

**PRICING IN A THREE-TIER MANUFACTURER-
RETAILER-CUSTOMER SYSTEM**

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Abstract

We present a multi-level pricing model with price sensitive consumer demand. A monopolistic manufacturer has the choice of either selling the item directly to the customer through a vertically integrated retailer or through a retailer who may act independently or respond to the manufacturer's price. We characterize the optimal pricing decisions and compare the resulting profit levels for the different channel structures and each of four specific demand functions. Results indicate that it is preferable for the manufacturer to sell through a vertically integrated retailer. The price offered to the consumer is also lower in this case. We also find that the division and magnitude of profits in a system with an independent retailer or a system with a retailer that responds to the manufacturer's price depends on the demand function.

1 Introduction

Pricing is one of most important decisions affecting the profitability of an enterprise. This decision is especially significant if the demand is price sensitive. This paper investigates how the structure of the distribution channel affects the pricing decision of a monopolistic manufacturer who is facing a downward sloping demand function. The pricing policy of a manufacturer will be quite different, depending upon whether it is selling the product directly to the consumer or to a retailer. Furthermore, the division of power between the manufacturer and the retailer will play an important role in the division of the profit between the two entities. A retailer, for example, may act independently setting its profit margin independent of the manufacturer. Alternatively, it may simply “follow” the price set by the manufacturer in arriving at the price it should charge to the consumer. Our analysis assumes that production is instantaneous and production cost is linear. For each order, both the manufacturer and the retailer incur a fixed order processing cost. The delivery frequency is pre-determined based on exogenous constraints, and a linear holding cost is incurred on stock not yet sold.

Our intention here is not to explore why certain power structure exists between the manufacturer and the retailer. Rather, we are interested in how prices are optimally set and profit shared given a specific power structure. The relationship between various prices and profits are also of interest since they provide important insights into the desirability of different divisions of power from the manufacturer’s, retailer’s and the entire system’s viewpoint. As a result, we intend to explore questions such as—Does the manufacturer gain by taking the retailing on itself? What is the impact of vertical integration on the total system profit? How much will the manufacturer gain by having a retailer which “follows” its price rather than one which acts independently in setting its margin?

We also explore the sensitivity of our results with respect to various cost parameters, such as—How does a change in the price offered by the manufacturer to the retailer affect the price offered by the retailer to the consumer? More generally, suppose due to improvement in production technology, the manufacturer can

now produce the commodity at a cheaper price. What fraction of this saving in production cost is passed on to the retailer and to the consumer?

The channel structure is equally important from the consumer point of view, since it effects the final price offered to the consumer. In this regard, a natural question that arises from the consumer's perspective is—Does the product price rise or fall due to vertical integration? A related question is—How does the power structure between manufacturer and the retailer affect the price paid by the consumer? We investigate these questions using four specific demand functions to characterize the *nature* and the *magnitude* of price changes due to differences in the division of power in the distribution channel.

The issues of pricing, channel coordination and distribution efficiency have been the subject of extensive research in economics, marketing and operations management literature (Choi, 1991; Dada and Srikanth, 1987; Dolan, 1987; Jeuland and Shugan, 1983, 1988; Kim and Hwang, 1989; Lal, 1990; Lal and Staelin, 1984; Lee and Rosenblatt, 1986; McGuire and Staelin, 1983; Monahan, 1984; Rosenblatt and Lee, 1985; Westfield, 1981). This research is closely related to Jeuland and Shugan (1983) who study the problem of coordination in channels of distribution. They explore the effect of cooperation between the manufacturer and the retailer by comparing an independent manufacturer-retailer structure to a vertically integrated channel and conclude that cooperation always leads to higher profit. The independent manufacturer-retailer structure in their model is symmetric since there is no holding cost paid on stock yet to be sold. In contrast, the manufacturer-retailer relationship is asymmetric in our model since the retailer has to pay holding cost. However, the conclusion that the system profit always increases due to coordination still holds. Furthermore, we analyze the leader-follower relationship between manufacturer and retailer and compare the results with those for the vertically integrated and independent manufacturer-retailer systems.

In a recent work, Choi(1991) analyzes a duopoly model where manufacturers sell their products through a common independent retailer. He compares several channel structures including the leader-follower and the independent manufacturer-retailer systems using linear and constant elasticity demand functions. For the

linear demand function, he shows that the retail prices are lower in the case of the independent manufacturer–retailer system compared to the leader–follower structure; while the reverse is true for the constant elasticity demand function. Our results are consistent with these observations. Moreover, we consider two other non-linear demand functions—exponential and algebraic. Our analysis shows that for the exponential demand function, there is no difference between the retail prices for the leader–follower and the independent manufacturer–retailer system while the results for algebraic demand function mirror those of the constant elasticity demand function.

The interaction between pricing and production/distribution efficiency has been studied by a number of authors. Monahan (1984) was first to illustrate how a monopolistic manufacturer operating as a “leader” can increase its profit by providing a discount. The price schedule devised by Monahan (1984) assumes a profit maximizing retailer facing a constant demand. The manufacturer tries to induce larger order quantities from the retailer to reduce its own order processing cost. To compensate for the increase in holding cost due to larger order size, the retailer receives a price discount. Overall, the retailer is no worse off and the manufacturer reaps all the benefits. Rosenblatt and Lee (1985) and Lee and Rosenblatt (1986) extend Monahan’s model by allowing the production frequency to be different than the ordering frequency of the retailer. They also allow a more equitable division of the resulting benefits between the manufacturer and retailer. Lal and Staelin (1984) and Dada and Srikanth (1987) propose pricing schemes that minimize the total system cost and provides a profit sharing mechanism. Our leader-follower model is similar to these models since the retailer must pay a holding cost and both manufacturer and retailer pay fixed order processing costs. However, unlike these models, we allow a price sensitive demand but keep the order frequency an exogeneous variable. In essence, the manufacturer here adjusts the price to induce a higher demand instead of a larger order size.

The rest of the paper is organized in the following manner. The following section introduces the basic model, the specific demand functions, and the decision framework. Section 3 provides the analysis for each channel structure. Since the nature of our analysis is similar for all demand functions, for brevity we present

the analysis for the constant elasticity demand function only. A sketch of the analysis for other demand functions can be found in the Appendix with results summarized in Tables 1, 2, and 3 for comparative purposes. Section 4 provides a qualitative discussion of the comparative summary of these results, (detailed in Table 4), answering the questions raised at the beginning of this paper. The paper ends with a few concluding remarks.

2 The Model

Consider a three-tier system consisting of a manufacturer, a retailer, and consumers. The manufacturer is the sole supplier of the product and the retailer is the only outlet through which consumer demand is satisfied. The shipment from manufacturer to retailer arrives at a fixed interval such that n orders arrive every year. The shipment interval is an exogenous variable determined by constraints such as shelf space and shelf life limitations, delivery frequency of other items, docking and handling constraints, and transportation capacity. Production of an order takes a much shorter time compared to the order interval; as a result, production can be considered instantaneous. Each unit is produced by the manufacturer at a cost of θ and is sold to the retailer at a price of P_m . The retailer, in turn, sells the product at a price of P_r per unit to the consumer. This model is concerned with the choice of decision variables P_m and P_r by manufacturer and retailer respectively to optimize their profit levels. The retailer incurs a fixed order processing cost S_r for each order. Similarly, the manufacturer incurs a fixed order filling cost of S_m for each order. Let i be the cost of carrying a dollar's worth of inventory for a year, then $h = \left(\frac{i}{2n}\right)$ is the average inventory cost per unit value of the item.

In this model, the contribution of the order processing and filling costs is the same irrespective of system type, since the order interval is fixed. These costs unnecessarily complicate the intuitive and geometric analysis of the model, and are therefore ignored in our discussion. Because these costs are important practical considerations, they are included in the mathematical descriptions of the analysis.

2.1 The Decision Framework

We analyze three distinct scenarios: (1) when the manufacturer serves as the price setter and the retailer responds; (2) when the manufacturer and retailer set their respective margins independently; and (3) when the manufacturer and retailer are part of the same vertically integrated system and act as a single decision maker. The decision making frameworks for each case are described below.

1. The Leader–Follower Manufacturer–Retailer System (Manufacturer Stackleberg):

The relationship between the manufacturer and the retailer is shown in Figure 1. The manufacturer sets the price P_m ; the retailer then responds by setting the selling price P_r to which the consumer responds according to the demand function $D(P_r)$. In effect, the manufacturer exercises greater control over the price setting process and the retailer has no choice but to accept the price specified by the manufacturer. This is similar to a Stackelberg game where the manufacturer acts as a leader.

The retailer can extract a high profit margin at the cost of a diminished demand, or it can set a low profit margin to induce a higher demand. For any price specified by the manufacturer, the retailer trades off between these two extremes, choosing a price that maximizes its total profit. This in turn, imposes a demand on the system.

The manufacturer, anticipating the response of a rational retailer, chooses its price to maximize its own profit. A high price charged by the manufacturer may leave the retailer with no other option but to charge an even higher price to the consumer, resulting in very low demand. On the other hand, a relatively low price charged by the manufacturer will undercut its profit margin but may yield an increased demand. However, this increase in demand may not be as significant since the retailer can reap most of the benefits by not passing this price break on to the consumer. The manufacturers' decision is thus more involved than that of retailer.

