THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

INTENSITY AND TIME SCALE EFFECTS OF ACCELEROGRAM

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April, 1968

IP-819

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INTRODUCTION

The uncertainties involved in predicting the earthquake forces a structure is called to stand in its life time, often make it necessary for the designer to consider a stronger earthquake accelerogram in his analysis than the ones recorded to date.

The same situation also arises when the spectral response of a system is to be compared for various recorded earthquakes. Since earthquake accelerograms do not have the same magnitudes, a modification of the accelerogram becomes necessary so that the spectrum intensity, defined as the area under the undamped velocity spectrum between the time intervals t_1 and t_2, are the same for all.(3,4)

It is also of interest to consider what effect modifying the time scale of an accelerogram has on the response spectra.

In this study it is assumed that spectral response curves are available for a system to a given accelerogram. The new accelerogram is to be prepared by multiplying either the intensity of the acceleration or the time scale of the given accelerogram by a constant.

The purpose of this brief discussion is to establish relations between response spectra of the modified accelerograms and spectra obtained from the original accelerogram.

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(3) Housner, G. W. "Behavior of Structures During Earthquakes," Journal of the Engineering Mechanics Division, ASCE, October 1959.

(4) Goel, S. C. and Berg, G. V. "Inelastic Earthquake Response of Tall Steel Frames," ASCE Structural Engineering Conference, Seattle, Washington, May 1967.

LOAD-DISPLACEMENT RELATION

The load-displacement curve for this study is a Ramberg-Osgood function[5] shown in Figure 1. This is a simple and a realistic model to represent actual structural member behavior.[6] Three parameters are employed, a characteristic or yield load q_y , a characteristic or yield displacement x_y , and an exponent r . The Ramberg-Osgood relation includes as special limiting cases the elastic case, obtained by setting $r = 1$, and the elasto-plastic case, obtained as r tends to infinity.

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(5) Jennings, P. C. "Periodic Response of a General Yieldings Structure," Journal of Engineering Mechanics Division, ASCE, Volume 90, Number EM2, April 1964.

(6) Kaldjian, M. J. "Moment-Curvature of Beams as Ramberg-Osgood Functions," Journal of the Structural Division, ASCE, Volume 93, Number ST5, October 1967.

