.81	9.11	11.40	13.69	15.98	18.27
45	46.5	5.03	7.58	10.13	12.69	15.24	17.79	20.34
45	61	5.99	9.03	12.08	15.12	18.16	21.20	24.24
45	75	8.20	12.37	16.53	20.69	24.85	29.01	33.17
60	0	4.96	7.48	10.00	12.52	15.03	17.55	20.07
60	31.5	5.38	8.10	10.83	13.55	16.28	19.01	21.73
60	46.5	5.98	9.02	12.05	15.09	18.12	21.15	24.19
60	61	7.13	10.74	14.36	17.98	21.59	25.21	28.82
60	75	9.76	14.71	19.65	24.60	29.55	34.50	39.45
75	0	6.90	10.40	13.90	17.40	20.89	24.39	27.89
75	31.5	7.47	11.26	15.05	18.84	22.63	26.42	30.21
75	46.5	8.32	12.53	16.75	20.97	25.18	29.40	33.62
75	61	9.91	14.93	19.96	24.98	30.01	35.03	40.06
75	75	13.56	20.44	27.32	34.19	41.07	47.95	54.83

Appendix F (continued)

Table b. Predicted Heights for Eyesight 20/16

Viewing angles		Height of letter Es viewed not perpendicularly, in mm						
Horz. ϕ	Vert. α	H'-3Sw	H'-2Sw	H'-Sw	H'	H'+Sw	H'+2Sw	H'+3Sw
0 deg	0 deg	2.82	4.24	5.66	7.08	8.50	9.92	11.34
0	31.5	3.05	4.59	6.13	7.67	9.21	10.74	12.28
0	46.5	3.40	5.11	6.82	8.53	10.25	11.96	13.67
0	61	4.05	6.09	8.13	10.17	12.21	14.25	16.29
0	75	5.54	8.33	11.13	13.92	16.71	19.50	22.29
30	0	3.03	4.56	6.08	7.61	9.13	10.66	12.19
30	31.5	3.28	4.93	6.59	8.24	9.89	11.54	13.20
30	46.5	3.65	5.49	7.33	9.17	11.01	12.85	14.69
30	61	4.35	6.54	8.74	10.93	13.12	15.31	17.50
30	75	5.96	8.96	11.96	14.95	17.95	20.95	23.95
45	0	3.35	5.04	6.73	8.42	10.11	11.80	13.49
45	31.5	3.63	5.46	7.29	9.12	10.95	12.78	14.60
45	46.5	4.04	6.08	8.11	10.15	12.18	14.22	16.25
45	61	4.82	7.24	9.67	12.09	14.52	16.94	19.37
45	75	6.59	9.91	13.23	16.55	19.87	23.19	26.51
60	0	3.99	6.00	8.00	10.01	12.02	14.03	16.04
60	31.5	4.32	6.49	8.67	10.84	13.02	15.19	17.37
60	46.5	4.81	7.23	9.65	12.07	14.49	16.91	19.33
60	61	5.73	8.61	11.50	14.38	17.26	20.15	23.03
60	75	7.84	11.79	15.73	19.68	23.63	27.58	31.52
75	0	5.54	8.33	11.13	13.92	16.71	19.50	22.29
75	31.5	6.00	9.03	12.05	15.07	18.09	21.12	24.14
75	46.5	6.68	10.05	13.41	16.77	20.14	23.50	26.87
75	61	7.96	11.97	15.98	19.99	24.00	28.00	32.01
75	75	10.90	16.38	21.87	27.36	32.84	38.33	43.81

Appendix F (continued)