2. The Independent Manufacturer–Retailer System (Cournot-Nash Equilibrium):

In this framework, the manufacturer and retailer act autonomously as shown in Figure 2. The manufacturer and retailer independently determine their profit margins. Both the retailer and manufacturer can determine whether to apply a high margin with a low resultant demand, or a low margin with a resultant high demand. Since an increase in individual profit margin results in a decrease in total demand, there exists an equilibrium condition, the Cournot-Nash Equilibrium, where the rate of change of profits with respect to the margin is equal to zero. In the absence of holding cost, the manufacturer and retailers' margins will be equal due to their symmetric roles (Jeuland and Shugan, 1983; Choi 1991). However, the holding cost brings about a fundamental asymmetry in their relationship since the retailer now has to pay holding cost on manufacturer's margin. In this sense, a unit increase in manufacturer's margin will have an amplified effect on the final price paid by the consumer and hence the resulting demand.

3. The Vertically Integrated System:

The manufacturer and retailer operate in total cooperation in a vertically integrated system, as shown in Figure 3. In effect, this is a two-tier system where the manufacturer and retailer can be considered to be a single decision maker. Since we assume that there is no transfer pricing between the manufacturer and the retailer, there is no need to specify a manufacturer's price in this model. The fixed order processing and order filling costs must be paid by this central agency which sets the price for the consumer so as to maximize its total profit. The decision making is similar to that for the retailer in the leader-follower system who is subjected to unit cost θ instead of P_m .

2.2 Demand Functions

We use four commonly used demand functions (Jeuland and Shugan, 1988) in our analysis. All demand functions are expressed as a function of P_r , the price offered by retailer to the consumer.

1. *Constant price-elasticity demand function*

$$D(P_r) = a(bP_r)^{-k},$$

where $a > 0$ and $b > 0$ are scaling constants for demand and price respectively and $k > 2$ is the price elasticity of demand.

2. *Exponential demand function*

$$D(P_r) = ae^{-bP_r},$$

where $a > 0$ and $b > 0$ are scaling constants for demand and price respectively.

3. *Linear demand function*

$$D(P_r) = a - bP_r,$$

where $a > 0$ is the maximum demand per year, $b > 0$ is the quantity decrease per unit increase in price and it is assumed that $P_r < \frac{a}{b}$.

4. *Algebraic demand function*

$$D(P_r) = (a + bP_r)^{-q},$$

where $a > 0$ and $b > 0$ are scaling constants for maximum demand and the quantity decrease per unit increase in price respectively, and $q > 2$.

3 Illustrative Analysis

In this section we present the analysis for the constant elasticity demand function

$$D(P_r) = a(bP_r)^{-k},$$

where a and b are the scaling constants, for demand and price respectively, and k is the price elasticity.

Section 3.1 analyzes the leader–follower manufacturer–retailer system deriving the optimal prices for both

the manufacturer and the retailer. Section 3.2 presents the analysis for the independent manufacturer–retailer system that derives the margins that result in the Cournot-Nash equilibrium. Finally, Section 3.3 presents an analysis for price determination in a vertically integrated system.

3.1 The Leader–Follower Manufacturer–Retailer System

The leader–follower manufacturer–retailer system features an independent retailer with the manufacturer taking on the role of the price setter. The result is a Stackleberg game, in which the retailer reacts to the price set by the manufacturer by setting the price to the consumer that will optimize its profits. Knowing the retailer’s optimal response to a given manufacturer’s price, the manufacturer can determine the demand and optimize its own profits.

Retailer’s Profit Maximization:

The objective of the retailer is to maximize its total profits which is given by the profit margin multiplied by the demand less any inventory carrying charges. Demand is a decreasing function of price, as is illustrated in Fig 4 with the constant elasticity demand function $D(P_r)$. For a given P_r and resultant demand D , the total revenue is the rectangle $ABIJ$. For a given P_m , the cost of each of D units of demand is the marginal cost $MC_r = P_m(1 + h)$, which means the total cost is the rectangle $DCIJ$. The retailer’s margin is thus $G_r = P_r - P_m(1 + h)$ and the shaded area $ABCD$, labelled Π_r , is the retailer’s profit which is to be maximized. Mathematically the profit function including order processing costs for the retailer under a constant elasticity demand function is given by

$$\begin{aligned}\Pi_r(P_r, P_m) &= \left(a(bP_r)^{-k}\right)(P_r - P_m) - \left(a(bP_r)^{-k}\right)P_m(1 + h) - nS_r \\ &= \left(a(bP_r)^{-k}\right)(P_r - P_m(1 + h)) - nS_r.\end{aligned}\tag{1}$$

From the first order conditions on the profit function and the fact that the demand function is concave, the

optimal retailer price, P_r^* , can be determined in terms of P_m to be

$$P_r^*(P_m) = P_m(1+h) \left(\frac{k}{k-1} \right). \quad (2)$$

Substituting this optimal retailer price into equation (1) gives the optimal total profit for the retailer as

$$\Pi_r^*(P_m) = \frac{P_m(1+h)}{(k-1)} a \left(bP_m(1+h) \left(\frac{k}{k-1} \right) \right)^{-k} - nS_r. \quad (3)$$

Equation (2) expresses P_r^* as a function of P_m only. Since the retailer will adopt P_r^* if the manufacturer adopts P_m , it follows that demand is determined by P_m . The curve $D(P_m)$ in Figure 4 represents demand as a function of P_m . This curve is termed the retailer's response function, (Choi, 1991), since it is based on the retailer's optimum choice of P_r^* for a given value of P_m .

Manufacturer's Profit Maximization:

The manufacturer seeks to set its price to maximize its own profits. For a given P_m , and hence demand D , the manufacturer's revenue is given in Figure 4 by the rectangle $EFIJ$. With unit cost θ , the manufacturing costs are given by the rectangle $HGIJ$, and hence the shaded area $EFGH$, labelled Π_m , represents the profits. The margin for the manufacturer is thus $P_m - \theta$. Mathematically the profit function including order filling costs for the manufacturer under a constant elasticity demand function is given by

$$\Pi_m(P_m, P_r) = \left(a(bP_r)^{-k} \right) (P_m - \theta(1+h)) - nS_m.$$

Substituting the value of P_r^* from equation (2) for P_r into this expression gives the manufacturer's profit function in terms of P_m only:

$$\Pi_m(P_m) = a \left(bP_m(1+h) \left(\frac{k}{k-1} \right) \right)^{-k} (P_m - \theta(1+h)) - nS_m \quad (4)$$

The manufacturer's optimal price can be determined from the first order conditions of this profit function to be

$$P_m^* = \left(\frac{k}{k-1} (\theta(1+h)) \right) (1+h)^{-1}$$

With this optimal value, P_m^* , the optimal value of P_r^* is obtained from equation (2) as

$$P_r^* = \frac{k^2}{(k-1)^2}(\theta(1+h)).$$

For subsequent comparisons of the alternative distribution channels, we need to convert from price to margin for both retailer and manufacturer. With $G_m = P_m - \theta$ and $G_r = P_r - (1+h)P_m$, the optimal margins can be expressed as follows:

$$G_r^* = \frac{k}{(k-1)^2}(\theta(1+h))$$

$$G_m^* = \left(\frac{1}{k-1}(\theta(1+h)) \right) (1+h)^{-1}.$$

Substituting the value of P_m^* into equations (3) and (4) provides explicit expressions for the manufacturer's, retailer's, and the total system's profits:

$$\Pi_m^* = a \left(\frac{\theta(1+h)}{k-1} \right) \left(\frac{b\theta(1+h)k^2}{(k-1)^2} \right)^{-k} (1+h)^{-1} - nS_m \quad (5)$$

$$\Pi_r^* = a \left(\frac{\theta(1+h)}{k-1} \right) \left(\frac{b\theta(1+h)k^2}{(k-1)^2} \right)^{-k} \left(\frac{k}{k-1} \right) - nS_r \quad (6)$$

$$\Pi_r^* + \Pi_m^* = a \left(\frac{\theta(1+h)}{k-2} \right) \left(\frac{bk^2\theta(1+h)}{(k-1)^2} \right)^{-k} \left(\frac{k(2+h)-1}{(k-1)(1+h)} \right) - n(S_m + S_r). \quad (7)$$

3.2 The Independent Manufacturer–Retailer System

This system features independent decision making by both the retailer and the manufacturer. The resulting solution is the Cournot-Nash equilibrium for the system as both the retailer and the manufacturer not only set their prices independently but at the same time. This implies that the division of profits between the manufacturer and the retailer are more equitable.

Equilibrium Conditions:

The objective of both the manufacturer and retailer is to maximize their profits without prior knowledge of the other's decision. Demand is a decreasing function of price, as illustrated in Fig 5 with the constant

elasticity demand function, $D(\tilde{P}_r)$. For a given \tilde{P}_r and the resultant demand D , the total revenue for the retailer is the rectangle $ABIJ$. For a given \tilde{P}_m , the cost of each of the D units of demand is the marginal cost, $MC_r = \tilde{P}_m(1+h)$ which reflects the additional cost of holding the inventory. This defines the retailer's margin, $\tilde{G}_r = \tilde{P}_r - \tilde{P}_m(1+h)$. The retailer's total cost is thus the rectangle $DCIJ$ and its profit is the shaded area $ABCD$, labelled $\tilde{\Pi}_r$. The manufacturer receives the per unit price of \tilde{P}_m from the retailer defining its revenue as the rectangle $EFIJ$. The cost of each unit is θ so the manufacturer's margin is $\tilde{G}_m = \tilde{P}_m - \theta$. The total cost is the rectangle $HGIJ$ and the manufacturer's profit is the shaded area $EFGH$, labelled $\tilde{\Pi}_m$. Mathematically, the total profits for the manufacturer and retailer are equal to their margin times the demand less applicable order filling and processing costs are

$$\tilde{\Pi}_m(\tilde{G}_r, \tilde{G}_m) = \tilde{G}_m a \left(b \left((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-k} - nS_m, \quad (8)$$

$$\tilde{\Pi}_r(\tilde{G}_r, \tilde{G}_m) = \tilde{G}_r a \left(b \left((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-k} - nS_r. \quad (9)$$

The manufacturer and the retailer maximize their profits by choosing their respective profit margins independently. The equilibrium conditions for this system consist of determining the manufacturer's and retailer's price such that neither can improve their profits in the long term by changing their decision. The equilibrium margin is defined for both the manufacturer and the retailer as the point at which the change in profits with respect to the margin is equal to zero. As can be seen in Fig 5, each additional unit claimed by the manufacturer results in an increase of $(1+h)$ units of total price whereas an additional unit claimed by the retailer results in an increase of only 1 unit of total price. Therefore the equilibrium margin for the retailer is $(1+h)$ times the margin for the manufacturer. Since demand is the same for both entities, the profit excluding ordering costs is also slightly higher for the retailer.