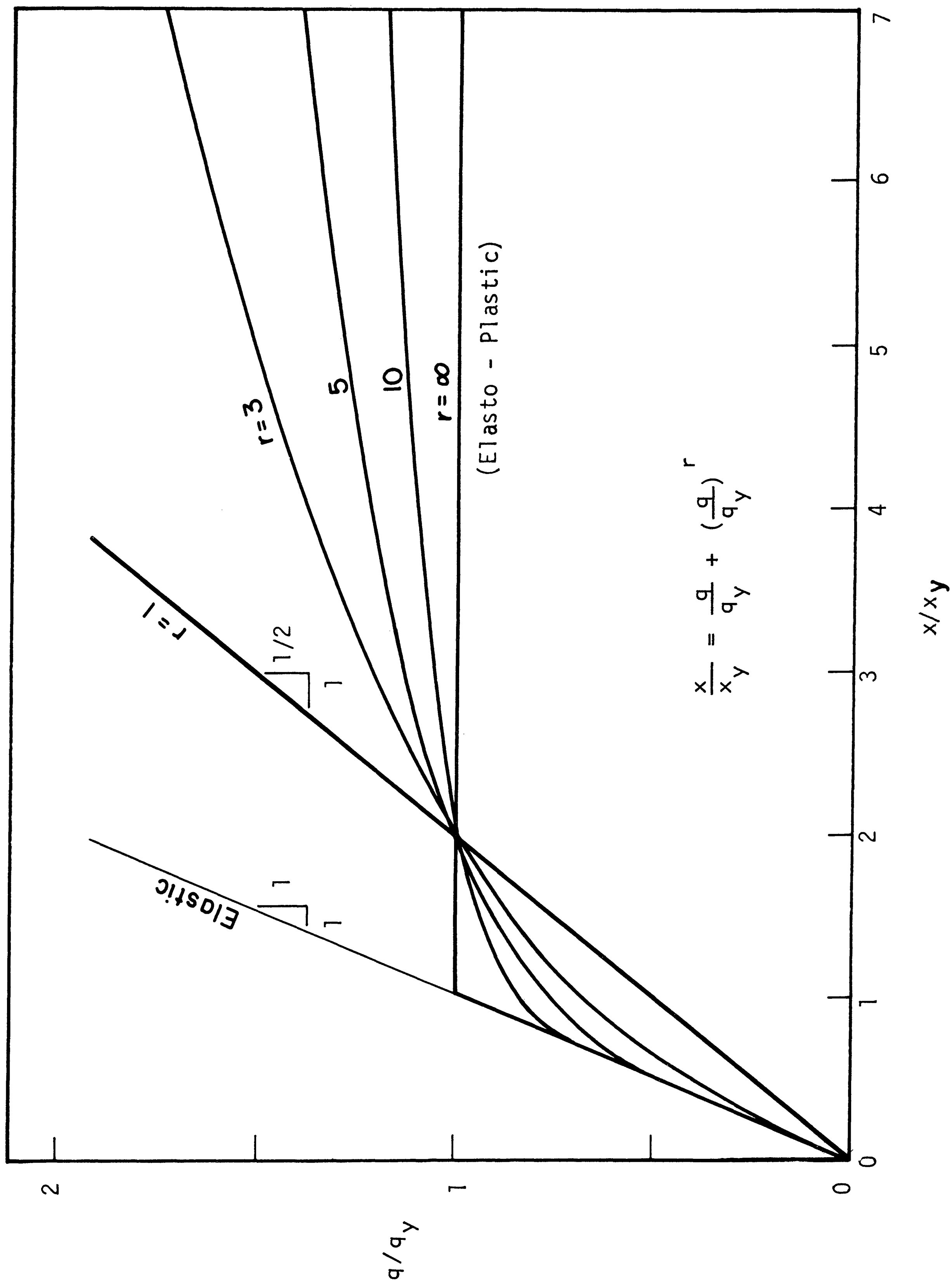

Figure 1. Ramberg-Osgood Functions.

THE EQUATION OF MOTION

Consider the differential equation,

$$\ddot{x} + 2\beta\omega_o\dot{x} + q(x) = -\ddot{y}(t) \tag{1}$$

where

x = relative displacement of mass to ground, a function of time

$y(t)$ = ground displacement, a function of time

$q(x)$ = restoring force per unit mass, a function of x

β = fraction of critical damping

ω_o = the undamped natural frequency of small oscillations

and differentiation with respect to time is denoted by dots. The numerical solution of this equation for various parameters, will result in the response spectra for the given accelerogram $\ddot{y}(t)$.

In dimensionless form Equation (1) becomes,[5]

$$\frac{d^2}{d\tau^2}\left(\frac{x}{x_y}\right) + 2\beta\frac{d}{d\tau}\left(\frac{x}{x_y}\right) + \frac{q}{q_y}\left(\frac{x}{x_y}\right) = -\frac{\ddot{y}(\tau/\omega_o)}{q_y} \tag{2}$$

where

q_y = yield or characteristic strength of spring per unit mass

x_y = yield or characteristic displacement of spring

$\omega_o^2 = q_y/x_y$

$\tau = \omega_o t$

and

$$\frac{q}{q_y}\left(\frac{x}{x_y}\right) \equiv \frac{q(x)}{q_y}$$

INTENSITY SCALE EFFECT

Consider now another system, with damping property β_1, undamped natural frequency ω_1, and force displacement relation $p_1(x)$. It is desired to find the response of the new system when the latter is subjected to an earthquake having acceleration intensities K_I times those of the original earthquake, the time characteristics remaining unchanged. K_I is a constant.

The differential equation for the new system is,

$$\ddot{z} + 2\beta_1\omega_1\dot{z} + p_1(z) = - K_I\, \ddot{y}(t) \tag{3}$$

where z is the relative displacement of the mass with respect ot the ground.

