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RESEARCH ON FUNCTIONS OF A COMPLEX VARIABLE

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ABSTRACT

The research program on functions of a complex variable has so far resulted in three papers: (1) Curve-families and Riemann surfaces, to be published shortly; (2) Extensions of the Gross Star Theorem, which appeared in the Michigan Mathematical Journal; and (3) Approximation by entire functions, which will be submitted for publication in the near future. Work has also been carried out on extending the Phragmen-Lindelöf principle by relating it to growth properties of entire functions.

RESEARCH ON FUNCTIONS OF A COMPLEX VARIABLE

INTRODUCTION

Two grants have been awarded by the National Science Foundation to the University of Michigan for research on functions of a complex variable, the first one (NSF-2286) covering the period from July 1, 1953, to July 1, 1954, and the second (NSF-G779) covering the period from July 1, 1954, to September 1, 1954. In both cases the research was to be carried out by Wilfred Kaplan, Associate Professor of Mathematics.

Professor Kaplan was on sabbatical leave for the academic year 1953-1954 and was able to devote the entire period from July 1, 1953, to September 1, 1954, to the research. In August, 1953, he left for Europe for a three-month visit, during which he conferred with mathematicians in England, Switzerland, Finland, Sweden, and Denmark on the subject of the research. Except for this period and brief vacations, the work was done in Ann Arbor.

RESEARCH COMPLETED

The research program was led to three separate papers which are in various stages of completion as described below:

1. Curve-Families and Riemann Surfaces

A new proof is given of the theorem: _For every abstract normal chordal system_ E _there exists a function_ $u(x,y)$ _harmonic for_ $x^2 + y^2 < 1$ _or for_ $x^2 + y^2 < \infty$ _such that the family of level curves of_ u _forms a chordal system isomorphic to_ E. The proof uses only the simplest properties of regular curve-families, and the uniformigation theorem of function theory.

Accordingly, it leads to a considerable simplification of the theory of curve-families.

This paper was announced at the Conference on Functions of a Complex Variable in Ann Arbor in June, 1953. It was written during the following months and will appear shortly in the proceedings of that conference.

2. Extensions of the Gross Star Theorem

Several theormes of the type of the Gross Star Theorem are proved. The following is typical: Let $w = \phi(z)$ be meromorphic for $|z| < 1$. Let B be a closed subset of $|z| = 1$ having capacity zero. Then each element of the inverse $\phi^{-1}(w)$ of $\phi(z)$ can be continued indefinitely in almost all directions for which $z = \phi^{-1}(w)$ approaches a point of B. The concept of "all directions" includes more general families of paths than rays.

This paper has appeared in the Michigan Mathematical Journal [vol 2 (1953-1954), pp 105-108].

3. Approximation by Entire Functions

In 1927 Carleman extended the classical Weierstrass approximation theroem to approximation of continuous functions $f(x)$ on the interval $[-\infty,\infty]$ by entire functions. This result is amplified and applied in several ways. It is shown that if $f(x)$ has a continuous derivative, then the approximating function $\phi(z)$ can be chosen so that $\phi'(x)$ also approximates $f'(x)$. It is shown that, if $f(z)$ is analytic in a domain D bounded by a single open curve C extending to infinity in both directions, and if f is sufficiently smooth on C, then f can be approximated uniformly in D by an entire function. Existence of solutions of the Dirichlet problem for the unit circle for a large class of nonintegrable boundary functions is established.

This paper requires some minor editing and will shortly be submitted to the Michigan Mathematical Journal.

RESEARCH IN PROGRESS

In addition to the work described, a considerable amount of time was devoted to a program of extending the Phragmen-Lindelöf principle by relating it to growth properties of entire functions. Some significant results were obtained, but more work is needed to round out the program and give a satisfactory general theory.