Since the retailer maximizes its profit by setting its margin, \tilde{G}_r , such that the change in its profits with respect to the margin is zero, its optimal margin, \tilde{G}_r^* is the solution to the following equation:

$$\tilde{G}_r \left(-kba \left(b \left((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-k-1} \right) + a \left(b \left((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-k} = 0. \quad (10)$$

Similarly, since the manufacturer maximizes its profit by setting its margin, \tilde{G}_m , such that the change in its

profits with respect to the margin is zero, \tilde{G}_m^* is the solution to the following equation:

$$\tilde{G}_m(1+h) \left(-kba \left(b \left((1+h) \left(\tilde{G}_m + \theta \right) + \tilde{G}_r \right) \right)^{-k-1} \right) + a \left(b \left((1+h) \left(\tilde{G}_m + \theta \right) + \tilde{G}_r \right) \right)^{-k} = 0. \quad (11)$$

Since the only difference between equation (10) and (11) is in the coefficient of the first term, and since both equations are set equal to zero, it follows that

$$\tilde{G}_r^* = \tilde{G}_m^*(1+h). \quad (12)$$

If we ignore the costs associated with ordering, the result can be equivalently expressed in terms of profits as

$$\tilde{\Pi}_r^* = \tilde{\Pi}_m^*(1+h). \quad (13)$$

The equilibrium margins for the manufacturer and retailer can be obtained by solving equations (10) and (11) to get

$$\begin{aligned} \tilde{G}_r^* &= \frac{1}{k-2}(\theta(1+h)), \\ \tilde{G}_m^* &= \left(\frac{1}{k-2}(\theta(1+h)) \right) (1+h)^{-1}. \end{aligned}$$

For subsequent comparisons of the alternative distribution channels, we need to convert from margin to price for both retailer and manufacturer. With $\tilde{G}_m = \tilde{P}_m - \theta$ and $\tilde{G}_r = \tilde{P}_r - (1+h)\tilde{P}_m$, the equilibrium prices can be expressed as follows:

$$\begin{aligned} \tilde{P}_r^* &= \frac{k}{k-2}(\theta(1+h)) \\ \tilde{P}_m^* &= \left(\frac{k-1}{k-2}(\theta(1+h)) \right) (1+h)^{-1}. \end{aligned}$$

Also, corresponding to the relationship between margins, the relationship between the manufacturer's and retailer's price is

$$\tilde{P}_m^* = \frac{1}{2} \left(\theta + \frac{\tilde{P}_r^*}{1+h} \right)$$

The total profits for the two entities and for the system as a whole can be obtained by substituting the values for \tilde{G}_m^* and \tilde{G}_r^* into equations (9) and (8). The results are as follows:

$$\begin{aligned}\tilde{\Pi}_m^* &= a \left(\frac{\theta(1+h)}{k-2} \right) \left(\frac{bk\theta(1+h)}{k-2} \right)^{-k} (1+h)^{-1} - nS_m \\ \tilde{\Pi}_r^* &= a \left(\frac{\theta(1+h)}{k-2} \right) \left(\frac{bk\theta(1+h)}{k-2} \right)^{-k} - nS_r \\ \tilde{\Pi}_m^* + \tilde{\Pi}_r^* &= a \left(\frac{\theta(1+h)}{k-2} \right) \left(\frac{bk\theta(1+h)}{k-2} \right)^{-k} \left(\frac{2+h}{1+h} \right) - n(S_m + S_r).\end{aligned}$$

3.3 The Vertically Integrated System

With the vertically integrated system, the manufacturing organization controls the retailer, and hence it is in effect selling directly to the consumer. There is therefore no transfer price between the manufacturer and the retailer, and thus only a single price, \hat{P}_r , to be determined.

System Profit Maximization:

Since the decision maker controls both the functions of the manufacturer and the retailer, it has to maintain an inventory. The holding costs can be included in the total costs by treating the actual per unit cost of goods as $\theta(1+h)$ instead of θ . The profits are thus equal to the margin, $\hat{G} = \hat{P}_r - \theta(1+h)$, multiplied by demand. In Figure 6, the area $ABCD$, labelled $\hat{\Pi}$, represents the profits for this system. Mathematically the total profit less order filling and processing costs for the vertically integrated system is

$$\begin{aligned}\hat{\Pi}(\hat{P}_r) &= \left(a(b\hat{P}_r)^{-k} \right) (\hat{P}_r - \theta) - \left(a(b\hat{P}_r)^{-k} \right) \theta(1+h) - n(S_m + S_r) \\ &= \left(a(b\hat{P}_r)^{-k} \right) (\hat{P}_r - \theta(1+h)) - n(S_m + S_r).\end{aligned}\tag{14}$$

Form the first order conditions, the optimal price, \hat{P}_r^* , is

$$\hat{P}_r^* = \frac{k}{k-1}(\theta(1+h)).$$

For subsequent comparisons of the alternative distribution channels, we need to convert from price to margin.

With $\hat{G} = \hat{P}_r - (1 + h)\theta$, the optimal margin can be expressed as follows:

$$\hat{G}^* = \frac{1}{k-1}(\theta(1+h))$$

Substituting \hat{P}_r^* into equation (14) gives the following explicit equation for the system profit:

$$\hat{\Pi} = a \left(\frac{\theta(1+h)}{k-1} \right) \left(\frac{bk\theta(1+h)}{k-1} \right)^{-k} - n(S_m + S_r).$$

4 Results for the Four Demand Functions

Table 1 and Table 2 summarize the prices and margins respectively for the three systems under the four demand functions. Table 3 summarizes the profits for both manufacturer and retailer as well as the total system profit for each channel structure. As previously mentioned, order processing and filling costs are not included in this table or the following discussions because they are common to all systems and are not effected by the pricing decisions.

One of the immediate factors that catches one's attention when examining the two tables is that of $(1 + h)$. The proliferation of this factor demonstrates the widespread effect of accounting for holding costs. This factor is present for two reasons. First, the minimum price that a unit can be sold to the consumer for is $\theta(1 + h)$. This is because each unit incurs a minimum average holding cost of θh as it passes through the system and thus the effective total cost of a product increases to $(1 + h)$ times its original cost. Therefore each price is given, not in terms of the unit cost θ , but instead, in terms of the effective minimum system cost, $\theta(1 + h)$. This term is also reflected in all margin and profit expressions.

The other reason may be found by inspection of the manufacturer's prices, margins and profits. Here, the inverse of the factor, $(1 + h)^{-1}$, is present because a unit increase in price charged by the manufacturer increases the effective marginal cost seen by the retailer by $1 + h$ units. This observation reveals an insight into the derivation of the manufacturer's price. In effect, the manufacturer determines the marginal cost for the retailer that maximizes the manufacturer's profit. It then sets its price by reducing this marginal cost

by the holding cost factor, $(1 + h)^{-1}$ so the retailer does in fact see the optimal marginal cost. As a result, the manufacturer absorbs the cost of holding the inventory.

The nature of the demand functions themselves also directly affect how the prices and margins are determined. The prices and margins under the constant elasticity demand function do not involve either of the two scaling constants, a and b . Upon closer inspection of the demand function, the reason becomes apparent. The two scaling parameters serve only to scale the actual demand available, and do not effect the optimal pricing decision. The actual demand seen by the system depends only on the price elasticity. Therefore the prices and margins are based only on the minimum selling price and the price elasticity. The exponential demand function is similar in that the scaling constant a has no effect on the determination of price and margin.

The linear demand function has a limit on the maximum price which is defined as $\frac{a}{b}$. Each price must now be between the minimum and maximum price and is formed as a convex linear combination of the two. Margins are thus a fraction of the difference between the maximum and minimum price. The determination of prices and margins under the algebraic demand function involves aspects of both the linear and constant elasticity demand functions. Similar to prices derived under the linear demand function, we see the optimal prices being a combination of $\frac{a}{b}$ and the minimum selling price. This similarity between the prices under linear and algebraic demand functions is not coincidental. However, the relationship between the two is quite intricate. If $b < 0$, then again we would have a convex combination of the two values. However, since $b > 0$, the intuitive definition for $\frac{a}{b}$ is lost, although the effect of this term is similar. The relationship between the constant elasticity and algebraic demand function is more obvious than the relationship between linear and algebraic demand functions. The effect of the exponent in the algebraic demand function is identical to the effect of the price elasticity in the constant elasticity demand function. If we set $a = 0$ and $b = a'b'^{-q}$, then the algebraic demand function has exactly the same form as the constant elasticity demand function.

5 Discussion

In Table 4 we present a summary of the qualitative comparison between the three systems under each of the four demand functions.