Table c. Predicted Heights for Eyesight 20/12.5

Viewing angles		Height of letter Es viewed not perpendicularly, in mm						
Horz. ϕ	Vert. α	H'-3Sw	H'-2Sw	H'-Sw	H'	H'+Sw	H'+2Sw	H'+3Sw
0 deg	0 deg	2.21	3.33	4.45	5.57	6.69	7.81	8.93
0	31.5	2.39	3.61	4.82	6.03	7.25	8.46	9.67
0	46.5	2.66	4.01	5.36	6.71	8.06	9.41	10.76
0	61	3.17	4.78	6.39	8.00	9.61	11.22	12.83
0	75	4.34	6.55	8.75	10.95	13.15	15.35	17.55
30	0	2.37	3.58	4.78	5.99	7.19	8.39	9.60
30	31.5	2.57	3.88	5.18	6.48	7.79	9.09	10.39
30	46.5	2.86	4.31	5.76	7.21	8.66	10.12	11.57
30	61	3.41	5.14	6.87	8.60	10.32	12.05	13.78
30	75	4.67	7.03	9.40	11.76	14.13	16.50	18.86
45	0	2.63	3.96	5.29	6.62	7.96	9.29	10.62
45	31.5	2.85	4.29	5.73	7.17	8.62	10.06	11.50
45	46.5	3.17	4.77	6.38	7.98	9.59	11.19	12.80
45	61	3.77	5.69	7.60	9.51	11.43	13.34	15.25
45	75	5.17	7.78	10.40	13.02	15.64	18.26	20.87
60	0	3.13	4.71	6.29	7.88	9.46	11.05	12.63
60	31.5	3.38	5.10	6.82	8.53	10.25	11.96	13.68
60	46.5	3.77	5.68	7.59	9.49	11.40	13.31	15.22
60	61	4.49	6.76	9.04	11.31	13.59	15.86	18.14
60	75	6.14	9.26	12.37	15.48	18.60	21.71	24.82
75	0	4.34	6.55	8.75	10.95	13.15	15.35	17.55
75	31.5	4.70	7.09	9.47	11.86	14.24	16.63	19.01
75	46.5	5.24	7.89	10.54	13.20	15.85	18.50	21.16
75	61	6.24	9.40	12.56	15.72	18.89	22.05	25.21
75	75	8.54	12.87	17.19	21.52	25.85	30.18	34.50