Introducing the dimensionless parameters:

$$\omega_1^2 = p_{1y}/z_y$$

$$\tau_1 = \omega_1 t$$

and

$$\frac{p_1}{p_{1y}}\left(\frac{z}{z_y}\right) \equiv \frac{p_1(z)}{p_{1y}}$$

in which p_{1y} and z_y are yield or characteristic spring strength and spring displacement respectively, Equation (3) can be written as,

$$\frac{d^2}{d\tau^2}\left(\frac{z}{z_y}\right) + 2\beta_1 \frac{d}{d\tau_1}\left(\frac{z}{z_y}\right) + \frac{p_1}{p_{1y}}\left(\frac{z}{z_y}\right) = - K_I\, \frac{y(\tau_1/\omega_1)}{p_{1y}} \tag{4}$$

Equations (4) and (2) can now be made identical by proper choice of parameters. This is done by letting

$$\beta_1 = \beta$$

$$\tau_1 = \tau$$

$$\omega_1 = \omega_o$$

$$\frac{p_1}{p_{1y}} \left(\frac{z}{z_y}\right) = \frac{q}{q_y} \left(\frac{x}{x_y}\right) \qquad (5)$$

and

$$K_I = p_{1y}/q_y = z_y/x_y$$

Using the above values in Equation (4) one can show that at any instant the following relations are valid:

$$\frac{d^2z}{d\tau_1^{\,2}} = \frac{z_y}{x_y}\,\frac{d^2x}{d\tau^2} = K_I\,\frac{d^2x}{d\tau^2}$$

$$\frac{dz}{d\tau_1} = \frac{z_y}{x_y}\,\frac{dx}{d\tau} = K_I\,\frac{dx}{d\tau} \qquad (6)$$

$$z = \frac{z_y}{x_y}\,x = K_I\,x$$

In terms of maximum values the above expressions become

$$|z|_{max} = K_I|x|_{max}$$

$$|\dot{z}|_{max} = K_I|\dot{x}|_{max} \qquad (7)$$

and

$$|\ddot{z} + K_I\ddot{y}|_{max} = K_I|\ddot{x} + \ddot{y}|_{max}$$

Thus the ordinates of the new response spectra are K_I times those obtained from the original accelerogram. Equation (7) is valid for linear as well as non-linear systems provided relations in Equation (5) are satisfied.

To illustrate the preceding, it is desired to use a given response spectra, Figure 2, in order to obtain the response spectra of an earthquake of acceleration intensity K_I times the one used for the given spectra. This is done by multiplying the ordinates of the curves in the above figures by K_I. On four-way-log plots this result is accomplished by shifting the curves vertically by $\log K_I$. Figure 3 shows the latter for $K_I = 3$ applied to the curves of Figure 2.