For the independent manufacturer–retailer system, the relationship between the manufacturer’s and retailer’s margins and profits are strictly defined by equations (12) and (13). These equations hold for all four demand functions. We intuitively expect the relationships between margins and profits for the leader–follower manufacturer–retailer system to be quite different. Because the manufacturer is the price setter, we expect that it should be able to reap a higher margin and profit than the retailer. But contrary to our intuition, the demand function determines who receives the larger share of profits. Similarly, we expect the retailer to have higher profits when acting independently than when forced to respond to the manufacturer. Again, we find that the result of profit comparisons is demand dependent. In fact the more profitable system is also demand dependent. This broadens the conclusions of Jeuland and Shugan (1983), who found that the profits increased moving from an independent manufacturer–retailer to a leader–follower manufacturer–retailer for constant elasticity demand function. It is interesting to note that the two systems under the exponential demand function exhibit the same solution. This implies that the division of power is irrelevant.

The question of whether the price offered to the consumer increases or decreases due to vertical integration is a topic well debated in the marketing literature. Westfield (1981) shows that whether the price offered by a vertically integrated system is lower than the price offered by a non-cooperating system depends on the price elasticity. This contradicts the results of Jeuland and Shugan (1983), who show that the price always decreases. The results presented in Table 4 show that the move to vertical integration results in a decrease in price for the four demand functions. Because a single decision is made in the vertically integrated system to maximize profit over the entire system, we intuitively expect the vertically integrated system to have higher total profits than the other two systems. Our results support this hypothesis as vertical integration increases the total system profits under all the demand functions.

Changes in the business environment frequently effect production unit costs. For example, a change in technology can result in a lower unit costs, or a shortage of raw material may increase unit costs. In an attempt to characterize such changes, we analyze the sensitivity of our results to changes in the unit cost. Returning to Table 2, we see that in all cases, some of the savings in unit cost are passed on down the system. However, the magnitude of change in the prices is dependent on the demand function. While savings are passed directly to the consumer under the exponential demand function, an erosion of the savings occurs as they are passed down under the linear demand function. In contrast, savings are magnified as they are passed down the channel under the constant elasticity and the algebraic demand functions. The change in retailer's price with respect to the changes in manufacturer's price in the independent and leader-follower systems also varies by demand function. In both systems, any decrease in price offered by the manufacturer causes a decrease in the retailer's price. However these reductions are again dependent on the demand function. The same conclusions may be drawn concerning the relative change in the retailer's price when the manufacturer's price changes as when the unit cost changes.

The benefits to each member of the distribution channel of using either the leader-follower manufacturer-retailer or independent manufacturer-retailer system is demand dependent. Both the relationships between individual profits and the system with the largest total profits is dependent on the demand function. This emphasizes the importance of understanding the nature of the demand function. However, the effect of vertical integration is consistent for all demand functions. Not only are system profits greater under vertical integration, but also the consumer pays less. The vertically integrated channel is therefore more efficient and hence more preferable than the leader-follower or independent systems.

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Appendix: Results For Various Demand Functions

In this section, we present analysis for the algebraic, constant elasticity, and linear demand functions. We analyze the independent manufacturer–retailer, leader–follower manufacturer–retailer, and the vertically integrated system to derive explicit equations for the prices and profits which are summarized in Table 4.

A.1 Analysis for Exponential Demand Functions

Demand in this section is represented by an exponential function which is

$$ae^{-bP_r},$$

where a and b are scaling constants for the demand and price respectively.

A.1.1 Leader–Follower Manufacturer–Retailer System

The total profit for the retailer is equal to its margin times the demand less inventory carrying charges and order processing costs, and is

$$\Pi_r(P_r, P_m) = (ae^{-bP_r})(P_r - P_m(1 + h)) - nS_r.$$

From the first order conditions, we arrive at an expression for P_r^* in terms of P_m :

$$P_r^*(P_m) = P_m(1 + h) + \frac{1}{b}.$$

The manufacturer's profit equation is

$$\Pi_m^*(P_m, P_r) = (ae^{-bP_r})(P_m - \theta) - nS_m.$$

Substituting our previously derived expression for P_r^* in terms of P_m , we compute the following optimal manufacturer's price from first order optimality conditions:

$$P_m^* = \left(\frac{1}{b} + \theta(1+h) \right) (1+h)^{-1}.$$

Substituting this value into the equation for P_r^* gives the following explicit expression for the optimal retailer's price:

$$P_r^* = \frac{2}{b} + \theta(1+h).$$

Since $G_r = P_r - (1+h)P_m$ and $G_m = P_m - \theta$ we can solve for explicit expressions of G_m^* and G_r^* which are:

$$\begin{aligned} G_r^* &= \frac{1}{b} \\ G_m^* &= \left(\frac{1}{b} \right) (1+h)^{-1}. \end{aligned}$$

Now we can explicitly evaluate the profit equations for the manufacturer, retailer and total system.

$$\begin{aligned} \Pi_m^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} (1+h)^{-1} - nS_m \\ \Pi_r^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} - nS_r \\ \Pi_m^* + \Pi_r^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} \left(\frac{2+h}{1+h} \right) - n(S_m + S_r). \end{aligned}$$

A.1.2 Independent Manufacturer-Retailer System

The total profits for the manufacturer and retailer are equal to their margin times the demand less any order filling or processing costs, and are

$$\begin{aligned} \tilde{\Pi}_m(\tilde{G}_m, \tilde{G}_r) &= \tilde{G}_m \left(ae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} \right) - nS_m \\ \tilde{\Pi}_r(\tilde{G}_r, \tilde{G}_m) &= \tilde{G}_r \left(ae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} \right) - nS_r. \end{aligned}$$

The manufacturer and the retailer maximize their profits by choosing their respective profit margins independently. The retailer maximizes its profit by setting its margin, \tilde{G}_r , such that the change in its profits with respect to the margin is zero:

$$G_r \left(-bae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} \right) + ae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} = 0.$$

Similarly, the manufacturer maximizes its profit by setting its margin, \tilde{G}_m , such that the change in its profits with respect to the margin is zero:

$$G_m(1+h) \left(-bae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} \right) + ae^{-b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r)} = 0.$$

Since these are a system of two equations in two unknowns we can now solve for \tilde{G}_r^* and \tilde{G}_m^* which are

$$\begin{aligned} \tilde{G}_r^* &= \frac{1}{b} \\ \tilde{G}_m^* &= \left(\frac{1}{b} \right) (1+h)^{-1}. \end{aligned}$$

Since $\tilde{G}_r = \tilde{P}_r - (1+h)\tilde{P}_m$ and $\tilde{G}_r = \tilde{P}_m - \theta$ we can solve for explicit expressions of \tilde{P}_m^* and \tilde{P}_r^* which are

$$\begin{aligned} \tilde{P}_r^* &= \frac{2}{b} + \theta(1+h) \\ \tilde{P}_m^* &= \left(\frac{1}{b} + \theta(1+h) \right) (1+h)^{-1}. \end{aligned}$$

We can also now explicitly evaluate the profit equations for the manufacturer, retailer and total system.

$$\begin{aligned} \Pi_m^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} (1+h)^{-1} - nS_m \\ \Pi_r^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} - nS_r \\ \Pi_m^* + \Pi_r^* &= \left(\frac{a}{b} \right) e^{-b\left(\frac{2}{b} + \theta(1+h)\right)} \left(\frac{2+h}{1+h} \right) - n(S_m + S_r). \end{aligned}$$

A.1.3 Vertically Integrated System

This section considers the case for vertical integration. The system profit equation is

$$\hat{\Pi}(\hat{P}_r) = \left(ae^{-b\hat{P}_r} \right) \left(\hat{P}_r - \theta(1+h) \right) - n(S_m + S_r).$$

From the first order conditions we arrive at the optimal value for \hat{P}_r^* which is

$$\hat{P}_r^* = \frac{1}{b} + \theta(1+h).$$

Since $\hat{G} = P_r - (1+h)\theta$ we can solve for an explicit expression of \hat{G}^* which is

$$\hat{G}^* = \frac{1}{b}$$

This means that the total profit for the vertically integrated system is

$$\hat{\Pi} = \left(\frac{a}{b}\right)e^{-b\left(\frac{1}{b} + \theta(1+h)\right)} - n(S_m + S_r).$$

A.2 Analysis for the Linear Demand Function

Demand in this section is represented by a linear function which is

$$a - bP_r,$$

where we have two parameters, a , the maximum demand, and b , the demand lost per increase in price.

A.2.1 Leader-Follower Manufacturer-Retailer System

The total profit for the retailer is equal to its margin times the demand less inventory carrying charges and order processing costs, and is

$$\Pi_r(P_r, P_m) = (a - bP_r)(P_r - P_m(1+h)) - nS_r.$$

From the first order conditions, we arrive at an expression for P_r^* in terms of P_m :

$$P_r^*(P_m) = \frac{1}{2}\left(P_m(1+h) + \frac{a}{b}\right).$$

The manufacturers profit equation is

$$\Pi_m(P_m, P_r) = (a - bP_r)(P_m - \theta) - nS_m.$$

Substituting our previously derived expression for P_r^* in terms of P_m , we compute the following optimal manufacturer's price from first order optimality conditions:

$$P_m^* = \left(\frac{1}{2} \left(\frac{a}{b} \right) + \frac{1}{2} (\theta(1+h)) \right) (1+h)^{-1}.$$

Substituting this value in to the equation for P_r^* gives the following explicit expression for the optimal retailer's price:

$$P_r^* = \frac{3}{4} \left(\frac{a}{b} \right) + \frac{1}{4} (\theta(1+h)).$$

Since $G_r = P_r - (1+h)P_m$ and $G_r = P_m - \theta$ we can solve for explicit expressions of G_m^* and G_r^* which are

$$G_r^* = \frac{1}{4} \left(\frac{a}{b} \right) - \frac{1}{4} (\theta(1+h))$$

$$G_m^* = \left(\frac{1}{2} \left(\frac{a}{b} \right) - \frac{1}{2} (\theta(1+h)) \right) (1+h)^{-1}.$$