APPENDIX G

Code of the Computation Program Developed in MatLab

```
%the number of viewing media and the matrix P of all parameters ;
syms k P ;
k = input('Total number of visual media? \n') ;
P = zeros(k*9, 6) ;

%calculate the on-axis legibility distance D0 ;
syms Ht Rhsw Sd Lb Cp D0 ;
format short g ;
Sd = input('The denominator of Snellen eyesight of the observer? \n') ;
for k = 1:k
    disp('For the visual media of number:') ;
    disp(k) ;
    Ht = input('The height of text to be viewed in mm? \n') ;
    Rhsw = input('The height-to-strokewidth ratio of the text? \n') ;
    Lb = input('The background luminance level of the text in cd/m2? \n') ;
    Cp = input('The luminance contrast percent of text? \n') ;
    D0 = 2443.5*Ht*(Rhsw^-1)*(Sd^-1)*(Lb^0.213)*(Cp^0.532)/1000 ;    % on-axis viewing distance in meter
    P((9*(k-1)+1):9*k,4) = D0 ;
end
```

```
% The geometries and angles of visual media
syms xc yc zc ;
syms xtl ytl ztl xtm ytm ztm xtr ytr ztr xml yml zml xmr ymr zmr xbl ybl zbl xbm ybm zbm xbr ybr zbr ;
syms h w deltaphi deltaalpha ;

for k = 1:k ;
    disp('For the visual media of number:') ;
    disp(k) ;
    h = input('height of the visual media in meters? \n') ;
    w = input('width of the visual media in meters? \n') ;
    xc = input('x-coordinate of the center point of visual media in meters? \n') ;
    yc = input('y-coordinate of the center point of visual media in meters? \n') ;
    zc = input('z-coordinate of the center point of visual media in meters? \n') ;
    deltaphi = input('initial horizontal viewing angle of the visual media in degrees? \n') ;
    deltaalpha = input('initial vertical viewing angle of the visual media in degrees? \n') ;

    % other 8 points coordinates in meters calculated from the center point;

    % top left point ;
    xtl = xc - (w/2)*cos(deltaphi) + (h/2)*sin(deltaalpha)*sin(deltaphi) ;
    ytl = yc + (w/2)*sin(deltaphi) + (h/2)*sin(deltaalpha)*cos(deltaphi) ;
    ztl = zc + (h/2)*cos(deltaalpha) ;
```

```
% top middle point ;
xtm = xc + (h/2)*sin(deltaalpha)*sin(deltaphi) ;
ytm = yc + (h/2)*sin(deltaalpha)*cos(deltaphi) ;
ztm = zc + (h/2)*cos(deltaalpha) ;

% top right point ;
xtr = xc + (w/2)*cos(deltaphi) + (h/2)*sin(deltaalpha)*sin(deltaphi) ;
ytr = yc - (w/2)*sin(deltaphi) + (h/2)*sin(deltaalpha)*cos(deltaphi) ;
ztr = zc + (h/2)*cos(deltaalpha) ;

% middle left point ;
xml = xc - (w/2)*cos(deltaphi) ;
yml = yc + (w/2)*sin(deltaphi) ;
zml = zc ;

% middle right point ;
xmr = xc + (w/2)*cos(deltaphi) ;
ymr = yc - (w/2)*sin(deltaphi) ;
zmr = zc ;

% bottme left point    ;
xbl = xc - (w/2)*cos(deltaphi) - (h/2)*sin(deltaalpha)*sin(deltaphi) ;
ybl = yc + (w/2)*sin(deltaphi) - (h/2)*sin(deltaalpha)*cos(deltaphi) ;
```

```
zbl = zc - (h/2)*cos(deltaalpha) ;

% bottom middle point ;
xbm = xc - (h/2)*sin(deltaalpha)*sin(deltaphi) ;
ybm = yc - (h/2)*sin(deltaalpha)*cos(deltaphi) ;
zbm = zc - (h/2)*cos(deltaalpha) ;

% bottom right point ;
xbr = xc + (w/2)*cos(deltaphi) - (h/2)*sin(deltaalpha)*sin(deltaphi) ;
ybr = yc - (w/2)*sin(deltaphi) - (h/2)*sin(deltaalpha)*cos(deltaphi) ;
zbr = zc - (h/2)*cos(deltaalpha) ;

% input all x, y and z coordinates to matrix p ;
P((9*(k-1) + 1),1) = xtl ;
P((9*(k-1) + 1),2) = ytl ;        %the coordinates of top left points ;
P((9*(k-1) + 1),3) = ztl ;
P((9*(k-1) + 2),1) = xtm ;
P((9*(k-1) + 2),2) = ytm ;        %the coordinates of top middle points ;
P((9*(k-1) + 2),3) = ztm ;
P((9*(k-1) + 3),1) = xtr ;
P((9*(k-1) + 3),2) = ytr ;        %the coordinates of top right points ;
P((9*(k-1) + 3),3) = ztr ;
P((9*(k-1) + 4),1) = xml ;
```

```
    P((9*(k-1) + 4),2) = yml ;         %the coordinates of middle left points ;
    P((9*(k-1) + 4),3) = zml ;
    P((9*(k-1) + 5),1) = xc ;
    P((9*(k-1) + 5),2) = yc ;          %the coordinates of center points ;
    P((9*(k-1) + 5),3) = zc ;
    P((9*(k-1) + 6),1) = xmr ;
    P((9*(k-1) + 6),2) = ymr ;         %the coordinates of middle right points ;
    P((9*(k-1) + 6),3) = zmr ;
    P((9*(k-1) + 7),1) = xbl ;
    P((9*(k-1) + 7),2) = ybl ;         %the coordinates of bottom left points ;
    P((9*(k-1) + 7),3) = zbl ;
    P((9*(k-1) + 8),1) = xbm ;