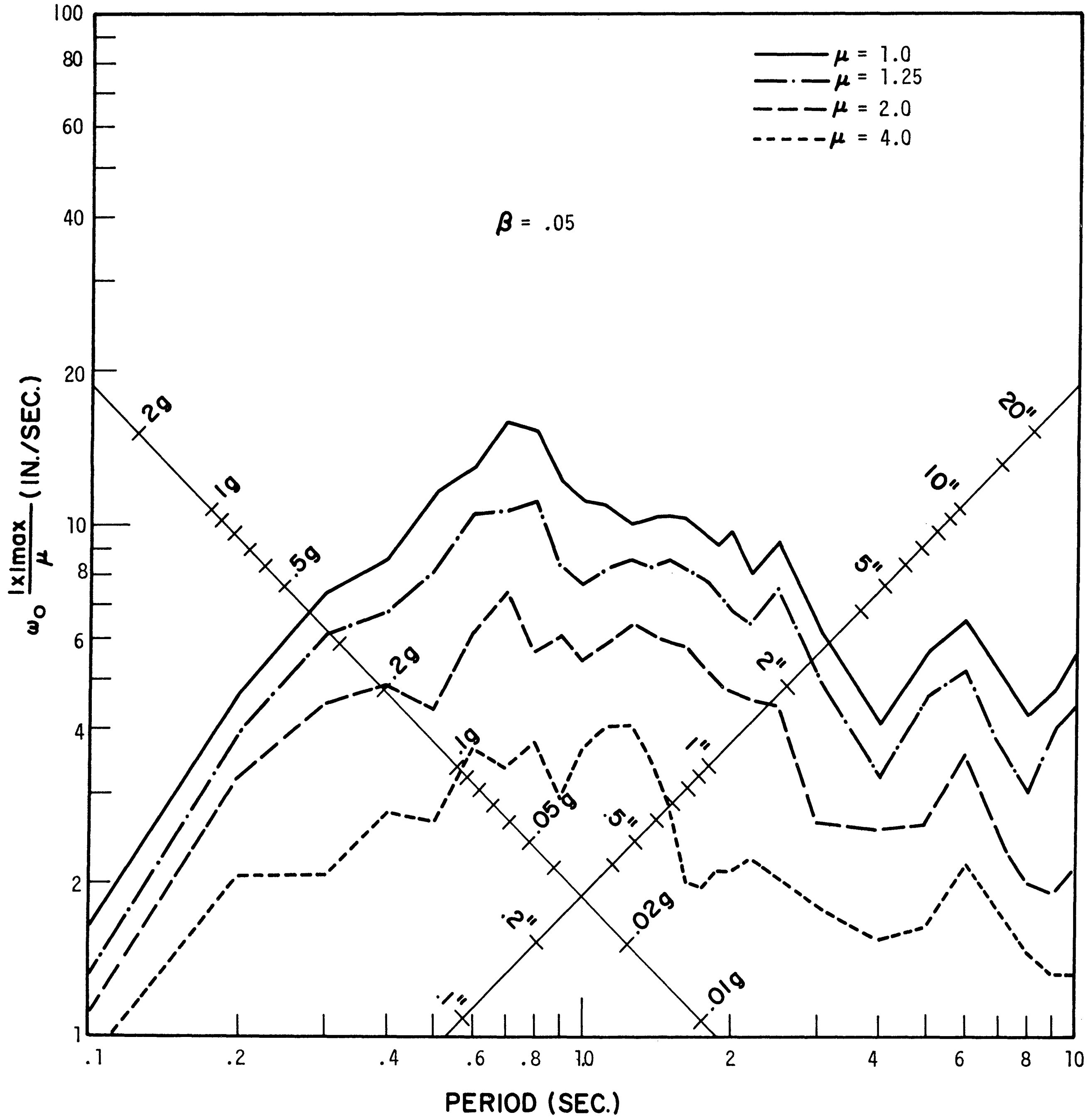

Figure 2. Response Spectra for the Elasto-Plastic System, Taft, July 21, 1952, S21°W. Constant Ductility Ratio "μ."