Now we can evaluate the profit equations for the manufacturer, retailer and total system.

$$\Pi_m^* = \frac{b}{8} \left(\frac{a}{b} - \theta(1+h) \right)^2 (1+h)^{-1} - nS_m \quad (15)$$

$$\Pi_r^* = \frac{b}{16} \left(\frac{a}{b} - \theta(1+h) \right)^2 - nS_r \quad (16)$$

$$\Pi_r^* + \Pi_m^* = \frac{b}{16} \left(\frac{a}{b} - \theta(1+h) \right)^2 \left(\frac{3+h}{1+h} \right) - n(S_m + S_r). \quad (17)$$

A.2.2 Independent Manufacturer–Retailer System

The total profits for the manufacturer and retailer are equal to their margin times the demand less any order filling or processing costs, and are

$$\tilde{\Pi}_m(\tilde{G}_r, \tilde{G}_m) = G_m \left(a - b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right) - nS_m$$

$$\tilde{\Pi}_r(\tilde{G}_r, \tilde{G}_m) = G_r \left(a - b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right) - nS_r.$$

The manufacturer and the retailer maximize their profits by choosing their respective profit margins independently. The retailer maximizes its profit by setting its margin, \tilde{G}_r , such that the change in its profits

with respect to the margin is zero:

$$G_r(-b) + a - b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r) = 0.$$

Similiarly, the manufacturer maximizes its profit by setting its margin, G_m , such that the change in its profits with respect to the margin is zero:

$$G_m(1+h)(-b) + a - b((1+h)(\tilde{G}_m + \theta) + \tilde{G}_r) = 0.$$

Since these are a system of two equations in two unknowns we can now solve for \tilde{G}_r^* and \tilde{G}_m^* which are

$$\begin{aligned}\tilde{G}_r^* &= \frac{1}{3}\left(\frac{a}{b}\right) - \frac{1}{3}(\theta(1+h)) \\ \tilde{G}_m^* &= \left(\frac{1}{3}\left(\frac{a}{b}\right) - \frac{1}{3}(\theta(1+h))\right)(1+h)^{-1}.\end{aligned}$$

Since $\tilde{G}_r = \tilde{P}_r - (1+h)\tilde{P}_m$ and $\tilde{G}_r = \tilde{P}_m - \theta$ we can solve for explicit expressions of \tilde{P}_m^* and \tilde{P}_r^* which are

$$\begin{aligned}\tilde{P}_r^* &= \frac{2}{3}\left(\frac{a}{b}\right) + \frac{1}{3}(\theta(1+h)) \\ \tilde{P}_m^* &= \left(\frac{1}{3}\left(\frac{a}{b}\right) + \frac{2}{3}(\theta(1+h))\right)(1+h)^{-1}.\end{aligned}$$

We can also now explicitly evaluate the profit equations for the manufacturer, retailer and total system.

$$\begin{aligned}\Pi_m^* &= \frac{b}{9}\left(\frac{a}{b} - \theta(1+h)\right)^2(1+h)^{-1} - nS_m \\ \Pi_r^* &= \frac{b}{9}\left(\frac{a}{b} - \theta(1+h)\right)^2 - nS_r \\ \Pi_m^* + \Pi_r^* &= \frac{b}{9}\left(\frac{a}{b} - \theta(1+h)\right)^2\left(\frac{2+h}{1+h}\right) - n(S_m + S_r).\end{aligned}$$

A.2.3 Vertically Integrated System

This section considers the case for vertical integration. The system profit equation is

$$\hat{\Pi}(\hat{P}_r) = (a - b\hat{P}_r)(\hat{P}_r - \theta(1+h)) - n(S_m + S_r).$$

From the first order conditions we arrive at the optimal value for \hat{P}_r^* which is

$$\hat{P}_r^* = \frac{1}{2}\left(\theta(1+h) + \frac{a}{b}\right).$$

Since $\hat{G} = P_r - (1 + h)\theta$ we can solve for an explicit expression of \hat{G}^* which is

$$\hat{G}^* = \frac{1}{2} \left(\frac{a}{b} \right) - \frac{1}{2} (\theta(1 + h))$$

This means that the total system profit for the vertically integrated retailer is

$$\hat{\Pi}^* = \frac{b}{4} \left(\frac{a}{b} - \theta(1 + h) \right)^2 - n(S_m + S_r).$$

A.3 Analysis for the Algebraic Demand Function

Demand in this section is represented by an algebraic function which is

$$(a + bP_r)^{-q},$$

where a and b are scaling constants for maximum demand and the quantity decrease per unit increase in price respectively.

A.3.1 Leader-Follower Manufacturer-Retailer System

The total profit for the retailer is equal to its margin times the demand less inventory carrying charges and order processing costs, and is

$$\Pi_r^A(P_r, P_m) = (a + bP_r)^{-q} (P_r - P_m(1 + h)) - nS_r.$$

From the first order conditions, we arrive at an expression for P_r^* in terms of P_m :

$$P_r^*(P_m) = \frac{\frac{a}{b} + P_m q(1 + h)}{(q - 1)}.$$

The manufacturer's profit equation is

$$\Pi_m(P_m, P_r) = (a + bP_r)^{-q} (P_m - \theta) - nS_m.$$

Substituting our previously derived expression for P_r^* in terms of P_m , we compute the following optimal manufacturer's price from first order optimality conditions:

$$P_m^* = \left(\frac{1}{q-1} \left(\frac{a}{b} \right) + \frac{q}{q-1} (\theta(1+h)) \right) (1+h)^{-1}.$$

Substituting this value in to the equation for P_r^* gives the following explicit expression for the optimal retailer's price:

$$P_r^* = \frac{2q-1}{(q-1)^2} \left(\frac{a}{b} \right) + \frac{q^2}{(q-1)^2} (\theta(1+h)).$$

Since $G_r = P_r - (1+h)P_m$ and $G_r = P_m - \theta$ we can solve for explicit expressions of G_m^* and G_r^* which are

$$\begin{aligned} G_r^* &= \frac{q}{(q-1)^2} \left(\frac{a}{b} \right) + \frac{q}{(q-1)^2} (\theta(1+h)) \\ G_m^* &= \left(\frac{1}{q-1} \left(\frac{a}{b} \right) + \frac{1}{q-1} (\theta(1+h)) \right) (1+h)^{-1}. \end{aligned}$$

Now we can evaluate the profit equations for the manufacturer, retailer and total system.

$$\begin{aligned} \Pi_m^* &= \frac{1}{q-1} \left(\frac{a}{b} + \theta(1+h) \right) \left(\left(\frac{bq^2}{(q-1)^2} \right) \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} (1+h)^{-1} - nS_m \\ \Pi_r^* &= \frac{1}{q-1} \left(\frac{a}{b} + \theta(1+h) \right) \left(\left(\frac{bq^2}{(q-1)^2} \right) \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} \frac{q}{(q-1)} - nS_r \\ \Pi_r^* + \Pi_m^* &= \frac{1}{q-1} \left(\frac{a}{b} + \theta(1+h) \right) \left(\frac{bq^2}{(q-1)^2} \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} \left(\frac{q(2+h)-1}{(q-1)(1+h)} \right) - n(S_m + S_r). \end{aligned}$$

A.3.2 Independent Manufacturer-Retailer System

The total profits for the manufacturer and retailer are equal to their margin times the demand less any order filling or processing costs, and are

$$\begin{aligned} \tilde{\Pi}_m^*(\tilde{G}_m, \tilde{G}_r) &= \tilde{G}_m \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q} - nS_m \\ \tilde{\Pi}_r^*(\tilde{G}_r, \tilde{G}_m) &= \tilde{G}_r \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q} - nS_r. \end{aligned}$$

The manufacturer and the retailer maximize their profits by choosing their respective profit margins independently. The retailer maximizes its profit by setting its margin, G_r , such that the change in its profits

with respect to the margin is zero:

$$G_r \left(-qb \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q-1} \right) + \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q} = 0.$$

Similarly, the manufacturer maximizes its profit by setting its margin, G_m , such that the change in its profits with respect to the margin is zero:

$$\tilde{G}_m (1+h) \left(-qb \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q-1} \right) + \left(a + b \left((1+h) (\tilde{G}_m + \theta) + \tilde{G}_r \right) \right)^{-q} = 0.$$

Since these are a system of two equations in two unknowns we can now solve for \tilde{G}_r^* and \tilde{G}_m^* which are

$$\begin{aligned} \tilde{G}_r^* &= \frac{1}{q-2} \left(\frac{a}{b} \right) + \frac{1}{q-2} (\theta(1+h)) \\ \tilde{G}_m^* &= \left(\frac{1}{q-2} \left(\frac{a}{b} \right) + \frac{1}{q-2} (\theta(1+h)) \right) (1+h)^{-1}. \end{aligned}$$

Since $\tilde{G}_r = \tilde{P}_r - (1+h) \tilde{P}_m$ and $\tilde{G}_r = \tilde{P}_m - \theta$ we can solve for explicit expressions of \tilde{P}_m^* and \tilde{P}_r^* which are

$$\begin{aligned} \tilde{P}_r^* &= \frac{2}{q-2} \left(\frac{a}{b} \right) + \frac{q}{q-2} (\theta(1+h)) \\ \tilde{P}_m^* &= \left(\frac{1}{q-2} \left(\frac{a}{b} \right) + \frac{q-1}{q-2} (\theta(1+h)) \right) (1+h)^{-1}. \end{aligned}$$

We can also now explicitly evaluate the profit equations for the manufacturer, retailer and total system.