    P((9*(k-1) + 8),2) = ybm ;         %the coordinates of bottom middle points ;
    P((9*(k-1) + 8),3) = zbm ;
    P((9*(k-1) + 9),1) = xbr ;
    P((9*(k-1) + 9),2) = ybr ;         %the coordinates of bottom right points ;
    P((9*(k-1) + 9),3) = zbr ;

    % input deltaphi and deltaalpha into matrix p
    P((9*(k-1)+1):9*k,5) = deltaphi*pi/180 ;    %the delta phi for all 9 points on each visual media ;
    P((9*(k-1)+1):9*k,6) = deltaalpha*pi/180 ; %the delta alpha for all 9 points on each visual media ;
end
```

```
% input the parameters to define the viewing plane
syms seta yv zv ;
seta = input('The angle of sloped viewing plane in degrees? if horizontal viewing plane, input zero. \n') ;
yv = input('The distance in y-coordinate in meters from original point to the start edge of the sloped viewing plane? \n') ;
zv = input('The height in meters of the eyes of the observers on the floor? \n') ;


% start to draw the ideal viewing space of texts presented on each visual medium ;

% define volume -50 meter ~ + 50 meter on x y z coordinates;
[x,y,z] = meshgrid(-50:1:50,-50:1:50,-50:1:50);

% define some angles ;
phi1 = (-90:1:90)*pi/180 ;    %the range of horizontal viewing angle phi ;
alpha1 = (-90:1:90)*pi/180 ;    %the range of vertical viewing angle alpha ;

%define the length of x y z ;
len1 = length(phi1) ;
len2 = length(alpha1) ;
xi = zeros(len1,len2) ;
yi = xi ;
zi = xi ;
```

```
% set the values of x, y, z for the contour of center point of first screen ;
for k = 1:k
    for j = 1:9
        for m = 1:len1
            for n = 1:len2
                phit = phi1(m) ; alphat = alpha1(n) ;
                    ksi = acos(cos(phit)*cos(alphat))*180/pi ;
                    if (ksi >= 0) && (ksi <= 65.7)
                        xi(m,n) = P((9*(k-1)+j),1) + P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^1.5*sin(phit +
                                P((9*(k-1)+j),5)) ;
                        yi(m,n) = P((9*(k-1)+j),2) + P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^0.5*(cos(alphat)*cos(phit
                                + P((9*(k-1)+j),5))*cos(P((9*(k-1)+j),6))-sin(alphat)*sin(P((9*(k-1)+j),6)));
                        zi(m,n) = P((9*(k-1)+j),3) + P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^0.5*(cos(alphat)*cos(phit+
                                P((9*(k-1)+j),5))*sin(P((9*(k-1)+j),6))+sin(alphat)*cos(P((9*(k-1)+j),6)));
                    elseif (ksi <= 82.8)
                        xi(m,n) = P((9*(k-1)+j),1) +
                                ((0.024*ksi-0.577)^-1)*P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^1.5*
                                sin(phit + P((9*(k-1)+j),5)) ;
                        yi(m,n) = P((9*(k-1)+j),2) +
                                ((0.024*ksi-0.577)^-1)*P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^0.5*
                                (cos(alphat)*cos(phit +
                                P((9*(k-1)+j),5))*cos(P((9*(k-1)+j),6))-sin(alphat)*sin(P((9*(k-1)+j),6)));
                        zi(m,n) = P((9*(k-1)+j),3) +
```

```
                    ((0.024*ksi-0.577)^-1)*P((9*(k-1)+j),4)*(cos(phit))^0.5*(cos(alphat))^0.5*
                    (cos(alphat)*cos(phit+P((9*(k-1)+j),5))*sin(P((9*(k-1)+j),6))+
                    sin(alphat)*cos(P((9*(k-1)+j),6)));
            else
                xi(m,n) = P((9*(k-1)+j),1) ;
                yi(m,n) = P((9*(k-1)+j),2) ;
                zi(m,n) = P((9*(k-1)+j),3) ;
            end
        end
    end

    % plot the contour of center point of the first screen ;
    v = z - y*tan(seta*pi/180)+ yv*tan(seta*pi/180) - zv ;   % Create volume data: defines the plane function
    figure(1) ; hold on ;
    contourslice(x,y,z,v,xi,yi,zi,[0 0]) ;
    view(0,90-seta);
    axis equal ;
    axis ([-10 10 -2 18]) ;
    xlabel('x') ;
    ylabel('y') ;
    zlabel('z') ;
  end
end
```

```
% display the parameters ;
disp(' ');
disp ('Results') ;
disp(' ');
disp ('list of the parameters of every 9 viewing points on each visual media');
disp(' ');
disp('            x0            y0            z0            D0            deltaphi            deltaalpha');
disp (P) ;
```

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