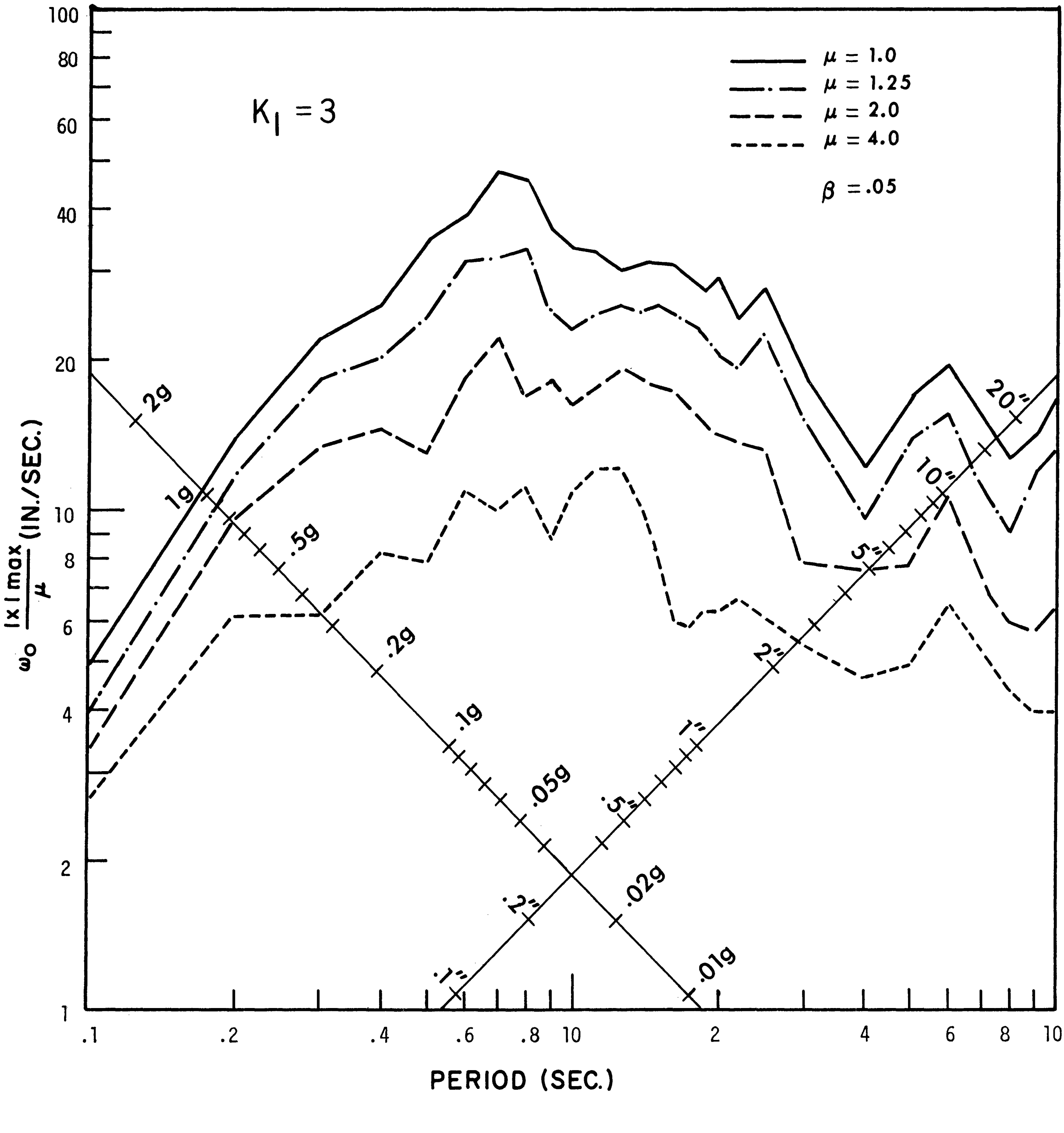

Figure 3. Response Spectra for the Elasto-Plastic System, 3 x Taft, July 21, 1952, S21°W. Constant Ductility Ratio "μ".

TIME SCALE EFFECT

As a second case consider a system with damping property β_2, undamped natural frequency ω_2 and a force-displacement relation $p_2(z)$. It is subjected to an accelerogram obtained by multiplying the time scale of $\ddot{y}(t)$ in Equation (1) by K_T, i.e., the duration of the modified accelerogram would be K_T times the duration of the original accelerogram.

The equation of motion for this can be written as,

$$\ddot{z} + 2\beta_2\omega_2\dot{z} + p_2(z) = - \ddot{y}(K_T t) \tag{8}$$

In dimensionless form this becomes,

$$\frac{d}{d\tau_2^2}\left(\frac{z}{z_y}\right) + 2\beta_2 \frac{d}{d\tau_2}\left(\frac{z}{z_y}\right) + \frac{p_2}{p_{2y}}\left(\frac{z}{z_y}\right) = - \ddot{y}\left(\frac{K_T}{\omega_2}\tau_2\right)/p_{2y} \tag{9}$$