$$\begin{aligned} \tilde{\Pi}_m^* &= \frac{1}{q-2} \left(\frac{a}{b} + \theta(1+h) \right) \left(\left(\frac{bq}{q-2} \right) \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} (1+h)^{-1} - nS_m \\ \tilde{\Pi}_r^* &= \frac{1}{q-2} \left(\frac{a}{b} + \theta(1+h) \right) \left(\left(\frac{bq}{q-2} \right) \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} - nS_r \\ \tilde{\Pi}_m^* + \tilde{\Pi}_r^* &= \frac{1}{q-2} \left(\frac{a}{b} + \theta(1+h) \right) \left(\left(\frac{bq}{q-2} \right) \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} \left(\frac{2+h}{1+h} \right) - n(S_m + S_r). \end{aligned}$$

A.3.3 Vertically Integrated System

This section considers the case for vertical integration. The system profit equation is as follows:

$$\hat{\Pi}(\hat{P}_r) = \left(a + b\hat{P}_r \right)^{-q} \left(\hat{P}_r - \theta(1+h) \right) - n(S_m + S_r).$$

From the first order conditions we arrive at the optimal value for \hat{P}_r^* which is

$$\hat{P}_r^* = \frac{1}{q-1} \left(\frac{a}{b} \right) + \frac{q}{q-1} (\theta(1+h)).$$

Since $\hat{G} = P_r - (1 + h)\theta$ we can solve for an explicit expression of \hat{G}^* which is

$$\hat{G}^* = \frac{1}{q-1} \left(\frac{a}{b} \right) + \frac{1}{q-1} (\theta(1+h))$$

This means that the total profit for the vertically integrated system is

$$\hat{\Pi} = \frac{1}{q-1} \left(\frac{a}{b} + \theta(1+h) \right) \left(\frac{bq}{q-1} \left(\frac{a}{b} + \theta(1+h) \right) \right)^{-q} - n(S_m + S_r).$$

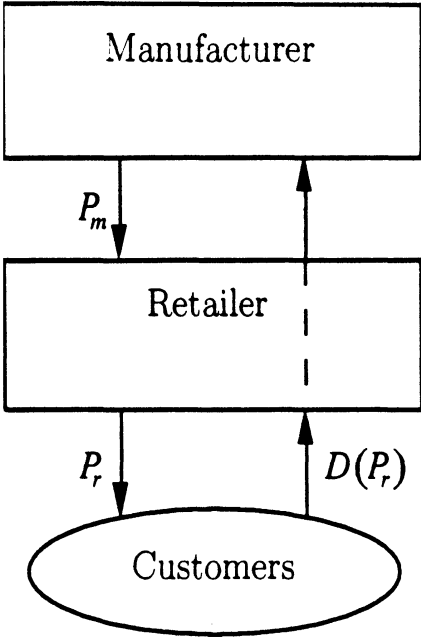


Figure 1. Leader-Follower Manufacturer-Retailer System

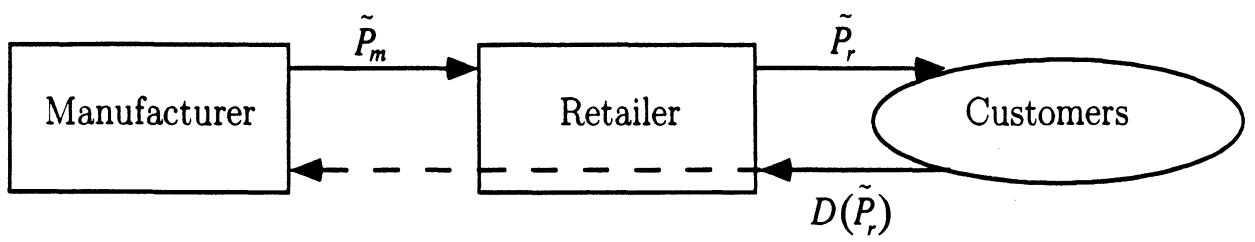


Figure 2. Independent Manufacturer-Retailer System

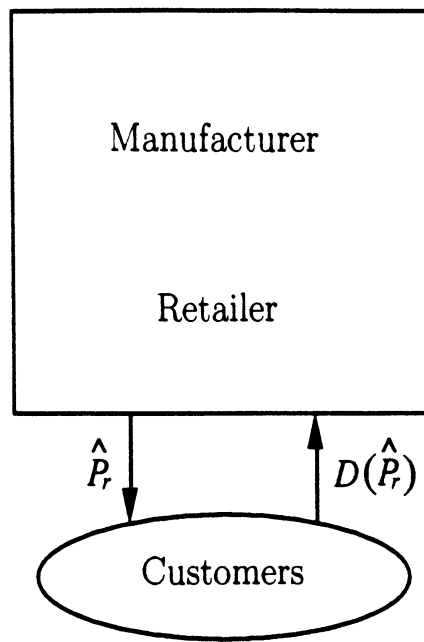


Figure 3. Vertically Integrated System

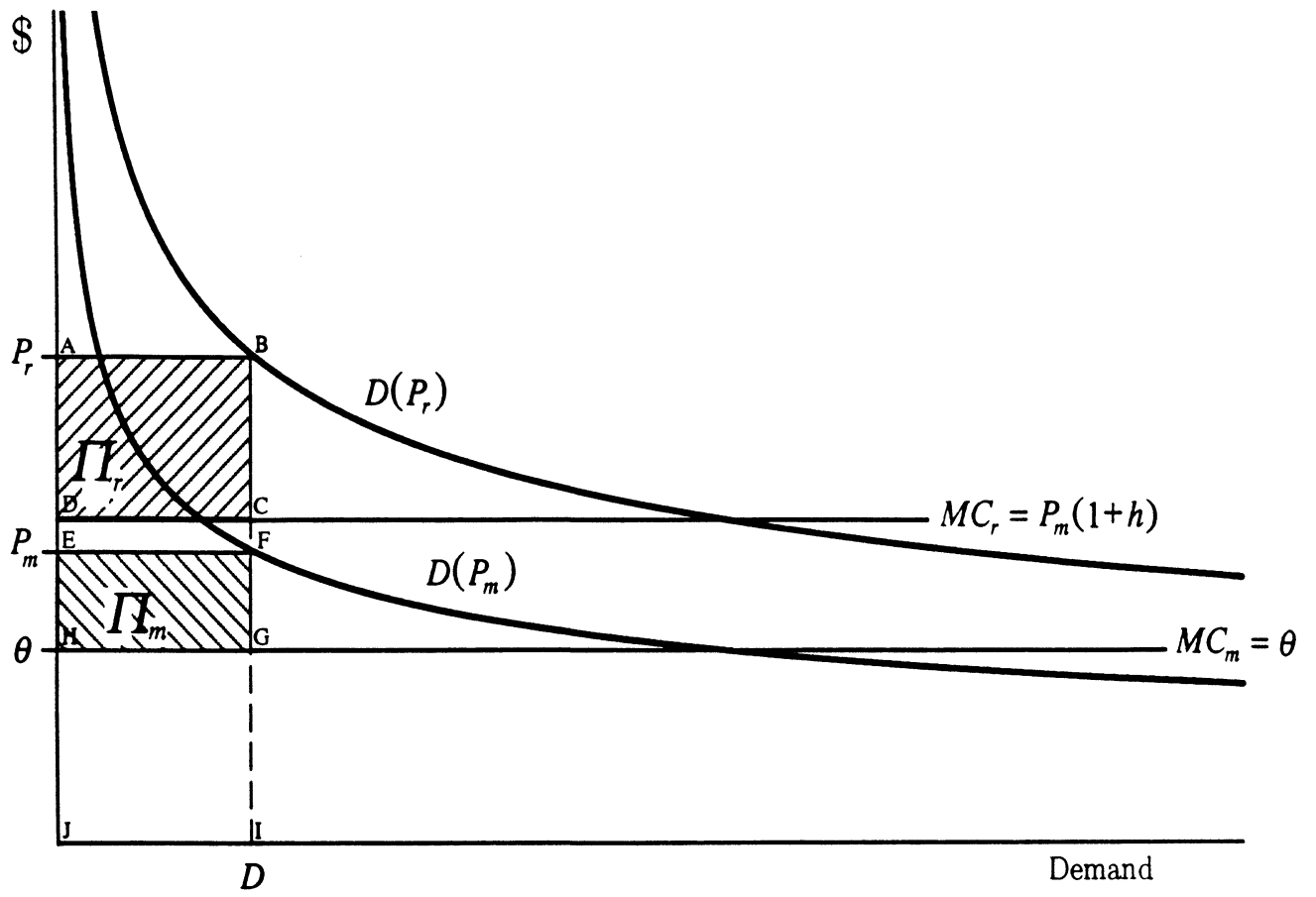


Figure 4. Price Determination for the Leader-Follower Manufacturer-Retailer System