where

$$\omega_2^2 = p_{2y}/z_y$$

$$\tau_2 = \omega_2 t$$

and

$$\frac{p_2}{p_{2y}}\left(\frac{z}{z_y}\right) \equiv \frac{p_2(z)}{p_{2y}}$$

Again by proper choice of parameters, i.e.,

$$\beta_2 = \beta$$

$$p_{2y} = q_y$$

$$\frac{p_2}{p_{2y}}\left(\frac{z}{z_y}\right) = \frac{q}{q_y}\left(\frac{x}{x_y}\right) \tag{10}$$

and

$$K_T = \omega_2/\omega_o = \tau_2/\tau = (x_y/z_y)^{1/2}$$

Equation (9) is made this time similar to Equation (2), and at any instant τ and corresponding τ_2 it can be shown to result in the equalities,

$$\frac{d^2}{d\tau_2^2}\left(\frac{z}{z_y}\right) = \frac{1}{K_T^2\, z_y} \cdot \frac{d^2 z}{d\tau^2} = \frac{1}{x_y} \cdot \frac{d^2 x}{d\tau^2}$$

$$\frac{d}{d\tau_2}\left(\frac{z}{z_y}\right) = \frac{1}{K_T\, z_y} \cdot \frac{dz}{d\tau} = \frac{1}{x_y} \cdot \frac{dx}{d\tau} \qquad (11)$$

$$\frac{z}{z_y} = \frac{x}{x_y}$$

The desired relations can now be found from Equation (11) to be,

$$|z|_{max} = |x|_{max}/K_T^2$$

$$|\dot{z}|_{max} = |\dot{x}|_{max}/K_T$$

$$|\ddot{z} + \ddot{y}|_{max} = |\ddot{x} + \ddot{y}|_{max} \qquad (12)$$

Thus to obtain the new acceleration spectra, the period scale of the original acceleration spectrum curves are divided by K_T . The new velocity spectra are obtained from the original velocity spectrum curves by dividing the period and the velocity scales by K_T . The new displacement spectra are obtained from the original displacement spectra by dividing the period scale by K_T and the displacement scale by K_T^2 .

On four-way-log plots the modified response spectra described above are readily obtained by shifting the original curves horizontally as well as vertically by $\log(1/K_T)$. Figure 4 shows the latter for $K_T = 0.5$ applied to spectra of Figures 2.