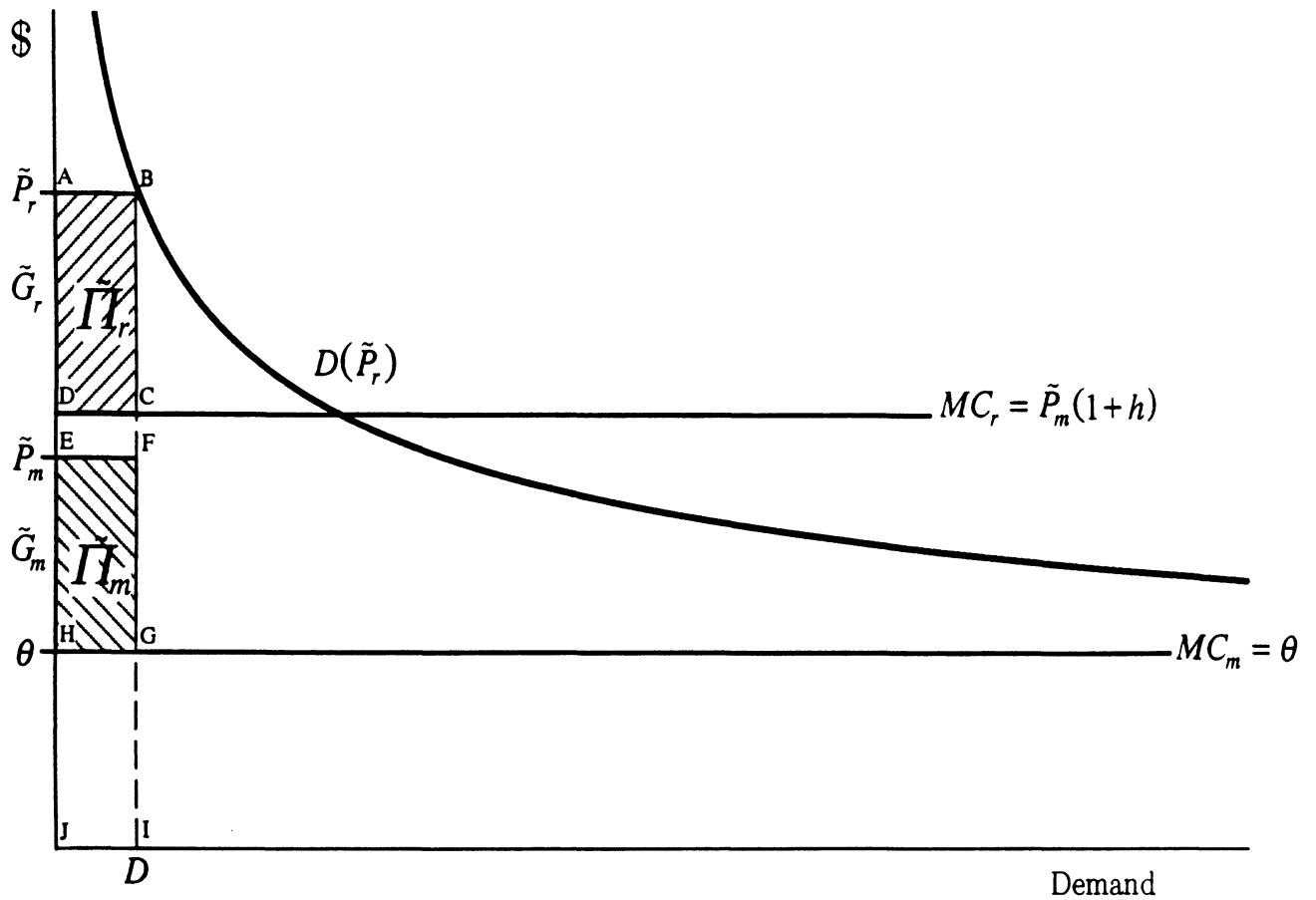


Figure 5. Price Determination for the Independent Manufacturer-Retailer System

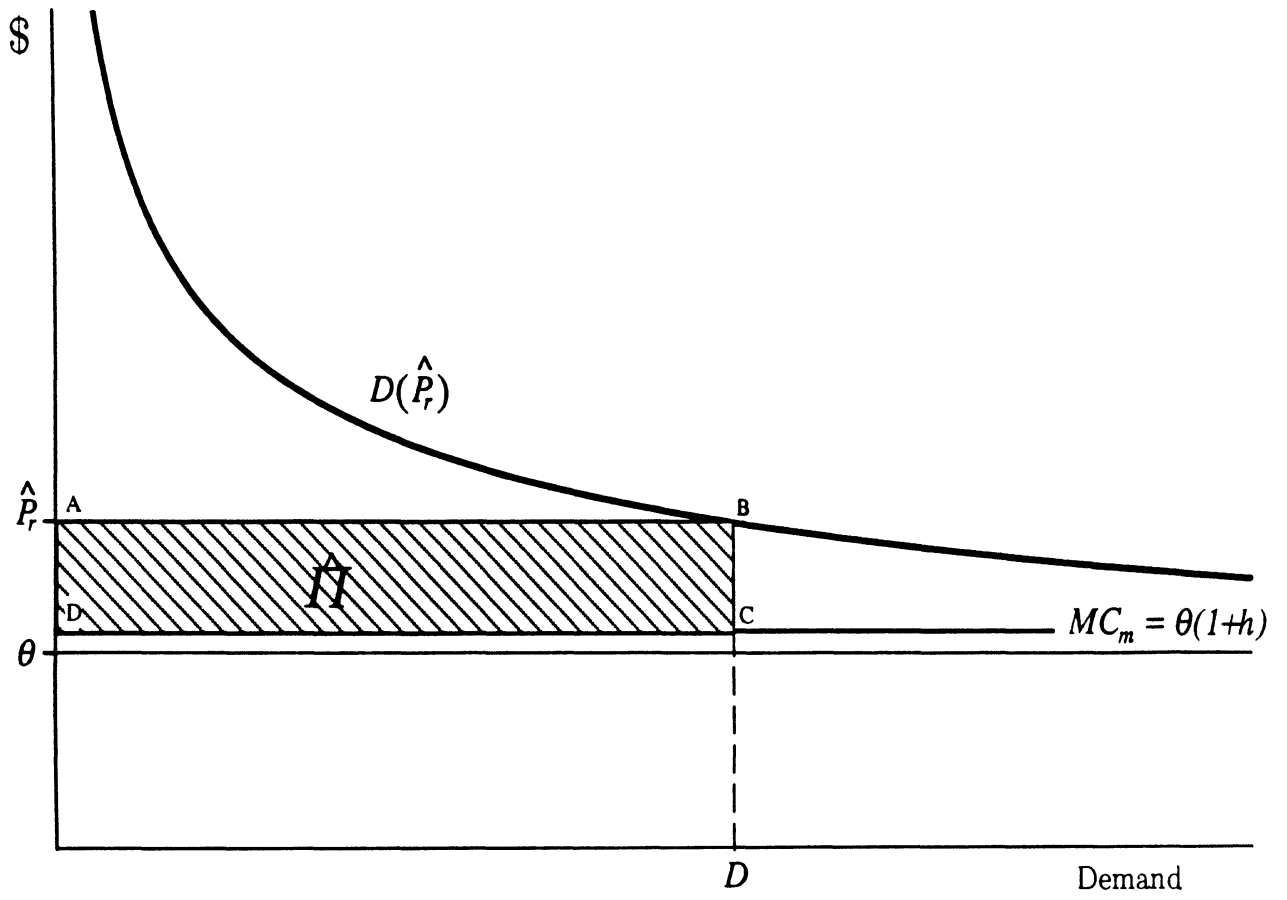


Figure 6. Price Determination for the Vertically Integrated Retailer System

Table 1: A Summary of Optimal Prices

Demand Type	Constant Elasticity $a(bP_r)^{-k}$	Exponential ae^{-bP_r}	Linear $a - bP_r$	Algebraic $(a + bP_r)^{-q}$
Leader-Follower Manufacturer-Retailer Prices				
P_r	$\frac{k^2}{(k-1)^2}(\theta(1+h))$	$\frac{2}{b} + \theta(1+h)$	$\frac{3}{4}(\frac{a}{b}) + \frac{1}{4}(\theta(1+h))$	$\frac{2q-1}{(q-1)^2}(\frac{a}{b}) + \frac{q^2}{(q-1)^2}(\theta(1+h))$
P_m	$(\frac{k}{k-1}(\theta(1+h)))(1+h)^{-1}$	$(\frac{1}{b} + \theta(1+h))(1+h)^{-1}$	$(\frac{1}{2}(\frac{a}{b}) + \frac{1}{2}(\theta(1+h)))(1+h)^{-1}$	$(\frac{1}{q-1}(\frac{a}{b}) + \frac{q}{q-1}(\theta(1+h)))(1+h)^{-1}$
Independent Manufacturer-Retailer Prices				
\tilde{P}_r	$\frac{k}{k-2}(\theta(1+h))$	$\frac{2}{b} + \theta(1+h)$	$\frac{2}{3}(\frac{a}{b}) + \frac{1}{3}(\theta(1+h))$	$\frac{2}{q-2}(\frac{a}{b}) + \frac{q}{q-2}(\theta(1+h))$
\tilde{P}_m	$(\frac{k-1}{k-2}(\theta(1+h)))(1+h)^{-1}$	$(\frac{1}{b} + \theta(1+h))(1+h)^{-1}$	$(\frac{1}{3}(\frac{a}{b}) + \frac{2}{3}(\theta(1+h)))(1+h)^{-1}$	$(\frac{1}{q-2}(\frac{a}{b}) + \frac{q-1}{q-2}(\theta(1+h)))(1+h)^{-1}$
Vertically Integrated Price				
\hat{P}_r	$\frac{k}{k-1}(\theta(1+h))$	$\frac{1}{b} + \theta(1+h)$	$\frac{1}{2}(\frac{a}{b}) + \frac{1}{2}(\theta(1+h))$	$\frac{1}{q-1}(\frac{a}{b}) + \frac{q}{q-1}(\theta(1+h))$
Customer Price Comparisons				
$P_r - \hat{P}_r$	$\frac{k}{(k-1)^2}(\theta(1+h))$	$\frac{1}{b}$	$\frac{1}{4}(\frac{a}{b}) - \frac{1}{4}(\theta(1+h))$	$\frac{q}{(q-1)^2}(\frac{a}{b}) + \frac{q}{(q-1)^2}(\theta(1+h))$
$\tilde{P}_r - \hat{P}_r$	$\frac{k}{(k-1)(k-2)}(\theta(1+h))$	$\frac{1}{b}$	$\frac{1}{6}(\frac{a}{b}) - \frac{1}{6}(\theta(1+h))$	$\frac{q}{(q-1)(q-2)}(\frac{a}{b}) + \frac{q}{(q-1)(q-2)}(\theta(1+h))$
$\tilde{P}_r - P_r$	$\frac{k}{(k-1)^2(k-2)}(\theta(1+h))$	0	$\frac{-1}{12}(\frac{a}{b}) - \frac{1}{12}(\theta(1+h))$	$\frac{q}{(q-1)^2(q-2)}(\frac{a}{b}) + \frac{q}{(q-1)^2(q-2)}(\theta(1+h))$