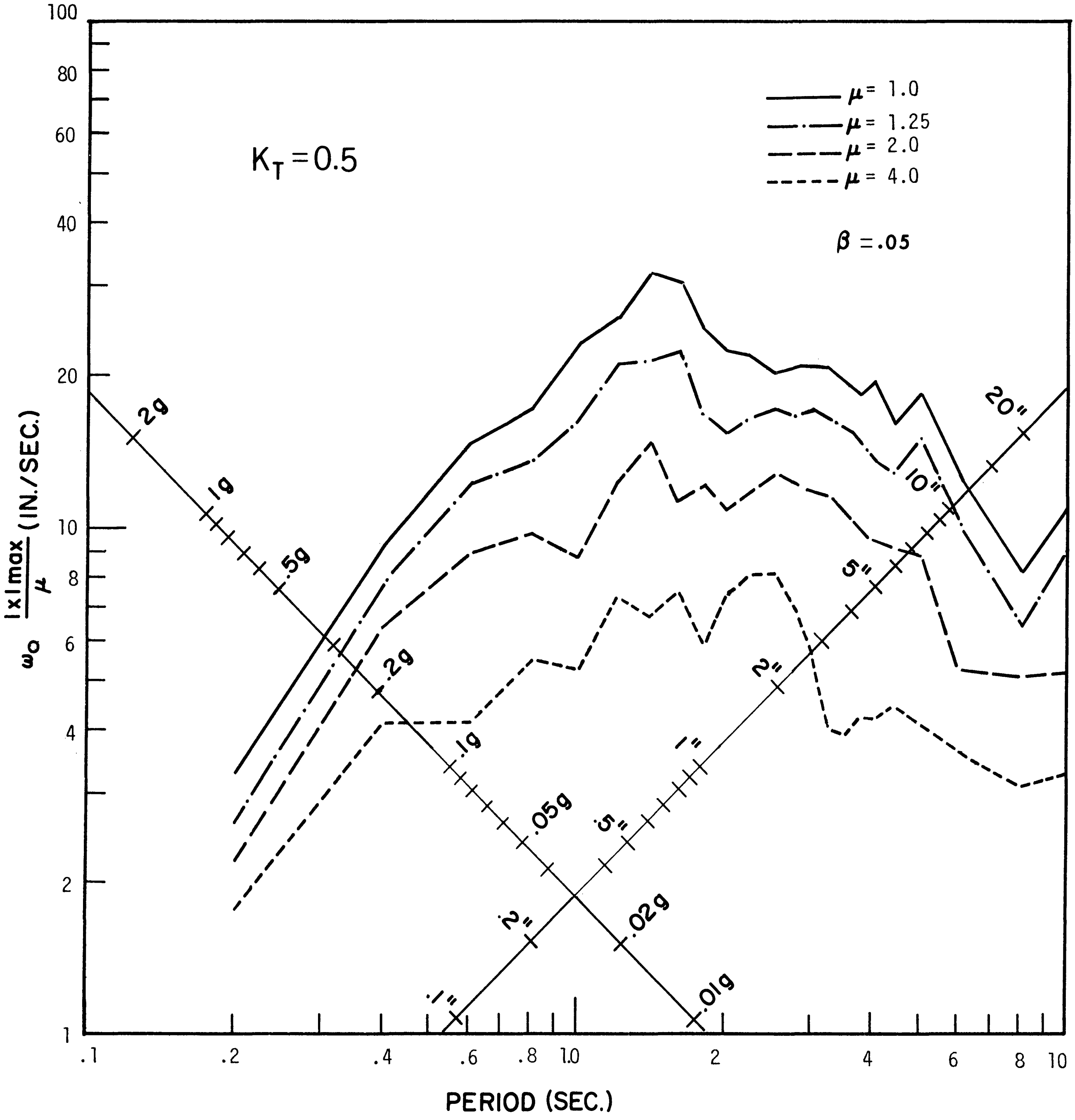

Figure 4. Response Spectra for the Elasto-Plastic System Taft (0.5 x time scale), July 21, 1952, S21°W. Constant Ductility Ratio "μ".

CONCLUSIONS

It has been shown that when an accelerogram is modified by multiplying the acceleration scale of a given earthquake accelerogram by an arbitrary constant K_I , the response spectra for the new accelerogram are K_I times the spectral values obtained from the original accelerogram.

On the other hand if the new accelerogram is obtained by multiplying the time scale by a factor K_T , all quantities involving time must be changed appropriately. The new acceleration, velocity and displacement response spectra are obtained in this case by dividing the corresponding original spectral values by 1, K_T, and K_T^2 , respectively, and dividing the period values by K_T .

When the intensity as well as the time scale of the accelerogram are modified simultaneously then the above two cases must be superimposed to obtain the desired response spectra.

The above three cases are shown plotted side by side in Figure 5 for curve $\mu = 2$ of Figure 2.