Table 2: A Summary of Optimal Margins

Demand Type	Constant Elasticity $a(bP_r)^{-k}$	Exponential ae^{-bP_r}	Linear $a - bP_r$	Algebraic $(a + bP_r)^{-q}$
Leader-Follower Manufacturer-Retailer Margins				
G_r	$\frac{k}{(k-1)^2}(\theta(1+h))$	$\frac{1}{b}$	$\frac{1}{4}\left(\frac{a}{b}\right) - \frac{1}{4}(\theta(1+h))$	$\frac{q}{(q-1)^2}\left(\frac{a}{b}\right) + \frac{q}{(q-1)^2}(\theta(1+h))$
G_m	$\left(\frac{1}{k-1}(\theta(1+h))\right)(1+h)^{-1}$	$\left(\frac{1}{b}\right)(1+h)^{-1}$	$\left(\frac{1}{2}\left(\frac{a}{b}\right) - \frac{1}{2}(\theta(1+h))\right)(1+h)^{-1}$	$\left(\frac{1}{q-1}\left(\frac{a}{b}\right) + \frac{1}{q-1}(\theta(1+h))\right)(1+h)^{-1}$
Independently Manufacturer-Retailer Margins				
\tilde{G}_r	$\frac{1}{k-2}(\theta(1+h))$	$\frac{1}{b}$	$\frac{1}{3}\left(\frac{a}{b}\right) - \frac{1}{3}(\theta(1+h))$	$\frac{1}{q-2}\left(\frac{a}{b}\right) + \frac{1}{q-2}(\theta(1+h))$
\tilde{G}_m	$\left(\frac{1}{k-2}(\theta(1+h))\right)(1+h)^{-1}$	$\left(\frac{1}{b}\right)(1+h)^{-1}$	$\left(\frac{1}{3}\left(\frac{a}{b}\right) - \frac{1}{3}(\theta(1+h))\right)(1+h)^{-1}$	$\left(\frac{1}{q-2}\left(\frac{a}{b}\right) + \frac{1}{q-2}(\theta(1+h))\right)(1+h)^{-1}$
Vertically Integrated Margin				
\hat{G}	$\frac{1}{k-1}(\theta(1+h))$	$\frac{1}{b}$	$\frac{1}{2}\left(\frac{a}{b}\right) - \frac{1}{2}(\theta(1+h))$	$\frac{1}{q-1}\left(\frac{a}{b}\right) + \frac{1}{q-1}(\theta(1+h))$

Table 3: A Summary of Optimal Profits

Demand Type	Constant Elasticity	Exponential	Linear	Algebraic
Demand Function	$a(bP_r)^{-k}$	ae^{-bP_r}	$a - bP_r$	$(a + bP_r)^{-q}$
Leader-Follower Manufacturer-Retailer Profits				
Π_r	$\mu \left(\frac{k}{k-1}\right)^{1-k}$	γ	$\frac{1}{16}\omega$	$\sigma \left(\frac{q}{q-1}\right)^{1-q}$
Π_m	$\mu \left(\frac{k}{k-1}\right)^{-k} (1+h)^{-1}$	$\gamma(1+h)^{-1}$	$\frac{1}{8}\omega(1+h)^{-1}$	$\sigma \left(\frac{q}{q-1}\right)^{-q} (1+h)^{-1}$
Independent Manufacturer-Retailer Profits				
$\tilde{\Pi}_r$	$\mu \left(\frac{k-1}{k-2}\right)^{1-k}$	γ	$\frac{1}{9}\omega$	$\sigma \left(\frac{q-1}{q-2}\right)^{1-q}$
$\tilde{\Pi}_m$	$\mu \left(\frac{k-1}{k-2}\right)^{1-k} (1+h)^{-1}$	$\gamma(1+h)^{-1}$	$\frac{1}{9}\omega(1+h)^{-1}$	$\sigma \left(\frac{q-1}{q-2}\right)^{1-q} (1+h)^{-1}$
Total System Profits				
$\Pi_r + \Pi_m$	$\mu \left(\frac{k}{k-1}\right)^{-k} \left(\frac{k(2+h)-1}{(k-1)(1+h)}\right)^{1-k}$	$\gamma \left(\frac{2+h}{1+h}\right)$	$\frac{1}{16}\omega \left(\frac{3+h}{1+h}\right)$	$\sigma \left(\frac{q}{q-1}\right)^{-q} \left(\frac{q(2+h)-1}{(q-1)(1+h)}\right)^{1-q}$
$\tilde{\Pi}_r + \tilde{\Pi}_m$	$\mu \left(\frac{k-1}{k-2}\right)^{1-k} \left(\frac{2+h}{1+h}\right)$	$\gamma \left(\frac{2+h}{1+h}\right)$	$\frac{1}{9}\omega \left(\frac{2+h}{1+h}\right)$	$\sigma \left(\frac{q-1}{q-2}\right)^{1-q} \left(\frac{2+h}{1+h}\right)$
$\hat{\Pi}$	μ	γe	$\frac{1}{4}\omega$	σ

Key : $\mu = a \left(\frac{\theta(1+h)}{k-1}\right) \left(bk \left(\frac{\theta(1+h)}{k-1}\right)\right)^{-k}$,

$\gamma = \left(\frac{a}{b}\right) e^{-b \left(\frac{a}{b} + \theta(1+h)\right)}$,

$\omega = b \left(\frac{a}{b} - \theta(1+h)\right)^2$,

$\sigma = \left(\frac{k+\theta(1+h)}{q-1}\right) \left(\frac{bq}{q-1} \left(\frac{a}{b} + \theta(1+h)\right)\right)^{-q}$.

Table 4: A Qualitative System Comparison

Demand Type	Constant Elasticity	Exponential	Linear	Algebraic
Independent Manufacturer–Retailer System				
Retailer vs Manufacturer Margin	$\hat{G}_r > \tilde{G}_m$	$\hat{G}_r > \tilde{G}_m$	$\hat{G}_r > \tilde{G}_m$	$\hat{G}_r > \tilde{G}_m$
Retailer vs Manufacturer Profits	$\hat{\Pi}_r > \tilde{\Pi}_m$	$\hat{\Pi}_r > \tilde{\Pi}_m$	$\hat{\Pi}_r > \tilde{\Pi}_m$	$\hat{\Pi}_r > \tilde{\Pi}_m$
Leader-Follower Manufacturer–Retailer System				
Retailer vs Manufacturer Margin	$G_r > G_m$	$G_r > G_m$	$G_r < G_m$	$G_r > G_m$
Retailer vs Manufacturer Profits	$\Pi_r > \Pi_m$	$\Pi_r > \Pi_m$	$\Pi_r < \Pi_m$	$\Pi_r > \Pi_m$
Independent vs Leader-Follower				
Retailer Price	$\hat{P}_r > P_r$	$\hat{P}_r = P_r$	$\hat{P}_r < P_r$	$\hat{P}_r > P_r$
Manufacturer Price	$\hat{P}_m > P_m$	$\hat{P}_m = P_m$	$\hat{P}_m < P_m$	$\hat{P}_m > P_m$
Retailer Margin	$\hat{C}_r < C_r$	$\hat{C}_r = C_r$	$\hat{C}_r > C_r$	$\hat{C}_r < C_r$
Manufacturer Margin	$\hat{C}_m > C_m$	$\hat{C}_m = C_m$	$\hat{C}_m < C_m$	$\hat{C}_m > C_m$
Retailer Profits	$\hat{\Pi}_r < \Pi_r$	$\hat{\Pi}_r = \Pi_r$	$\hat{\Pi}_r > \Pi_r$	$\hat{\Pi}_r < \Pi_r$
Manufacturer Profits	$\hat{\Pi}_m < \Pi_m$	$\hat{\Pi}_m = \Pi_m$	$\hat{\Pi}_m < \Pi_m$	$\hat{\Pi}_m < \Pi_m$
System Profits	$\hat{\Pi}_r + \hat{\Pi}_m < \Pi_r + \Pi_m$	$\hat{\Pi}_r + \hat{\Pi}_m = \Pi_r + \Pi_m$	$\hat{\Pi}_r + \hat{\Pi}_m > \Pi_r + \Pi_m$	$\hat{\Pi}_r + \hat{\Pi}_m < \Pi_r + \Pi_m$
Effects of Vertical Integration				
Retailer Price	$\hat{P}_r < \tilde{P}_r, P_r$	$\hat{P}_r < \tilde{P}_r, P_r$	$\hat{P}_r < \tilde{P}_r, P_r$	$\hat{P}_r < \tilde{P}_r, P_r$
System Profit Comparison	$\hat{\Pi} > \tilde{\Pi}_r + \tilde{\Pi}_m, \Pi_r + \Pi_m$	$\hat{\Pi} > \tilde{\Pi}_r + \tilde{\Pi}_m, \Pi_r + \Pi_m$	$\hat{\Pi} > \tilde{\Pi}_r + \tilde{\Pi}_m, \Pi_r + \Pi_m$	$\hat{\Pi} > \tilde{\Pi}_r + \tilde{\Pi}_m, \Pi_r + \Pi_m$

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