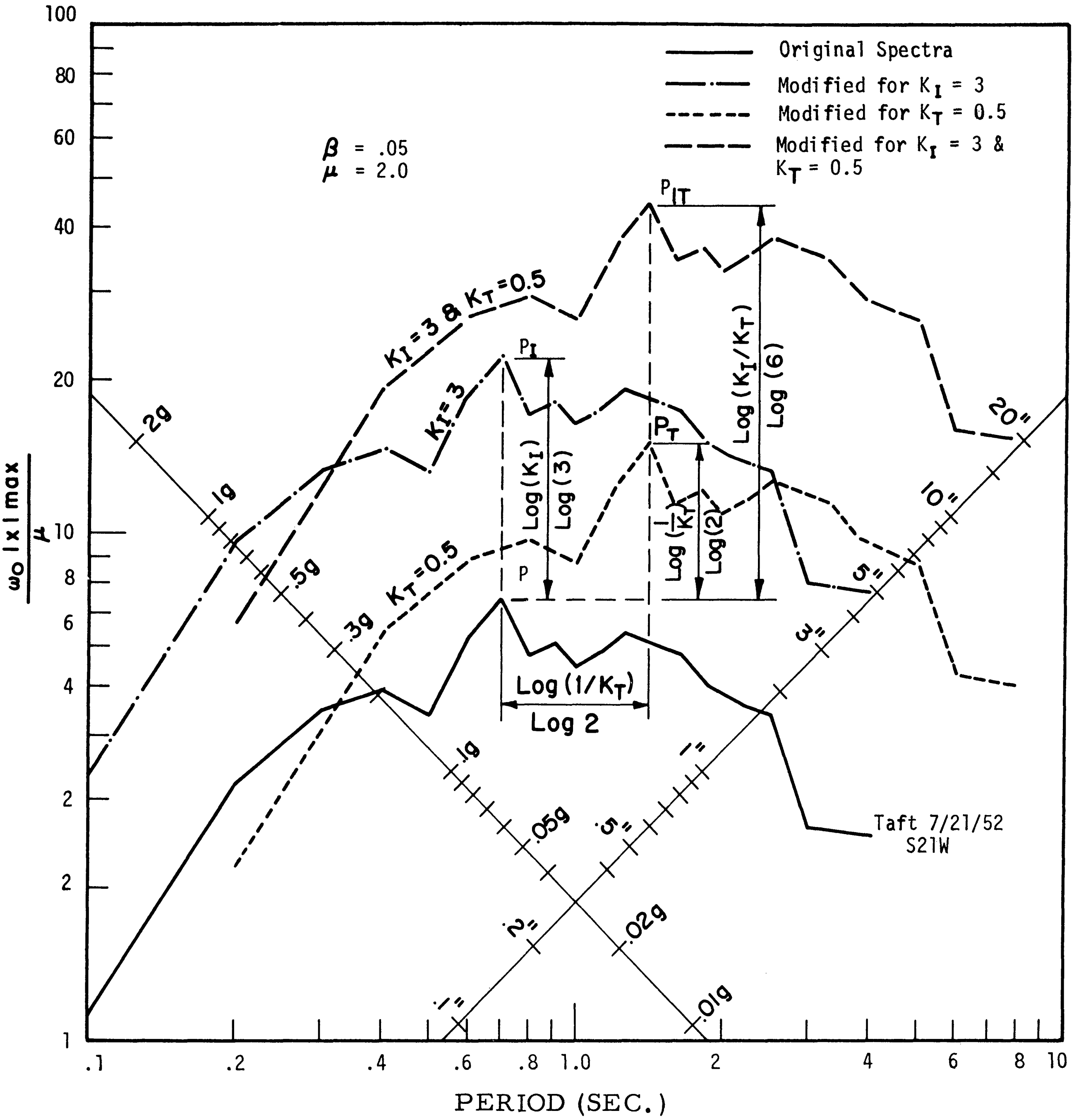

Figure 5. Intensity and Time Scale Effects of Accelerogram on Response Spectra.

ACKNOWLEDGMENTS

This research was carried out at the University of Michigan as part of AISI Project 119 on earthquake response studies.

The writers are indebted to Glen V. Berg, who initiated the project and to Robert D. Hanson who reviewed this report.

NOMENCLATURE

K_I, K_T	a constant
q, p_1, p_2	restoring force per unit mass
q_y, p_{1y}, p_{2y}	yield or characteristic strength of spring per unit mass
r	an exponent
t	time
x, z	relative displacement of mass to ground
x_y, z_y	yield or characteristic displacement of spring
y	ground displacement
β, β_1, β_2	fraction of critical damping
μ	ductility ratio = $\lvert x \rvert_{max}/x_y$
τ, τ_1, τ_2	a time parameter
ω_o, ω_1, ω_2	undamped natural frequency of small oscillations

Note: Differentiation with respect to time is denoted by dots.

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